

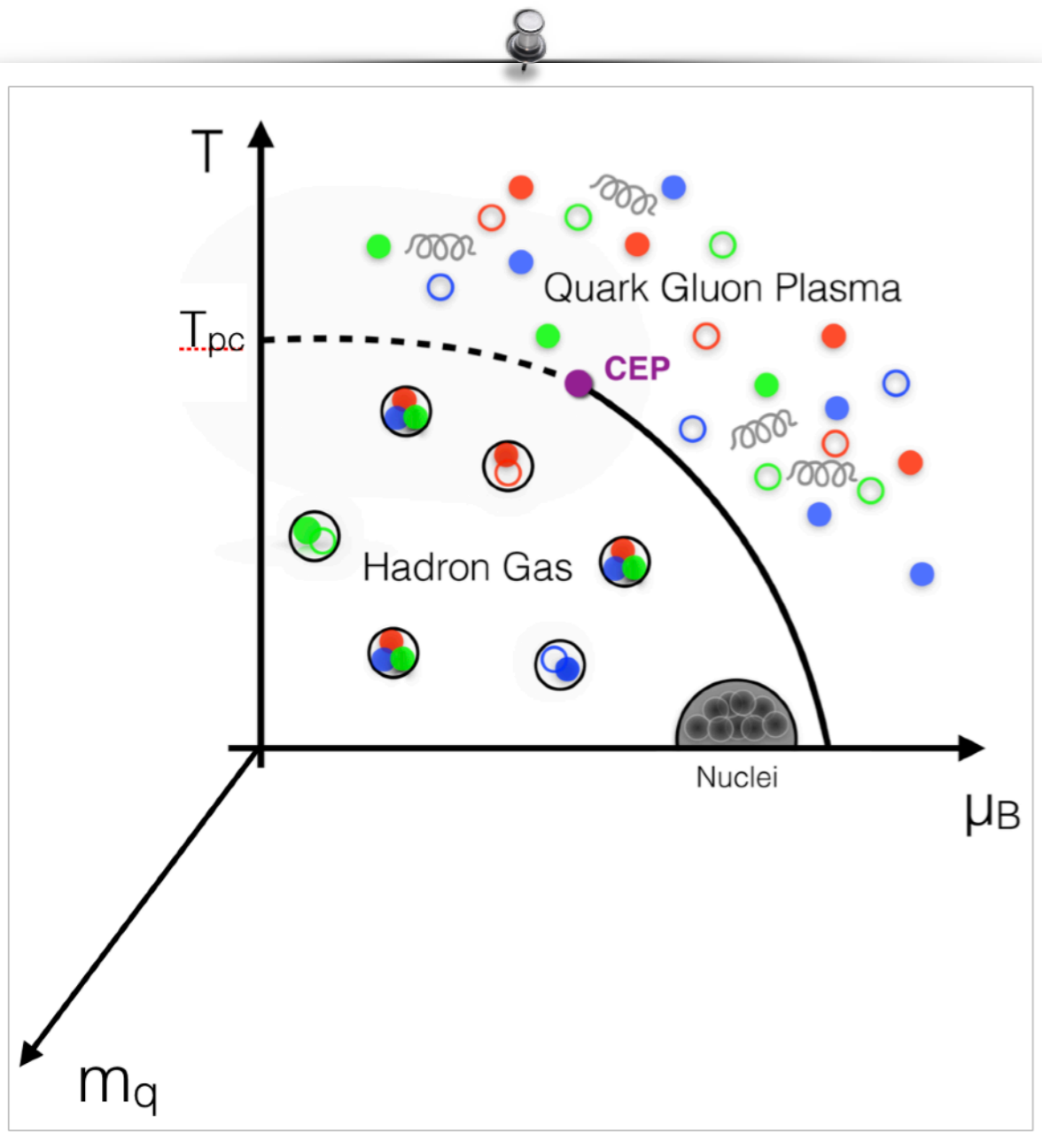
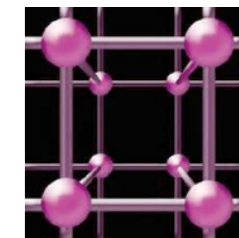
Chiral crossover and phase transition in (2+1)-flavor QCD

Heng-Tong Ding (丁亨通)

Central China Normal University

International Workshop on Partonic and Hadronic Transport Approaches for
Relativistic Heavy Ion Collisions

11-12 May, 2019@Dalian



📌 Chiral crossover at zero and small μ_B

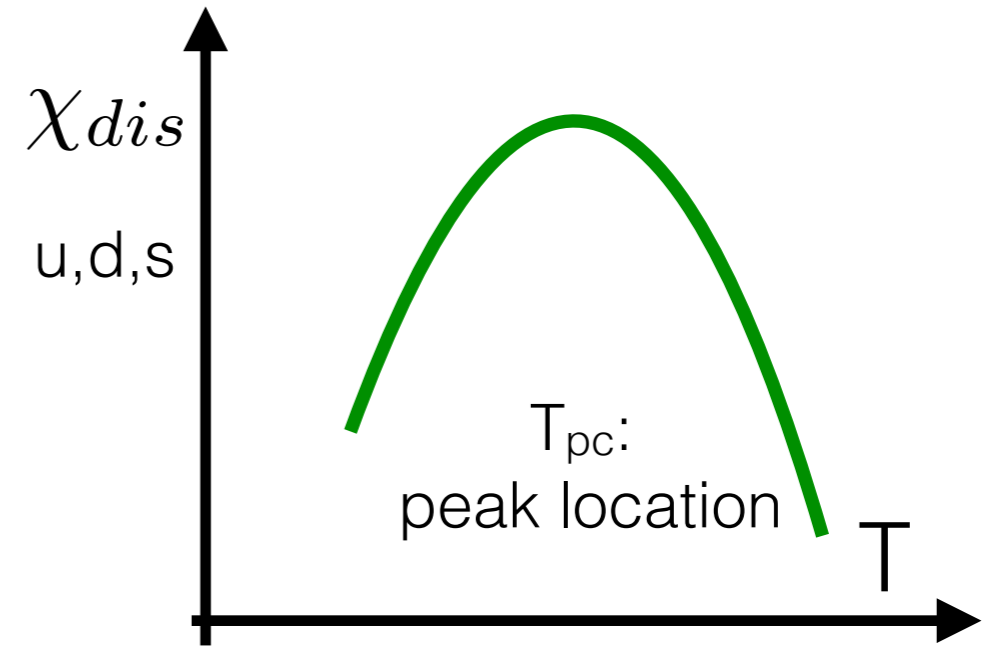
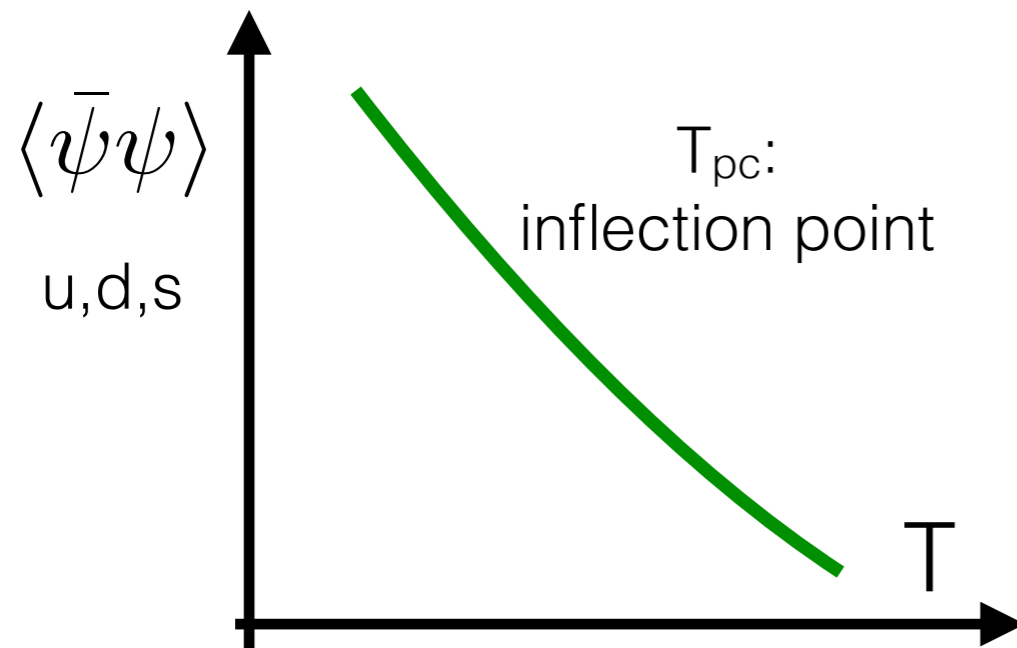
A. Bazavov, HTD, P. Hegde et al.
[HotQCD], arXiv:1812.08235

📌 Chiral phase transition temperature

HTD, P. Hegde, O. Kaczmarek et al.
[HotQCD], arXiv:1903.04801

Crossover transition temperature T_{pc} in the real world

📌 Crossover nature of the transition



📌 Chiral phase transition: most likely 2nd order, 3d $O(4)$

Ejiri et al., PRD 80(2009)094505,
HotQCD, arXiv:1903.04801

...

📌 A well-defined **chiral crossover transition temperature**: based on scaling properties of QCD

HTD, P. Hegde, O. Kaczmarek et al.
[HotQCD], arXiv:1903.04801

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T, \mu_B) \sim m^{1/\delta} f_G$

chiral susceptibility: $\chi^\Sigma(T, \mu_B) \sim m^{1/\delta-1} f_\chi$

m: quark mass
f: scaling functions
 δ, β : critical exponents

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T, \mu_B) \sim m^{1/\delta} f_G$

chiral susceptibility: $\chi^\Sigma(T, \mu_B) \sim m^{1/\delta-1} f_\chi$

m: quark mass
f: scaling functions
 δ, β : critical exponents

Taylor expansions:

$$\Sigma(T, \mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^\Sigma(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n} \quad \chi(T, \mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^\chi(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n}$$

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T, \mu_B) \sim m^{1/\delta} f_G$

chiral susceptibility: $\chi^\Sigma(T, \mu_B) \sim m^{1/\delta-1} f_\chi$

m: quark mass
f: scaling functions
 δ, β : critical exponents

Taylor expansions:

$$\Sigma(T, \mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^\Sigma(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n} \quad \chi(T, \mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^\chi(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n}$$

$$\begin{matrix} \partial_T \chi^\Sigma(T) \\ \partial_T C_0^\chi(T) \\ C_2^\chi(T) \end{matrix} \sim m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$$

$$\begin{matrix} \partial_T^2 C_0^\Sigma(T) \\ \partial_T C_2^\Sigma(T) \end{matrix} \sim m^{1/\delta-2/\beta\delta} f''_G(z)$$

Scaling behavior of chiral observables

chiral condensate: $\Sigma(T, \mu_B) \sim m^{1/\delta} f_G$

chiral susceptibility: $\chi^\Sigma(T, \mu_B) \sim m^{1/\delta-1} f_\chi$

m: quark mass
f: scaling functions
 δ, β : critical exponents

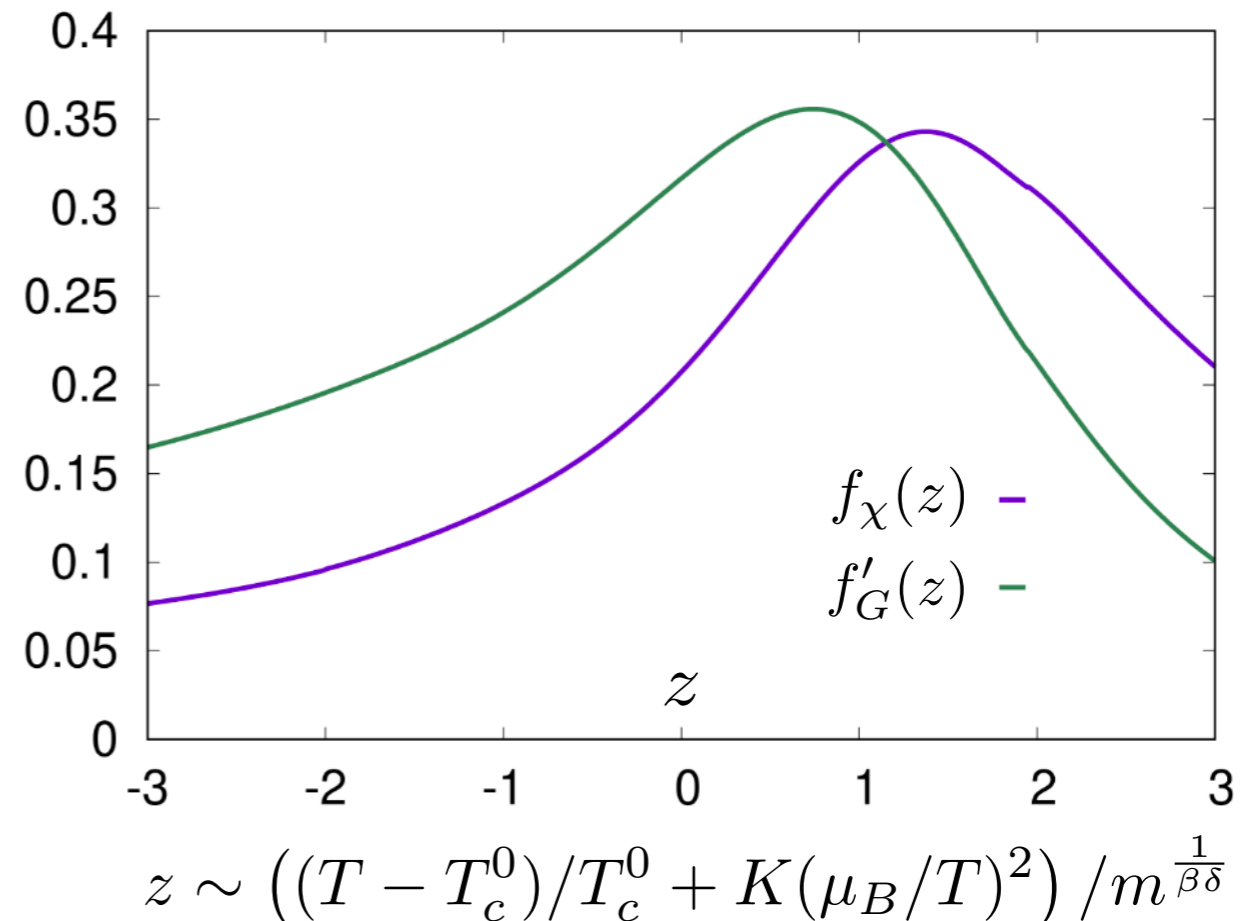
Taylor expansions:

$$\Sigma(T, \mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^\Sigma(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n}$$

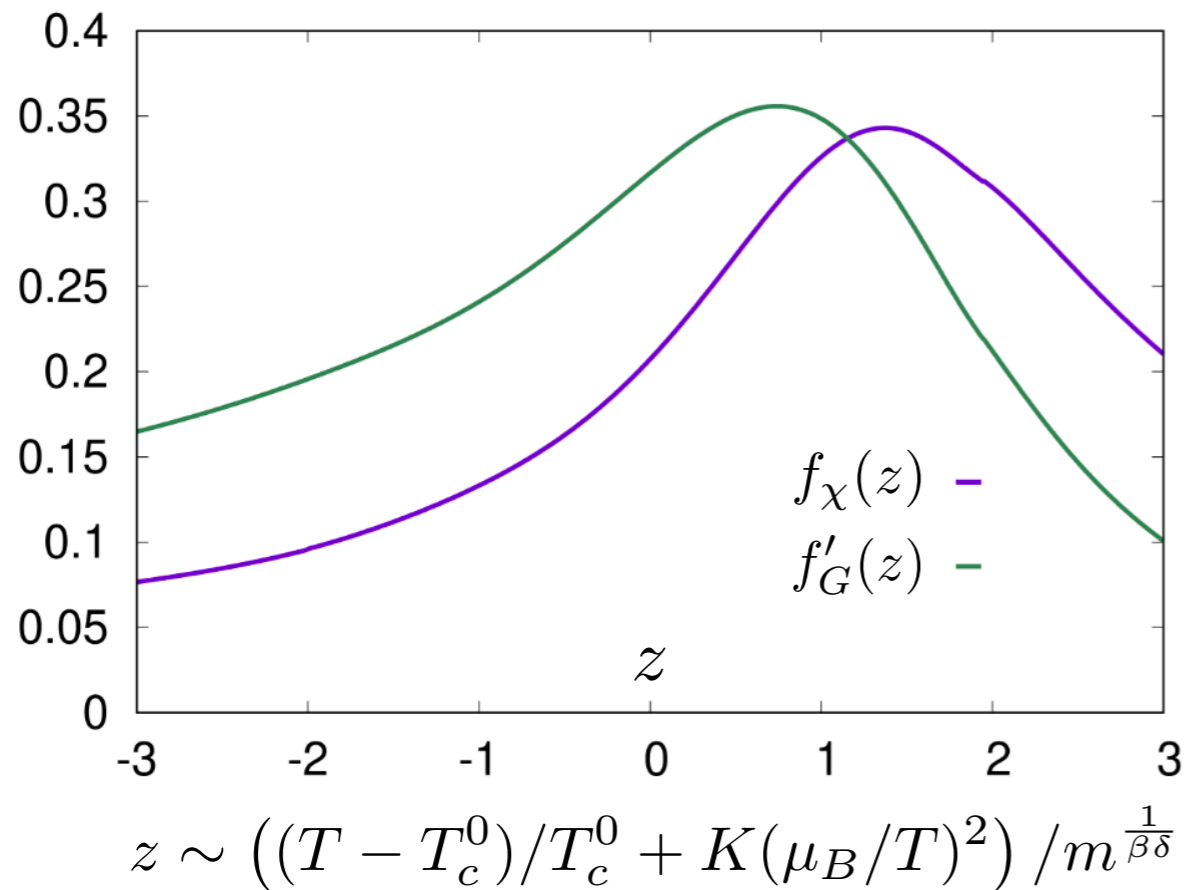
$$\chi(T, \mu_X) = \sum_{n=0}^{\infty} \frac{C_{2n}^\chi(T)}{(2n)!} \left(\frac{\mu_X}{T}\right)^{2n}$$

$$\begin{pmatrix} \partial_T \chi^\Sigma(T) \\ \partial_T C_0^\chi(T) \\ C_2^\chi(T) \end{pmatrix} \sim m^{1/\delta-1-1/\beta\delta} f'_\chi(z)$$

$$\begin{pmatrix} \partial_T^2 C_0^\Sigma(T) \\ \partial_T C_2^\Sigma(T) \end{pmatrix} \sim m^{1/\delta-2/\beta\delta} f''_G(z)$$



Well-defined notation of chiral crossover transition temperature



$$\begin{aligned} \partial_T \chi^\Sigma(T) \\ \partial_T C_0^\chi(T) \\ C_2^\chi(T) \end{aligned} \sim m^{1/\delta - 1 - 1/\beta\delta} f'_\chi(z)$$

$$\begin{aligned} \partial_T^2 C_0^\Sigma(T) \\ \partial_T C_2^\Sigma(T) \end{aligned} \sim m^{1/\delta - 2/\beta\delta} f''_G(z)$$

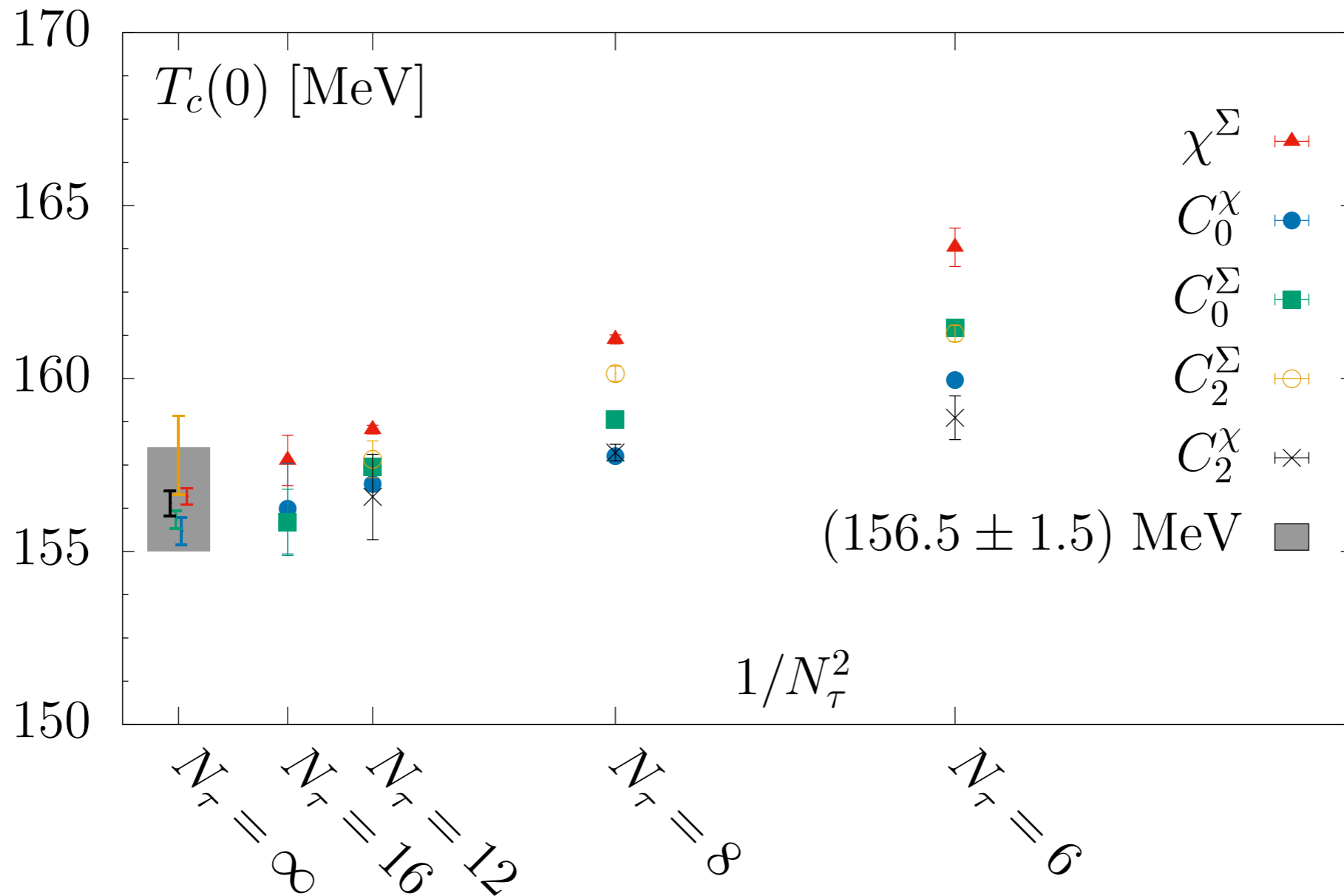
5 conditions to extract T_{pc} : maxima of f_χ and f'_G

$$\partial_T \chi^\Sigma(T) = 0 \quad \partial_T C_0^\chi(T) = 0 \quad C_2^\chi(T) = 0 \quad \partial_T^2 C_0^\Sigma(T) = 0 \quad \partial_T C_2^\Sigma(T) = 0$$

$m=0$: all these susceptibilities diverge at a unique T

$m \neq 0$: non-unique temperatures, crossover

QCD transition with $m_\pi = 140$ MeV at $\mu_B = 0$

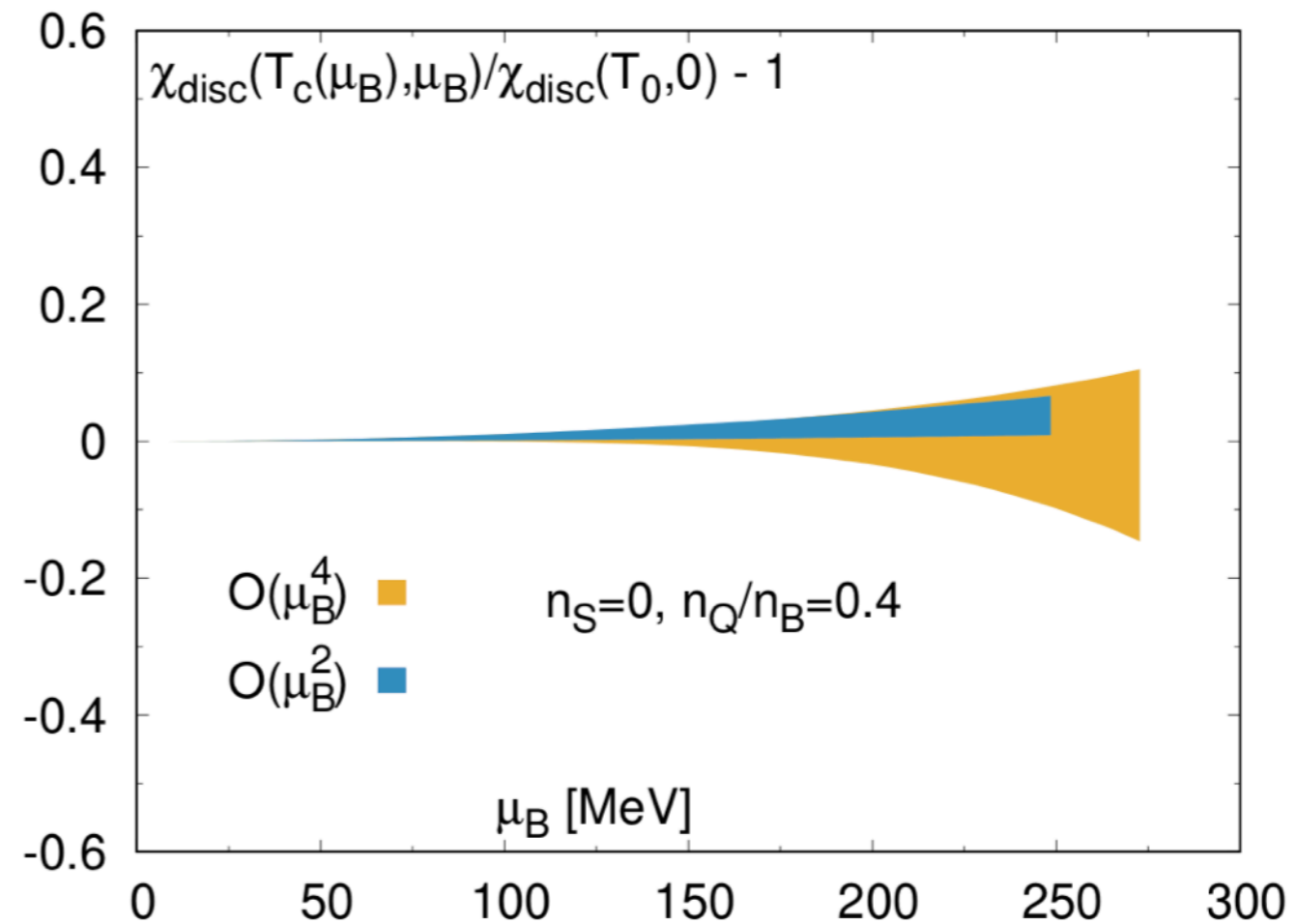
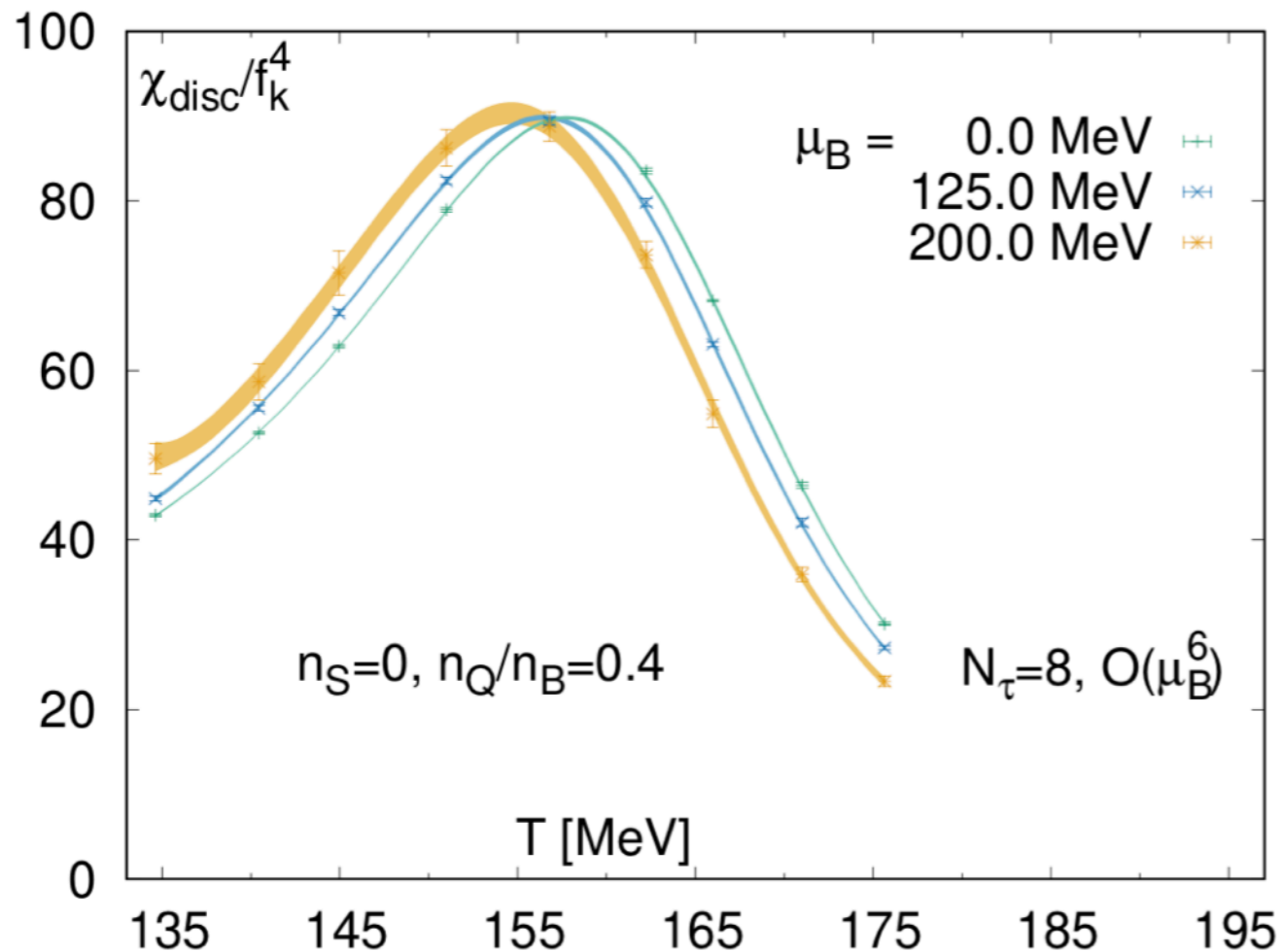


Higher precision in the continuum limit:

$$T_{pc} = 156.5(1.5) \text{ MeV}$$

HotQCD, arXiv:1812.08235

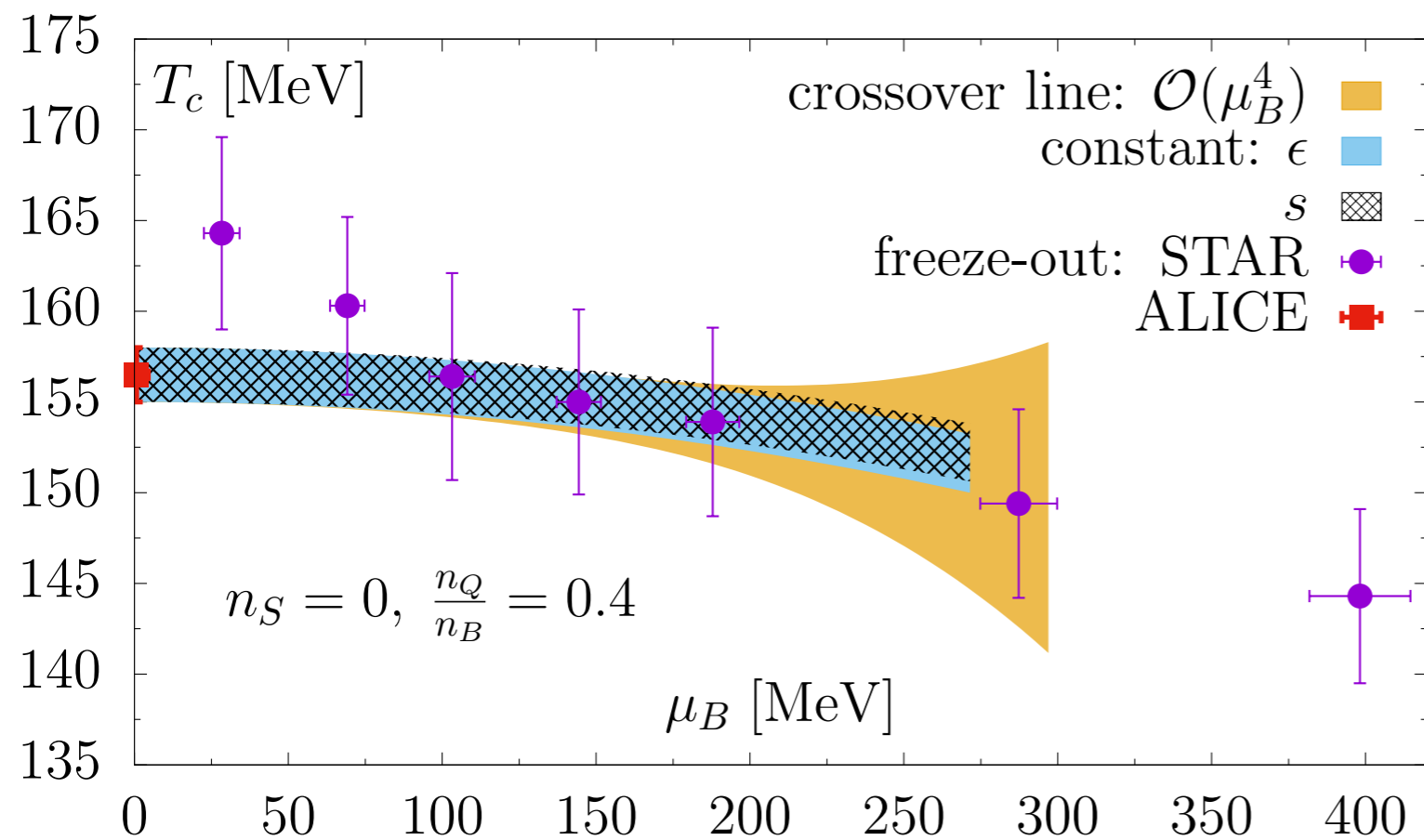
Order Parameter Susceptibility at $\mu_B \neq 0$



No indication of a stronger phase transition at larger μ_B

Crossover, line of constant physics & freeze-out

$$T(\mu_B) = T(0) \left(1 - \kappa_2 \left(\frac{\mu_B}{T} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T} \right)^4 + \mathcal{O} \left(\frac{\mu_B}{T} \right)^6 \right)$$



curvature of crossover line

$$\kappa_2 = 0.0123 \pm 0.003$$

$$\kappa_4 = 0.000131 \pm 0.0041$$

curvature at constant b:

$$0.006 \leq \kappa_2^b \leq 0.012, \quad b = P, \epsilon, s$$

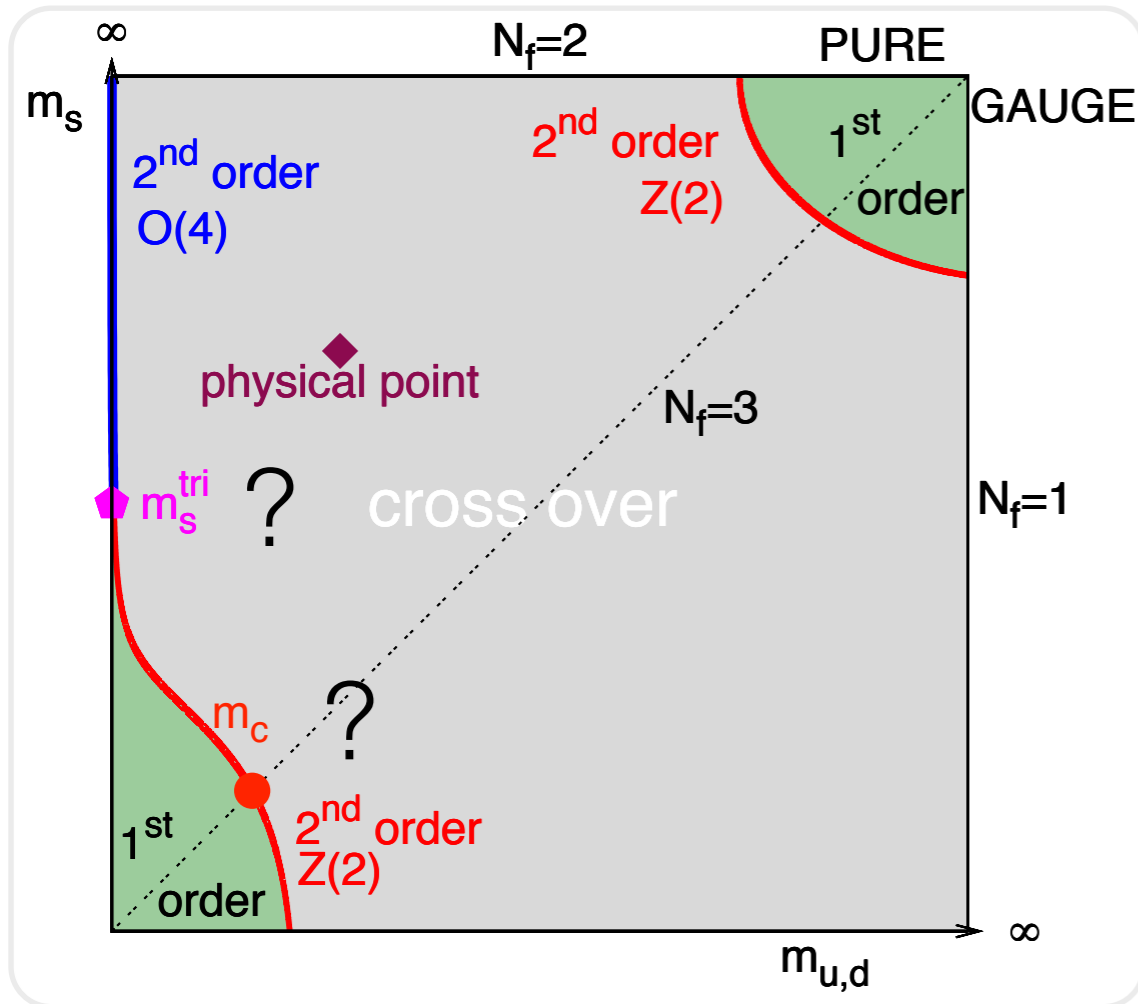
Bielefeld-BNL-CCNU, PRD95 (2017) no.5, 054504

A. Bazaovo, HTD et al. [HotQCD], arXiv:1812.08235

c.f. Wei-Jie Fu's talk on results from FRG

QCD phase diagram in the quark mass plane

Columbia plot:



At physical point: crossover,

$$T_{pc} = 156.5(1.5) \text{ MeV}$$

HotQCD, arXiv:1812.08235

N_f=2(+1): U_A(1) remains broken at T_{χSB}

JLQCD '13,'14,'15, HotQCD '13,'14

Critical lines of second order transition

Pisarski & Wilczek PRD '84

N_f=2: O(4) universality class

Kogut & Sinclair, PRD '06

N_f=3: Ising universality class

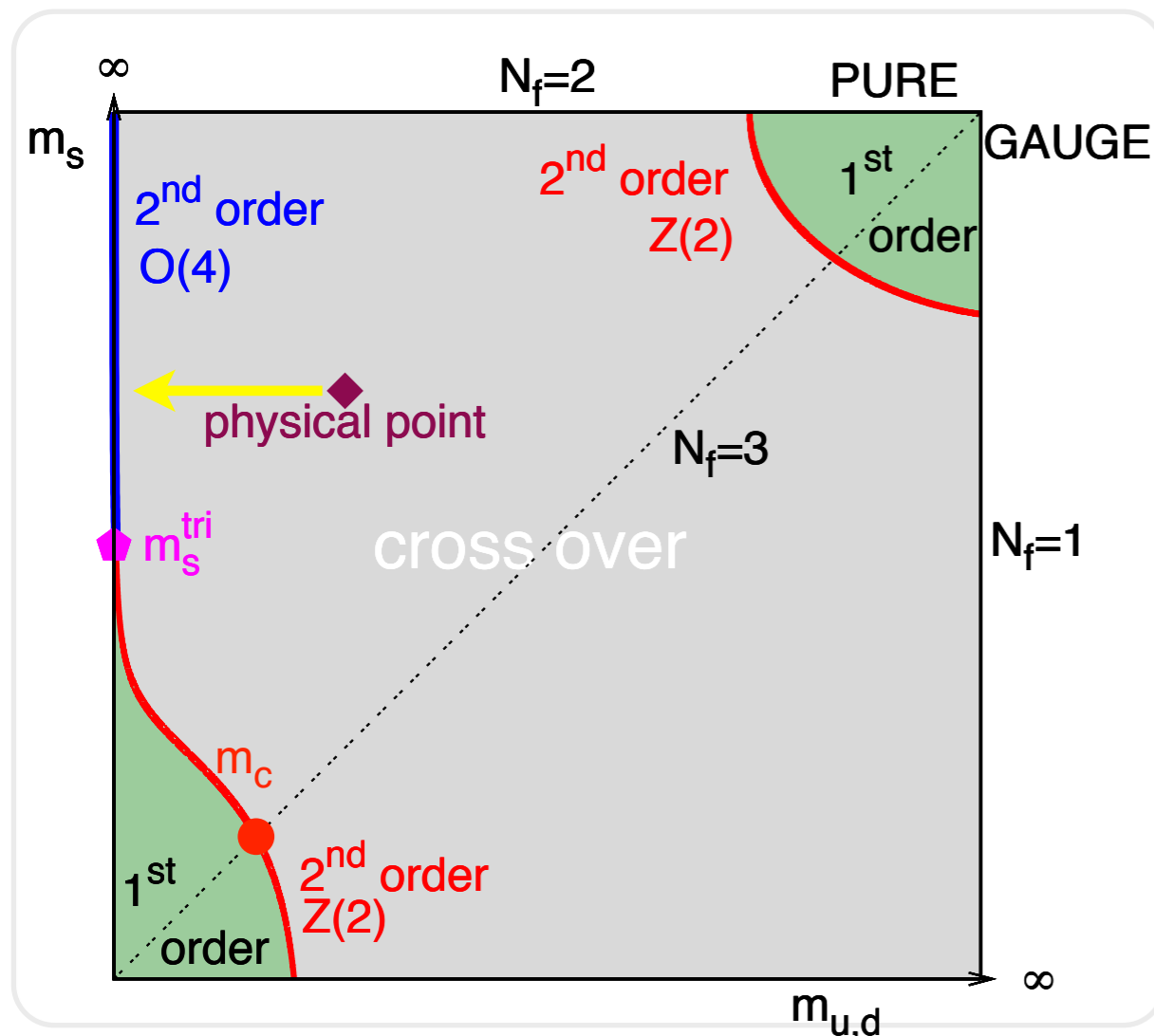
Karsch, Laermann, Schmidt PLB '04,...

Towards the chiral limit:

N_f=2+1 QCD: m_s^{tri} ? m_s^{phy}

Fundamental scale of QCD: chiral T_c ?

Towards chiral limit of (2+1)-flavor QCD



📌 HISQ/tree action

📌 **$N_f=2+1$:**

$$m_u = m_d \rightarrow 0$$

$$m_s = m_s^{\text{phy}}$$

☑ $N_t = 6, 8, 12$

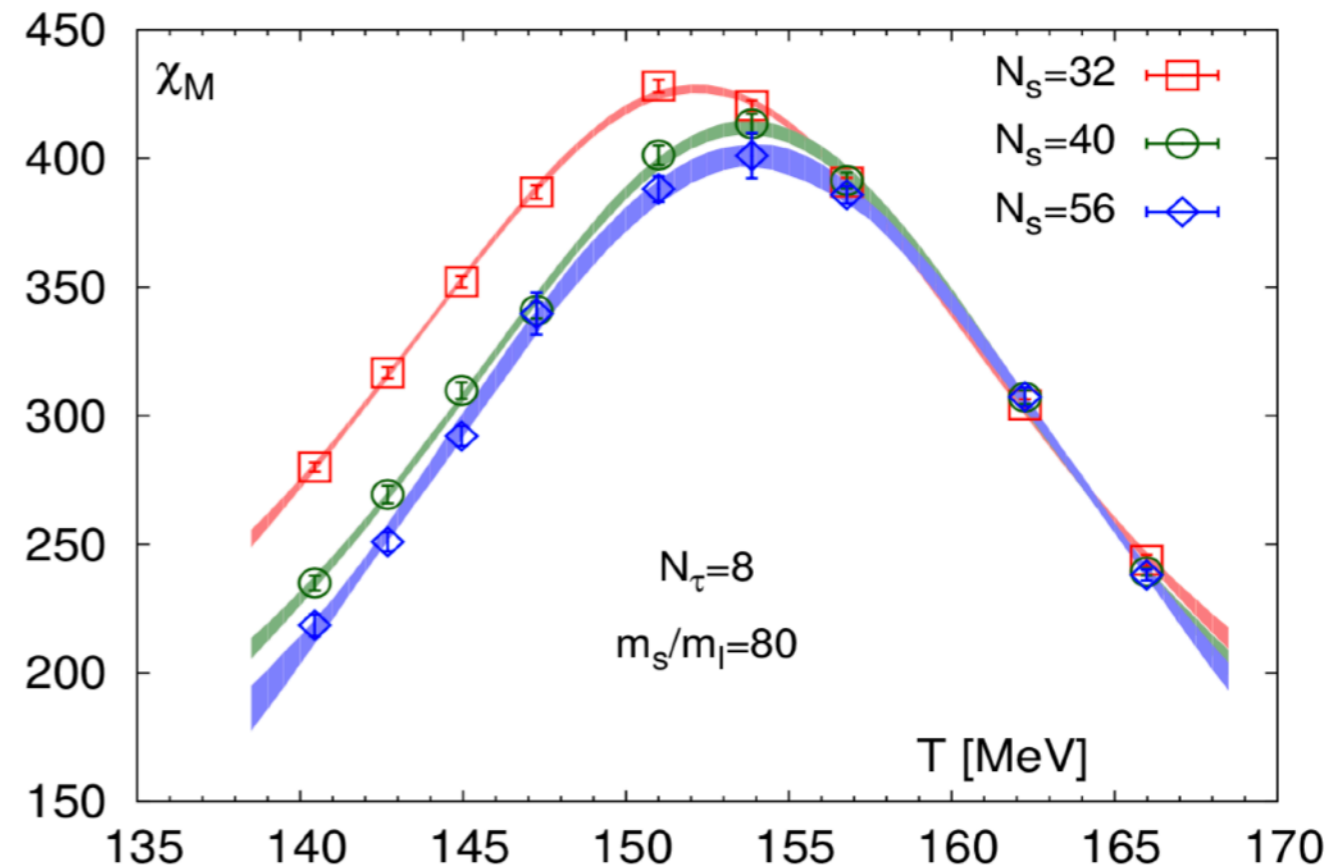
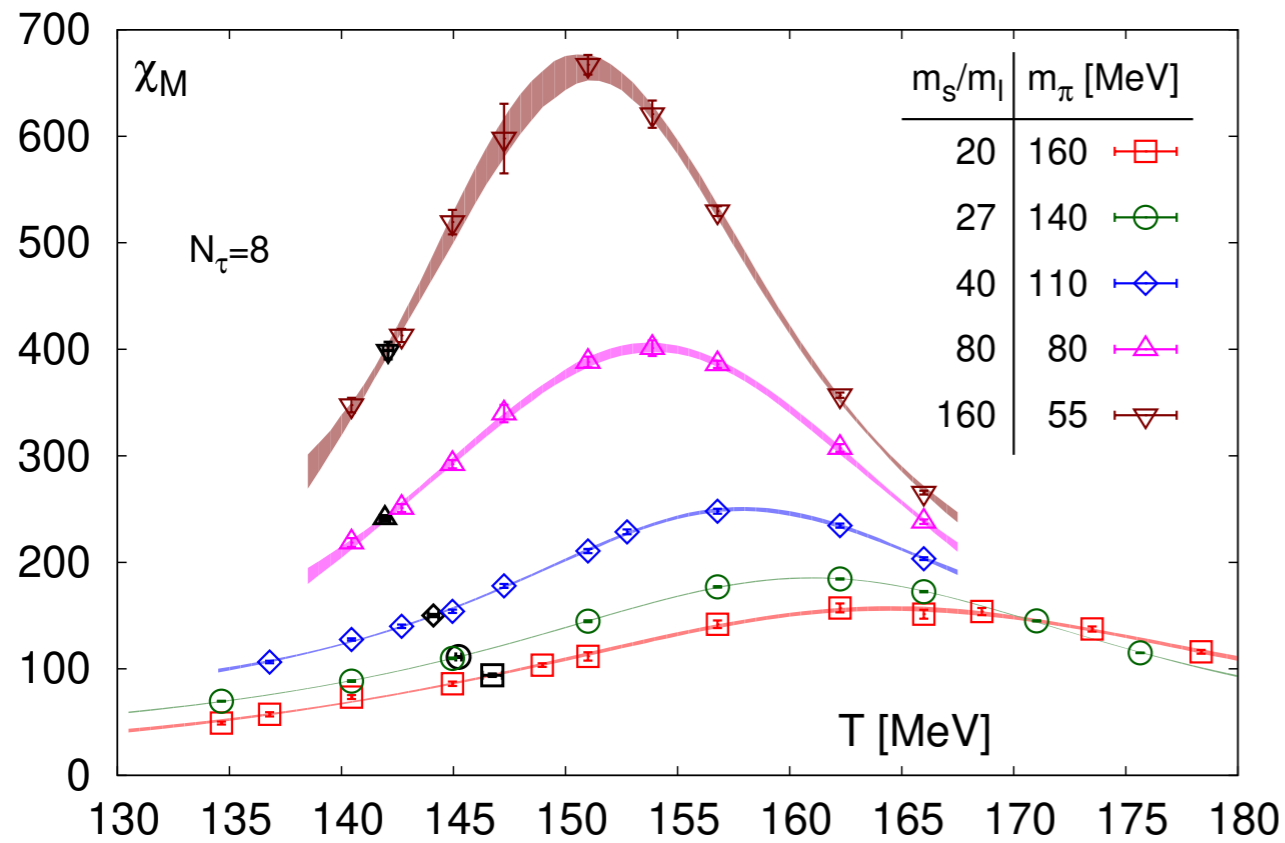
☑ $m_s^{\text{phy}} / m_l = 20, 27, 40, 60, 80, 160$

$m_\pi \approx 160, 140, 110, 90, 80, 55$ MeV

☑ $7 \geq N_s / N_t \geq 4 \Leftrightarrow 5 \gtrsim m_\pi L \gtrsim 3$

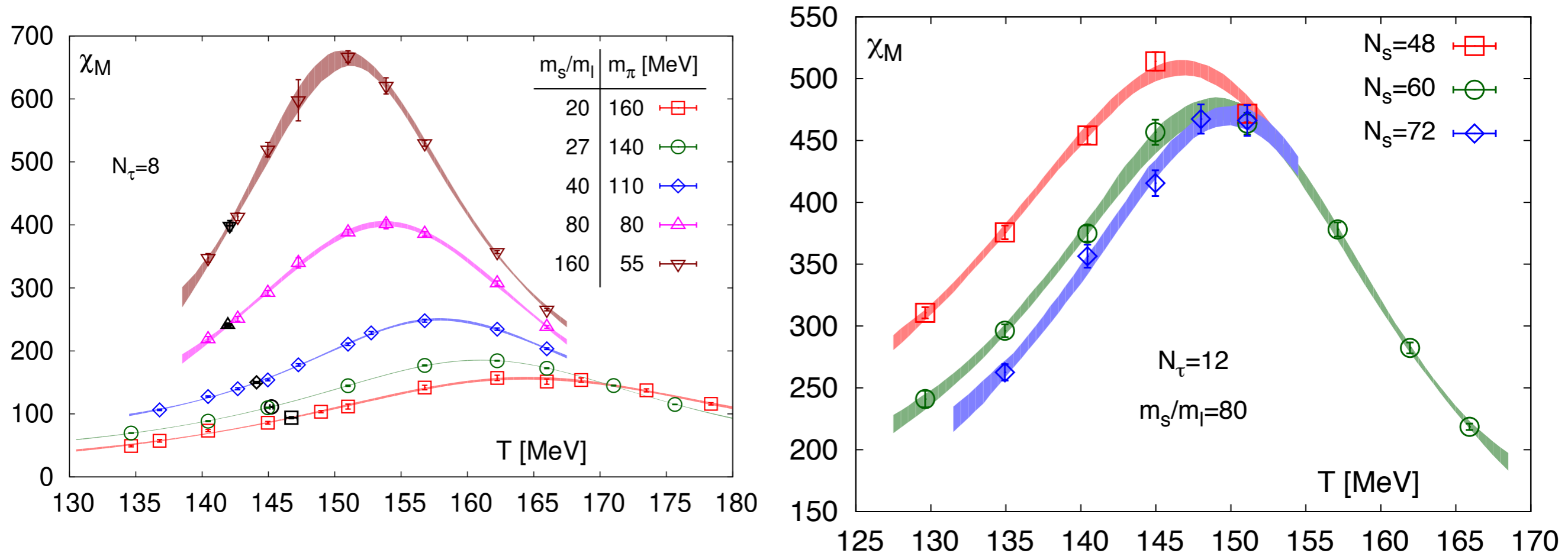
This allows us to perform infinite volume, continuum and then chiral extrapolation!

Quark mass and volume dependences of chiral susceptibility



- 📌 Susceptibility increases as $m_l^{1/\delta-1} + \text{const}$, here $\delta \approx 4.8$
- 📌 Peak height of susceptibility slightly changes with Volume
- 📌 Consistent with a continuous phase transition with $O(N)$ universality class in the chiral limit of m_l

Quark mass and volume dependences of chiral susceptibility



- 📌 Susceptibility increases as $m_l^{1/\delta-1} + \text{const}$, here $\delta \approx 4.8$
- 📌 Peak height of susceptibility slightly changes with Volume
- 📌 Consistent with a continuous phase transition with $O(N)$ universality class in the chiral limit of m_l

chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$$f(m, T) = h^{1+1/\delta} f_s(z) + f_{\text{reg}}, \quad z = t/h^{1/\beta\delta}$$

h : external field, t : reduced temperature, β, δ : universal critical exponents

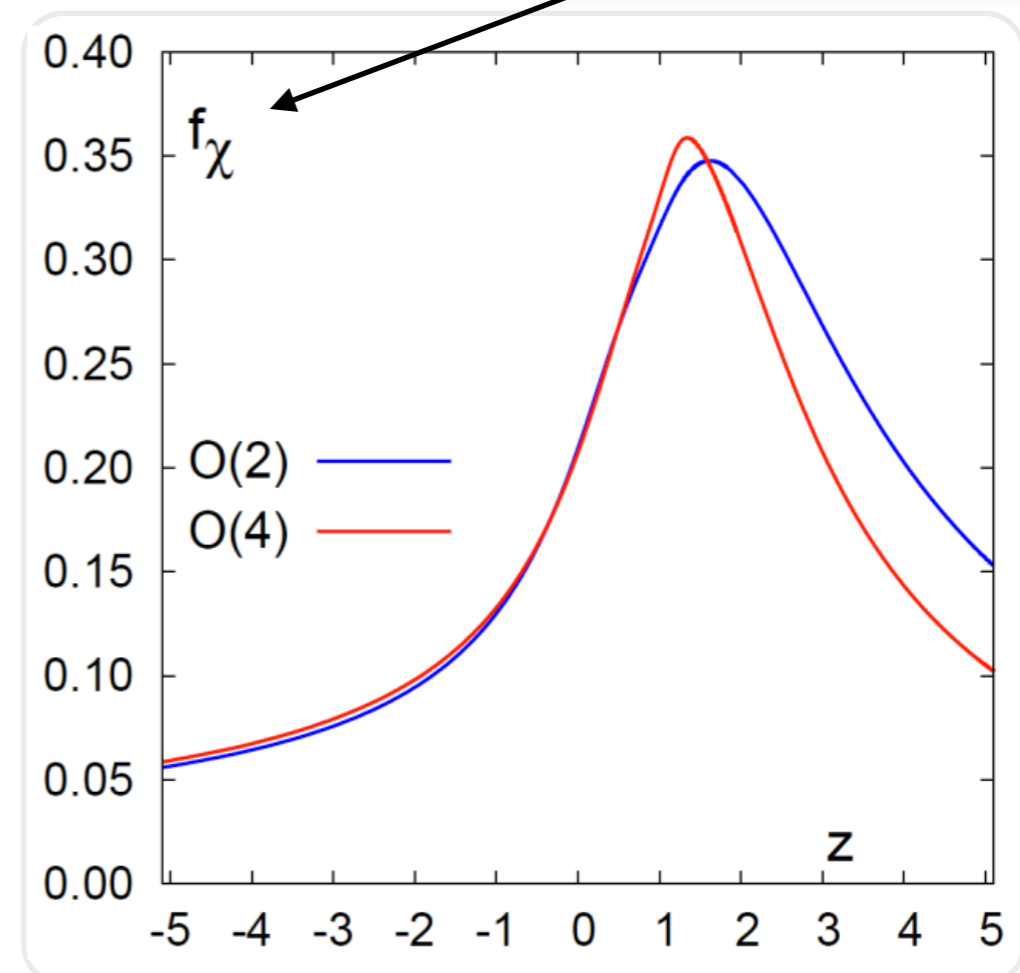
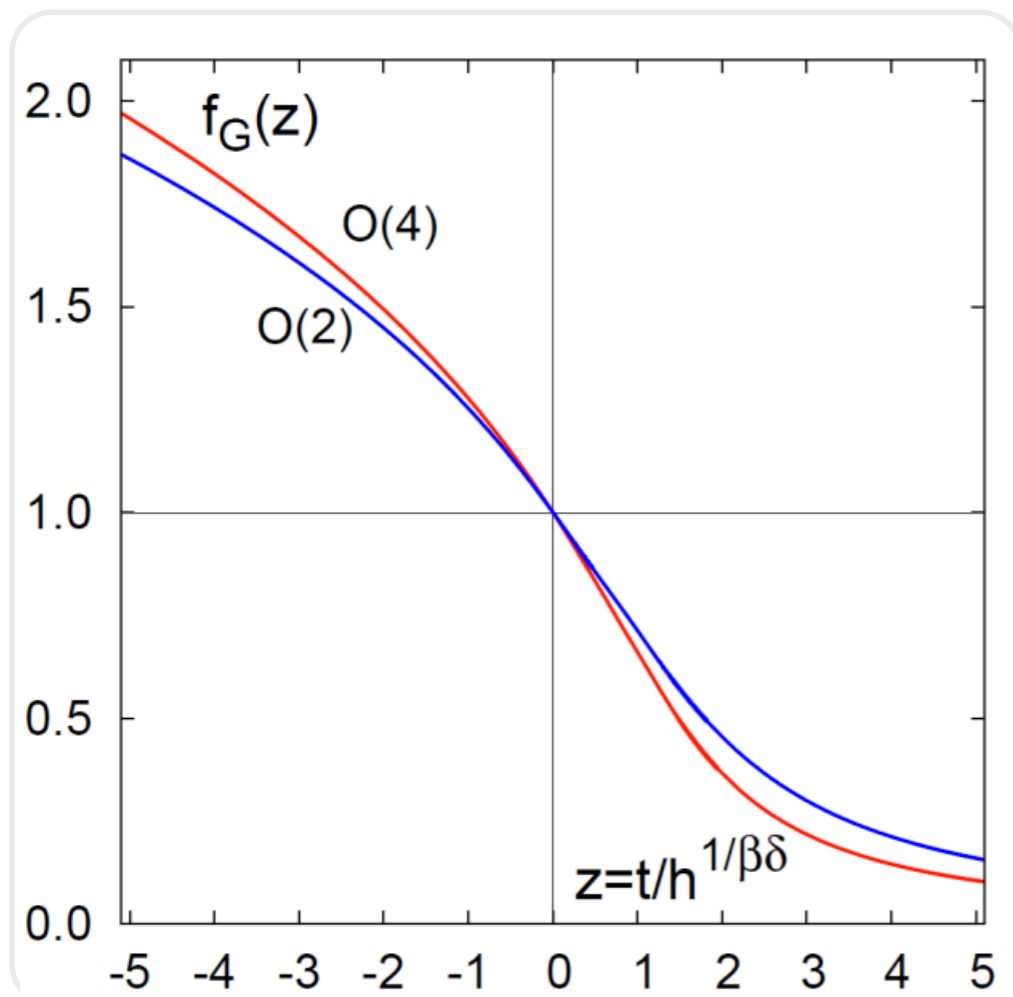
$f_s(z)$: universal scaling function, O(N) etc.

$$h = \frac{|m_l|}{h_0 m_s}$$

$$t = \frac{T - T_c}{T_c}$$

Magnetic Equation of State (MEoS):

$$M = -\partial f_s(t, h) / \partial H = h^{1/\delta} f_G(z) \quad \chi_M = \partial M / \partial H = \frac{h^{1/\delta}}{H} \left(f_G(z) - \frac{z}{\beta} \frac{df_G(z)}{dz} \right)$$



Chiral phase transition temperature T_c^0

$$M = -\partial f_s(t, h) / \partial H = h^{1/\delta} f_G(z)$$

$$\chi_M = \partial M / \partial H = \frac{h^{1/\delta}}{H} \frac{1}{\delta} \left(f_G(z) - \frac{z}{\beta} \frac{df_G(z)}{dz} \right)$$

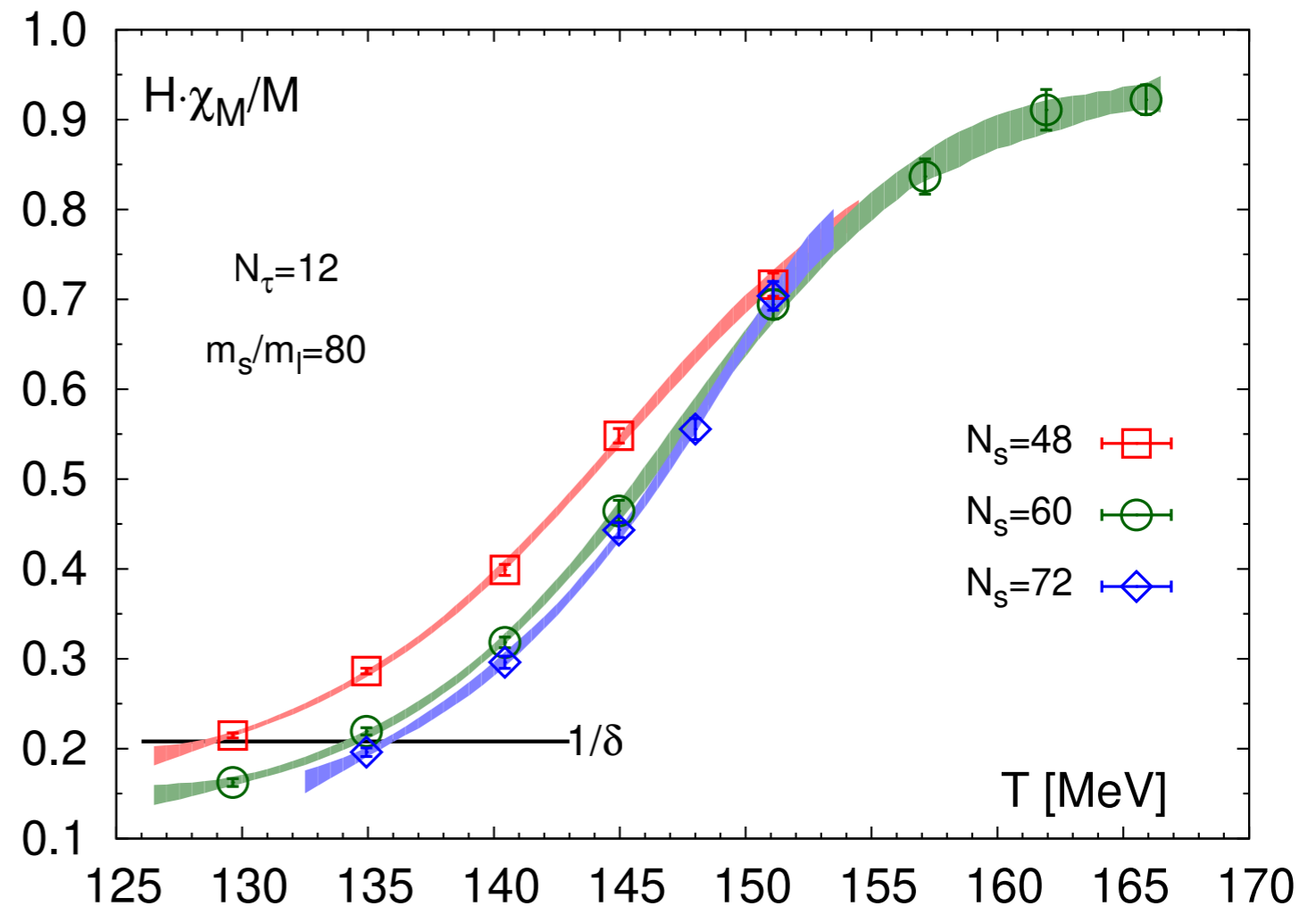
$z=0$ at $T=T_c^0$

$$H \chi_M / M \rightarrow 1/\delta @ T_c^0$$

H: m_l/m_s

M: chiral condensate

χ_M : chiral susceptibility



A novel approach to estimate T_c^0

📌 Pseudo-critical temperature at H

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}} \right)$$

$$z = \frac{1}{t_0} \frac{T - T_c^0}{T_c^0} \left(\frac{H}{h_0} \right)^{-1/\beta\delta} = z_0 \frac{T - T_c^0}{T_c^0} H^{-1/\beta\delta}$$

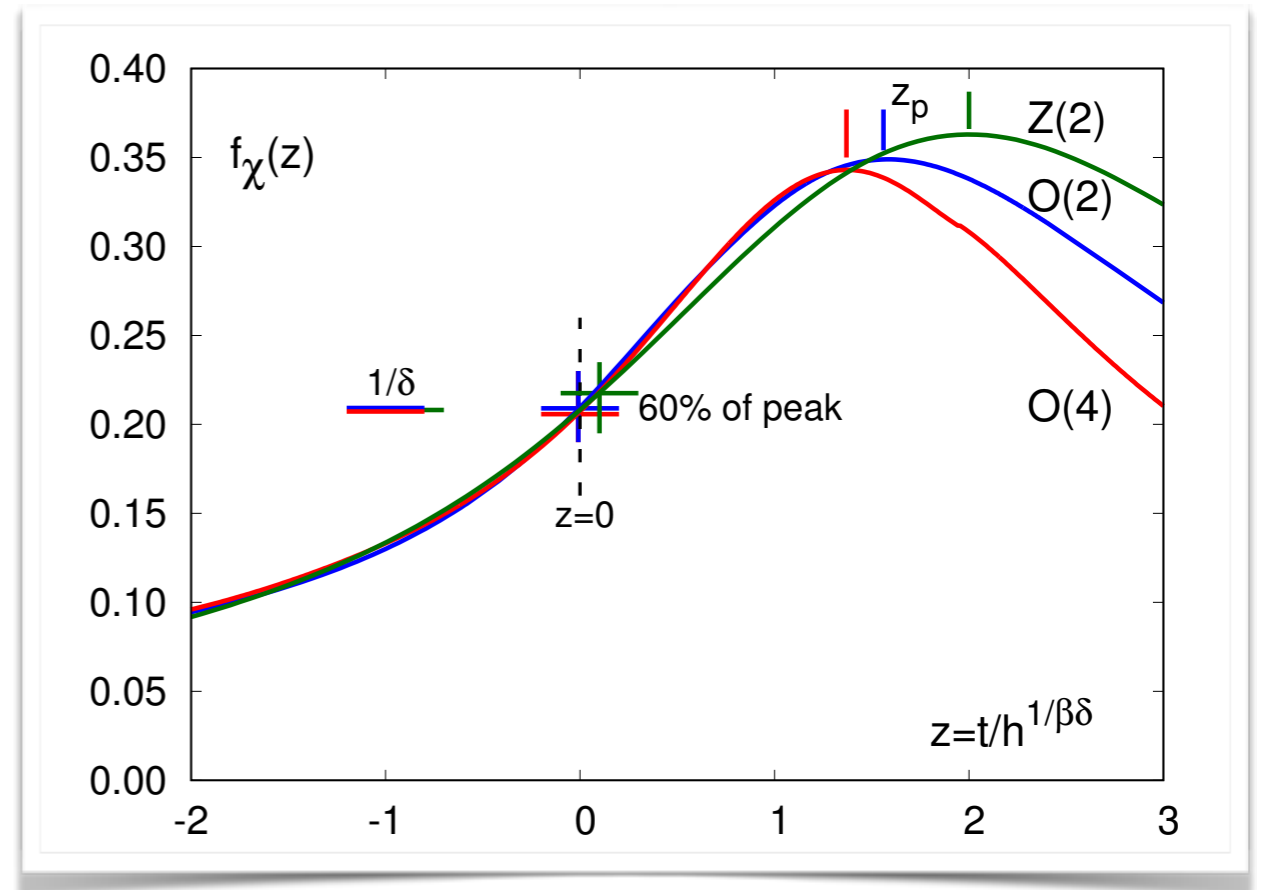
📌 Estimate of the chiral transition T_c^0

$$\frac{H\chi_M(T_\delta, H, L)}{M(T_\delta, H, L)} = \frac{1}{\delta} \longleftrightarrow z(T_\delta) = 0$$

$$\chi_M(T_{60}, H) = 0.6\chi_M^{max} \longleftrightarrow z(T_{60}) \approx 0$$

☑️ small quark mass dependence

☑️ small variations among universality classes



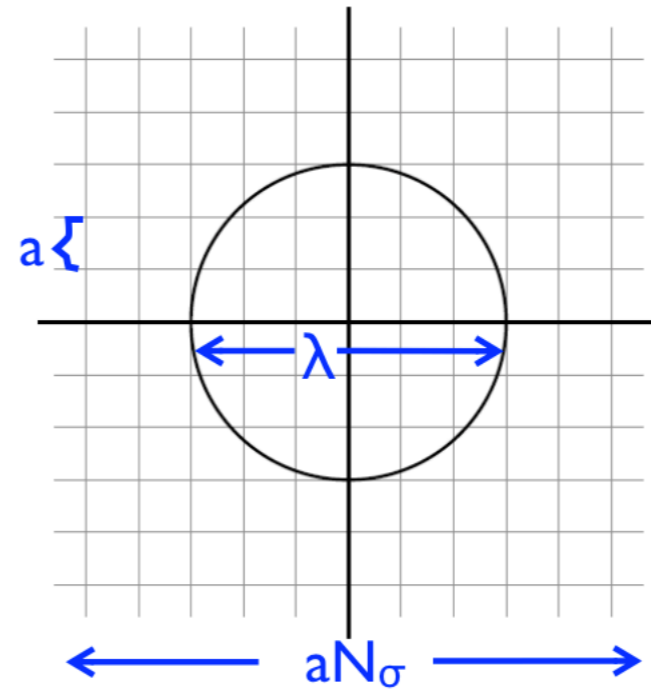
z_p : peak location of the susceptibility

z_{60} : location of 60% of peak height from left

	δ	z_p	z_{60}^-
Z(2)	4.805	2.00(5)	0.10(1)
O(2)	4.780	1.58(4)	-0.005(9)
O(4)	4.824	1.37(3)	-0.013(7)

Things need to be taken care of

- Thermodynamic limit
- Continuum limit
- Chiral limit

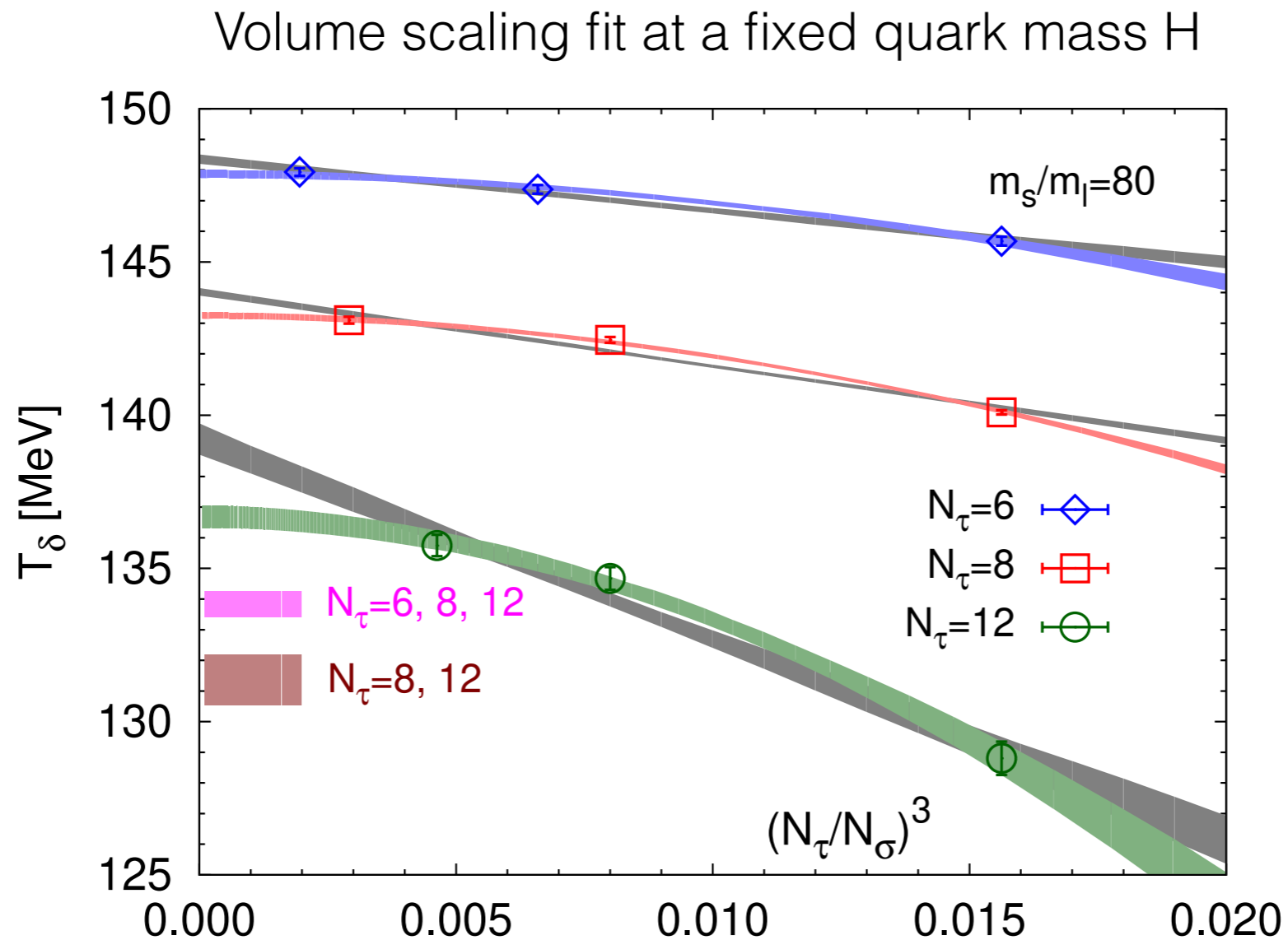


$$T_X(H, L) = T_c^0 \left(1 + \left(\frac{z_X(z_L)}{z_0} \right) H^{1/\beta\delta} \right) + c_X H^{1-1/\delta+1/\beta\delta}$$

Singular Regular

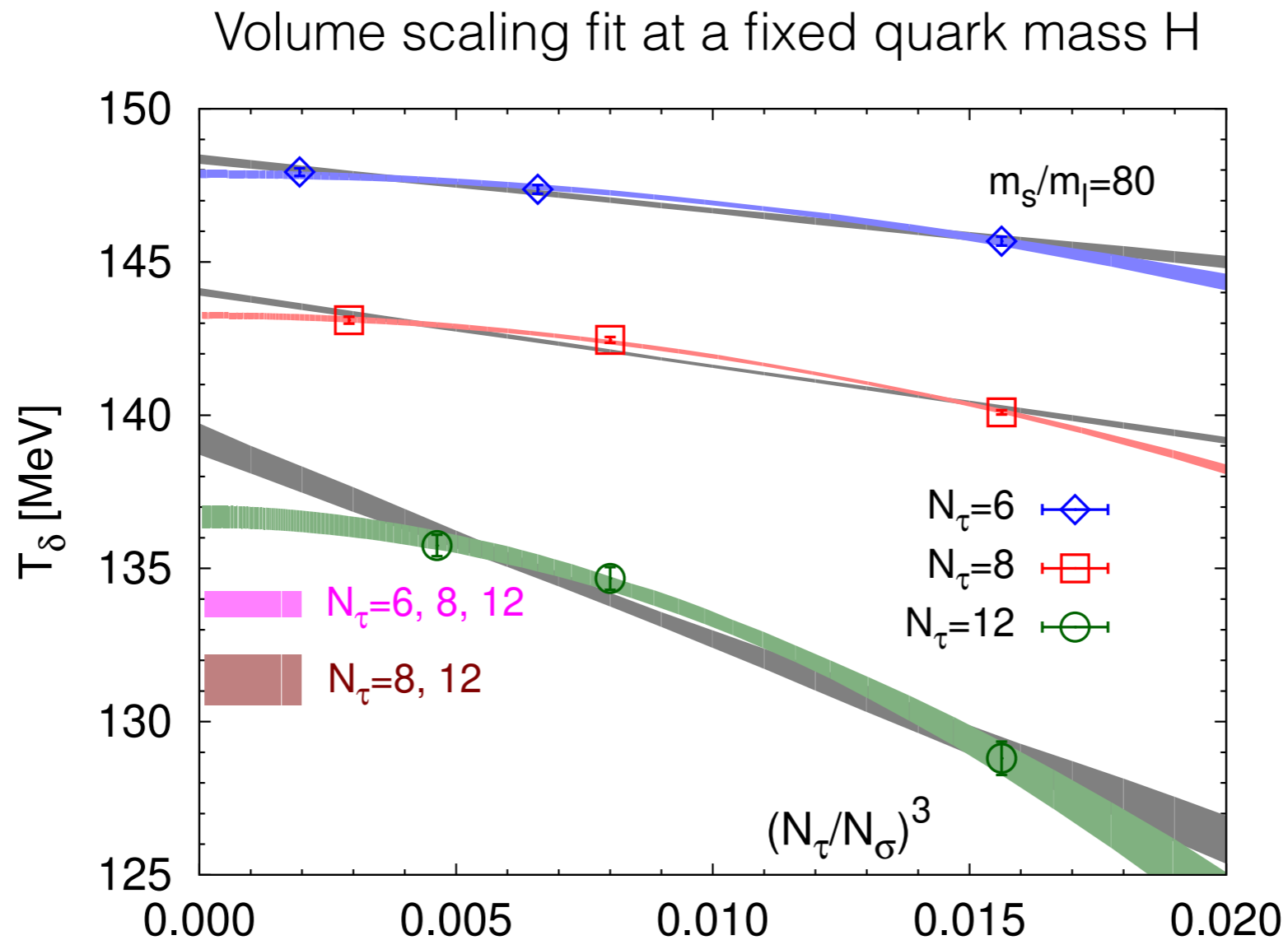
$$X=60, \delta$$

T_δ : Infinite V limit \rightarrow continuum limit \rightarrow chiral limit



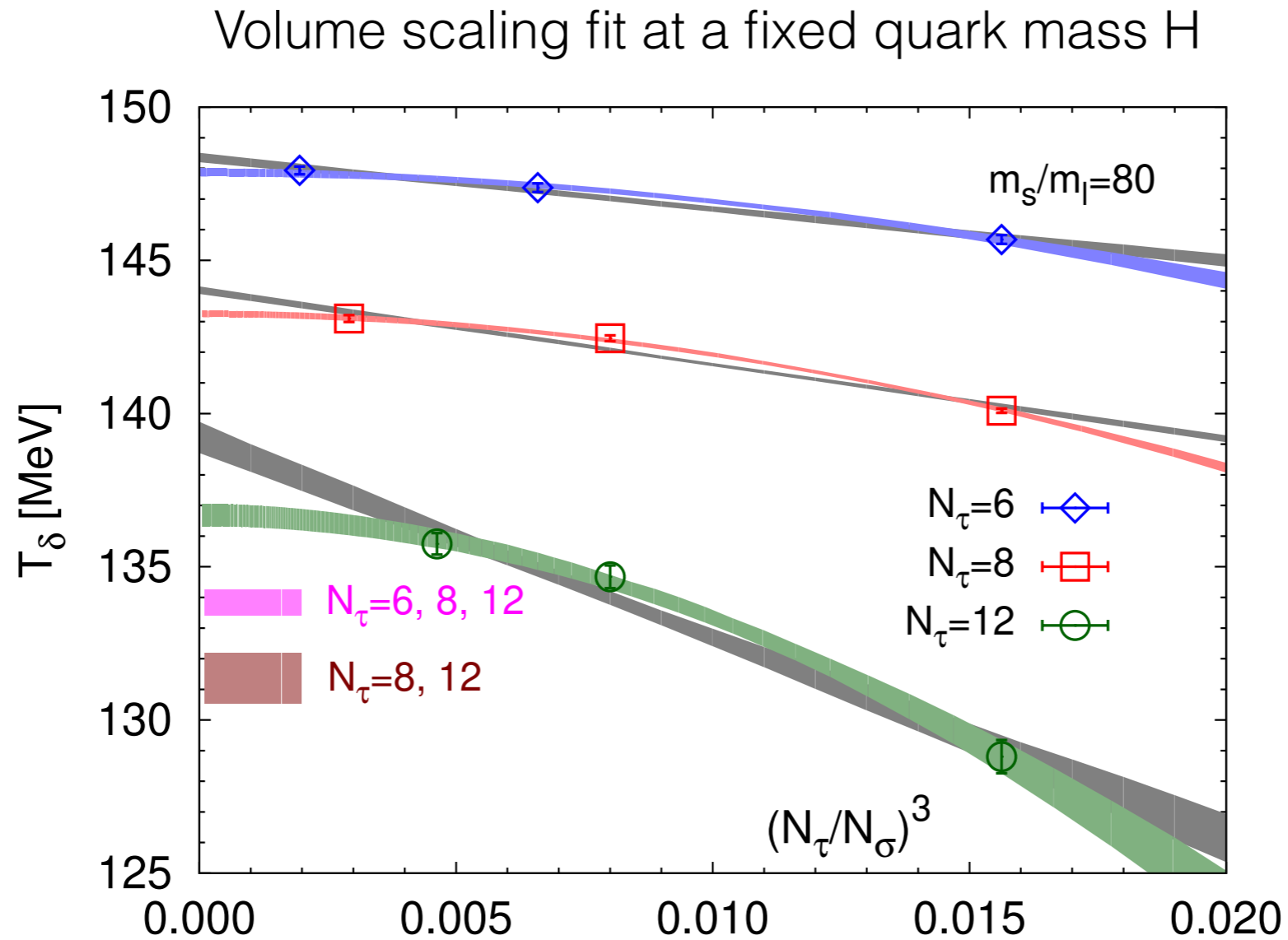
	$T_\delta(H,V,a)$ [MeV] with $N_t=6,8\&12$	$T_\delta(H,V,a)$ [MeV] with $N_t=8\&12$
$V \rightarrow \infty, a \rightarrow 0, H=1/80$	133.8(4)	131.4(8)

T_δ : Infinite V limit \rightarrow continuum limit \rightarrow chiral limit



	$T_\delta(H,V,a)$ [MeV] with $N_t=6,8\&12$	$T_\delta(H,V,a)$ [MeV] with $N_t=8\&12$
$V \rightarrow \infty, a \rightarrow 0, H=1/80$	133.8(4)	131.4(8)
$V \rightarrow \infty, a \rightarrow 0, H=1/40$	136.9(5)	135.5(8)

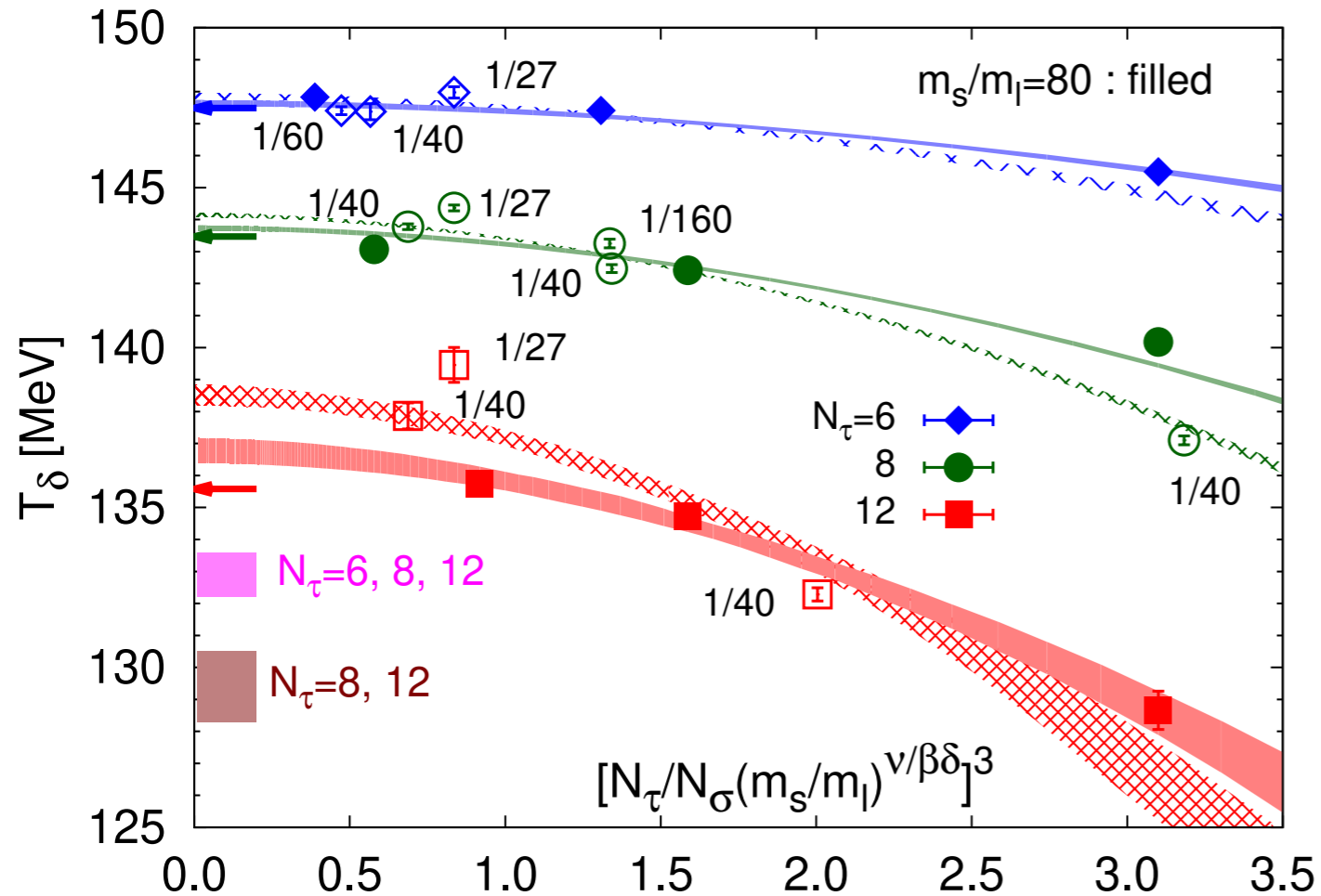
T_δ : Infinite V limit \rightarrow continuum limit \rightarrow chiral limit



	$T_\delta(H,V,a)$ [MeV] with $N_t=6,8\&12$	$T_\delta(H,V,a)$ [MeV] with $N_t=8\&12$
$V \rightarrow \infty, a \rightarrow 0, H=1/80$	133.8(4)	131.4(8)
$V \rightarrow \infty, a \rightarrow 0, H=1/40$	136.9(5)	135.5(8)
$V \rightarrow \infty, a \rightarrow 0, H \rightarrow 0$	132.8(1.4)	130.6(2.4)

T_δ : Infinite V limit \rightarrow chiral limit \rightarrow continuum limit

Joint volume scaling fit with all quark masses

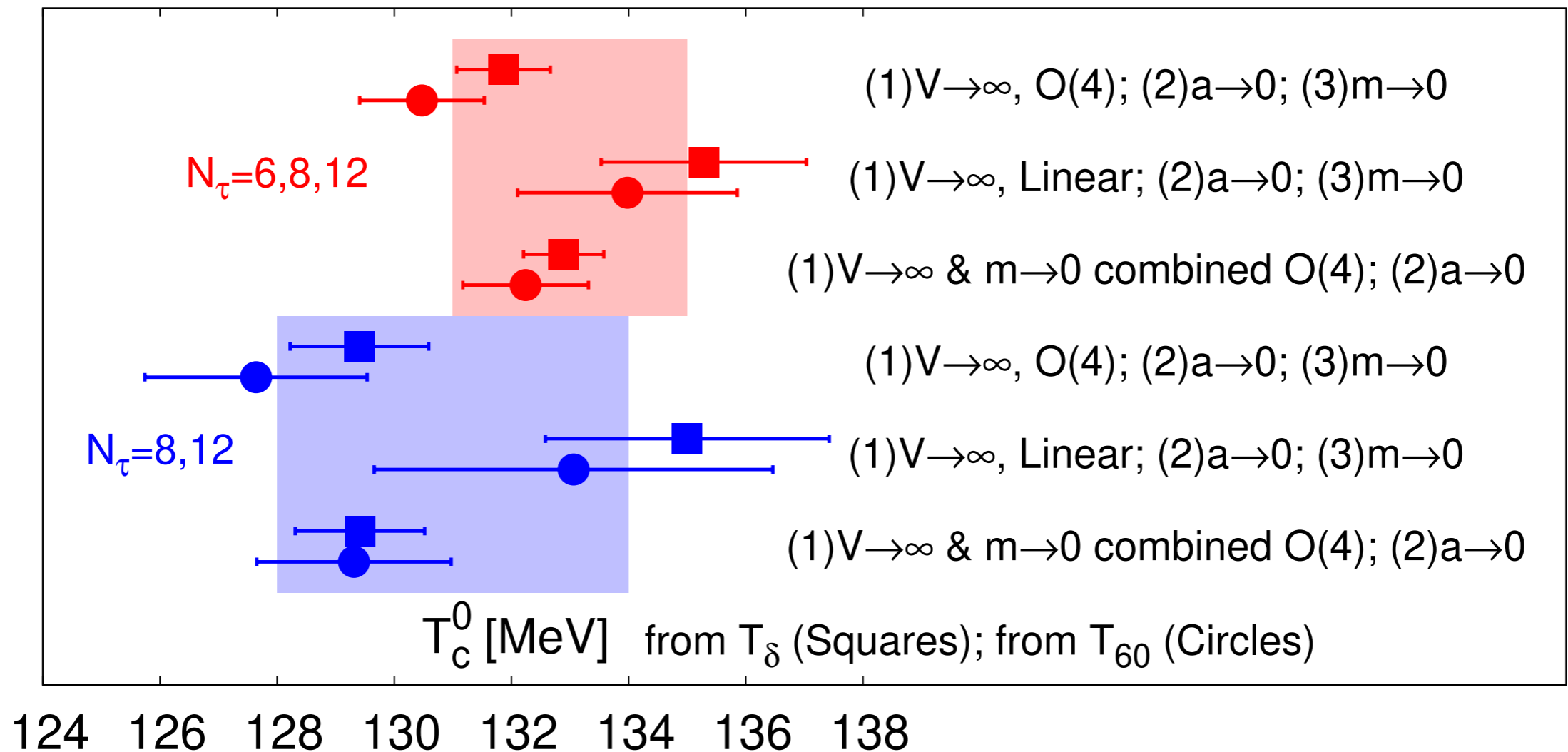


$T_\delta(H, V, a)$ [MeV]

	Nt=6,8&12	Nt=8&12
$V \rightarrow \infty, H \rightarrow 0, a \rightarrow 0$	132.9(6)	128.6(1.1)
$V \rightarrow \infty, a \rightarrow 0, H \rightarrow 0$	132.8(1.4)	130.6(2.4)

Chiral and continuum limits are Interchangeable

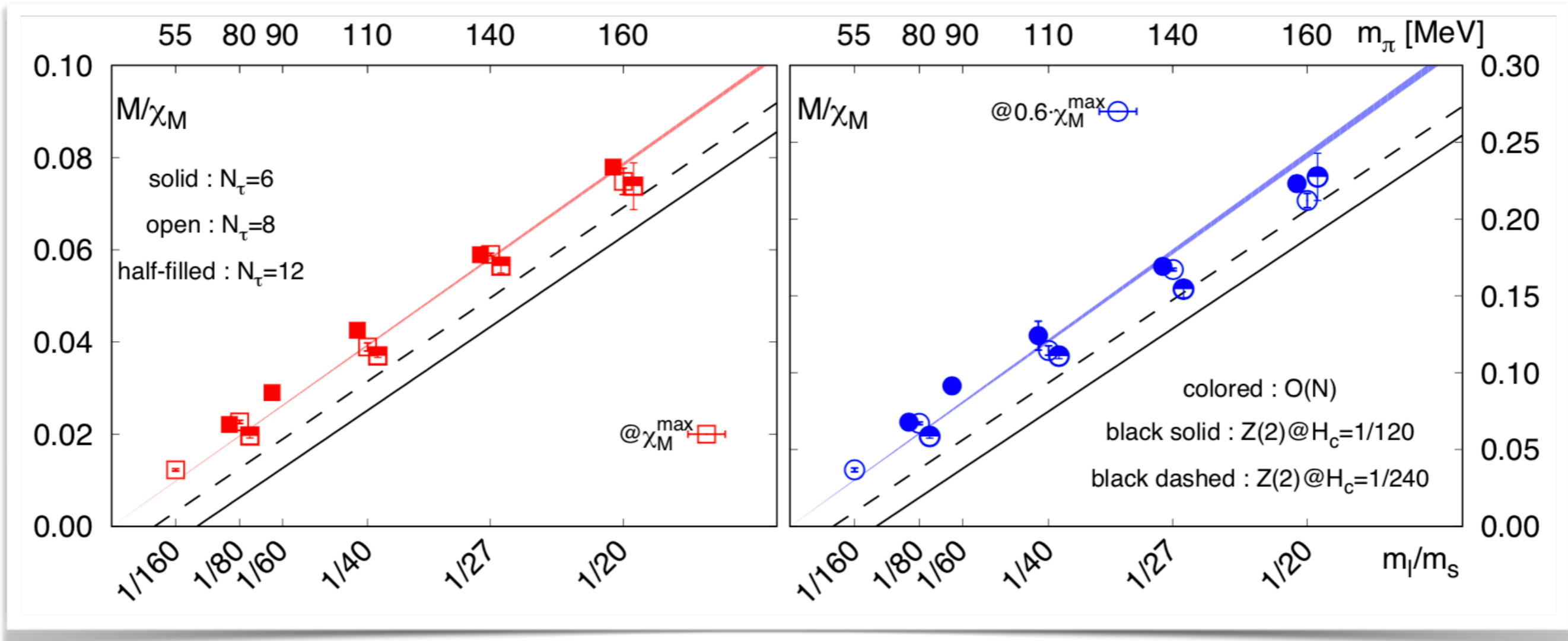
$$T_c^0 = 132_{-6}^{+3} \text{ MeV}$$



📌 T_{60} and T_δ give consistent results

📌 About 25 MeV lower than T_{pc} at the physical point!

Consistency of QCD chiral phase transition with $O(N)$ universality class



S.-T. Li(李胜泰), Lattice 2018, A. Lahiri, QM 2018

$$M/\chi_M = \frac{m_l - m_l^{\text{critical}}}{m_s^{\text{phys}}} \frac{f_M}{f_{\chi_M}}$$

Conclusions

- ☑ chiral crossover temperature is determined with better precision, i.e. $T_{pc} = 156.5(1.5)$ MeV
- ☑ The chiral T_c of $N_f=2+1$ QCD is 132^{+3}_{-6} MeV, and the $O(N)$ universality class of the chiral phase transition is favored

谢谢!

Thanks for your attention!

Congratulations to Prof. Che-Ming Ko
for his
50 years fruitful scientific career!

Lattice 2019

37th series

June 16-22, 2019
Wuhan, China

~ 400 participants annually

<http://lattice2019.ccnu.edu.cn>

You are welcome
to join us!

Lattice 
2019 June 16-22, 2019
WUHAN CHINA
[Http://lattice2019.ccnu.edu.cn](http://lattice2019.ccnu.edu.cn)

**37th International
Symposium on
Lattice
Field Theory**

TOPICS:

- Algorithms and Machines
- Applications Beyond QCD
- Chiral Symmetry
- Hadron Spectroscopy and Interactions
- Hadron Structure
- Nonzero Temperature and Density
- Physics Beyond the Standard Model
- Standard Model Parameters and Renormalization
- Theoretical Developments
- Vacuum Structure and Confinement
- Weak Decays and Matrix Elements

INTERNATIONAL ADVISORY COMMITTEE

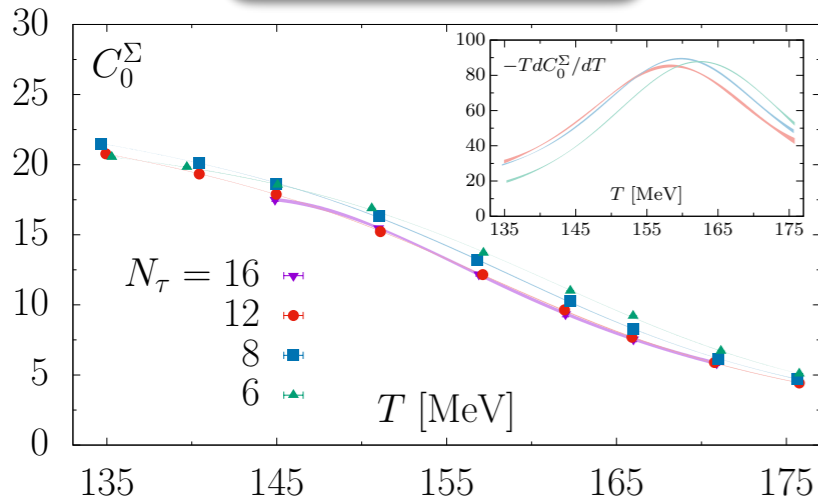
- Gert Aarts (Swansea U.)
- Constantia Alexandrou (U. of Cyprus/Cyprus Inst.)
- Sinya Aoki (Kyoto U.)
- Yasumichi Aoki (KEK/RIKEN)
- Gunnar Bali (Regensburg U.)
- Vitaly Boryakov (IHEP, Protvino)
- Massimo D'Elia (U. of Pisa)
- William Detmold (MIT)
- Takumi Doi (RIKEN)
- Aida El-Khadra (UIUC)
- George Fleming (Yale U.)
- Elvira Gamiz (U. of Granada)
- Prasad Hegde (IISc, Bangalore)
- Taku Izubuchi (BNL/RIKEN)
- Frithjof Karsch (BNL/Bielefeld U.)
- Weonjong Lee (Seoul National U.)
- Laurent Lellouch (CNRS/Aix-Marseille U.)
- Huey-Wen Lin (Michigan State U.)
- Nilmani Mathur (TIFR)
- Robert Mawhinney (Columbia U.)
- Tereza Mendes (São Paulo U.)
- Kostas Orginos (W&M/Jlab)
- Sinéad Ryan (Trinity College Dublin)
- Chris Sachrajda (U. of Southampton)
- Carsten Urbach (Bonn U.)
- Matthew Wingate (U. of Cambridge)
- Ross Young (U. of Adelaide)

LOCAL ORGANIZING COMMITTEE

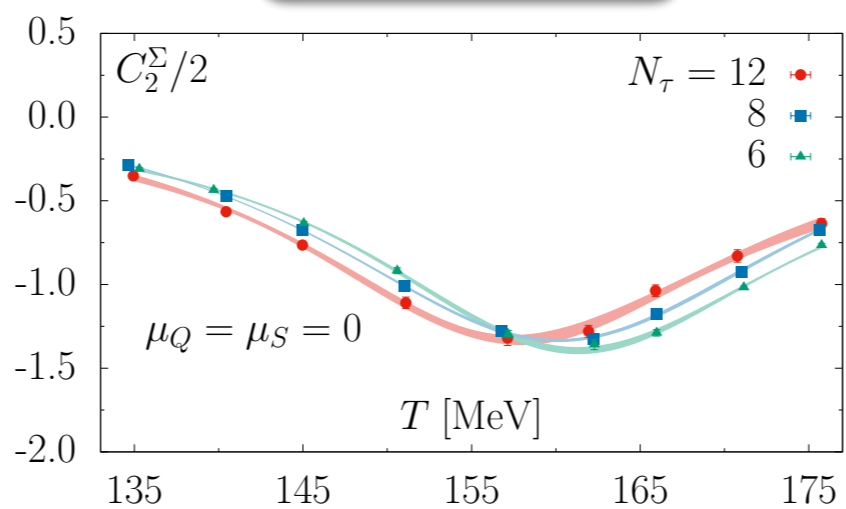
- Heng-Tong Ding (Chair, CCNU)
- Ying Chen (IHEP, CAS)
- Xu Feng (Peking U.)
- Ziwen Fu (Sichuan U.)
- Ming Gong (IHEP, CAS)
- Longcheng Gui (Hunan Normal U.)
- Olaf Kaczmarek (Bielefeld U./CCNU)
- Ning Li (Xi'an Technological U.)
- David Lin (NCTU, Hsinchu)
- Chuan Liu (Peking U.)
- Liuming Liu (IMP, CAS)
- Yubin Liu (Nankai U.)
- Zhaofeng Liu (IHEP, CAS)
- Jian-Ping Ma (ITP, CAS)
- Swagato Mukherjee (BNL/Tsinghua U.)
- Yibo Yang (ITP, CAS)
- Liangkai Wu (Jiangsu U.)
- Jianbo Zhang (Zhejiang U.)



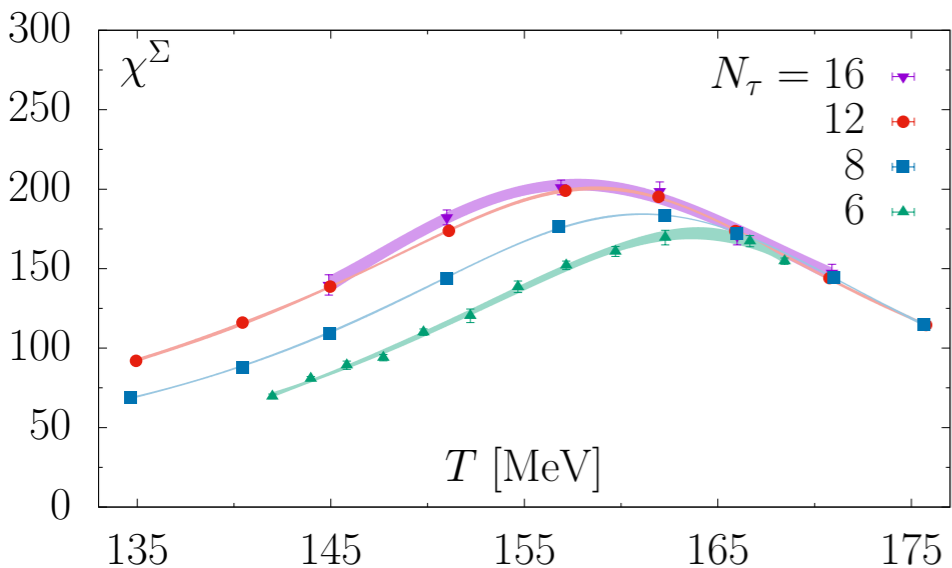
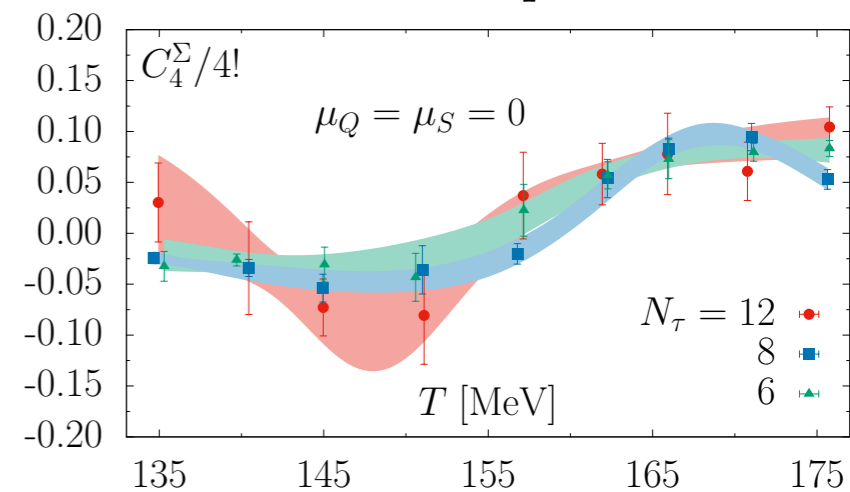
$$\partial_T^2 C_0^\Sigma(T) = 0$$



$$\partial_T C_2^\Sigma(T) = 0$$

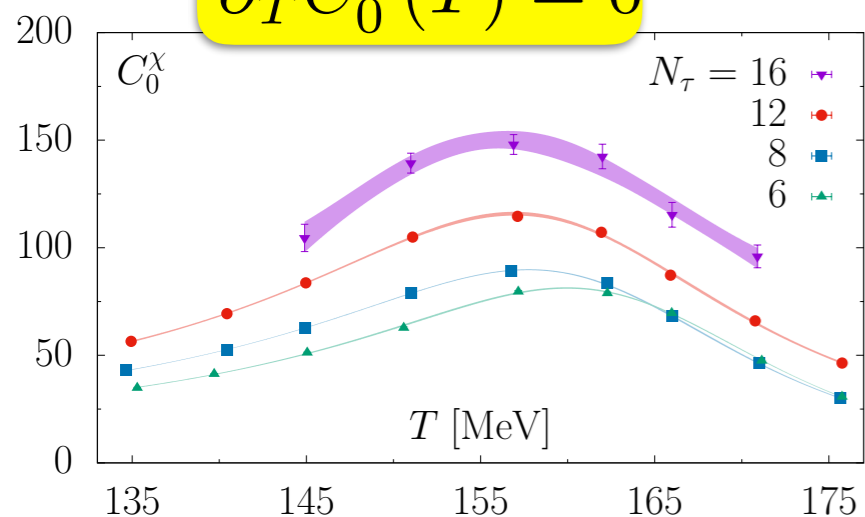


$$C_4^\Sigma(T = T_{pc}) = 0$$

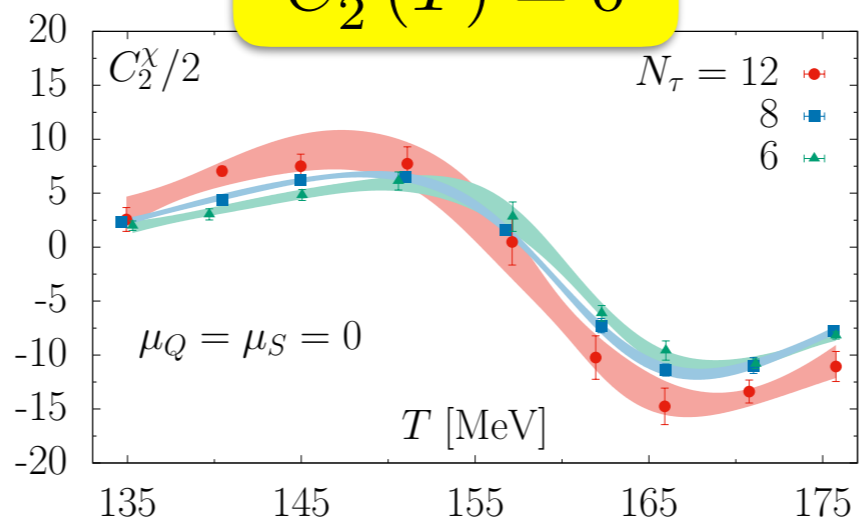


$$\partial_T \chi^\Sigma(T) = 0$$

$$\partial_T C_0^\chi(T) = 0$$



$$C_2^\chi(T) = 0$$



$$\partial_T C_4^\chi(T = T_{pc}) = 0$$

