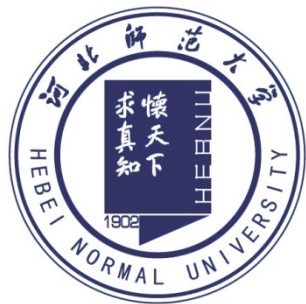


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Approaches for Relativistic Heavy Ion Collisions

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Thermal behaviors of light-flavor mesons at
low temperatures in Chiral EFT



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Outline:

1. Introduction
2. Chiral perturbation theory and η - η' mixing
3. S -wave meson-meson scattering and scalar resonances
4. Thermal behaviors of light pseudoscalar and scalar mesons at low temperatures
5. Summary

Introduction

Lowest QCD scalar resonances

$f_0(500)/\sigma$: precise $\pi\pi$ scattering data + dispersive technique + chiral EFT. Well determined pole positions !

[Xiao,Zheng, NPA01] [Caprini,Colangelo,Leutwyler,PRL06]
[Garcia-Martin, et al., PRD11]

$f_0(980)$: $\pi\pi$ scattering data + (dispersive technique,Unitarized chiral EFT). Confirmed pole in the complex energy plane !

[Garcia-Martin, et al., PRD11] [Oller, Oset, Pelaez, PRD99]

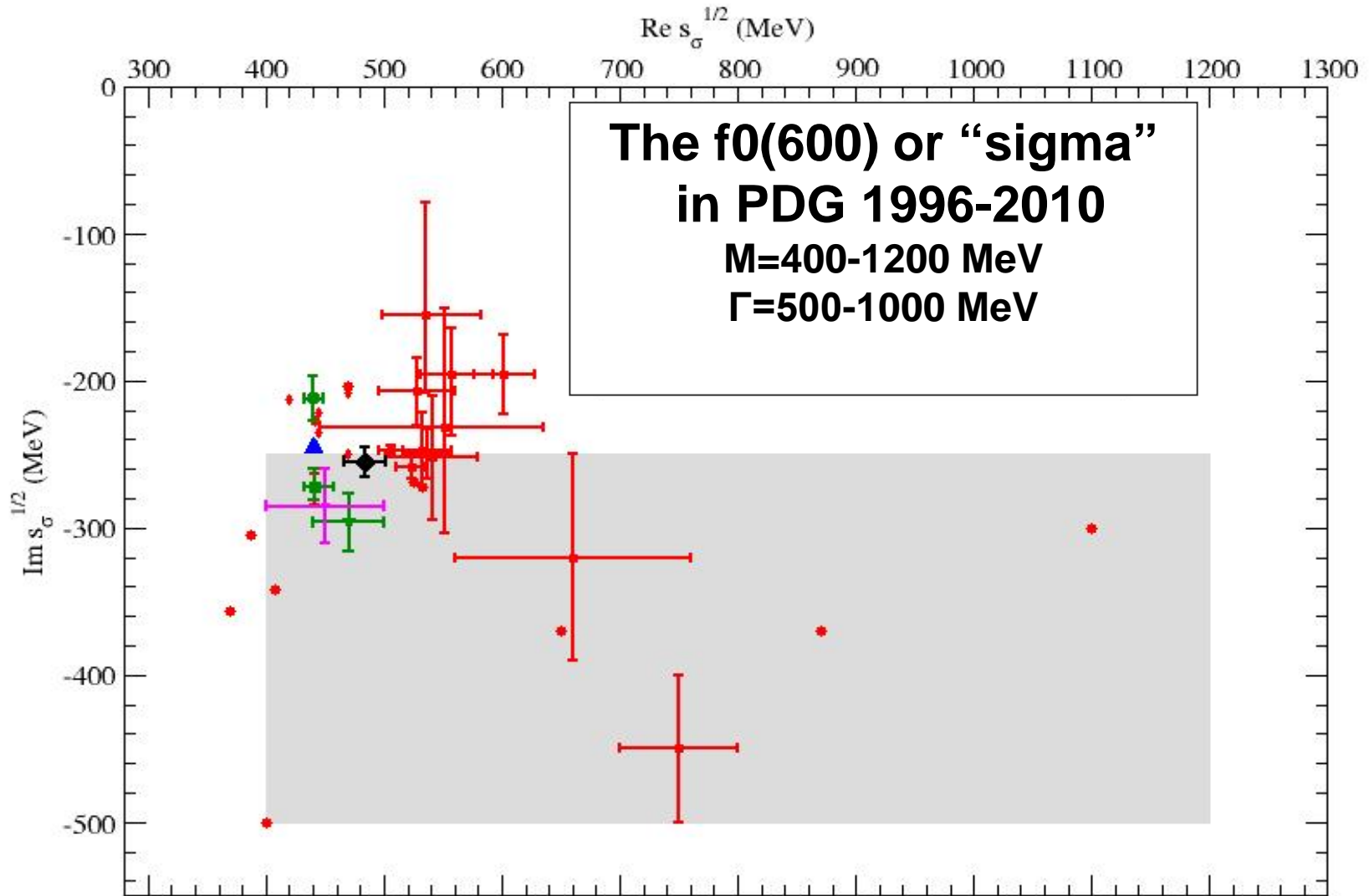
$K^*_0(700)/\kappa$: πK scattering data+ (dispersive technique,Unitarized chiral EFT). Confirmed pole in the complex energy plane !

[Zheng, Zhou, et al.,] [Descotes-Genon, Moussallam, EPJC06]
[Pelaez, Rodas, PRD16]

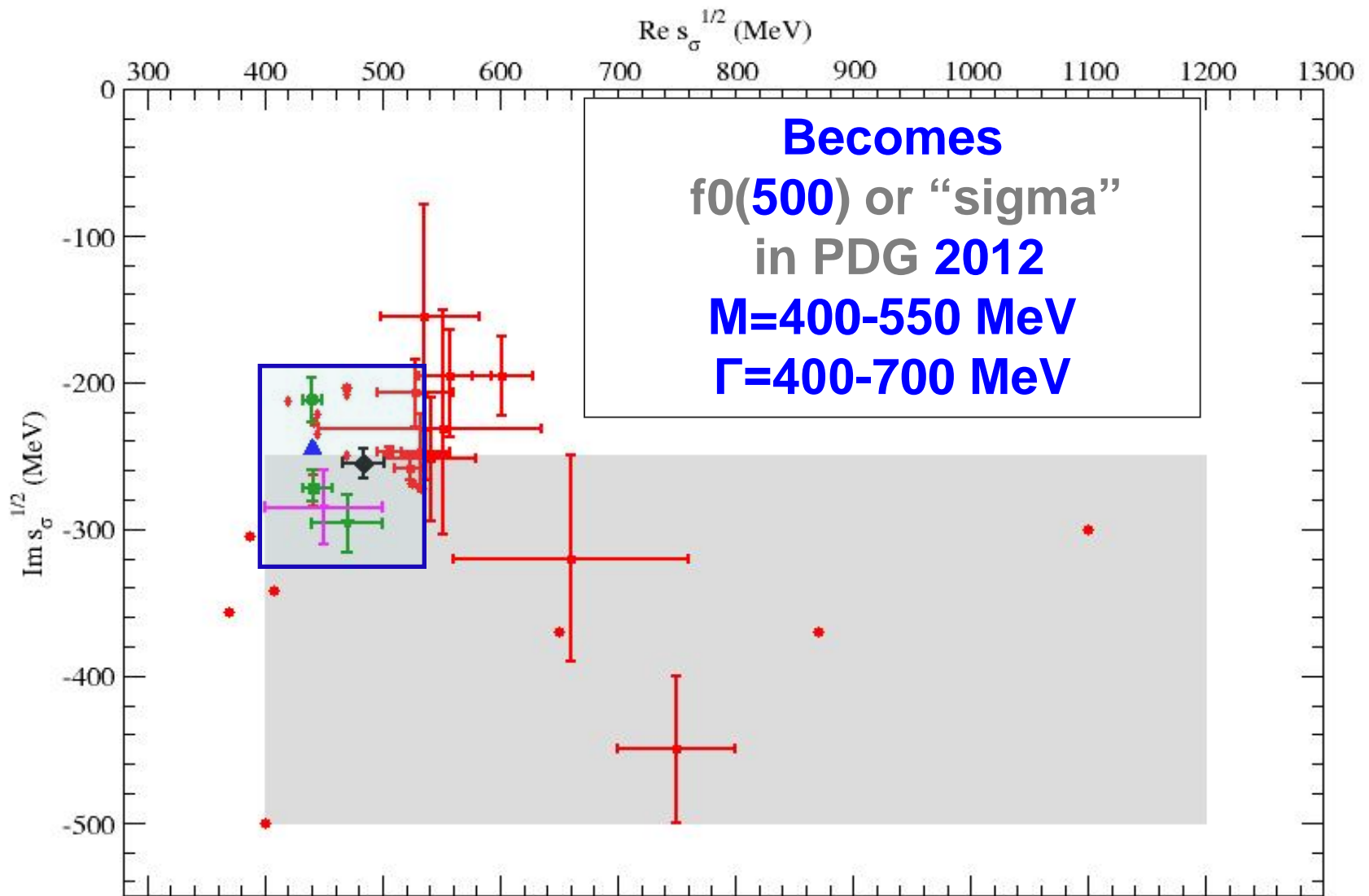
$a_0(980)$: Absence of the $\pi\eta$ scattering data. Lattice can help.

[ZHG, L.Liu, Meissner, Oller, Rusetsky, PRD17]

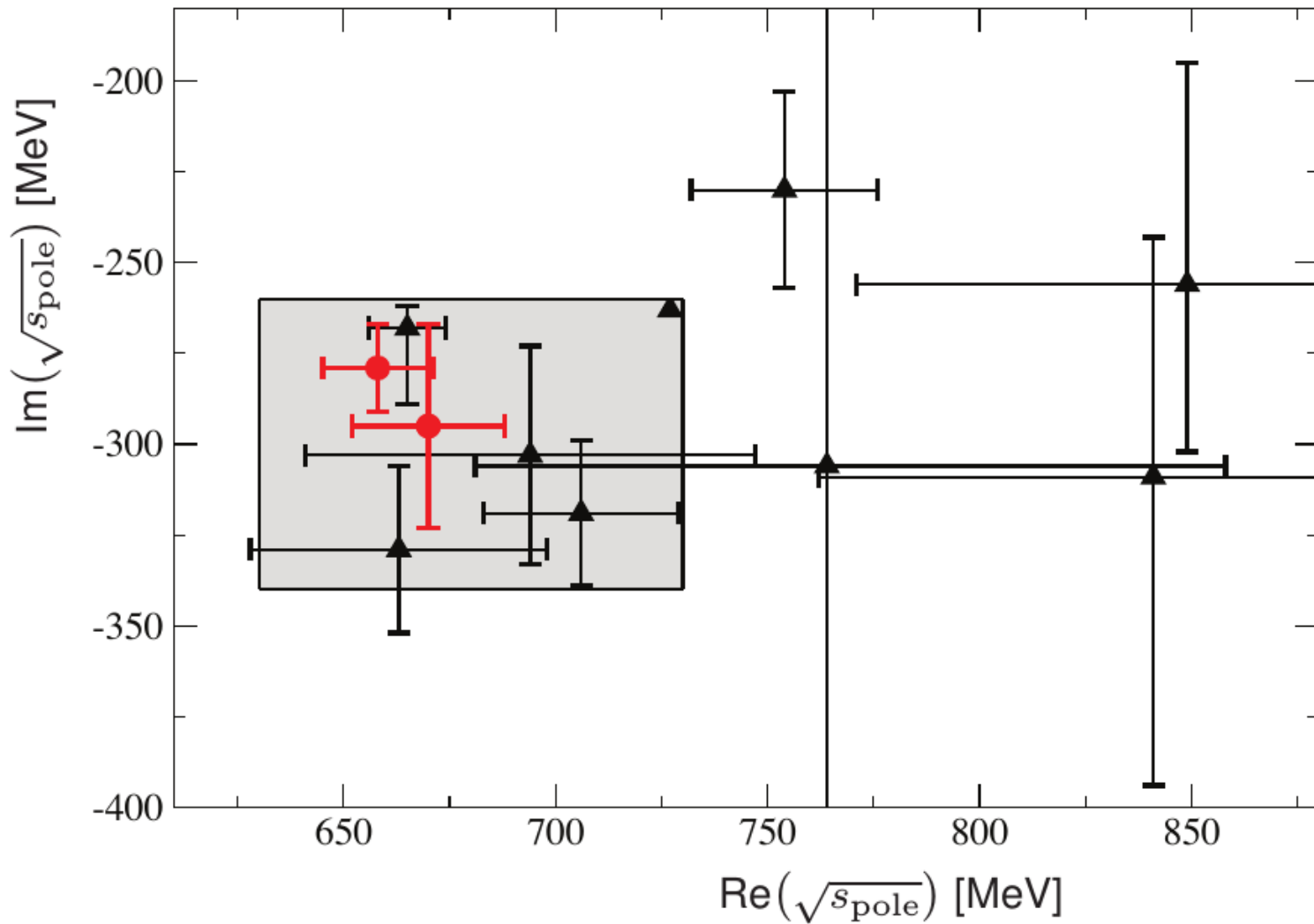
Lightest QCD scalar resonance: σ



From J. Pelaez



From J.Pelaez



Kappa or $K^*(700)$ from PDG

$f_0(980)$ and $a_0(980)$:

theoretically less clear, due to the nearby $K\bar{K}$ - K threshold.

Coupled-channel analysis needed !

[Garcia-Martin, et al., PRD11] [Oller, Oset, Pelaez, PRD99]

[ZHG, L.Liu, Meissner, Oller, Rusetsky, PRD17]

In this talk, we focus on the thermal behaviors of these scalar resonances at finite temperatures.

- Chiral EFT is used to describe sigma, kappa, $f_0(980)$ and $a_0(980)$ simultaneously.
- The QCD $U_A(1)$ anomaly is tentatively addressed by examining the thermal masses of the eta and eta'.
- Scattering and relevant inputs from Exp/Lattice are nicely reproduced in our approach.
- The thermal behaviors of both light pseudoscalar and scalar mesons are pure predictions !

Chiral perturbation theory and η - η' mixing

The relevant dynamical d.o.f: π, K, η, η'

$SU_L(3) \otimes SU_R(3) \rightarrow SU_V(3)$: Goldstone π, K, η_8 [$SU(3)$ χ PT]
[Gasser and Leutwyler, NPB'85]

$N_C \rightarrow \infty$: $M_{\eta_0}^2 \sim \mathcal{O}(1/N_C)$, $\therefore \eta_0$ becomes Goldstone
[Witten, NPB'79]

$U(3)$ χ PT : π, K, η_8 and η_0

A consistent power counting scheme in $U(3)$ χ PT:

$$\delta \sim p^2 / \Lambda_\chi^2 \sim m_{\pi,K,\eta}^2 / \Lambda_\chi^2 \sim 1/N_C$$

In this case, it is essential to generalize from $SU(3)$ to $U(3)$ ChPT

$$\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \longrightarrow \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

Leading order in the δ counting:

$$\mathcal{L}_2 = \frac{F^2}{4} \langle u_\mu u^\mu \rangle + \frac{F^2}{4} \langle \chi_+ \rangle + \frac{F^2}{3} M_0^2 \ln^2 \det u$$

Leads to a
massive η_0

NLO: $\mathcal{O}(N_C p^4)$ and $\mathcal{O}(N_C^0 p^2)$

$$\begin{aligned}\mathcal{L}^{(\delta^1)} = & L_5 \langle u_\mu u^\mu \chi_+ \rangle + L_8/2 \langle \chi_+ \chi_+ + \chi_- \chi_- \rangle \\ & + F^2 \tilde{\Lambda}_1 \langle u_\mu \rangle \langle u^\mu \rangle + F^2 \tilde{\Lambda}_2 \ln(\det U) \langle U^\dagger \chi - \chi^\dagger U \rangle + \dots\end{aligned}$$

NNLO: $\mathcal{O}(N_C^{-2} p^0)$, $\mathcal{O}(N_C^{-1} p^2)$, $\mathcal{O}(N_C^0 p^4)$ and $\mathcal{O}(N_C p^6)$

$$\begin{aligned}\mathcal{L}^{(\delta^2)} = & \tilde{v}_0^{(4)} X^4 + \tilde{v}_1^{(2)} X^2 \langle u_\mu u^\mu \rangle + L_4 \langle u_\mu u^\mu \rangle \langle \chi_+ \rangle \\ & + C_1 \langle u_\rho u^\rho h_{\mu\nu} h^{\mu\nu} \rangle + \dots\end{aligned}$$

[Herrera-Siklody, Latorre, Pascual, Taron, NPB'97]

[Bijnens, Colangelo, Ecker, JHEP'99], [Jiang, Ge, Wang, '14]

δ counting in U(3) χ PT: a systematical expansion to describe eta-eta' mixing

η - η' mixing at LO:
$$\frac{F^2}{4} \langle u_\mu u^\mu \rangle + \frac{F^2}{4} \langle \chi_+ \rangle + \frac{F^2}{12} M_0^2 X^2$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \eta_8 \partial^\mu \eta_8 + \frac{1}{2} \partial_\mu \eta_1 \partial^\mu \eta_1 - \frac{4\bar{m}_K^2 - \bar{m}_\pi^2}{3} \eta_8 \eta_8 - \frac{2\bar{m}_K^2 + \bar{m}_\pi^2 + 3M_0^2}{3} \eta_1 \eta_1 - \frac{4\sqrt{2}(\bar{m}_K^2 - \bar{m}_\pi^2)}{3} \eta_1 \eta_8$$

with

$$\bar{m}_\pi^2 = 2Bm_{ud}, \quad \bar{m}_K^2 = B(m_s + m_{ud})$$

\Rightarrow

$$\eta_8 = c_\theta \bar{\eta} + s_\theta \bar{\eta}', \quad (c_\theta = \cos \theta \quad s_\theta = \sin \theta)$$

$$\eta_1 = -s_\theta \bar{\eta} + c_\theta \bar{\eta}',$$

The leading order calculation leads to the conventional one-mixing angle scheme.

$\bar{\eta}$ - $\bar{\eta}'$ will get mixing again due to higher order effects.

General parameterizations of $\bar{\eta}$, $\bar{\eta}'$ bilinear terms

$$\begin{aligned}
 \mathcal{L} = & \frac{\delta_1}{2} \partial_\mu \partial_\nu \bar{\eta} \partial^\mu \partial^\nu \eta + \frac{\delta_2}{2} \partial_\mu \partial_\nu \bar{\eta}' \partial^\mu \partial^\nu \eta' + \delta_3 \partial_\mu \partial_\nu \bar{\eta} \partial^\mu \partial^\nu \eta' \\
 & + \frac{1 + \delta_{\bar{\eta}}}{2} \partial_\mu \bar{\eta} \partial^\mu \eta + \frac{1 + \delta_{\bar{\eta}'}}{2} \partial_\mu \bar{\eta}' \partial^\mu \eta' + \delta_k \partial_\mu \bar{\eta} \partial^\mu \eta' \\
 & - \frac{m_\eta^2 + \delta_{m_\eta^2}}{2} \bar{\eta} \eta - \frac{m_{\eta'}^2 + \delta_{m_{\eta'}^2}}{2} \bar{\eta}' \eta' - \delta_{m^2} \bar{\eta} \eta'.
 \end{aligned}$$

δ_i 's are calculated up to NNLO in δ -counting U(3) ChPT

LO from \mathcal{L}^{δ^0} : $\delta_i = 0$,

NLO from \mathcal{L}^{δ^1} :

$$\delta_1^{NLO} = \delta_2^{NLO} = \delta_3^{NLO} = 0,$$

$$\delta_{\bar{\eta}}^{NLO} = \frac{8L_5}{3F^2} [m_\pi^2 (-c_\theta^2 - 4\sqrt{2}c_\theta s_\theta + s_\theta^2) + 2m_K^2 (2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)] + s_\theta^2 \Lambda_1,$$

$$\delta_{\bar{\eta}'}^{NLO} = \frac{8L_5}{3F^2} [m_\pi^2 (c_\theta^2 + 4\sqrt{2}c_\theta s_\theta - s_\theta^2) + 2m_K^2 (c_\theta^2 - 2\sqrt{2}c_\theta s_\theta + 2s_\theta^2)] + c_\theta^2 \Lambda_1,$$

$$\delta_k^{NLO} = -\frac{16L_5}{3F^2} (m_K^2 - m_\pi^2) (\sqrt{2}c_\theta^2 - c_\theta s_\theta - \sqrt{2}s_\theta^2) - c_\theta s_\theta \Lambda_1,$$

.....

NNLO from \mathcal{L}^{δ^2} and chiral loops:

$$\delta_1^{NNLO} = \frac{32C_{12}}{3F^2} [m_\pi^2(-c_\theta^2 - 4\sqrt{2}c_\theta s_\theta + s_\theta^2) + 2m_K^2(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)],$$

$$\delta_{\eta}^{NNLO} =$$

$$\frac{8L_4}{F^2}(2m_K^2 + m_\pi^2) + \dots$$

$$+ \frac{128L_5L_8}{3F^4} [m_\pi^4(-c_\theta^2 - 4\sqrt{2}c_\theta s_\theta + s_\theta^2) + m_K^4(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)] + \dots$$

$$+ \frac{16C_{14}}{3F^4} [3m_\pi^4 + 4m_K^4(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2) - 4m_K^2m_\pi^2(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)] + \dots$$

$$+ \frac{c_\theta^2}{F^2} A_0(m_\pi^2) + \dots,$$

.....

$$A_0(m^2) = -\frac{m^2}{16\pi^2} \log \frac{m^2}{\mu^2}$$

[X.K.Guo, ZHG, Oller, Sanz-Cillero, JHEP'15]

- We also give the relation to the popular two-mixing angle scheme:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \frac{1}{F} \begin{pmatrix} F_8 \cos \theta_8 & -F_0 \sin \theta_0 \\ F_8 \sin \theta_8 & F_0 \cos \theta_0 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} .$$

$$\begin{aligned} F_{\eta}^8 &= \cos \theta_8 F_8 & F_{\eta'}^8 &= \sin \theta_8 F_8 & \langle 0 | A_{\mu}^a | \eta^{(\prime)} \rangle &= ip_{\mu} F_{\eta^{(\prime)}}^a \\ F_{\eta}^0 &= -\sin \theta_0 F_0 & F_{\eta'}^0 &= \cos \theta_0 F_0 & & \end{aligned}$$

Now the mixing parameters (F_0 F_8 θ_0 θ_8) are calculated in terms of the chiral low energy constants up to NNLO !

First complete NNLO calculation of eta-eta' mixing !

Eta-eta' mixing in ChPT in literature

Some previous (partial) lower-order calculations:

LO in δ -expansion, Georgi, PRD'94

LO in p^2 and NLO in $1/N_c$, Peris, PLB'94.

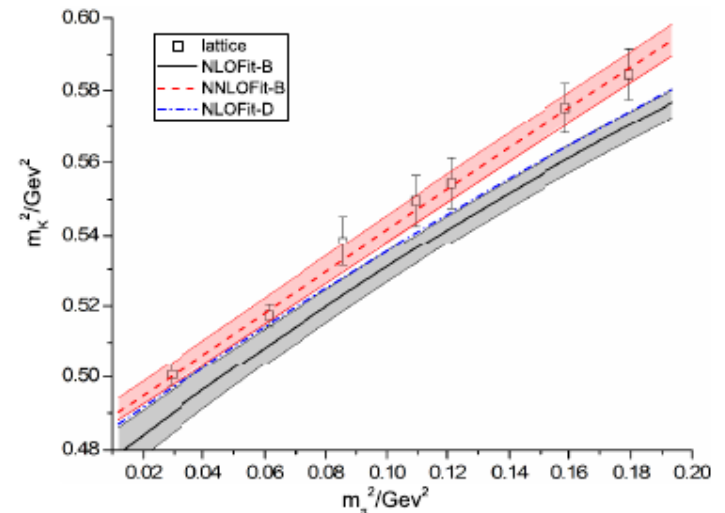
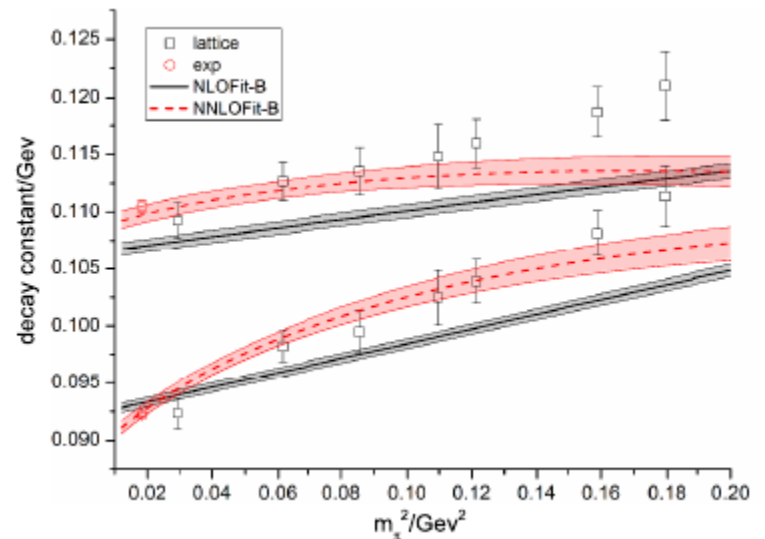
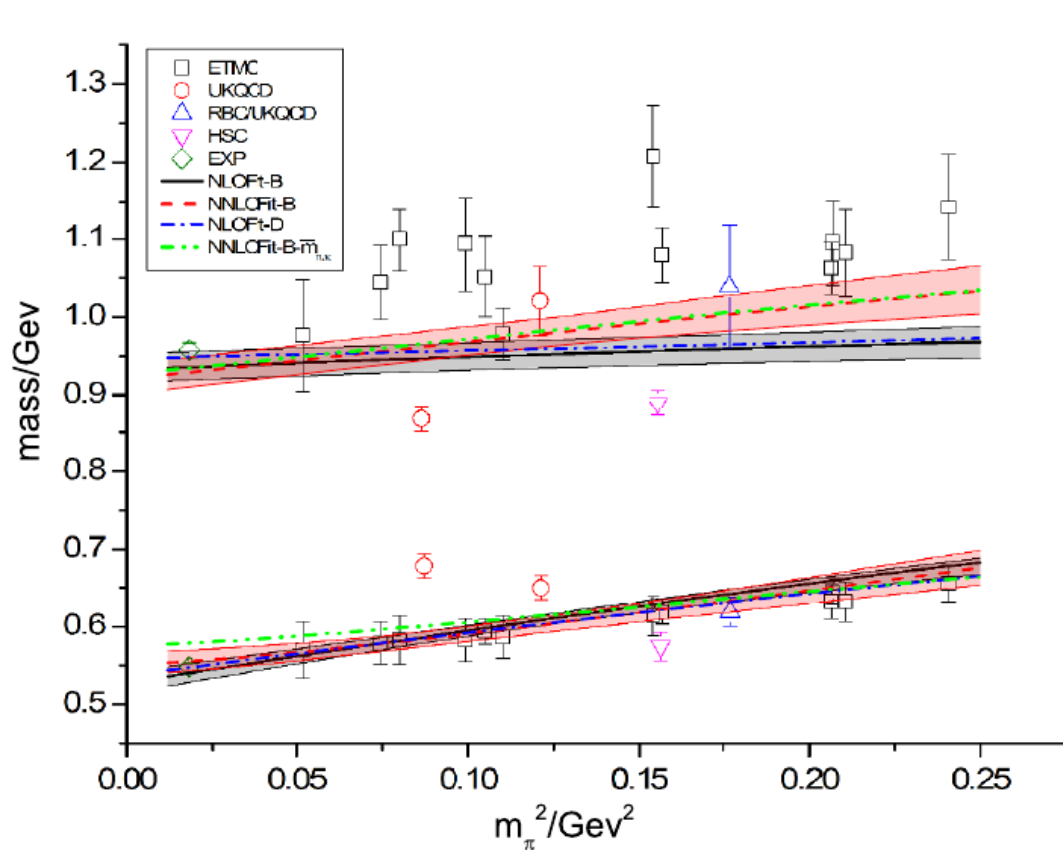
NLO in p^2 , LO in $1/N_c$, Gerard, Kou, PLB'05; Degrande, Gerard JHEP'09; Mathieu, Vento PLB'10.

NLO in p^2 and $1/N_c$, Herrera-Siklody *et al.*, PLB'98

One-loop calculation+resonance exchange (partial NNLO),
Z.H.Guo,Oller, PRD'11

FKS formalism, Feldmann,Kroll,Stech,PRD'98

etc



[X.K.Guo, ZHG, Oller, Sanz-Cillero, JHEP'15]

Nice reproduction of the lattice results with reasonable chiral LECs !

S-wave meson-meson scattering and Scalar resonances

ChPT is reliable at threshold energy region.

E.g. Leading order amplitudes of pi-eta, K-Kbar, pi-eta' scattering

$$\begin{aligned}T_{J=0}^{I=1,\pi\eta\rightarrow\pi\eta}(s)^{(2)} &= \frac{(c_\theta - \sqrt{2}s_\theta)^2 m_\pi^2}{3F_\pi^2}, \\T_{J=0}^{I=1,\pi\eta\rightarrow K\bar{K}}(s)^{(2)} &= \frac{c_\theta(3m_\eta^2 + 8m_K^2 + m_\pi^2 - 9s) + 2\sqrt{2}s_\theta(2m_K^2 + m_\pi^2)}{6\sqrt{6}F_\pi^2}, \\T_{J=0}^{I=1,\pi\eta\rightarrow\pi\eta'}(s)^{(2)} &= \frac{(\sqrt{2}c_\theta^2 - c_\theta s_\theta - \sqrt{2}s_\theta^2)m_\pi^2}{3F_\pi^2}, \\T_{J=0}^{I=1,K\bar{K}\rightarrow K\bar{K}}(s)^{(2)} &= \frac{s}{4F_\pi^2}, \\T_{J=0}^{I=1,K\bar{K}\rightarrow\pi\eta'}(s)^{(2)} &= \frac{s_\theta(3m_{\eta'}^2 + 8m_K^2 + m_\pi^2 - 9s) - 2\sqrt{2}c_\theta(2m_K^2 + m_\pi^2)}{6\sqrt{6}F_\pi^2}, \\T_{J=0}^{I=1,\pi\eta'\rightarrow\pi\eta'}(s)^{(2)} &= \frac{(\sqrt{2}c_\theta + s_\theta)^2 m_\pi^2}{3F_\pi^2},\end{aligned}$$

There is no resonance in such amplitudes !

[ZHG,Oller, PRD'11] [ZHG,Oller,Ruiz de Elvira PRD'12,PLB'12]

[ZHG,Liu,Meissner,Oller,Rusetsky,PRD'17]

Unitarization: Algebraic approximation of N/D (a variant version of K-matrix)

$$T_J(s) = \frac{N(s)}{1 + G(s) N(s)}$$

- **The s-channel unitarity is exact. The crossed-channel dynamics is included in a perturbative manner.**

- **Unitarity condition:** $\text{Im}G(s) = -\rho(s)$

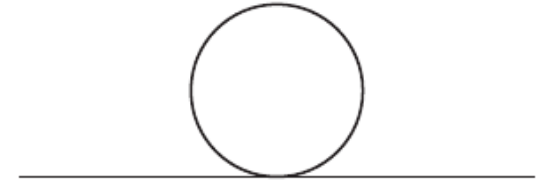
$$G(s) = a^{SL}(s_0) - \frac{s - s_0}{\pi} \int_{4m^2}^{\infty} \frac{\rho(s')}{(s' - s)(s' - s_0)} ds'$$

- **$N(s)$: given by the partial wave chiral amplitudes**

$$\mathcal{V}_{J,D_1\phi_1 \rightarrow D_2\phi_2}^{(S,I)}(s) = \frac{1}{2} \int_{-1}^{+1} d \cos \varphi P_J(\cos \varphi) V_{D_1\phi_1 \rightarrow D_2\phi_2}^{(S,I)}(s, t(s, \cos \varphi)).$$

For eta-eta' mixing, temperatures enter via the loop functions

$$A_0(m^2) = -m^2 \ln \frac{m^2}{\mu^2} - \int_0^\infty dp \frac{8p^2}{E_p} \frac{1}{e^{\frac{E_p}{T}} - 1}$$



For meson-meson scattering, temperatures enter via $G(s)$

$$\begin{aligned} G(s) &= i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_1^2)[(P - q)^2 - m_2^2]} \\ &= i \int \frac{d^3 \vec{q}}{(2\pi)^3} \int \frac{dq_0}{2\pi} \frac{1}{q_0^2 - E_1^2} \frac{1}{(P_0 - q_0)^2 - E_2^2} \end{aligned}$$



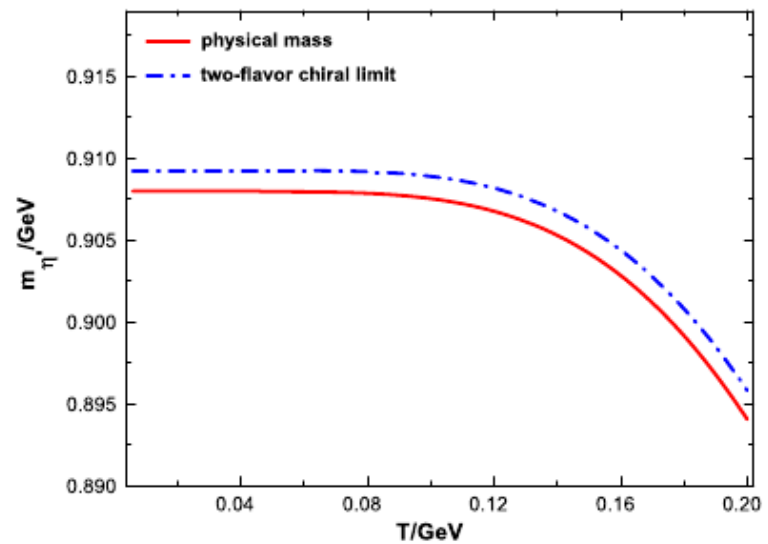
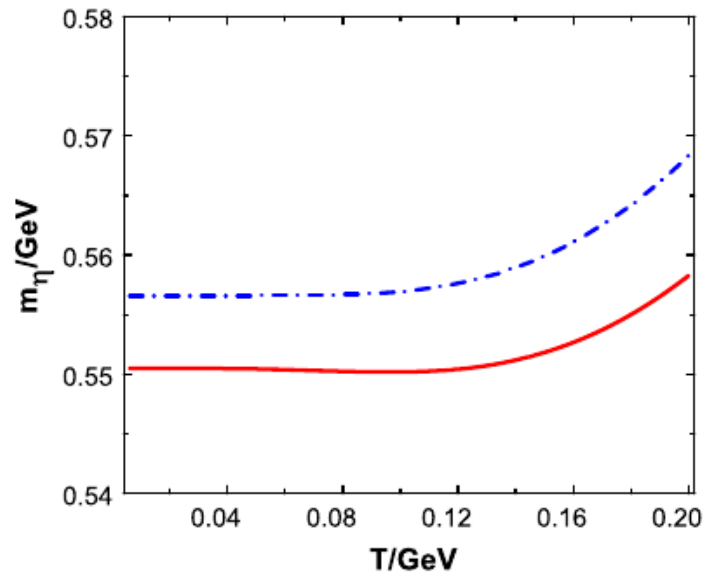
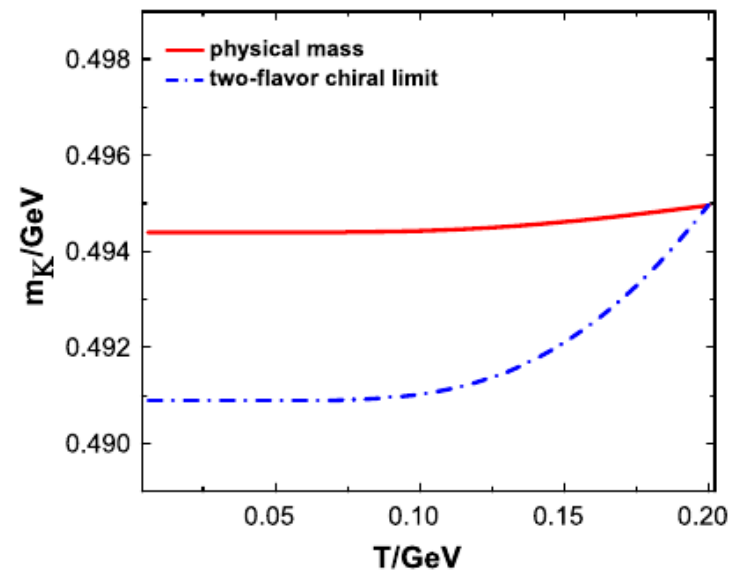
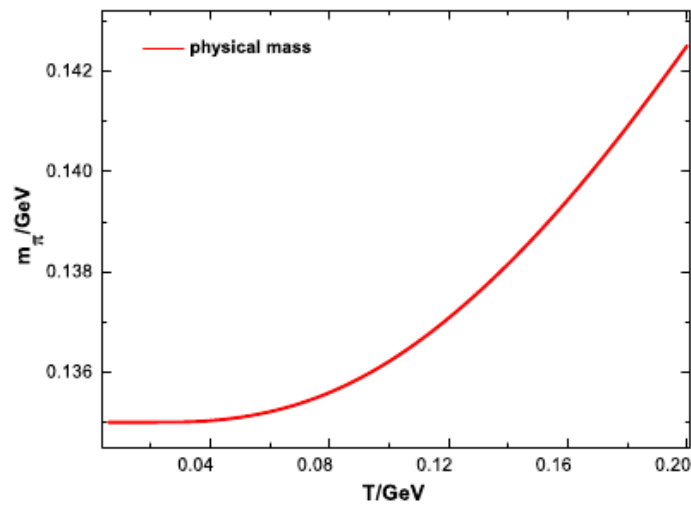
$$q_0 \rightarrow i\omega_n = i2\pi nT, \quad dq_0 \rightarrow i2\pi T$$

$$= \int \frac{d^3 \vec{q}}{(2\pi)^3} \sum_{n=-\infty}^{n=+\infty} T \frac{1}{\omega_n^2 + E_1^2} \frac{1}{(P_0 - i\omega_n)^2 - E_2^2}$$

Standard Matsubara techniques to calculate $G(s)$.

Be careful about the complex extrapolation !

Thermal behaviors of light pseudoscalar mesons and scalar resonances at low temperatures



[Gu, Duan, ZHG, PRD'18]

Relevant channels to study σ , $f_0(980)$, κ , $a_0(980)$

- $IJ=10$: $\pi\eta$, $K\bar{K}$, $\pi\eta'$

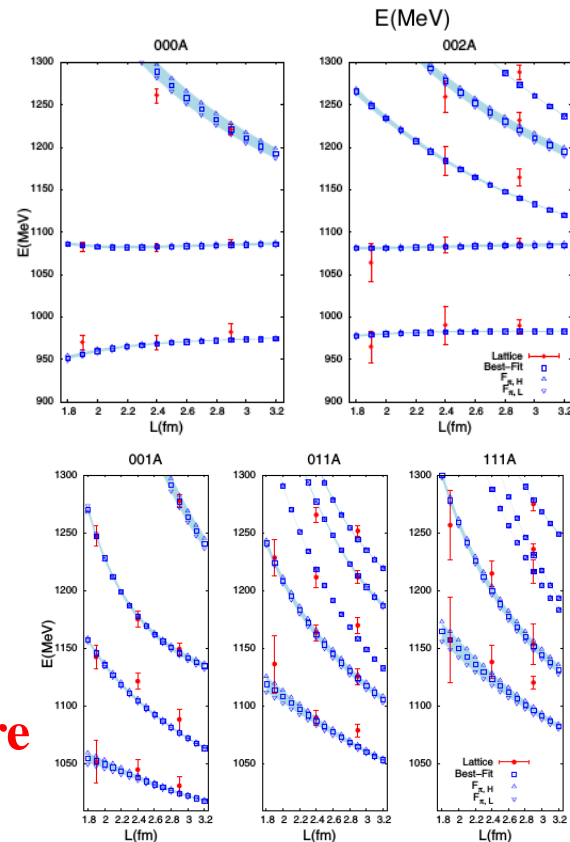
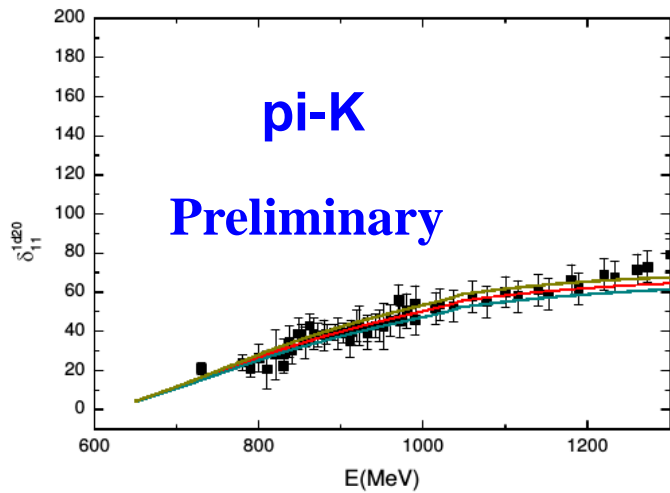
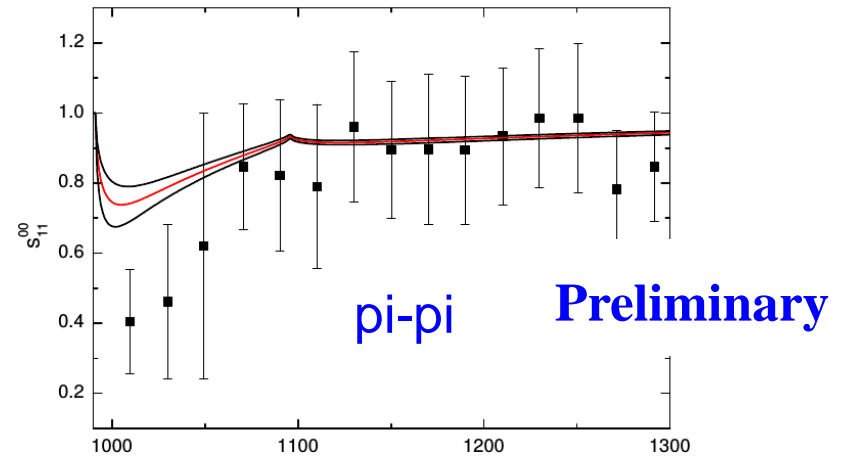
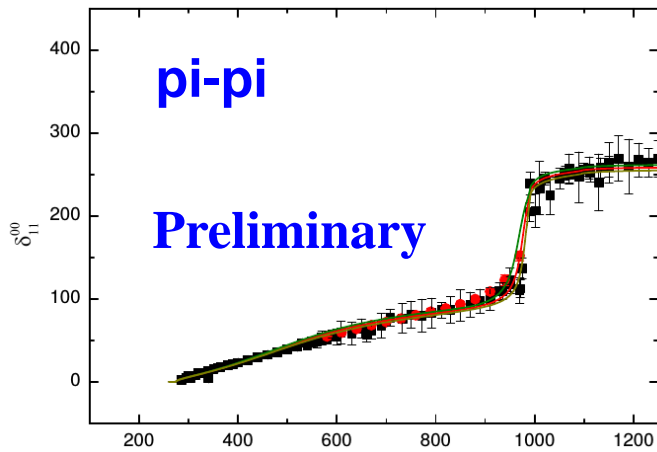
$a_0(980)$

- $IJ=\frac{1}{2} 0$: $K\pi$, $K\eta$, $K\eta'$

κ

- $IJ=00$: $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$

σ , $f_0(980)$

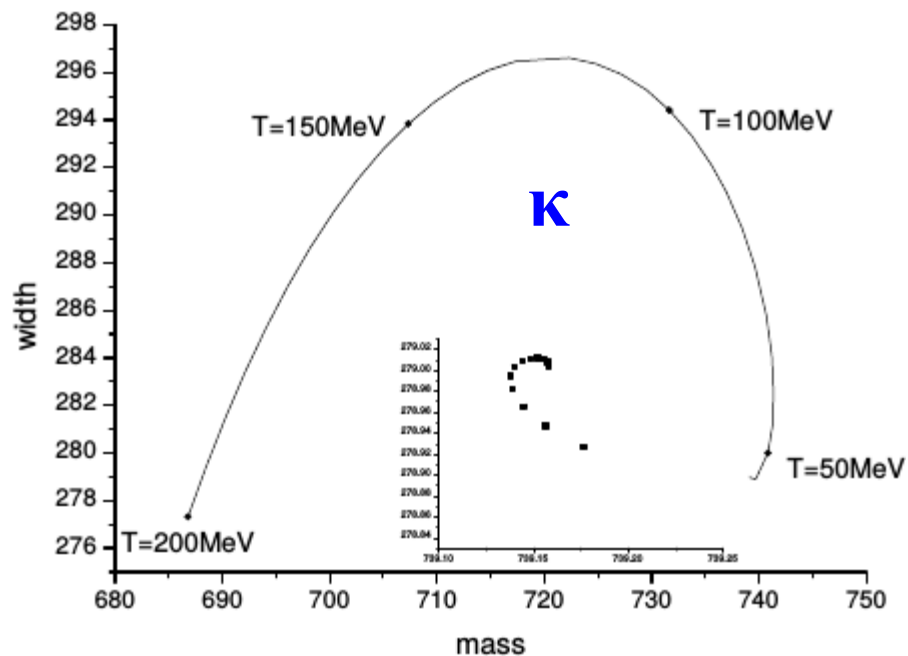
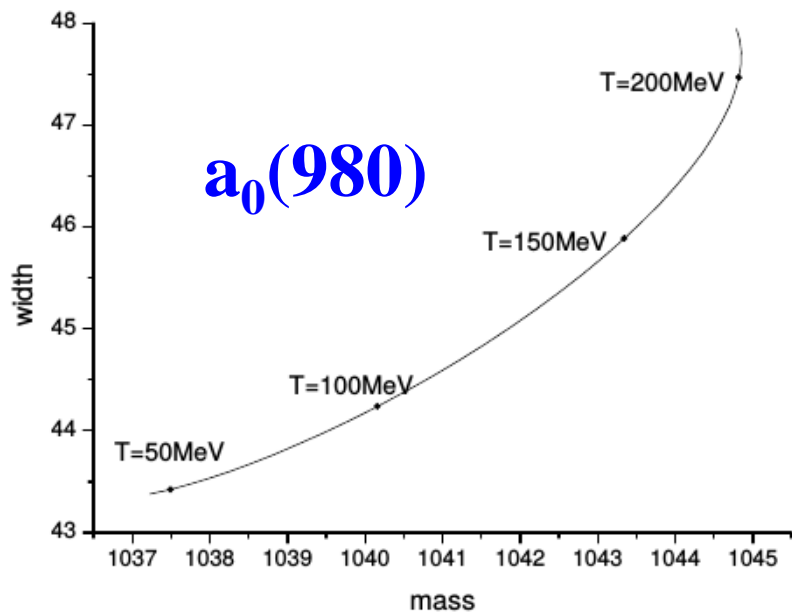
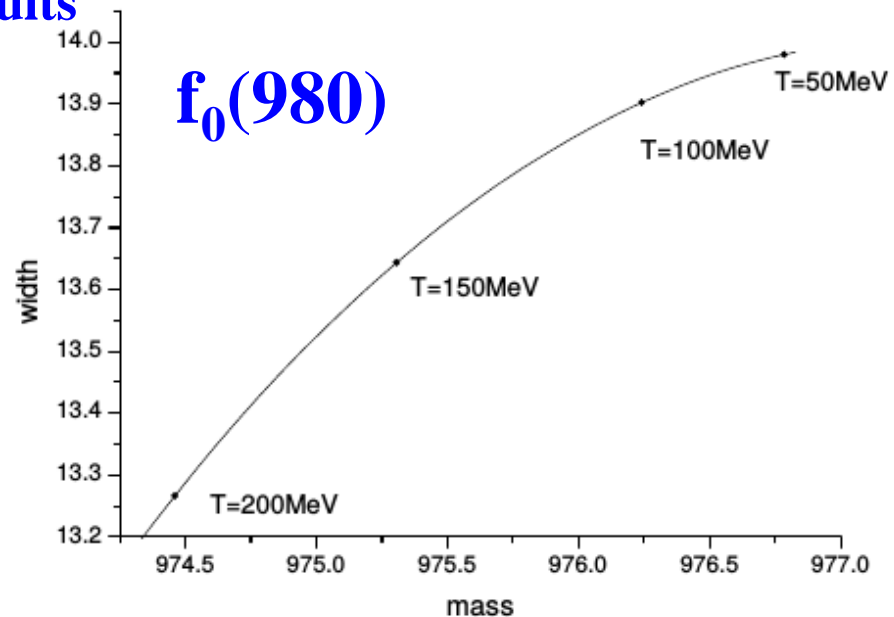
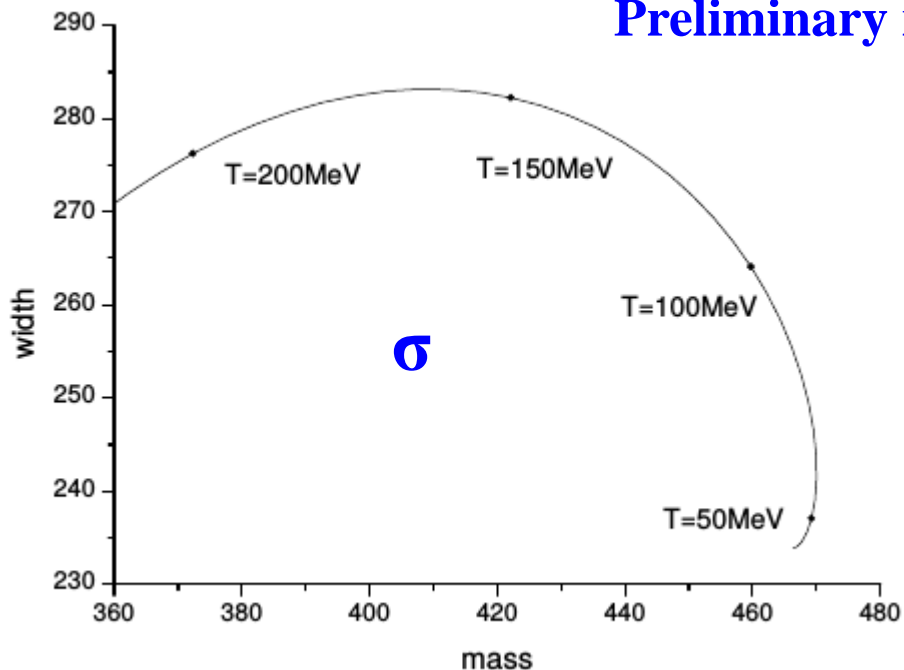


pi-eta
 [ZHG, et. al., PRD' 17]

Lattice data
 [Dudek, et. al., PRD' 16]

Preliminary fits to Exp and Lattice scattering observables at zero temperature

Preliminary results



Summary

- **Chiral perturbation theory provides a reliable framework to study the interactions of π , K , η and η' and also the possible resonances in their scattering at zero temperature.**
- **The finite-temperature effects are pure predictions in chiral perturbation theory.**
- **Our preliminary results show interesting thermal behaviors of light scalar resonances of QCD !**

谢谢大家!