Workshop on partonic and hadronic transport approaches for relativistic heavy-ion collisions, Dalian, May 11-12th, 2019

Transport simulations of intermediate-energy heavy-ion collisions

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Content

1. Nuclear matter EOS and nucleon mean-field potential
2. BUU and QMD transport approaches
3. Achievements from transport approaches
4. Transport comparison/evaluation project - status report

Intermediate-energy HIC: nucleon/hadron DOF dominate rather than QGP

Nuclear matter EOS and nucleon mean-field potential

$\rho=\rho_n+\rho_p$ $\delta=(\rho_n-\rho_p)/\rho$

Energy per nucleon in asymmetric matter | Energy per nucleon in symmetric matter | Symmetry energy

$$E(\rho,\delta) \approx E_0(\rho) + E_{sym}(\rho)\delta^2$$

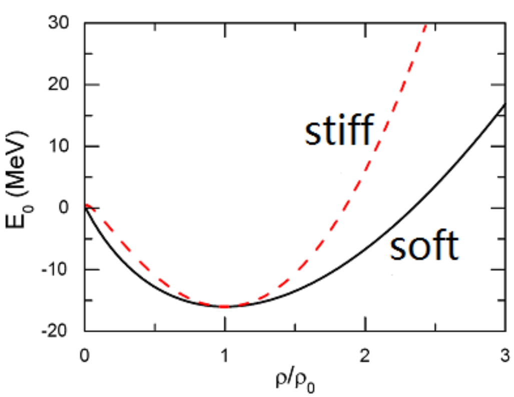
nucleon MF potential in asymmetric matter | nucleon MF potential in symmetric matter | Symmetry potential

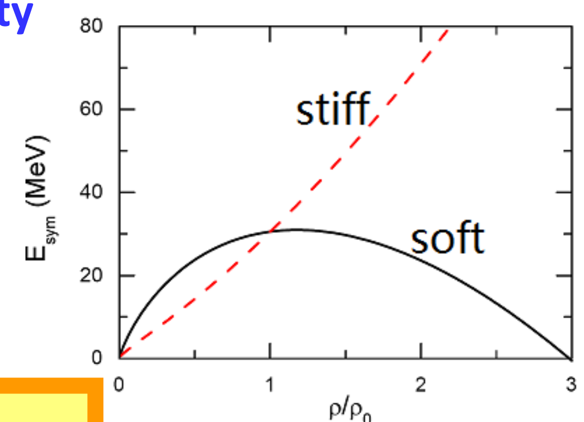
$$U_{n/p}(\rho,\delta) \approx U_0(\rho) \pm U_{sym}(\rho)\delta$$

Intermediate-energy HIC ⟷ (Transport approach) ⟷ EOS of asymmetric nuclear matter ⟷ (General relativity) ⟷ Nstar, GW, …

EOS of asymmetric nuclear matter ⟷ (Many-body theory) ⟷ Nuclear force

Nuclear force ⟷ (Many-body theory) ⟷ Nuclear structure, SHE, …





BUU transport approach

Boltzmann-Uehling-Uhlenback equation:

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m} \cdot \nabla_r - \nabla_r U \cdot \nabla_p\right) f(\vec{r}, \vec{p}; t) = I_{coll}[f; \sigma_{12}]$$

Collision term with quantum statistics

$$\begin{aligned} I_{coll} &= \frac{1}{(2\pi)^6} \int dp_2 dp_3 d\Omega |v - v_2| \frac{d\sigma_{12}^{med}}{d\Omega} (2\pi)^3 \delta(p + p_2 - p_3 - p_4) \\ &\times [f_3 f_4 (1 - f)(1 - f_2) - f f_2 (1 - f_3)(1 - f_4)] \end{aligned}$$

Derivation: real-time Green's function formulism; von-Neumann equation with density matrix; higher-order cutoff from TDHF; …

test-particle (TP) method: parallel events

C.Y. Wong, PRC 25, 1460 (1982); G.F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988).

Point particle or finite size (triangular, Gaussian)

$$f(\vec{r}, \vec{p}; t) = \frac{1}{N_{TP}} \sum_{i=1}^{N_{TP}A} g(\vec{r} - \vec{r}_i(t)) \tilde{g}(\vec{p} - \vec{p}_i(t))$$

Equations of motion from pseudoparticle method:

$$d\vec{r}_i / dt = \nabla_{\vec{p}_i} H; \qquad d\vec{p}_i / dt = -\nabla_{\vec{r}_i} H.$$

QMD transport approach

single-particle wave function:

$$\phi_i(\vec{r};t) = \frac{1}{(2\pi L)^{4/3}} \exp\left[-\frac{(\vec{r}-\vec{r}_i(t))^2}{4L} + \frac{i\vec{p}_i(t)\cdot\vec{r}}{\hbar}\right]$$

Wigner function (phase-space distribution):

$$\begin{aligned} f_i(\vec{r},\vec{p}) &= \frac{1}{(2\pi\hbar)^3}\int \phi_i^*(\vec{r}-\vec{s}/2)\phi_i(\vec{r}+\vec{s}/2)\exp(-i\vec{p}\cdot\vec{s})d^3s \\ &= \frac{1}{(\pi\hbar)^3}\exp\left[-\frac{(\vec{r}-\vec{r}_i)^2}{2L} - \frac{2L(\vec{p}-\vec{p}_i)^2}{\hbar^2}\right], \end{aligned}$$

Many-body Hamiltonian $H = \sum_i T_i + \frac{1}{2}\sum_{i\neq j} V_{ij}$

$\langle V_{ij}\rangle$ **from Hartree calculation**

Equations of motion

$$\frac{d\vec{r}_i}{dt} = \frac{\vec{p}_i}{m} + \frac{1}{2}\sum_{j,j\neq i}\frac{\partial\langle V_{ij}\rangle}{\partial\vec{p}_i} = \frac{\partial\langle H\rangle}{\partial\vec{p}_i},$$

$$\frac{d\vec{p}_i}{dt} = -\frac{1}{2}\sum_{j,j\neq i}\frac{\partial\langle V_{ij}\rangle}{\partial\vec{r}_i} = -\frac{\partial\langle H\rangle}{\partial\vec{r}_i}.$$

Ch. Hartnack et al., PRC 495, 303 (1989); J. Aichelin, Phys. Rep. 202, 233 (1988).

AMD and FMD: wave function antisymmetrized

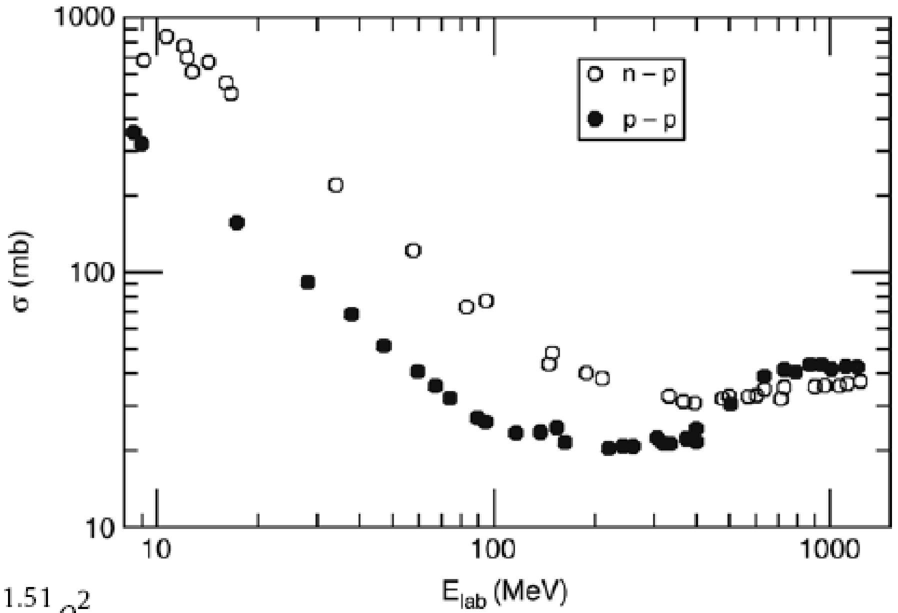
Parameterized NN scattering cross section

In free space:

$$\sigma_{np}^{\rm free} = -70.67 - 18.18\beta^{-1} + 25.26\beta^{-2} + 113.85\beta \ ({\rm mb}),$$

$$\sigma_{pp}^{\rm free} = 13.73 - 15.04\beta^{-1} + 8.76\beta^{-2} + 68.67\beta^{4} \ ({\rm mb}),$$

$$\beta \equiv v/c$$



In symmetric nuclear matter:

$$\sigma_{np}^{\rm medium} = \left[31.5 + 0.092 abs(20.2 - E_{\rm lab}^{0.53})^{2.9}\right] \cdot \frac{1.0 + 0.0034E_{\rm lab}^{1.51}\rho^{2}}{1.0 + 21.55\rho^{1.34}} \ ({\rm mb}),$$

$$\sigma_{pp}^{\rm medium} = \left[23.5 + 0.0256(18.2 - E_{\rm lab}^{0.5})^{4}\right] \cdot \frac{1.0 + 0.1667E_{\rm lab}^{1.05}\rho^{3}}{1.0 + 9.704\rho^{1.2}} \ ({\rm mb}).$$

or

$$\sigma_{NN}^{\rm medium} = \left(1 + \alpha\frac{\rho}{\rho_0}\right)\sigma_{NN}^{\rm free}$$

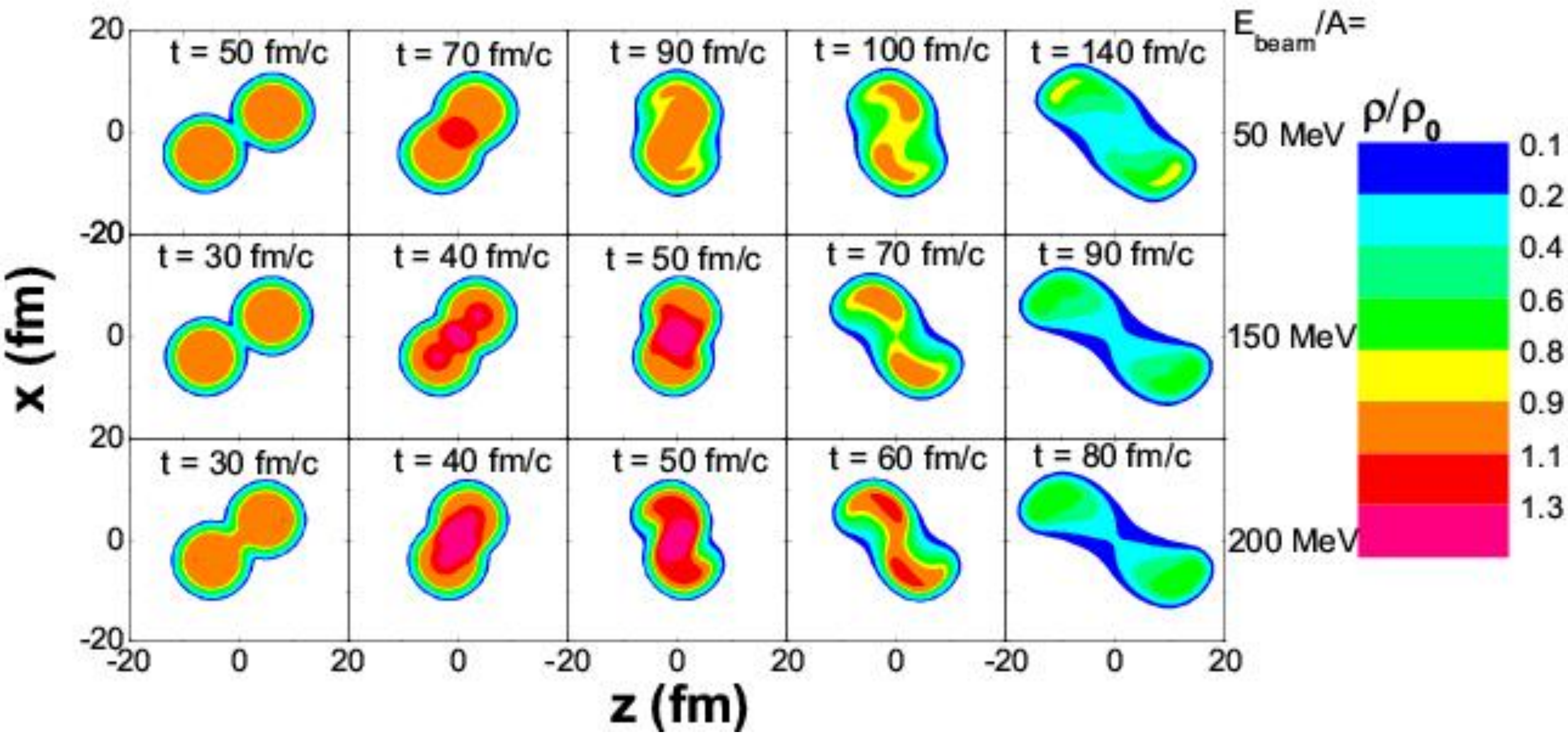
$$\alpha \approx -0.2$$

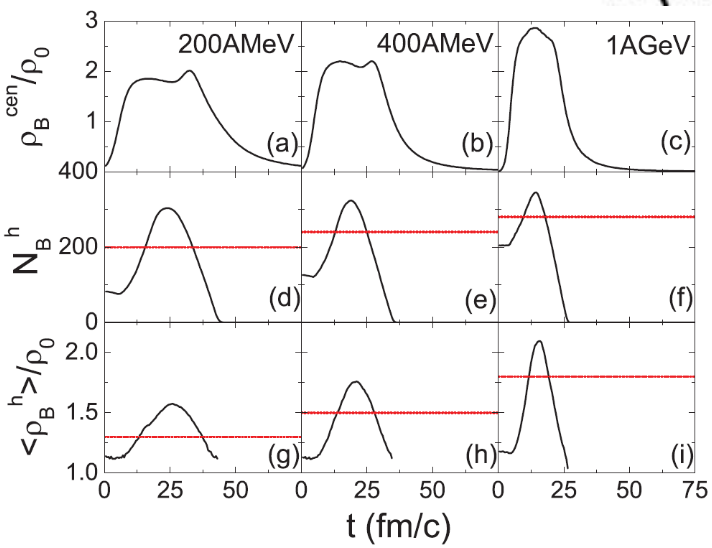
From effective mass:

$$\frac{d\sigma}{d\Omega} = \frac{L^3}{v_{\rm rel}}\frac{2\pi}{\hbar}|t|^2 D_f \qquad v_{rel} = \left|\frac{p_1}{m_1^*} - \frac{p_2}{m_2^*}\right|$$

$$\Rightarrow \quad R_{\rm medium} \equiv \sigma_{NN}^{\rm medium}/\sigma_{NN}^{\rm free} = (\mu_{NN}^*/\mu_{NN})^2$$

Intermediate-energy heavy-ion collisions





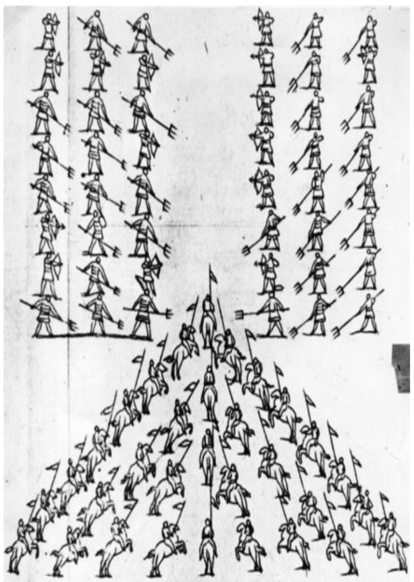
Energy regime: 50 A MeV ~ 2 A GeV
Maximum density reached: 1.2~3 ρ_0
Produced particles: pions, Δ, kaons, hyperons

Nuclear EOS, E_{sym}

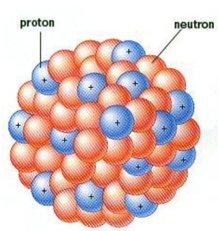
Heavy-ion experiments ⟷ **Mean-field potential**

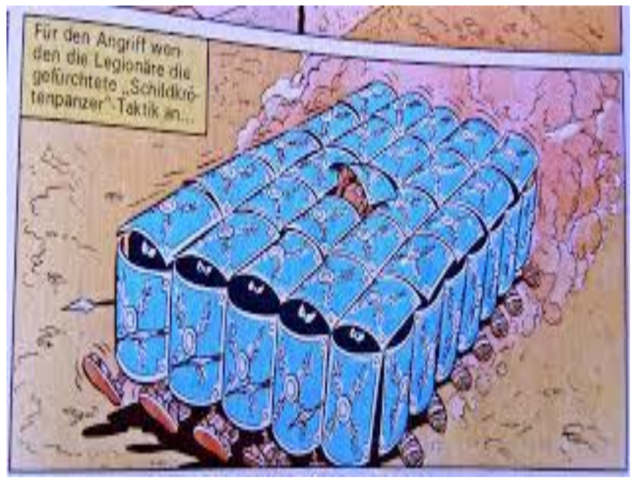
How reliable?

Transport simulations



Initialization





Mean Field :

attractive

Low Energy

$$\vec{p}_i(t+\Delta t) = \vec{p}_i(t) - \nabla U[f(\vec{r}, \vec{p}; t)]$$

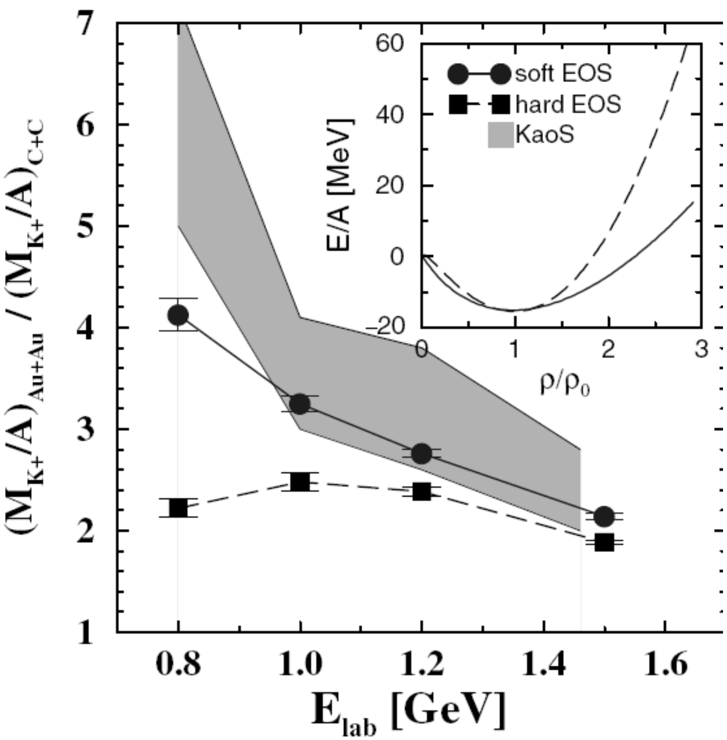
NN collisions:

repulsive

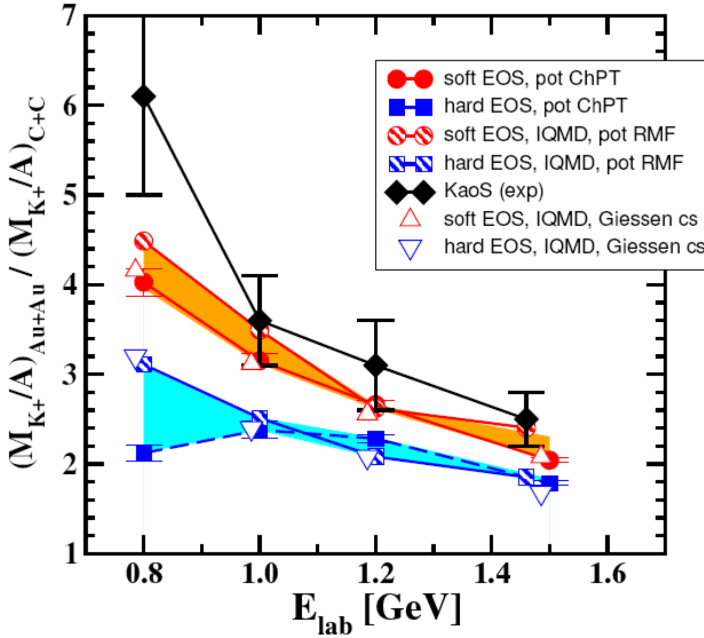
Pauli Blocking

High Energy

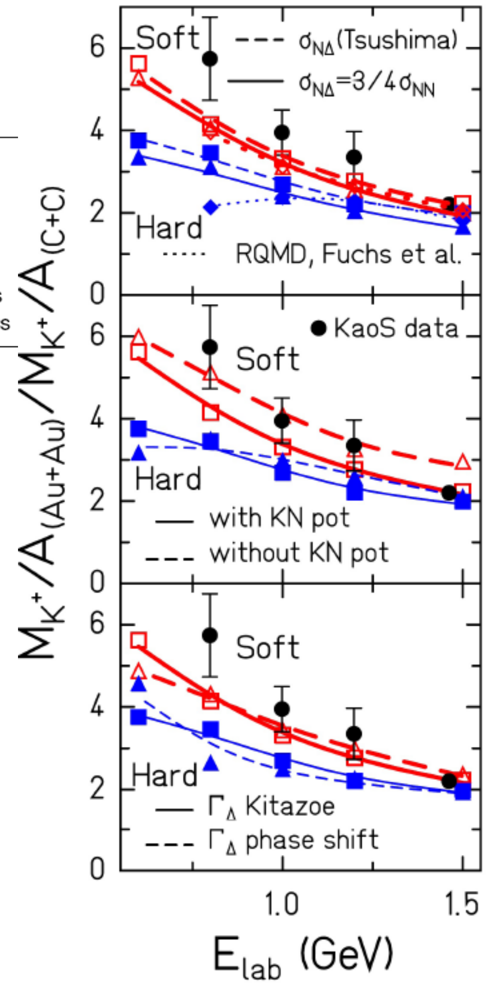
Probe of symmetric NM EOS: kaon production



J. Aichelin and C.M. Ko, PRL 55, 2661 (1985)
C. Fuchs et al., PRL 86, 1974 (2001)



C. Fuchs and H.H. Wolter, EPJA 30, 5 (2006)



Ch. Hartnack, H. Oeschler, and J. Aichelin, PRL 96, 012302 (2006)

$$B + B \rightarrow B + Y + K^+ \qquad \pi + B \rightarrow Y + K^+$$

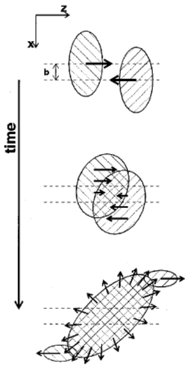
$$\Delta + N \rightarrow N + K^+ + \Lambda$$

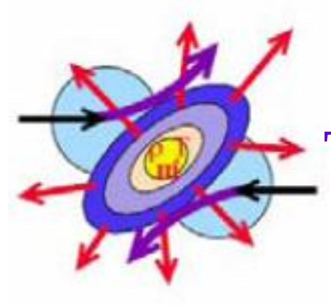
Probes of symmetric NM EOS: collective flows

Slope of Transverse flow/directed flow (v_1)

Elliptic flow(v_2)

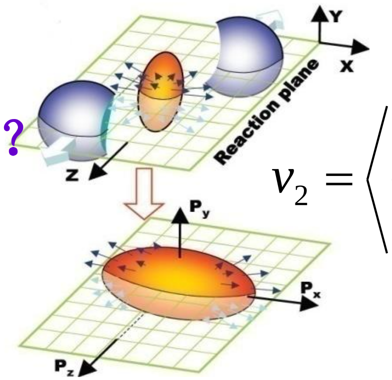
$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$$



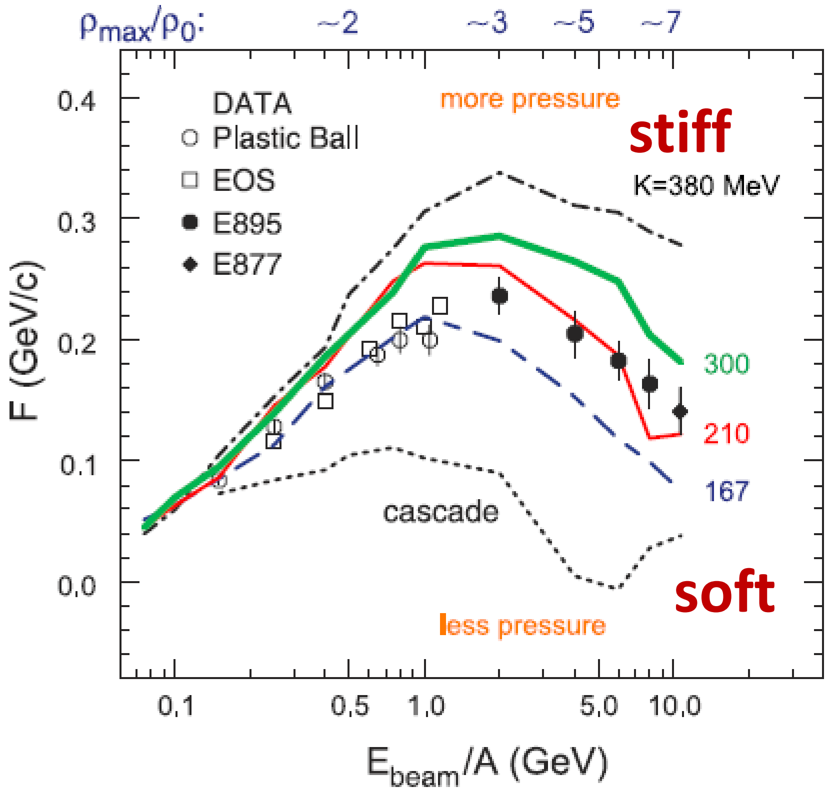


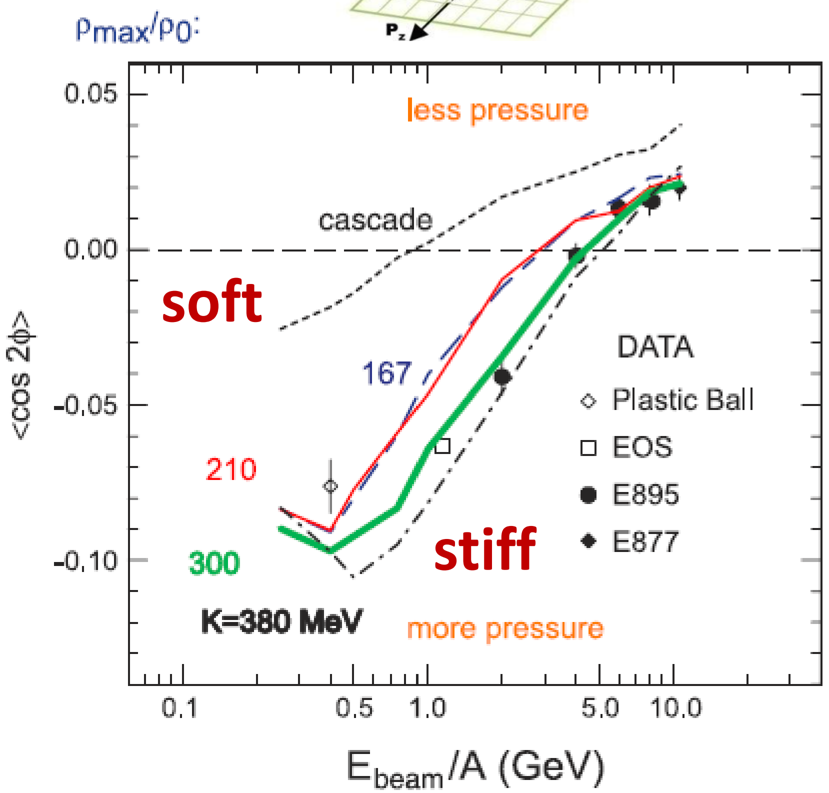
Model dependence?

Theoretical uncertainty？



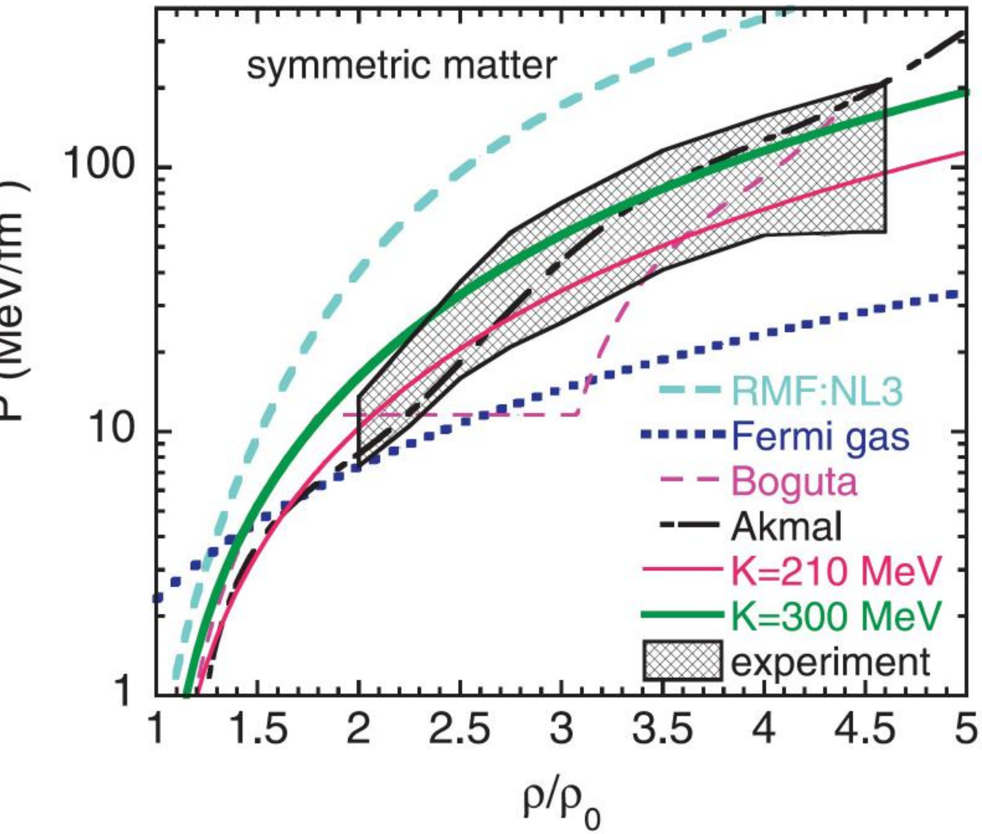
$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_T^2} \right\rangle$$

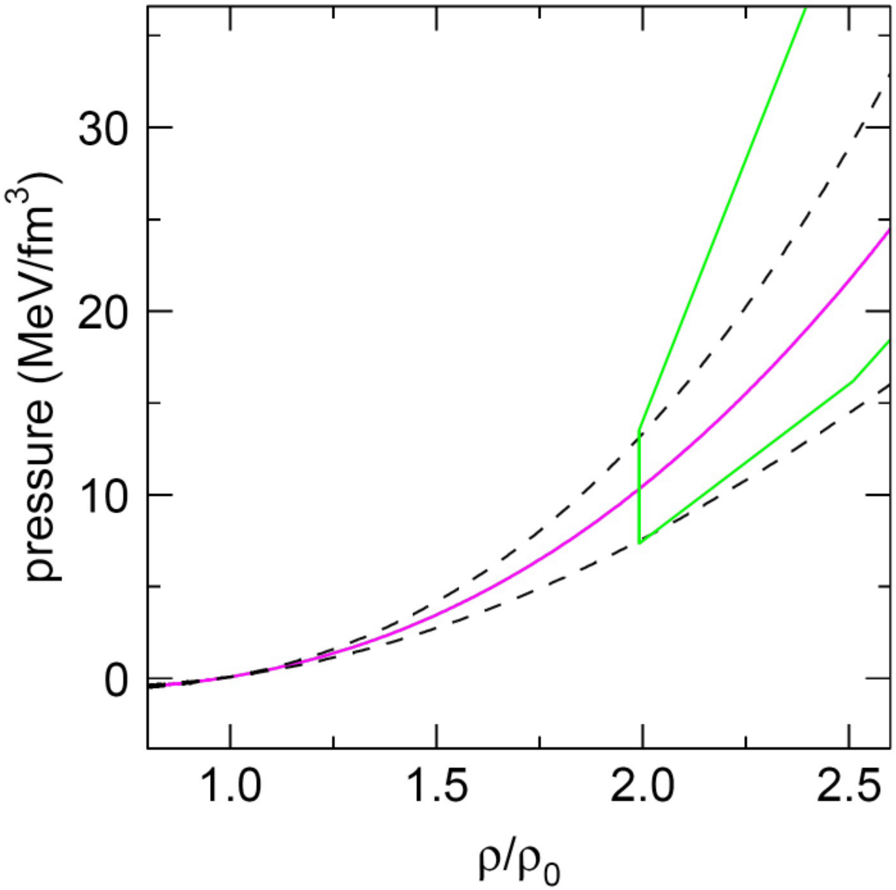




P. Danielewicz, R. Lacey, and W.G. Lynch, Science (2002)

Probes of symmetric NM EOS: collective flows





P. Danielewicz, R. Lacey,
and W.G. Lynch, Science (2002)

More latest constraint from IQMD-FOPI:
A. LeFèvre, Y. Leifels, W. Reisdorf, J. Aichelin,
and Ch. Hartnack, NPA 945, 112 (2016).

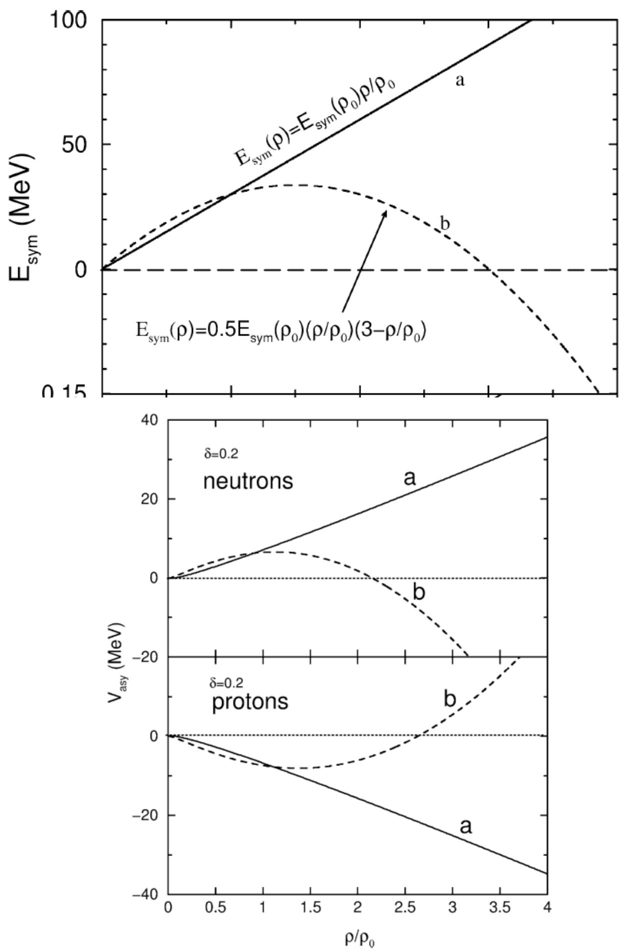
Probes of E_{sym}: π^-/π^+ ratio

$$n + n \longrightarrow p + \Delta^- \qquad \Delta^- \longrightarrow n + \pi^-$$

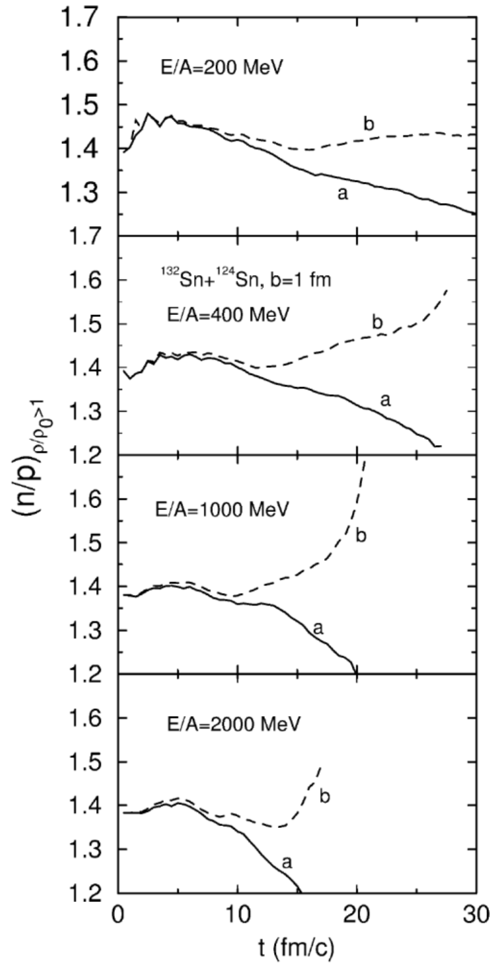
$$p + p \longrightarrow n + \Delta^{++} \qquad \Delta^{++} \longrightarrow p + \pi^+$$

B.A. Li, NPA, (2002)

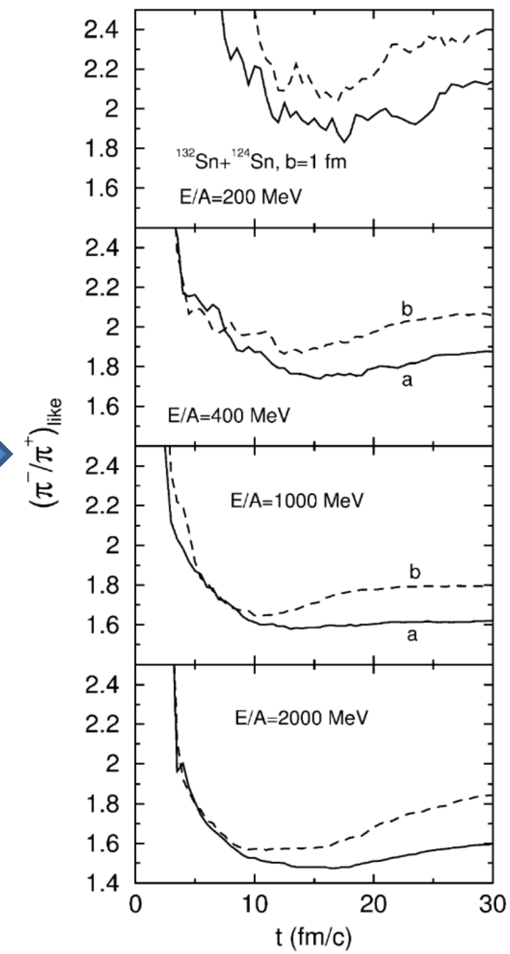
E_{sym} or U_{sym} at $\rho>\rho_0$



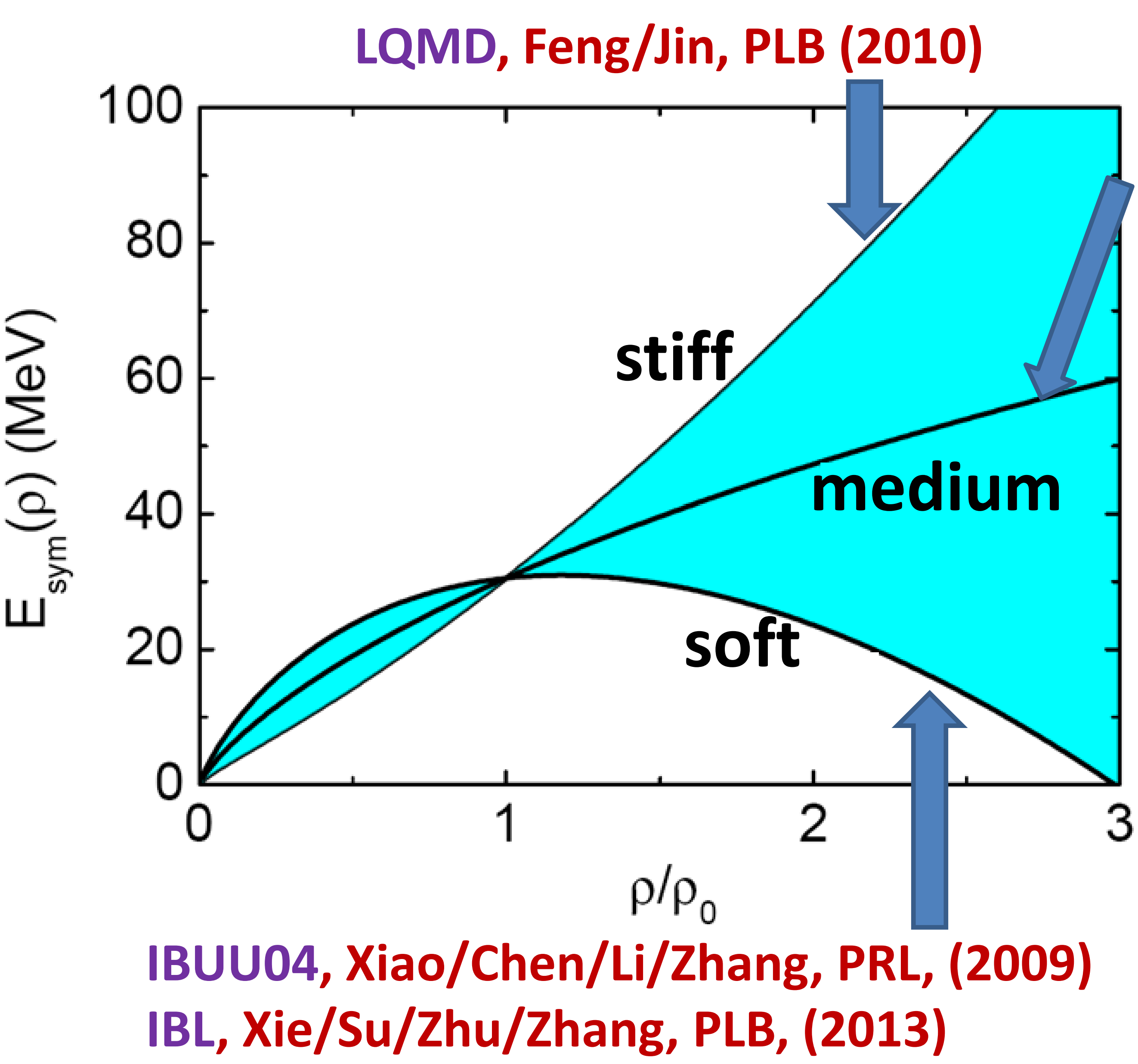
n/p at $\rho>\rho_0$



π^-/π^+ ratio



Divergence of Esym from FOPI π^-/π^+ data



TuQMD, Cozma/Trauntmann/Li, PRC (2013)

Pion s&p-wave from thermal model Xu/Ko/Oh, PRC (2010) Xu/Chen/Li/Ko/Ma, PRC (2013)

Pion s-wave from pBUU Hong/Pawel, PRC (2014)

Pion threshold effect from RVUU Song/Ko, PRC (2015)

Energy conservation from TuQMD Cozma, PLB (2016)

Clustering and Pauli Blocking from JAM/AMD Ikeno/Ono/Nara/Onoshi, PRC (2016)

Pion s&p-wave from RVUU Zhang/Ko (2017,2018)

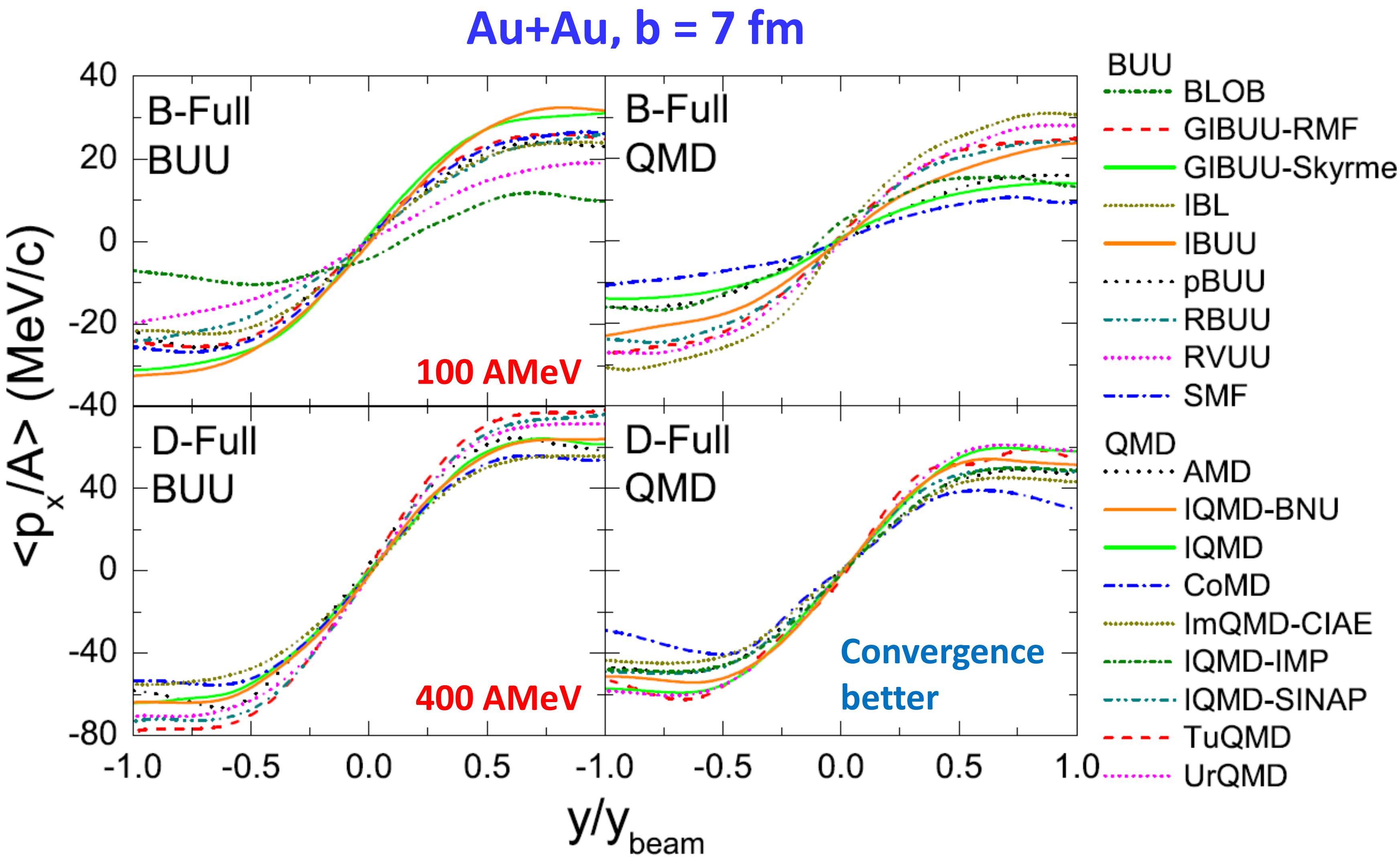
…

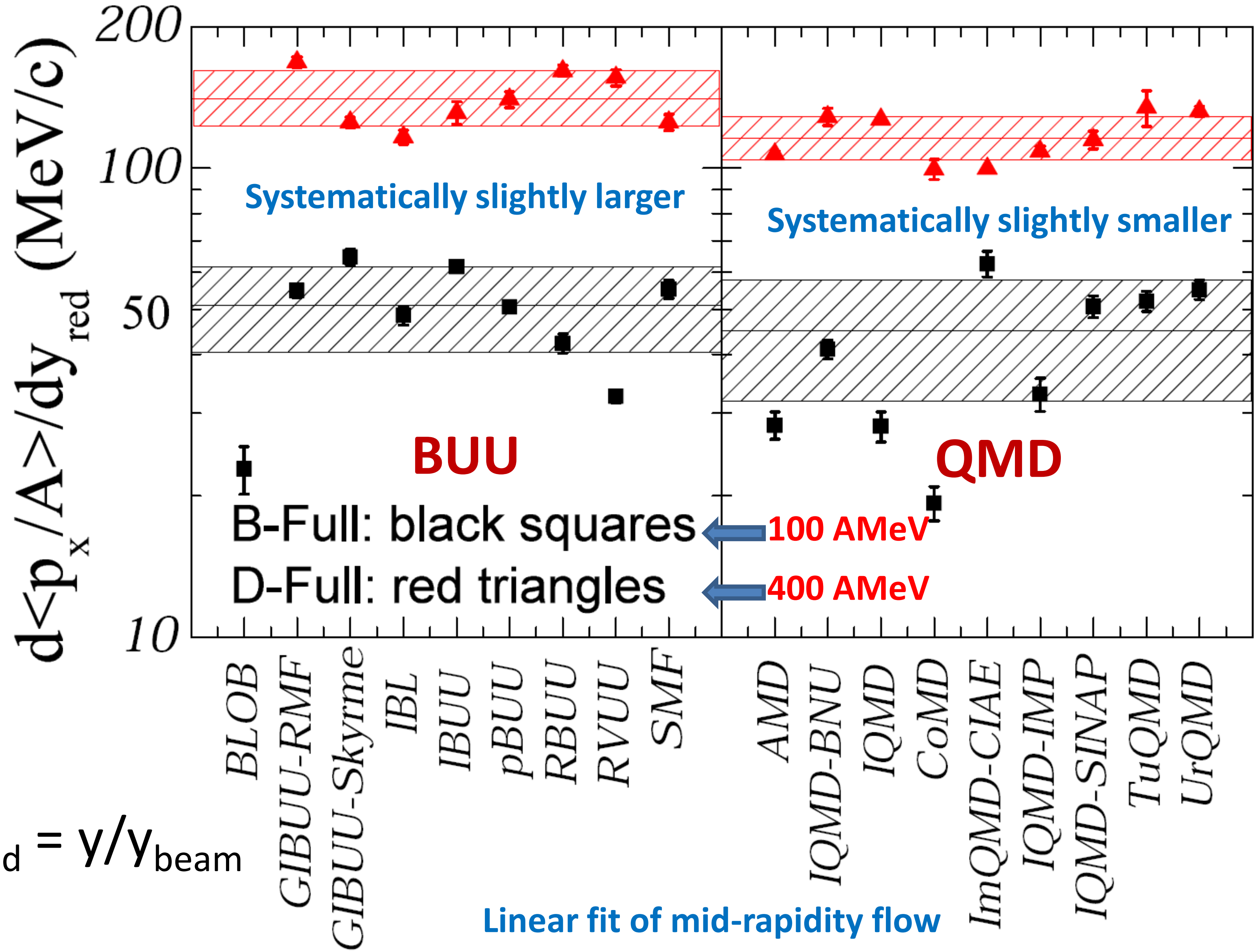
Transport Comparison/Evaluation Project

- **Trento I (2004): energy 1-2 AGeV, particle production p, π, K**
- **Trento II (2009): energy 100, 400 AMeV, not finished**
- **Transport2014 (2014): Mainly 100 AMeV, also 400 AMeV. stability, stopping, and flow, NN scatterings**
- **Transport2017 (2017): Box calculation of NN scatterings, mean-field evolution, and production of pion-like particles**
- **Transport2019 (2019): production of pion-like particles at 270 AMeV, …**

Transport2014 in Shanghai

Transverse flow





$y_{red} = y/y_{beam}$

Linear fit of mid-rapidity flow

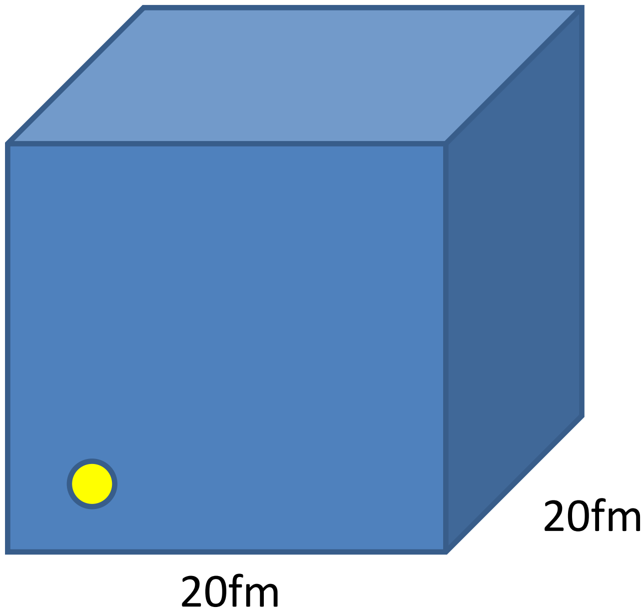
Theoretical uncertainties of flow parameter:

about 30% at 100 AMeV, 13% at 400 AMeV

Transport2017 at MSU

Transport2017 at MSU

Box calculations with periodic boundary conditions



•Details of periodic boundary conditions

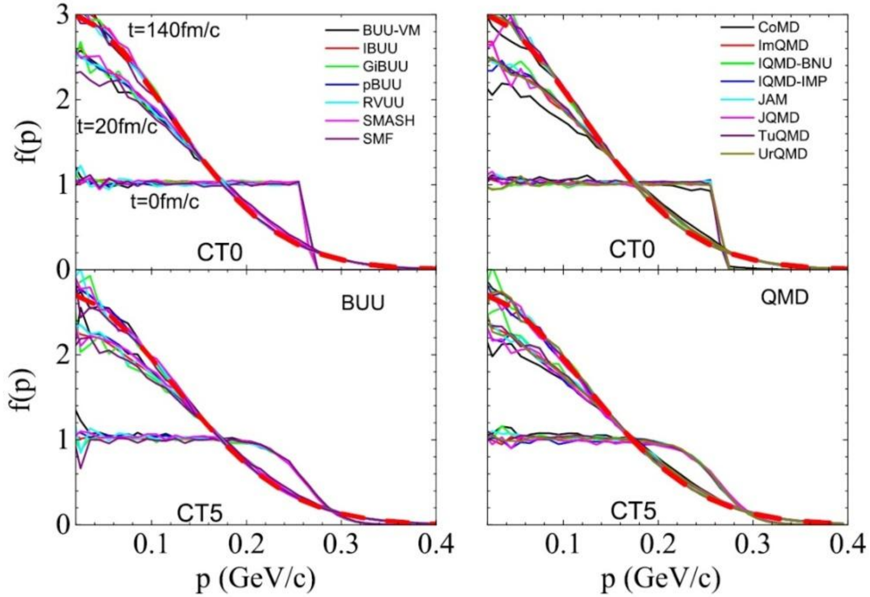
1. a box of volume $V = L_1*L_2*L_3$, where the system is confined.
2. The position of the center of box is $(L_1/2, L_2/2, L_3/2)$.
3. In order to keep all particles inside the box, a particle leaving the box has to enter it on the opposite side, keeping the same momentum.

•Initialization:

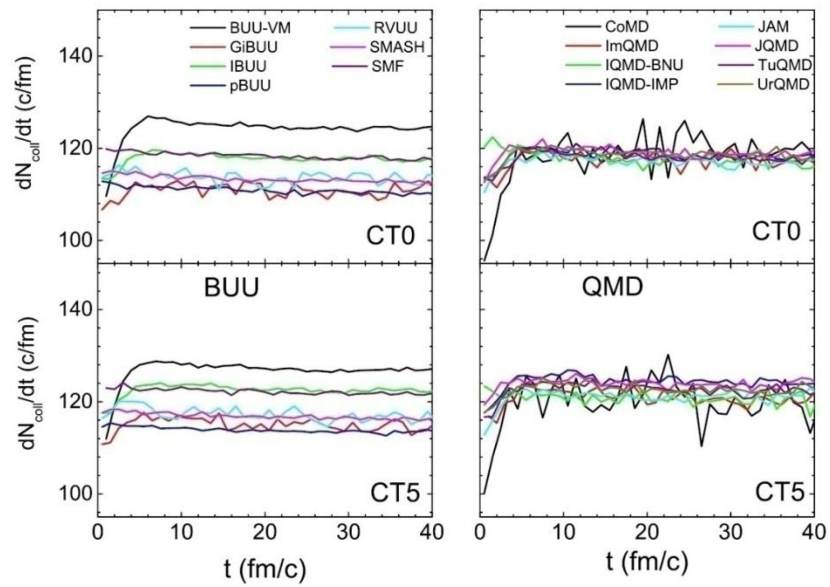
Uniform density ρ_0=0.16 fm^{-3}, with isospin asymmetry equal to zero. With the above size of the box this corresponds to 1280 nucleons, 640 neutrons and 640 protons. Particle positions are initialized randomly from 0 to L_k.

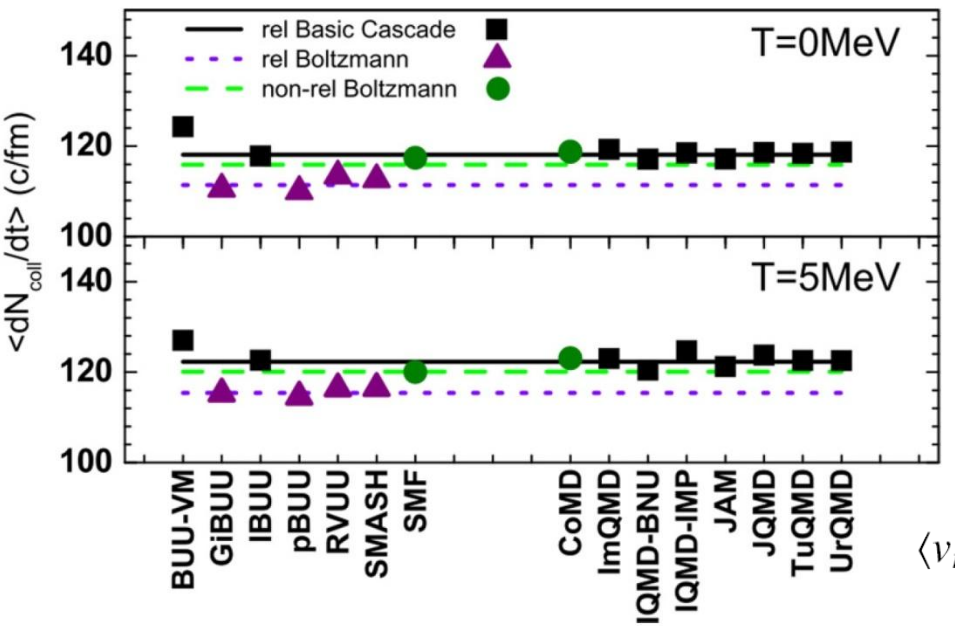
Only NN scatterings without Pauli blocking

Time evolution of momentum distribution



Time evolution of collision rate





$$\frac{dN_{coll}}{dt} = \frac{1}{2} A\rho\sigma\langle v_{rel}\rangle$$

non-relativisitic Boltzmann

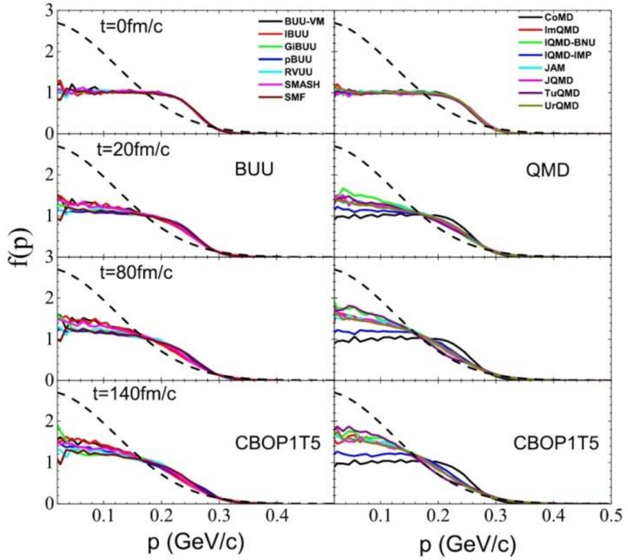
$$\langle v_{rel}\rangle = (4/\sqrt{5\pi})(p_F/m)$$

Relativistic Boltzmann

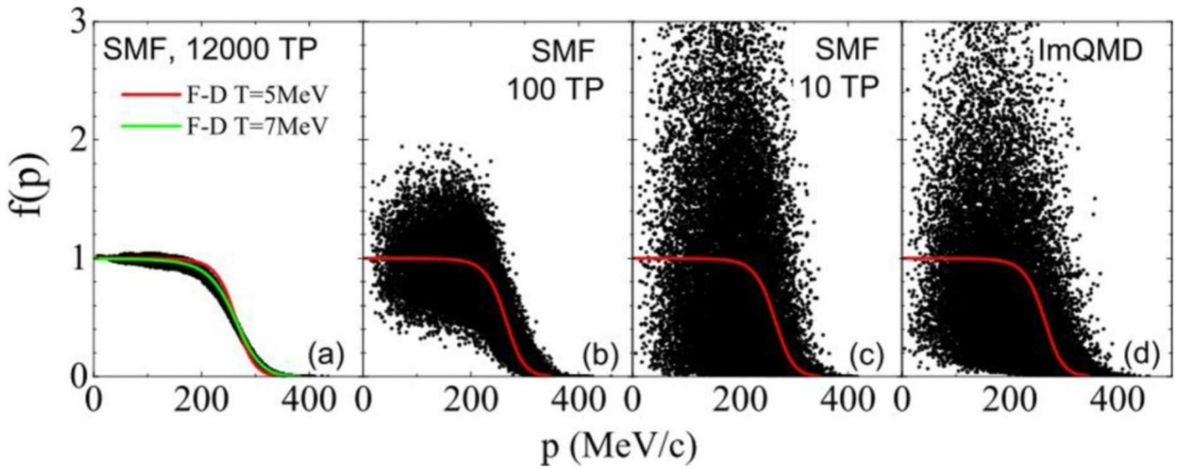
$$\langle v_{rel}\rangle = \frac{1}{4m^4 T_B K_2^2(m/T_B)} \int_{2m}^{\infty} d\sqrt{s}\, s(s-4m^2) K_1(\sqrt{s}/T_B)$$

NN scatterings with Pauli blocking

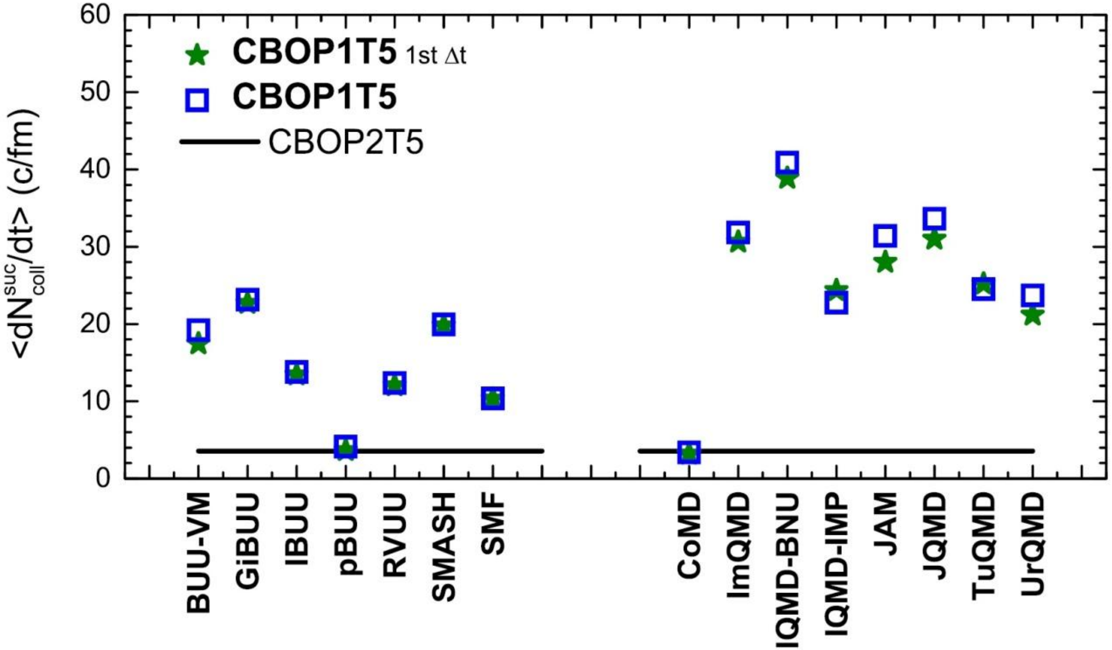
Time evolution of momentum distribution



Momentum occupation at 1st time step

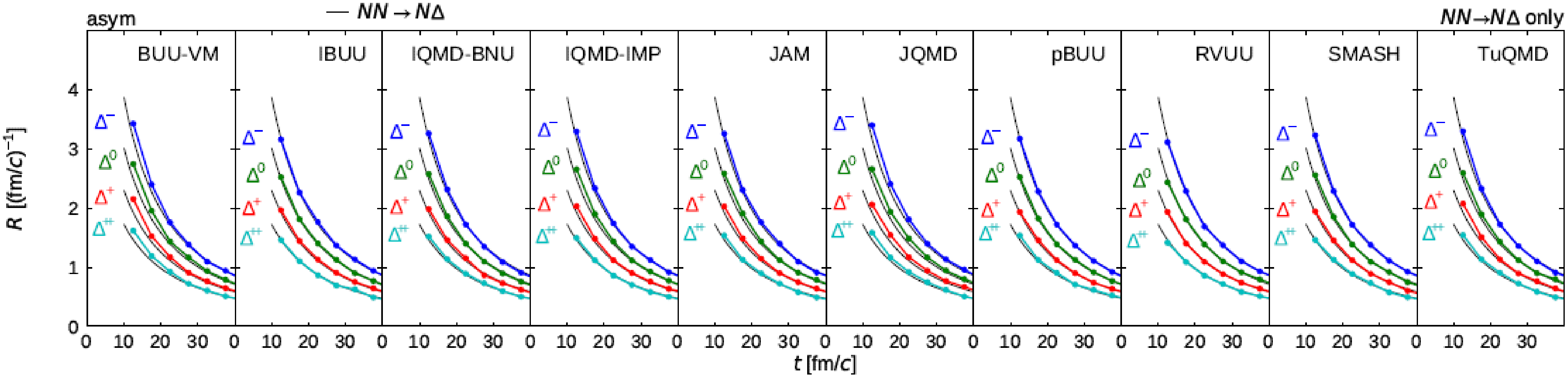


Pauli blocking rate: $1-(1-f_1)(1-f_2)$



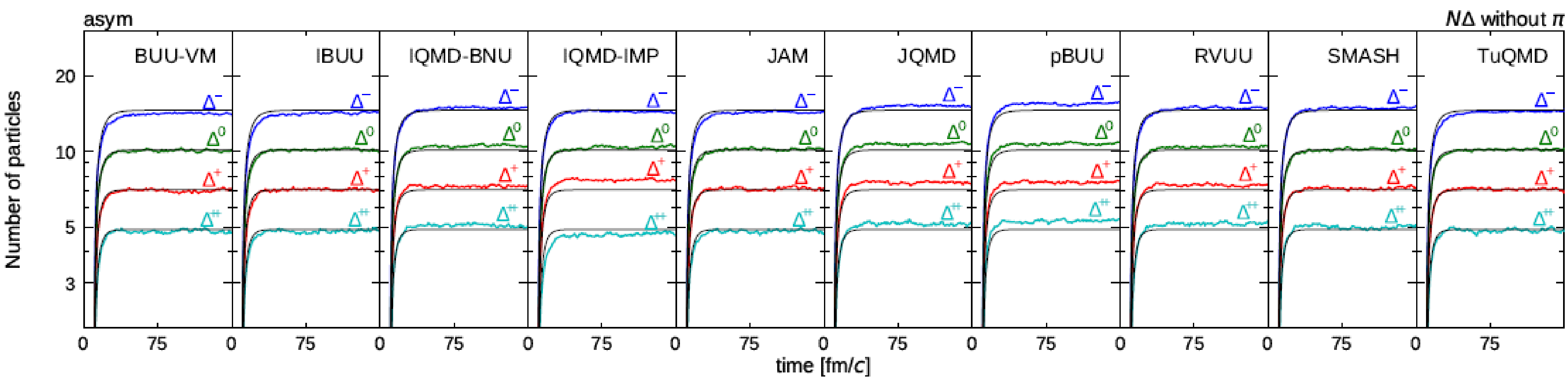
With Pauli blocking,
the successful collision rates
are much overestimated
in QMD codes than in BUU codes,
due to larger fluctuations in QMD.

N+N->N+Δ and elastic B+B<->B+B



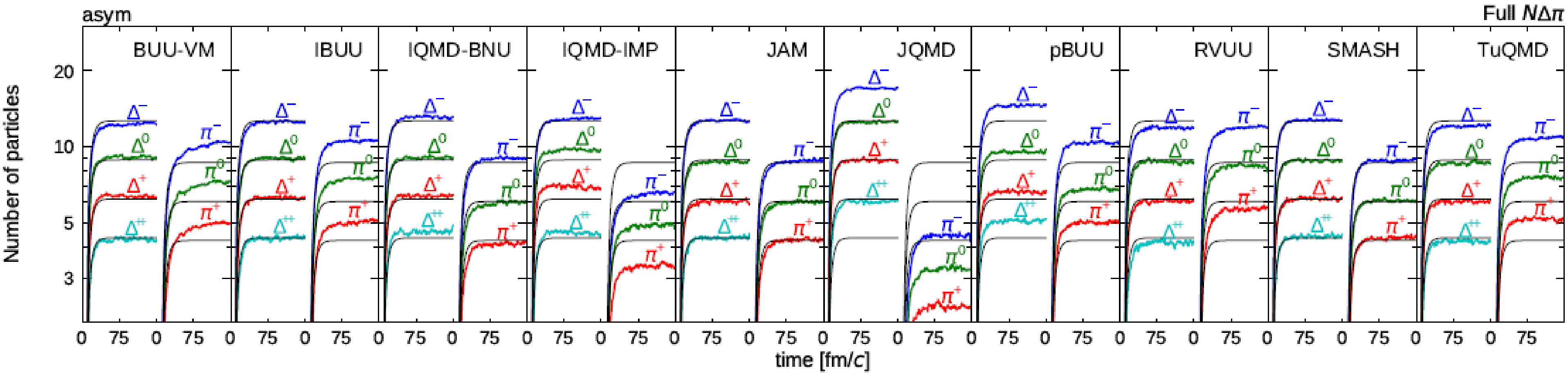
N+N<->N+Δ and elastic B+B<->B+B

detailed balance

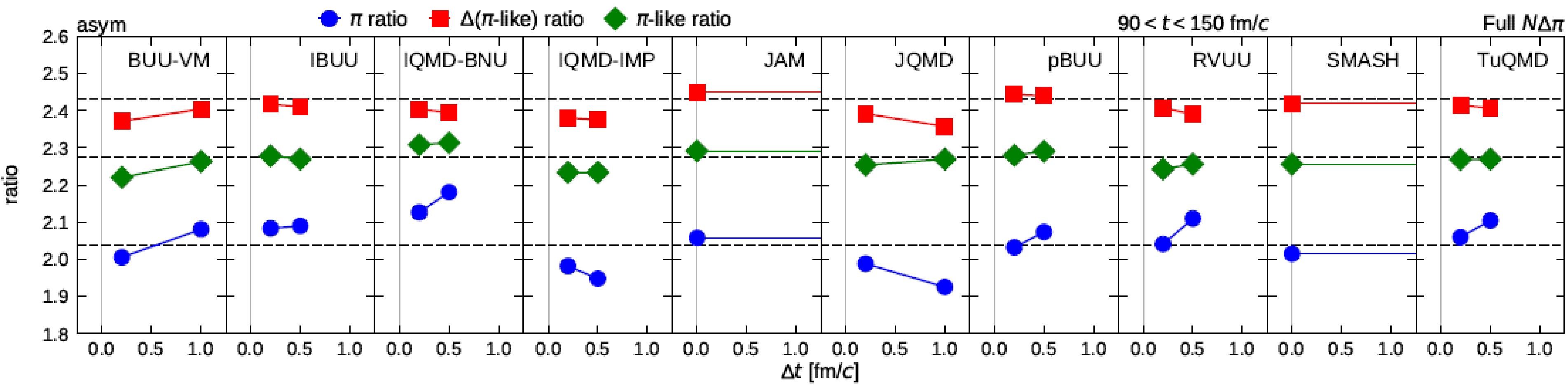


Blacking solid lines: theoretical limits from reaction rate equations/statistical model

N+N<->N+Δ, and Δ<->N+π, and elastic B+B<->B+B



Situation becomes worse with pions.



Sequence of N+N<->N+Δ and Δ<->N+π affects pion multiplicity;
Higher-order correlations lead to isospin violation in geometrical collision treatment (full ensemble method as a cure).

In progress

- Box-Vlasov calculation
- HIC-pion calculation

To be done list

- Box-Vlasov calculation with isospin
- Box-Vlasov calculation in spinodal region
- Box calculation with momentum-dependent MF
- …

Concluding remarks

Accurate knowledge of nuclear force/EOS extracted from intermediate-energy HIC needs well calibrated transport approaches.

Transport codes that (partially) participated in transport comparison/evaluation project

Boltzmann-Uehling-Uhlenbeck approach	Quantum Molecular Dynamics approach
Boltzmann-Langevin One Body (BLOB)	Antisymmetrized Molecular Dynamics (AMD)
BUU by Budapest/Rossendorf group (BUU-BR)	Constrained Molecular Dynamics (CoMD)
BUU by VECC and McGill University (BUU-VM)	Improved QMD at CIAE (ImQMD-CIAE)
BUU by Giessen group (GiBUU)	Isospin-dependent QMD (IQMD)
Hadron String Dynamics (HSD)	Isospin-dependent QMD at BNU (IQMD-BNU)
Isospin-dependent Boltzmann-Langevin (IBL)	Isospin-dependent QMD at IMP (IQMD-IMP)
Isospin-dependent BUU (IBUU)	Isospin-dependent QMD at SINAP (IQMD-SINAP)
Pawel's BUU (pBUU)	jet AA microscopic (JAM)
Relativistic BUU (RBUU)	QMD at Japan Atomic Energy Research Institute (JQMD)
Relativistic Vlasov-Uehling-Uhlenbeck (RVUU)	Tübingen QMD(TuQMD)
Simulating Many Accelerated Strongly-interacting Hadron (SMASH)	Ultra-relativistic QMD (UrQMD)
Stochastic Mean-Field (SMF)	

Acknowledgement

Lie-Wen Chen, Maria Colonna, Pawel Danielewicz,
Che Ming Ko, Akira Ono, Betty Tsang,
Yong-Jia Wang, Hermann Wolter, and Ying-Xun Zhang

and all code correspondents

Publications from transport comparison/evaluation project:

E.E. Kolomeitsev et al., J. Phys. G 31, S741 (2005)
J. Xu et al., Phys. Rev. C 93, 044609 (2016)
Y.X. Zhang et al., Phys. Rev. C 97, 034625 (2018)
A. Ono et al., arXiv: 1904.02888 [nucl-th]
J. Xu, Prog. Part. Nucl. Phys. 106, 312 (2019)

Thank you!

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See you again in the workshop that we celebrate Prof. Che Ming Ko's 60-year scientific career.