Hadrons in medium

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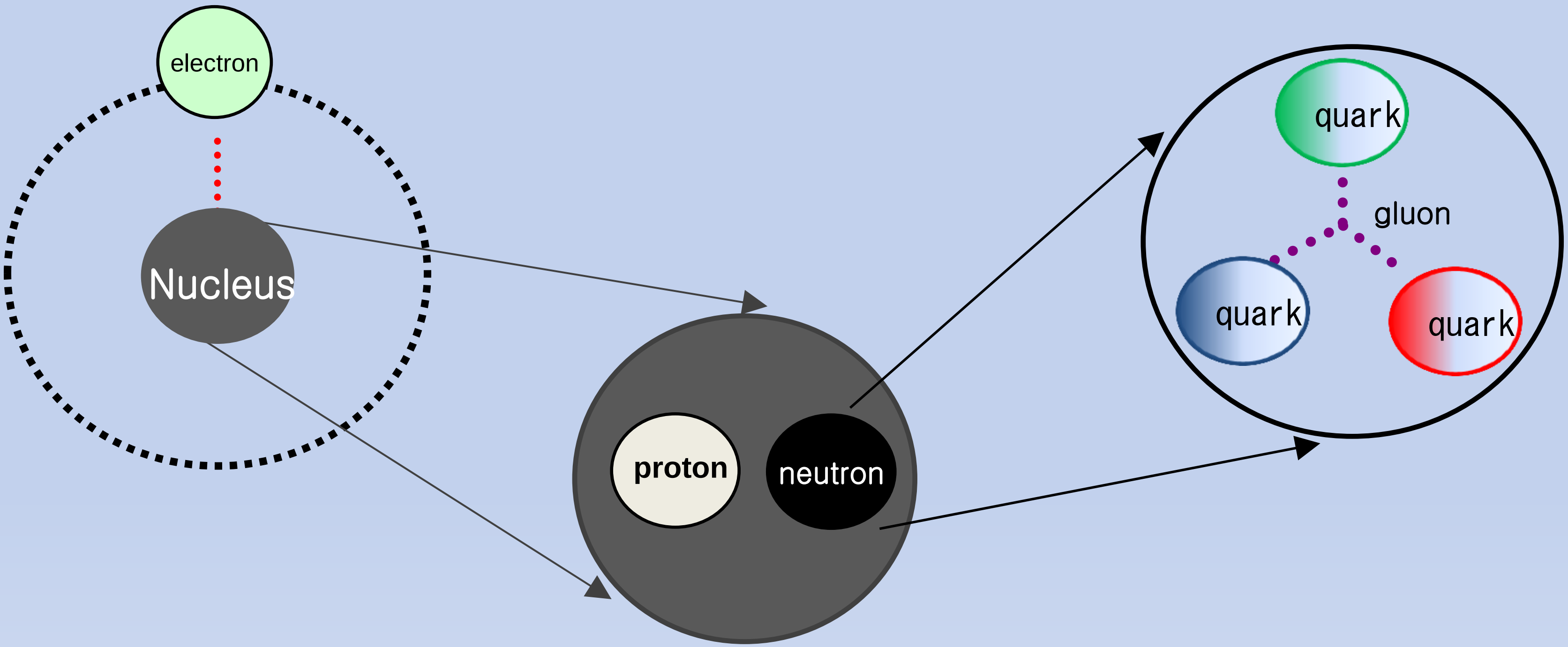
1. Few words on Confinement, chiral symmetry breaking and UA(1) effect from the 80's and 90's
2. Few words on QCD sum rule and Prof. Che-Ming Ko
3. K*, K1 meson
4. Conclusion

I: Few words on confinement, Chiral symmetry breaking and UA(1) effect

Understanding the mass of a composite object



Mass of an Atom

Nucleon: 99.95 %

electron: 0.05 %

EM binding< 0.00001 %

Nucleus

Nucleons: 99%

Nuclear binding < 1 %

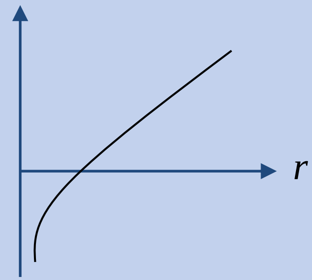
Nucleon

Quark < 5 %

The rest ??

QCD → { Confinement / $U_A(1)$ breaking / χ-sym breaking }

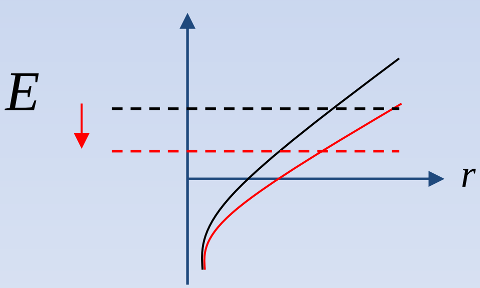
☞ Simple example for meson mass $$H = 2m_q + \frac{p^2}{m_q} + V(r)$$

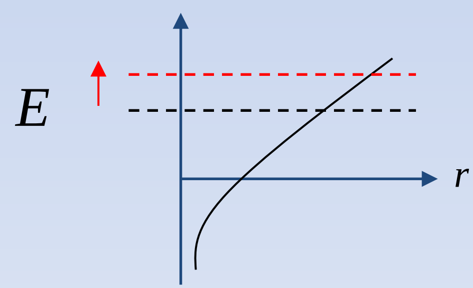


$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r} + \sigma \times r$$

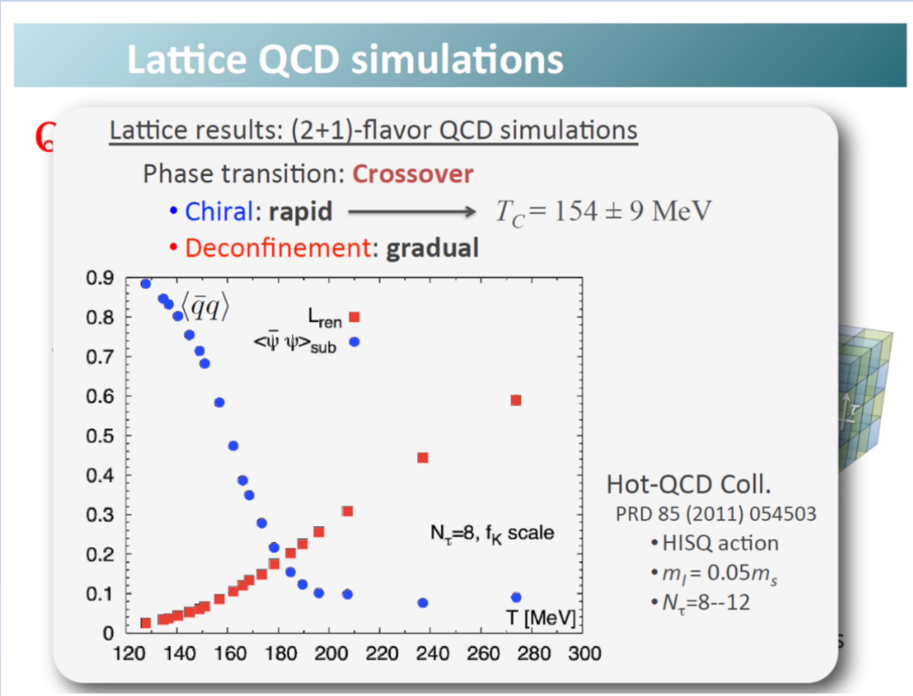
decrease in σ

decrease in m_q





Chiral symmetry restoration at finite T and ρ



W. Weise

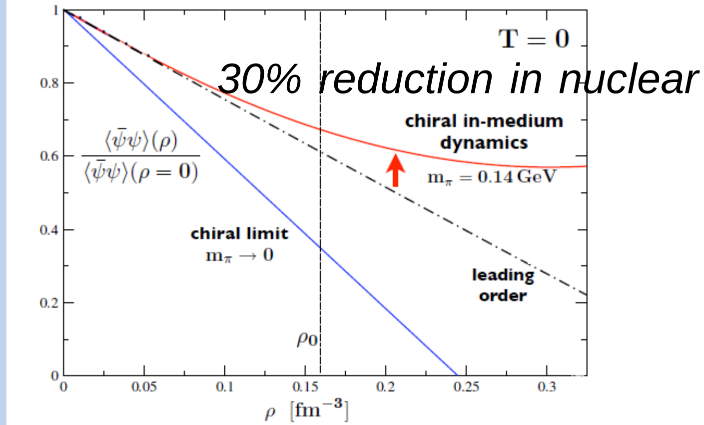


Fig. 4. Density dependence of the chiral condensate in symmetric nuclear matter [13]. Dot-dashed curve: leading order term using $\sigma_N = 50$ MeV. Upper curve: full in-medium chiral dynamics result at three-loop order. Lower curve: chiral limit with vanishing pion mass.

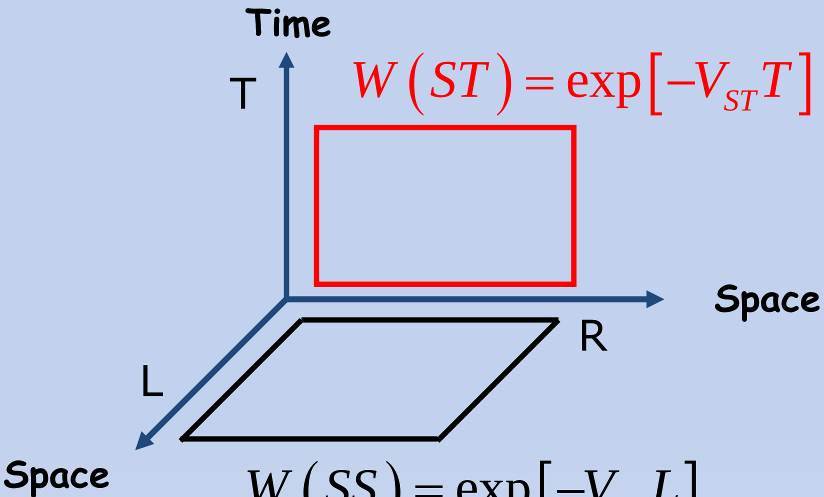
→ What will happen to hadron masses : A bridge between QCD and experiment ?

1. *Soft modes, scalar meson: Hatsuda, Kunihiro (85,87)*
2. *Pseudoscalar mesons: Bernard, Jaffe, Meissner (88), Klimt, Lutz, Vogel, Weise (90)*
3. *Brown-Rho: 91*
4. *Vector mesons: Hatsuda, Lee (92) ……*

→ *nuclear target provides a good environment to test effects of restoration*

Confinement and Deconfinement at finite T

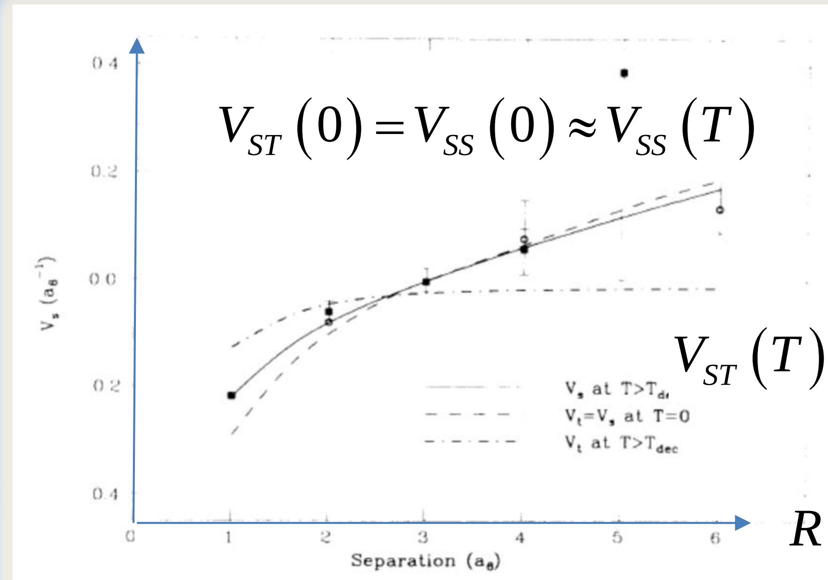
☞ Wilson Loops and potential



$$W(ST) = \exp[-V_{ST}T]$$

$$W(SS) = \exp[-V_{SS}L]$$

Manousakis, Polonyi PRL 1987



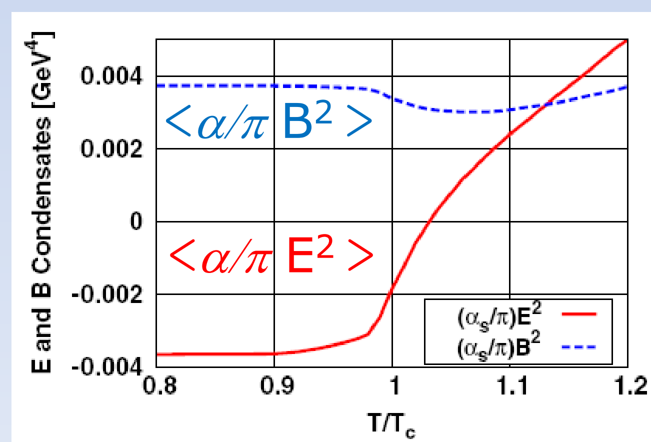
☞ Local operators

OPE for Wilson lines: Shifman NPB73 (80)
Dosch, Simonov PLB339 (88)

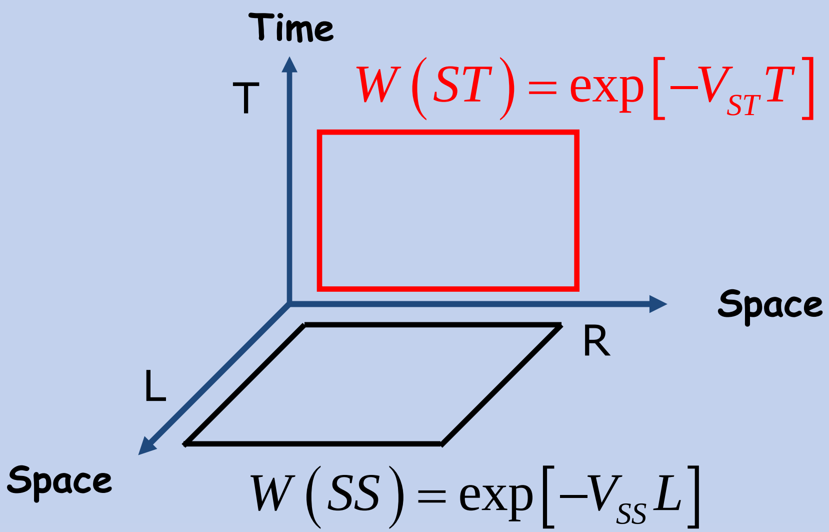
W(S-T) = 1- ⟨α/π E^2⟩ $(ST)^2$ +…

W(S-S) = 1- ⟨α/π B^2⟩ $(SS)^2$ +…

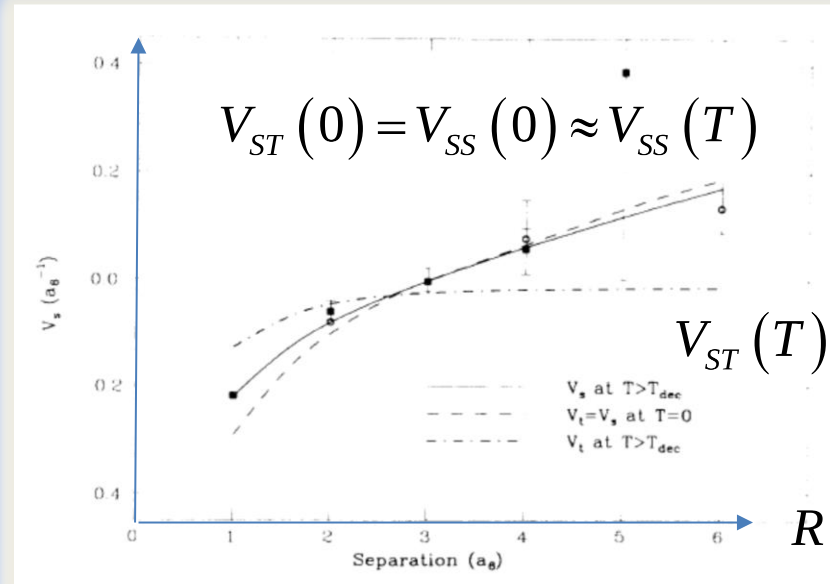
Morita, SHLee PRL 2008, PRD 2009



☞ Wilson Loops and potential



Manousakis, Polonyi PRL 1987



☞ Local operators

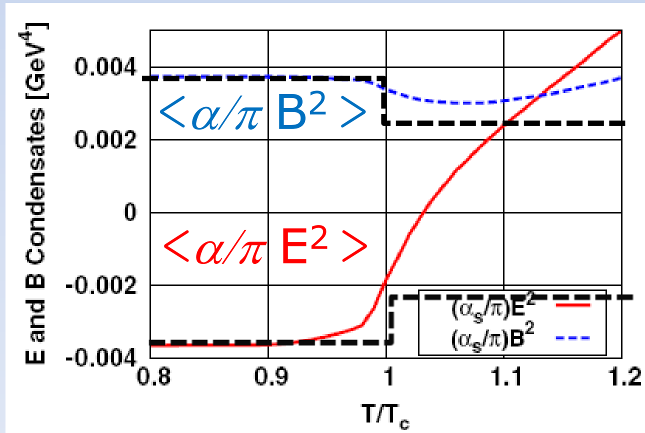
OPE for Wilson lines: Shifman NPB73 (80)
Dosch, Simonov PLB339 (88)

$W(S\text{-}T) = 1 - \langle \alpha/\pi\, E^2 \rangle (ST)^2 + \ldots$

$W(S\text{-}S) = 1 - \langle \alpha/\pi\, B^2 \rangle (SS)^2 + \ldots$

SHLee PRD40 (89): ------------
Non-perturbative Gluon condensate above Tc

Morita, SHLee PRL 2008, PRD 2009

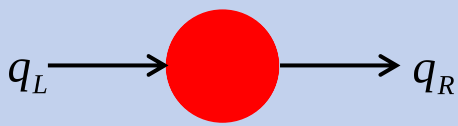


Chiral symmetry breaking ($m \to 0$) : order parameter

- Quark condensate

$$\mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R \to \mathrm{SU}(N_F)_V$$

☞ $$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle = -\lim_{x\to 0} \langle \mathrm{Tr}[S(x,0)] \rangle = -\lim_{x\to 0} \left\langle \frac{1}{2} \mathrm{Tr}\left[S(x,0) - i\gamma^5 S(x,0) i\gamma^5 \right] \right\rangle$$



Chiral rotation $q \to \exp\left(i\gamma^5 \tau^a \alpha^a\right) q$

☞ Casher Banks formula: nontrivial zero mode (λ =0) contribution

$$d\mu = dA e^{-S_{Glue}} \det\left[\not{D} + m \right] \quad \to \quad i\not{D}\psi_\lambda = \lambda \psi_\lambda \quad \text{where} \quad \psi_\lambda(0) = \langle 0 | \lambda \rangle$$

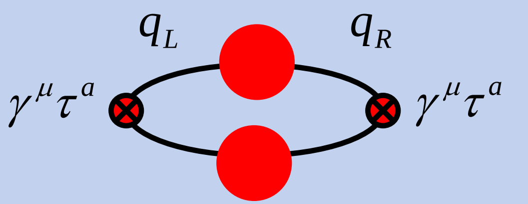
$$\langle \bar{q}(0) q(0) \rangle = \left\langle -\mathrm{Tr}\left[\left(0 \middle| \frac{1}{\not{D}+m} \middle| 0 \right) \right] \right\rangle = \left\langle \int d\lambda \psi_\lambda^+(0) \psi_\lambda(0) \frac{m}{m^2+\lambda^2} \right\rangle \xrightarrow{m\to 0} \langle \pi\rho(\lambda=0) \rangle$$

• Other order parameters: V - A correlator + more

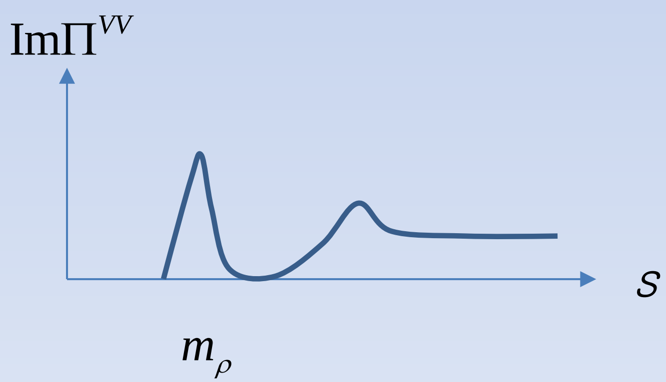
☞ $$\Pi^{VV}-\Pi^{AA}=\frac{1}{V}\int d^4x\,\left[\,\left\langle\bar{q}\gamma^\mu\tau^a q(x),\bar{q}\gamma^\mu\tau^a q(0)\right\rangle-\left\langle\bar{q}\tau^a i\gamma^5\gamma^\mu q(x),\bar{q}\tau^a i\gamma^5\gamma^\mu q(0)\right\rangle\right]$$

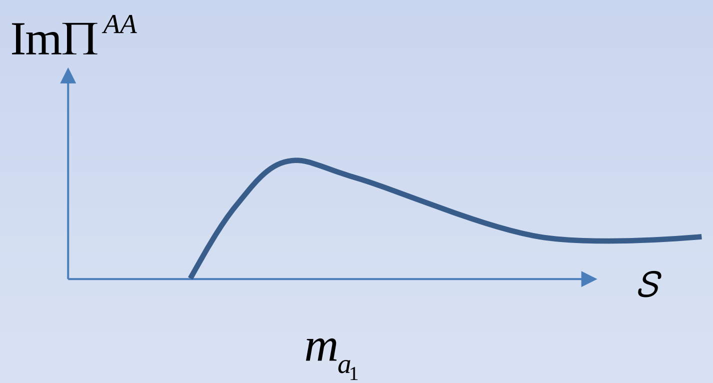
$$=-\frac{1}{2}\mathrm{Tr}\left[\,\gamma^\mu\left(S(x,0)-i\gamma^5 S(x,0)i\gamma^5\right)\gamma^\mu\left(S(0,x)-i\gamma^5 S(0,x)i\gamma^5\right)\right]\propto\left\langle\rho^2(\lambda=0)\right\rangle$$

Lee, S. Cho 2013



$$\propto\left\langle(\bar{q}q)^2\right\rangle$$



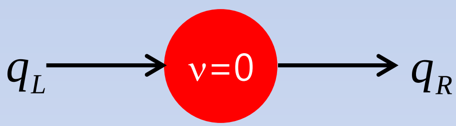


$U_A(1)$ effect : effective order parameter (Lee, Hatsuda 96)

$\mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R \to \mathrm{SU}(N_F)_{L+R=V}$	$\mathrm{U}_A(1)$ Breaking

☞ Topologically trivial

$$Z = Z_{\nu=0} + ...$$

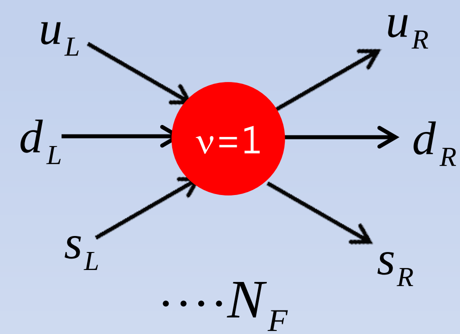


$$\langle \bar{q}q \rangle \propto \int dA e^{-S_{Glue}} \det[\not{D}+m]\left(\frac{1}{\not{D}+m}\right)$$

☞ Topologically non-trivial

$$Z = ... + Z_{\nu=\pm 1} + ...$$

$$\nu = \frac{\alpha_s}{4\pi}\int d^4x \left(G\tilde{G}\right) = n_R - n_L$$



$$\left\langle (\bar{u}u)(\bar{d}d)... \right\rangle$$

$$\propto \int dA e^{-S_{Glue}} \det[\not{D}+m]\left(\frac{1}{\not{D}+m}\right)\left(\frac{1}{\not{D}+m}\right)...$$

- $\sigma - \eta$ *correlator : SU(2) case*

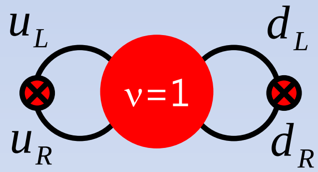
☞ $$\Pi^{\sigma\sigma} - \Pi^{\eta\eta} = \frac{1}{V}\int d^4x e^{ikx}\left[\left\langle \bar{q}q(x), \bar{q}q(0)\right\rangle - \left\langle \bar{q}i\gamma^5 q(x), \bar{q}i\gamma^5 q(0)\right\rangle\right]$$

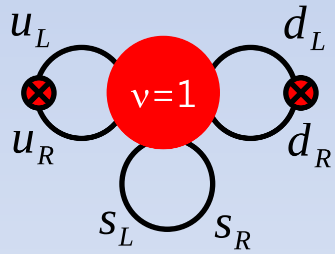
☞ 't Hooft Interaction $$V = 2c\left(\sigma^2 + \pi^2 - \eta^2 - \alpha^2\right)$$

$$\Pi^{\sigma\sigma} - \Pi^{\eta\eta} \propto 4c$$

☞ Quark picture $$\det\left[\bar{q}_R q_L\right] = \left(\bar{u}_R u_L \times \bar{d}_R d_L - \bar{u}_R d_L \times \bar{d}_R u_L\right)$$

$$\Pi^{\sigma\sigma} - \Pi^{\eta\eta} \propto$$



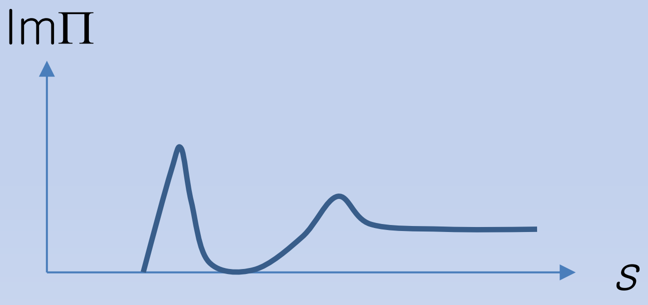


SU(3)

Few Important points to note about hadron masses :

1. Changes at 1) finite temperature, 2) finite density, 3) Chiral symmetric limit of QCD are all different
2. However, changes that only depend on order parameters are universal
3. $\langle \bar{q}q \rangle$ can not be directly related to physical observable in a model independent way
4. Could consider

$$\langle VV - AA \rangle$$



→ Comparison of whole spectrum is not needed

(Glozeman: Chiral symmetry is restored for excited states)

→ Ground states should have small intrinsic width to be experimentally observable

II: QCD sum rules and Prof. Che-Ming Ko

QCD sum rules approach

$$\Pi(\varpi, q) = i \int d^4x e^{iqx} \langle J(x) J(0) \rangle$$ where $J = \bar{\psi}\Gamma\psi,\ \psi\psi\psi$

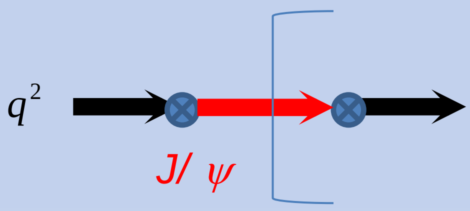
- Correlation can be calculated directly from QCD at large Q^2

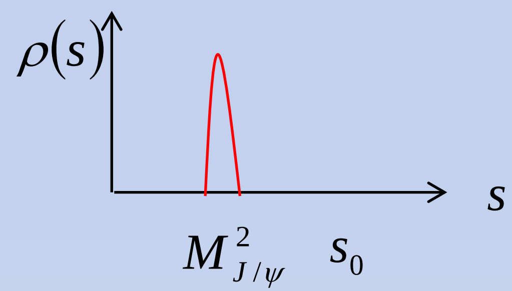
- How are they used?

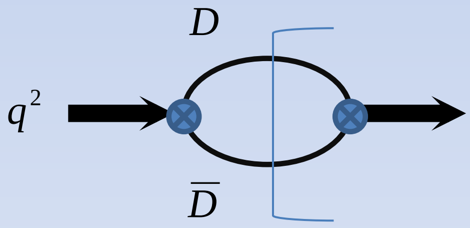
Traditionally: Imaginary part of the vector correlator

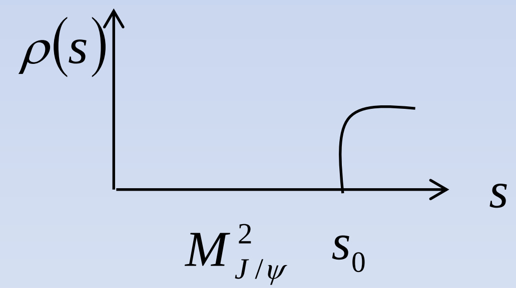
- *Dispersion relation*

$$\Pi(q) = \int ds \frac{\rho(s)}{s - q^2} \qquad \text{where} \qquad \rho(s) = \frac{1}{\pi} \mathrm{Im}\Pi(s)$$







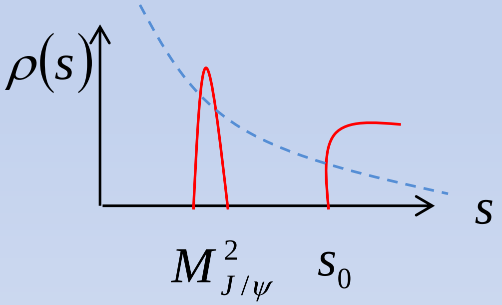


iii) Matching: QCD sum rule for pole

- *Borel transformed Dispersion relation*

$$B.T[\Pi(q)] = \mathrm{M}^{OPE}(M^2) = \sum_n \frac{C_n(m,M)}{n!(M^2)^n}\langle G^n\rangle = \int ds e^{-s/M^2}\rho(s)$$

$$\rho(s) = f\delta(s - M^2_{J/\psi}) + c\theta(s - s_0)$$

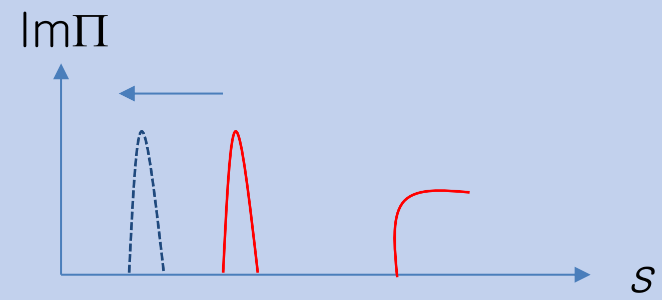


- *Predicted* $\Delta = M_{J/\psi} - M_{\eta_c} \approx 100$ MeV *before experiment* → *non trivial result*

Contribution from Prof. Ko : light vector meson

1. *Vector meson sum rule: Hatsuda and Lee (1992) PRC 46 (1992) r34*

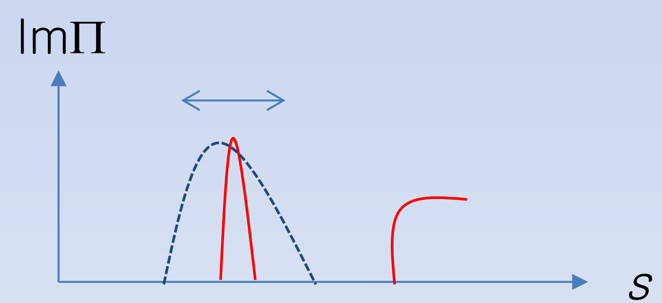
$$8\pi \mathrm{Im}\Pi^R(u>0^-) = \delta(u^2)\rho_{sc} + F\delta(u^2-m_V^2) + (1+\frac{\alpha_s}{\pi})\theta(u^2-S_0)$$



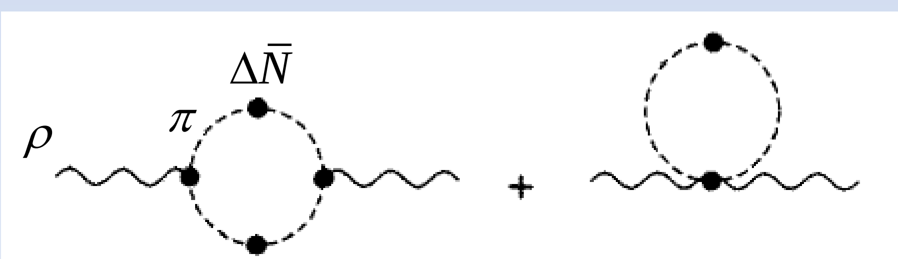
2. *Importance of new structure in medium: M. Asakawa, C.M. Ko NPA560 (1993) 399*

→ *Very important as chiral symmetry restoration effect only affects vacuum change*

→ *Future work, how will this affect chiral order parameter ?*



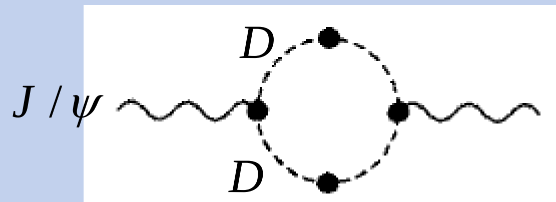
$$8\pi\, \mathrm{Im}\, \Pi_L(s) = F'\frac{S(s)}{s}\theta(s_0-s) + \left(1+\frac{\alpha_s}{\pi}\right)\theta(s-s_0).$$



Contribution from Prof. Ko : Heavy vector meson

1. *Inclusion of D meson loop effects in Charmonium sum rules in the medium*

   SHLee, C.M. Ko PRC 67 (2003) 038202



$$\Delta m_{J/\psi} = -8 + 3 \text{ MeV},$$
$$\Delta m_{\psi(3686)} = -100 - 30 \text{ MeV},$$
$$\Delta m_{\psi(3770)} = -140 + 15 \text{ MeV},$$

2. *QCD sum rules for heavy quark system in medium*

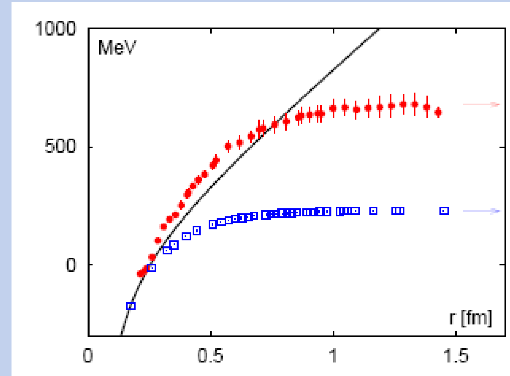
→ Can be searched for in future anti proton – nuclear target experiment

Contribution from Prof. Ko : Heavy quark Finite temperature potential

Controversy on potential at finite temperature: Lee, Morita, Song, C.M. Ko PRD 89 (2014) 094015

☞ Method 1: Solve Schrodinger equation

$$\left[2m-\frac{1}{m}\nabla^2+V(r,T)\right]\Psi(r,T)=M_{J/\psi}\Psi(r,T)$$



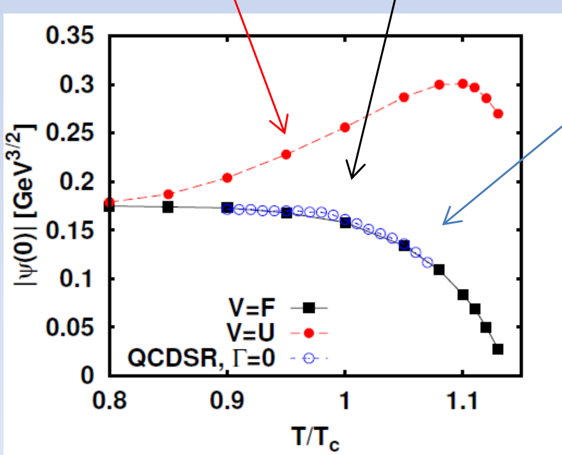
U(T,r) at 1.3 Tc

F(T,r) at 1.3 Tc

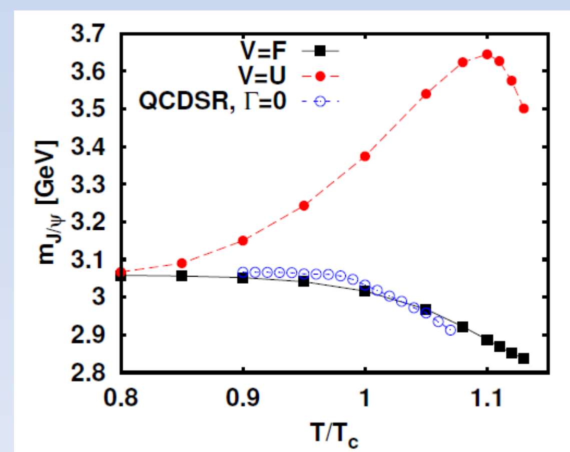
☞ Method 2: From QCD sum rule ⟨ [illegible]

☞ Result ● V= U ■ V= F ○ Sum rule result

$|\Psi(0)|$

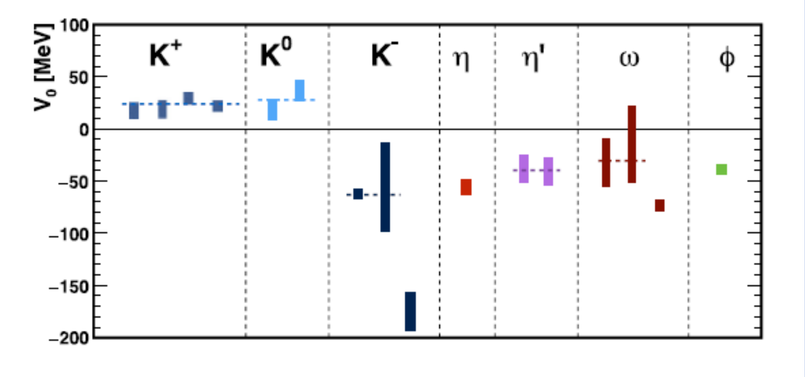


$M_{J/\psi}$

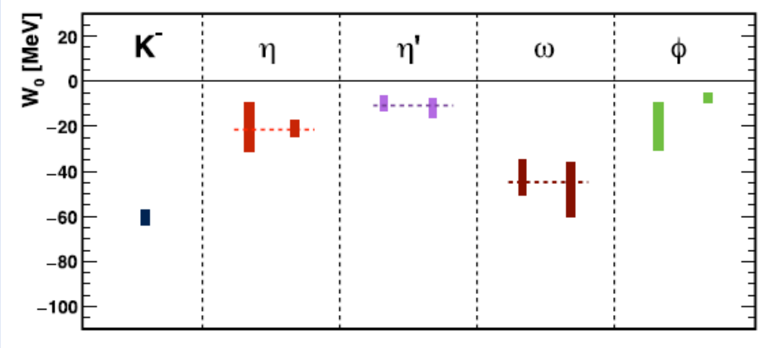


Summary by V. Metag (PPNP97 (2017)199)

☞ Downward mass shift at nuclear matter ↓



☞ Width increase at nuclear matter ↓



☞ Lesson from experiment

1. Look at small width hadrons (<100 MeV)
2. Can look at excitation energy

☞ Lesson from Theory

1. Look chiral partners

III: K* and K1 in nuclear medium

K* and K1

$J^{PC}=1^{--}$	Mass	Width	$J^{PC}=1^{++}$	Mass	Width
ρ	770	150.	a_1	1260	250-600
ω	782	8.49	f_1	1285	24.2
ϕ	1020	4.266	f_1	1420	54.9
$K^*(1^-)$	892	50.3	$K_1(1^+)$	1270	90

- (ρ, a_1) are chiral partners but have too large vacuum width

$$\rho \to \left(\bar{q}_R\gamma_\mu \tau q_R + \bar{q}_L\gamma_\mu \tau q_L\right) \qquad a_1 \to \left(\bar{q}_R\gamma_\mu \tau q_R - \bar{q}_L\gamma_\mu \tau q_L\right)$$

- Coupling to quark currents

$$\omega \to \left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) \quad \phi \to \left(\bar{s}\gamma_\mu s\right) \qquad K^* \to \left(\bar{q}\gamma_\mu s\right), \ \left(\bar{s}\gamma_\mu q\right)$$

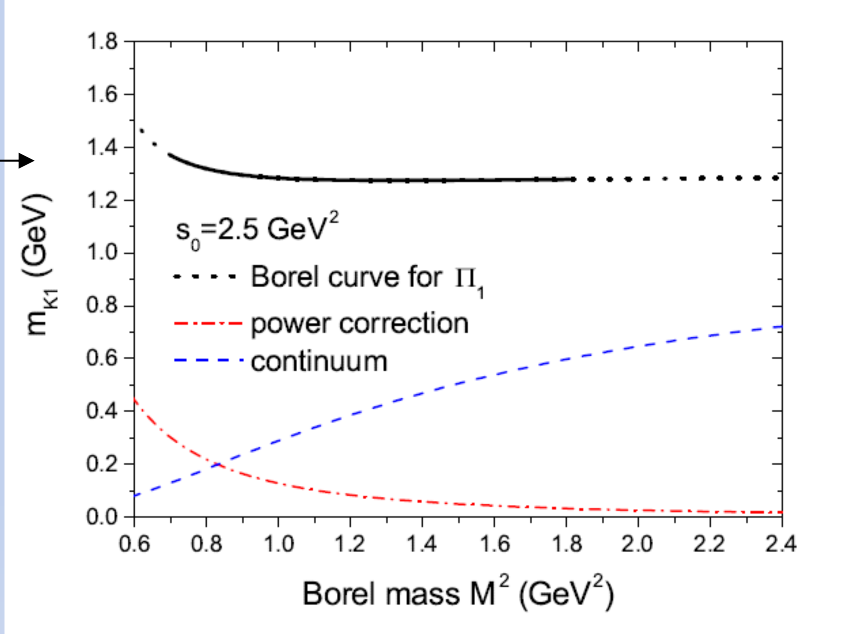
→ What about quark content of K_1 ?

- Are $\left(K^*, K_1\right)$ chiral partners ?

K1(1270) sum rules in medium *(Song, Hatsuda, Lee, PLB792 (2019) 160)*

☞ *currents*

$$K_1(1270) \to \left(\bar{u}\gamma_\mu\gamma^5 s\right)$$



☞ *Hence, strong coupling to currents*

$$K^* \to \left(\bar{u}\gamma_\mu s\right) \qquad K_1(1270) \to \left(\bar{u}\gamma_\mu\gamma^5 s\right)$$

• *Chiral Partner ?*

☞ Chiral partner

$$\Pi^{\rho\rho} - \Pi^{a_1 a_1} = \left\langle \left(\bar{u}_R \gamma_\mu u_R\right)\left(\bar{u}_L \gamma_\mu u_L\right) - \left(\bar{u}_R \gamma_\mu u_R\right)\left(\bar{d}_L \gamma_\mu d_L\right) \right\rangle \propto \left\langle \bar{q} q \right\rangle^2 \sim \left(m_{a_1} - m_\rho\right) \approx 490 \text{ MeV}$$

$$= \; -\frac{1}{2}\mathrm{Tr}\left[\gamma^\mu \left(S_{q,s}(x,0) - i\gamma^5 S_{q,s}(x,0) i\gamma^5\right) \gamma^\mu \left(S_q(0,x) - i\gamma^5 S_q(0,x) i\gamma^5\right)\right]$$

$$\Pi^{K^* K^*} - \Pi^{K_1 K_1} = \left\langle \left(\bar{u}_R \gamma_\mu s_R\right)\left(\bar{s}_L \gamma_\mu u_L\right) \right\rangle \qquad \propto \left\langle \bar{q} q \right\rangle \left\langle \bar{s} s \right\rangle \sim \left(m_{K_1} - m_{K^*}\right) \approx 378 \text{ MeV}$$

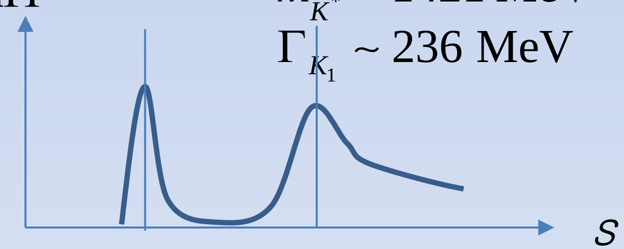
Smaller mass splitting

☞ Distinct spectral density → can understand how chiral symmetry restoration is realized in nature

$\mathrm{Im}\Pi^{K^* K^*}$

$m_{K^*} = 1421$ MeV

$\Gamma_{K_1} \sim 236$ MeV



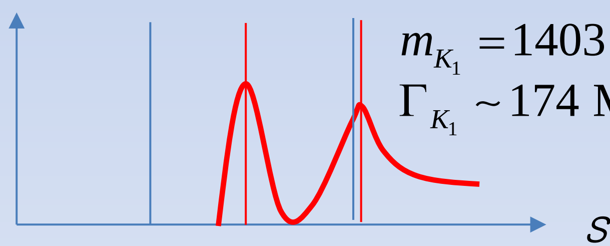
$m_{K^*} = 892$ MeV

$\Gamma_{K^*} \sim 47$ MeV

$\mathrm{Im}\Pi^{K_1 K_1}$

$m_{K_1} = 1403$ MeV

$\Gamma_{K_1} \sim 174$ MeV



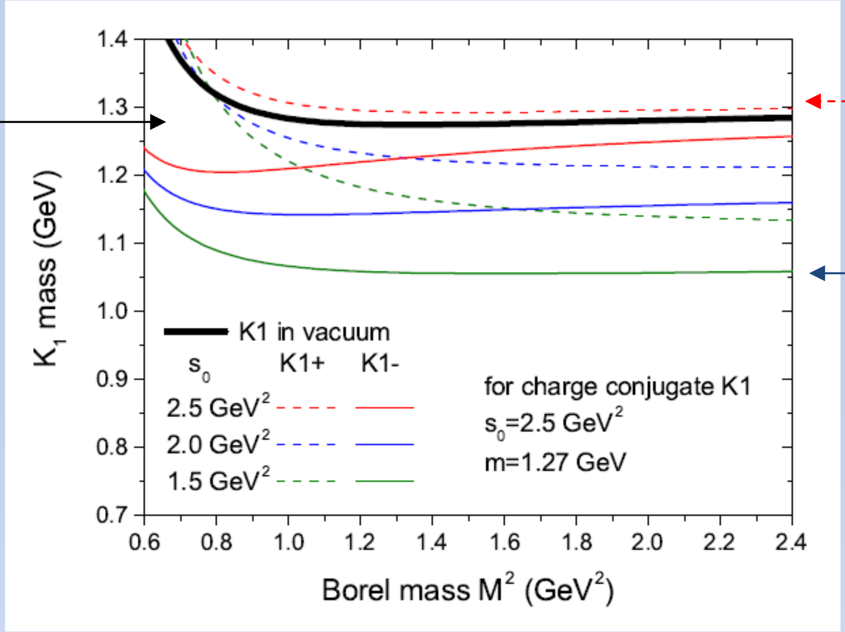
$m_{K_1} = 1272$ MeV

$\Gamma_{K_1} \sim 90$ MeV

- *Expected mass shift from sum rules*

☞ *current* $K_1^- \rightarrow \left(\bar{u}\gamma_\mu\gamma^5 s\right)$ $K_1^+ \rightarrow \left(\bar{s}\gamma_\mu\gamma^5 u\right)$ $K^{*-} \rightarrow \left(\bar{u}\gamma_\mu s\right)$ $K^{*+} \rightarrow \left(\bar{s}\gamma_\mu u\right)$

K_1 vacuum mass



K_1^+ medium mass

K_1^- medium mass

☞ *Hence, mass shift at nuclear matter*

$$\Delta m\left(K_1^-\right) \approx -208 \text{ MeV} \qquad \Delta m\left(K_1^+\right) \approx +32 \text{ MeV}$$

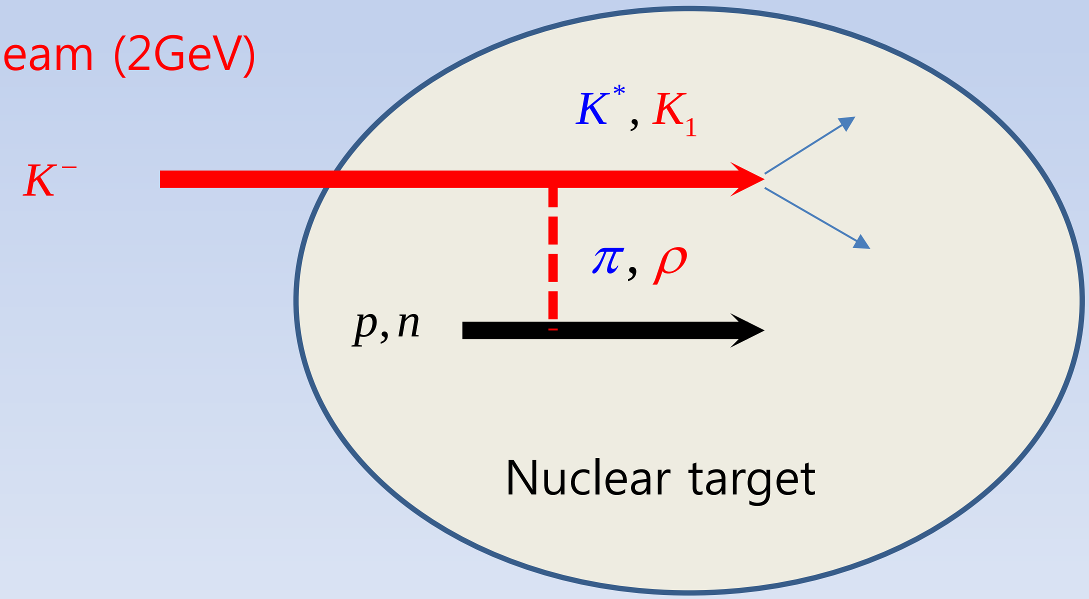
• *Possible future experiment*

→ K1 excitation energy measurement at Jparc

Decay mode of K_1 (Γ=90MeV)

Decay mode	Fraction
K1(1270)→ K ρ	42 %
K1(1270)→ K* π	16 %

Kaon beam (2GeV)



$$K_1^- \rightarrow \begin{pmatrix} \rho^0 K^- \\ \rho^- \bar{K}^0 \end{pmatrix} \quad \begin{pmatrix} \pi^0 K^{*-} \\ \pi^- \bar{K}^{*0} \end{pmatrix}$$

$$\bar{K}_1^0 \rightarrow \begin{pmatrix} \rho^+ K^- \\ \rho^0 \bar{K}^0 \end{pmatrix} \quad \begin{pmatrix} \pi^+ K^{*-} \\ \pi^0 \bar{K}^{*0} \end{pmatrix}$$

Summary

1. Prof. Che-Ming Ko also made important contributions in QCD sum rules at finite temperature and density
2. K* K_1 are chiral partners could be done at J-PARC

Prof. Che-Ming Ko congratulations on your impressive 50 years of research and looking forward to your insights and idea in future collaboration