The mass splitting of vector mesons

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Motivation

- The average value of the mass of K^{*0}, $K^{*\pm}$:

$$m_{k^{*0}} = 895.55 \pm 0.20\ \mathrm{MeV}$$
$$m_{k^{*\pm}} = 891.76 \pm 0.25\ \mathrm{MeV}$$

$$\Rightarrow \quad m_{k^{*0}} - m_{k^{*\pm}} \sim 4\ \mathrm{MeV}$$

- The mass splitting between K^{*0} and $K^{*\pm}$ from PDG:

$$m_{k^{*0}} - m_{k^{*\pm}} = 6.7 \pm 1.2\ \mathrm{MeV}$$

- The $K^{*\pm}$ in τ decay mass:

$$m_{k^{*\pm}} = 895.5 \pm 0.8\ \mathrm{MeV}$$

- The experimental value of the mass splitting:

$$(m_{k^{*0}} - m_{k^{*\pm}})_{\mathrm{expt}} \sim 0 \text{ to } 8\ \mathrm{MeV}$$

- Both exp. & theo. calculations are necessary to clarify the puzzling situation.

M. Tanabashi et al. [ParticleDataGroup], Phys. Rev. D 98, no. 3, 030001 (2018)
D. Epifanov et al. [Belle Collaboration], Phys. Lett. B 654, 65 (2007)

Theoretical results

- The mass splittings between the isospin multiplets are caused by two effects: (i) $m_u \neq m_d$; (ii) the electromagnetic interactions inside hadrons.

$$(m_{k^{*0}} - m_{k^{*\pm}})_{\text{theory}} = (m_{k^{*0}} - m_{k^{*\pm}})_{\text{QM}} + (m_{k^{*0}} - m_{k^{*\pm}})_{\text{EM}}$$

- Chiral constituent quark model

$$\begin{aligned} (m_{k^{*0}} - m_{k^{*\pm}})_{\text{QM}} &= 3.07 \pm 0.18\ \text{MeV} \\ (m_{k^{*0}} - m_{k^{*\pm}})_{\text{EM}} &= -1.76\ \text{MeV} \end{aligned} \quad \Rightarrow \quad (m_{k^{*0}} - m_{k^{*\pm}})_{\text{ChQM}} \sim 1.3\ \text{MeV}$$

- Chiral Perturbation Theory or Heavy Meson Effective Theory

$$(m_{k^{*0}} - m_{k^{*\pm}})_{\text{ChPT}} \sim 4.5\ \text{MeV}$$

D. N. Gao, B. A. Li, and M. L. Yan, Phys. Rev. D 56 (1997) 4115, D. N. Gao and M. L. Yan, Eur. Phys. J. A 3 (1998) 293.
J. Bijnens, P. Gosdzinsky and P. Talavera, Nucl. Phys. B 501, 495 (1997)

The coupled channel Inverse Amplitude Method

In the case of two coupled channels, the *T* matrix elements are related to *S* matrix elements

$$S = 1 + \begin{bmatrix} 2i\sigma_1 T_{11} & 2i\sqrt{\sigma_1\sigma_2}T_{12} \\ 2i\sqrt{\sigma_1\sigma_2}T_{21} & 2i\sigma_2 T_{22} \end{bmatrix}$$

The unitarity relation means the *T* matrix elements satisfy

$$\mathrm{Im}T = T\Sigma T^* \Longrightarrow \mathrm{Im}T^{-1} = -\Sigma \qquad \Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

In the chiral pertubation theory

$$T \simeq T_2 + T_4 + T_6 + \ldots$$

thus

$$T \simeq T_2(T_2 - \mathrm{Re}T_4 - iT_2\Sigma T_2)^{-1}T_2$$

If the amplitude satisfy exact perturbative unitarity, i.e.:

$$\begin{aligned} \mathrm{Im}T_2 &= 0 \\ \mathrm{Im}T_4 &= T_2\Sigma T_2 \\ &\ldots \end{aligned} \quad \Longrightarrow \quad \boxed{T \simeq T_2(T_2 - T_4)^{-1}T_2}$$

Effective Lagrangian

- The leading order chiral lagrangian

$$\mathcal{L}_2 = \frac{F_0^2}{4}\mathrm{Tr}[D_\mu U (D^\mu U)^\dagger] + \frac{F_0^2}{4}\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)$$

Include the isospin breaking effects

$$M = \begin{pmatrix} m^2_{0K^+} - m^2_{0K^0} + m^2_{0\pi^+} & 0 & 0 \\ 0 & m^2_{0K^0} - m^2_{0K^+} + m^2_{0\pi^+} & 0 \\ 0 & 0 & m^2_{0K^+} + m^2_{0K^0} - m^2_{0\pi^+} \end{pmatrix}$$

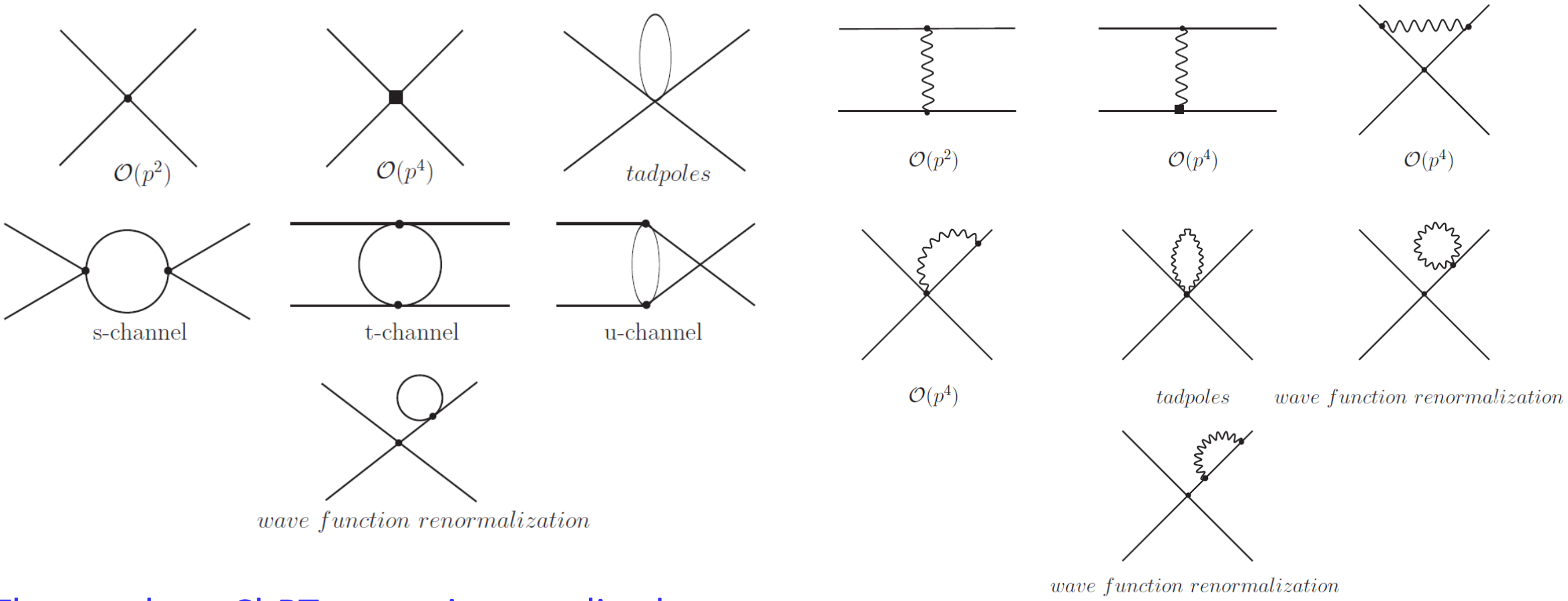
- The $\mathcal{O}(p^4)$ chiral Lagrangian

$$\begin{aligned}\mathcal{L}_4 \;=\; & L_1\{\mathrm{Tr}[D_\mu U(D^\mu U)^\dagger]\}^2 + L_2\mathrm{Tr}[D_\mu U(D_\nu U)^\dagger]\mathrm{Tr}[D^\mu U(D^\nu U)^\dagger] \\ & + L_3\mathrm{Tr}[D_\mu U(D^\mu U)^\dagger D_\nu U(D^\nu U)^\dagger] + L_4\mathrm{Tr}[D_\mu U(D^\mu U)^\dagger]\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger) \\ & + L_5\mathrm{Tr}[D_\mu U(D^\mu U)^\dagger(\chi U^\dagger + U\chi^\dagger)] + L_6[\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)]^2 \\ & + L_7[\mathrm{Tr}(\chi U^\dagger - U\chi^\dagger)]^2 + L_8\mathrm{Tr}(\chi U^\dagger\chi U^\dagger + U\chi^\dagger U\chi^\dagger) \\ & - iL_9\mathrm{Tr}[f^R_{\mu\nu}D^\mu U(D^\nu U)^\dagger + f^L_{\mu\nu}(D^\mu U)^\dagger D^\nu U]\end{aligned}$$

J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. D 59, 074001 (1999)
A. Gomez Nicola and J. R. Pelaez, Phys. Rev. D 65, 054009 (2002)

Meson-meson scattering amplitudes

Generic Feynman diagrams that have to be evaluated in meson-meson scattering up to $\mathcal{O}(p^4)$ and $\mathcal{O}(\alpha_{EM})$



The one-loop ChPT scattering amplitude:

$$T(s,t,u) = T^2_{tree}(s,t,u) + T^4_{tree}(s,t,u) + T^4_{loop}(s,t,u) + T^4_{tadpole}(s,t,u) + T^4_z(s,t,u) + T_{EM}(s,t,u)$$

Partial wave amplitudes for ρ

For ρ^+: I=1, I_3=+1

$$|\pi\pi\rangle = \frac{1}{\sqrt{2}}|\pi^+(\vec{q})\pi^0(-\vec{q})\rangle - \frac{1}{\sqrt{2}}|\pi^0(\vec{q})\pi^+(-\vec{q})\rangle$$

$$|KK\rangle = |K^+(\vec{q})\bar{K}^0(-\vec{q})\rangle$$

$$\begin{bmatrix} \pi^+\pi^0 \to \pi^+\pi^0 & \pi^+\pi^0 \to K^+\bar{K}^0 \\ K^+\bar{K}^0 \to \pi^+\pi^0 & K^+\bar{K}^0 \to K^+\bar{K}^0 \end{bmatrix}$$

- ππ → ππ scattering:

$$T^1(s,t,u) = T_{\pi^+\pi^0\to\pi^+\pi^0}(s,t,u) - T_{\pi^+\pi^0\to\pi^+\pi^0}(s,u,t)$$

- ππ → *KK*/*KK* → ππ scattering:

$$T^1(s,t,u) = \sqrt{2}T_{\pi^+\pi^0\to K^+\bar{K}^0}(s,t,u)$$

- *KK* → *KK* scattering:

$$T^1(s,t,u) = T_{K^+K^-\to K^0\bar{K}^0}(t,s,u)$$

For ρ^0: I=1, I_3=0

$$|\pi\pi\rangle = \frac{1}{\sqrt{2}}|\pi^+(\vec{q})\pi^-(-\vec{q})\rangle - \frac{1}{\sqrt{2}}|\pi^-(\vec{q})\pi^+(-\vec{q})\rangle$$

$$|KK\rangle = \frac{1}{\sqrt{2}}|K^+(\vec{q})K^-(-\vec{q})\rangle + \frac{1}{\sqrt{2}}|K^0(\vec{q})\bar{K}^0(-\vec{q})\rangle$$

$$\begin{bmatrix} \pi^+\pi^- \to \pi^+\pi^- & \pi^+\pi^- \to K^+K^- \\ K^+K^- \to \pi^+\pi^- & K^+K^- \to K^+K^-/K^+K^- \to K^0\bar{K}^0 \end{bmatrix}$$

- ππ → ππ scattering:

$$T^1(s,t,u) = T_{\pi^+\pi^-\to\pi^+\pi^-}(s,t,u) - T_{\pi^+\pi^-\to\pi^+\pi^-}(s,u,t)$$

- ππ → *KK*/*KK* → ππ scattering:

$$T^1(s,t,u) = T_{\pi^+\pi^-\to k^+k^-}(s,t,u) - T_{\pi^+\pi^-\to k^+k^-}(s,u,t)$$

- *KK* → *KK* scattering:

$$T^1(s,t,u) = T_{K^+K^-\to K^+K^-}(s,t,u) + T_{K^+K^-\to K^0\bar{K}^0}(s,t,u)$$

Partial wave amplitudes for K^*

For K^{*+}: I=1/2, I_3=+1/2

$$|K\pi\rangle = \sqrt{\frac{2}{3}}|\pi^+(\vec{q})K^0(-\vec{q})\rangle - \frac{1}{\sqrt{3}}|\pi^0(\vec{q})K^+(-\vec{q})\rangle$$

$$|K\eta\rangle = |K^+(\vec{q})\eta(-\vec{q})\rangle$$

$$\begin{bmatrix} K^+\pi^0 \to K^+\pi^0/K^0\pi^+ \to K^0\pi^+ & K^+\pi^0 \to K^+\eta \\ K^+\eta \to K^+\pi^0 & K^+\eta \to K^+\eta \end{bmatrix}$$

- $K\pi \to K\pi$ scattering:

$$T^{\frac{1}{2}}(s,t,u) = 2T_{K^0\pi^+\to K^0\pi^+}(s,t,u) - T_{K^+\pi^0\to K^+\pi^0}(s,t,u)$$

- $K\pi \to K\eta/K\eta \to K\pi$ scattering:

$$T^{\frac{1}{2}}(s,t,u) = -\sqrt{3}T_{K^+\pi^0\to K^+\eta}(s,t,u)$$

- $K\eta \to K\eta$ scattering:

$$T^{\frac{1}{2}}(s,t,u) = T_{K^+\eta\to K^+\eta}(s,t,u)$$

For K^{*0}: I=1/2, I_3=-1/2

$$|K\pi\rangle = -\sqrt{\frac{2}{3}}|\pi^-(\vec{q})K^+(-\vec{q})\rangle + \frac{1}{\sqrt{3}}|\pi^0(\vec{q})K^0(-\vec{q})\rangle$$

$$|K\eta\rangle = |K^0(\vec{q})\eta(-\vec{q})\rangle$$

$$\begin{bmatrix} K^+\pi^- \to K^+\pi^-/K^0\pi^0 \to K^0\pi^0 & K^0\pi^0 \to K^0\eta \\ K^0\eta \to K^0\pi^0 & K^0\eta \to K^0\eta \end{bmatrix}$$

- $K\pi \to K\pi$ scattering:

$$T^{\frac{1}{2}}(s,t,u) = 2T_{\pi^+\pi^-\to K^+K^-}(t,s,u) - T_{K^0\pi^0\to K^0\pi^0}(s,t,u)$$

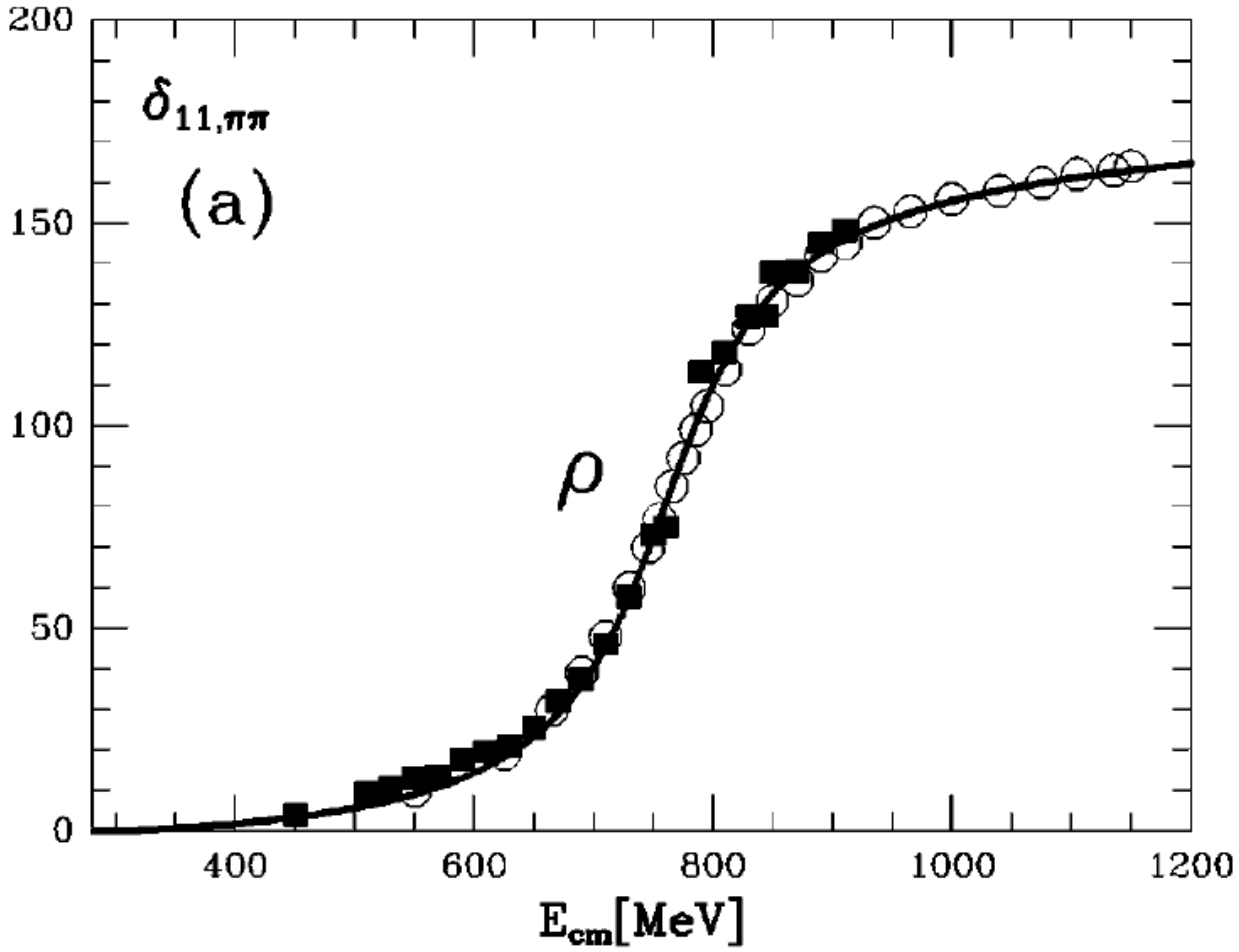
- $K\pi \to K\eta/K\eta \to K\pi$ scattering:

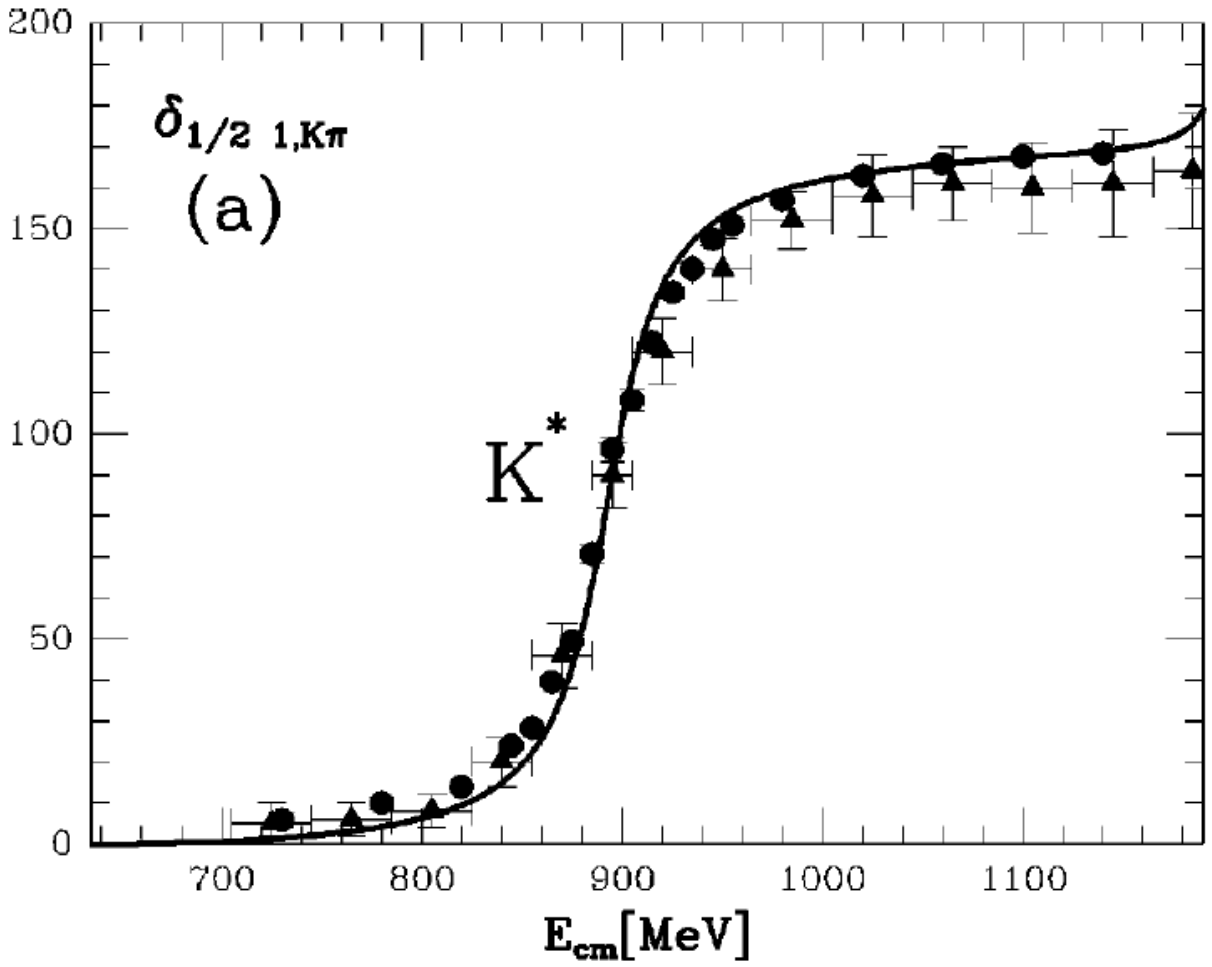
$$T^{\frac{1}{2}}(s,t,u) = \sqrt{3}T_{K^0\pi^0\to K^0\eta}(s,t,u)$$

- $K\eta \to K\eta$ scattering:

$$T^{\frac{1}{2}}(s,t,u) = T_{K^0\eta\to K^0\eta}(s,t,u)$$

Phase shifts





$$\delta_{IJ}(M_R) = 90^0 \quad , \quad \Gamma_R = \frac{1}{M_R}\left(\frac{d\delta_{IJ}}{ds}\right)^{-1}_{s=M_R^2}$$

$$M_\rho = 775.7^{+4.3}_{-3.3}\ \text{MeV}$$

$$M_{K^*} = 889 \pm 5\ \text{MeV}$$

J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. D 59, 074001 (1999) A. Gomez Nicola and J. R. Pelaez, Phys. Rev. D 65, 054009 (2002)

Low energy constants

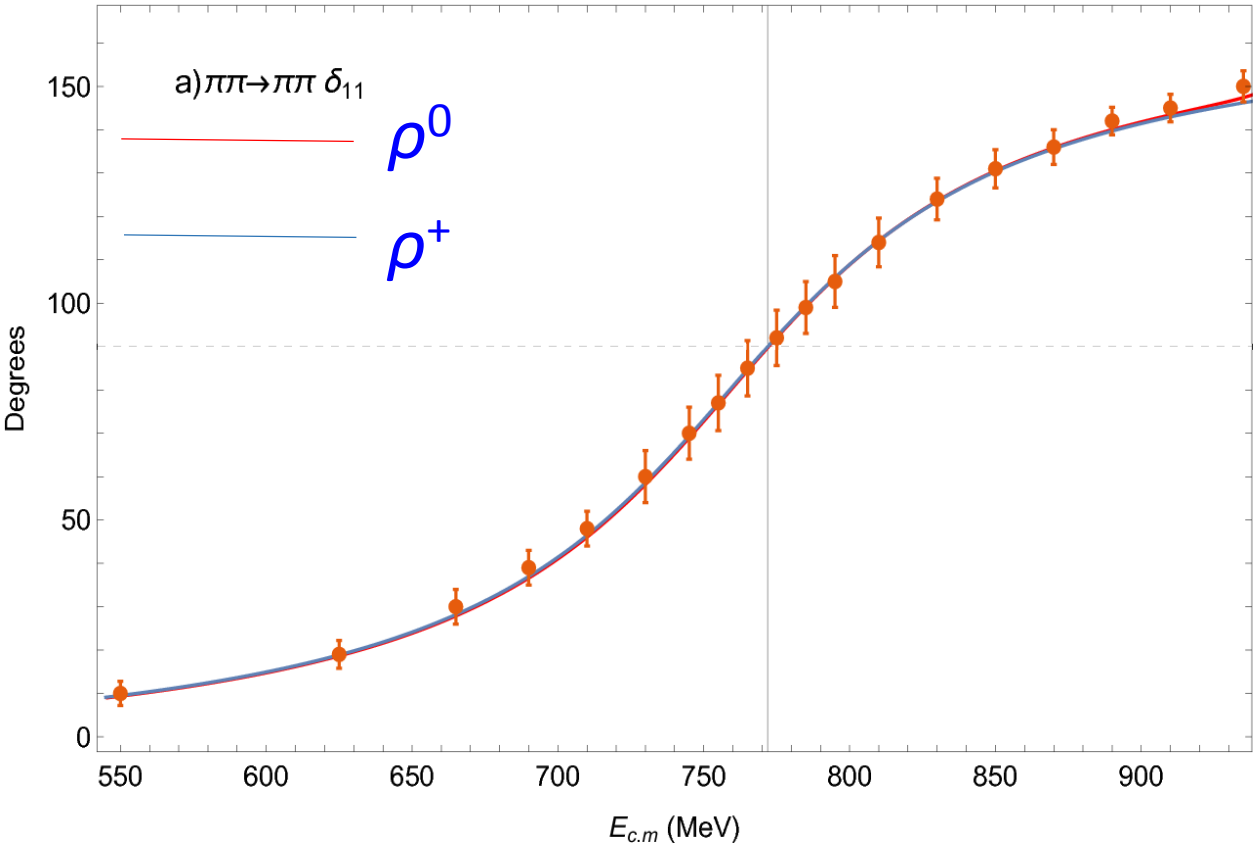
	Ref	Our Fit	ChPT
$L^r_1 \times 10^3$	0.56 ± 0.10	0.601 ± 0.002	0.64 ± 0.06
$L^r_2 \times 10^3$	1.21 ± 0.10	1.30 ± 0.01	0.59 ± 0.04
$L^r_3 \times 10^3$	-2.79 ± 0.14	-2.81 ± 0.01	-2.80 ± 0.20
$L^r_4 \times 10^3$	-0.36 ± 0.17	-0.48 ± 0.03	0.76 ± 0.18
$L^r_5 \times 10^3$	1.4 ± 0.5	2.0 ± 0.1	0.50 ± 0.07
$L^r_6 \times 10^3$	0.07 ± 0.08	6.96 ± 1.52	0.49 ± 0.25
$L^r_7 \times 10^3$	-0.44 ± 0.15	-0.43 ± 0.15	-0.19 ± 0.08
$L^r_8 \times 10^3$	0.78 ± 0.18	0.78 ± 0.13	0.17 ±0.11
$L^r_9 \times 10^3$		1.06 ± 0.67	5.93 ± 0.43
χ^2/d.o.f	$\mathcal{O}(1)$	1.63	0.5

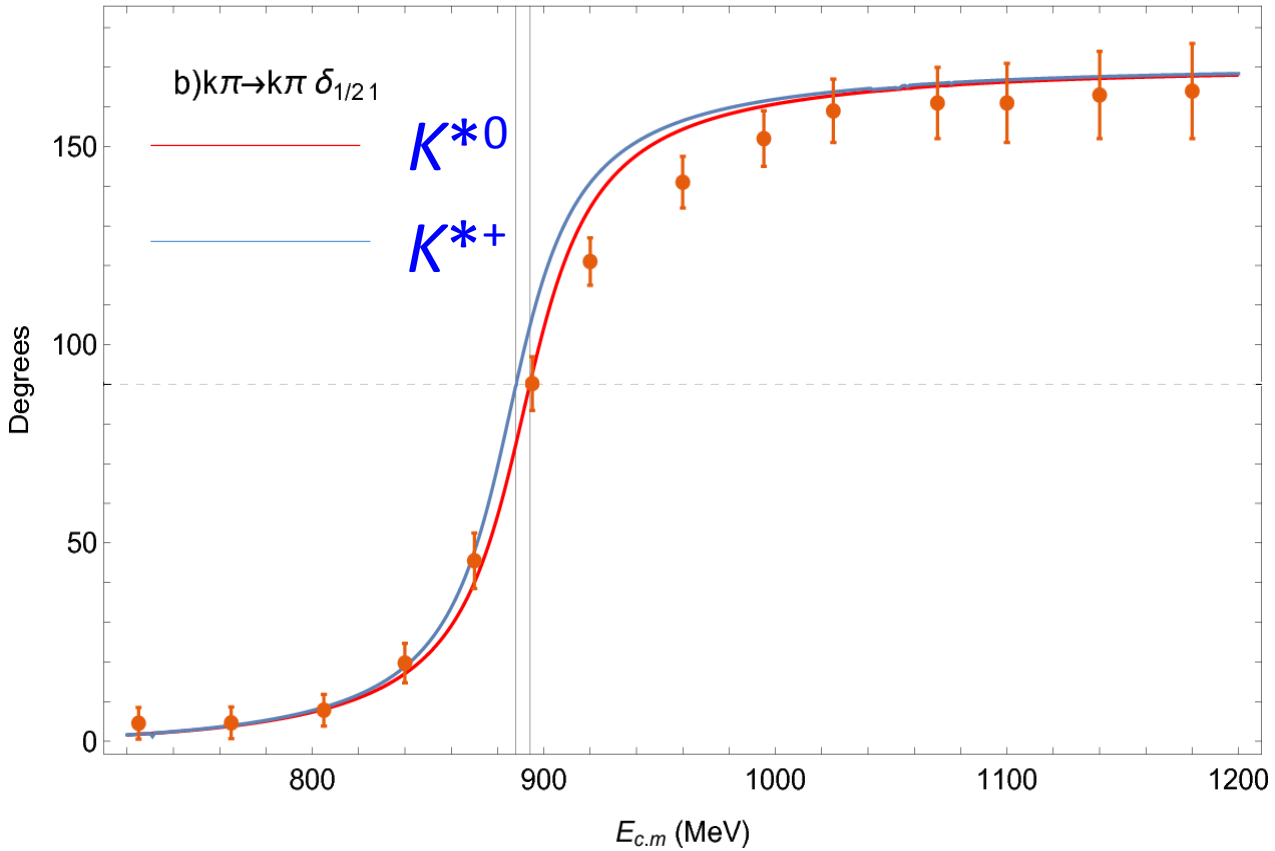
Fitting scheme:

- General free fit without any restrictions on the L^r_i.
- Θ cut is π/3 for the electromagnetic contributions in the amplitudes.
- L^r_6 may be too large, a better fit should be performed.

A. Gomez Nicola and J. R. Pelaez, Phys. Rev. D 65, 054009 (2002)
J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64 (2014) 149

Preliminary results





$$m_{\rho^0} = m_{\rho^\pm} = 773.4\,\mathrm{MeV}$$

$$m_{\rho^0} - m_{\rho^\pm} \sim 0\,\mathrm{MeV}$$

$$m_{k^{*\pm}} = 888.4\,\mathrm{MeV}$$

$$m_{k^{*0}} = 894.5\,\mathrm{MeV}$$

$$m_{k^{*0}} - m_{k^{*\pm}} \sim 6.1\,\mathrm{MeV}$$

Summary

- Within the framework of SU(3) Chiral Perturbation Theory, we calculate the isospin breaking effects on the vector meson mass splittings via the meson meson scatterings to one loop level.

- It should be possible that BESIII can provide precise measurement of the K^* mass splitting and help clarify the inconsistent situation exposed by the PDG analysis.

Thanks for your attention!