



天津大学  
Tianjin University

# Correlations between exotic hadrons and threshold effects

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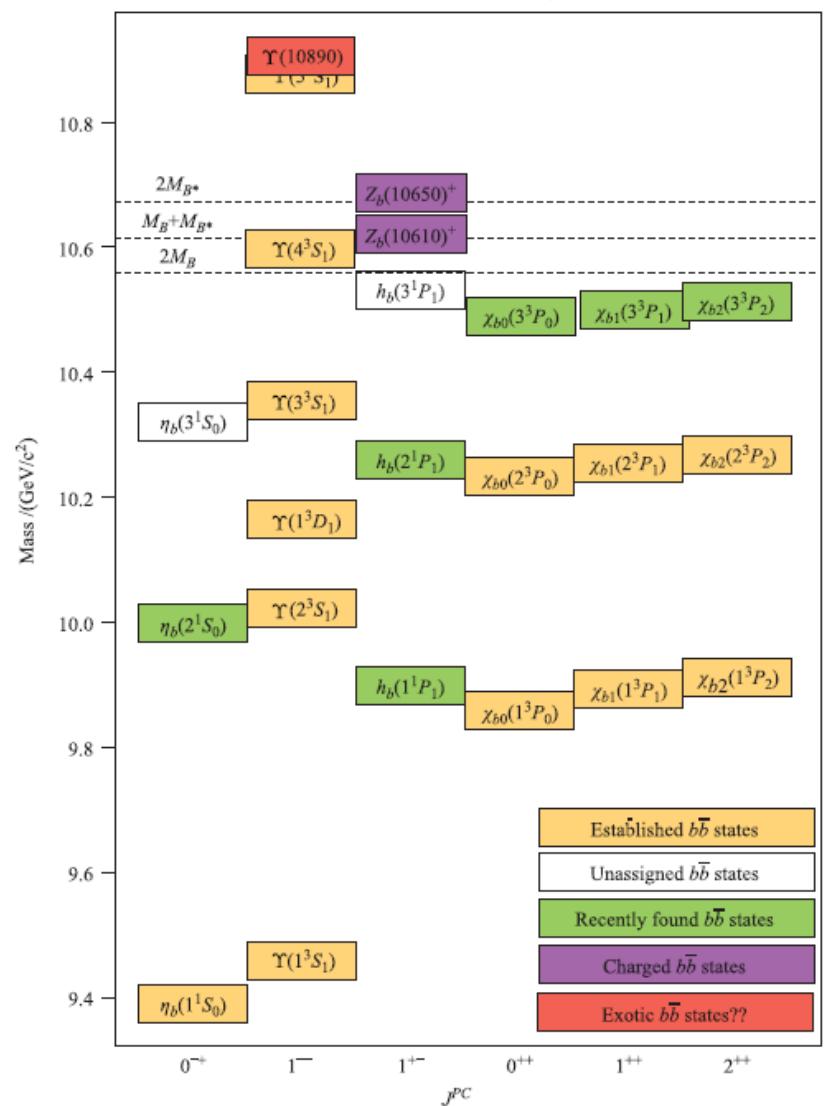
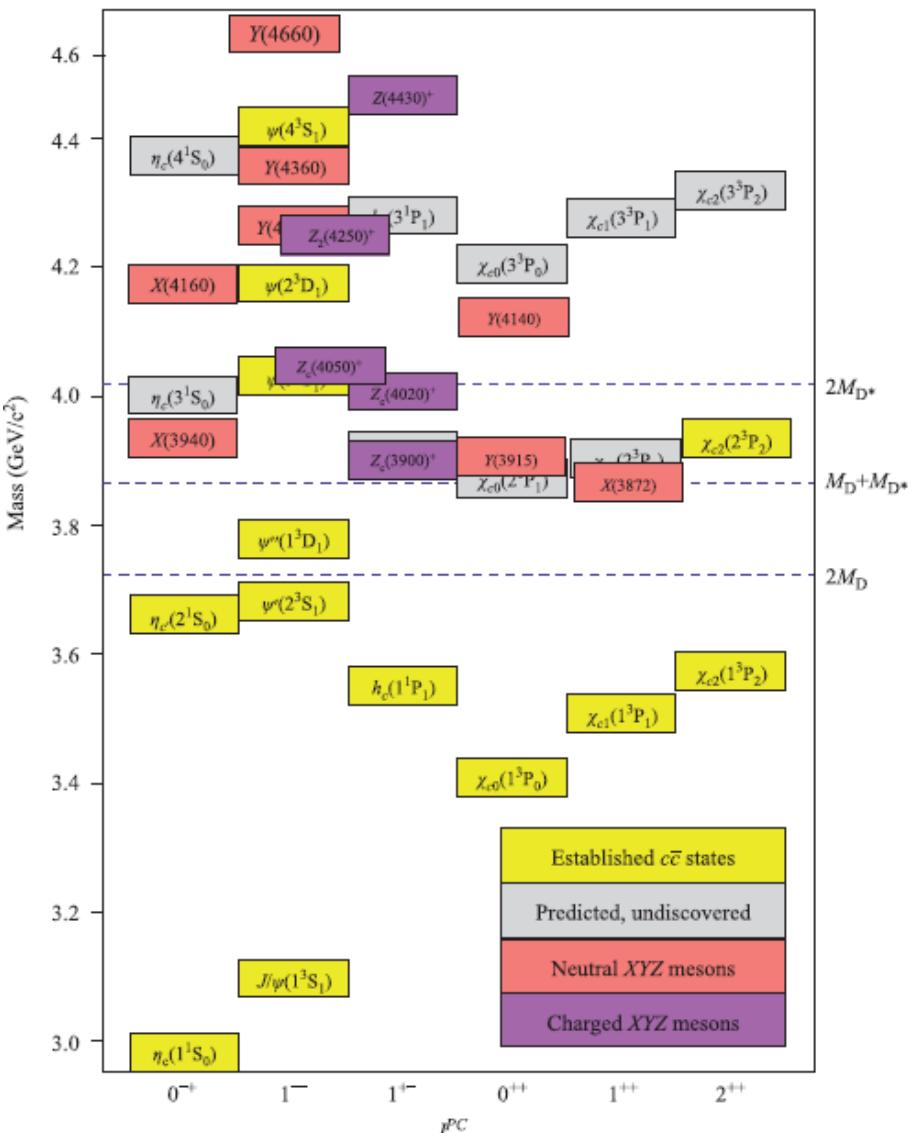
第十八届中高能核物理大会，湖南师范大学，长沙，2019年6月

# Outline

- **Brief introduction to exotic hadrons**
- **Cusp effect**
- **Triangle singularity (TS) phenomena**
- **Distinguish kinematic singularities from dynamic poles**
- **Summary**

# Dozens of Exotic Hadron are observed since 2003

S. Olsen, Front.Phys.10,121



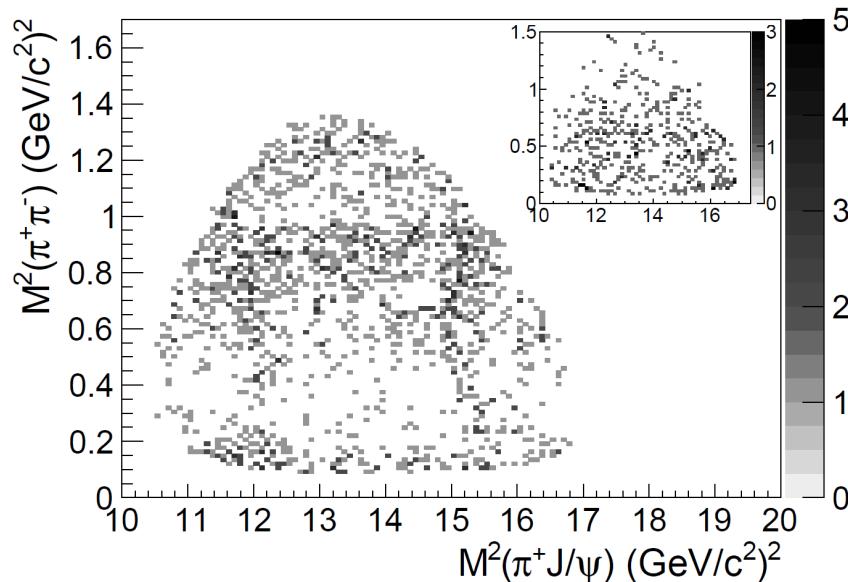
Renaissance of Hadron Spectroscopy!

# Theoretical Interpretation

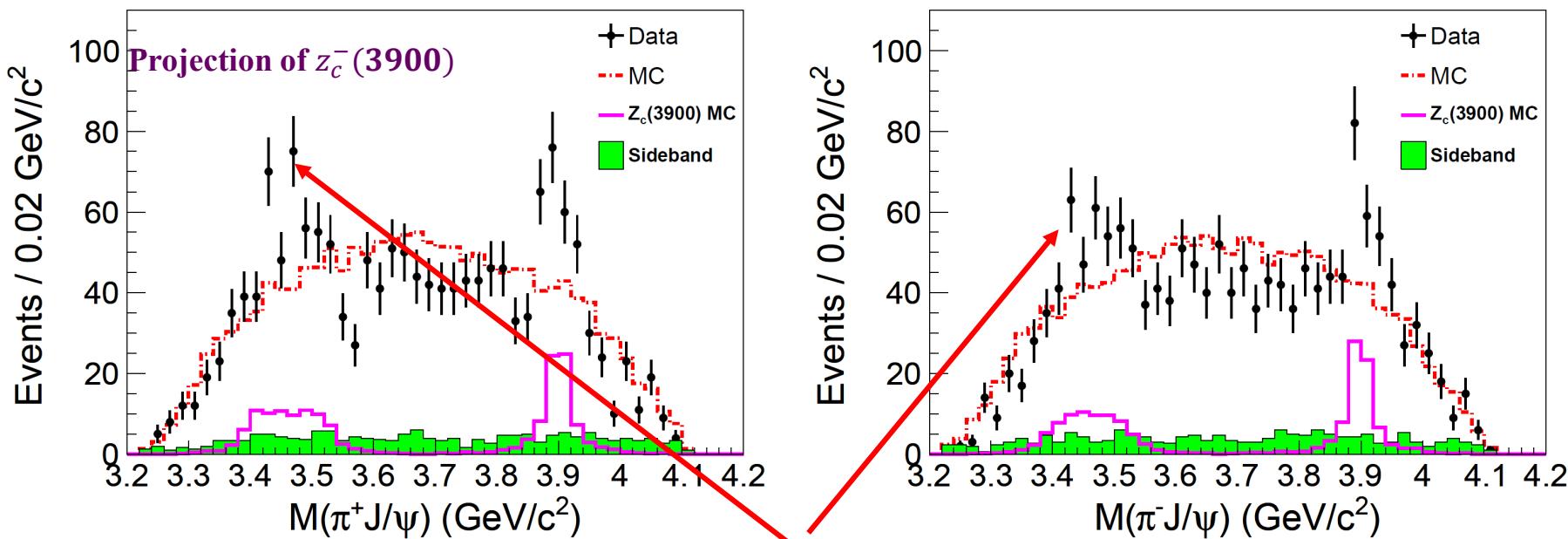
- ✓ Hadronic molecule
  - ✓ Tetraquark, Pentaquark
  - ✓ Hybrid
  - ✓ Hadrocharmonium
  - ✓ Threshold effect (cusp, triangle singularity, ...) (*Non-resonance interpretation*)
- Genuine resonance interpretations*

“Resonance-like” structure    $\stackrel{?}{=}$    Genuine particle

# “Resonance-like” structure $\neq$ Genuine particle



BESIII



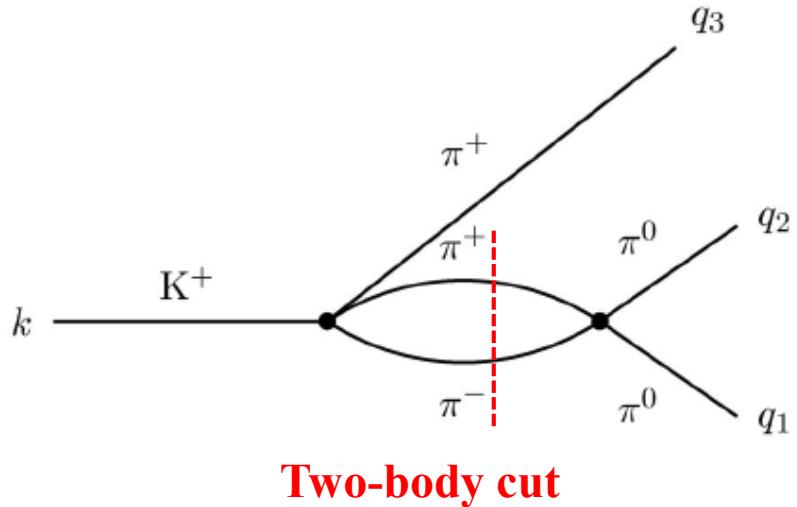
“Reflection” in Dalitz plot

# Cusp effect

E.P. Wigner, “*On the Behavior of Cross Sections Near Thresholds*”, PR73, 1002 (1948)

See also Feng-Kun Guo’s talk

Induced by the charge-exchange rescattering  $\pi^+\pi^- \rightarrow \pi^0\pi^0$



Two-body cut

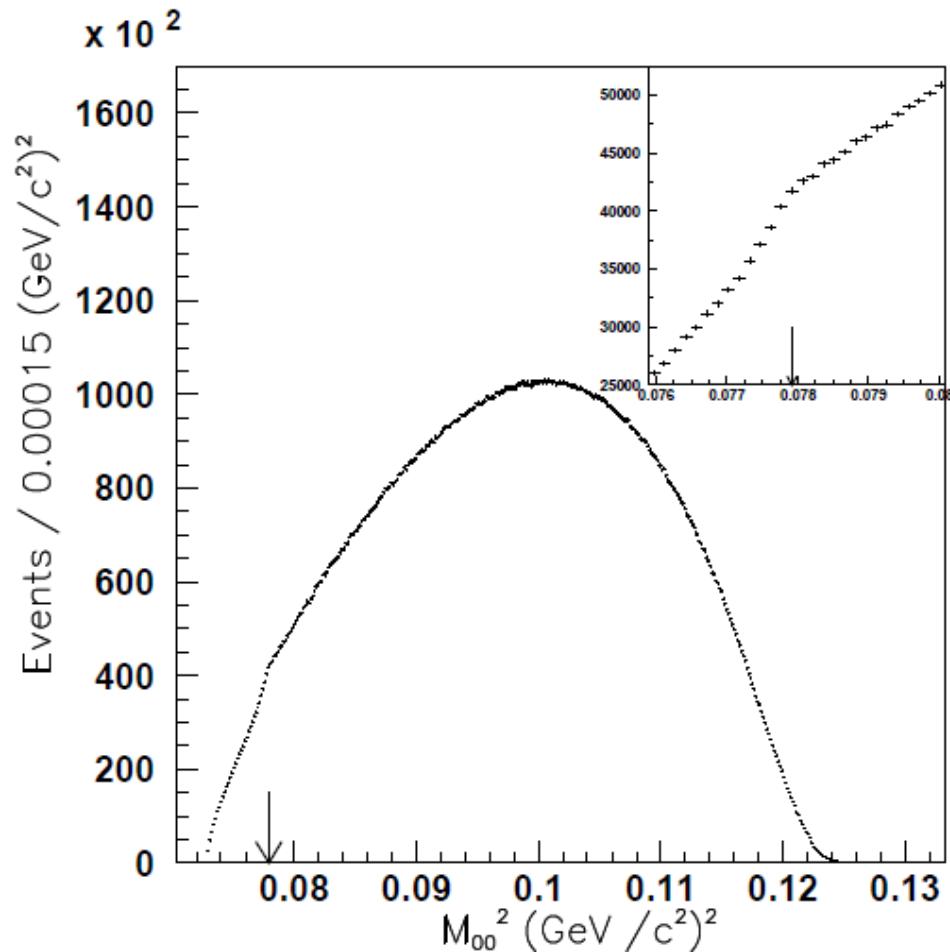
Budini & Fonda, PRL6,419(1961);  
Cabibbo, PRL93,121801(2004);

Branching ratio

$K^+ \rightarrow \pi^+\pi^-\pi^+$  ((5.59 ± 0.04)%)

much larger than

$K^+ \rightarrow \pi^0\pi^0\pi^+$  ((1.761 ± 0.022)%)



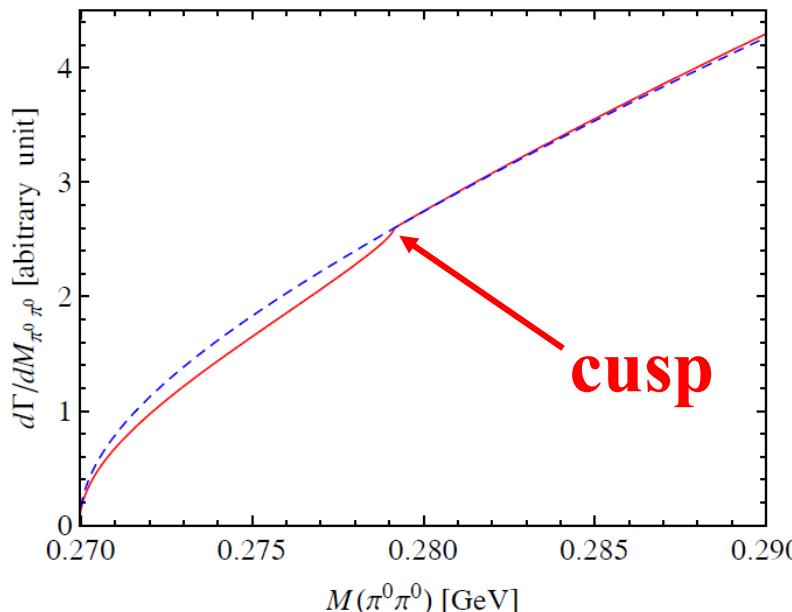
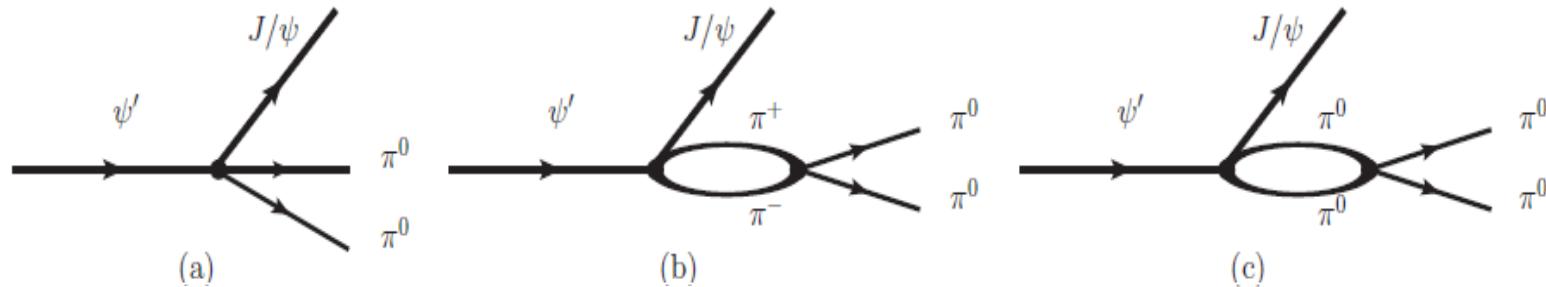
NA48/2, PLB633,173 (2006)  
 $6 \times 10^7$  events

# Cusp effect

Could be used to extract the  $\pi\pi$  scattering length

$K \rightarrow 3\pi$ , one of the most accurate experiments

Heavy quarkonium dipion transitions:  $\psi' \rightarrow J/\psi \pi^0 \pi^0$  ,  
 $Y(3S) \rightarrow Y(2S) \pi^0 \pi^0$  (better)



X.H Liu, F.K. Guo, E. Epelbaum,  
EPJC73,2284(2013)

Difficult to observe: high statistics; perfect energy resolution; clear background

# Cusp effect

- Possible correlation with some XYZ states:  $Z_b(10610/10650)$ ,  $Z_c(3900)$ ,  $Z_c(4020)$

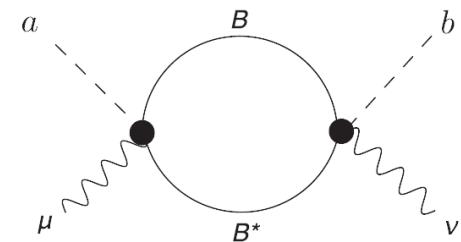
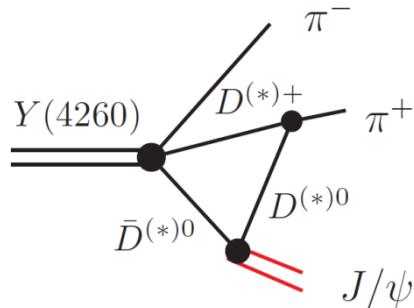
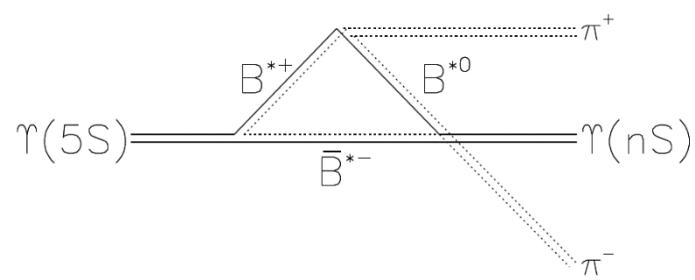
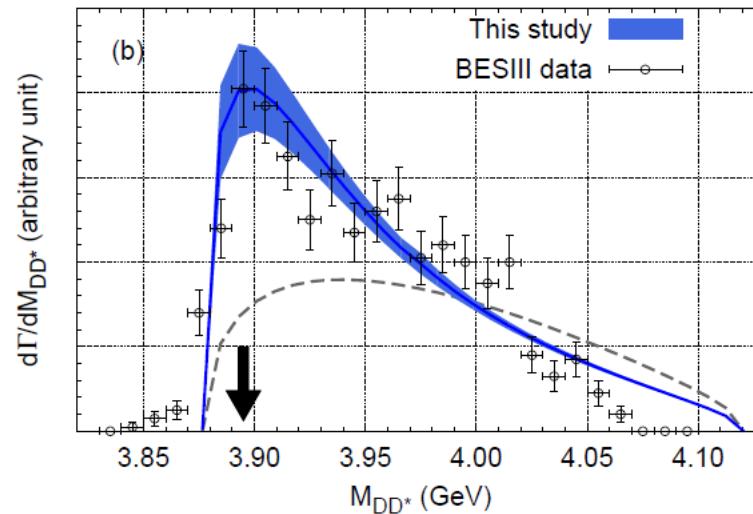
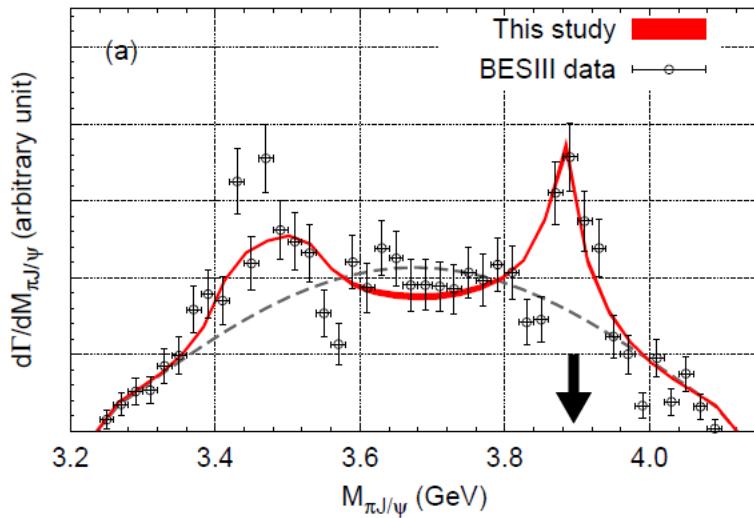


FIG. 1. Coupled channels in  $\Upsilon\pi$  scattering.

**D.V. Bugg,  
EPL96, 11002(2011)**

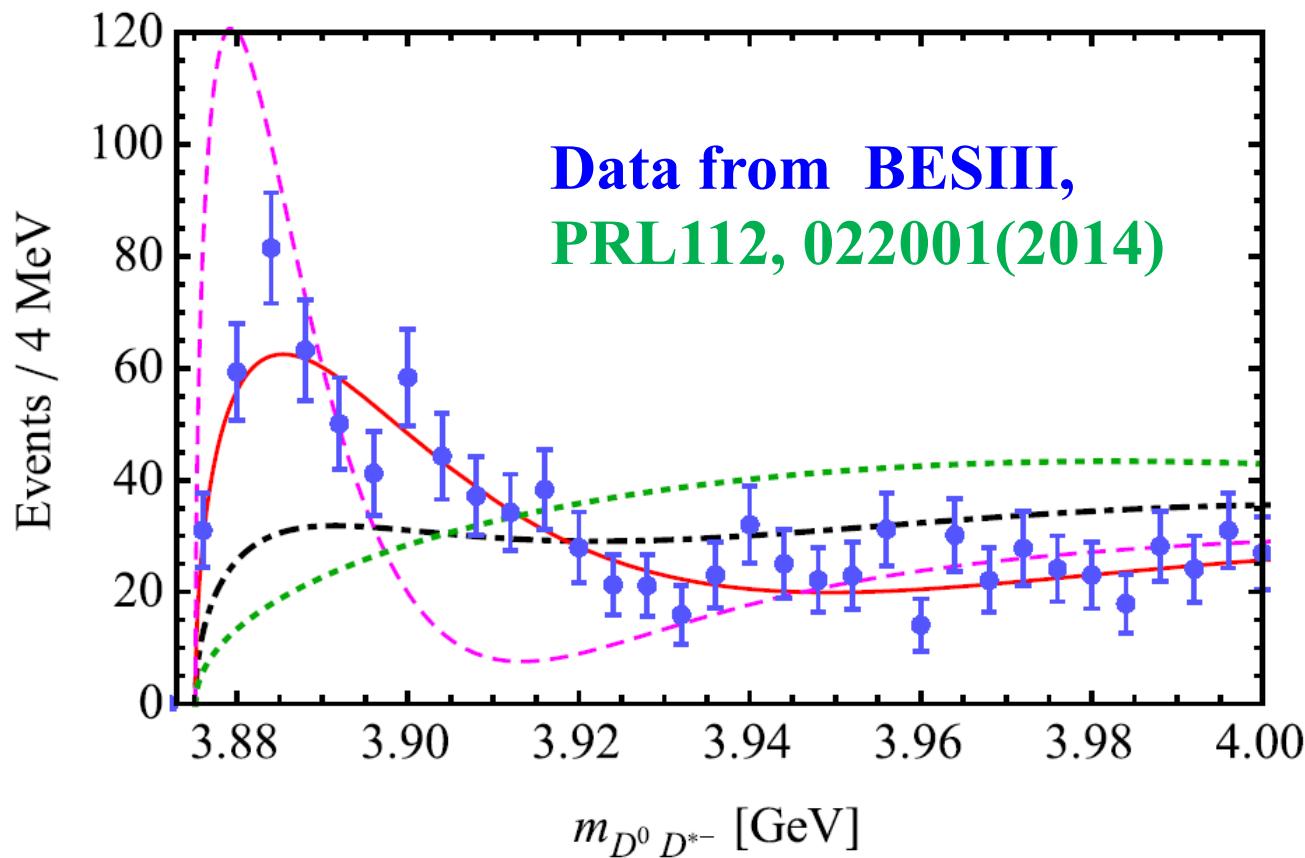
**D.Y. Chen, X. Liu,  
PRD88, 11002(2013)**

**E. Swanson,  
PRD91, 034009(2015)**

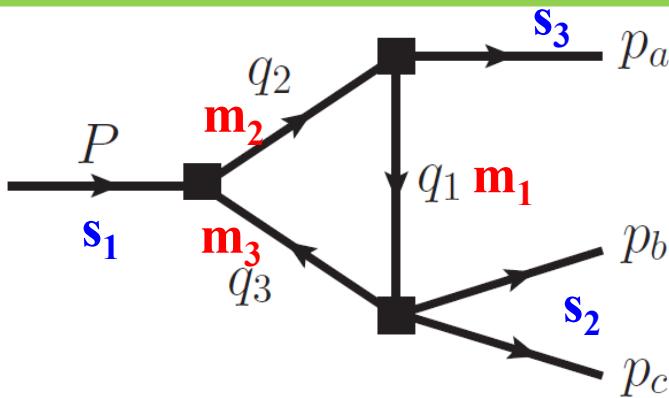


# Cusp effect

- A sharp peak **cannot** be resulted by a pure threshold cusp in the elastic channel [Guo, Hanhart, Wang, Zhao, PRD91, 051504(2015)]:  $Z_c(3900)$  was also observed in the  $DD^*$  invariant mass distributions



# Triangle Singularity Mechanism



$$P^2 = s_1, (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

$$\Gamma_3(s_1, s_2, s_3) = \frac{-1}{16\pi^2} \int_0^1 \int_0^1 \int_0^1 da_1 da_2 da_3 \frac{\delta(1 - a_1 - a_2 - a_3)}{D - i\epsilon}$$

$$D = \sum_{i,j=1}^3 a_i a_j Y_{ij}, \quad Y_{ij} = \frac{1}{2} [m_i^2 + m_j^2 - (q_i - q_j)^2]$$

✓ Singularity in the complex space

Necessary conditions (Landau Equation)

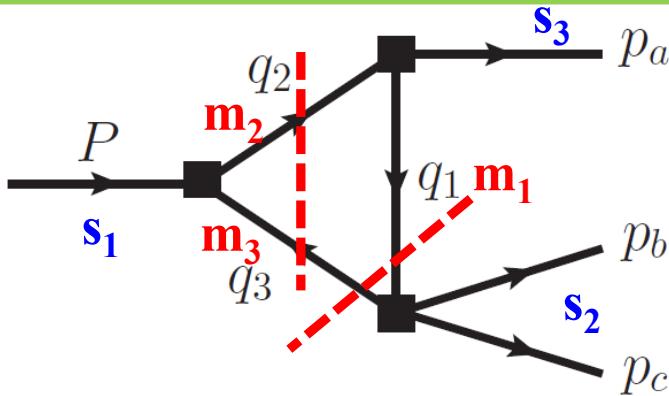
$$D = 0,$$

either  $a_j = 0$  or  $\frac{\partial D}{\partial a_j} = 0$ .

Leading singularity

Landau, Nucl.Phys.13,181(1959)

# Triangle Singularity Mechanism



$$P^2 = s_1, (p_b + p_c)^2 = s_2$$

$$p_a^2 = s_3$$

✓ Singularity in the complex space

The position of the singularity is obtained by solving

$$\det[Y_{ij}] = 0$$

Normal Threshold

$$s_2^\pm = (m_1 + m_3)^2 + \frac{1}{2m_2^2} [(m_1^2 + m_2^2 - s_3)(s_1 - m_2^2 - m_3^2) - 4m_2^2 m_1 m_3]$$

$s_1, s_3, m_{1,2,3}$  fixed

$$\pm \lambda^{1/2}(s_1, m_2^2, m_3^2) \lambda^{1/2}(s_3, m_1^2, m_2^2)], \quad \lambda(x, y, z) \equiv (x - y - z)^2 - 4yz$$

Anomalous Threshold

$$s_1^\pm = (m_2 + m_3)^2 + \frac{1}{2m_1^2} [(m_1^2 + m_2^2 - s_3)(s_2 - m_1^2 - m_3^2) - 4m_1^2 m_2 m_3]$$

$$\pm \lambda^{1/2}(s_2, m_1^2, m_3^2) \lambda^{1/2}(s_3, m_1^2, m_2^2)].$$

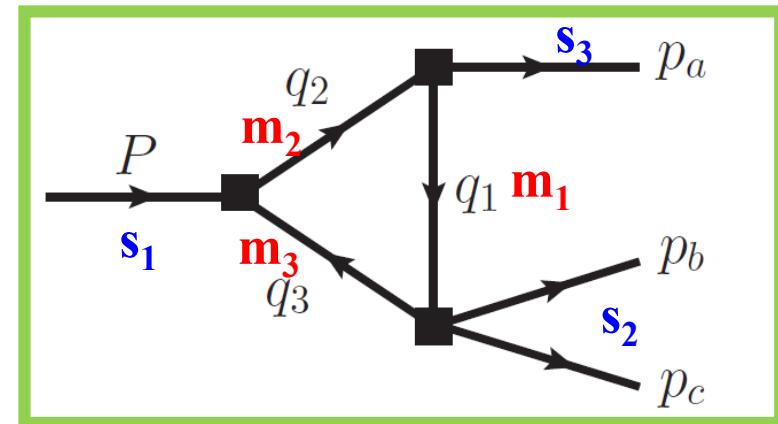
$s_2, s_3, m_{1,2,3}$  fixed

# Triangle Singularity Mechanism

## Single dispersion representation

$$\Gamma_3(s_1, s_2, s_3) = \frac{1}{\pi} \int_{(m_1+m_3)^2}^{\infty} \frac{ds'_2}{s'_2 - s_2 - i\epsilon} \sigma(s_1, s'_2, s_3)$$

$$\sigma(s_1, s_2, s_3) = \sigma_+ - \sigma_-$$



$$\begin{aligned} \sigma_{\pm}(s_1, s_2, s_3) &= \frac{-1}{16\pi\lambda^{1/2}(s_1, s_2, s_3)} \log[-s_2(s_1 + s_3 - s_2 + m_1^2 + m_3^2 - 2m_2^2) \\ &\quad - (s_1 - s_3)(m_1^2 - m_3^2) \pm \lambda^{1/2}(s_1, s_2, s_3)\lambda^{1/2}(s_2, m_1^2, m_3^2)]. \end{aligned}$$

## Work in the kinematical region

$$s_1 \leq (m_2 + m_3)^2, \quad s_3 \leq (m_2 - m_1)^2 \quad 0 < s_2 < (m_1 + m_3)^2$$

By analytic continuation, it can be extended into the over threshold region    Fronsdal&Norton,J.Math.Phys.5,100(1964)

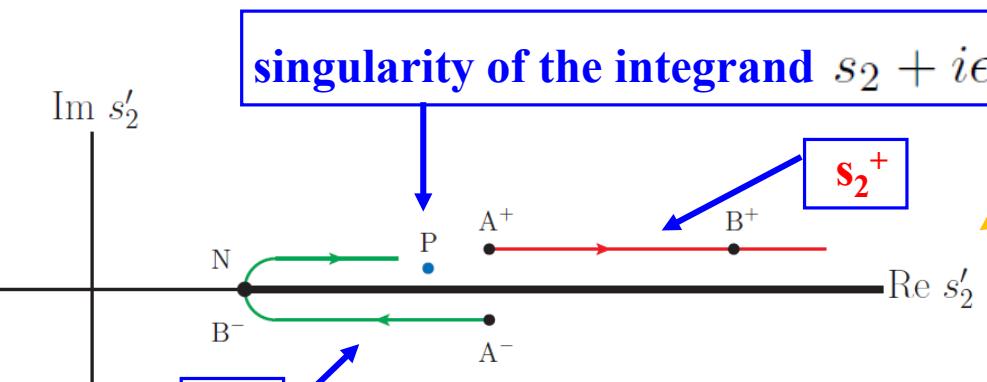
$$s_1 \geq (m_2 + m_3)^2, \quad (m_1 + m_3)^2 \leq s_2 \leq (\sqrt{s_1} - \sqrt{s_3})^2, \quad 0 \leq \sqrt{s_3} \leq m_2 - m_1$$

Branch points of the log function     $s_2^{\pm}$

# Triangle Singularity Mechanism

Locations of  $s_2^\pm$  in the  $s_2'$ -plane can be determined by

$$s_2^\pm(s_1 + i\epsilon) = s_2(s_1) + i\epsilon \frac{\partial s_2^\pm}{\partial s_1}$$



Trajectory of the branch points  $s_2^-$ ,  $s_2^+$  with  $s_1$  increases from normal threshold to infinity in the complex  $s_2'$ -plane

$$A^\pm s_{1N} s_1 = (m_2 + m_3)^2, \quad B^\pm : s_1 = (m_2 + m_3)^2 + \frac{m_3}{m_1} [(m_2 - m_1)^2 - s_3] = s_{1C}$$

$$A^- : s_2^- = (m_1 + m_3)^2 + \frac{m_3}{m_2} [(m_2 - m_1)^2 - s_3] - i\epsilon = s_{2C}$$

$$B^- : s_2^- = (m_1 + m_3)^2 = s_{2N}$$

$s_2^-$  and P will pinch the integral contour

This pinch singularity is the triangle singularity (TS)

# TS Kinematic Region

## Normal threshold and critical point

$$s_{1N} = (m_2 + m_3)^2, \quad s_{1C} = (m_2 + m_3)^2 + \frac{m_3}{m_1} [(m_2 - m_1)^2 - s_3],$$

$$s_{2N} = (m_1 + m_3)^2, \quad s_{2C} = (m_1 + m_3)^2 + \frac{m_3}{m_2} [(m_2 - m_1)^2 - s_3],$$

$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$

$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

When  $s_2 = s_{2N}$

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2 + m_3)} [(m_2 - m_1)^2 - s_3],$$

When  $s_1 = s_{1N}$

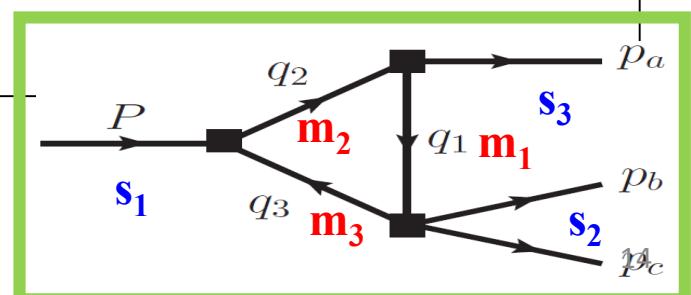
Discrepancy between anomalous and normal threshold

Largest discrepancy

$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \boxed{\frac{m_3}{2m_2(m_1 + m_3)} [(m_2 - m_1)^2 - s_3]}.$$

How to amplify the discrepancy between normal and anomalous threshold?

Liu, Oka, Zhao, PLB753,297 (2016)

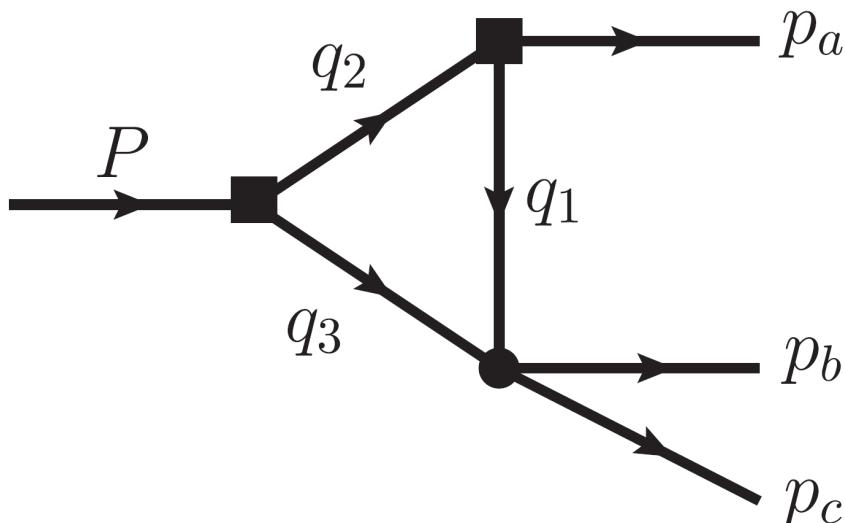


# Triangle Singularity Mechanism

“The kinematic conditions for the existence of singularities on the physical boundary are equivalent to the condition that the relevant Feynman diagram be interpretable as a picture of an energy and momentum-conserving process occurring in space-time, with all internal particles real, on the mass shell and moving forward in time.” —*Coleman-Norton theorem*

Coleman&Norton, Nuovo Cimento 38,5018 (1965)

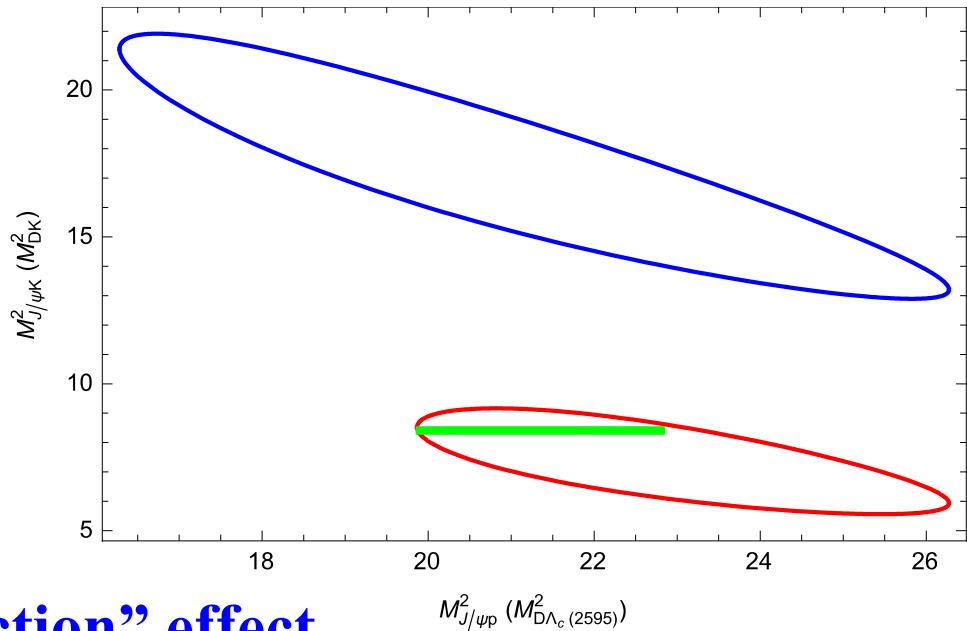
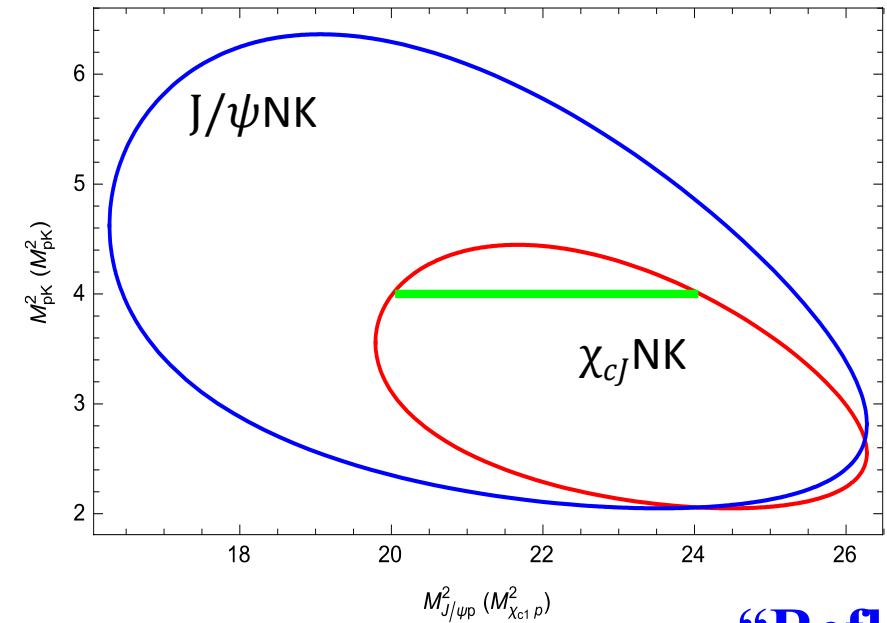
Fronsdal&Norton,J.Math.Phys.5, 100(1964)



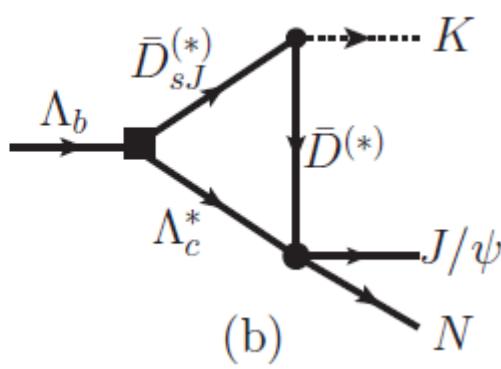
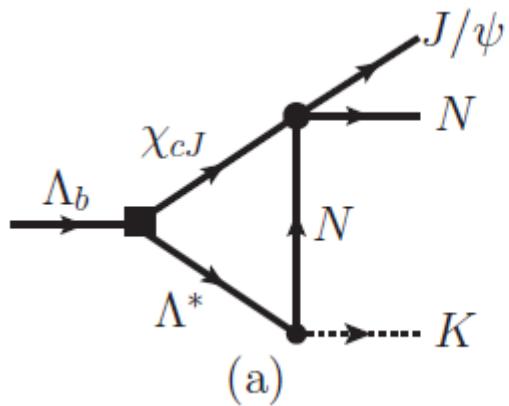
Width effect

Complex mass scheme:  $m_2 = m - i\Gamma/2$

# TS mechanism: Dalitz plot



**“Reflection” effect**



# Triangle Singularity Mechanism

*Early study in 1960s*

- Connections between kinematic singularities of the S-matrix elements and resonance-like peaks: e.g. Peierls mechanism

R.F.Peierls, PRL6,641(1961);

R.C.Hwa, PhysRev130,2580(1963);

C.Goebel,PRL13,143(1964);

P.Landshoff&S.Treiman Phys.Rev.127,649(1962);

.....

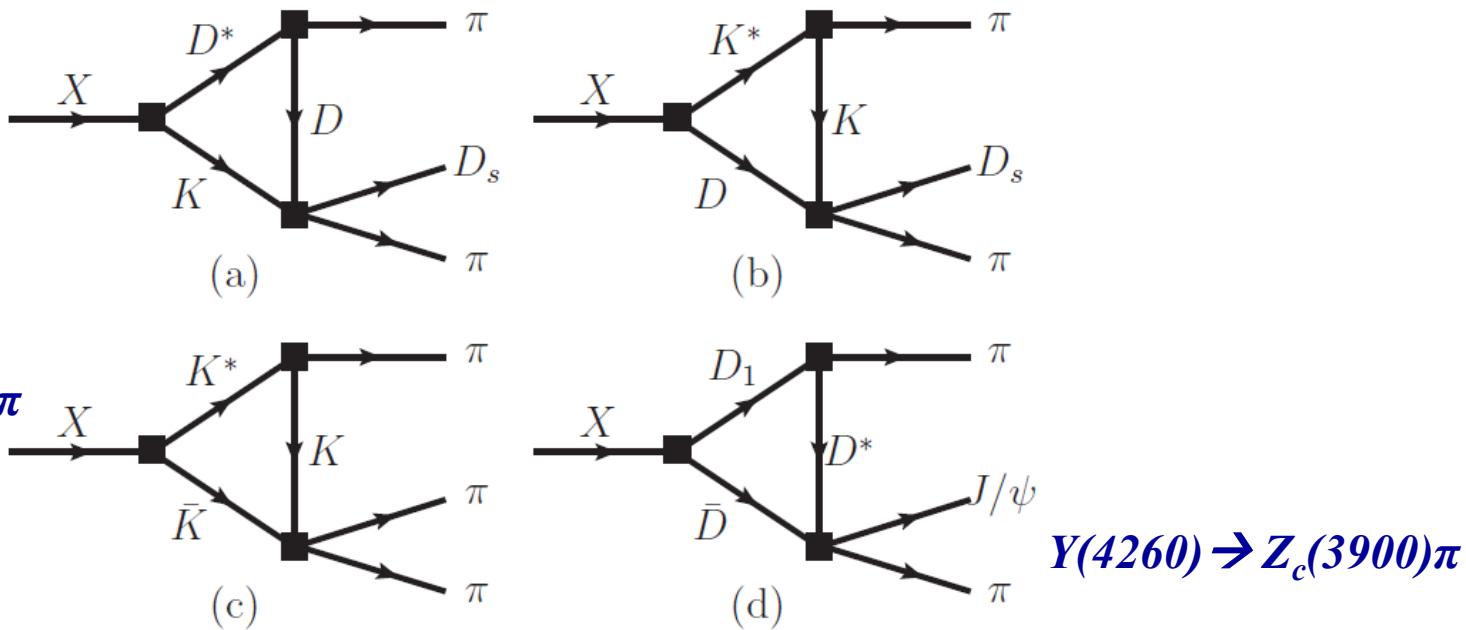
- Some disadvantages:

- ✓ Few experiments to search for the effects;
- ✓ Low statistics;
- ✓ For the elastic scattering process, the effect of the triangle diagram is nothing more than a multiplication of the singularity from the tree diagram by a phase factor, according to the so-called Schmid theorem

C.Schmid,Phys.Rev.154,1363(1967);

I. J. R. Aitchison & C. Kacser, Phys.Rev.173,1700(1968); A.V. Anisovich,  
PLB345,321(1995)

# Triangle Singularity Phenomena



Wu, Liu, Zhao & Zou, PRL108,081803(2012)

Wang,Hanhart,Zhao,PRL111,132003(2013)

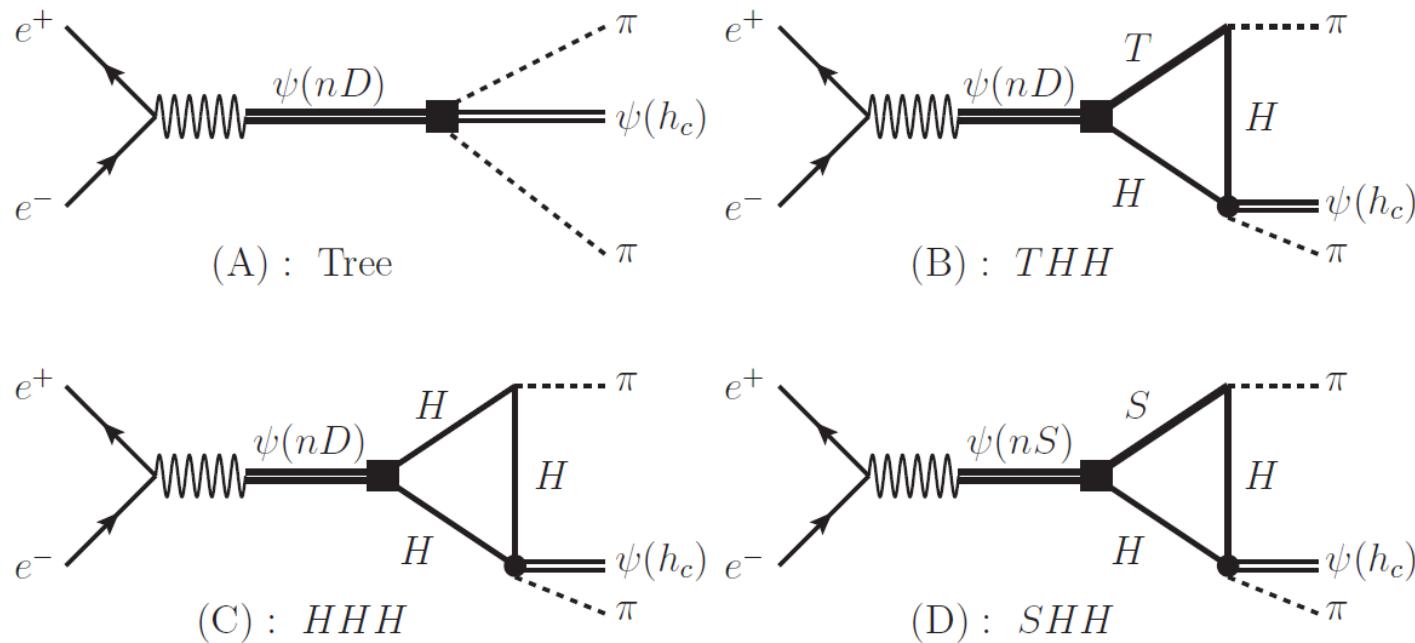
## Kinematic region of ATS

[MeV]	Fig. 3(a)	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
$\Delta_{s_1}^{\max}$	0.089	96	49	16
$\Delta_{s_2}^{\max}$	0.087	62	38	15

$$\Delta_{s_1} = \sqrt{s_1} - \sqrt{s_{1N}},$$

$$\Delta_{s_2} = \sqrt{s_2} - \sqrt{s_{2N}}.$$

# Dipion transitions via open-charm loops

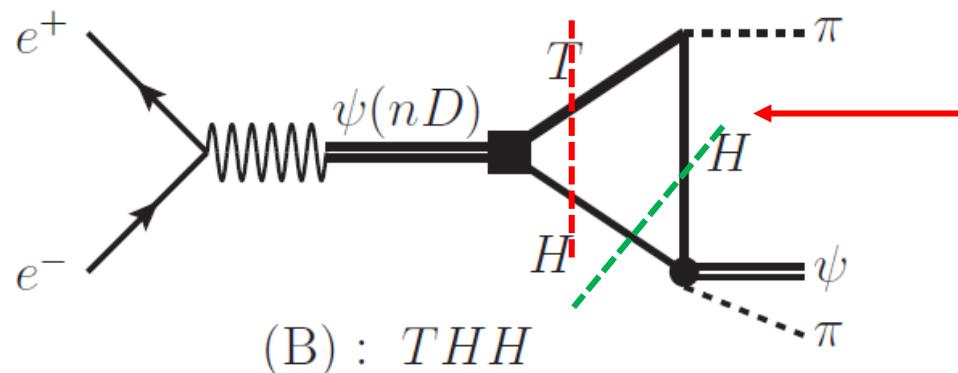


**Doublets with light degrees of freedom**  
 $j^P = 1/2^-$ ,  $1/2^+$ ,  $3/2^+$

$$\begin{aligned}
 H_a &= \frac{1 + \gamma}{2} [\mathcal{D}_{a\mu}^* \gamma^\mu - \mathcal{D}_a \gamma_5] , \\
 S_a &= \frac{1 + \gamma}{2} [\mathcal{D}'_\mu{}^a \gamma_\mu \gamma_5 - \mathcal{D}_0^* a] , \\
 T_a^\mu &= \frac{1 + \gamma}{2} \left\{ \mathcal{D}_{2a}^{\mu\nu} \gamma_\nu \right. \\
 &\quad \left. - \sqrt{\frac{3}{2}} \mathcal{D}_{1a\nu} \gamma_5 \left[ g^{\mu\nu} - \frac{1}{3} \gamma^\nu (\gamma^\mu - v^\mu) \right] \right\} ,
 \end{aligned}$$

**Model based on heavy hadron ChPT (HHChPT)**

# THH Loop



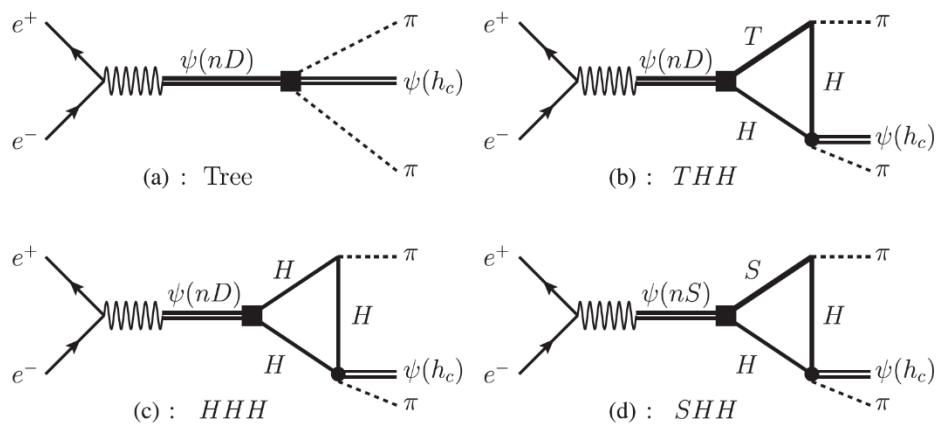
Triangle singularity (TS)  
may occur under special  
kinematic configurations

- I)  $\{D_1 D [D^*]\},$
- II)  $\{D_1 D^* [D^*]\},$
- III)  $\{D_2 D^* [D]\},$
- IV)  $\{D_2 D^* [D^*]\},$

By means of heavy quark symmetry

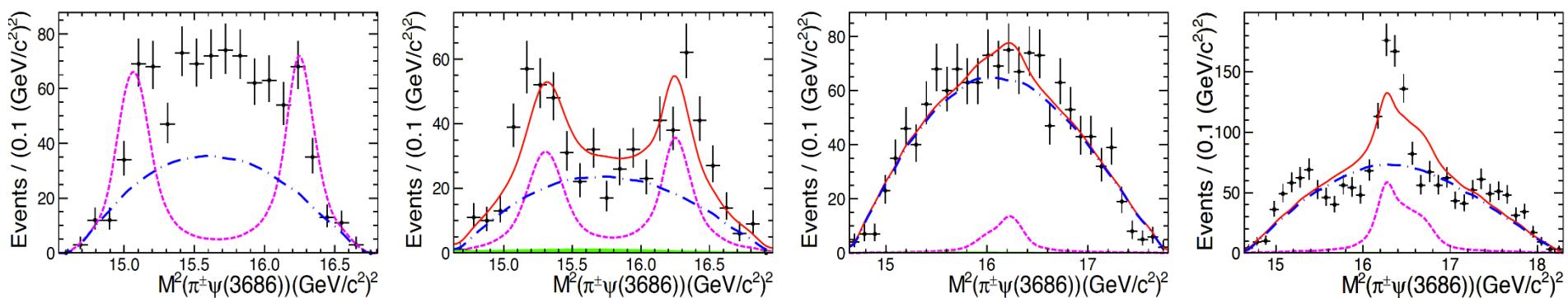
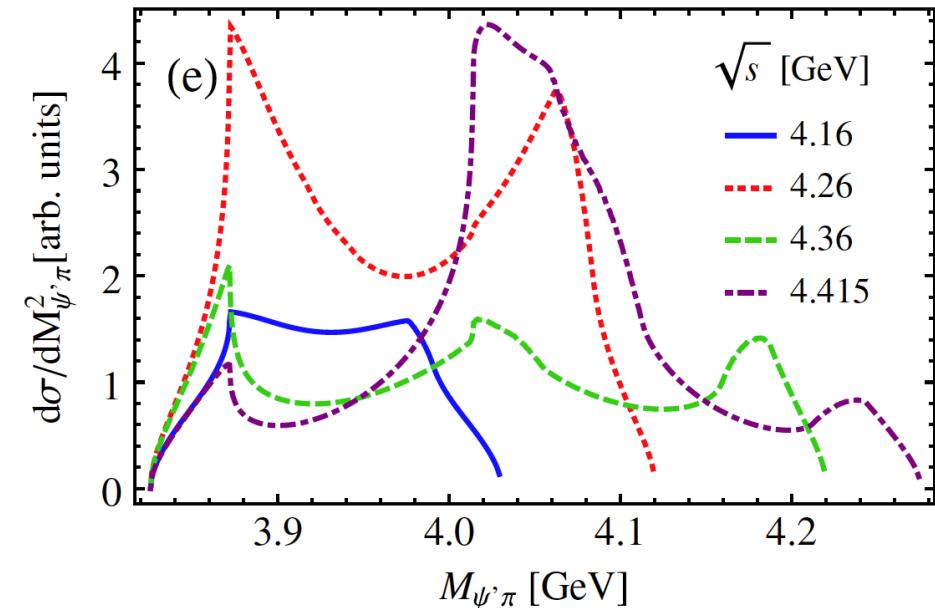
$$\mathcal{M}^I : \mathcal{M}^{II} : \mathcal{M}^{III} : \mathcal{M}^{IV} = 1 : \frac{1}{2} : -\frac{1}{5} : \frac{3}{10} .$$

# TS mechanism and structures in $e^+e^- \rightarrow \psi(3686)\pi\pi$



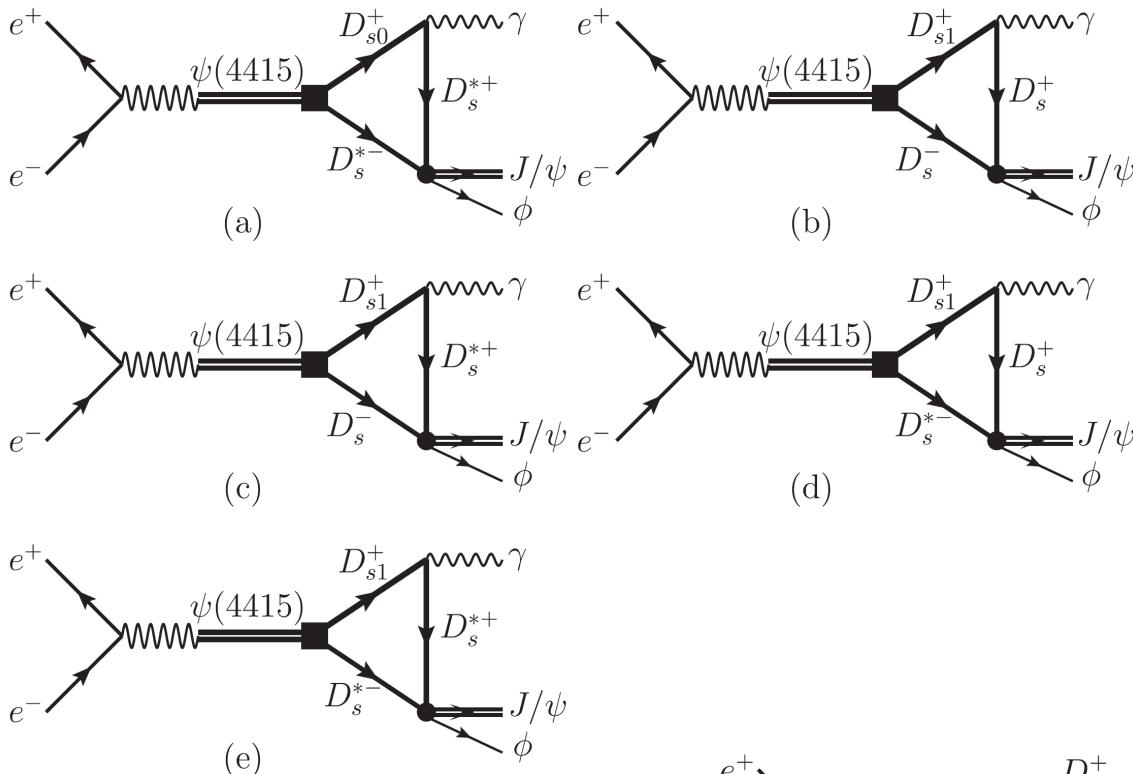
X.H. Liu, PRD90,074004(2014)

BESIII, arXiv:1703.08787



Theoretical predictions are consistent with the observed  $\psi(3686)\pi$  invariant mass distributions at various CM energies

# Searching for charmoniumlike states with hidden $s\bar{s}$

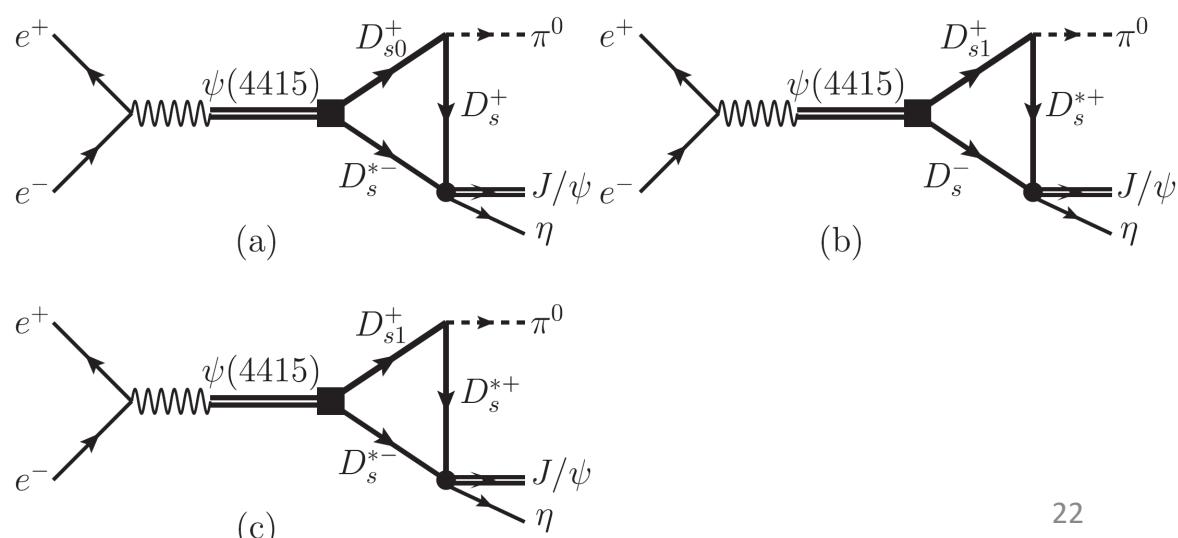


**D<sub>sJ</sub>'s are narrow,  
width effect is not  
obvious**

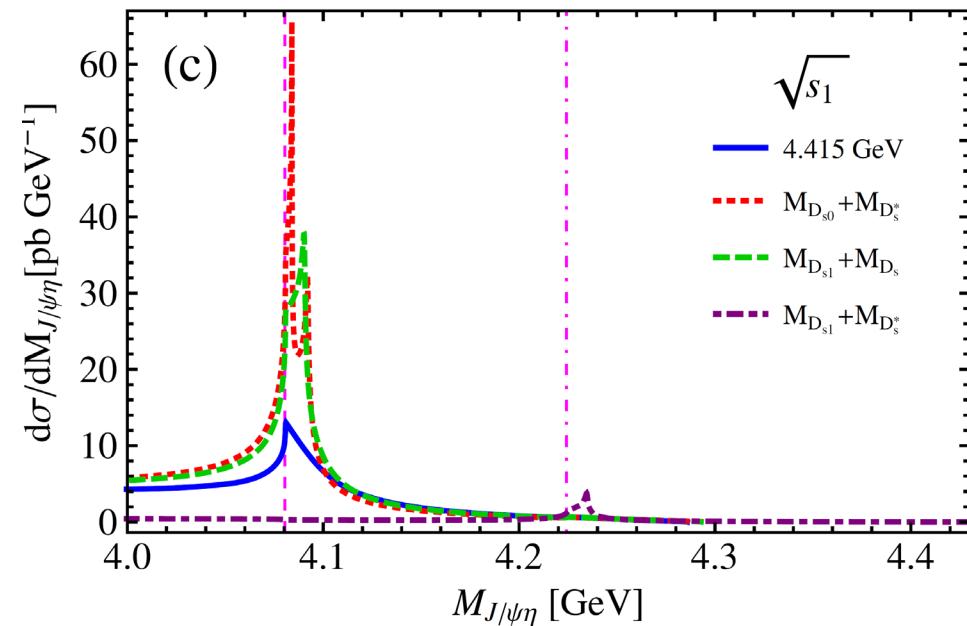
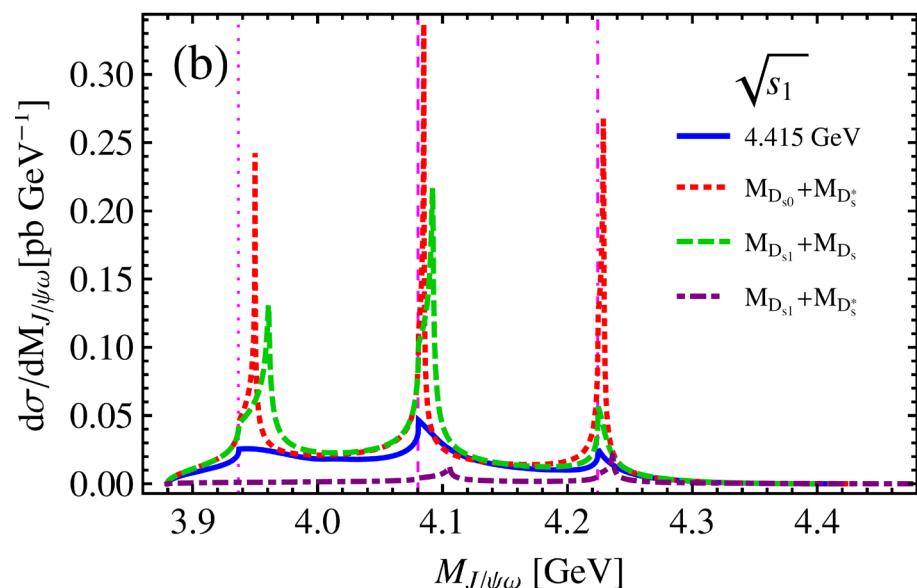
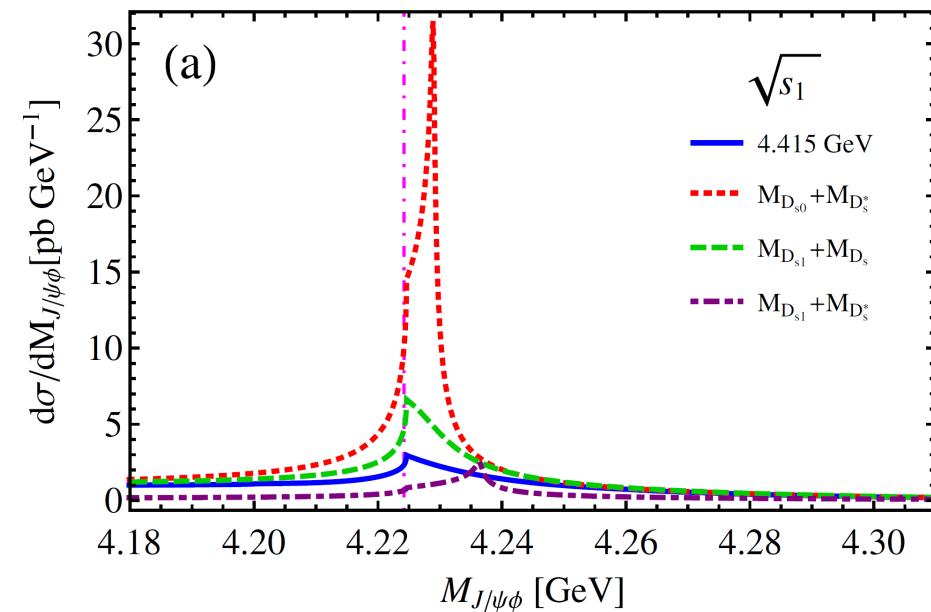
Peaks around  $D_s D_s^*$  and  
 $D_s^* D_s^*$  thresholds  
(C=-1)



Production of partners of  
 $\text{Y}(4160)$  and  $\text{Y}(4274)$   
(observed in  $J/\psi \phi$ , C=+1)



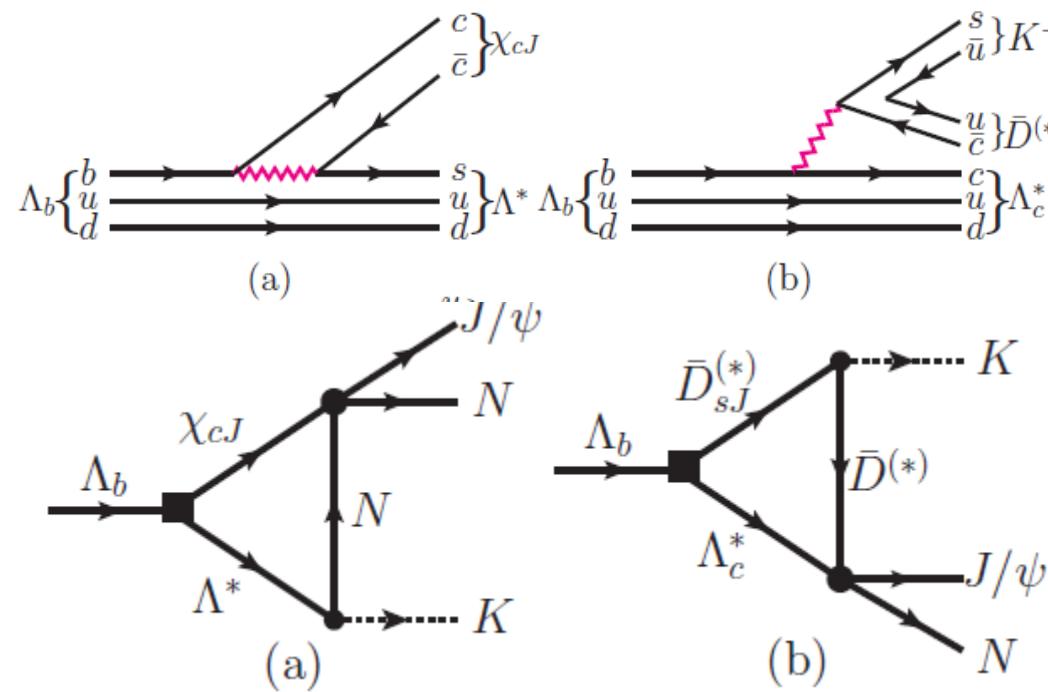
# Searching for charmoniumlike states with hidden $s\bar{s}$



X.H. Liu, M.Oka,  
PRD93, 054032(2016)

Lineshape is sensitive  
to the CM energy

# TS mechanism and the heavy pentaquark “Pc”

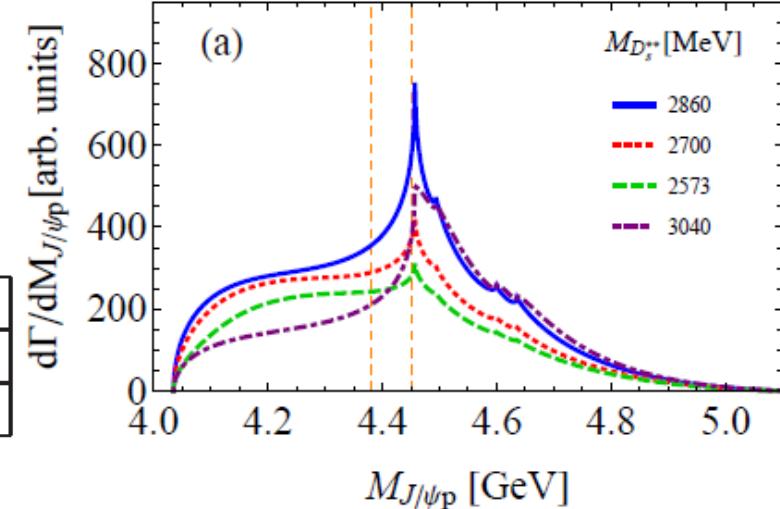
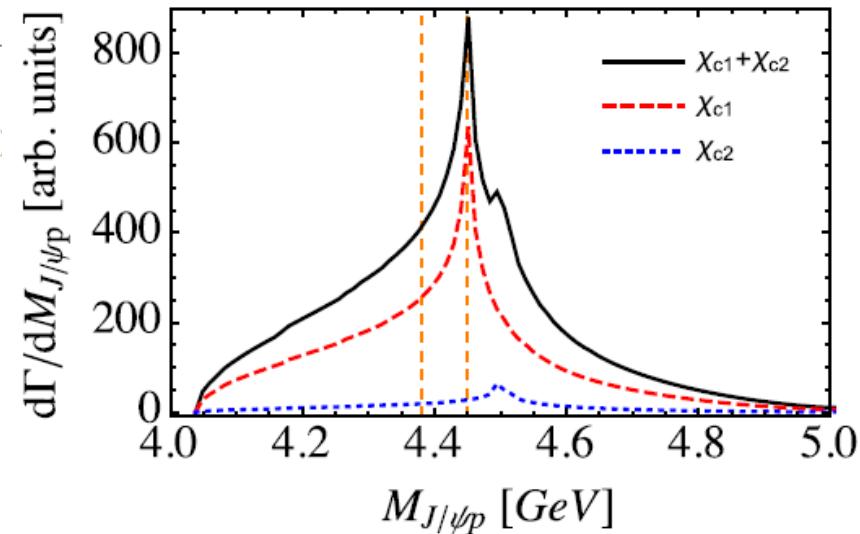


Thresholds [GeV]	$\chi_{c0}(1P)$ 0 <sup>+</sup>	$\chi_{c1}(1P)$ 1 <sup>+</sup>	$\chi_{c2}(1P)$ 2 <sup>+</sup>
$p$ 1/2 <sup>+</sup>	4.353	4.449	4.494

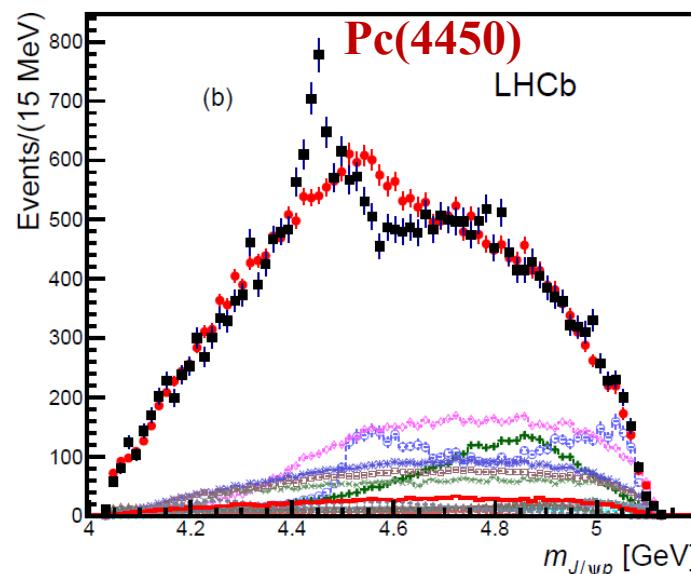
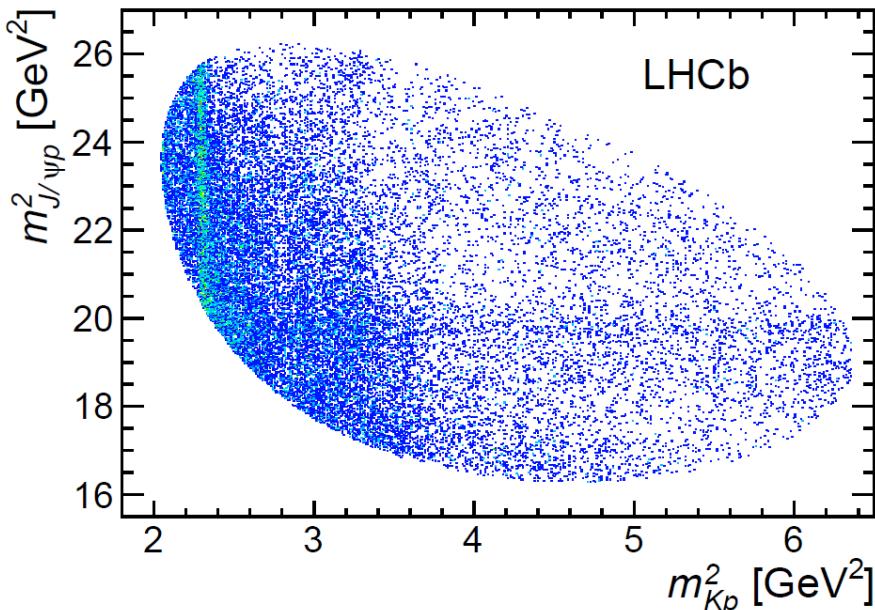
Thresholds [GeV]	$\Lambda_c(2286)$ 1/2 <sup>+</sup>	$\Lambda_c(2595)$ 1/2 <sup>-</sup>	$\Lambda_c(2625)$ 3/2 <sup>-</sup>
$\bar{D}(1865)$ 0 <sup>-</sup>	4.151	4.457	4.493
$\bar{D}^*(2007)$ 1 <sup>-</sup>	4.293	4.599	4.635

X.H.Liu, Q.Wang, Q.Zhao, arXiv:1507.05359

F.K.Guo, Ulf-G. Meissner, , W.Wang, Z.Yang,  
arXiv:1507.04950

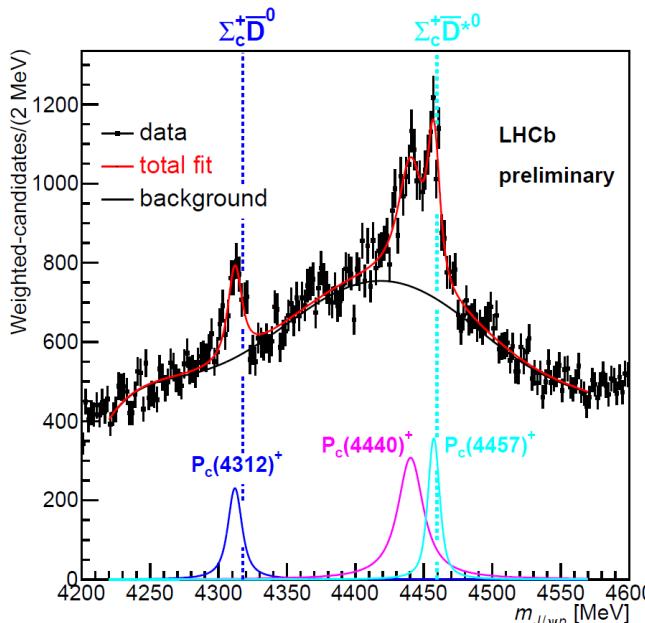


# Heavy pentaquark “Pc” observed in LHCb



LHCb, arXiv: 1507.03414

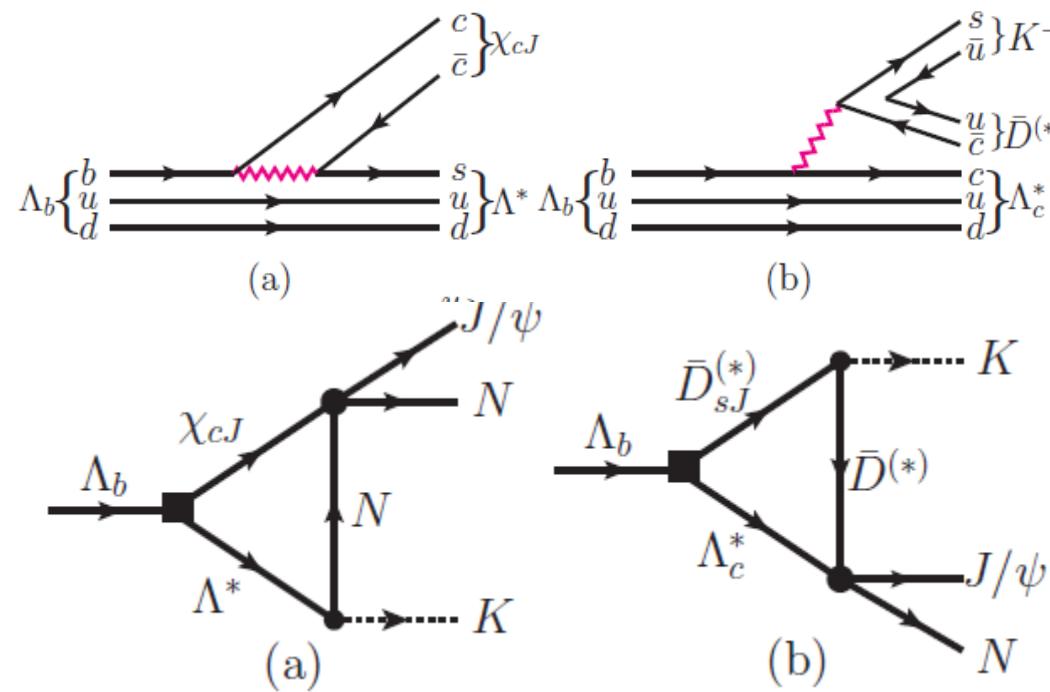
PRL115, 072001 (2015)



State	$M$ [MeV]	$\Gamma$ [MeV]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	( $< 27$ )	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	( $< 49$ )	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	( $< 20$ )	$0.53 \pm 0.16^{+0.15}_{-0.13}$

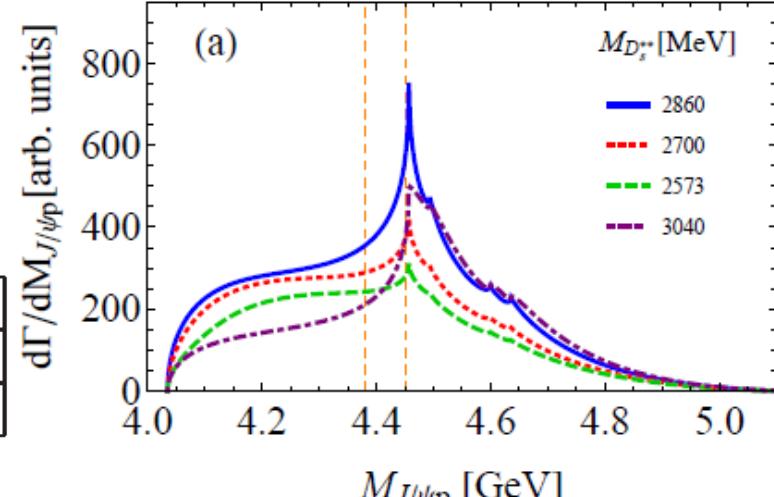
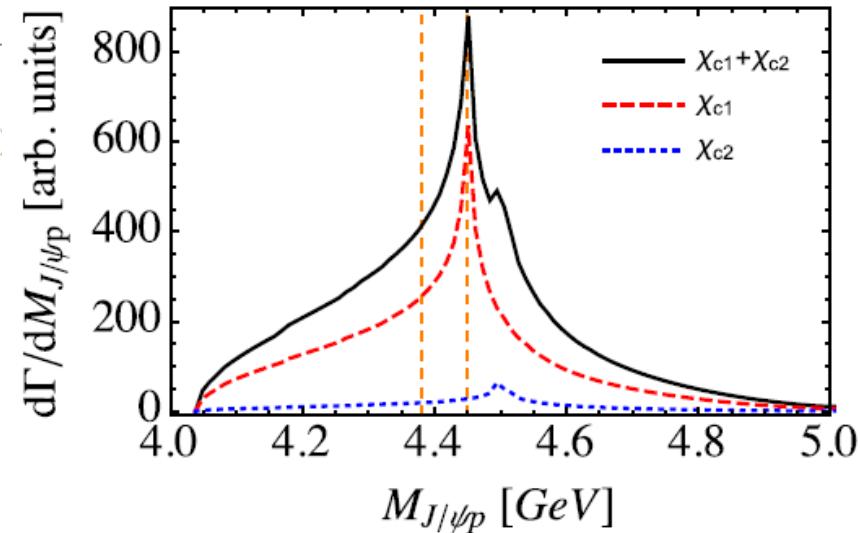
LHCb, arXiv: 1904.03947

# TS mechanism and the heavy pentaquark “Pc”



Thresholds [GeV]	$\chi_{c0}(1P)$ 0 <sup>+</sup>	$\chi_{c1}(1P)$ 1 <sup>+</sup>	$\chi_{c2}(1P)$ 2 <sup>+</sup>
$p$ 1/2 <sup>+</sup>	4.353	4.449	4.494

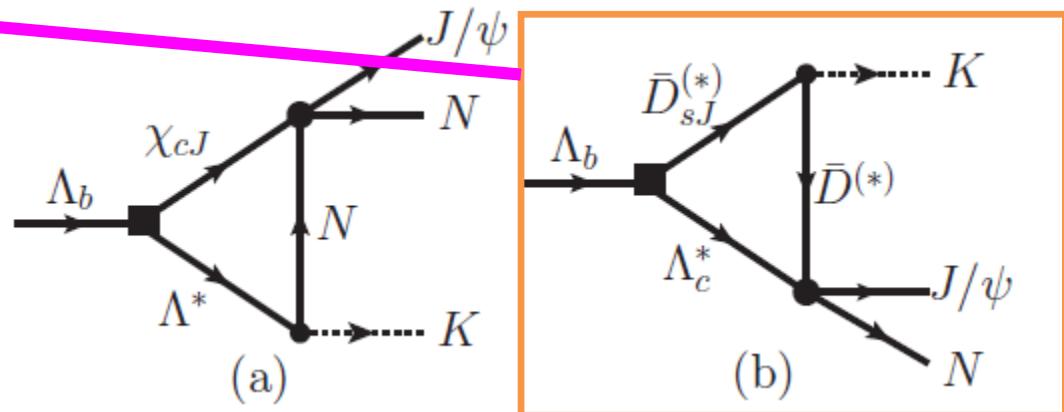
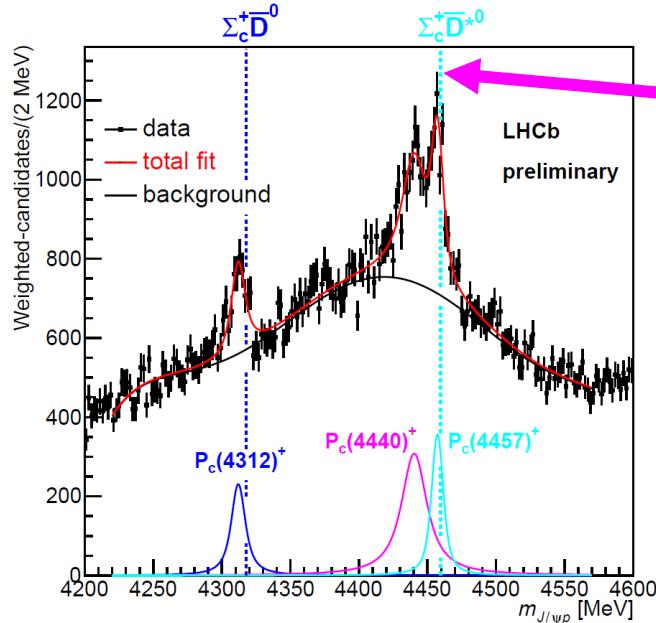
Thresholds [GeV]	$\Lambda_c(2286)$ 1/2 <sup>+</sup>	$\Lambda_c(2595)$ 1/2 <sup>-</sup>	$\Lambda_c(2625)$ 3/2 <sup>-</sup>
$\bar{D}(1865)$ 0 <sup>-</sup>	4.151	4.457	4.493
$\bar{D}^*(2007)$ 1 <sup>-</sup>	4.293	4.599	4.635



Liu, Wang, Zhao, arXiv:1507.05359

Guo et al, arXiv:1507.04950

# TS mechanism and the heavy pentaquark “Pc”



Liu, Wang, Zhao, arXiv:1507.05359

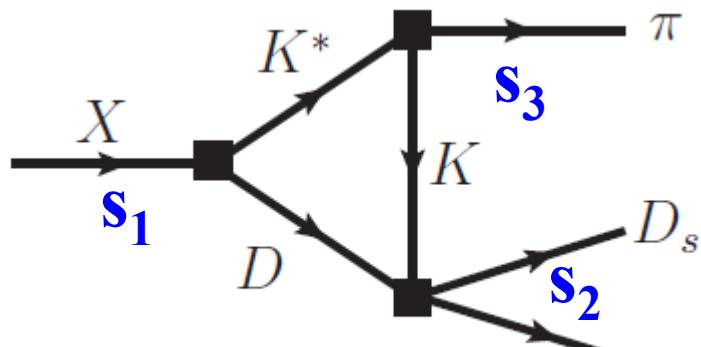
LHCb, arXiv: 1904.03947

State	$M$ [ MeV ]	$\Gamma$ [ MeV ]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	( $< 27$ )	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	( $< 49$ )	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	( $< 20$ )	$0.53 \pm 0.16^{+0.15}_{-0.13}$

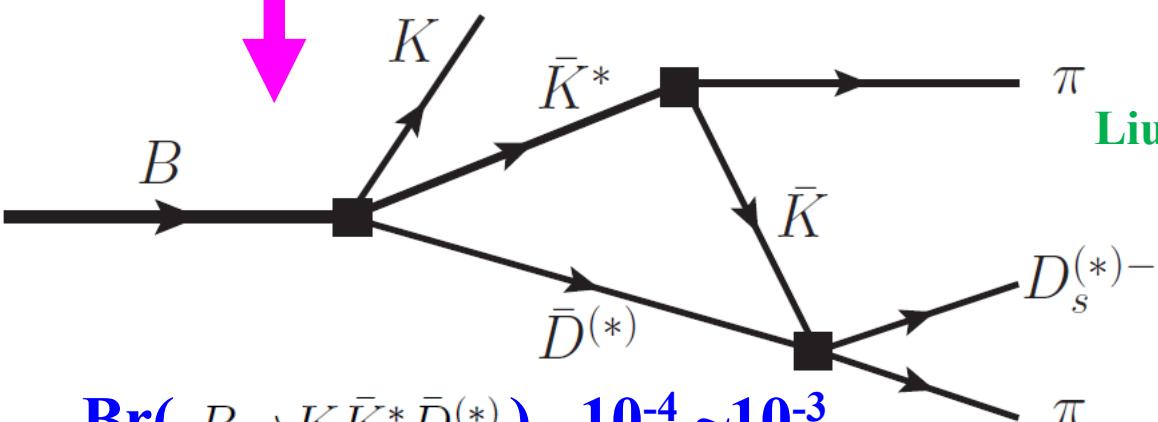
Thresholds [GeV]	$\Lambda_c(2286)$ $1/2^+$	$\Lambda_c(2595)$ $1/2^-$	$\Lambda_c(2625)$ $3/2^-$
$\bar{D}(1865)$ $0^-$	4.151	4.457	4.493
$\bar{D}^*(2007)$ $1^-$	4.293	4.599	4.635

# **Distinguish Kinematic Singularities from Dynamic Poles**

# Criterion: Movement of TS peak

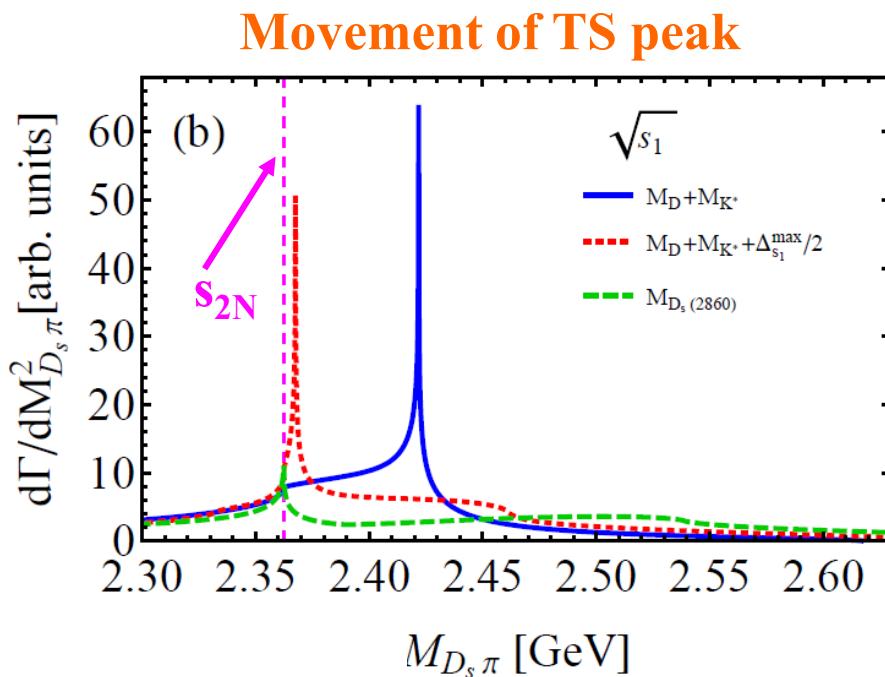


(b)



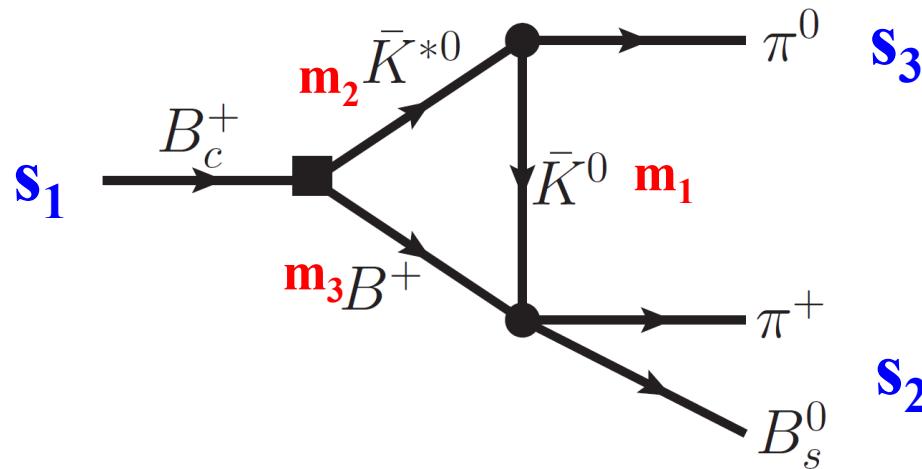
$\text{Br}(B \rightarrow K \bar{K}^* \bar{D}^{(*)}) \sim 10^{-4} \sim 10^{-3}$

$\text{Br}(B \rightarrow K D_s^{(*)-} \pi \pi) \sim 10^{-4}$



[MeV]	Fig. 3(a)	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
$\Delta_{s1}^{\max}$	0.089	96	49	16
$\Delta_{s2}^{\max}$	0.087	62	38	25

# Possible exotic structure “X(5777)” around *BK*-threshold



Particle	$B_c$	$B_s$	$B$	$K^*$	$K$	$\pi$
Mass [GeV]	6.276	5.367	5.279	0.892	0.498	0.135

## Kinematic region of triangle singularity (TS)

$$\Delta_{s_1}^{\max} = \sqrt{s_{1C}} - \sqrt{s_{1N}} \approx \frac{m_3}{2m_1(m_2 + m_3)}[(m_2 - m_1)^2 - s_3], \quad \sim 119 \text{ MeV}$$

$$\Delta_{s_2}^{\max} = \sqrt{s_{2C}} - \sqrt{s_{2N}} \approx \frac{m_3}{2m_2(m_1 + m_3)}[(m_2 - m_1)^2 - s_3]. \quad \sim 71 \text{ MeV}$$

$\sqrt{s_1}$ : 6.175 GeV ~ 6.294 GeV

$\sqrt{s_2}$ : 5.848 GeV ~ 5.777 GeV

The mass of  $B_c \sim 6.276$  GeV perfectly falls into the TS kinematic region!

# *DK* interaction

See also Li-Sheng Geng's talk

Scattering length $a_0$ [fm]	LQCD [1]	LQCD [2]	
$ l=0$	$-0.86 \pm 0.03$	$-1.33(20)$	strong
$ l=1$	$0.07 \pm 0.03 + 0.17 i$		weak

[1]L. Liu et al., PRD87,014508 (2013)

[2]D. Mohler et al., PRL111,222001(2013)

See also Chen et al., 1609.08928 for a review

Heavy quark symmetry

$D_{s0}(2317)/D_{s1}(2460)$  , isoscalar hadronic molecule of  $DK/D^*K$

# *BK* interaction

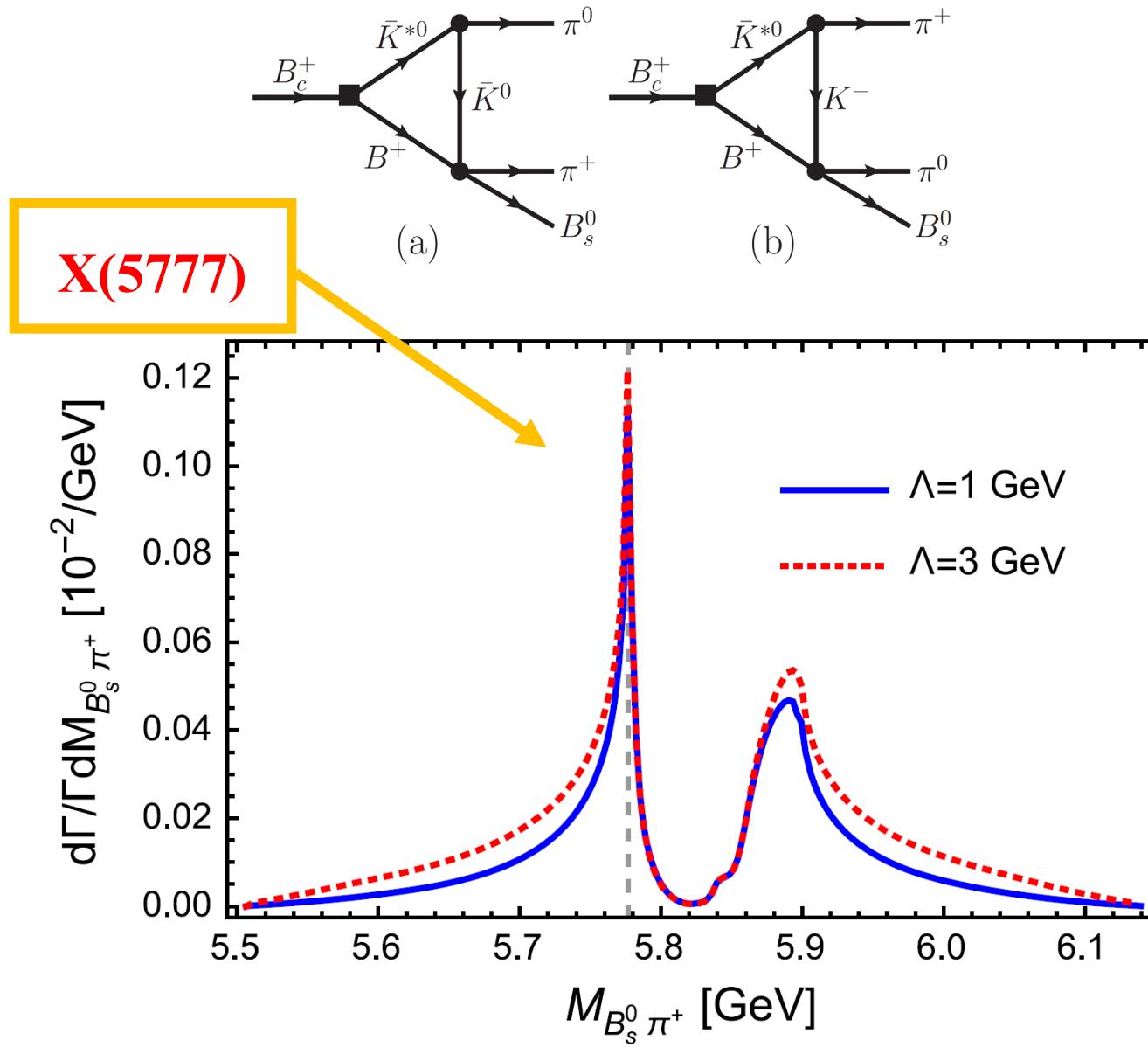
Scattering length  $a_0$  ( $|l=1\rangle \sim 0.02 + 0.23 i$  [fm]

Geng et al.,  
PRD89,014026(2014);EPJC77,94(2017)

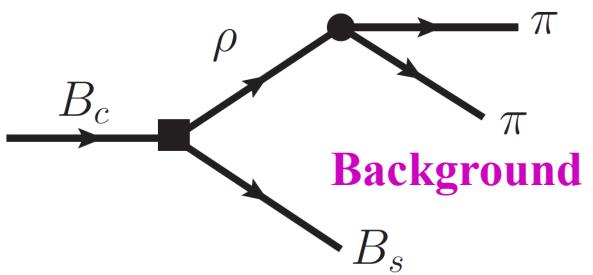
$B_{s0}/B_{s1}$ , isoscalar hadronic molecule of  $BK/B^*K$ , not observed yet

The relative weak interaction ( $|l=1\rangle$  does not support the existence of an isovector hadronic molecule.

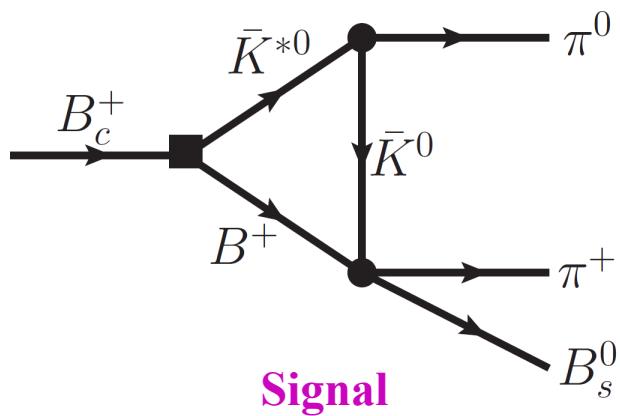
# Invariant mass distribution of $B_s\pi$



# Background Analysis



+



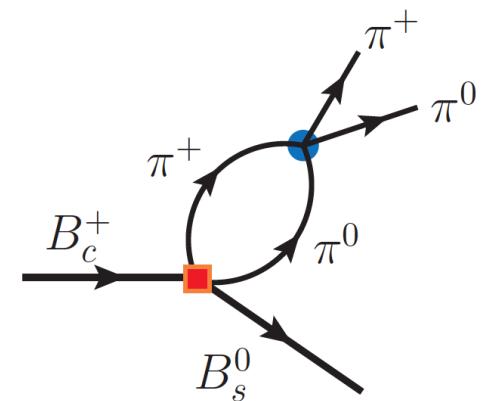
$$\frac{Br(B_c \rightarrow B\bar{K}^*)}{Br(B_c \rightarrow B_s\rho)} \sim \frac{1}{10}$$

$$\mathcal{A}(B_c^+ \rightarrow B_s^0 \pi^+ \pi^0) = e^{i\theta} \mathcal{A}_\rho^{\text{tree}} + \mathcal{A}^{\text{loop}} \mathcal{F}(s_{\pi\pi})$$

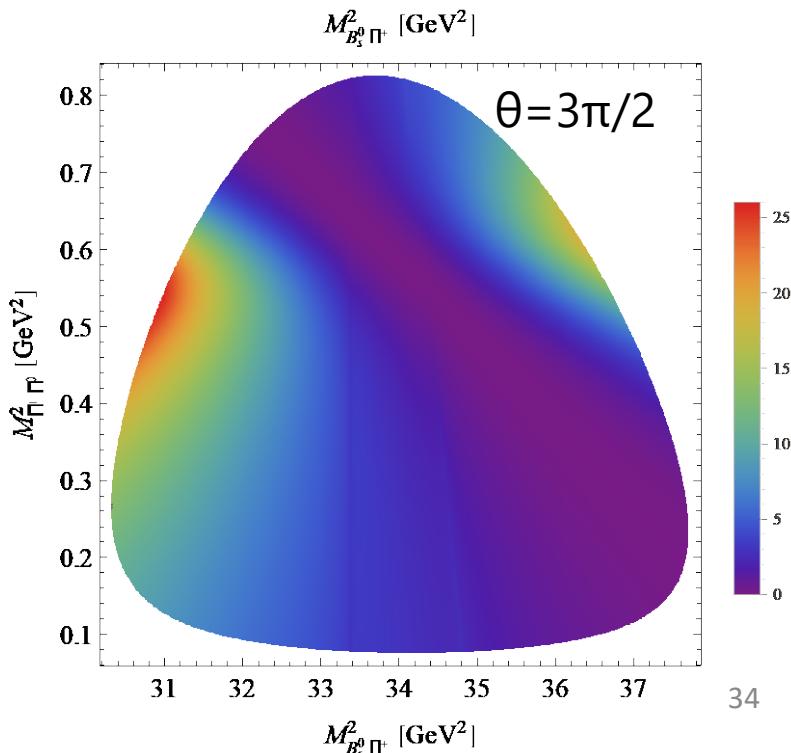
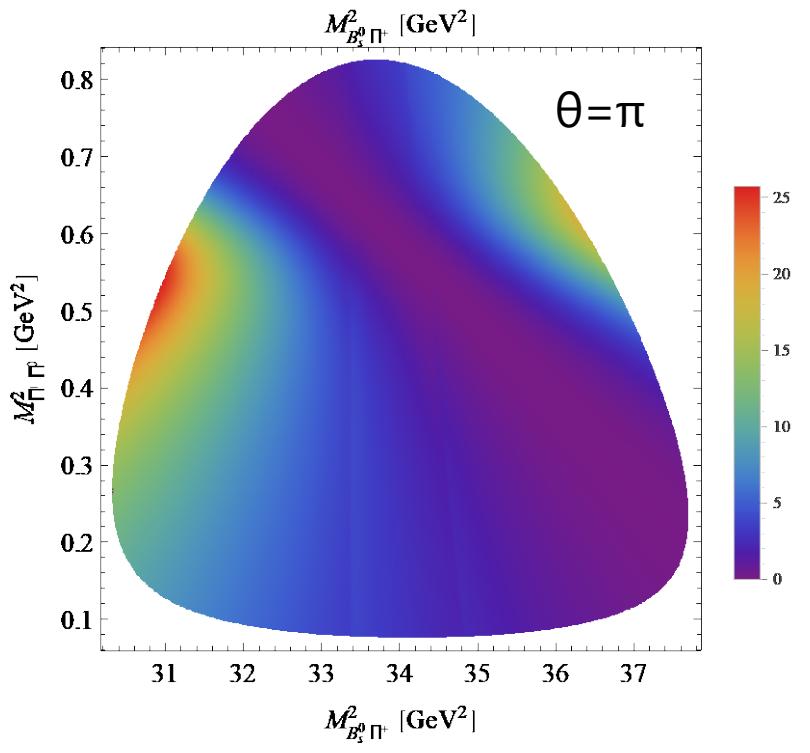
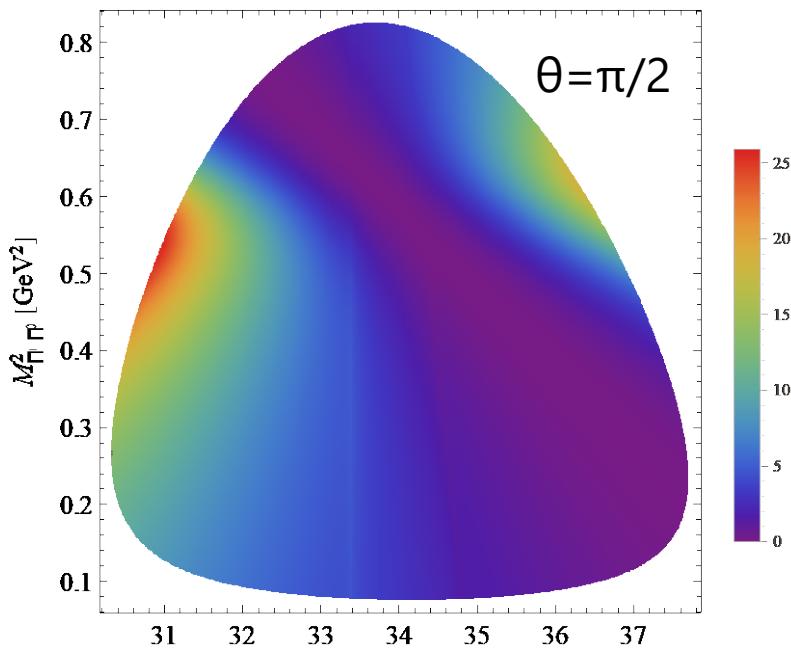
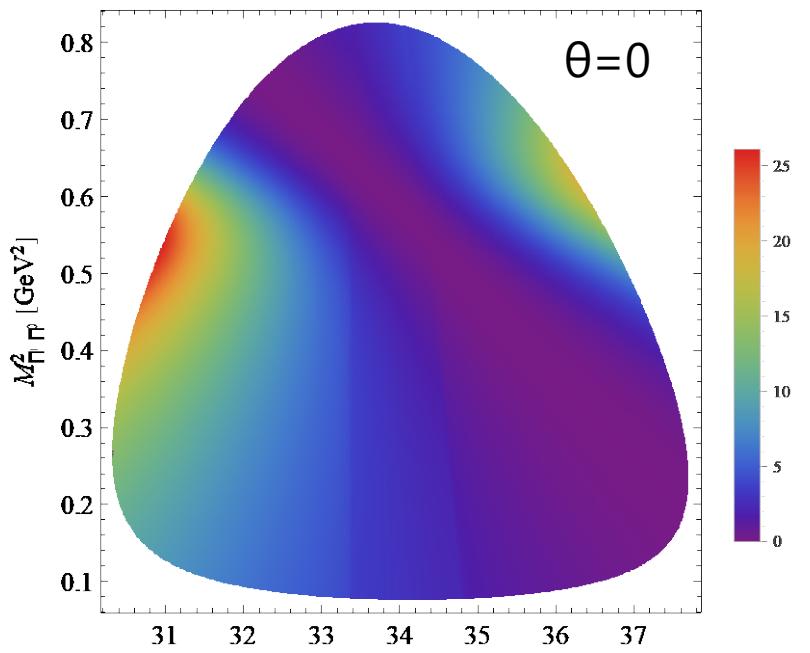
Relative phase

Account for  $\pi\pi$  FSI

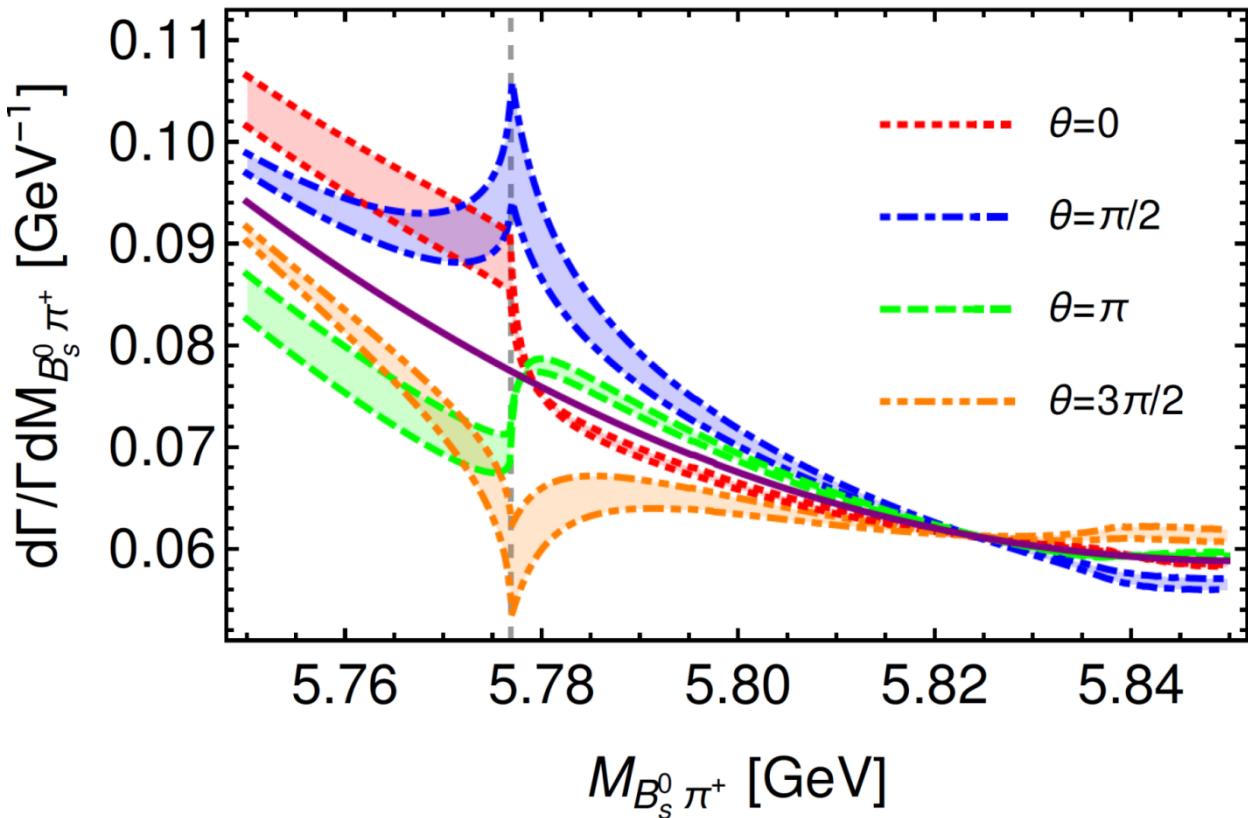
VMD model



$$\mathcal{F}(s_{\pi\pi}) \simeq \frac{s_{\pi\pi} - \overset{\circ}{m}_\rho^2}{s_{\pi\pi} - m_\rho^2 + im_\rho\Gamma_\rho} \quad \overset{\circ}{m}_\rho \simeq 0.81 \text{ GeV}$$



# Background Analysis



$a_1 = 1.14$ , be fixed  
according to  $\text{Br}(B_c \rightarrow B_s \pi)$

$a_2 = -0.31 \sim -0.51$

Rescattering diagrams  
contribute 4.4~7.5%

Estimation of experimental feasibility:

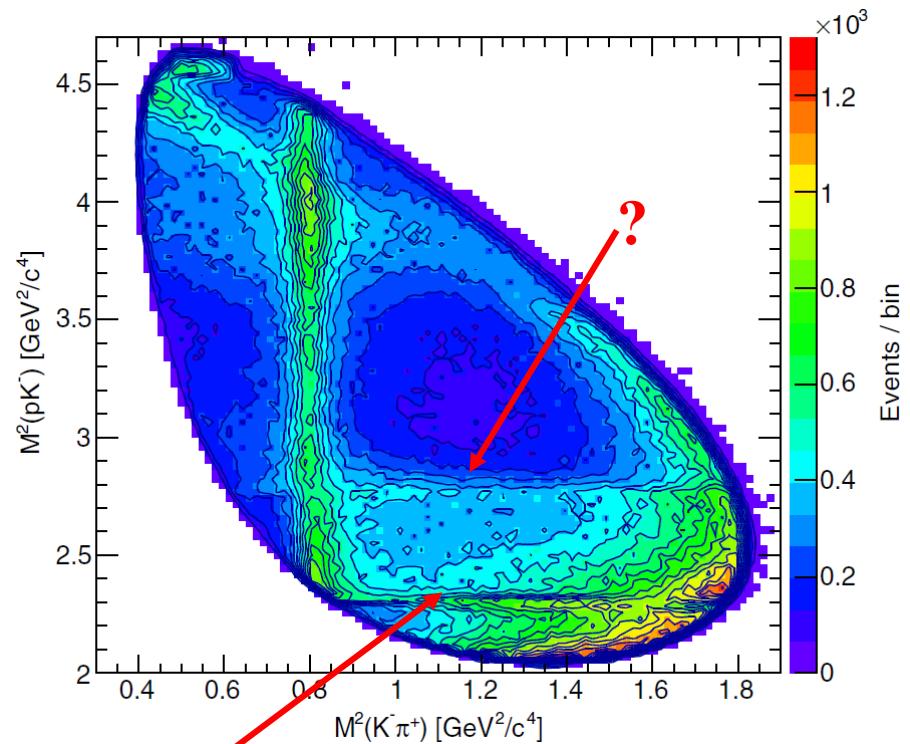
If requiring the signal significance  $N_s/\sqrt{N_b} = 5$ , around  $9.6 \times 10^5$   $B_c$  events are needed to observe the TS phenomena

# Observation of “X(1663)”

Dalitz plot for  $\Lambda_c \rightarrow p K^- \pi^+$

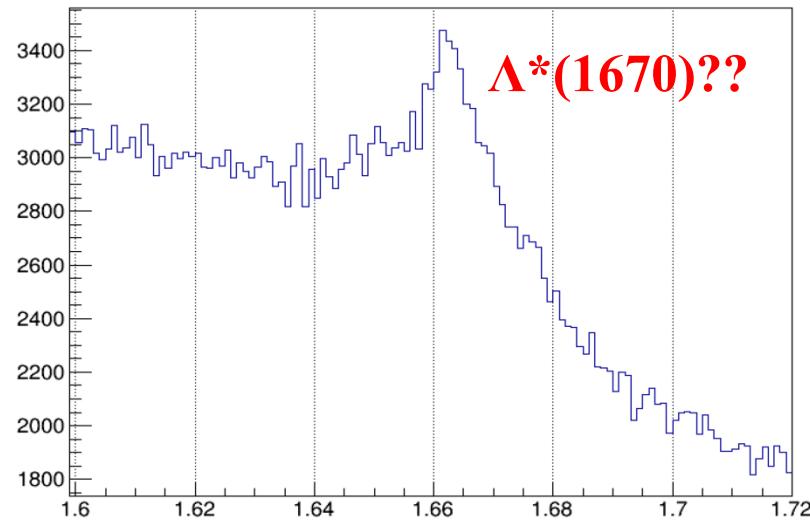
Belle, PRL117,011801(2016)

$1.452 \times 10^6$  events



$\Lambda(1520)$

■ 1D projection --  $M(pK^-)$



✓ Bin width: 1 MeV

✓  $M \approx 1663$  MeV

✓  $\Gamma \approx 10$  MeV

✓  $\Lambda\eta$  threshold: 1663.545 MeV

From C.P. Shen's talk, no published result concerning “X(1663)”

# Observation of “X(1663)”

## Hyperons around 1663 MeV [PDG]

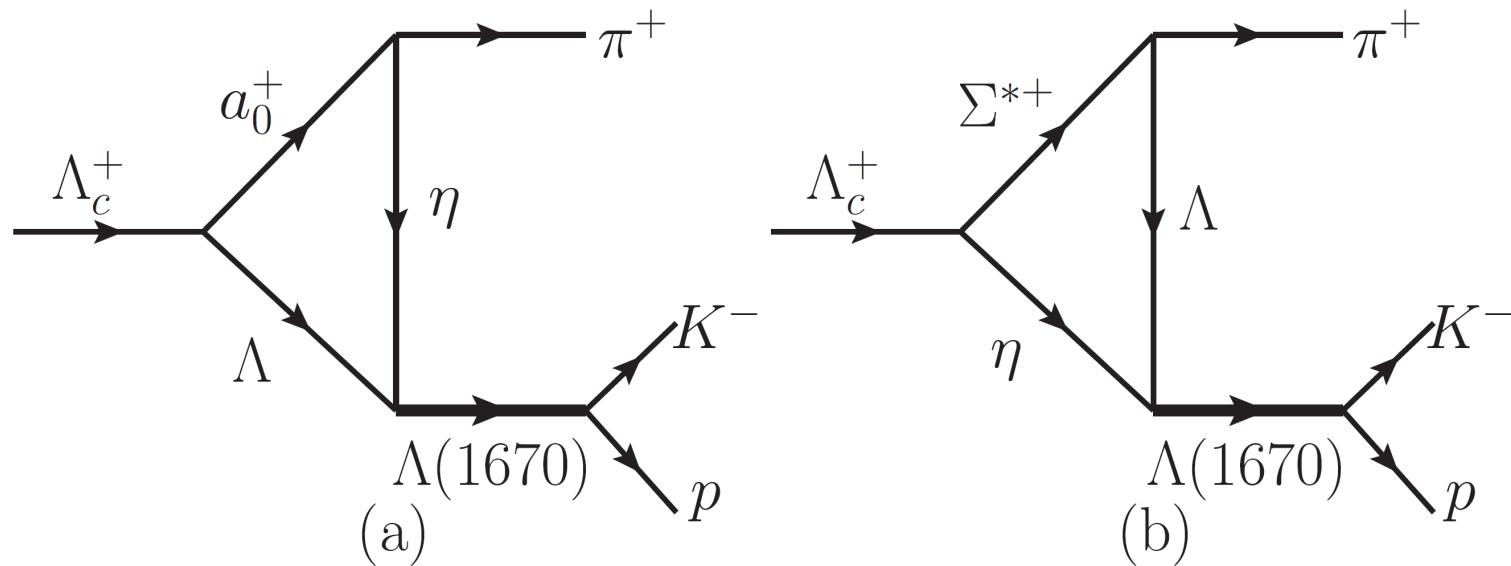
[MeV]	Mass	Width	J <sup>¶</sup> P
X(1663)	1663	~10	?
$\Lambda^*(1670)$	1660 to 1680 ≈1670	25 to 50 ≈35	1/2-
$\Lambda^*(1690)$	1685 to 1695 ≈1690	50 to 70 ≈60	3/2-
$\Sigma^*(1660)$	1630 to 1690 ≈1660	40 to 200 ≈100	1/2+
$\Sigma^*(1670)$	1665 to 1685 ≈1670	40 to 80 ≈60	3/2-

No established hyperons correspond to this “X(1663)”

Two groups claim there is a narrow  $\Lambda^*$  with  $J=3/2$ :

- Liu & Xie [PRC85, 038201; PRC86, 055202]  
 $J^{\wedge}P=3/2-(D03)$ ,  $M=1668.5 \pm 0.5$  MeV,  $\Gamma=1.5 \pm 0.5$  MeV
- Kamano *et al.* [PRC90, 065204; PRC92, 025205]  
 $J^{\wedge}P=3/2+(P03)$ ,  $M=1671+2-8$  MeV,  $\Gamma=10+22-4$  MeV

# Contributions from rescattering processes



✓ Cabibbo-favored process

✓ Strong couplings

✓ Exp. value:  $Br(\Lambda_c \rightarrow \Lambda \eta \pi^+) \sim (2.2 \pm 0.5)\%$

$Br(\Lambda_c \rightarrow \Sigma(1385) \eta \rightarrow \Lambda \eta \pi^+) \sim (1.06 \pm 0.32)\%$

# TS location

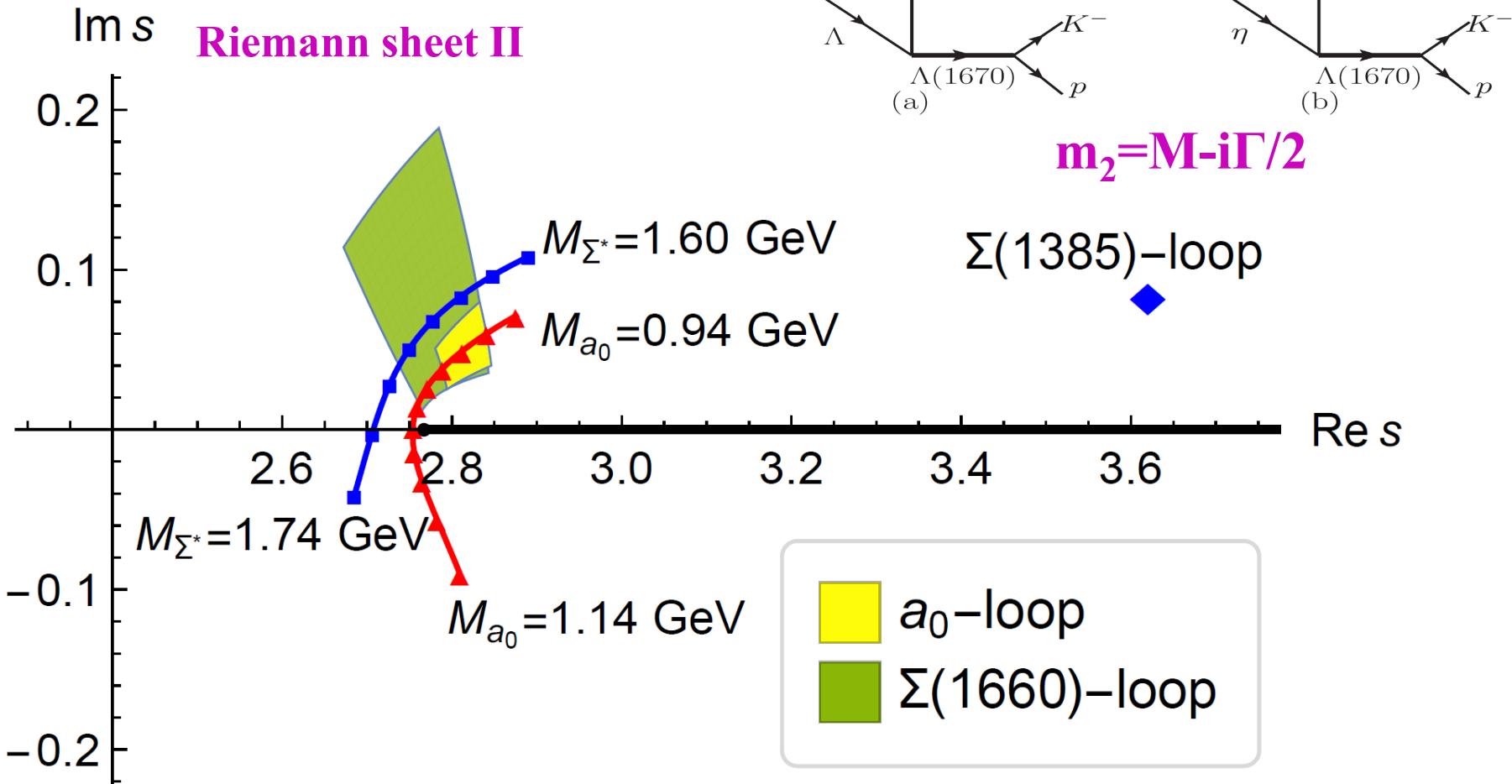
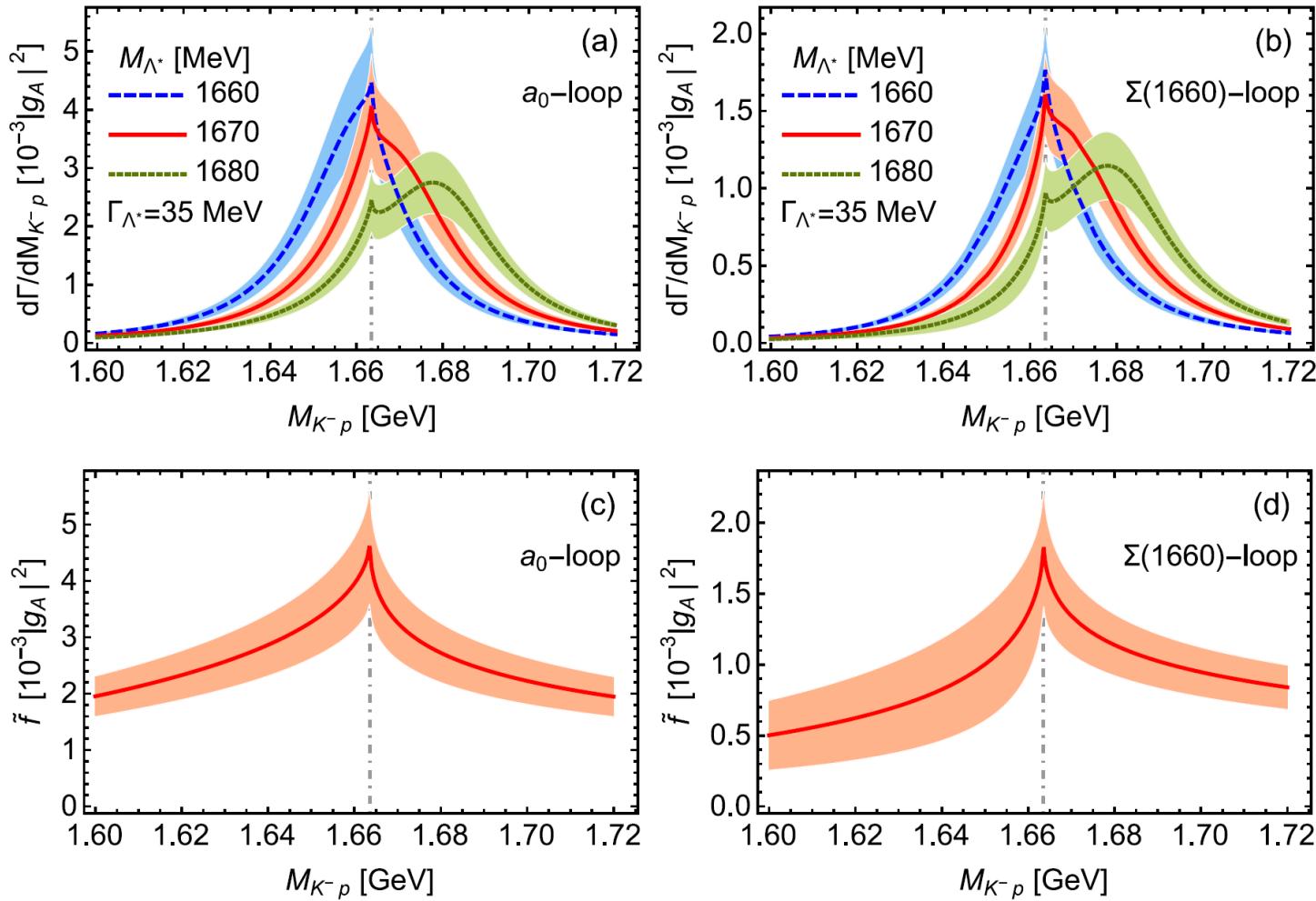


FIG. 2: The TS location of  $\mathcal{T}(s, m_2^2)$  in the complex  $s$ -plane. The thick line on the real axis represents the unitary cut starting from  $s_{\text{th}}$ . The trajectory marked with triangle (box) is obtained by varying  $M_{a_0}$  ( $M_{\Sigma^*}$ ) and fixing  $\Gamma_{a_0} = 75 \text{ MeV}$  ( $\Gamma_{\Sigma^*} = 100 \text{ MeV}$ ).

# Invariant Mass Distributions



$$\tilde{f}(M_{K^-p}) = \left| \frac{s - M_{\Lambda^*}^2 + iM_{\Lambda^*}\Gamma_{\Lambda^*}}{M_{\Lambda^*}\Gamma_{\Lambda^*}} \right|^2 \times \frac{d\Gamma}{dM_{K^-p}}$$

# Summary

- Kinematic singularities can behave themselves as peaks in the invariant mass distribution, which implies that non-resonance interpretation for some resonance-like structures is possible.
- Being different from the genuine resonances, the TS mechanism is a process-dependent mechanism, and sensitive to the kinematic configurations.

**Model independent but Process dependent.**

- Necessary before claiming that a resonance-like structure is a genuine particle.

Thanks!