Nuclear Science Computing Center at CCNU

Chiral crossover and phase transition in (2+1)-flavor QCD

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Central China Normal University

International Workshop on Partonic and Hadronic Transport Approaches for Relativistic Heavy Ion Collisions

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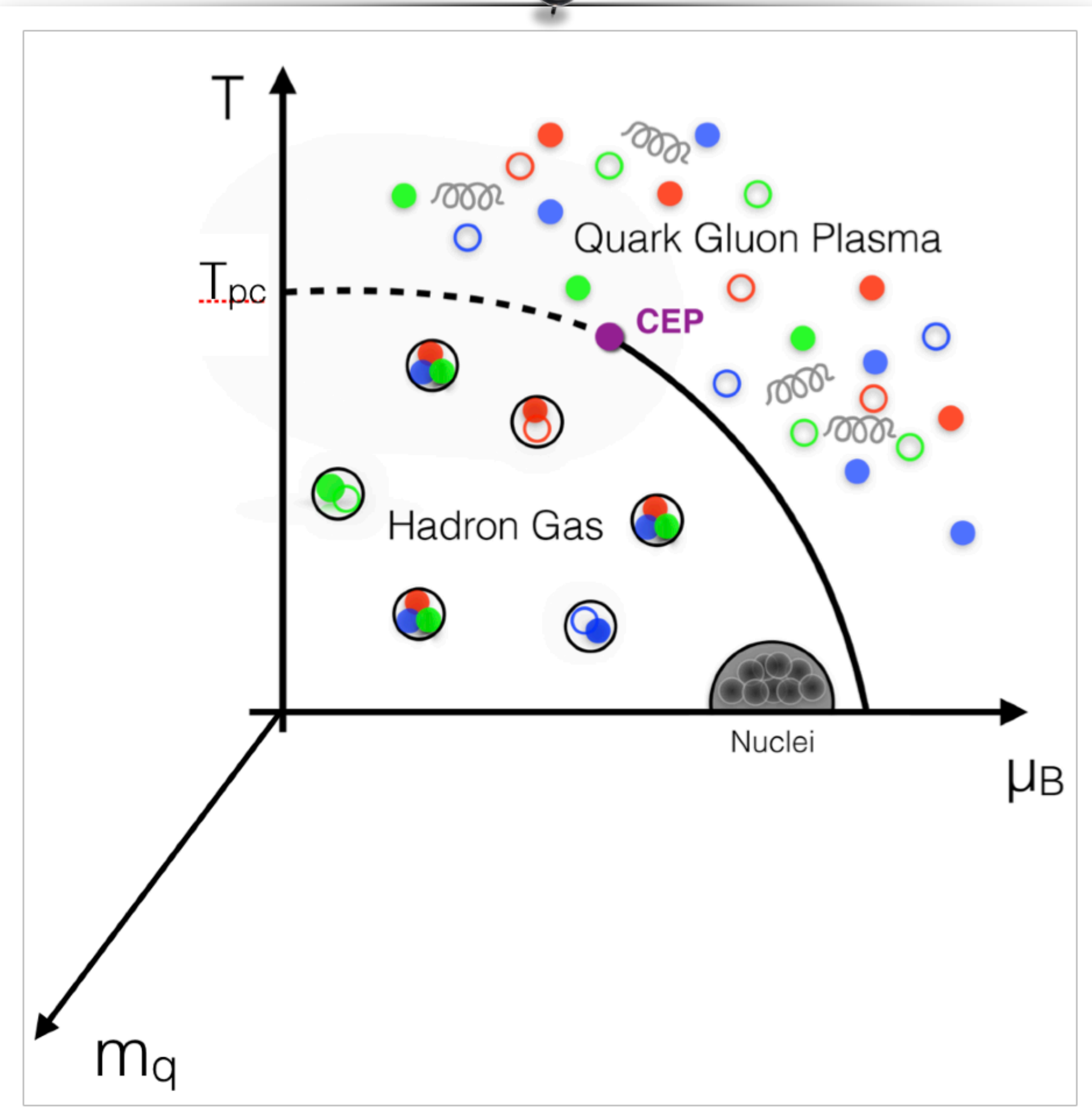
Outline: QCD phase structure



HTD, F. Karsch, S. Mukherjee, arXiv: 1504.05274
Int.J.Mod.Phys. E24 (2015) no.10, 1530007

- Chiral crossover at zero and small μ_B

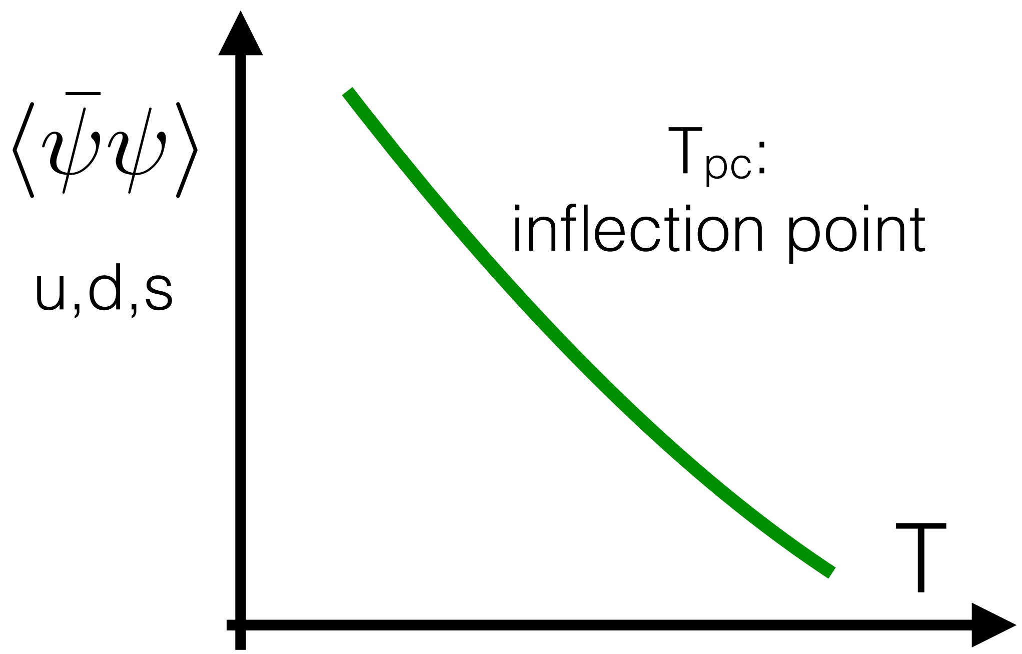
  A. Bazavov, HTD, P. Hegde et al.
  [HotQCD], arXiv:1812.08235

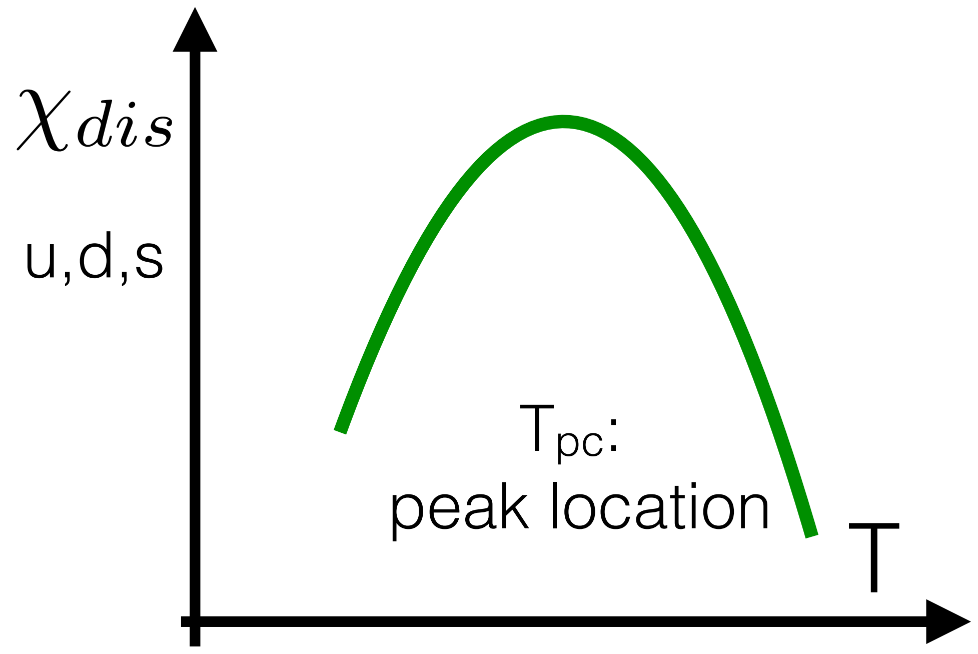
- Chiral phase transition temperature

  HTD, P. Hegde, O. Kaczmarek et al.
  [HotQCD], arXiv:1903.04801

Crossover transition temperature T_{pc} in the real world

- Crossover nature of the transition





- Chiral phase transition: most likely 2nd order, 3d O(4)

Ejiri et al., PRD 80(2009)094505,
HotQCD, arXiv:1903.04801
…

- A well-defined **chiral crossover transition temperature**: based on scaling properties of QCD

HTD, P. Hegde, O. Kaczmarek et al.
[HotQCD], arXiv:1903.04801

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T, \mu_B) \sim m^{1/\delta} f_G$

chiral susceptibility: $\chi^{\Sigma}(T, \mu_B) \sim m^{1/\delta - 1} f_\chi$

m： quark mass
f: scaling functions
δ,β: critical exponents

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Taylor expansions:

$$\Sigma(T, \mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\Sigma}(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n} \qquad \chi(T, \mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^{\chi}(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n}$$

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$\partial_T \chi^{\Sigma}(T)$
$\partial_T C_0^{\chi}(T)$
$C_2^{\chi}(T)$

$$\sim \quad m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$$

$\partial_T^2 C_0^{\Sigma}(T)$
$\partial_T C_2^{\Sigma}(T)$

$$\sim \quad m^{1/\delta-2/\beta\delta} f''_G(z)$$

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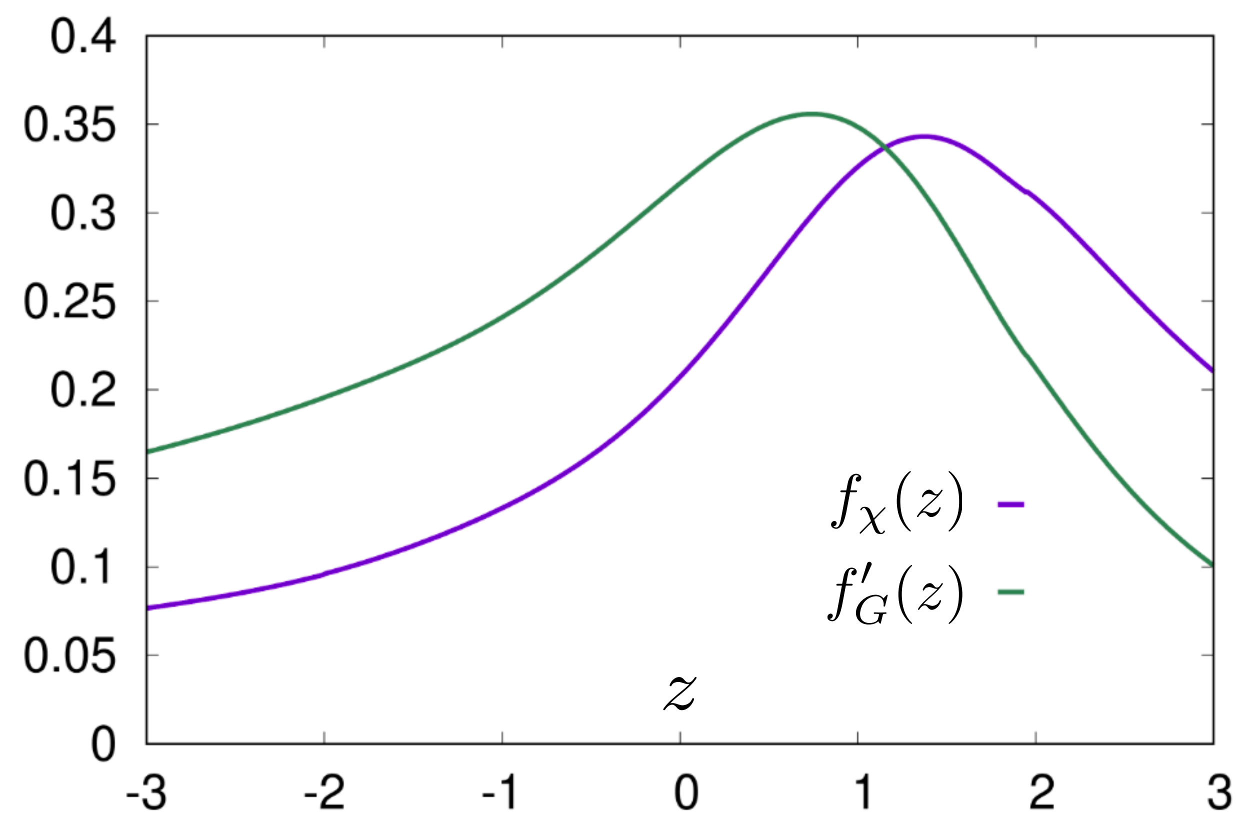
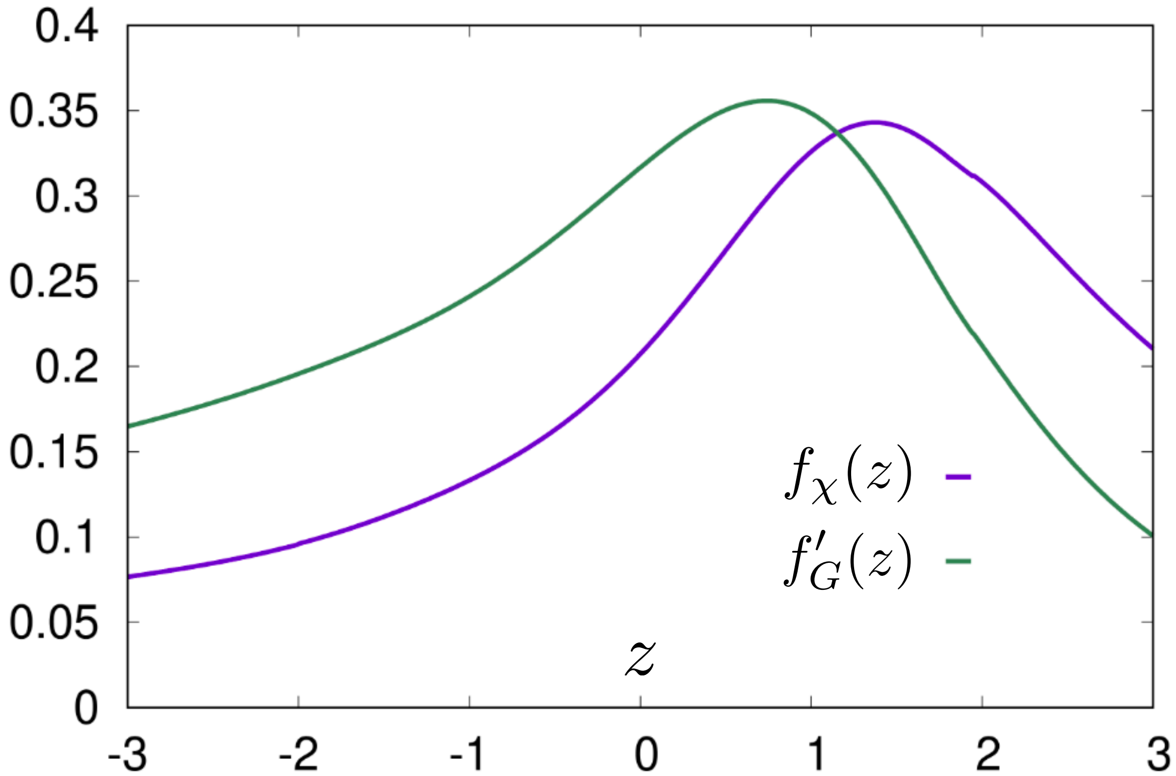
$\partial_T \chi^{\Sigma}(T)$, $\partial_T C_0^{\chi}(T)$, $C_2^{\chi}(T)$ $\sim \; m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$

$\partial_T^2 C_0^{\Sigma}(T)$, $\partial_T C_2^{\Sigma}(T)$ $\sim \; m^{1/\delta-2/\beta\delta} f''_G(z)$



$z \sim \left((T-T_c^0)/T_c^0 + K(\mu_B/T)^2\right)/m^{\frac{1}{\beta\delta}}$

Well-defined notation of chiral crossover transition temperature



$$z \sim \left((T-T_c^0)/T_c^0 + K(\mu_B/T)^2\right)/m^{\frac{1}{\beta\delta}}$$

$$\partial_T \chi^\Sigma(T),\ \partial_T C_0^\chi(T),\ C_2^\chi(T) \sim m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$$

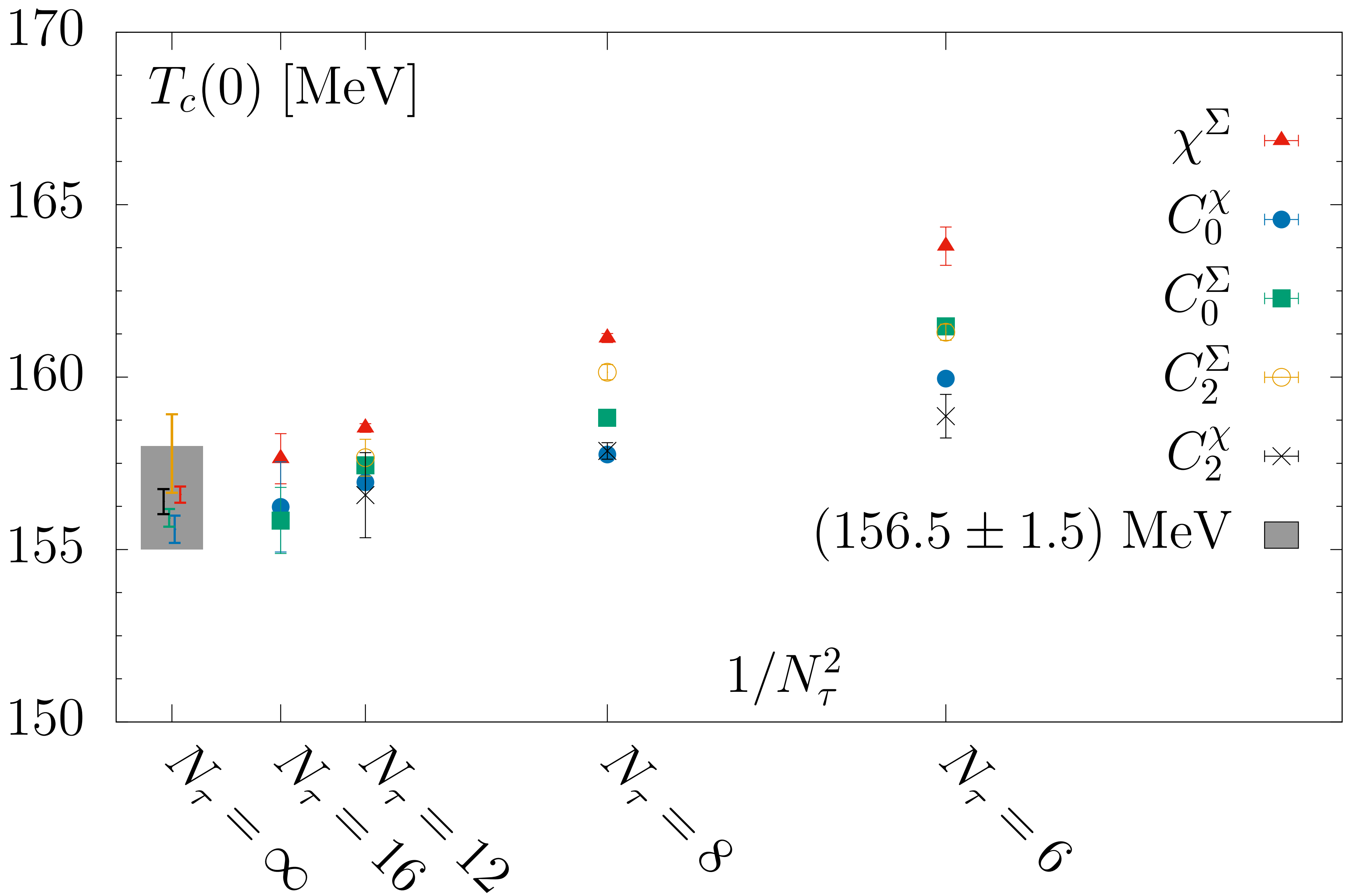
$$\partial_T^2 C_0^\Sigma(T),\ \partial_T C_2^\Sigma(T) \sim m^{1/\delta-2/\beta\delta} f''_G(z)$$

- 5 conditions to extract T_{pc}: maxima of f_χ and f'_G

$\partial_T \chi^\Sigma(T) = 0$ $\partial_T C_0^\chi(T) = 0$ $C_2^\chi(T) = 0$ $\partial_T^2 C_0^\Sigma(T) = 0$ $\partial_T C_2^\Sigma(T) = 0$

- m=0: all these susceptibilities diverge at a unique T
- m=/=0: non-unique temperatures, crossover

QCD transition with m_π =140 MeV at μ_B=0

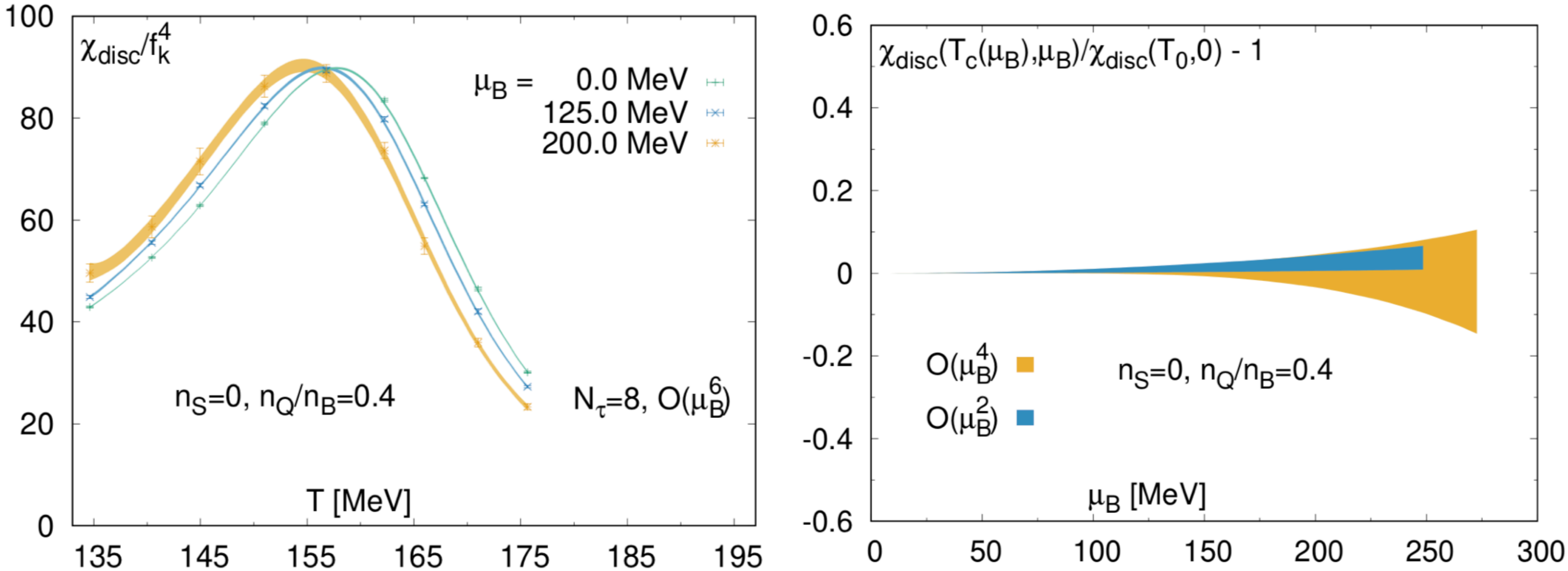


Higher precision in the continuum limit:

T_{pc} =156.5(1.5)MeV

HotQCD, arXiv:1812.08235

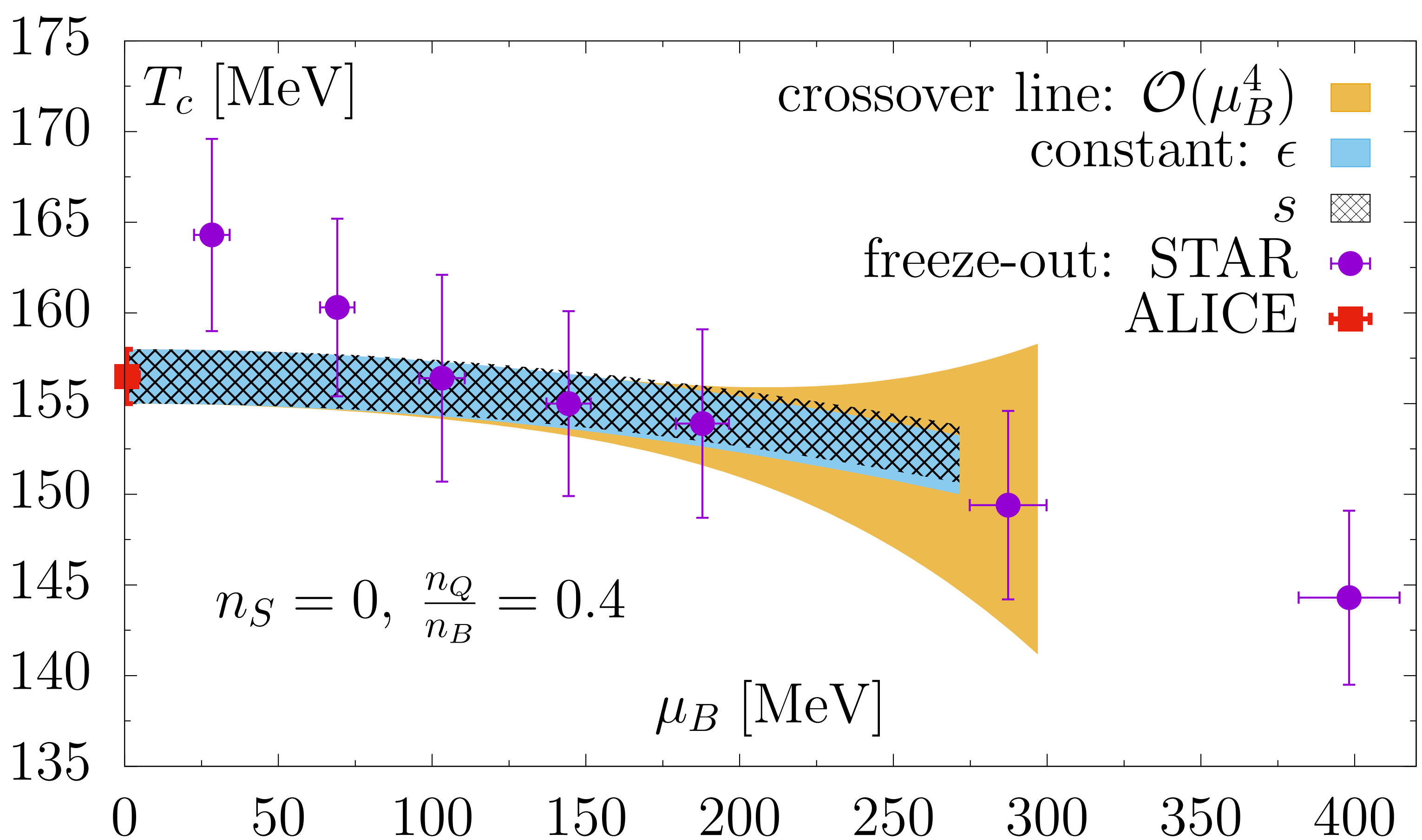
Order Parameter Susceptibility at μB=/=0



No indication of a stronger phase transition at larger μ_B

Crossover, line of constant physics & freeze-out

$$T(\mu_B) = T(0)\left(1 - \kappa_2\left(\frac{\mu_B}{T}\right)^2 - \kappa_4\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}\left(\frac{\mu_B}{T}\right)^6\right)$$



A. Bazaovo, HTD et al. [HotQCD], arXiv:1812.08235

curvature of crossover line

$$\kappa_2 = 0.0123 \pm 0.003$$

$$\kappa_4 = 0.000131 \pm 0.0041$$

curvature at constant b:

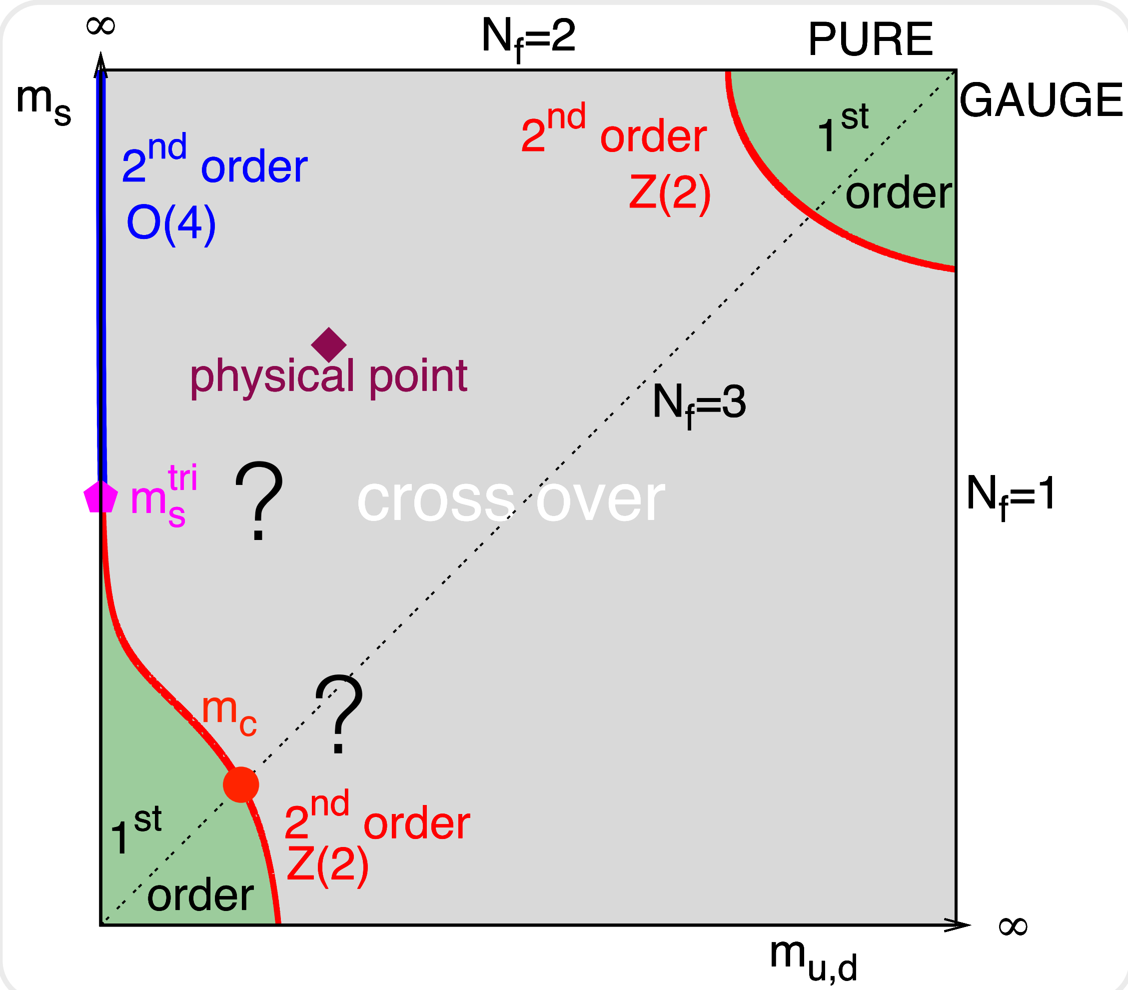
$$0.006 \leq \kappa_2^b \leq 0.012, \quad b = P, \epsilon, s$$

Bielefeld-BNL-CCNU, PRD95 (2017) no.5, 054504

c.f. Wei-Jie Fu's talk on results from FRG

QCD phase diagram in the quark mass plane

Columbia plot:

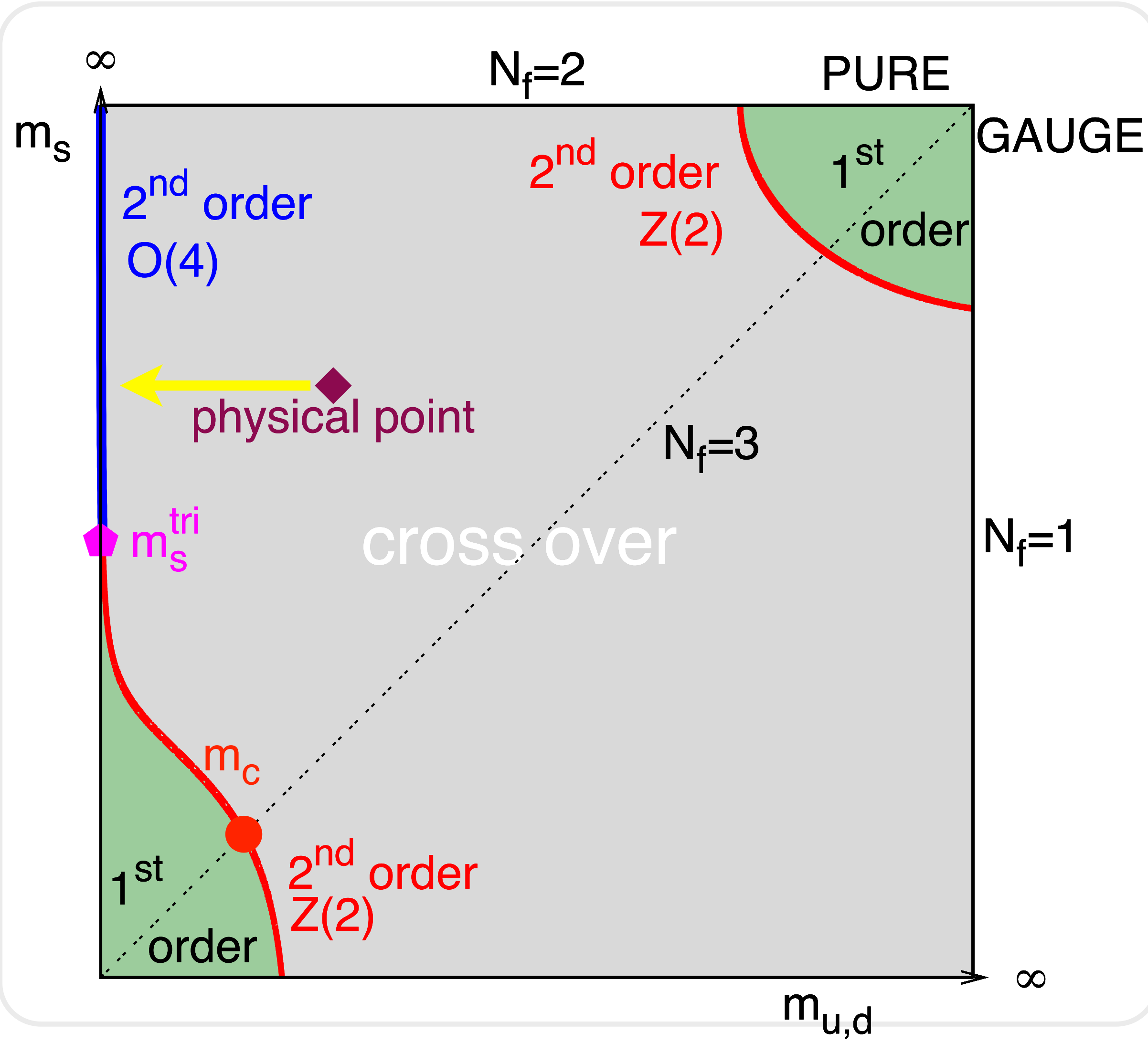


- At physical point: crossover, T_{pc} = 156.5(1.5) MeV HotQCD, arXiv:1812.08235
- N_f=2(+1): $U_A(1)$ remains broken at $T_{\chi SB}$ JLQCD '13,'14,'15, HotQCD '13,'14
- Critical lines of second order transition Pisarski & Wilczek PRD '84
  - N_f=2: O(4) universality class Kogut & Sinclair, PRD '06
  - N_f=3: Ising universality class Karsch, Laermann, Schmidt PLB '04,...

Towards the chiral limit:

- N_f=2+1 QCD: m_s^{tri} ? m_s^{phy}
- Fundamental scale of QCD: chiral T_c ?

Towards chiral limit of (2+1)-flavor QCD



- HISQ/tree action
- **N_f=2+1:**

$m_u = m_d \rightarrow 0$

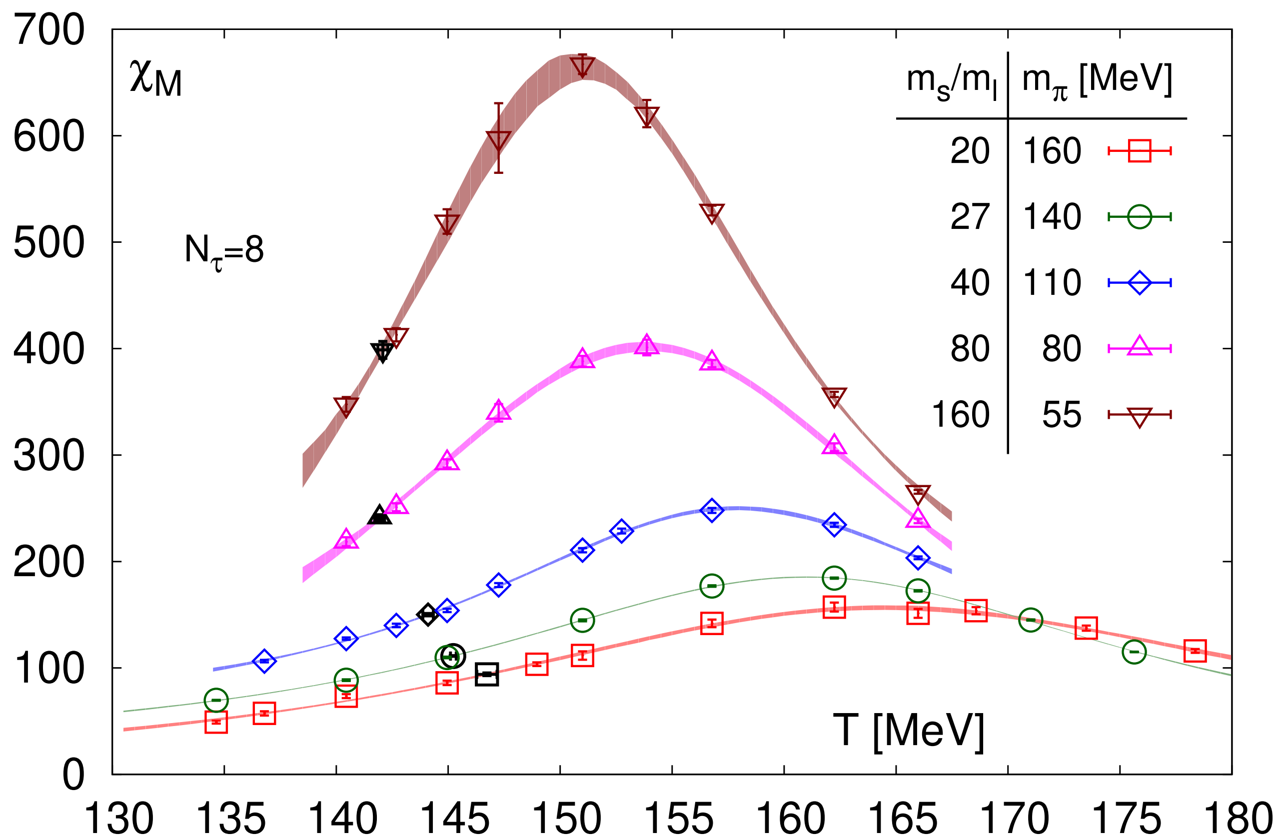
$m_s = m_s^{phy}$

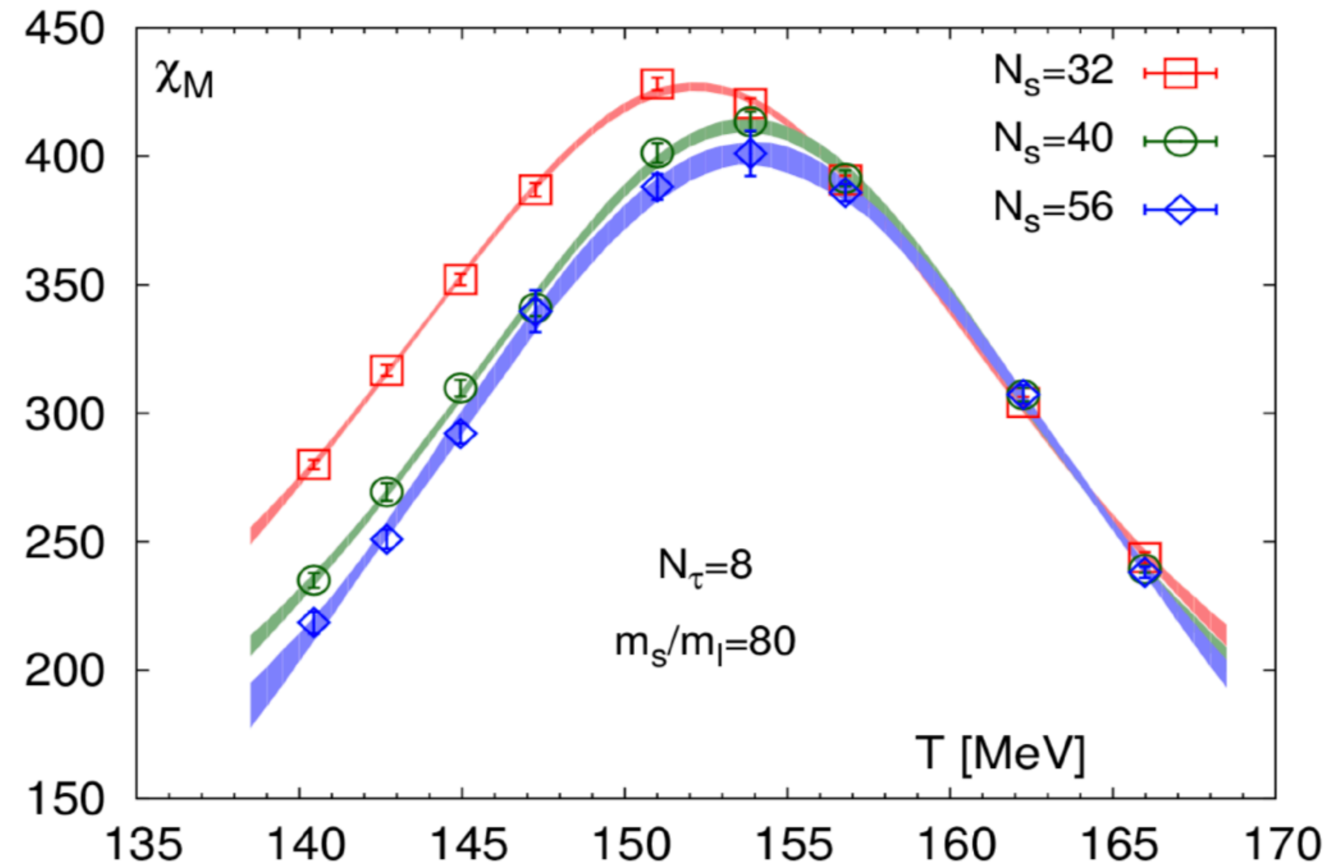
- ☑ Nt=6,8,12
- ☑ $m_s^{phy}/\mathbf{m_l}$ = 20, 27, 40, 60, 80,160

  $m_\pi \approx$160,140,110,90,80,55 MeV
- ☑ $7 \geq N_s/N_t \geq 4 \;\Leftrightarrow\; 5 \gtrsim m_\pi L \gtrsim 3$

This allows us to perform
infinite volume, continuum and then chiral extrapolation!

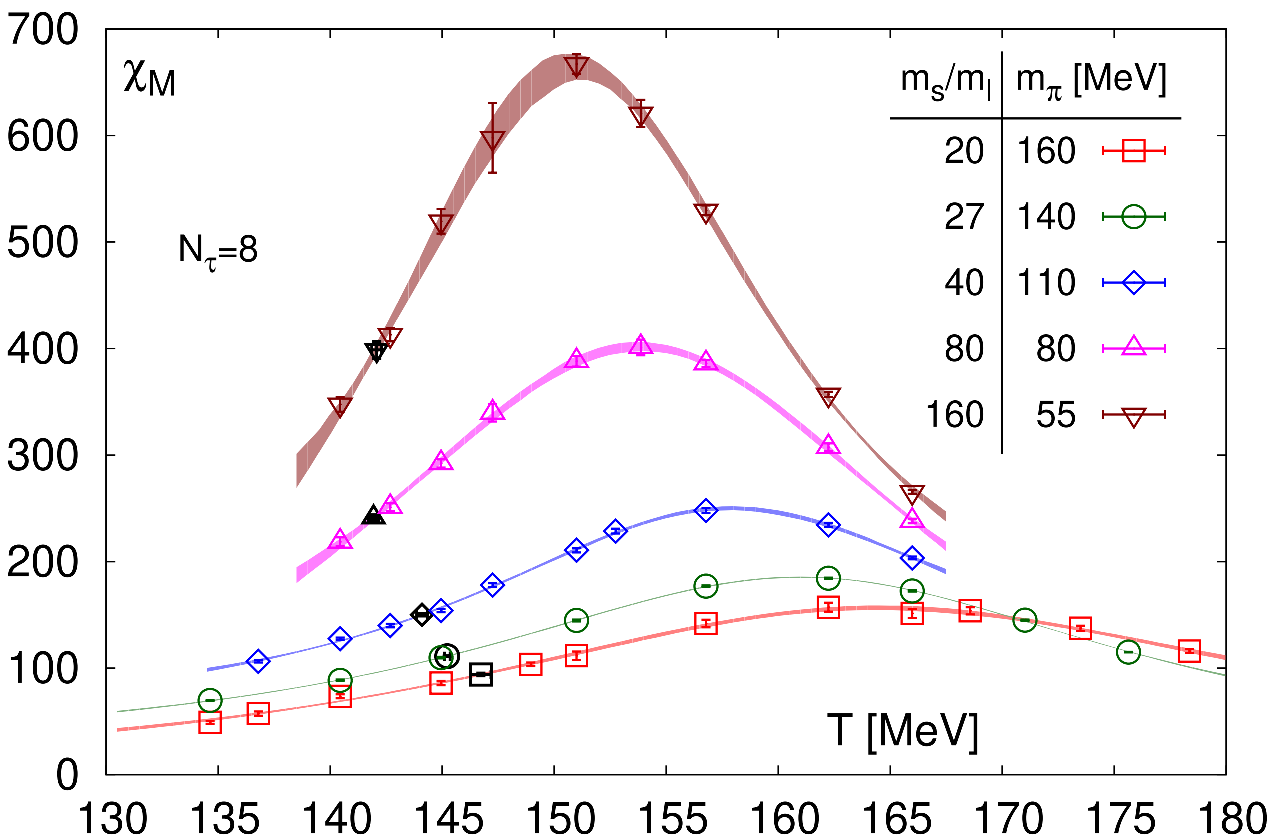
Quark mass and volume dependences of chiral susceptibility

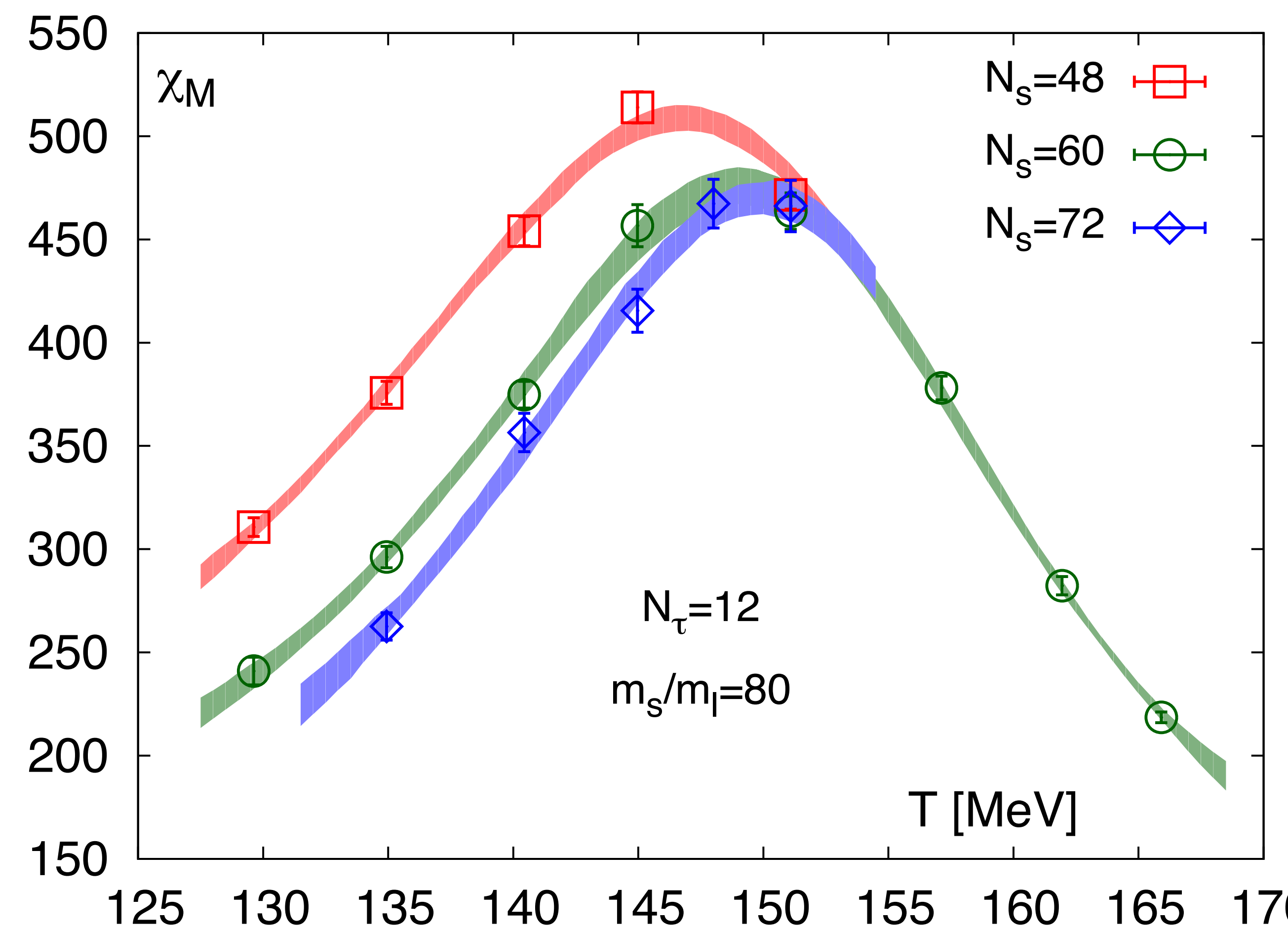




- Susceptibility increases as $m_l^{1/\delta-1}$+const, here $\delta \simeq 4.8$
- Peak height of susceptibility slightly changes with Volume
- Consistent with a continuous phase transition with O(N) universality class in the chiral limit of m_l

Quark mass and volume dependences of chiral susceptibility





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chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$$f(m,T)=h^{1+1/\delta}\, f_s(z) + f_{reg}\ , \qquad z=t/h^{1/\beta\delta}$$

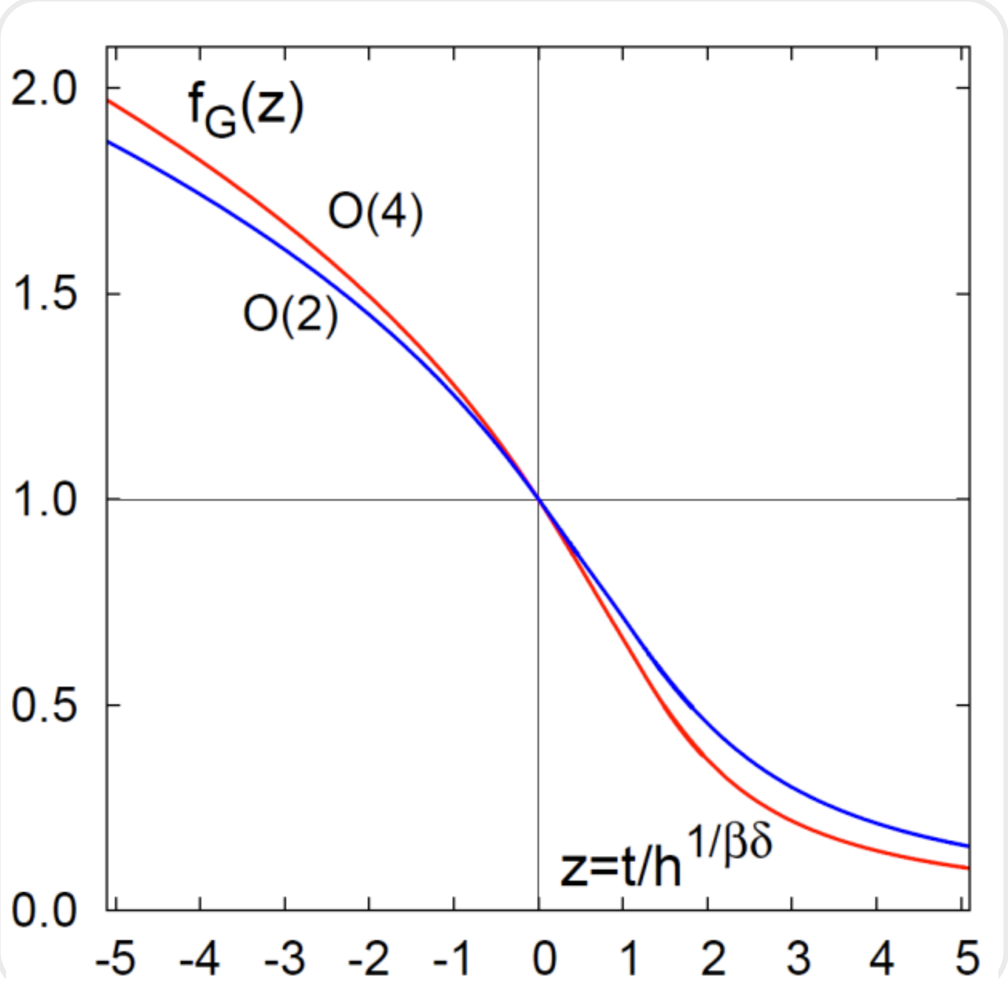
h: external field, t: reduced temperature, β,δ: universal critical exponents

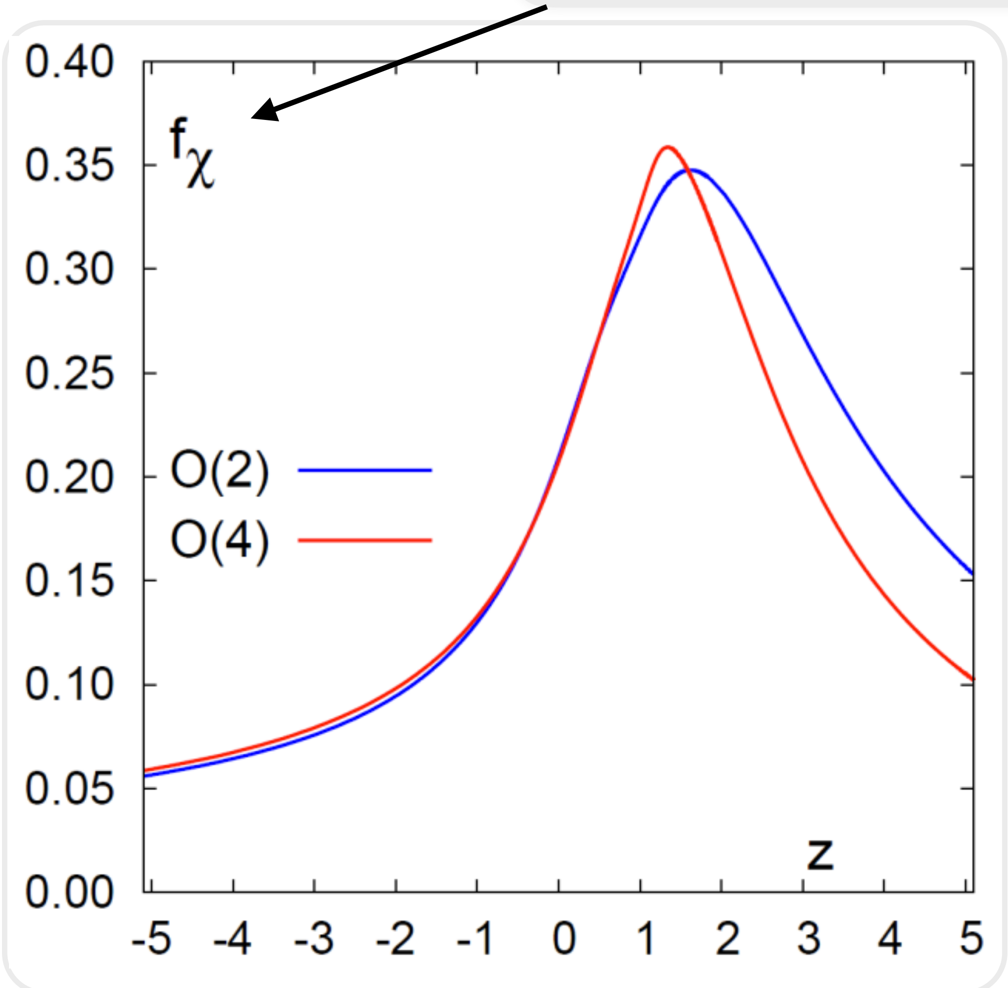
$f_s(z)$: universal scaling function, O(N) etc.

$$h=\frac{1}{h_0}\frac{m_l}{m_s} \qquad t=\frac{1}{t_0}\frac{T-T_c}{T_c}$$

Magnetic Equation of State (MEoS):

$$M=-\partial f_s(t,h)/\partial H = h^{1/\delta} f_G(z) \qquad \chi_M=\partial M/\partial H=\frac{h^{1/\delta}}{H}\,\frac{1}{\delta}\left(f_G(z)-\frac{z}{\beta}\frac{df_G(z)}{dz}\right)$$





Chiral phase transition temperature T_c^0

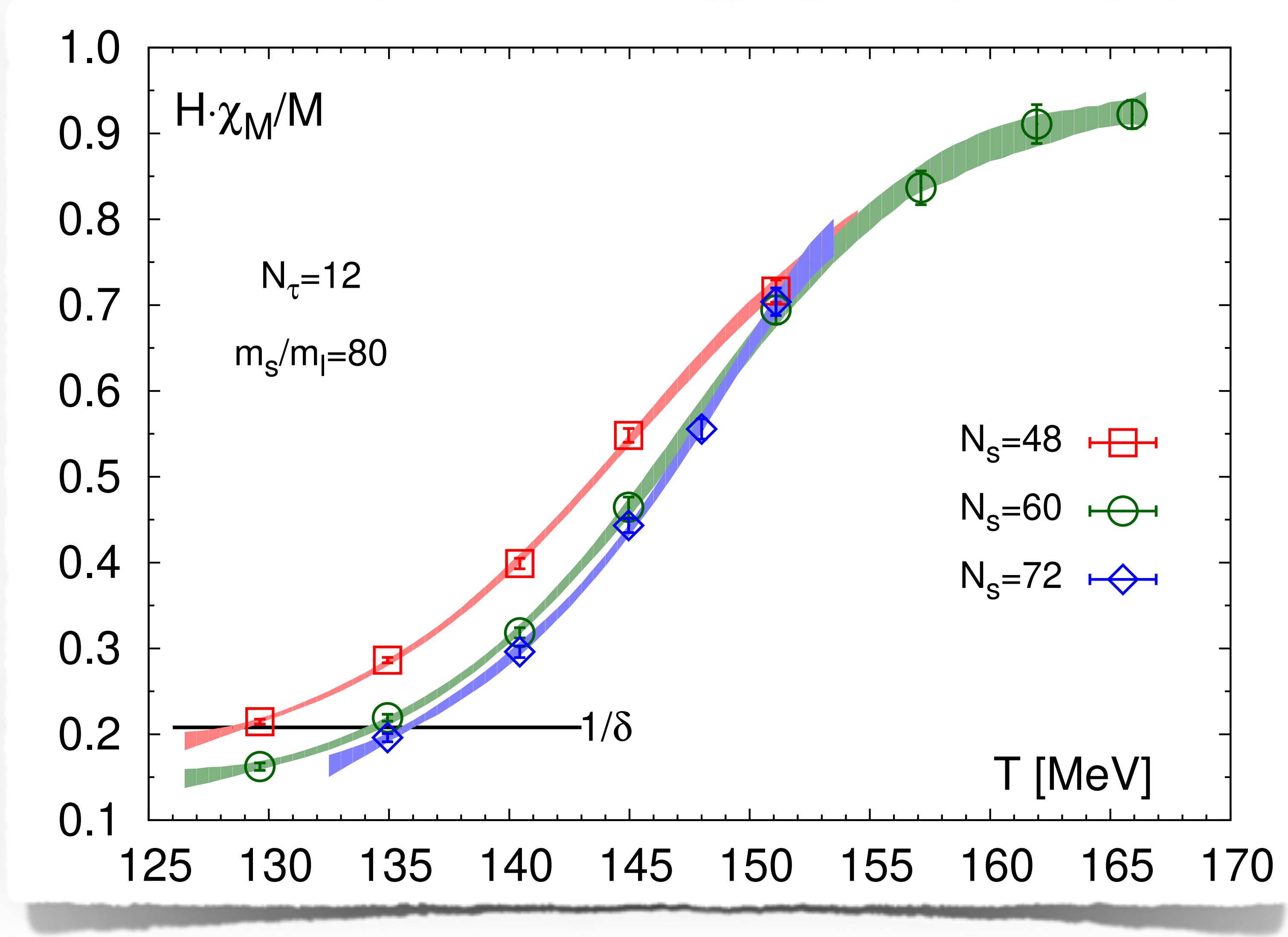
$$M = -\partial f_s(t,h)/\partial H = h^{1/\delta} f_G(z)$$

$$\chi_M = \partial M/\partial H = \frac{h^{1/\delta}}{H}\frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta}\frac{df_G(z)}{dz}\right)$$

z=0 at $T=T_c^0$

$H\chi_M/M \to 1/\delta$ @ T_c^0

H: m_l/m_s
M: chiral condensate
χ_M: chiral susceptibility



A novel approach to estimate T_c^0

- Pseudo-critical temperature at H

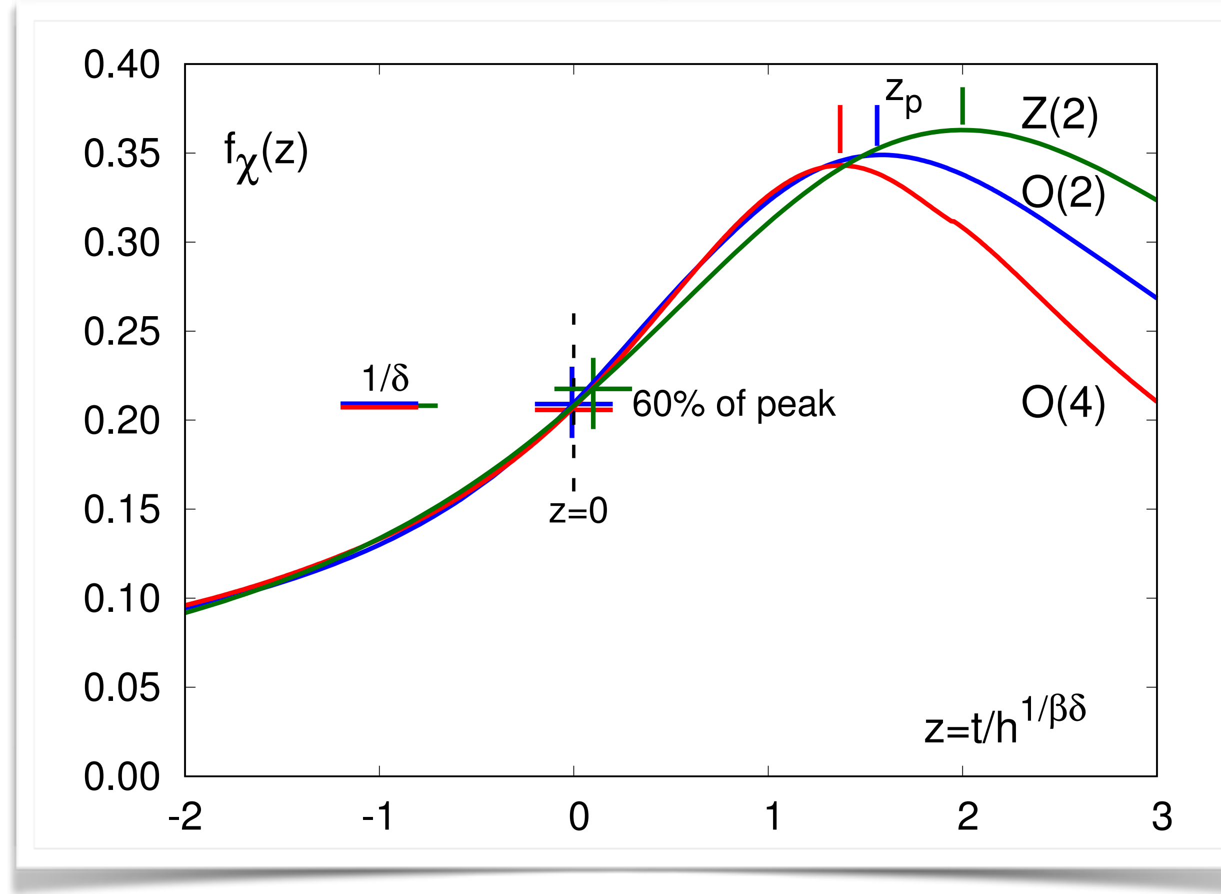
$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}}\right)$$

$$z = \frac{1}{t_0}\frac{T - T_c^0}{T_c^0}\left(\frac{H}{h_0}\right)^{-1/\beta\delta} = z_0 \frac{T - T_c^0}{T_c^0} H^{-1/\beta\delta}$$

- Estimate of the chiral transition T_c^0

$$\frac{H\chi_M(T_\delta, H, L)}{M(T_\delta, H, L)} = \frac{1}{\delta} \quad \Longleftrightarrow \quad z(T_\delta)=0$$

$$\chi_M(T_{60}, H) = 0.6\chi_M^{max} \quad \Longleftrightarrow \quad z(T_{60})\simeq 0$$



z_p: peak location of the susceptibility

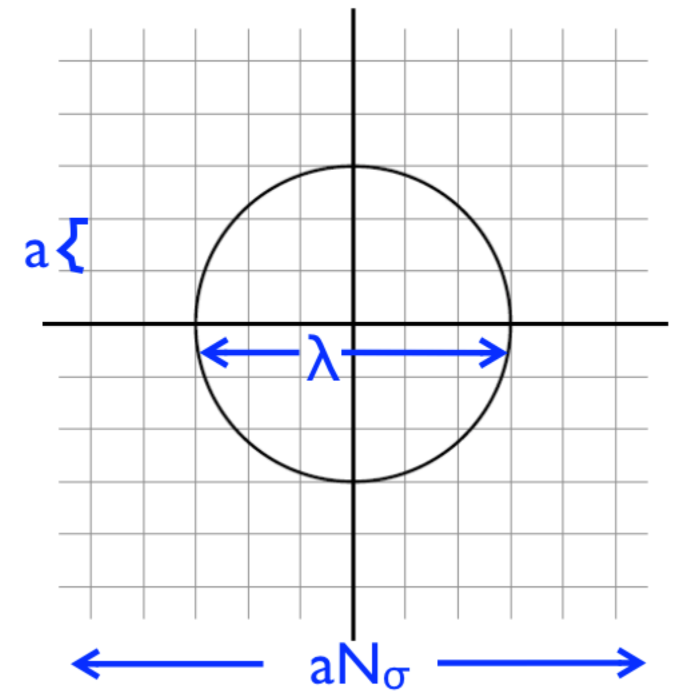
z_{60}: location of 60% of peak height from left

	δ	z_p	z_{60}^-
Z(2)	4.805	2.00(5)	0.10(1)
O(2)	4.780	1.58(4)	-0.005(9)
O(4)	4.824	1.37(3)	-0.013(7)

- ☑ small quark mass dependence
- ☑ small variations among universality classes

Things need to be taken care of

- Thermodynamic limit
- Continuum limit
- Chiral limit



$$T_X(H,L) = T_c^0 \left(1 + \left(\frac{z_X(z_L)}{z_0}\right) H^{1/\beta\delta}\right) + c_X H^{1-1/\delta+1/\beta\delta}$$

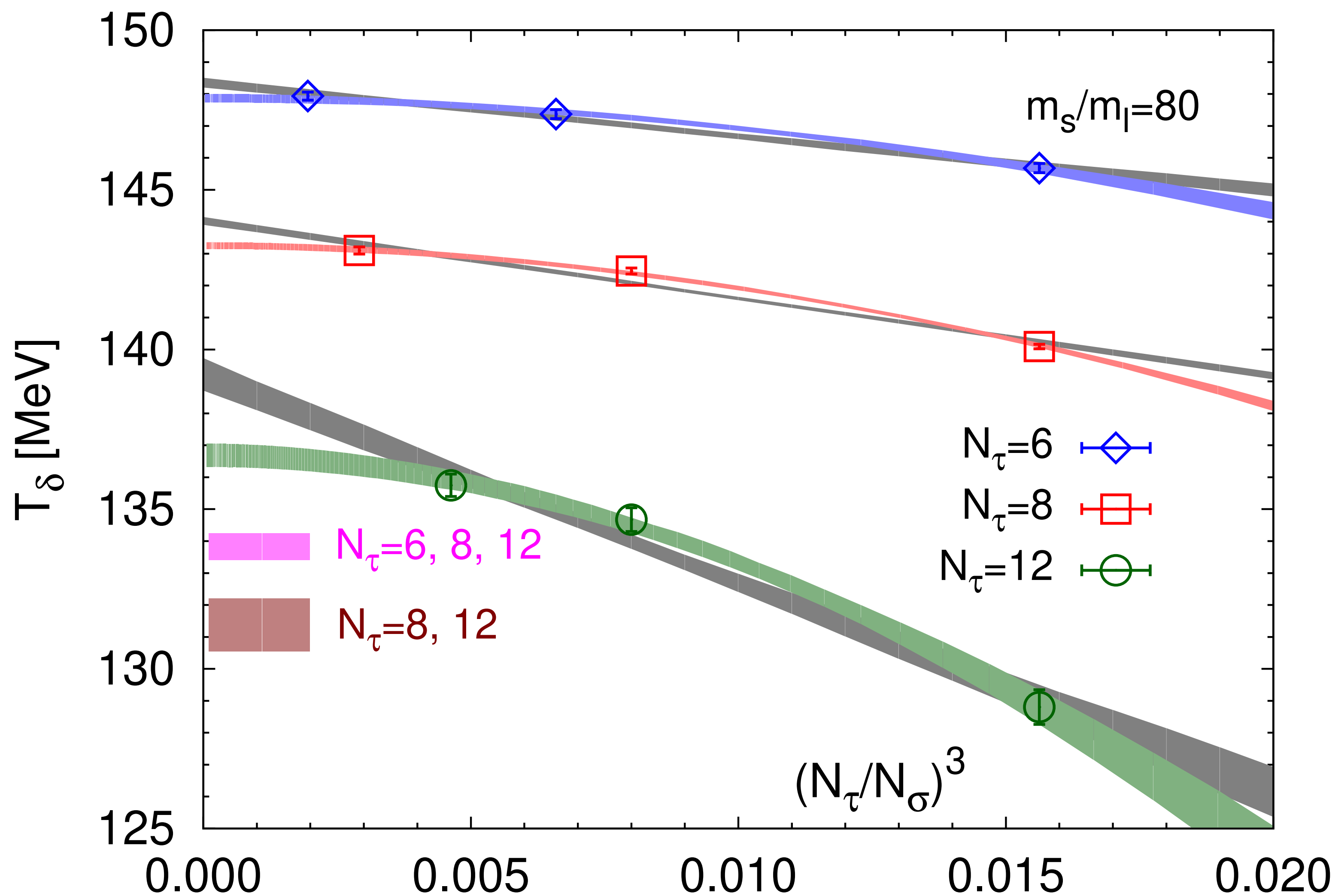
Singular

Regular

X=60,δ

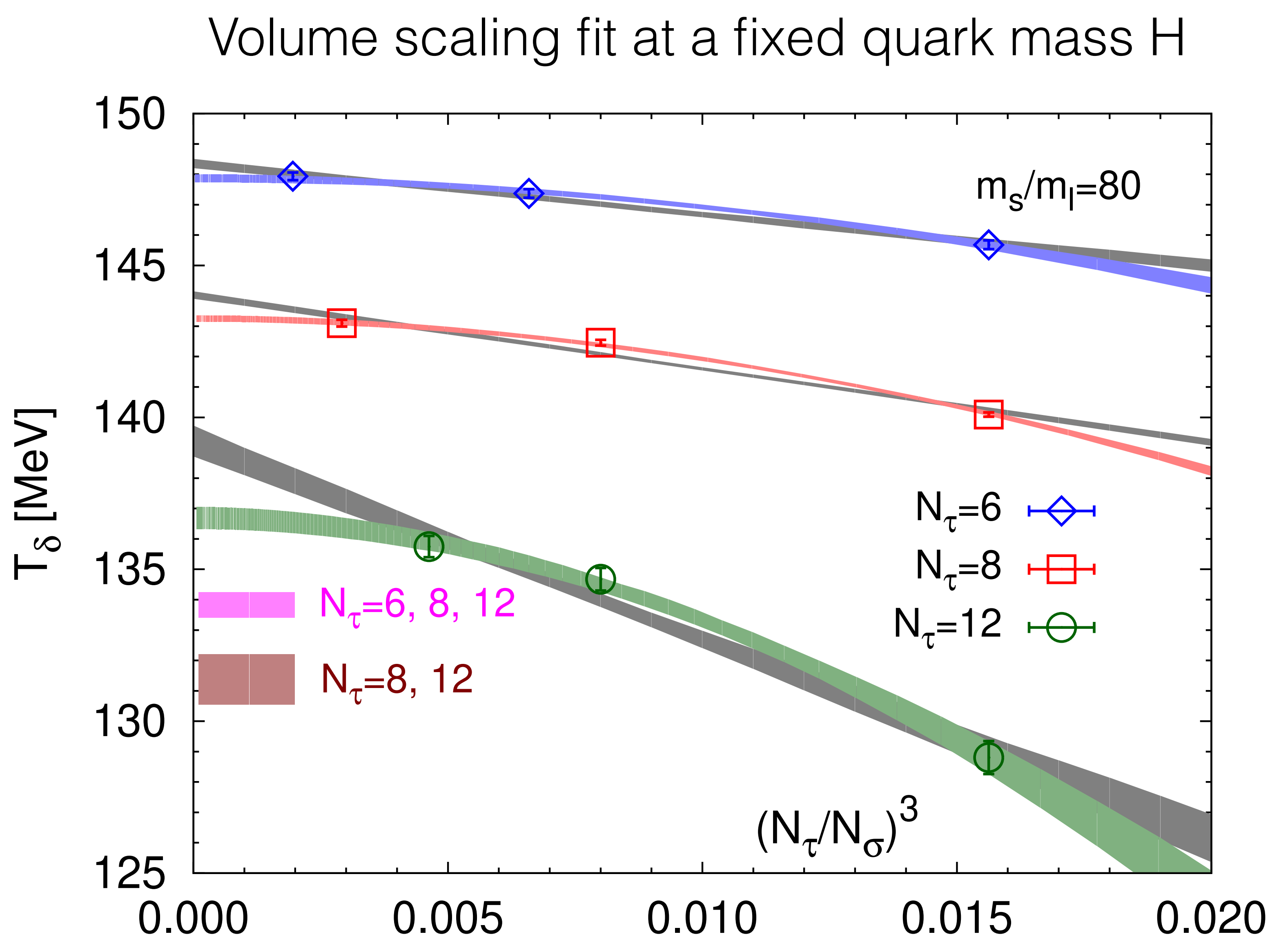
T_δ: Infinite V limit → continuum limit → chiral limit

Volume scaling fit at a fixed quark mass H



	T_δ(H,V,a) [MeV] with Nt=6,8&12	T_δ(H,V,a) [MeV] with Nt= 8&12
V→∞, a→0, H=1/80	133.8(4)	131.4(8)

T_δ: Infinite V limit → continuum limit → chiral limit



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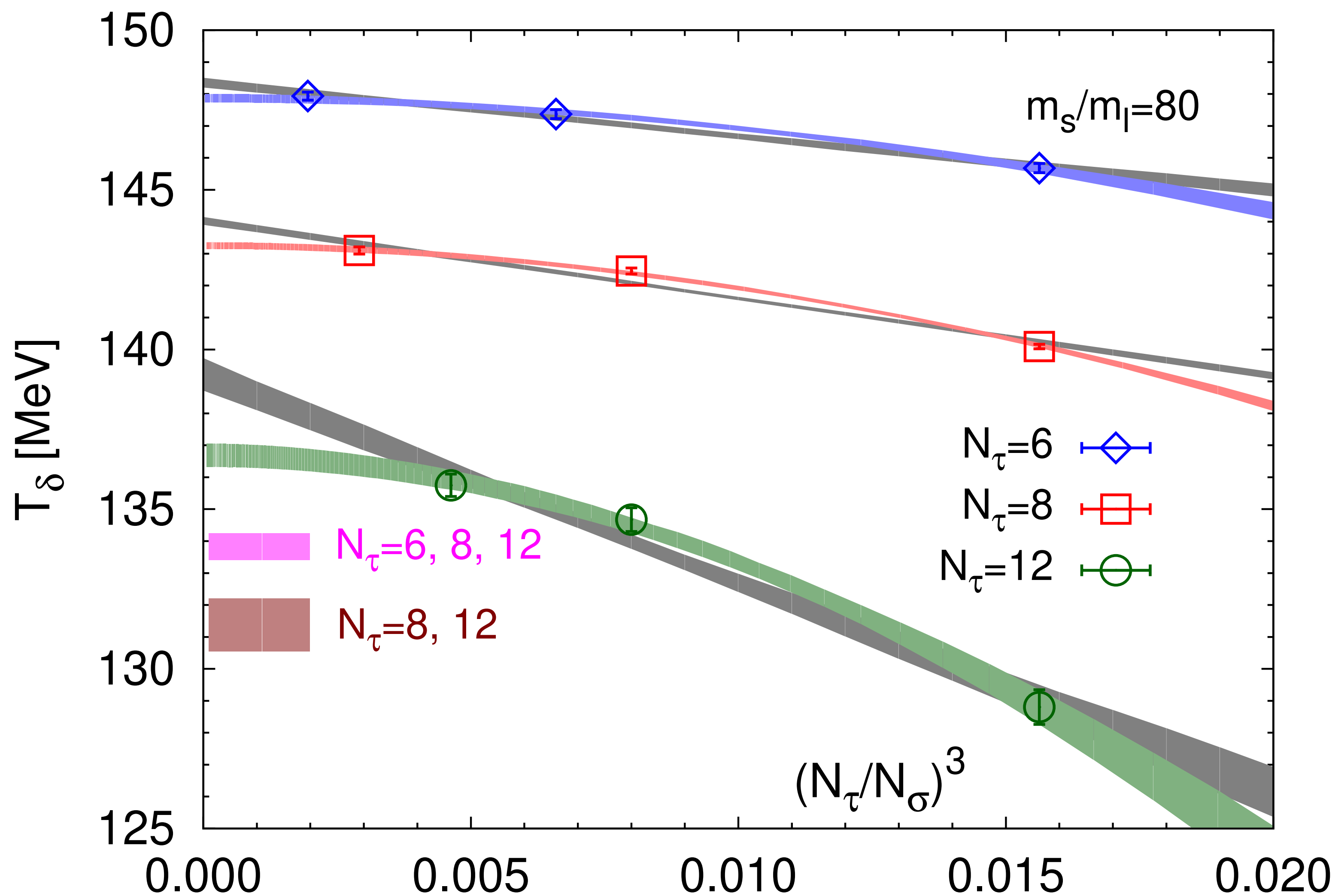
V→∞, a→0, H=1/80	133.8(4)	131.4(8)
V→∞, a→0, H=1/40	136.9(5)	135.5(8)

T_δ: Infinite V limit → continuum limit → chiral limit

Volume scaling fit at a fixed quark mass H



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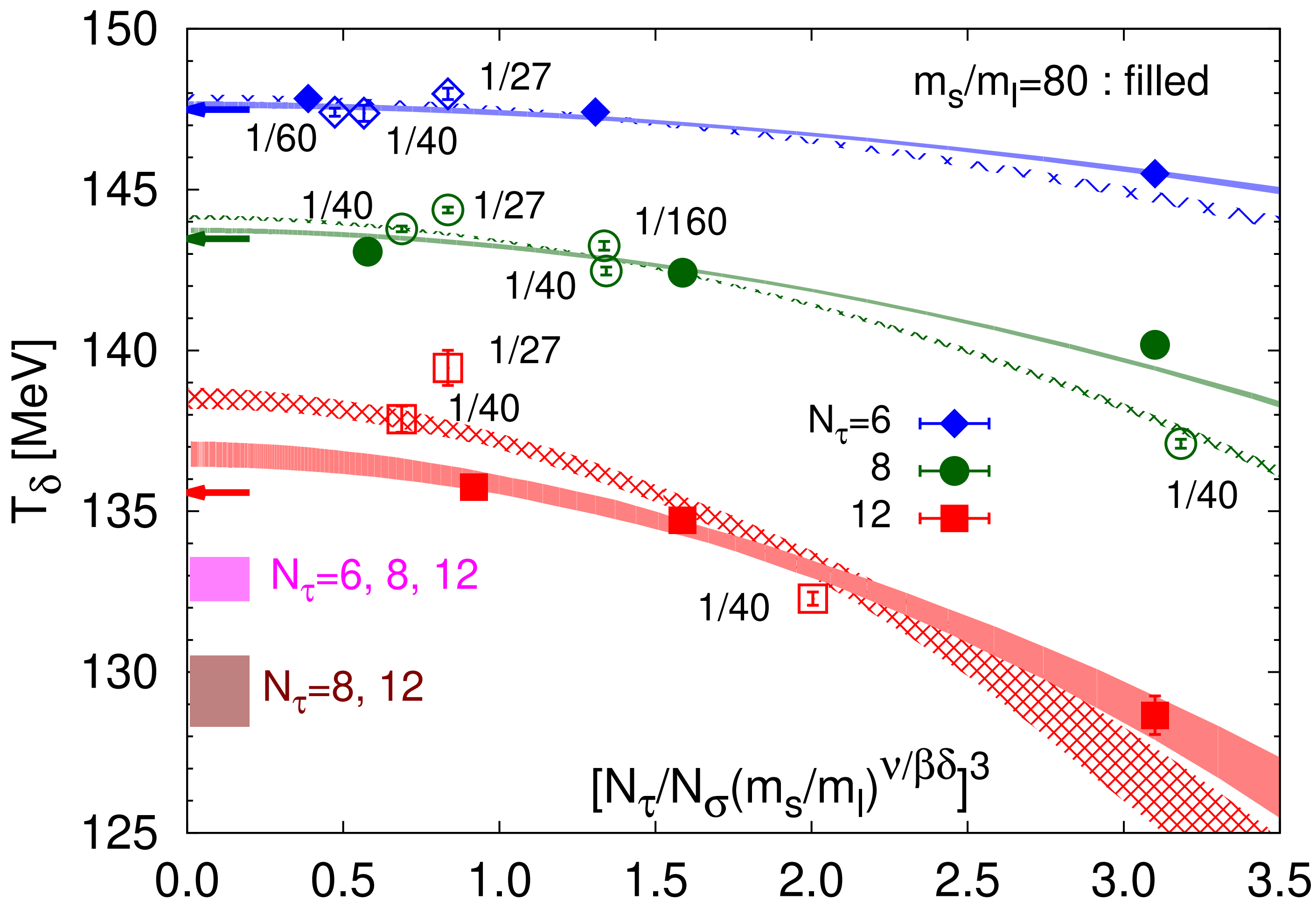
V→∞, a→0, H=1/80	133.8(4)	131.4(8)
V→∞, a→0, H=1/40	136.9(5)	135.5(8)
V→∞, a→0, H→0	132.8(1.4)	130.6(2.4)

T_δ: Infinite V limit → chiral limit → continuum limit

Joint volume scaling fit with all quark masses

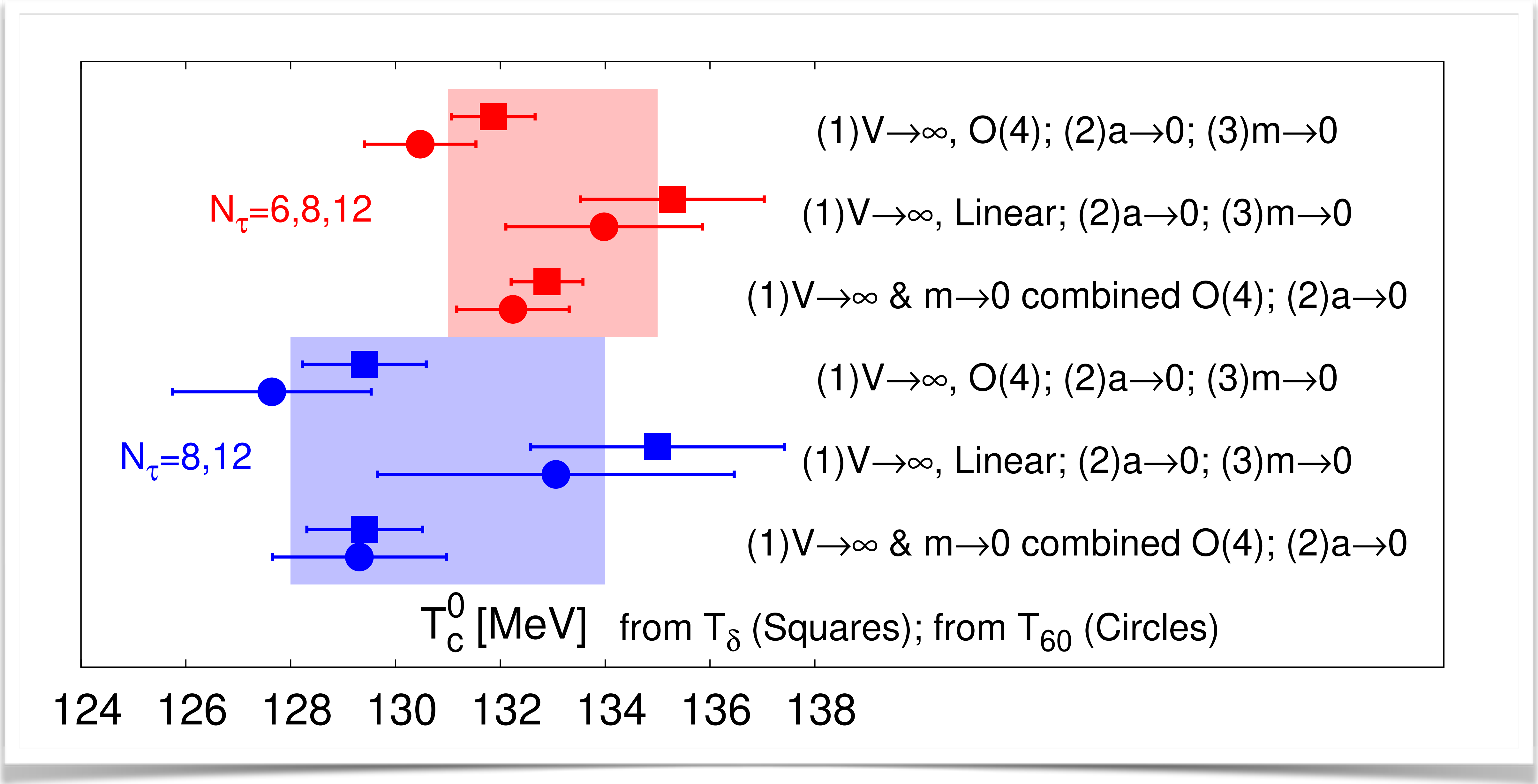


$T_\delta(H,V,a)$ [MeV]

	Nt=6,8&12	Nt=8&12
V→∞, H→0, a→0	132.9(6)	128.6(1.1)
V→∞, a→0, H→0	132.8(1.4)	130.6(2.4)

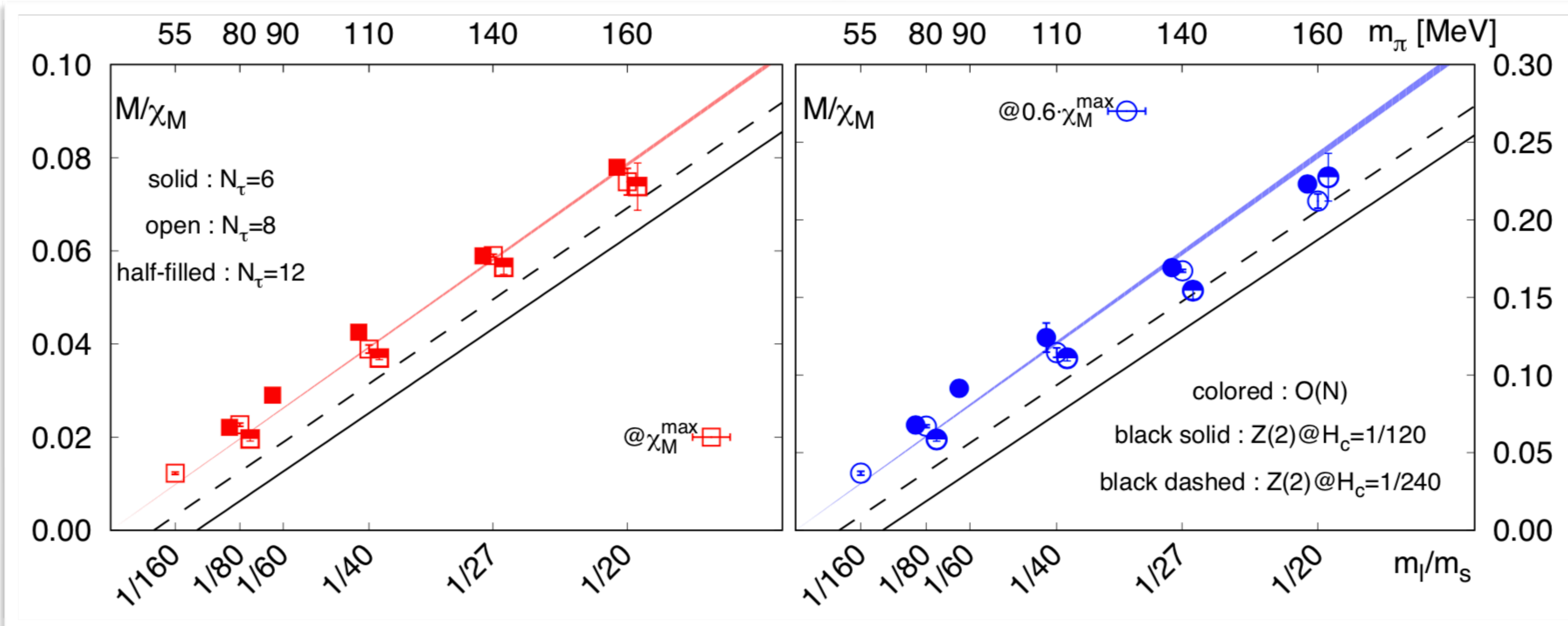
Chiral and continuum limits are Interchangeable

$$T_c^0 = 132^{+3}_{-6}\ \mathrm{MeV}$$



- T_{60} and T_δ give consistent results
- About 25 MeV lower than T_{pc} at the physical point!

Consistency of QCD chiral phase transition with O(N) universality class



S.-T. Li(李胜泰), Lattice 2018, A. Lahiri, QM 2018

$$M/\chi_M = \frac{m_l - m_l^{critical}}{m_s^{phys}} \frac{f_M}{f_{\chi_M}}$$

Conclusions

- ☑ chiral crossover temperature is determined with better precision, i.e. T_{pc} = 156.5(1.5) MeV

- ☑ The chiral Tc of Nf=2+1 QCD is 132^{+3}_{-6} MeV, and the O(N) universality class of the chiral phase transition is favored

谢谢!

Thanks for your attention!

Congratulations to Prof. Che-Ming Ko
for his
50 years fruitful scientific career!

Lattice 2019

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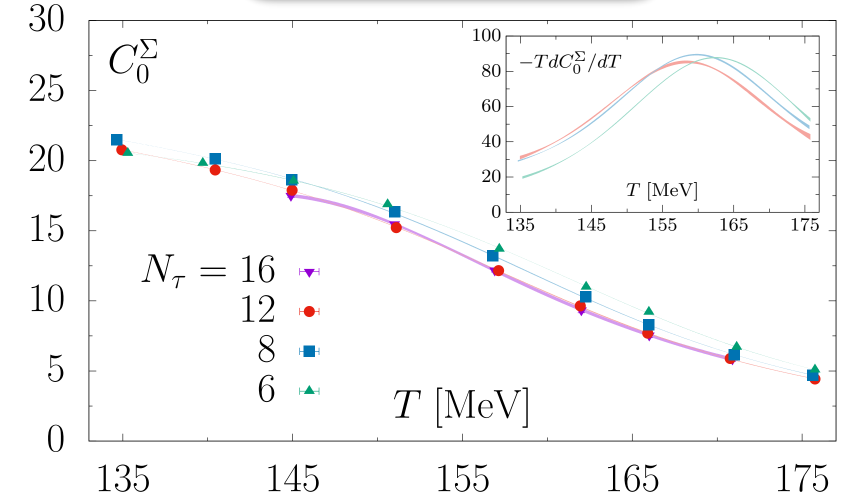
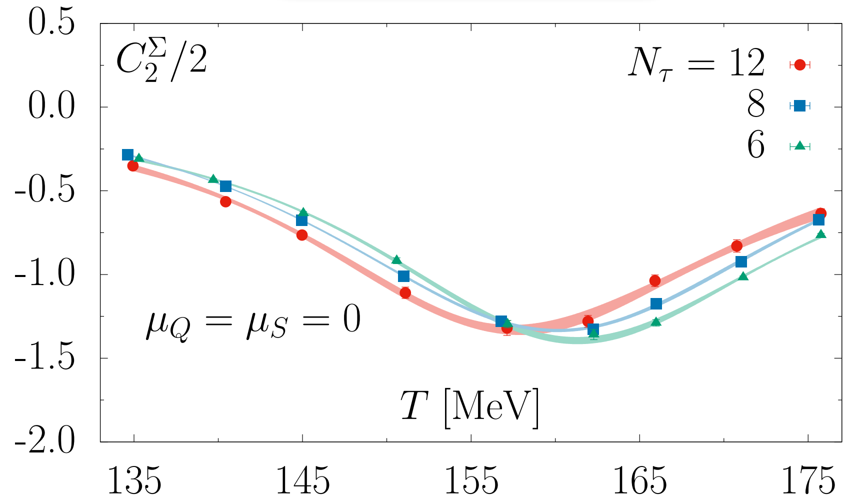
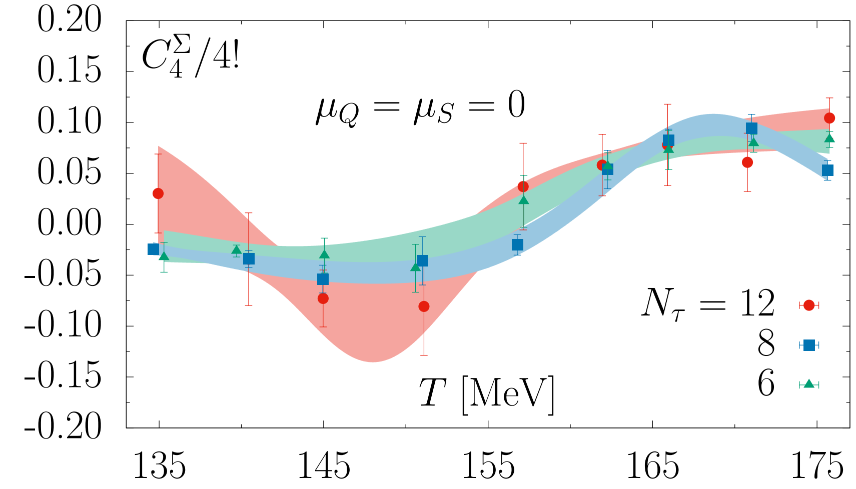
$$\partial_T^2 C_0^\Sigma(T) = 0$$

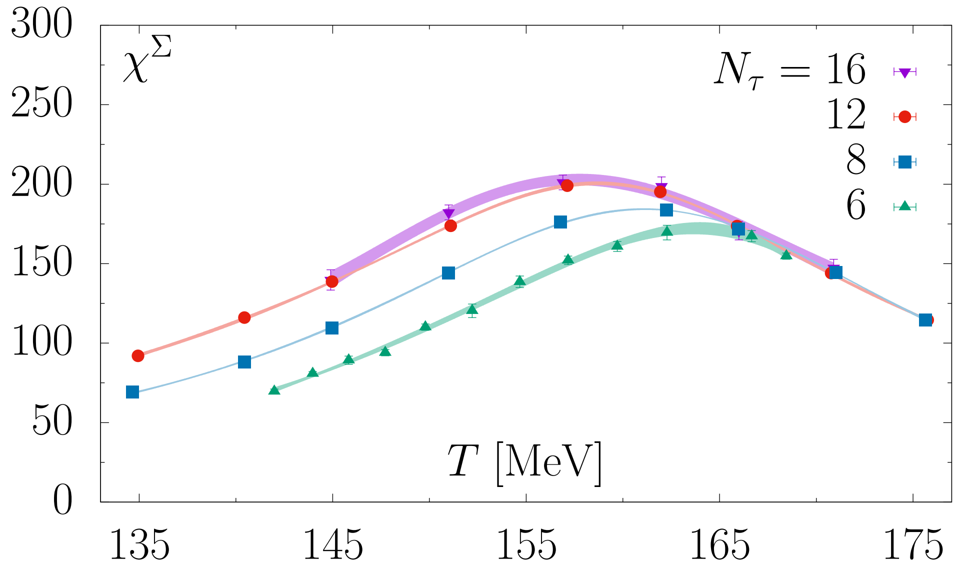


$$\partial_T C_2^\Sigma(T) = 0$$



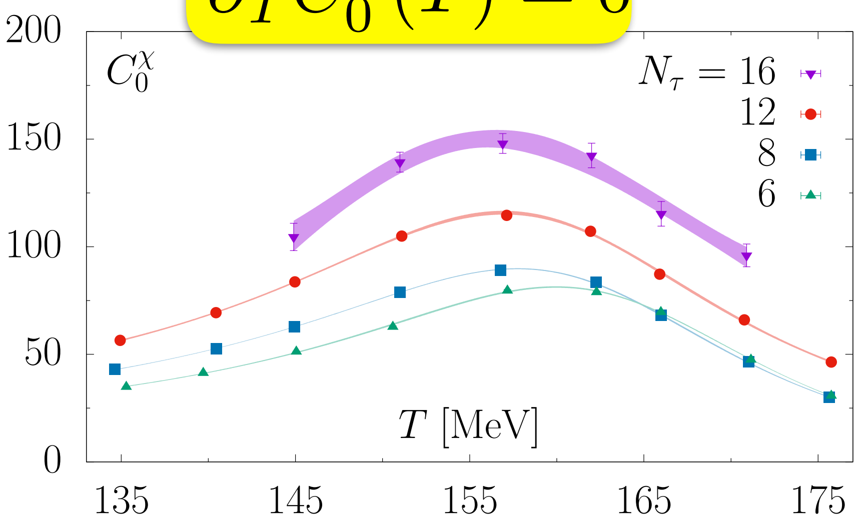
$$C_4^\Sigma(T = T_{pc}) = 0$$



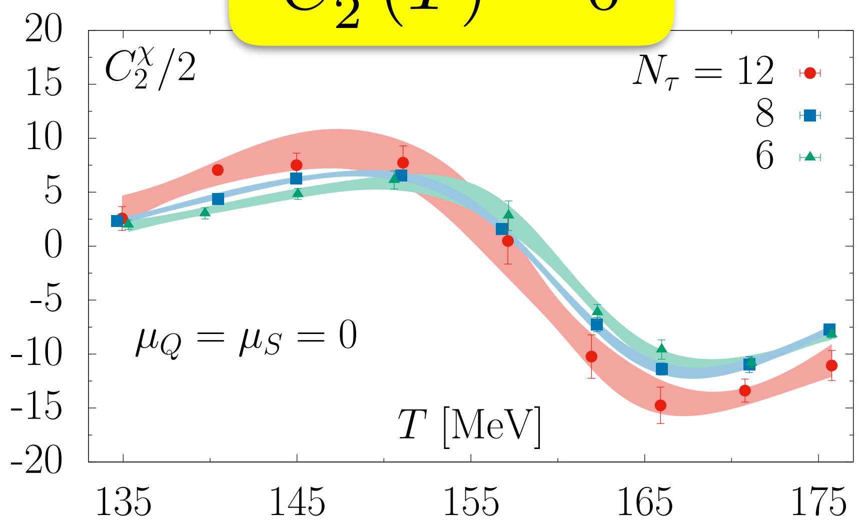


$$\partial_T \chi^\Sigma(T) = 0$$

$$\partial_T C_0^\chi(T) = 0$$



$$C_2^\chi(T) = 0$$



$$\partial_T C_4^\chi(T = T_{pc}) = 0$$