Studying QCD Phase Diagram by Combining AMPT with FRG

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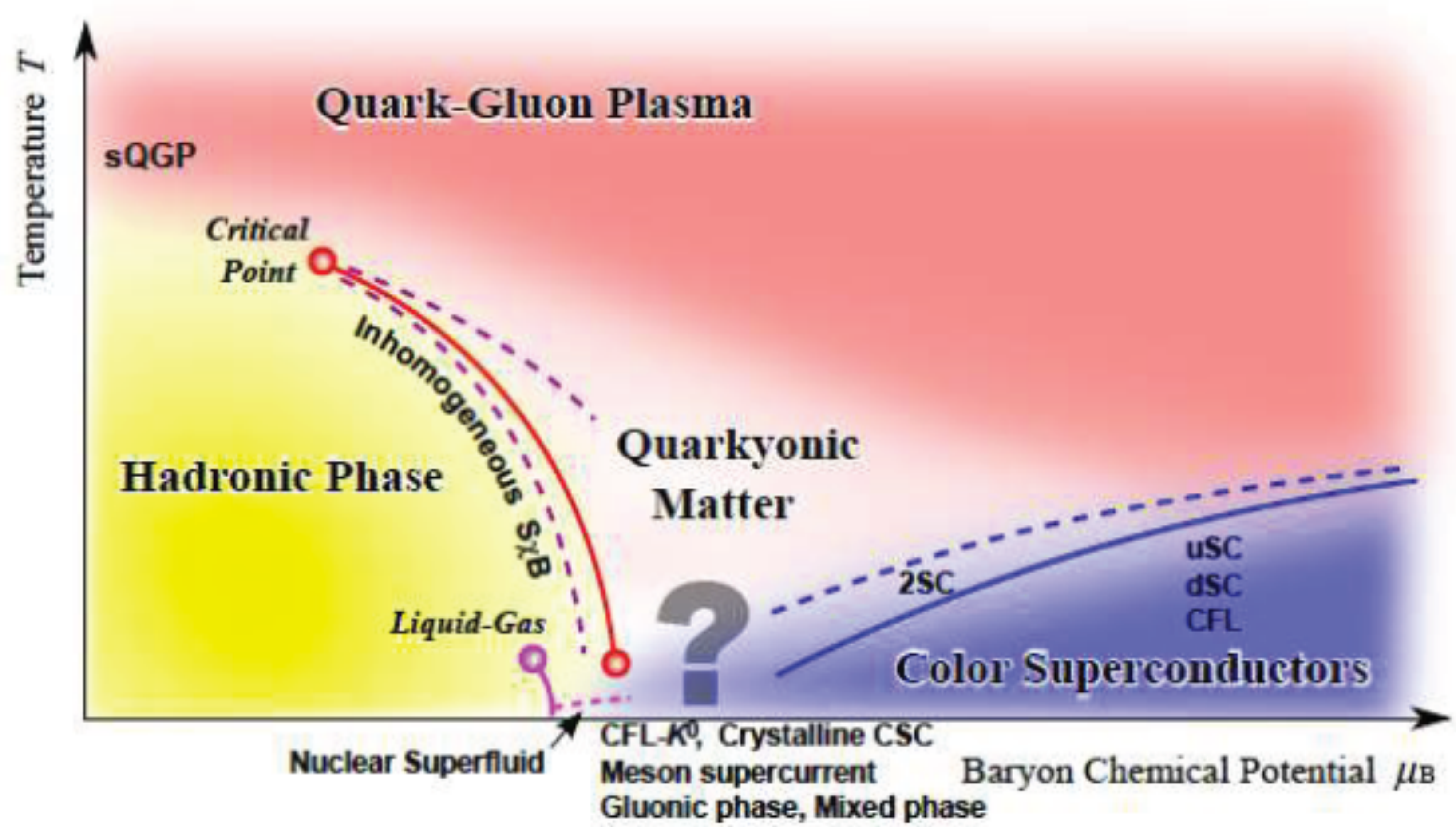
International Workshop on Partonic and Hadronic Transport Approaches for Relativistic Heavy Ion Collisions, Dalian, May 11-12, 2019

Outline

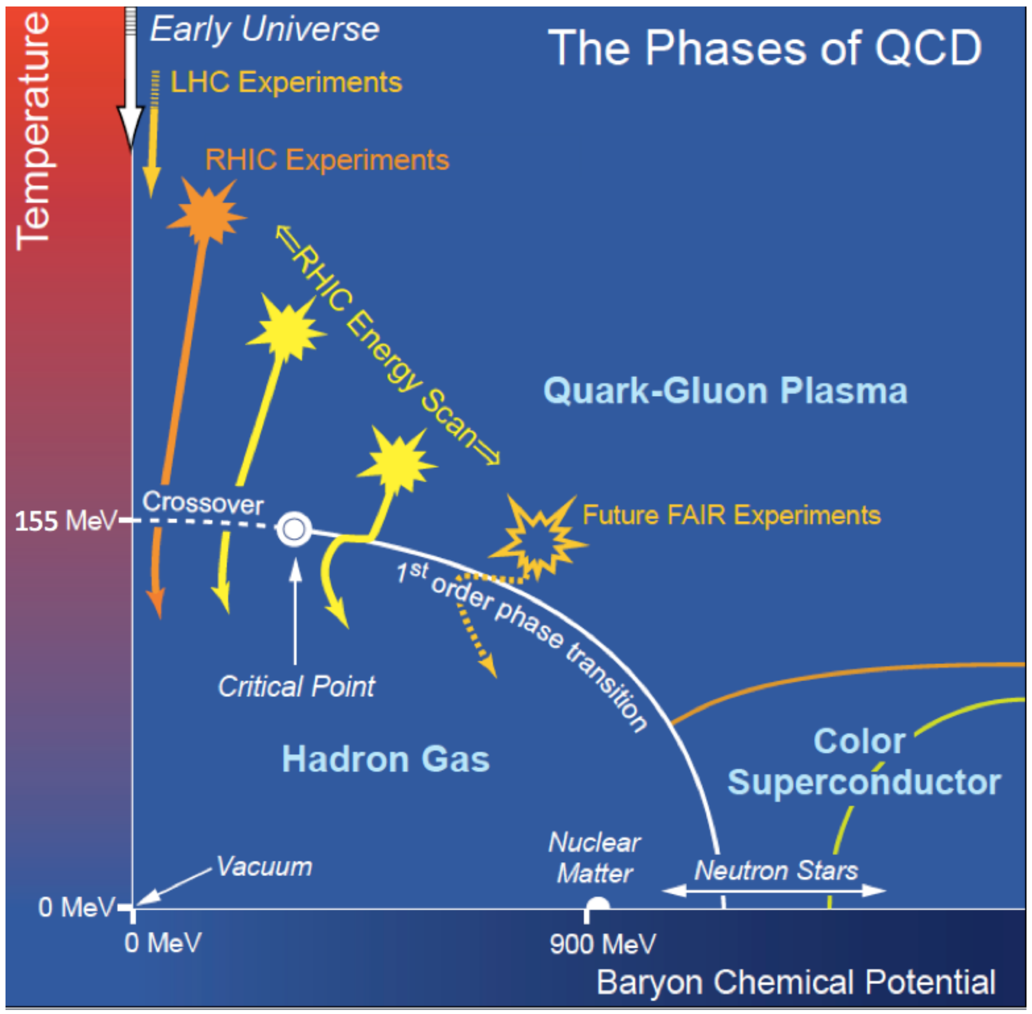
- Experimental signals of CEP
- AMPT results on related observables
- FRG results on QCD phase diagram
- Plan of combining AMPT with FRG

QCD phase diagram



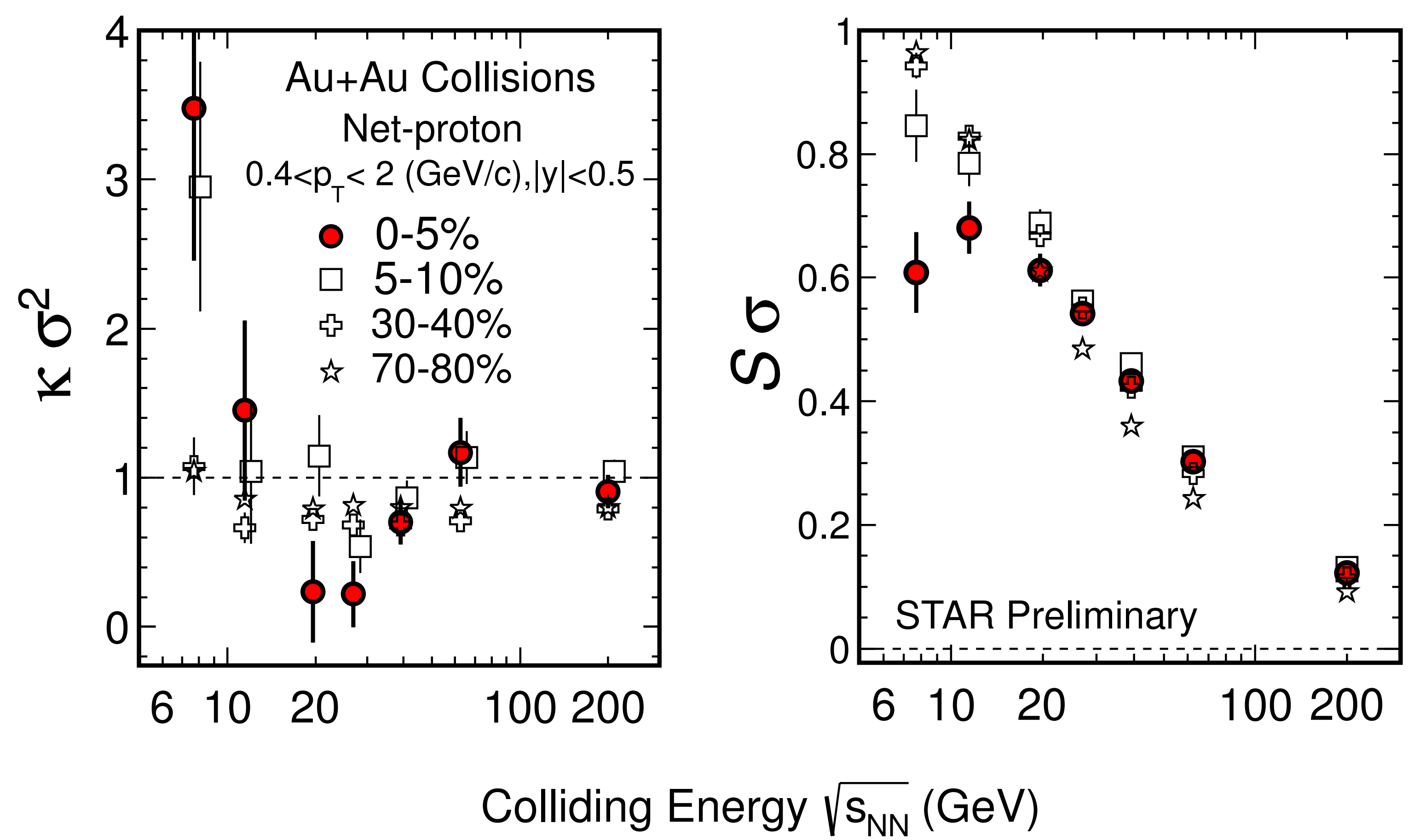
- QCD phase diagram in the T-muB plane.
- Critical end point (or critical point) is a key feature of QCD phase structure.
- Experimental programs: RHIC-BES, FAIR, NICA, HIAF.
- Some hints from RHIC-BES experiment: net-baryon (proton) cumulants, directed flow, HBT radii, light nuclei……

High-order net-proton cumulants



The Hot QCD White Paper (2015)

RHIC:

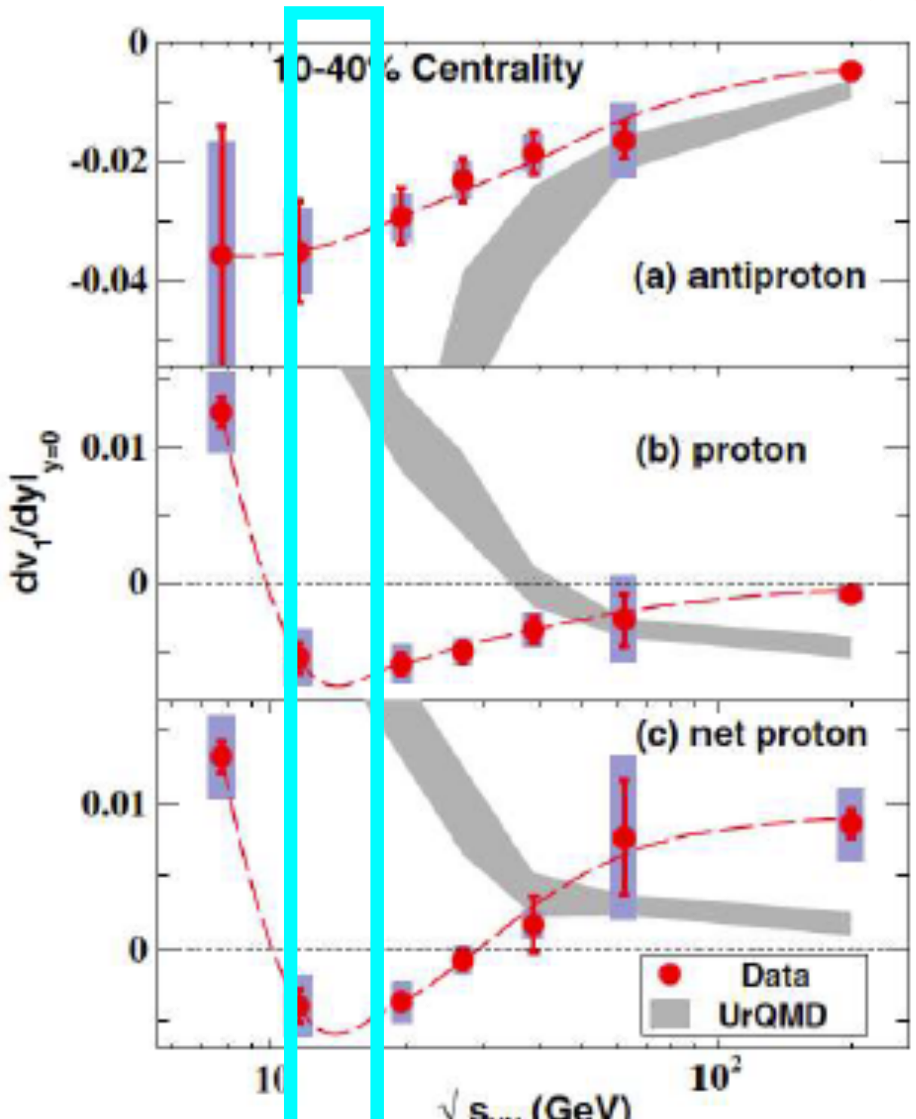


X.Luo (STAR), PoS CPOD2014, 019 (2014)

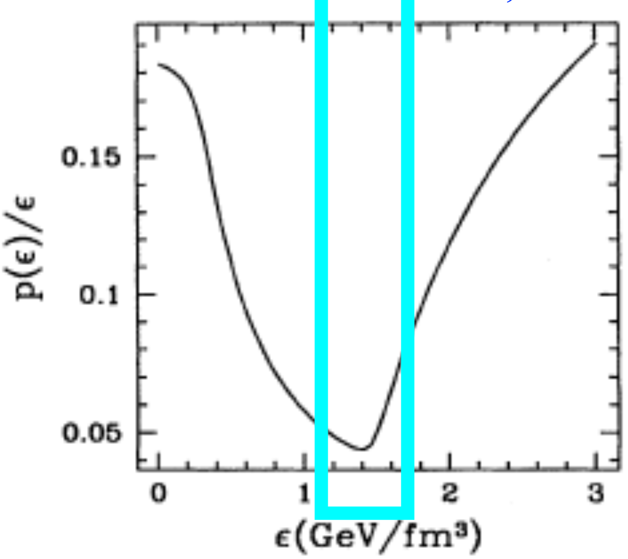
cf. Xiaofeng Luo's talk

- Non-monotonic energy dependence of the kurtosis => hint of entering critical region.

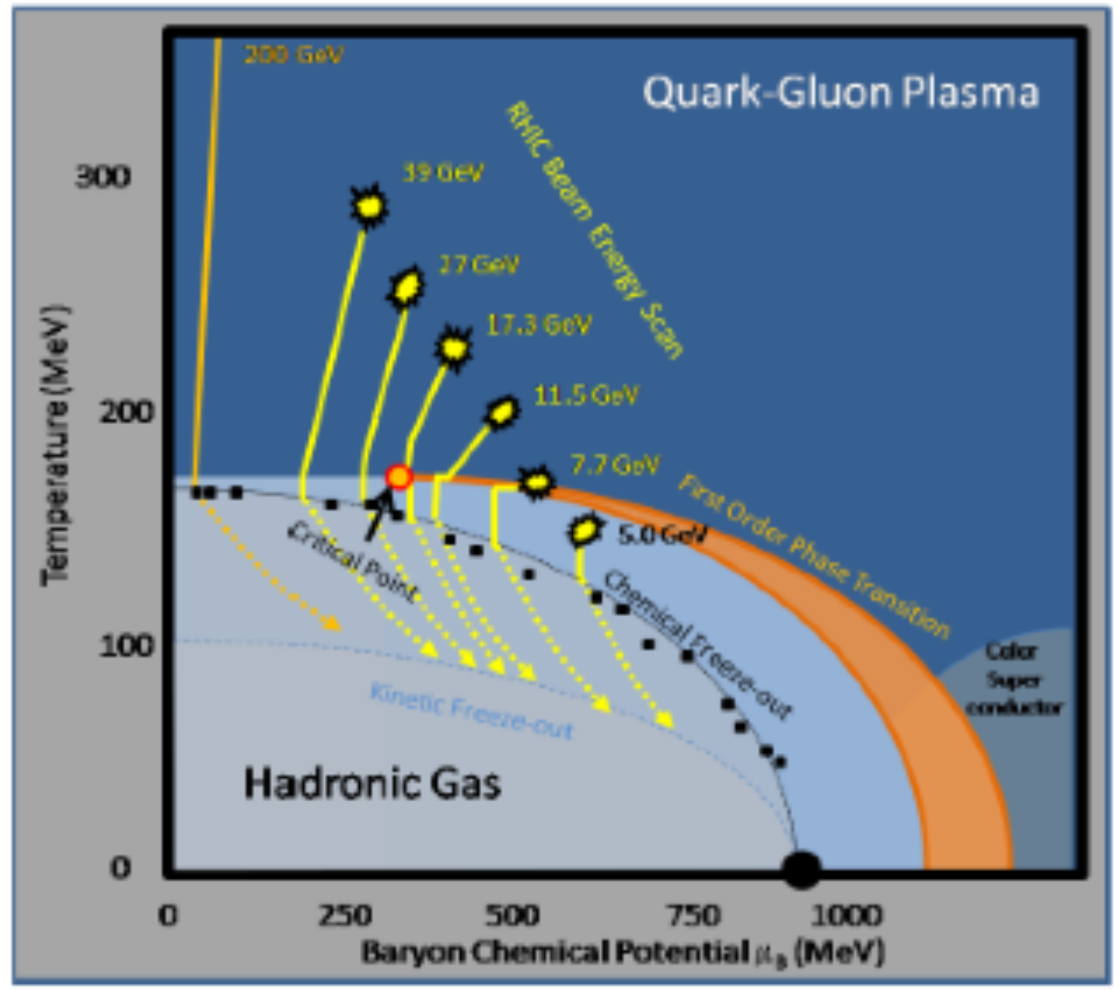
Directed flow



STAR Collaboration, PRL 112, 162301 (2014).



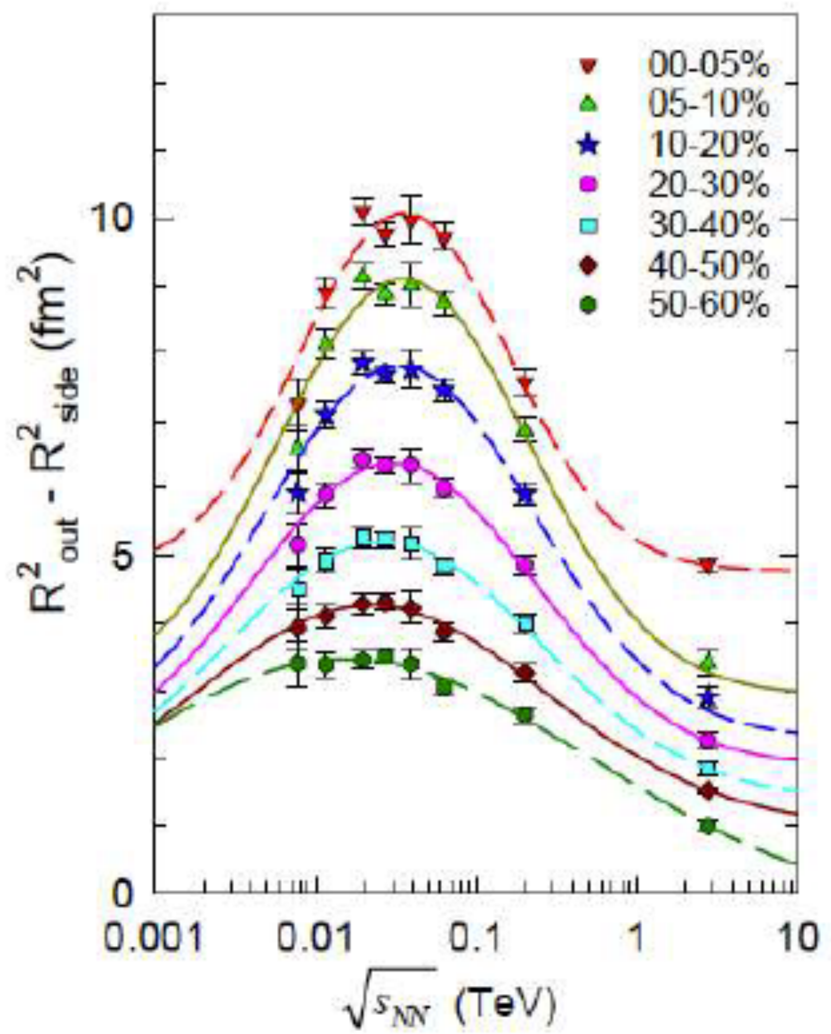
C.M. Hung, E.V. Shuryak, PRL 75, 4003 (1995).



Softest point collapse

- Non-monotonic energy dependence of directed flow => softest point of EoS, related to the first-order transition.

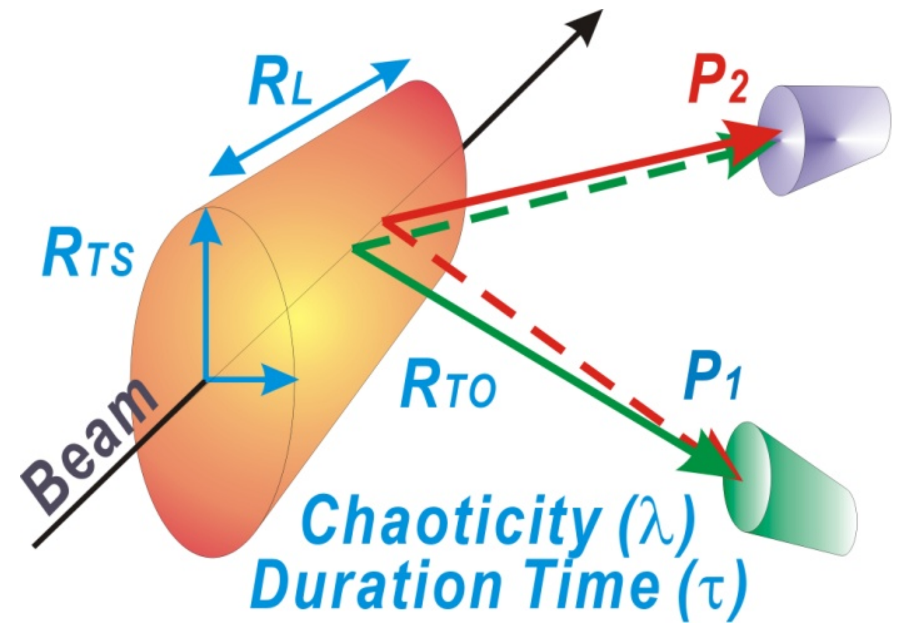
HBT radii



$$R^2_{side} = \frac{R^2_{geo}}{1 + \frac{m_T}{T}\beta_T^2}$$

$$R^2_{out} = \frac{R^2_{geo}}{1 + \frac{m_T}{T}\beta_T^2} + \beta_T^2(\Delta\tau)^2$$

$$R^2_{long} \approx \frac{T}{m_T}\tau^2$$



$$\left(R^2_{out} - R^2_{side}\right) \propto \Delta\tau^2$$

Roy A. Lacey , PRL 114, 142301 (2015).

- Non-monotonic energy dependence (a maximum for R^2_{out}-R^2_{side}) =>critical end point?

Light nuclei

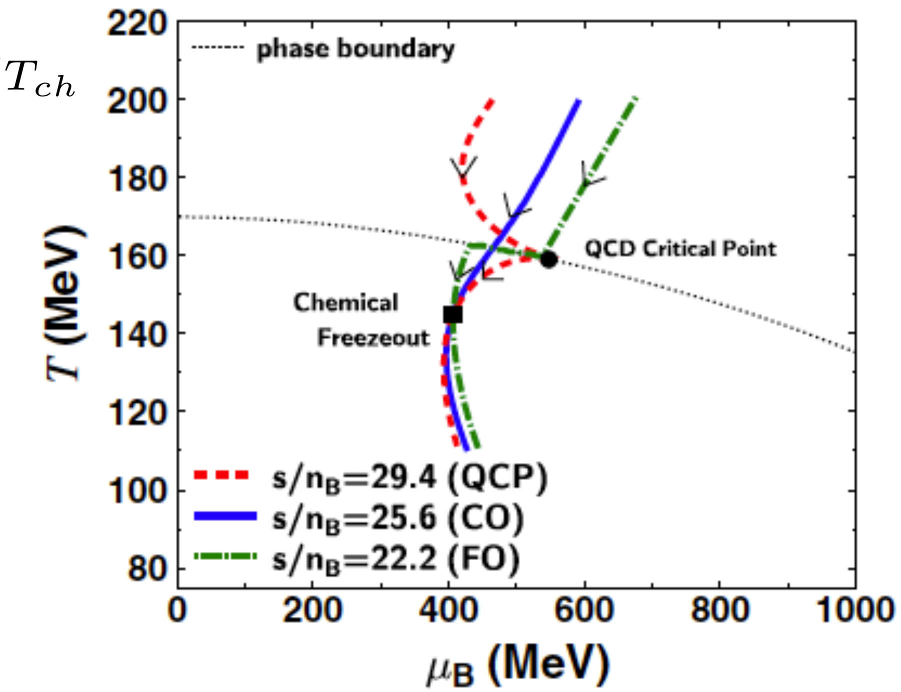
A. Andronic et al., PLB 697, 203 (2011)

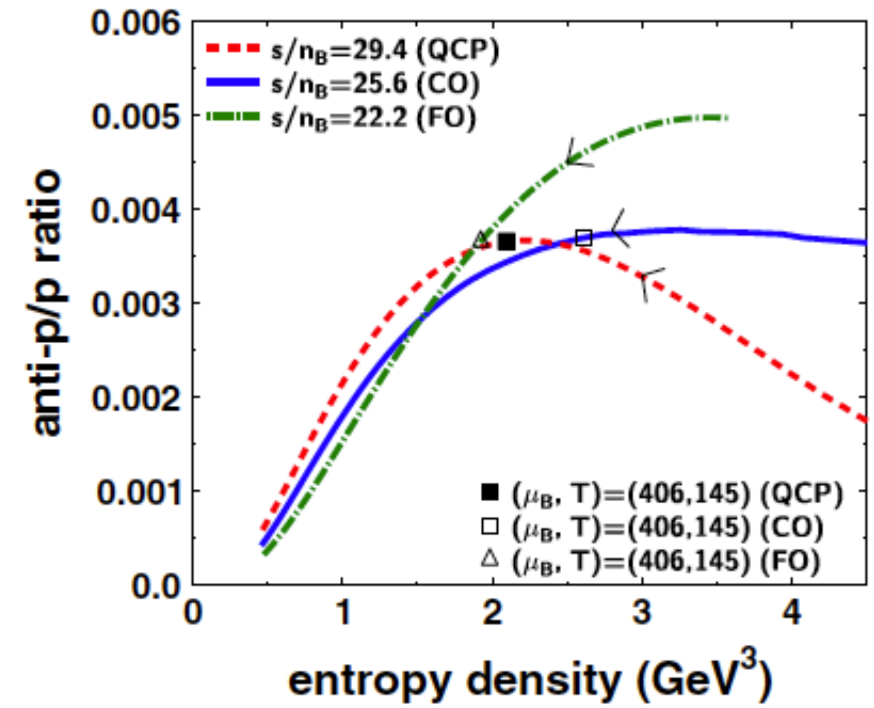
$$N_i = \frac{g_i V}{2\pi^2} m_i^2 T_{ch} K_2\left(\frac{m_i}{T_{ch}}\right) e^{\mu_i/T_{ch}}$$

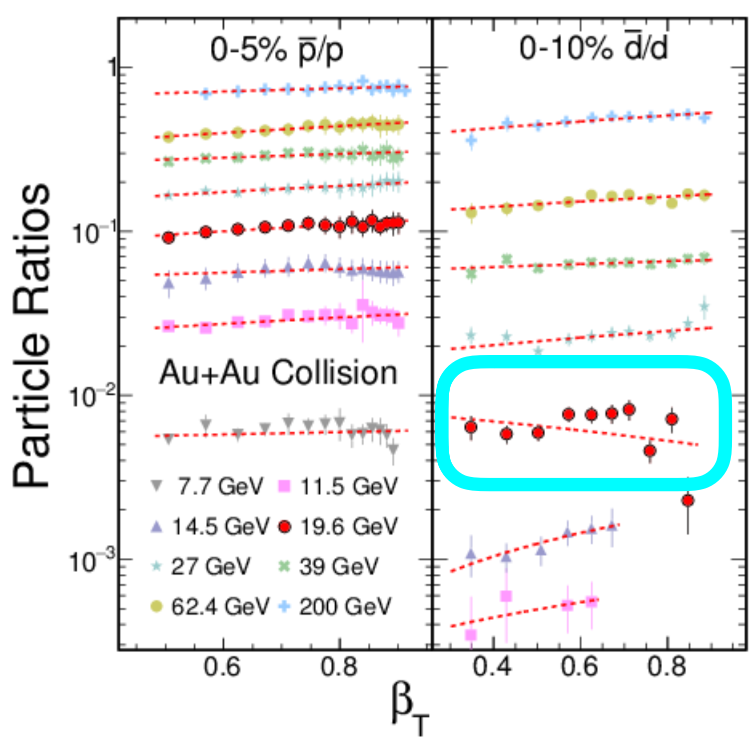
$$\bar{p}/p \sim \exp(-2\mu_B/T)$$

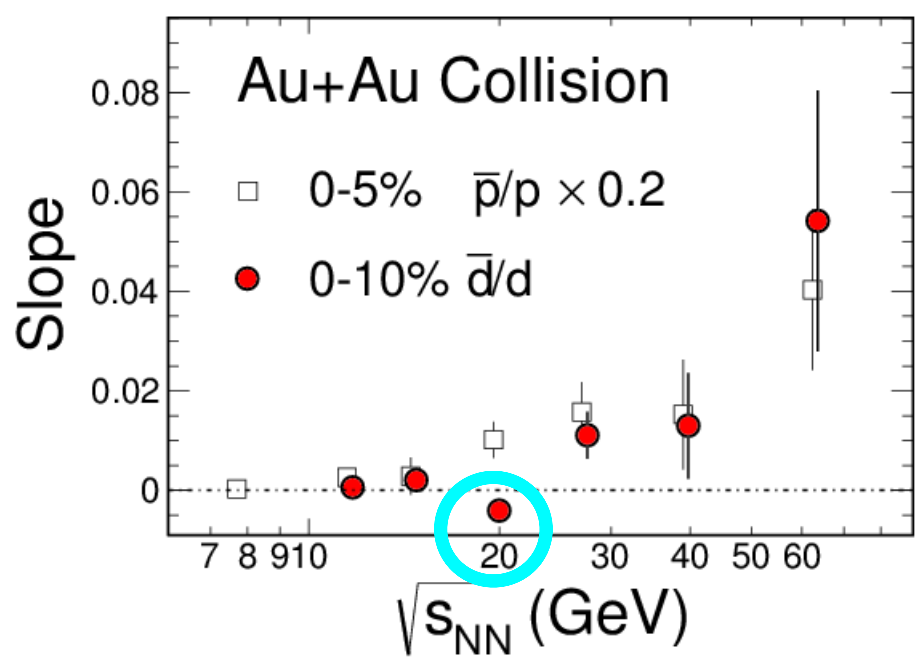
$$\bar{d}/d \sim \exp(-4\mu_B/T)$$

M. Asakawa et al, PRL 101, 122302 (2008).









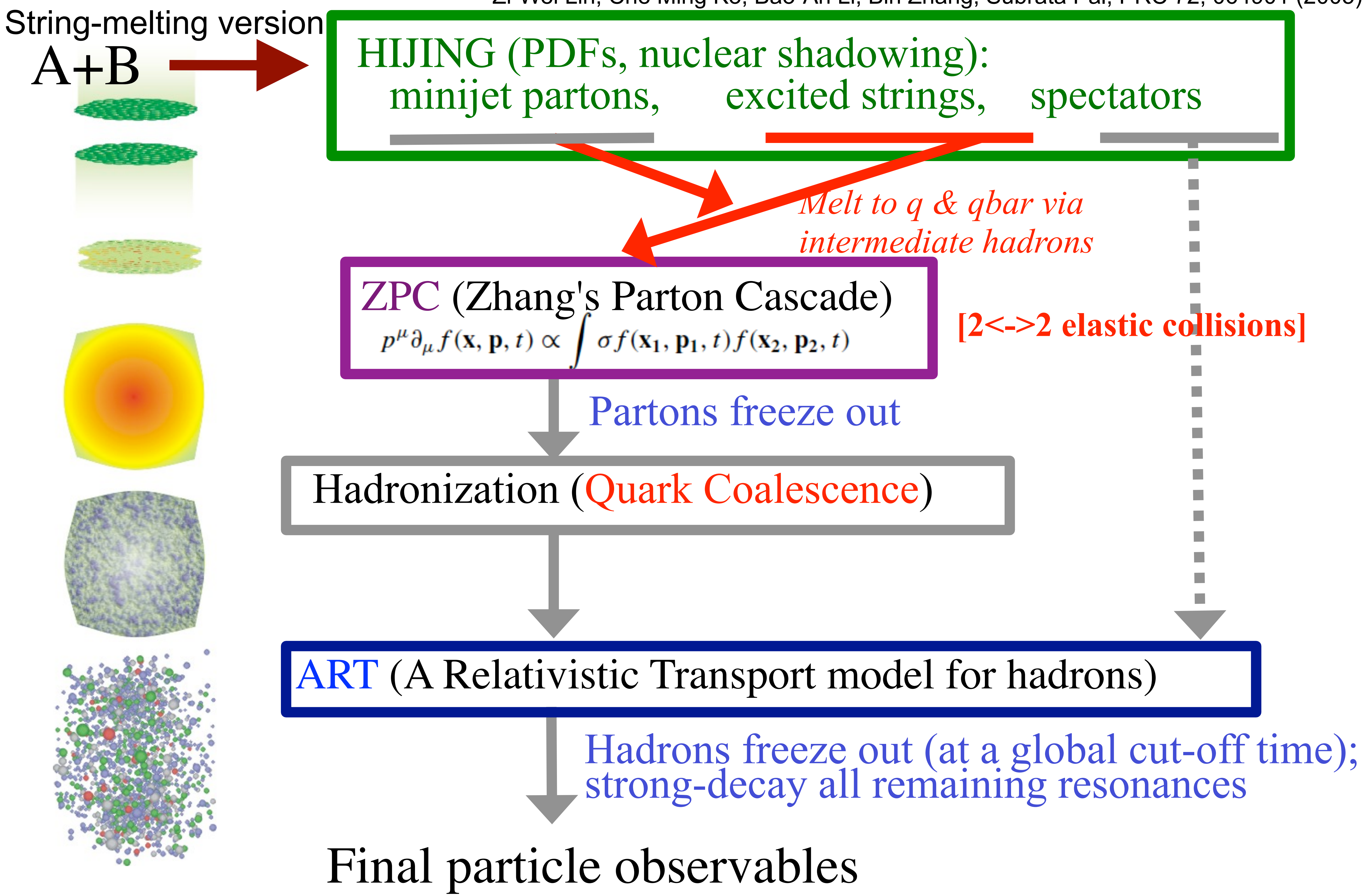
N. Yu, D Zhang, X.F. Luo, arXiv:1812.04291.

also cf. Lie-Wen Chen's talk

- A negative slope of dbar/d vs β_T
  =>focusing effect of QCD critical point.

A multiphase transport (AMPT) model

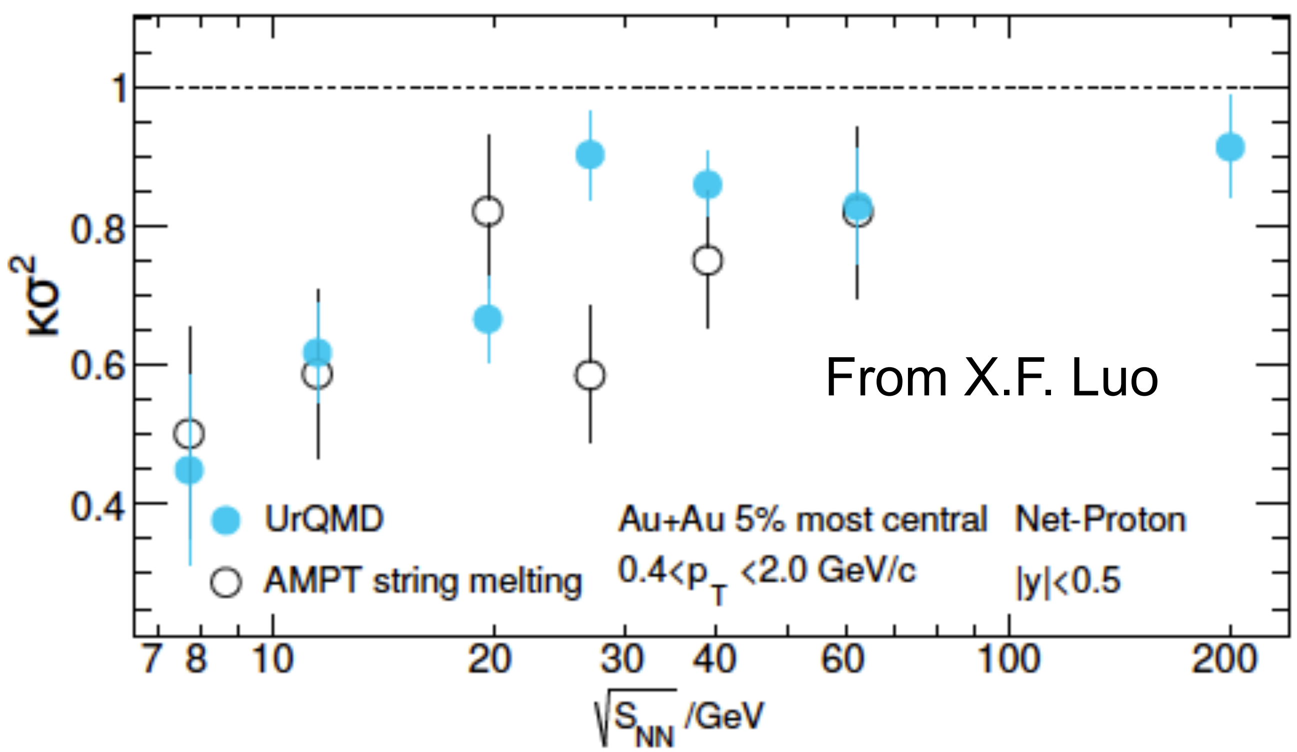
Zi-Wei Lin, Che Ming Ko, Bao-An Li, Bin Zhang, Subrata Pal, PRC 72, 064901 (2005)

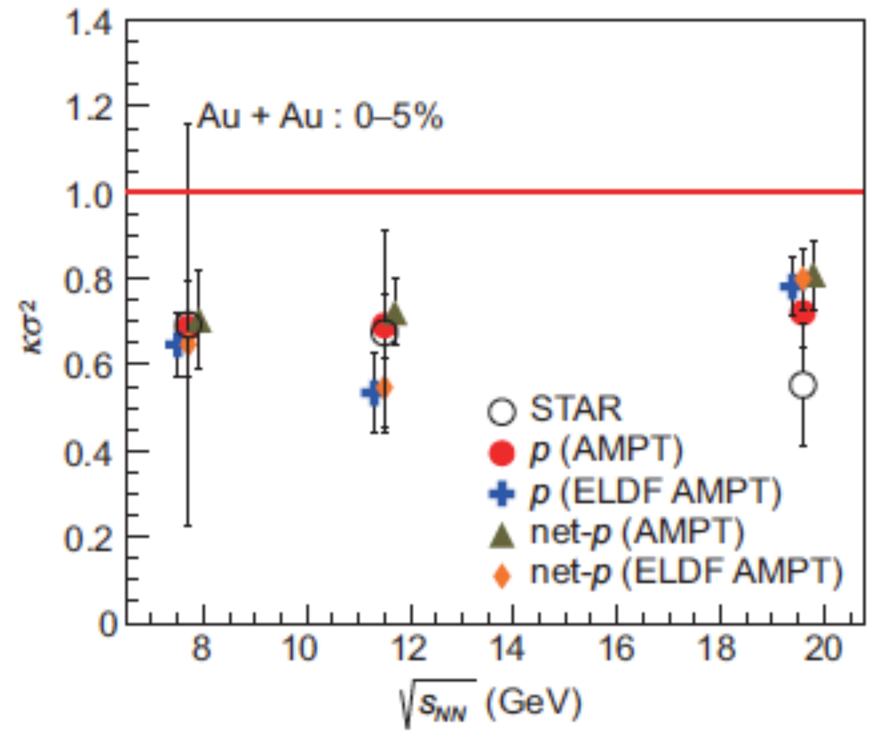


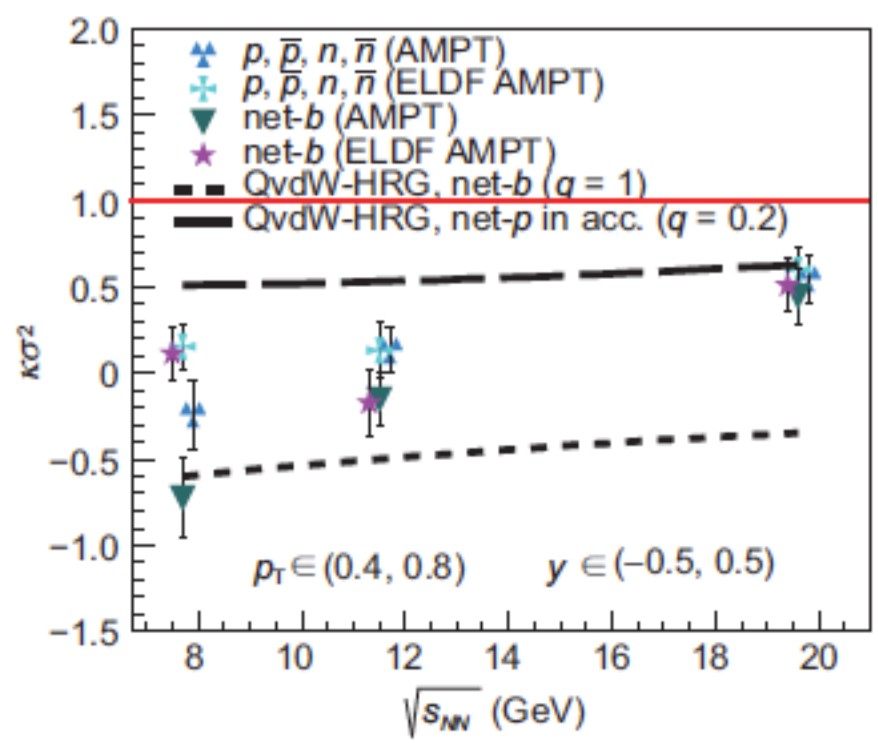
AMPT results on net-B cumulant

ELDF AMPT

Original AMPT





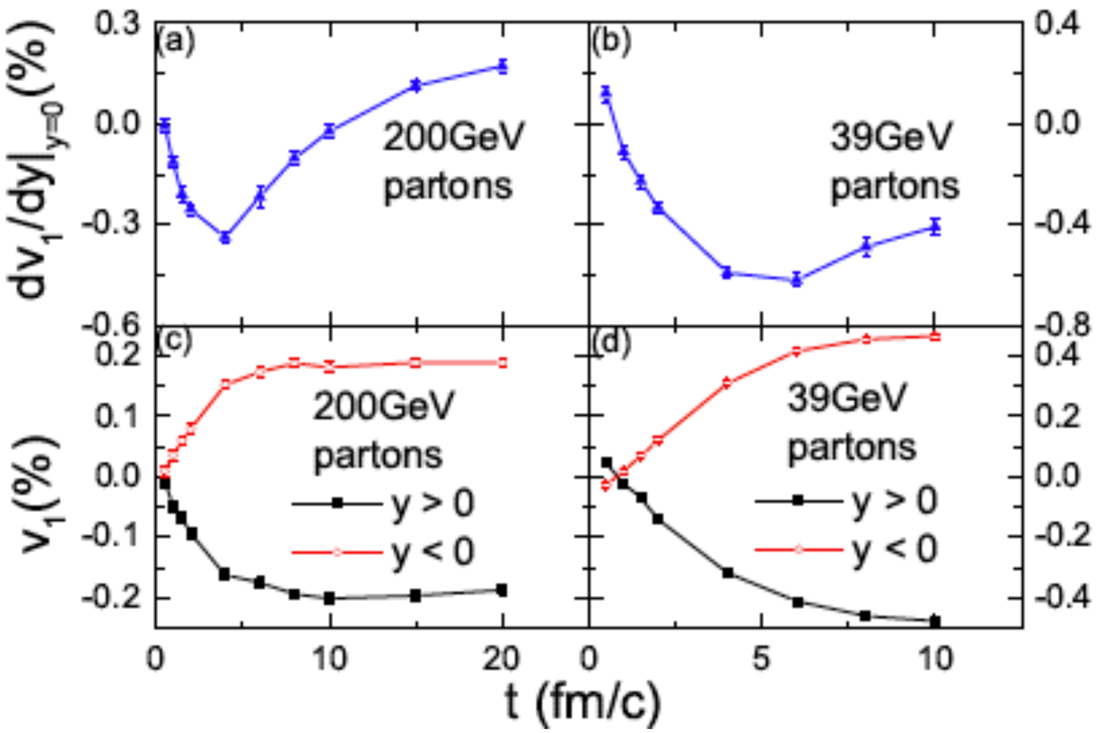


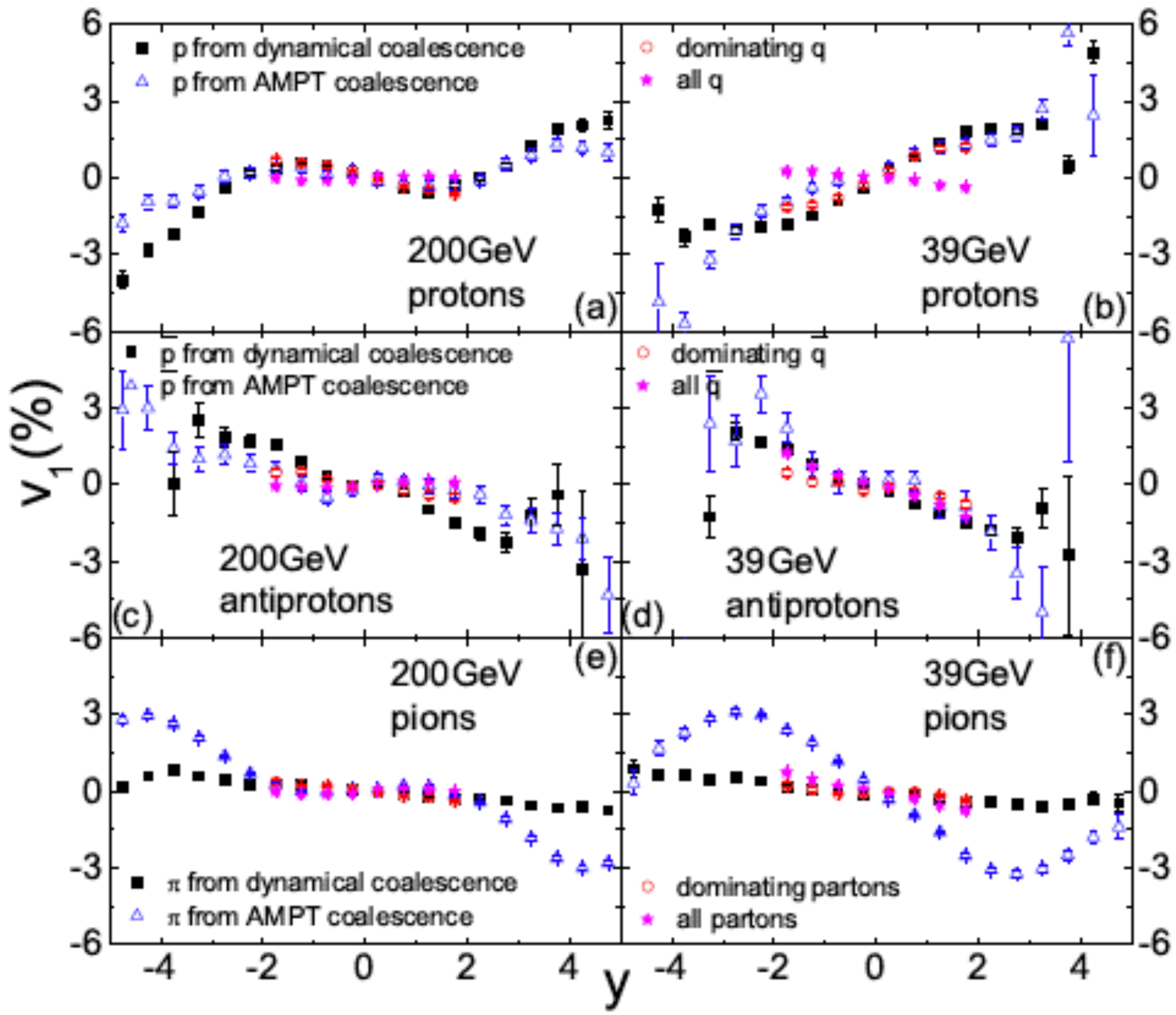
- Non-monotonic energy dependence of kurtosis is not seen from original AMPT and Extended-Local-Density-Fluctuation (ELDF) AMPT.
- Physics of the critical point is missing in current AMPT.

Xiaohai Jin et al., Sci. China Phys. Mech. Astron. 62 (2019) 11012.

AMPT results on directed flow

Chong-Qiang Guo, Chun-Jian Zhang, Jun Xu, Eur. Phys. J. A 53 (2017) 233.

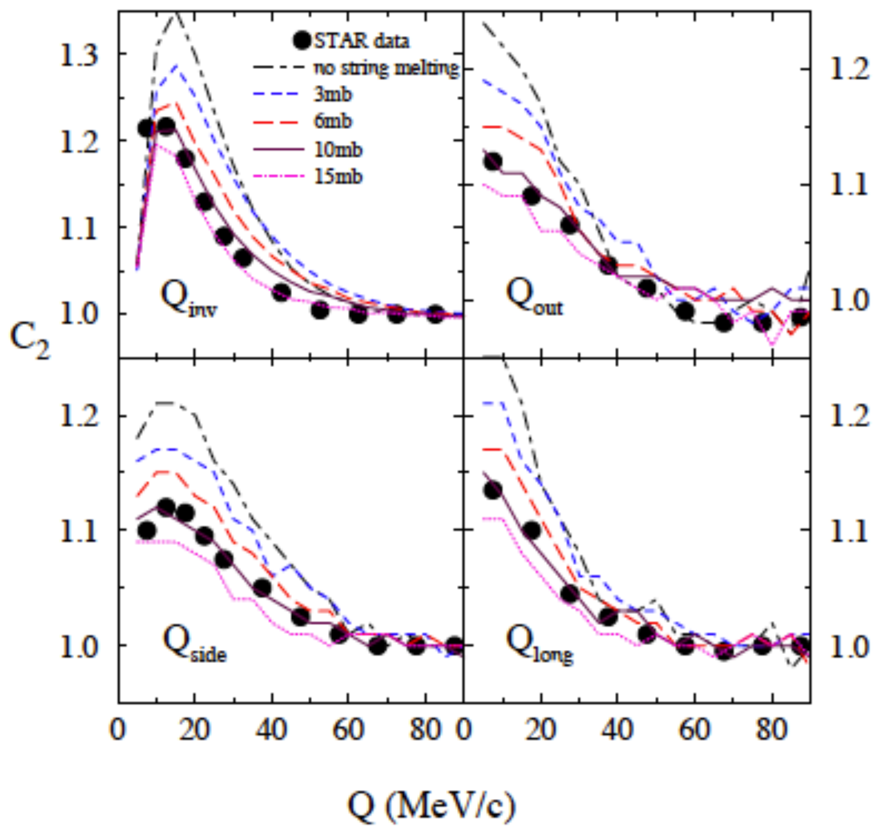




- Original AMPT: dv_1/dy depends on partonic evolution time and hadronization.
- We can learn about EOS & QCD phase transition from experimental data.

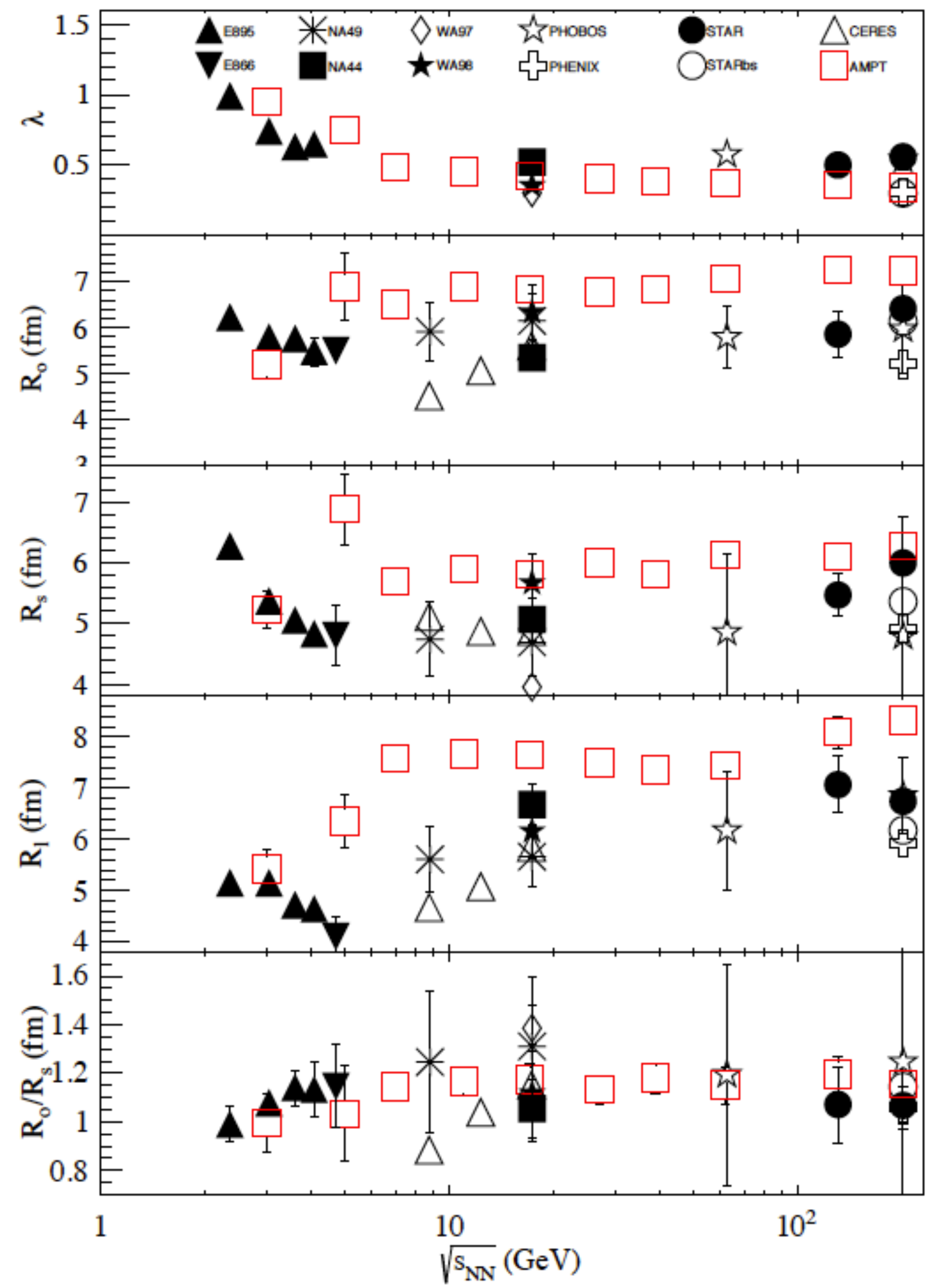
AMPT results on HBT radii

Zi-Wei Lin, C.M. Ko, Subrata Pal, Phys. Rev. Lett. 89 (2002) 152301



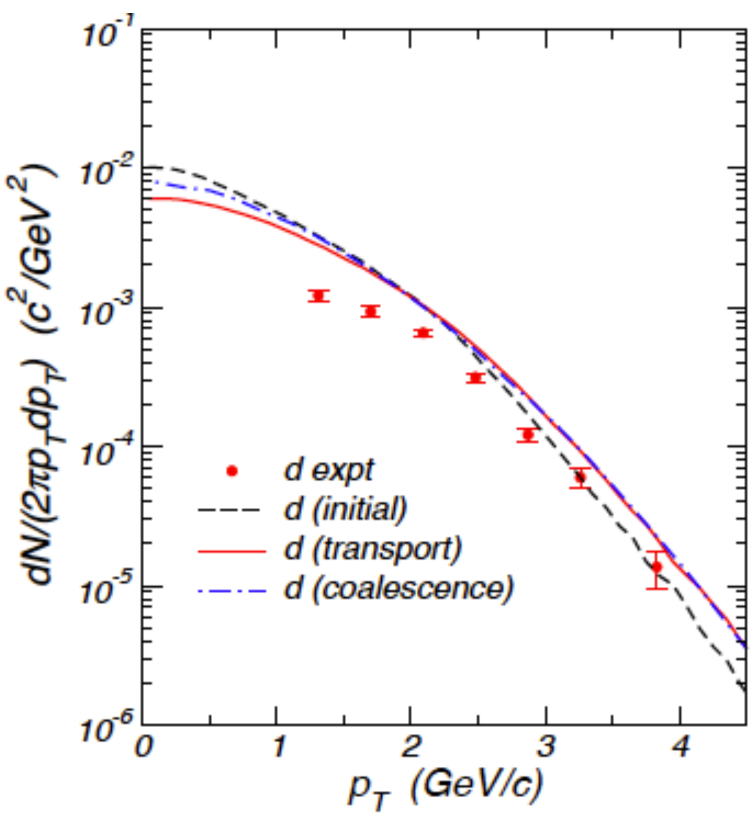
- Lin et al(2002): HBT is sensitive to partonic interactions and freeze-out volume.
- Zhang et al(2014): R_{out} & R_{side} are greater than data; R_{out}/R_{side} is consistent with data.
- Proper treatment of parton interaction, QCD phase transition, and critical point is needed.

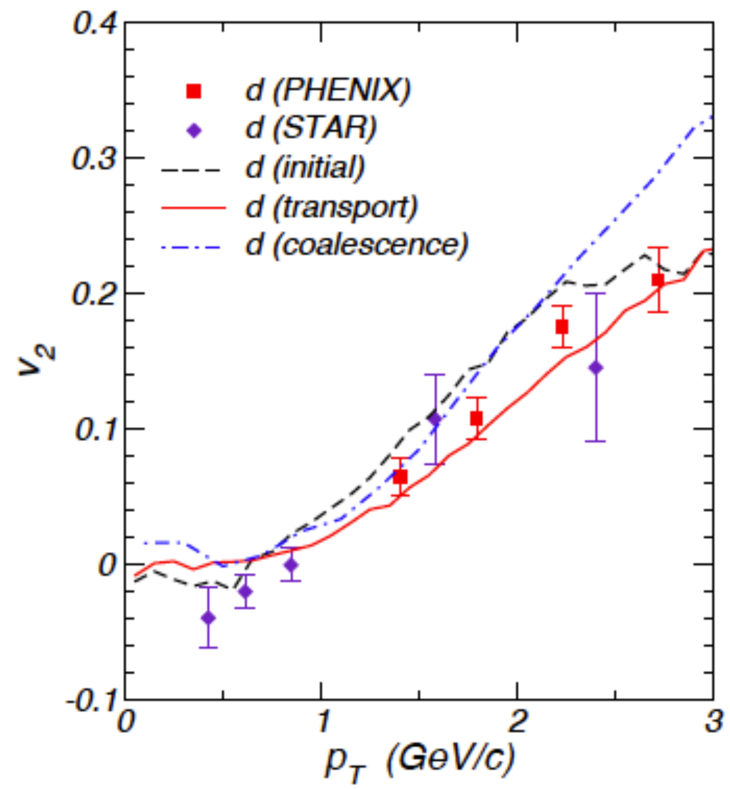
Zheng-Qiao Zhang, S. Zhang, Y.G. Ma , Chin. Phys. C 38 (2014) 014102



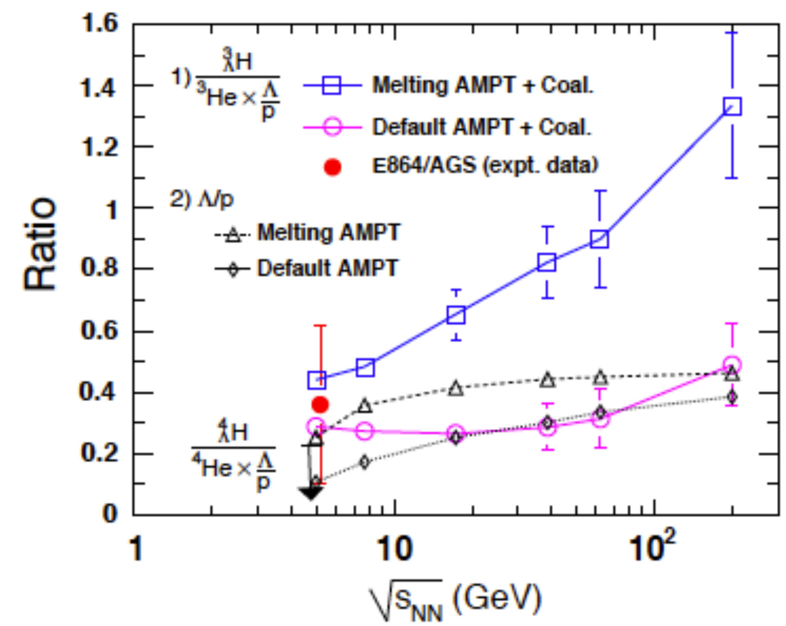
AMPT results on light nuclei

Yongseok Oh, Zi-Wei Lin, Che Ming Ko,
Phys.Rev.C80, 064902, 2009





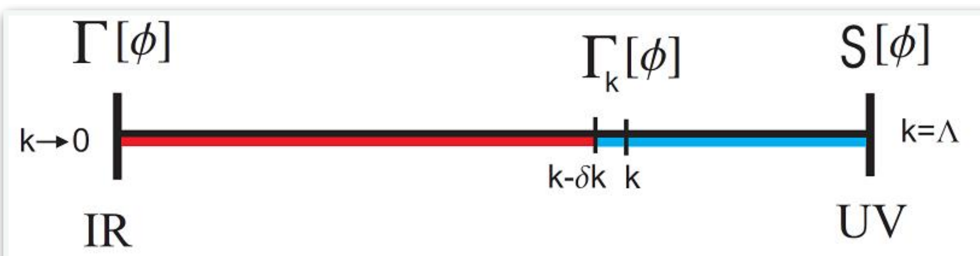
Song Zhang et al., Phys.Lett. B684 (2010) 224-227



- Oh, Lin, and Ko (2009): Blast Wave model + ART model well describe deuterons' spectra and elliptic flow.
- Song Zhang et al (2010) studied hyper-light nuclei production with AMPT.
- Focusing effect of QCD critical point is expected to be necessary in order to reproduce the negative slope of dbar/d vs β_T.

First principle-based FRG

Functional renormalization group (FRG)



Rebosonized QCD Effective action:

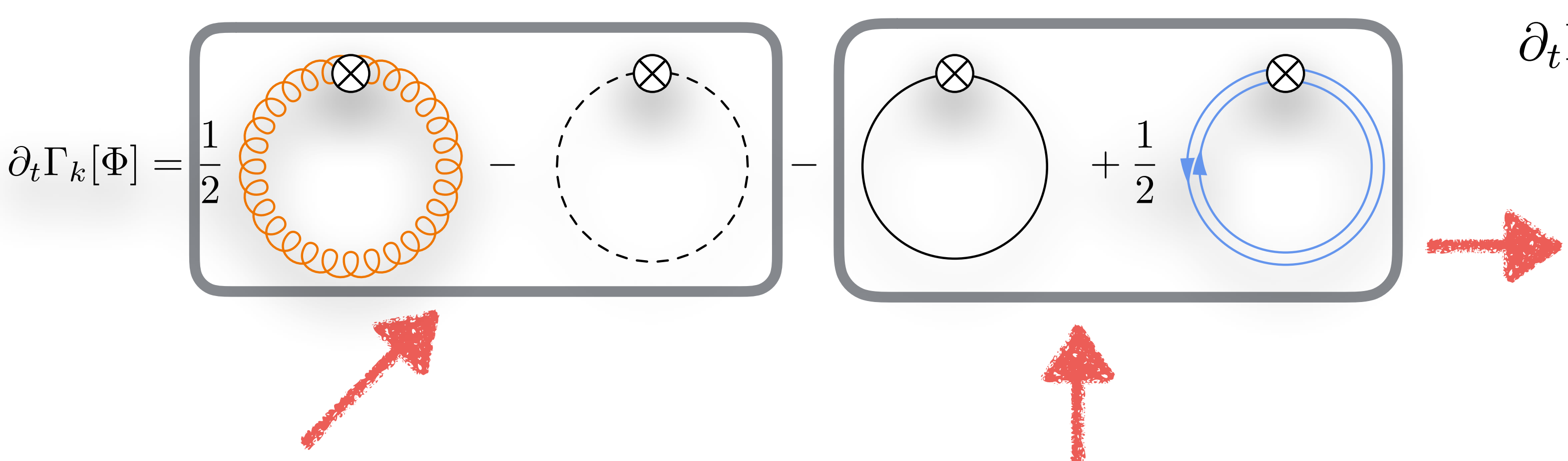
$$\Gamma_k = \int_x \Big\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + Z_c (\partial_\mu \bar{c}^a) D^{ab}_\mu c^b + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2$$

$$+ Z_q \bar{q}\,(\gamma_\mu D_\mu + m_0^s)\, q$$

$$- \lambda_q \Big[(\bar{q}\,\tau^0 q)^2 + (\bar{q}\,\vec{\tau} q)^2 \Big] + h_k\, \bar{q}\, \left(\tau^0 \sigma + \vec{\tau}\cdot\vec{\pi}\right)\, q$$

$$+ \frac{1}{2} Z_\phi (\partial_\mu \phi)^2 + V_k(\rho) - c_\sigma\, \sigma \Big\},$$

Flow equation:



glue sector

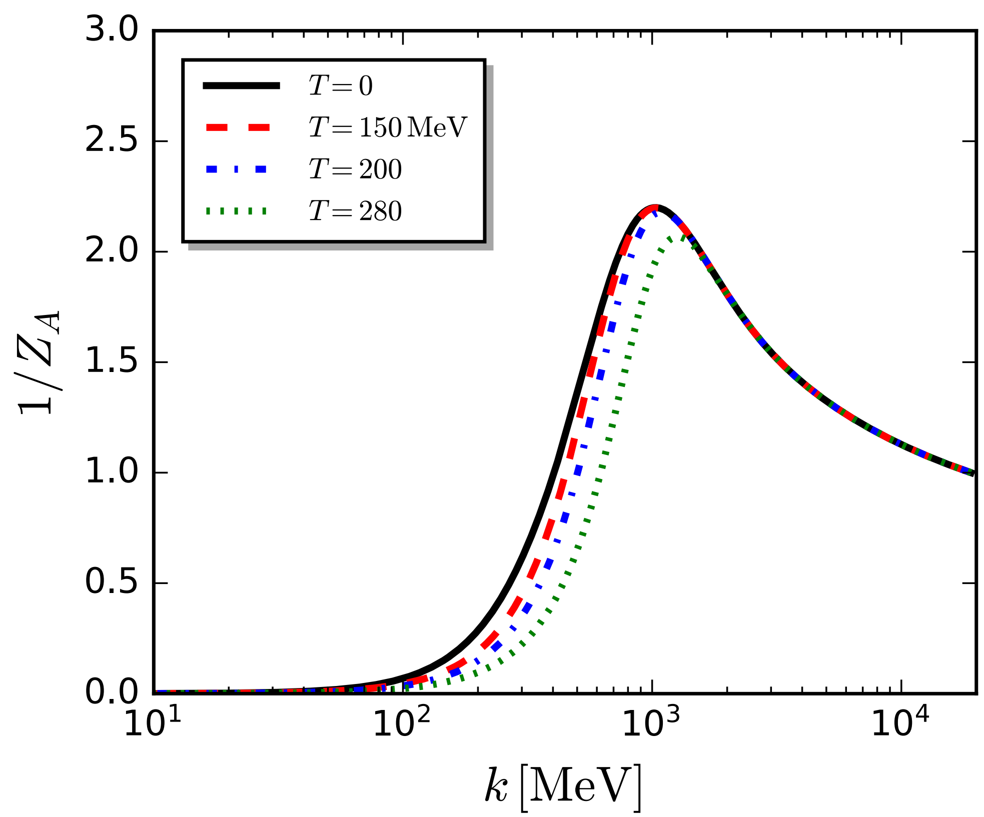
matter part

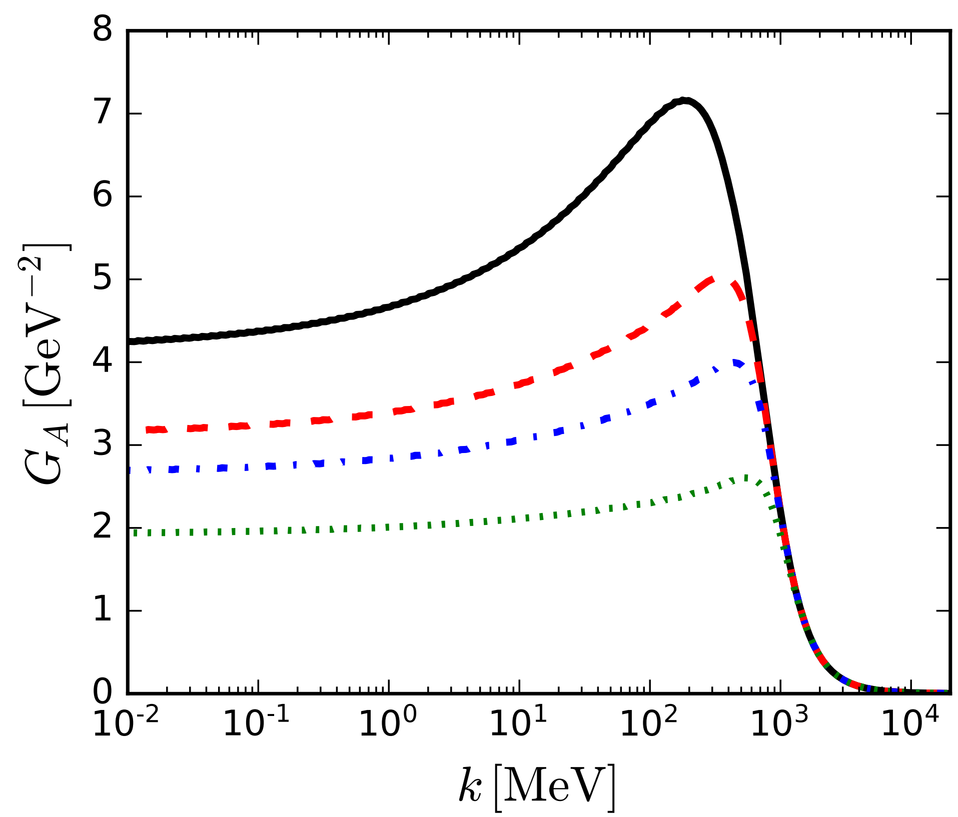
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr}\Big[\partial_t R_k \big(\Gamma_k^{(2)}[\Phi] + R_k\big)^{-1} \Big]$$

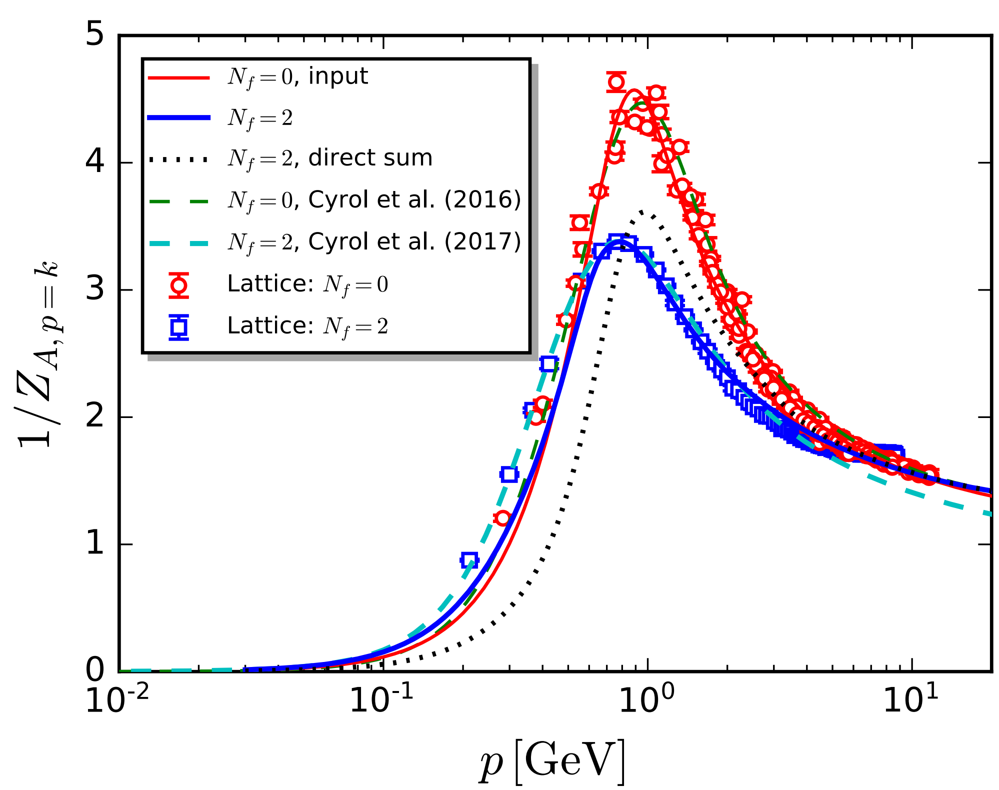
$$= \frac{1}{2} \mathrm{Tr}\big(G_k^{AA}[\Phi] \partial_t R_k^A\big) - \mathrm{Tr}\big(G_k^{c\bar{c}}[\Phi] \partial_t R_k^c\big)$$

$$- \mathrm{Tr}\big(G_k^{q\bar{q}}[\Phi] \partial_t R_k^q\big) + \frac{1}{2} \mathrm{Tr}\big(G_k^{\phi\phi}[\Phi] \partial_t R_k^\phi\big),$$

QCD interaction: gluon propagator at finite temperature

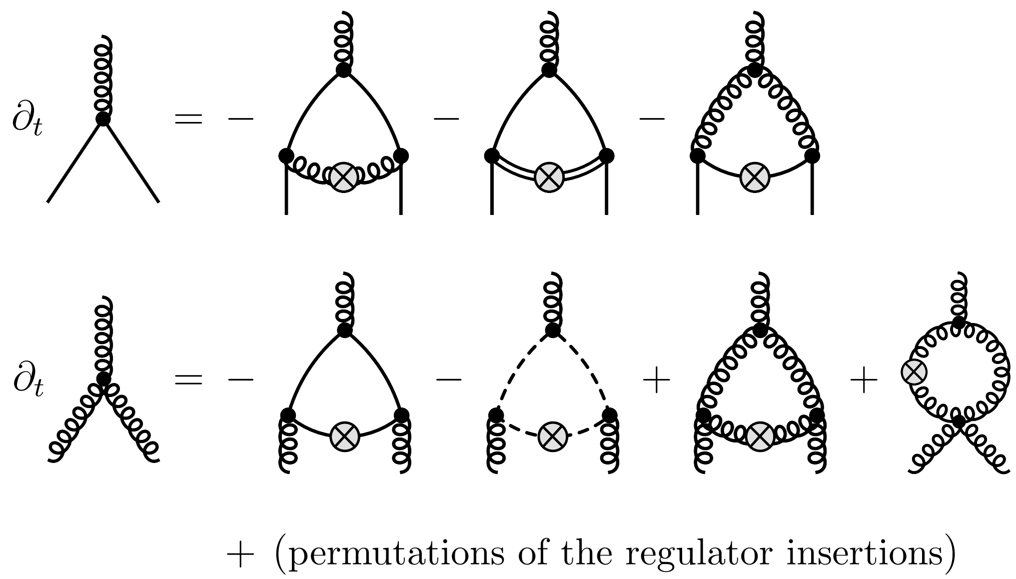


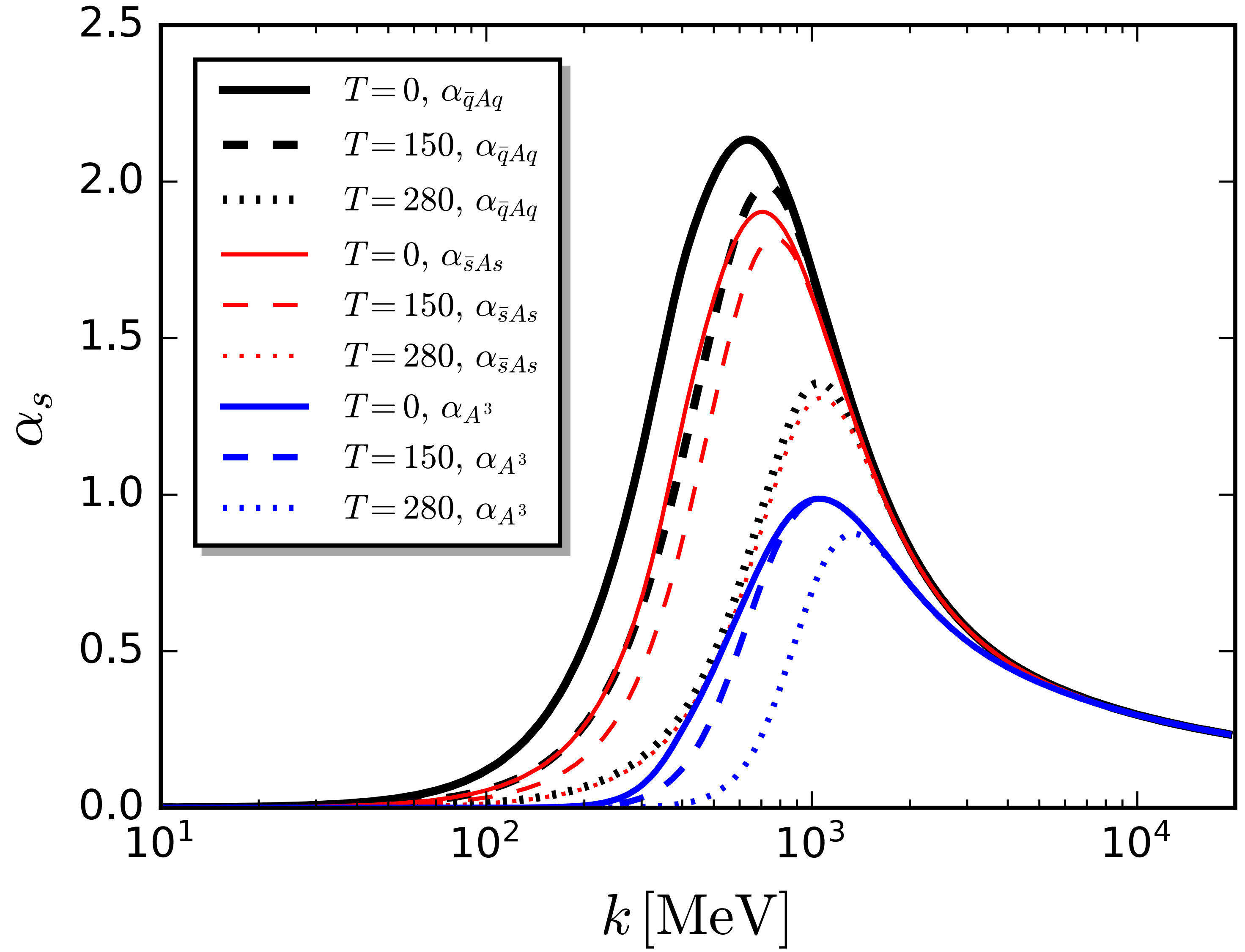




WF, J.M. Pawlowski, F. Rennecke, arXiv:1905.xxxxx.

QCD strong couplings among quarks and gluons



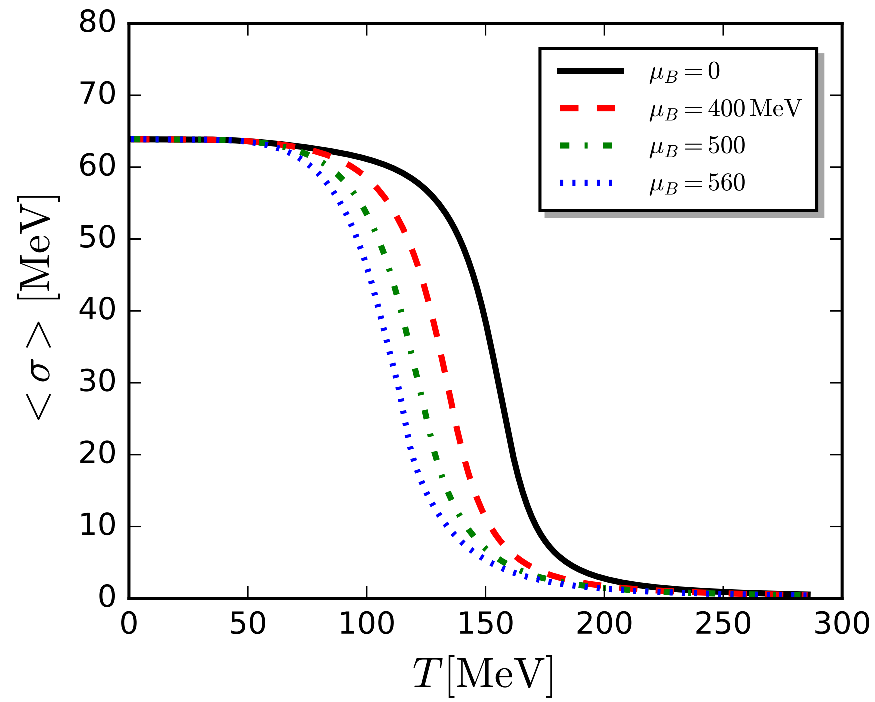


Crucial for proper treatment of QCD interactions in AMPT

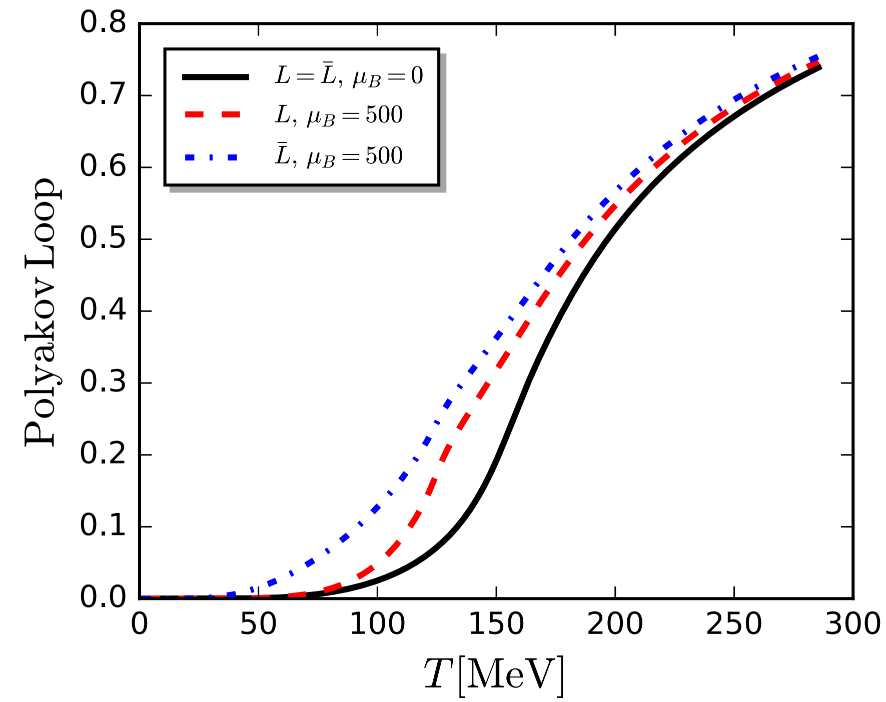
WF, J.M. Pawlowski, F. Rennecke, arXiv:1905.xxxxx.

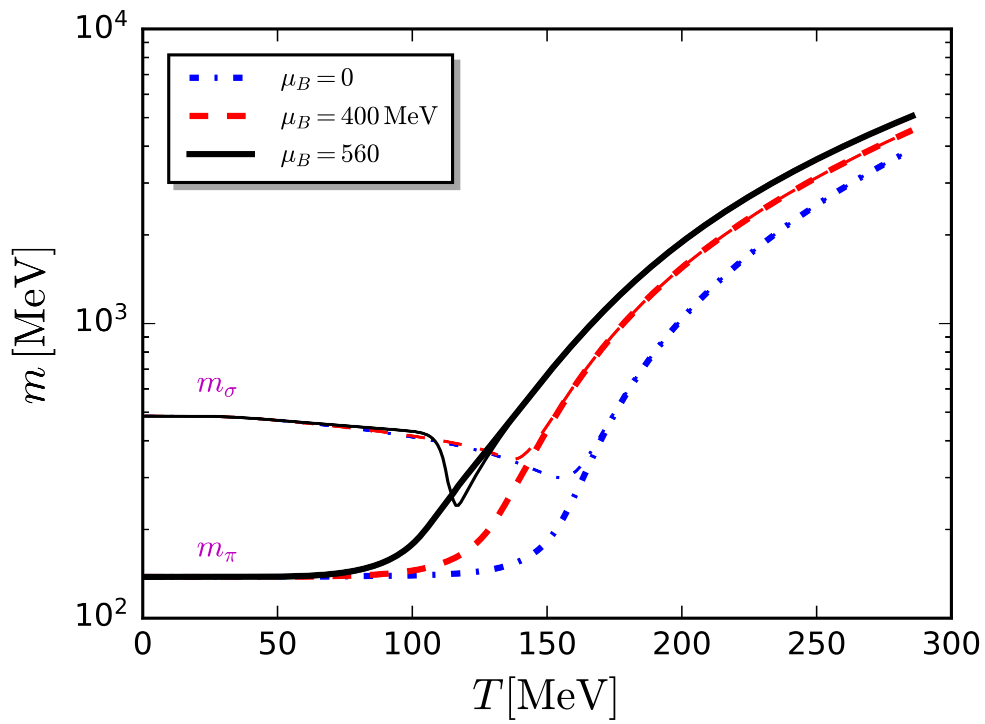
QCD phase transitions well described by FRG

Chiral phase transition



Deconfinement phase transition

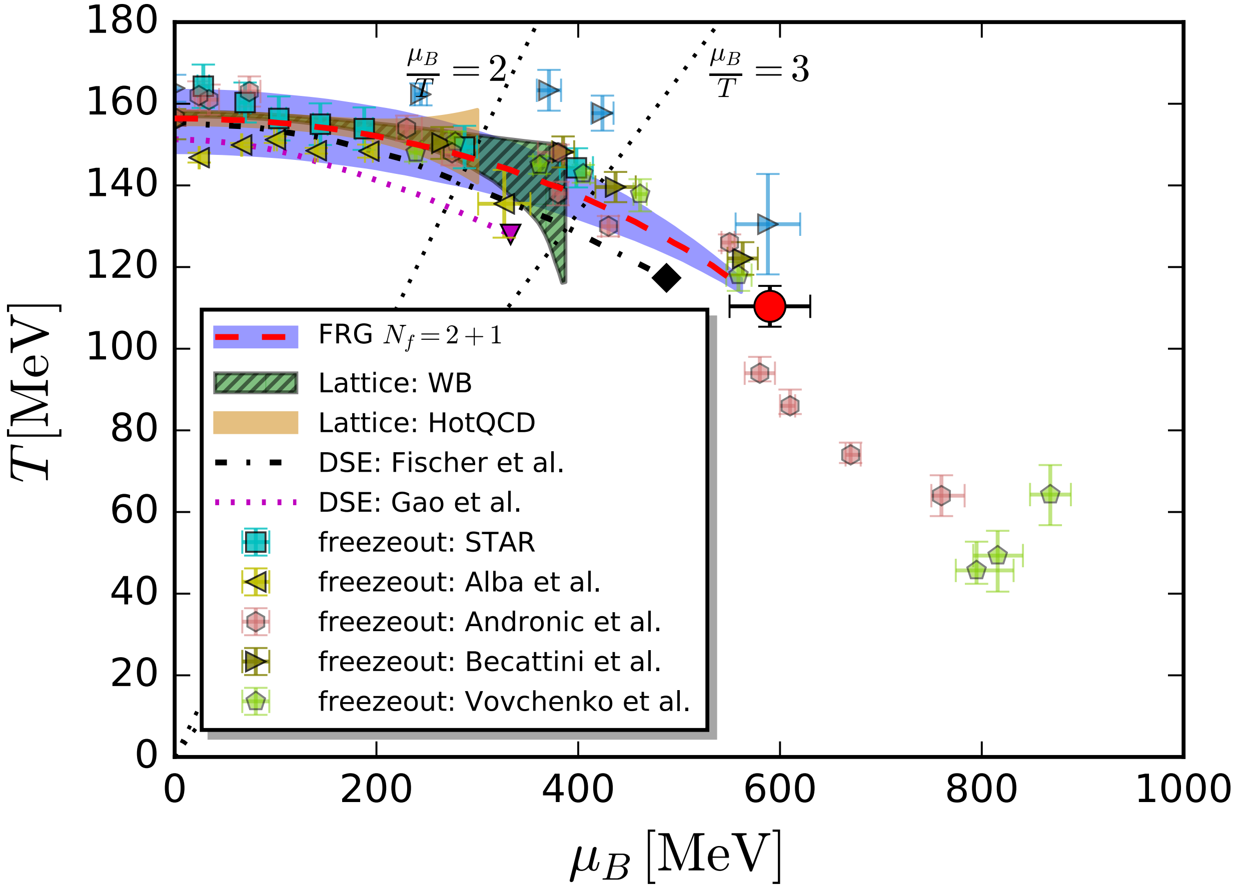




CEP

WF, J.M. Pawlowski, F. Rennecke, arXiv:1905.xxxxx.

Phase diagram and curvature



$$(T_{\mathrm{CEP}}, \mu_{B\,\mathrm{CEP}}) = (110 \pm 5, 590^{+30}_{-20})\,\mathrm{MeV}$$

WF, J.M. Pawlowski, F. Rennecke, arXiv:1905.xxxxx.

FRG is applicable at high muB and well suited for search of CEP.

FRG curvature of the phase boundary:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \lambda \left(\frac{\mu_B}{T_c}\right)^4 + \cdots,$$

$$\kappa = 0.0167 \pm 0.0025$$

Lattice result:

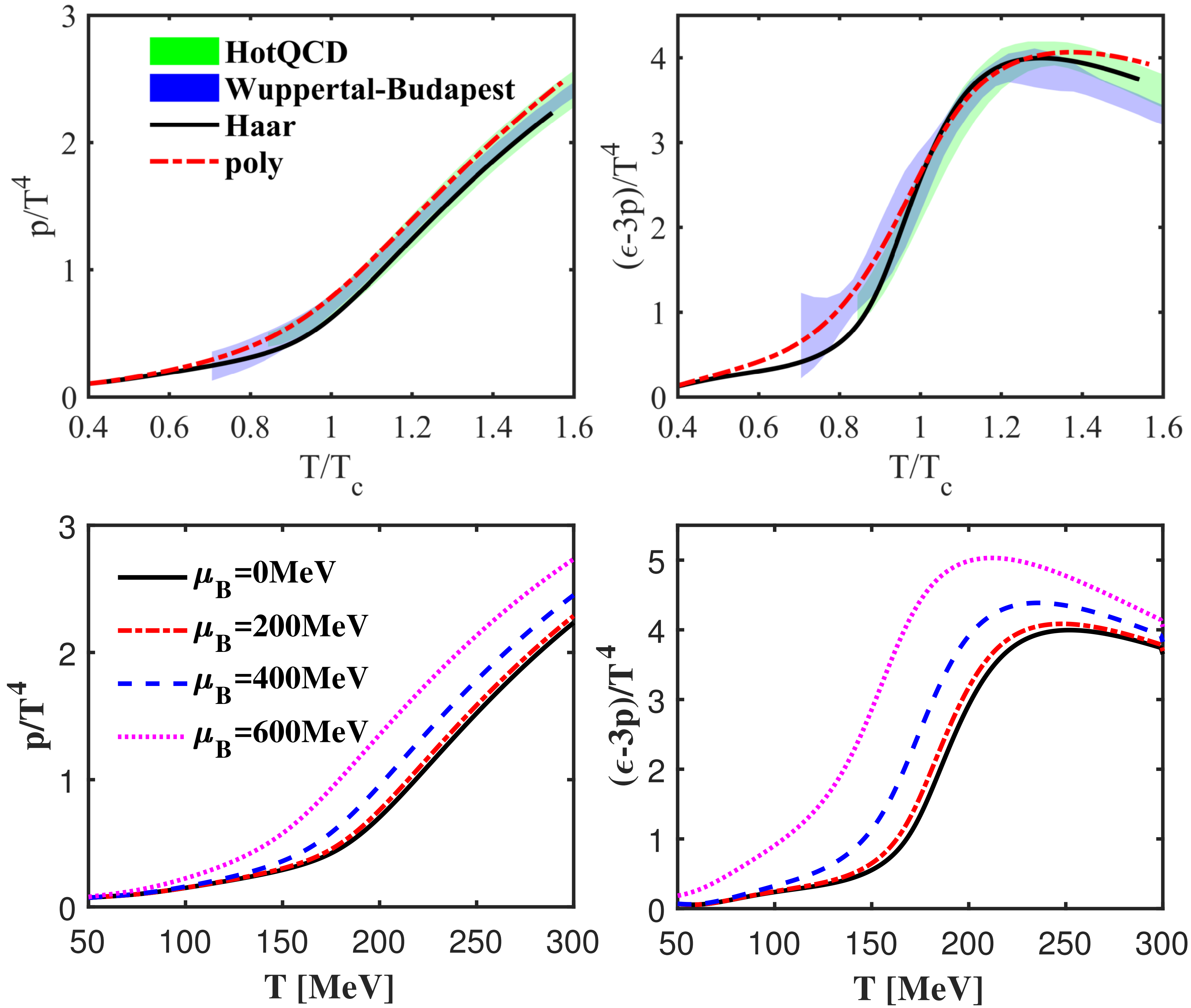
$$\kappa = 0.0149 \pm 0.0021$$

R. Bellwied *et al.* (WB), arXiv:1507.07510.

$$\kappa = 0.016 \pm 0.006$$

A. Bazavov *et al.* (HotQCD), arXiv:1812.08235.

QCD equation of state

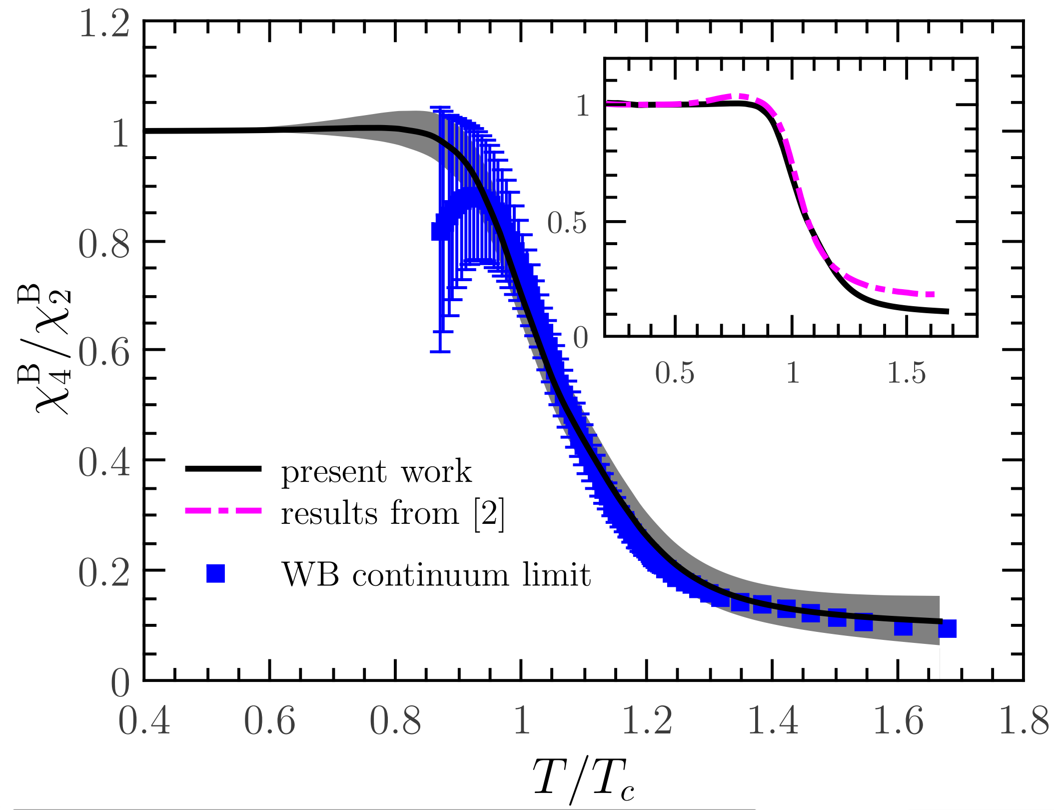


FRG results at high muB

R.Wen, C. Huang, WF, arXiv:1809.04233, to appear in PRD.

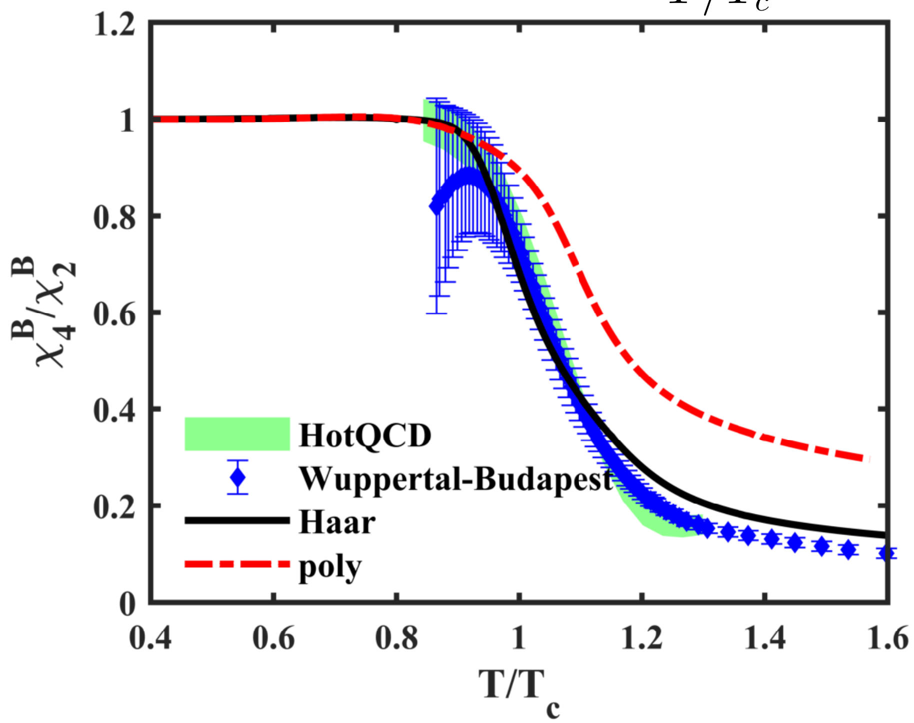
Baryon number fluctuations from FRG

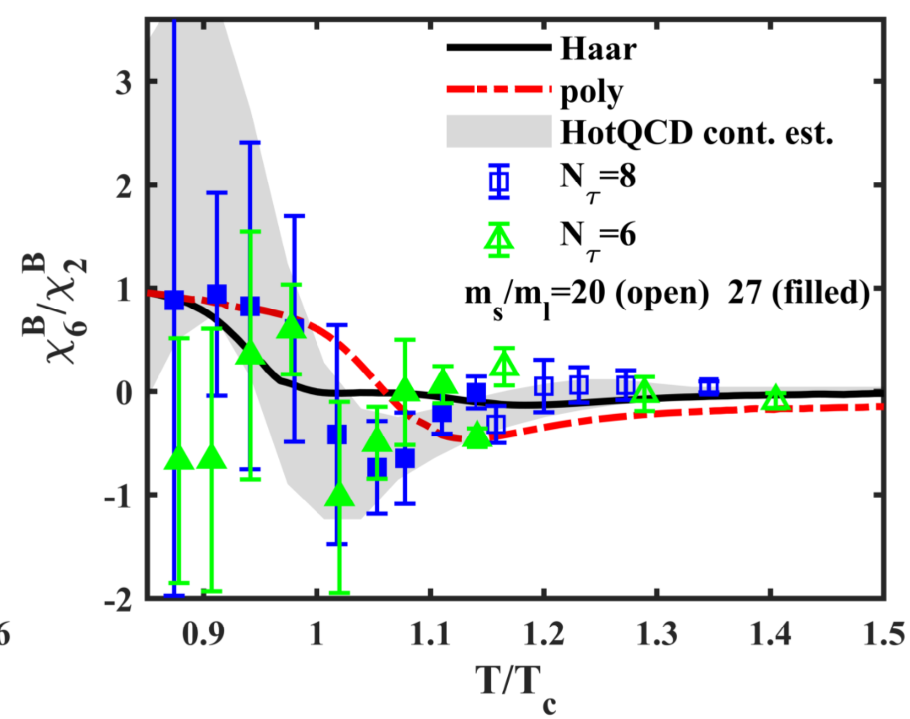
WF, J.M. Pawlowski, F. Rennecke, Bernd-Jochen Schaefer, PRD 94 (2016) 116020



$N_f = 2$

Combining AMPT and FRG allows us to calculate dynamical evolution of the fluctuations.





$N_f = 2 + 1$

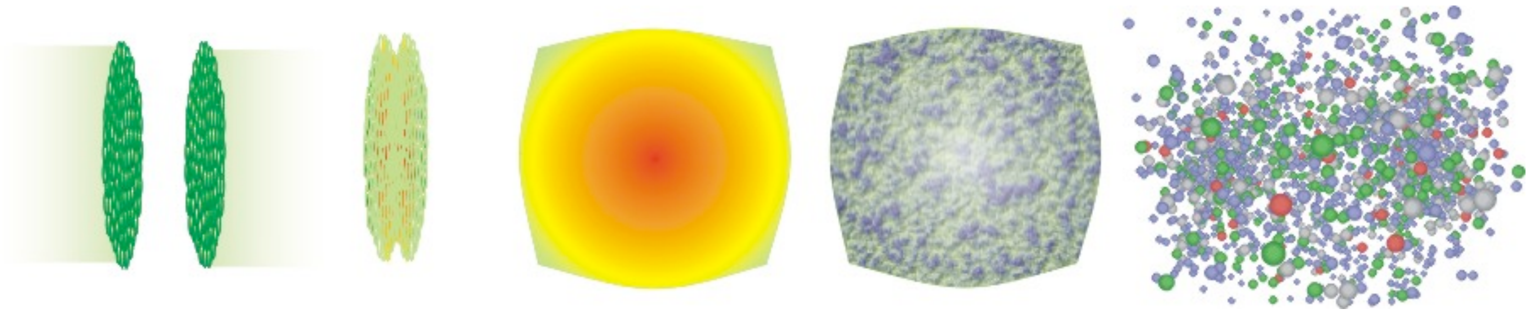
R.Wen, C. Huang, WF, arXiv:1809.04233, to appear in PRD.

Plan of combining AMPT with FRG

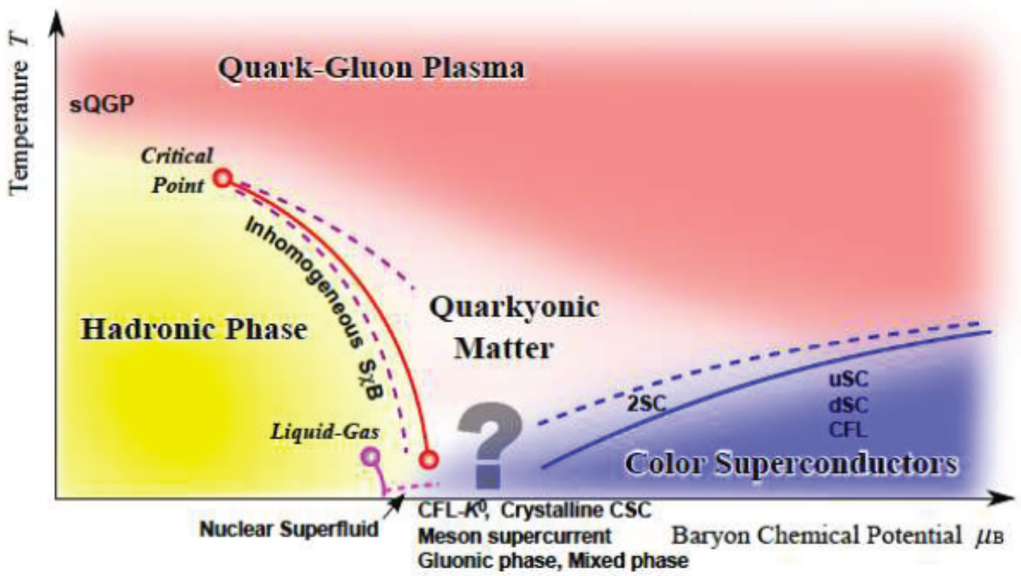
- Model parton collisions with FRG at finite temperature and chemical potential.
- Relate chiral and deconfinement order parameters to parton potential in AMPT.
- Relate sigma mode near CEP to interaction potential.
- Improve the hadronization in AMPT with FRG phase transition.
- Study hadronic modification of CEP signals.

AMPT and FRG complement each other

- **AMPT**: a good **dynamical** model including important evolution stages of heavy-ion collisions, can directly compare with experimental observables.



- **FRG**: a non-perturbative **static** QCD approach, well describes QCD phase transitions, and consistent with lattice QCD.

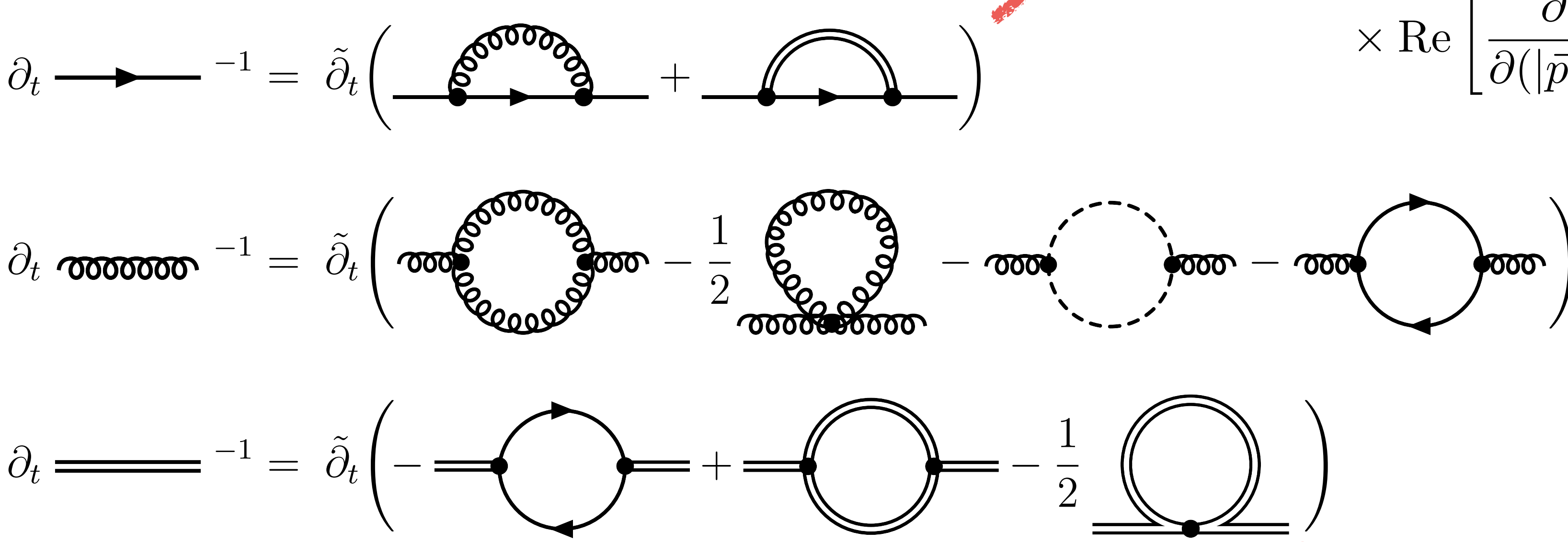


Thank you very much for your attention!

Backup

Propagators and Anomalous Dimensions

Flow equation:



Quark anomalous dimension:

$$\eta_{q,k}(p_0,\vec{p}) = \frac{1}{4Z_{q,k}(p_0,\vec{p})}$$

$$\times \,\mathrm{Re}\left[\frac{\partial}{\partial(|\vec{p}|^2)}\mathrm{tr}\left(i\vec{\gamma}\cdot\vec{p}\left(-\frac{\delta^2}{\delta\bar{q}(p)\delta q(p)}\partial_t\Gamma_k\right)\right)\right],$$

Glue sector:

$$\eta_A = \eta^{\mathrm{QCD}}_{A,T=0} + \Delta\eta^{\mathrm{glue}}_{A,T} + \Delta\eta^{q}_{A,T}\,,$$

with

$$\eta^{\mathrm{QCD}}_{A,k;T=0} = -\frac{\partial_t Z^{\mathrm{QCD}}_{A,k=0}(p=k)}{Z^{\mathrm{QCD}}_{A,k=0}}\,.$$

Meson anomalous dimension:

$$\eta_{\phi,k}(p_0,\vec{p}) = -\frac{1}{Z_{\phi,k}}\frac{1}{\delta_{ij}}\frac{\partial}{\partial(|\vec{p}|^2)}\frac{\delta^2\partial_t\Gamma_k}{\delta\pi_i(-p)\delta\pi_j(p)}\,,$$

Dynamical Hadronization

Introduce a scale-dependent meson field:

$$\langle \partial_t \hat{\phi}_k \rangle = \dot{A}_k \, \bar{q}\tau q + \dot{B}_k \, \phi + \dot{C}_k \, \hat{e}_\sigma \, ,$$

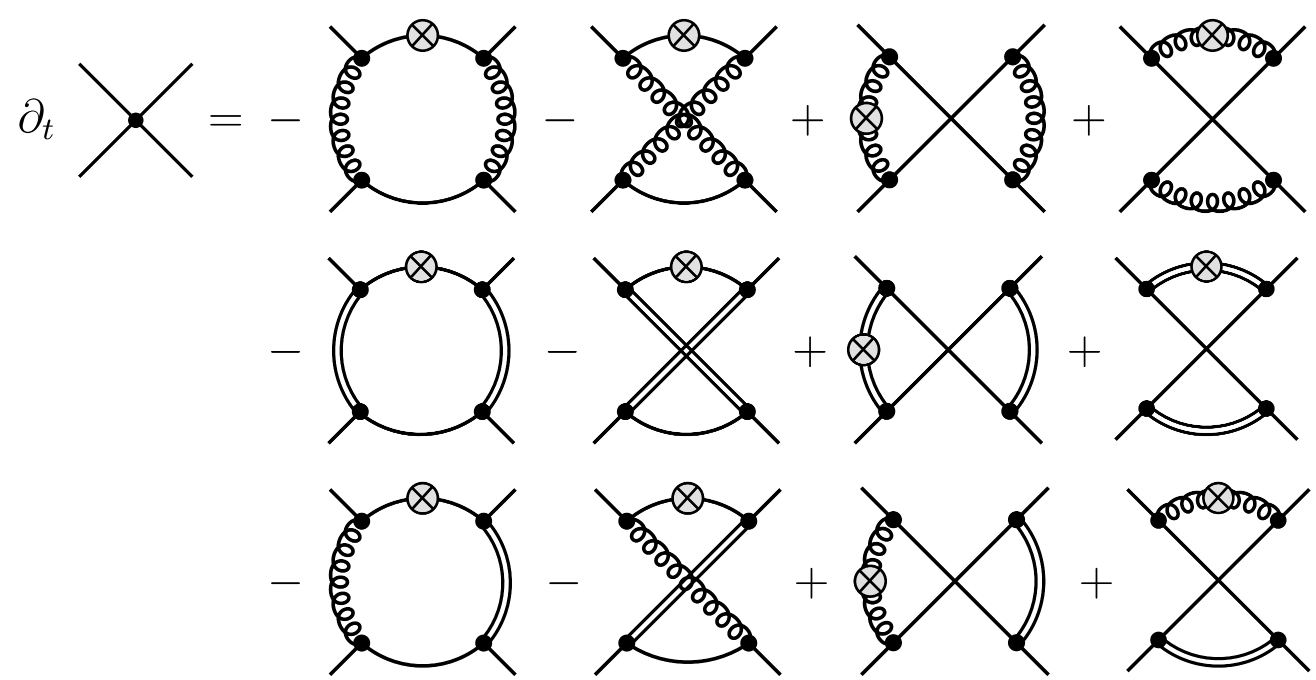
The Wetterich equation is modified:

$$\partial_t \Gamma_k[\Phi] + \int \langle \partial_t \hat{\phi}_{k,i} \rangle \frac{\delta \Gamma_k[\Phi]}{\delta \phi_i}$$

$$= \frac{1}{2} \mathrm{Tr}\, G_k[\phi] \, \partial_t R_k + \frac{1}{2} \mathrm{Tr}\, G_{\phi\varphi_j}[\Phi] \frac{\delta \langle \partial_t \hat{\phi}_k \rangle}{\delta \varphi_j} \, R_\phi \, ,$$

The four-fermion couplings:

$$\partial_t \bar{\lambda}_q - 2\,(1+\eta_q)\,\bar{\lambda}_q - \bar{h}\,\dot{\bar{A}} = \frac{1}{4} \overline{\mathrm{Flow}}^{(4)}_{(\bar{q}q)(\bar{q}q)} \, ,$$



$+$ (gluons $\leftrightarrow$ mesons)

$+$ (permutations of the regulator insertions)

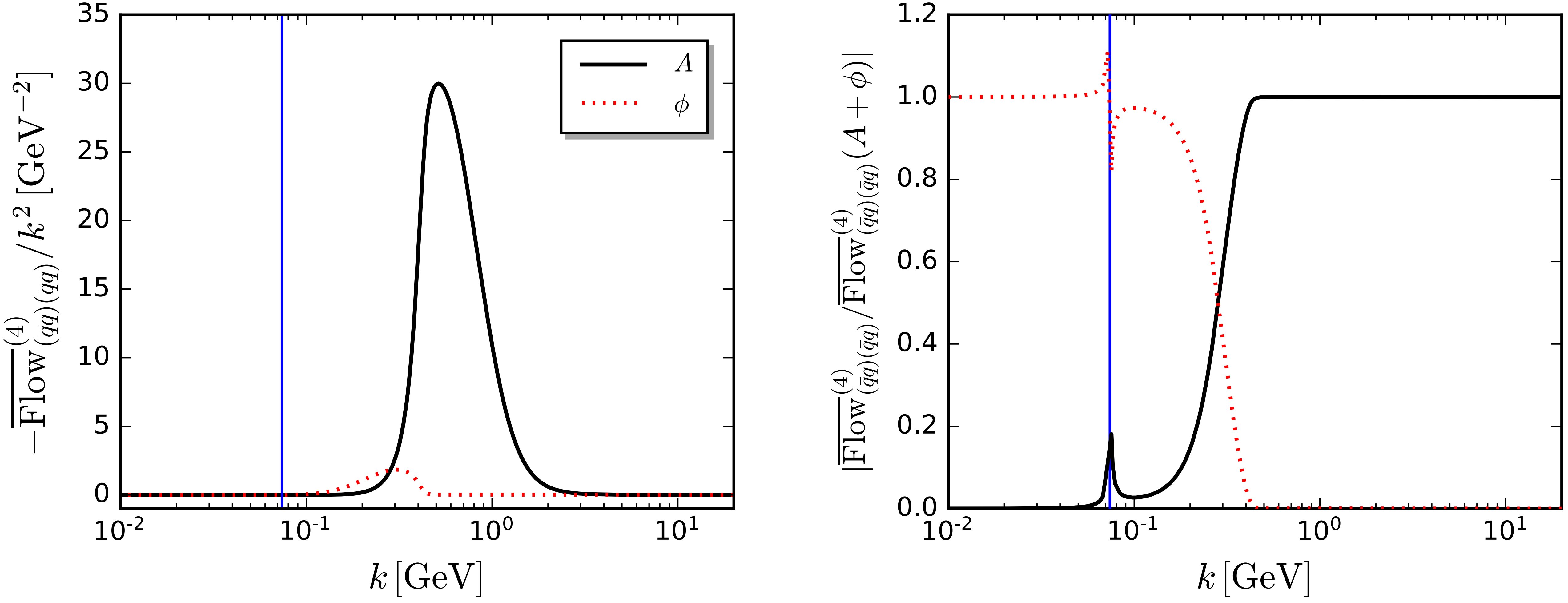
Demanding

$$\bar{\lambda}_q \equiv 0 \, , \qquad \forall k \, .$$

The hadronization function reads

$$\dot{\bar{A}} = -\frac{1}{\bar{h}} \overline{\mathrm{Flow}}^{(4)}_{(\bar{q}q)(\bar{q}q)/4} \, .$$

Flow of 4-quark Coupling—Gluon versus Meson



WF, J.M. Pawlowski, F. Rennecke, arXiv:1905.xxxxx.

Yukawa coupling

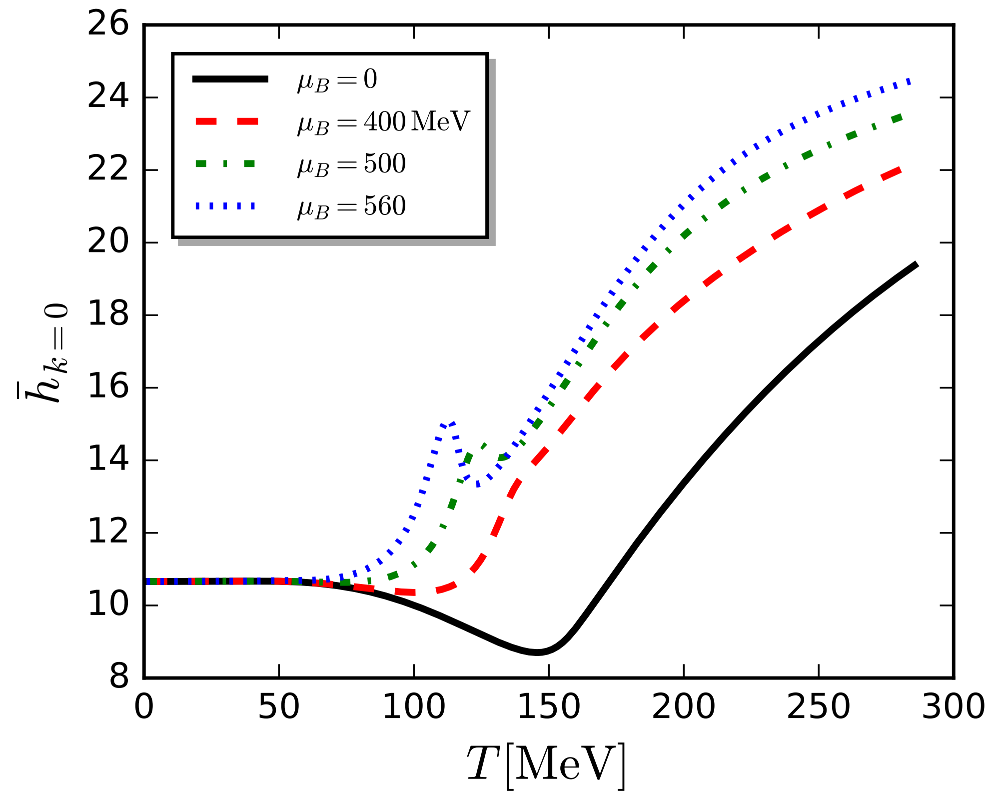
Two different Yukawa coupling:

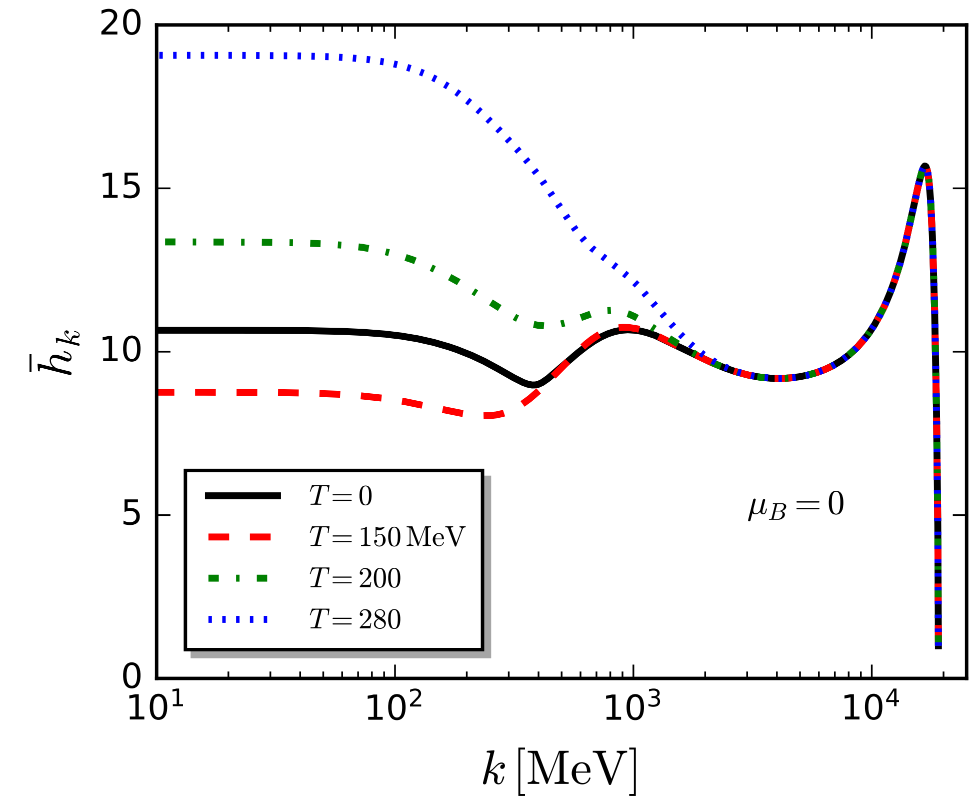
$$h_\pi = h(\rho_0) = \Gamma^{(3)}_{(\bar{q}\vec{\tau}q)\vec{\pi}}[\Phi_0]\,,$$

$$h_\sigma = h(\rho_0) + \rho\, h'(\rho_0) = \Gamma^{(3)}_{(\bar{q}\tau^0 q)\sigma}[\Phi_0]\,.$$

The flow equation:

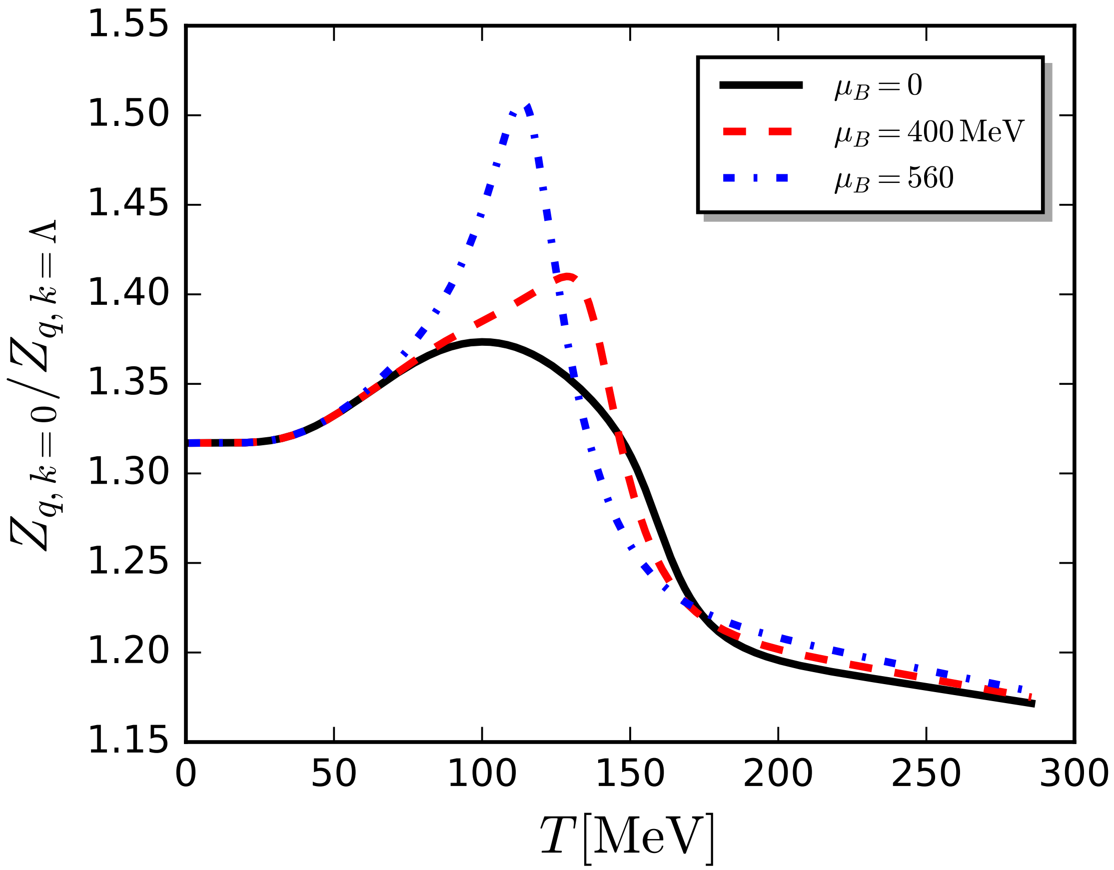
$$\partial_t \bar{h} = \left(\frac{1}{2}\eta_{\phi,k} + \eta_{q,k}\right)\bar{h} - \bar{m}_\pi^2\,\dot{\bar{A}} + \frac{1}{\bar{\sigma}}\mathrm{Re}\,\overline{\mathrm{Flow}}^{(2)}_{(\bar{q}\tau^0 q)}\,,$$

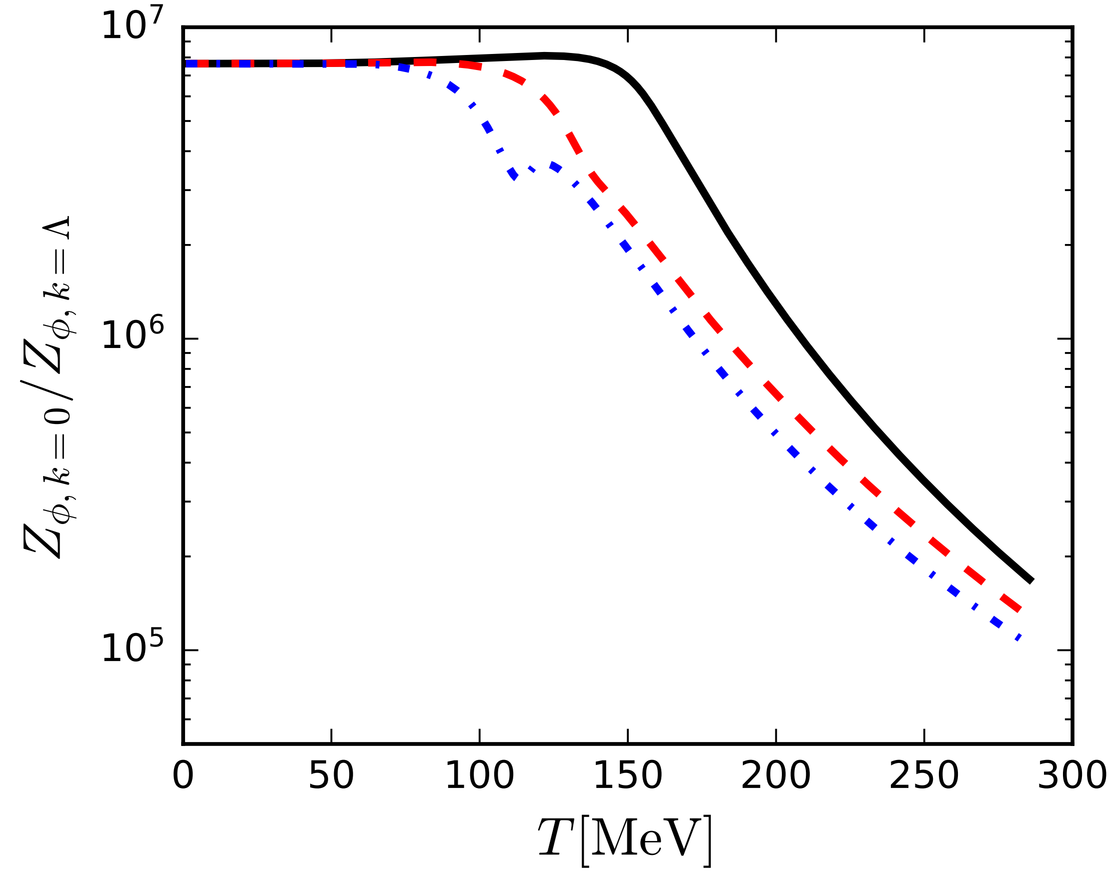




WF, J.M. Pawlowski, F. Rennecke, arXiv:1905.xxxxx.

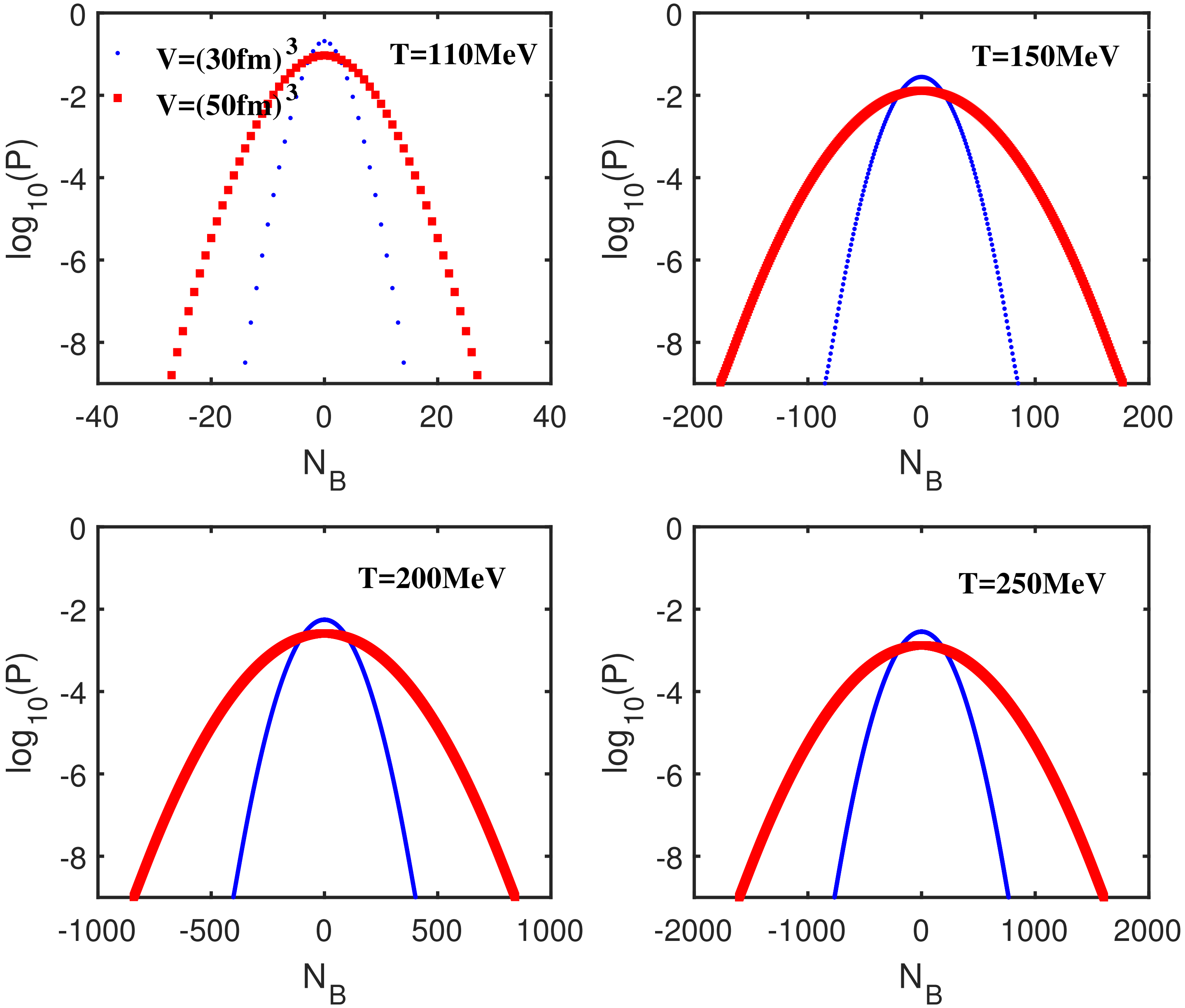
Quark and Meson Wave Function Renormalization





WF, J.M. Pawlowski, F. Rennecke, arXiv:1905.xxxxx.

Baryon Number Probability Distribution



K.-x. Sun, R. Wen, WF, PRD 98, 074028, (2018)

Probability distribution:

canonical partition function

$$P(N_B; T, V, \mu_B) = \frac{Z(T, V, N_B)}{\mathcal{Z}(T, V, \mu_B)} \exp\left(\frac{\mu_B N_B}{T}\right),$$

grand canonical partition function

Some Prospects on Combining AMPT with FRG

1. Partonic dynamics with FRG at finite temperature and chemical potential.

- Running strong coupling at different T and muB.
- Gluon thermal Debye screening mass.
- Polyakov potential.

⇒

- Interaction potential between partons.
- Cross section.

FRG **AMPT**

2. Quark potential encoded with the chiral and deconfinement order parameter.

- **Chiral**: Constitent quark mass.
- **Confinement**: Polyakov loop potential .

⇒ Quark potential ⇒ Boltzmann transport equation with a background potential

FRG **AMPT**

Some Prospects on Combining AMPT with FRG

3. Interaction potential related to the sigma mode near the CEP.

- Sigma mode is most relevant collective mode near the CEP, which can be calculate in the FRG.
- Interaction potential related to the sigma mode in the AMPT.

4. Improvement of the hadronization with FRG phase boundary in AMPT.

- Phase boundary calculated in the FRG can be used as the criteria for the hadronization in the AMPT.

5. Influence of hadronic dynamics on the CEP signal.

- Net baryon distribution calculated in the FRG in the partonic phase can be evolved through the hadronic phase.
- Then, the influence of the hadronic phase on the CEP signal can be investigated.