SKYRME MODEL STUDY OF LIGHT BARYON PROPERTIES IN A STRONG MAGNETIC FIELD

Phys.Rev. D92 (2015) 111503 (R)
Phys.Lett. B765 (2017) 109
Phys.Rev. D99 (2019) 034019

@Changsha
June 24, 2019

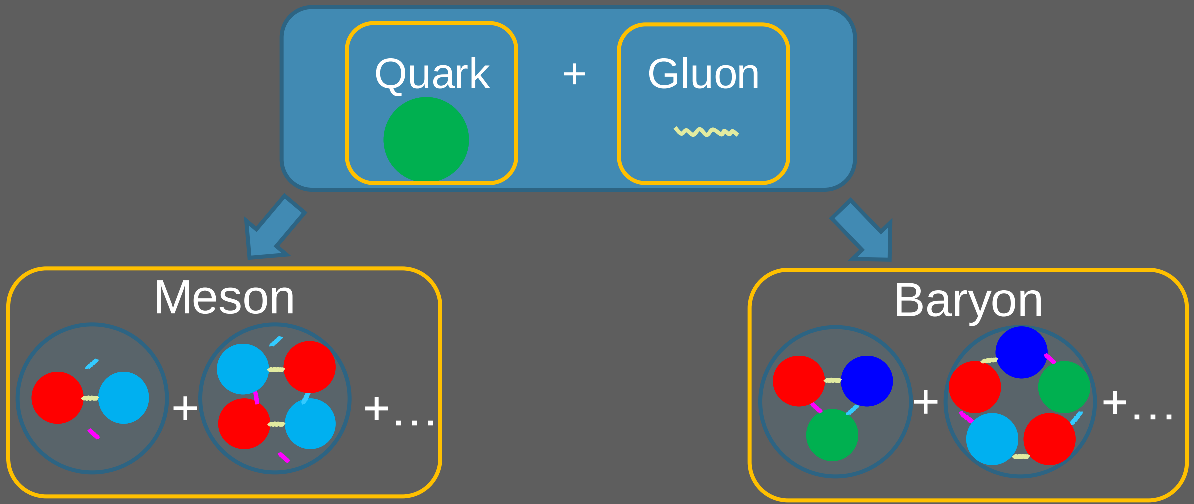
BingRan He(何秉然)
Nanjing Normal University

Outline

- ***Introduction***
- Skyrme model
- Nucleon and Delta in the magnetic background
- Summary

Introduction

Hadron are made by quarks and gluons



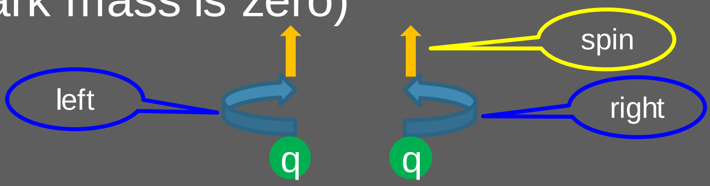
The dynamics of quarks and gluons are described by Quantum chromodynamics (QCD)

- QCD have two important features:
  - Quark confinement
  - Asymptotic freedom

- In low energy region the perturbative calculation for QCD is impossible, alternatively:
  - Lattice QCD (non-perturbative calculation)
  - Effective models (chiral perturbation theory, quark model, etc…)

Introduction

- QCD has several symmetries:
  - Chiral symmetry (when quark mass is zero)

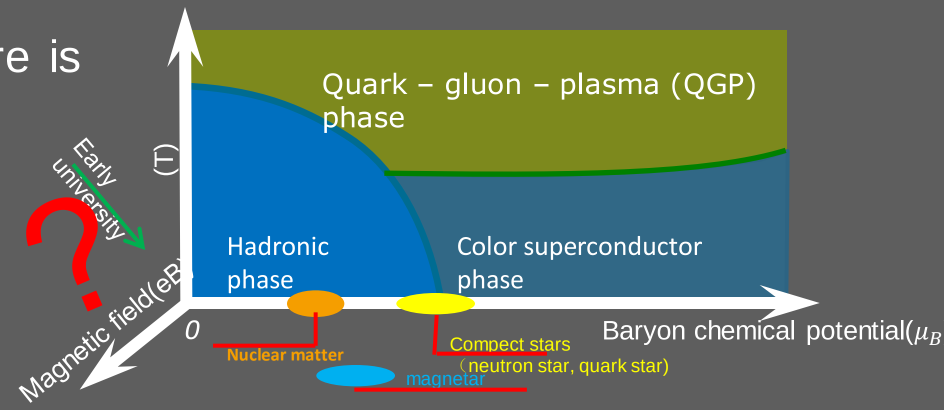


- In the low energy region, people construct effective models in hadron level by mimic the symmetries of QCD:
  - The size of hadron is not considered:
    - Chiral perturbation theory (ChPT): π meson
    - Hidden local symmetry (HLS): π, ρ, ω meson
    - Chiral baryon model
    - …
  - The size of baryon is considered:
    - Soliton models
    - MIT bag model
    - Chiral bag model

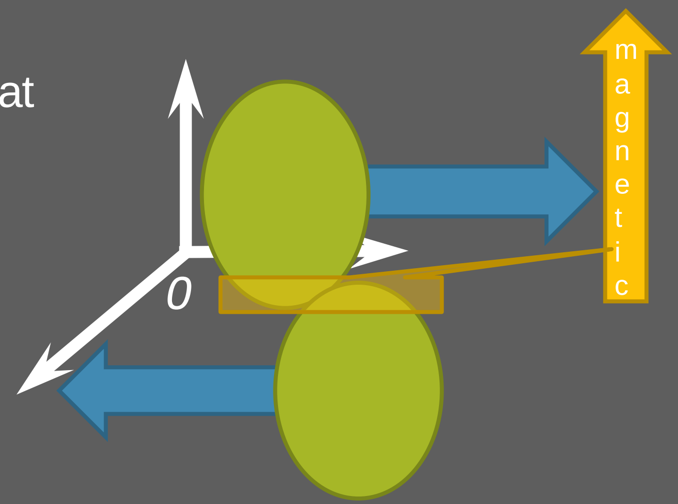
In the finite density & magnetic region, the size effect of baryon is very important

The hadron properties in strong magnetic field background

- In the early university, the temperature is high and the magnetic field is strong



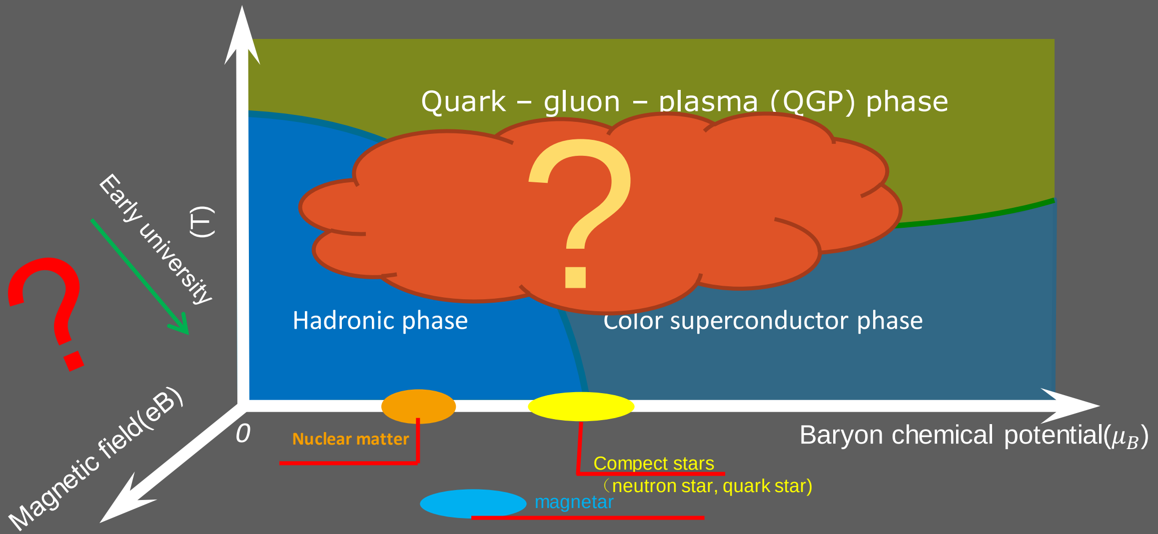
- Recently, the heavy ion collider found that there exists strong magnetic field (15 times of π mass, about 10^19 Gauss)



Problem

- So far, the baryon properties in the strong magnetic field background is still unclear
- With the introduce of magnetic field, QCD phase diagram has a new dimension of freedom, which leave us lots of challenges

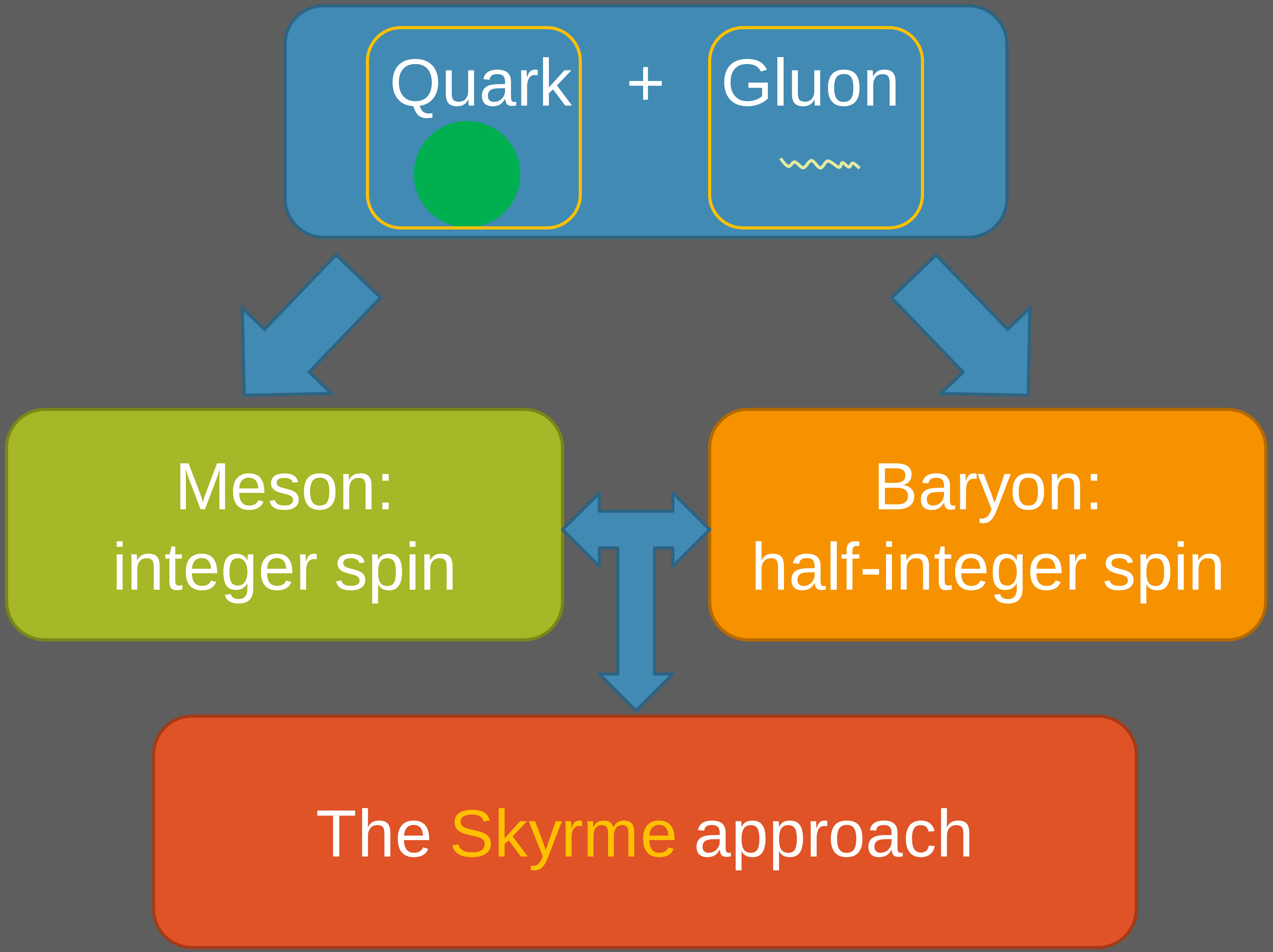
QCD Phase Diagram



- Lattice QCD methods (based on the first principle of QCD calculation) are improper to study density matter
- Lattice QCD methods are hard to predict the size effects of baryons (form factor, charge radius etc.)

A. Abdel-Rehim, C. Alexandrou, M. Constantinou, K. Hadjiyiannakou, K. Jansen, and G. Koutsou, Proc. Sci., LATTICE2014 (2015) 148

- To understand the dynamics of magnetars, we need to use effective models to investigate the QCD dynamics in both density and magnetic region

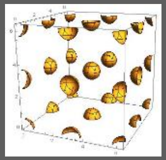

Quark
+
Gluon
Meson:
integer spin
Baryon:
half-integer spin
The Skyrme approach

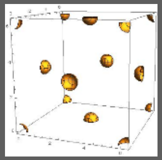
Introduction

- The soliton model: baryon is identified as the topological solution of mesons

T. H. R. Skyrme, Nucl. Phys. 31, 556 (1962)

- The skyrmion crystal model: put skyrmion together we can construct skyrmion crystal, which help us to study the dense effects





Igor R. Klebanov, Nucl. Phys. B262 (1985) 133

H. Dong, T. T. S. Kuo, H. K. Lee, R. Machleidt, and M. Rho, Phys. Rev. C 87, 054332 (2013)

D. Suenaga, B. -R. He, Y. -L. Ma, M. Harada
Phys.Rev. D91 (2015) 3, 036001
Phys.Rev. C89 (2014) 6, 068201

Model	Soliton mass [MeV]	$\sqrt{\langle r^2 \rangle_B}$ [fm]
Skyrmion(π)	939	0.68
Experiment	939	0.72

G. S. Adkins, C. R. Nappi, and E.Witten, Nucl. Phys. B228, 552 (1983).

- We can use several parameters to get the baryon states which are consistent with experiment values

Introduction

PHYSICAL REVIEW D **85**, 114038 (2012)

Anomaly-induced charges in baryons

Minoru Eto,[1,*] Koji Hashimoto,[2,†] Hideaki Iida,[2,‡] Takaaki Ishii,[3,§] and Yu Maezawa[2,‖]

[1]*Department of Physics, Yamagata University, Yamagata 990-8560, Japan*
[2]*Mathematical Physics Laboratory, RIKEN Nishina Center, Saitama 351-0198, Japan*
[3]*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, United Kingdom*

(Received 4 September 2011; published 21 June 2012)

Baryon number is not conserved when the magnetic field is nonzero.

Introduction

PHYSICAL REVIEW D **85**, 114038 (2012)

Anomaly-induced charges in baryons

Minoru Eto,[1,*] Koji Hashimoto,[2,†] Hideaki Iida,[2,‡] Takaaki Ishii,[3,§] and Yu Maezawa[2,||]

[1]*Department of Physics, Yamagata University, Yamagata 990-8560, Japan*
[2]*Mathematical Physics Laboratory, RIKEN Nishina Center, Saitama 351-0198, Japan*
[3]*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 0WA, United Kingdom*
(Received 4 September 2011; published 21 June 2012)

Baryon number is ~~not~~ conserved when the magnetic field is nonzero.

Outline

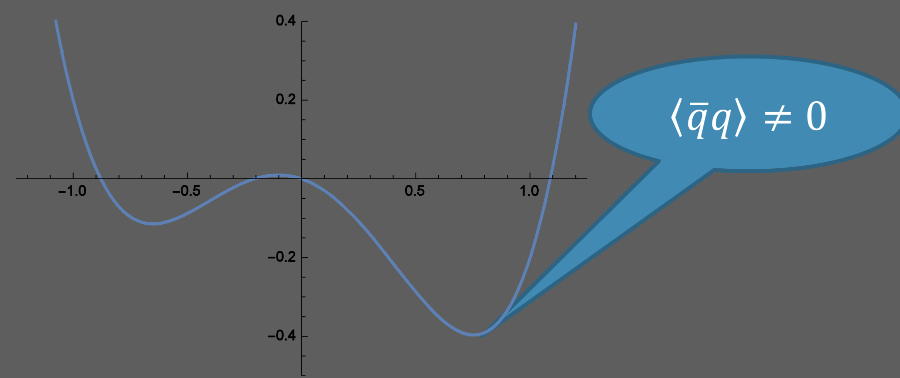
- Introduction
- ***Skyrme model***
- Nucleon and Delta in the magnetic background
- Summary

The chiral symmetry

The chiral symmetry:

Spontaneously breaking of chiral symmetry:

$$\langle \bar{q}q \rangle = \langle \overline{q_L} q_R + \overline{q_R} q_L \rangle \neq 0$$



The effective theory based on chiral symmetry:

- Nonlinear sigma model
- Chiral perturbation theory

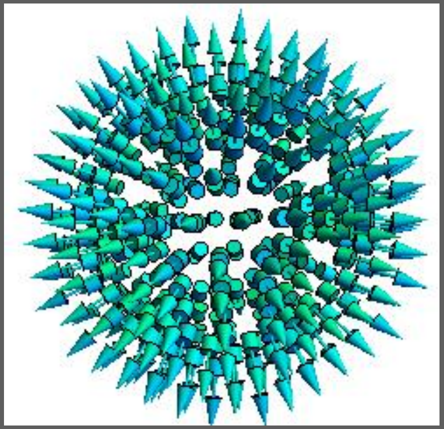
The Skyrme model

The nonlinear sigma model:

- Only pion is included
- Chiral symmetry is spontaneously broken
- Pion is the Nambu–Goldstone boson of chiral symmetry breaking

The Skyrme model only contains pion:

- The space group $SO(3)$ is mapping to isospin group $SU(2)$, which ensure the model have a non trivial topological solution
- The baryon is identified as the topological solution of mesonic model
- The Skyrme term, generating repulsive force to prevent the soliton shrinks



T. H. R. Skyrme, Nucl. Phys. 31, 556 (1962)

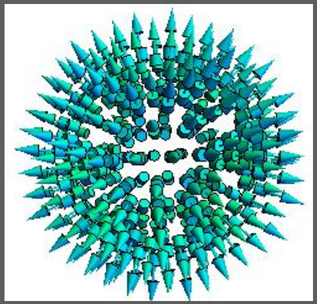
The Skyrme model

The model:

Even intrinsic parity

Odd intrinsic parity

$$\Gamma = \int d^4x \mathcal{L} + \Gamma_{\mathrm{WZW}}$$



Skyrme term, repulsive force

The Lagrangian:

$$\begin{aligned}\mathcal{L} &= \frac{f_\pi^2}{16}\mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2}\mathrm{Tr}([U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2) \\ &+ \frac{m_\pi^2 f_\pi^2}{16}\mathrm{Tr}(U + U^\dagger - 2)\end{aligned}$$

Pion mass term

T. H. R. Skyrme, Nucl. Phys. 31, 556 (1962)

$$\begin{aligned} U &= \xi_L^\dagger \xi_R = e^{2i\frac{\pi(x)}{f_\pi}} \\ \xi_{L,R} &\to \xi_{L,R}\cdot g_{L,R}^\dagger \\ \xi_{L,R} &= e^{\mp i\frac{\pi(x)}{f_\pi}} \end{aligned}$$

$$\pi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 \end{pmatrix}$$

The baryon number current

WZW term with external field

represents the chiral anomaly effects

$$\Gamma_{\mathrm{WZW}}[A_\mu] = \int d^4x j_B^\mu A_\mu$$

- Baryon number current is acquired by functional derivative the Wess-Zumino action with corresponding gauge field for U(1) baryon number

$$\begin{aligned} j_B^\mu &= \frac{1}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\,\{-i(\alpha_\nu\alpha_\rho\alpha_\sigma+\beta_\nu\beta_\rho\beta_\sigma)-3(\partial_\nu\mathcal{L}_\rho\alpha_\sigma+\partial_\nu\mathcal{R}_\rho\beta_\sigma)+3i(\mathcal{L}_\nu\alpha_\rho\alpha_\sigma-\mathcal{R}_\nu\beta_\rho\beta_\sigma) \\ &\quad +2(\partial_\nu\mathcal{R}_\rho U^\dagger\mathcal{L}_\sigma U-\partial_\nu\mathcal{R}_\rho\mathcal{R}_\sigma)-2(\partial_\nu\mathcal{L}_\rho U\mathcal{R}_\sigma U^\dagger-\partial_\nu\mathcal{L}_\rho\mathcal{L}_\sigma)+2i(U\mathcal{R}_\nu U^\dagger\mathcal{L}_\rho\alpha_\sigma \\ &\quad +U^\dagger\mathcal{L}_\nu U\mathcal{R}_\rho\beta_\sigma)+i(\mathcal{R}_\nu\mathcal{R}_\rho\mathcal{R}_\sigma-\mathcal{L}_\nu\mathcal{L}_\rho\mathcal{L}_\sigma)\}|_{\mathcal{V}_{B\mu}\to 0} \end{aligned}$$

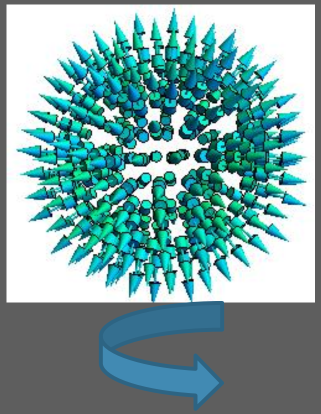
$$\alpha_\mu = \frac{1}{i}(\partial_\mu U)\,U^\dagger\,,\quad \beta_\mu = \frac{1}{i}U^\dagger\partial_\mu U$$

The baryon number current (when external fields are zero)

$$j_B^\mu = \frac{1}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\,\{-i(\alpha_\nu\alpha_\rho\alpha_\sigma+\beta_\nu\beta_\rho\beta_\sigma)\}|_{\mathcal{V}_{B\mu}\to 0}$$

The semi-classical quantization of skyrmion

Rotation in isospin space and spatial space



$$\hat{U} = A(U(R))A^\dagger$$

$$A^{-1}\dot{A} = \frac{i}{2}\omega_a \tau_a$$

$$(R^{-1}\dot{R})_{ij} = -\epsilon_{ijk}\Omega_k$$

Isospin space

Spatial space

The isospin and spin

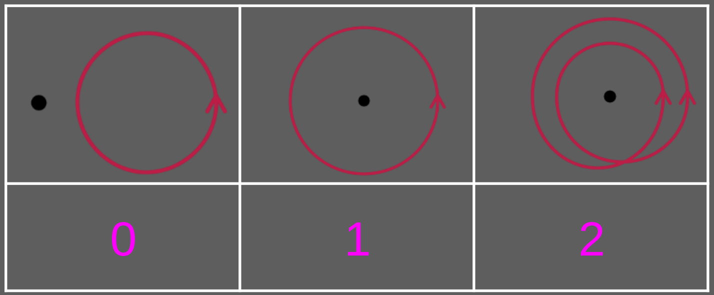
- Isospin

$$I_a = \frac{\partial \hat{\mathscr{L}}_{\text{total}}}{\partial \omega_a}\bigg|_{\mathcal{V}_{B\mu}\to 0}$$

- Spin

$$J_k = \frac{\partial \hat{\mathscr{L}}_{\text{total}}}{\partial \Omega_k}\bigg|_{\mathcal{V}_{B\mu}\to 0}$$

The properties of skyrmion



The winding number

$$N(B) = \int d^3x \frac{1}{24\pi^2} \epsilon^{ijk} \, \mathrm{Tr}(U^\dagger \partial_i U U^\dagger \partial_j U U^\dagger \partial_k U)$$

- Winding number, the topological number of skyrmion, corresponding to baryon number

Mass for Skyrmion

$$\mathcal{H} = \omega_a I_a + \Omega_i J_i - \mathcal{L}$$

$$M_\Psi \equiv \langle \Psi | \int dV \mathcal{H} | \Psi \rangle$$

The charge radius(root-mean-square (rms) radius)

- Charge radius of the baryon-number(winding number) current

$$\langle r^2 \rangle_B^{1/2} = \sqrt{\int_0^\infty d^3 r r^2 j_B^0(r)}$$

- Charge radius of the energy(soliton mass)

$$\langle r^2 \rangle_E^{1/2} = \sqrt{\frac{1}{M_{\mathrm{sol}}} \int_0^\infty d^3 r r^2 M_{\mathrm{sol}}(r)}$$

The properties of skyrmion

G.S. Adkins, C.R. Nappi, Nucl. Phys. B 233 (1984) 109

Quantity	Prediction	Experiment
M_N	Input	938.9 MeV
M_Δ	Input	1232 MeV
m_π	Input	138 MeV
$\langle R_p^2 \rangle_E^{1/2}$	0.865 fm	0.84-0.87 fm
$\langle R_n^2 \rangle_E$	-0.278 fm^2	-0.116 fm^2
μ_p	1.97	2.79
μ_n	-1.24	-1.91
μ_p/μ_n	-1.59	-1.46
g_A	0.65	1.23
$g_{\pi NN}$	11.9	13.5
$g_{\pi N\Delta}$	17.8	20.3

Outline

- Introduction
- Skyrme model
- ***Nucleon and Delta in the magnetic background***
- Summary

The model

Field type	Operator	Physics field
Pseudo-scalar	$F(r)$	π

Lagrangian (q=u,d)

Skyrmion term, generate repulsive force

π mass term

$$\mathscr{L} = \frac{f_\pi^2}{16}\mathrm{Tr}(D_\mu U^\dagger D^\mu U) + \frac{1}{32g^2}\mathrm{Tr}([U^\dagger D_\mu U, U^\dagger D_\nu U]^2) + \frac{m_\pi^2 f_\pi^2}{16}\mathrm{Tr}(U + U^\dagger - 2)$$

covariant derivative

$$D_\mu U = \partial_\mu U - i\mathcal{L}_\mu U + iU\mathcal{R}_\mu$$

external field

$$\mathcal{L}_\mu = \mathcal{R}_\mu = eQ_{\mathrm{B}}\mathcal{V}_{B\mu} + eQ_{\mathrm{E}}H_\mu$$

magnetic field

$$H_\mu = -\frac{1}{2}Byg_\mu{}^1 + \frac{1}{2}Bxg_\mu{}^2$$

electric charge matrix

$$Q_{\mathrm{E}} = \frac{1}{6}\mathbb{1} + \frac{1}{2}\tau_3$$

baryon number matrix

$$Q_{\mathrm{B}} = \frac{1}{3}\mathbb{1}$$

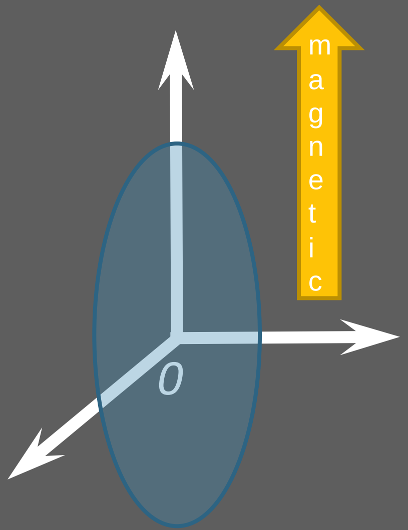
The symmetries of the model

$$SU(2)_{\text{flavor}} \times SO(3)_{\text{space}} \times U(1)_V \Rightarrow U(1)_{\text{flavor}} \times SO(2)_{\text{space}} \times U(1)_V$$

The ansatz

$$
\begin{aligned}
x &= c_\rho r \sin(\theta)\cos(\varphi) \\
y &= c_\rho r \sin(\theta)\sin(\varphi) \\
z &= c_z r \cos(\theta)
\end{aligned}
$$

$$U = \cos(F(r))\mathbb{1} + \frac{i\sin(F(r))}{r}\left(\frac{\tau_1}{c_\rho}x + \frac{\tau_2}{c_\rho}y + \frac{\tau_3}{c_z}z\right)$$



G. Holzwarth and B. Schwesinger, Rep. Prog. Phys. 49, 825 (1986)

Model parameters

- The parameters are determined by using the masses of proton(neutron) and delta.

G. S. Adkins, C. R. Nappi, and E.Witten, Nucl. Phys. B228, 552 (1983).

f_π	108 MeV
m_π	138 MeV
g	4.84

The baryon number current

WZW term with external field

$$\Gamma_{\mathrm{WZW}}[A_\mu] = \int d^4x\, j_B^\mu A_\mu$$

- Baryon number current is acquired by functional derivative the Wess-Zumino action with corresponding gauge field for U(1) baryon number

The baryon number current

$$\begin{aligned} j_B^\mu \;=\; & \frac{1}{48\pi^2}\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\big\{-i(\alpha_\nu\alpha_\rho\alpha_\sigma+\beta_\nu\beta_\rho\beta_\sigma)-3(\partial_\nu\mathcal{L}_\rho\alpha_\sigma+\partial_\nu\mathcal{R}_\rho\beta_\sigma)+3i(\mathcal{L}_\nu\alpha_\rho\alpha_\sigma-\mathcal{R}_\nu\beta_\rho\beta_\sigma) \\ & +2(\partial_\nu\mathcal{R}_\rho U^\dagger\mathcal{L}_\sigma U-\partial_\nu\mathcal{R}_\rho\mathcal{R}_\sigma)-2(\partial_\nu\mathcal{L}_\rho U\mathcal{R}_\sigma U^\dagger-\partial_\nu\mathcal{L}_\rho\mathcal{L}_\sigma)+2i(U\mathcal{R}_\nu U^\dagger\mathcal{L}_\rho\alpha_\sigma \\ & +U^\dagger\mathcal{L}_\nu U\mathcal{R}_\rho\beta_\sigma)+i(\mathcal{R}_\nu\mathcal{R}_\rho\mathcal{R}_\sigma-\mathcal{L}_\nu\mathcal{L}_\rho\mathcal{L}_\sigma)\big\}|_{\mathcal{V}_{B\mu}\to 0} \end{aligned}$$

$$\alpha_\mu = \frac{1}{i}(\partial_\mu U)U^\dagger, \quad \beta_\mu = \frac{1}{i}U^\dagger\partial_\mu U$$

The baryon number

$$N_B = \int dV\, j_B^0 = \frac{\sin(2F)\left(eBc_\rho^2 r^2 D_{33}+6\right)-12F}{12\pi}\Bigg|_{F(0)=\pi}^{F(\infty)=0} = 1$$

- The correct boundary condition is given by requiring $\int dV \mathrm{j}_\mathrm{B}^0 = 1$

The Gell-Mann-Nishijima formula

Iso-vector current

$$j_{\mathcal{V}}^{a,\mu} = \frac{\partial(\hat{\mathscr{L}}_{\text{total}})}{\partial(\delta(\mathcal{V}_\mu^a))}|_{\mathcal{V}_{B\,\mu}\to 0,\delta(\mathcal{V}_\mu^a)\to 0}$$

- Iso-vector current is acquired by functional derivative the total action with corresponding SU(2) iso-vector gauge field

The conserved charge corresponding to 3rd component of Iso-vector current

$$\begin{aligned} N_{\mathcal{V}^{3,0}} &= \int dV\, j_{\mathcal{V}}^{3,0} \\ &= -\frac{\sin(2F)\left(eBc_\rho^2 r^2 D_{33}\right)}{24\pi}\Big|_{F(0)=\pi}^{F(\infty)=0} + I_3 \\ &= I_3 \end{aligned}$$

The Gell-Mann-Nishijima formula

$$N_E = \int dV \left(\frac{j_B^0}{2} + j_{\mathcal{V}}^{3,0}\right) = \frac{N_B}{2} + I_3$$

- Where the electric charge is conserved in the magnetic field background

The general relation between nucleons and delta isobars magnetic moment

spin

iso spin

	$J_3 = 3/2$	$J_3 = 1/2$
$\mu_{\Delta^{++}}$	$\frac{3}{5}(4\mu_p + \mu_n) + 3\mu_I$	$\frac{1}{5}(4\mu_p + \mu_n) + 3\mu_I$
$\mu_{\Delta^{+}}$	$\frac{3}{5}(3\mu_p + 2\mu_n) + \mu_I$	$\frac{1}{5}(3\mu_p + 2\mu_n) + \mu_I$
$\mu_{\Delta^{0}}$	$\frac{3}{5}(2\mu_p + 3\mu_n) - \mu_I$	$\frac{1}{5}(2\mu_p + 3\mu_n) - \mu_I$
$\mu_{\Delta^{-}}$	$\frac{3}{5}(\mu_p + 4\mu_n) - 3\mu_I$	$\frac{1}{5}(\mu_p + 4\mu_n) - 3\mu_I$

- The magnetic moment of a Delta isobar state is constructed by two parts, one part is related to the strength of spin and another part is related to the strength of iso-spin
- The magnetic moment relates to spin part is a combination of proton and neutron magnetic moment μ_p and μ_n
- The magnetic moment relates to iso-spin part μ_I can be determined numerically as about μ_I =-0.045 μ_N which is much smaller than the magnitude of μ_p and μ_n

The general relation between nucleons and delta isobars magnetic moment

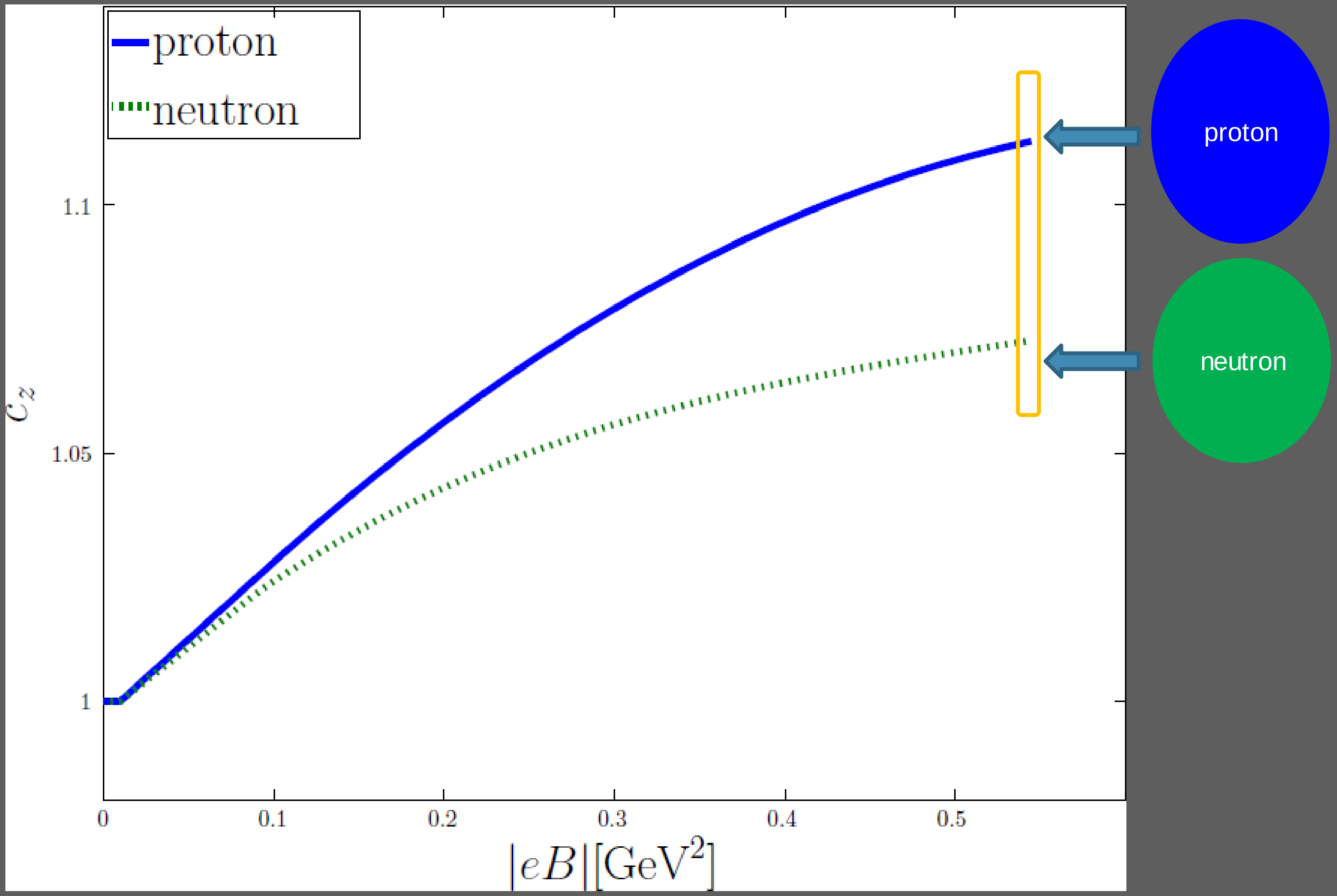
The theoretical prediction and experimental result of Delta isobars magnetic moment:

	$J_3 = 3/2$	$J_3 = 1/2$	Exp.[PDG]
$\mu_{\Delta^{++}}$	5.42	1.72	5.6 ± 1.9
$\mu_{\Delta^{+}}$	2.69	0.87	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$
$\mu_{\Delta^{0}}$	-0.05	0.01	
$\mu_{\Delta^{-}}$	-2.78	-0.84	

The experimental result of Delta isobars magnetic moment satisfies relation:

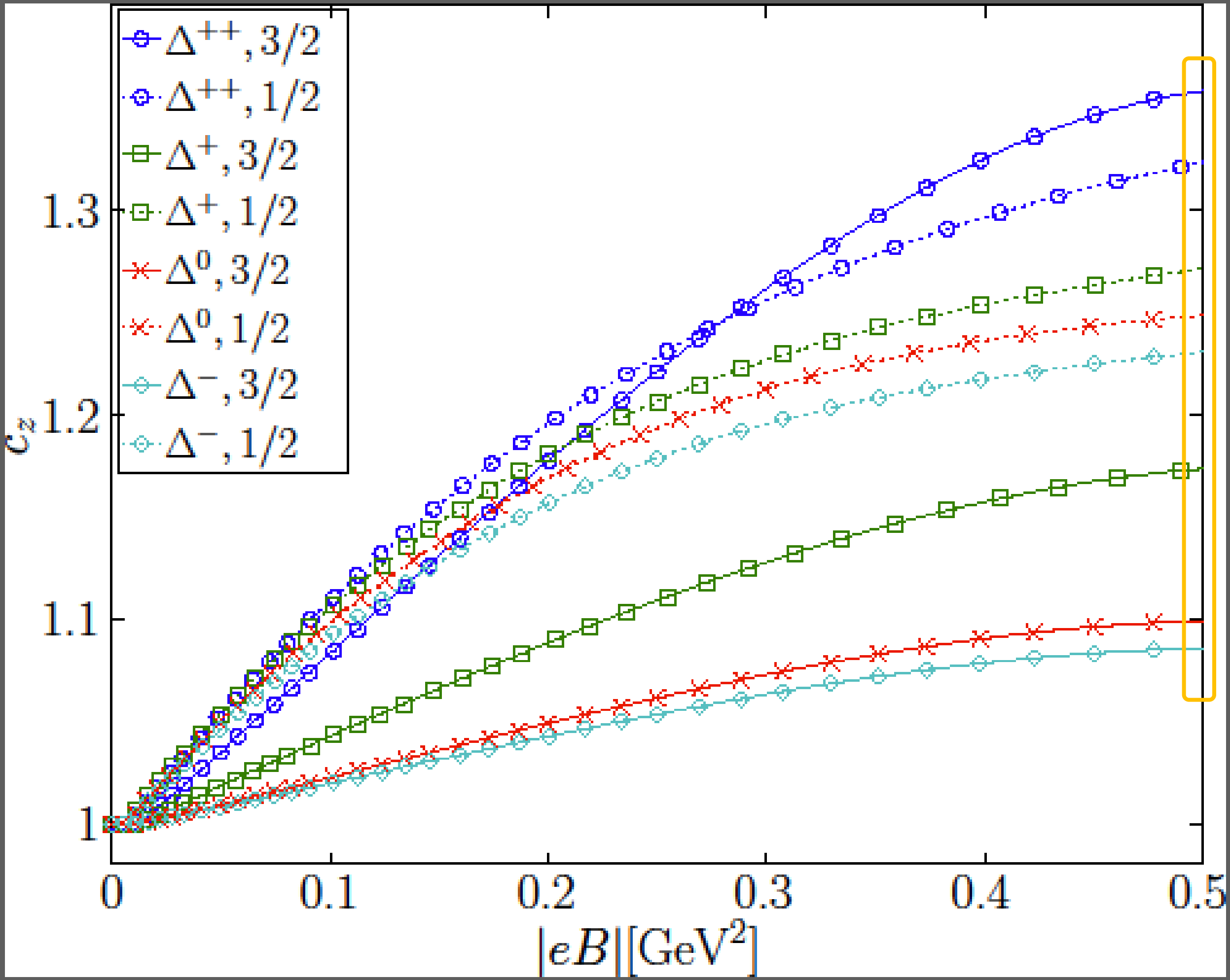
$$3\mu_{\Delta^+, J_3=3/2} - \mu_{\Delta^{++}, J_3=3/2} : \mu_p + \mu_n = 3 : 1.056 \simeq 3 : 1$$

The shape of nucleons are stretched



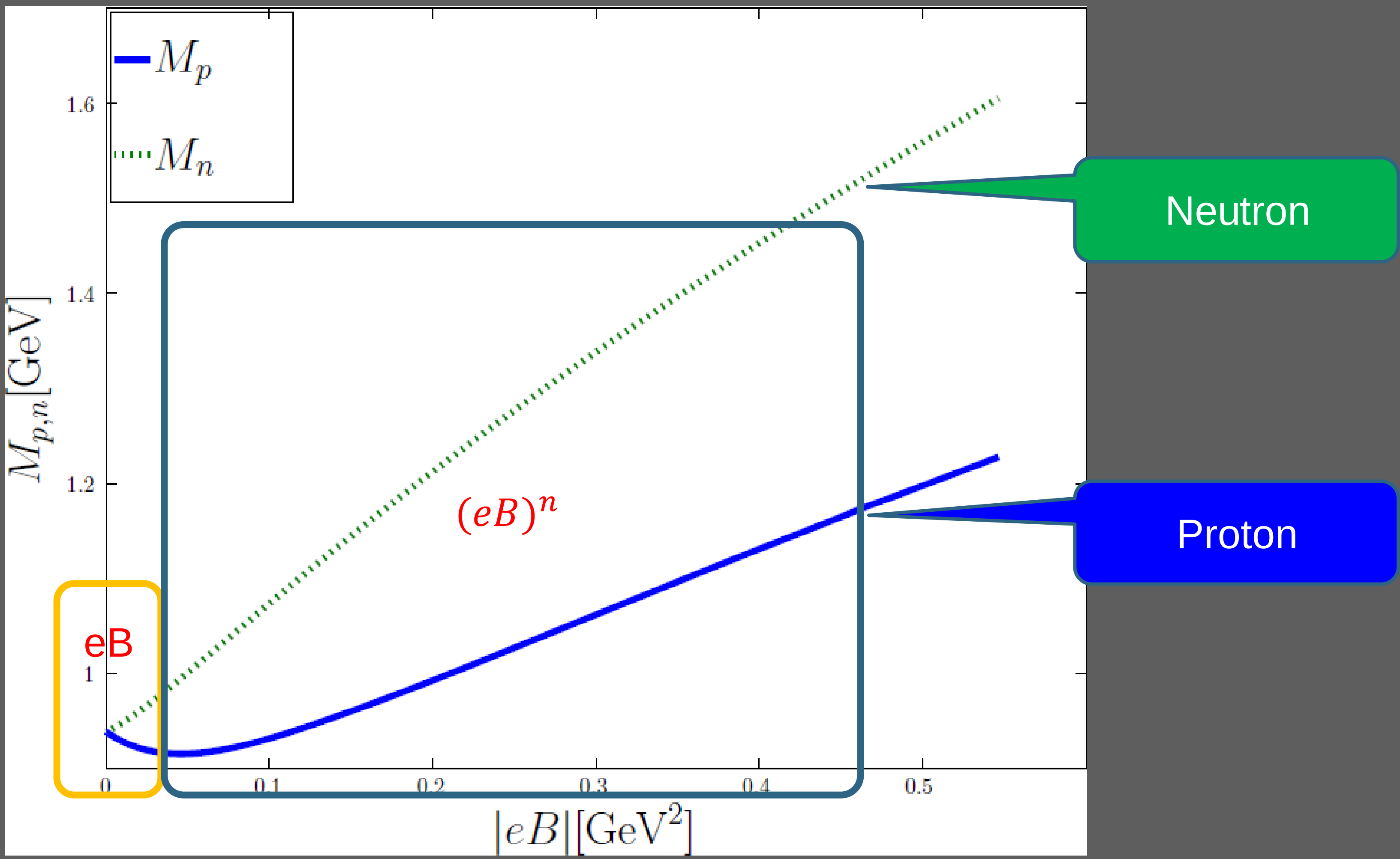
The ansatz is axially symmetric
when magnetic field is non-zero

The shape of Delta isobars are stretched



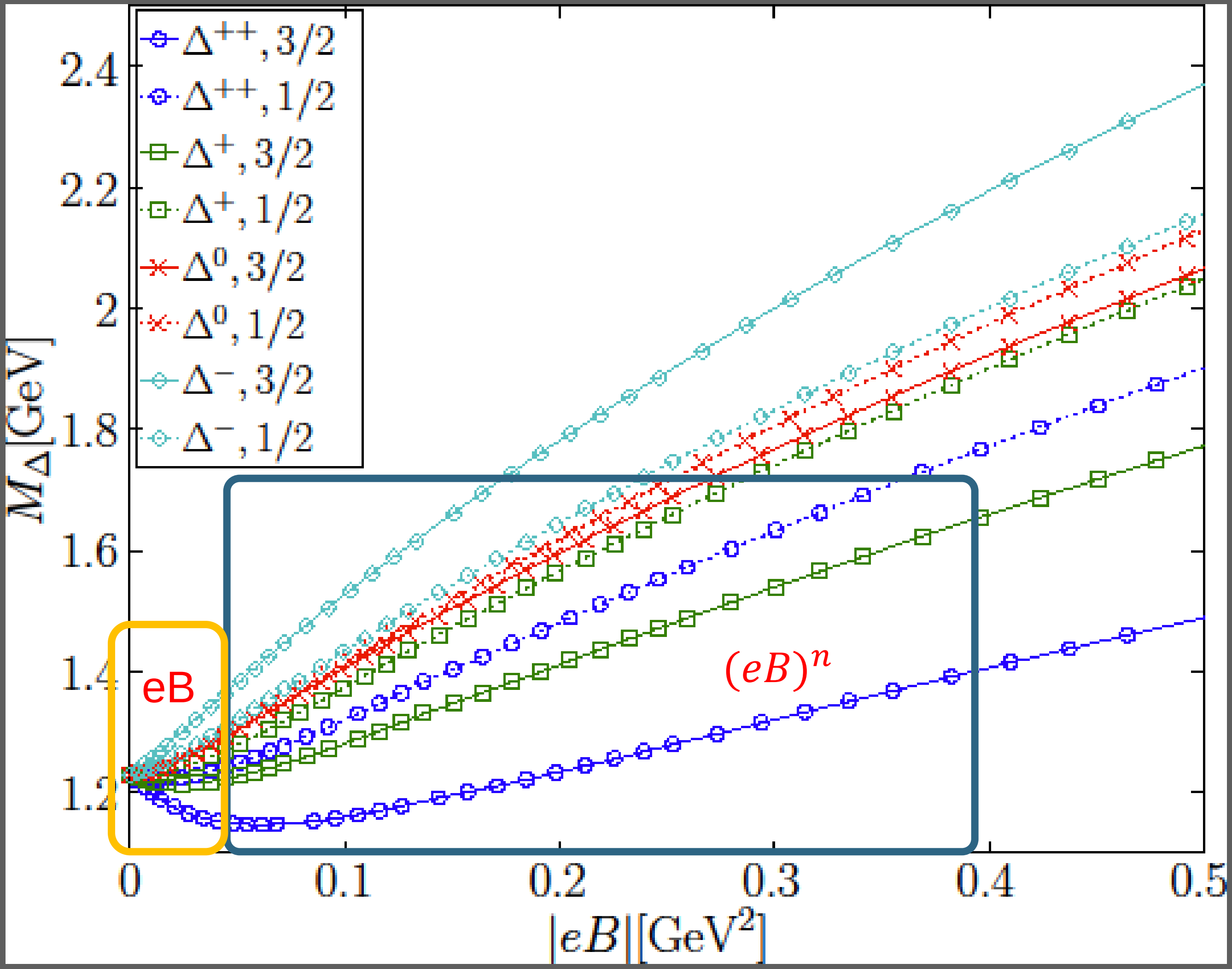
The ansatz is axially symmetric
when magnetic field is non-zero

The magnetic response of nucleons mass



The mass of baryon is changed

The magnetic response of Delta isobars mass

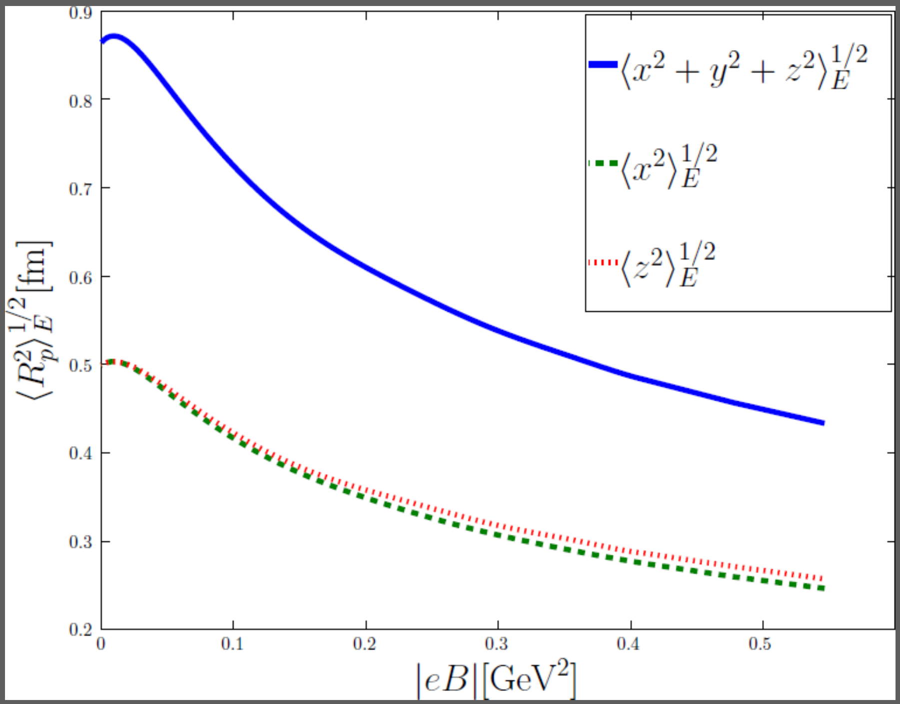


The mass of baryon is changed

The anisotropy of proton electric charge radius

The proton root mean square(RMS) electric charge radii:

$$\langle R_p^2\rangle_E^{1/2} \equiv \langle p|\int dV\, R^2 \rho_E |p\rangle^{1/2}$$



The electric charge radius of proton is changed

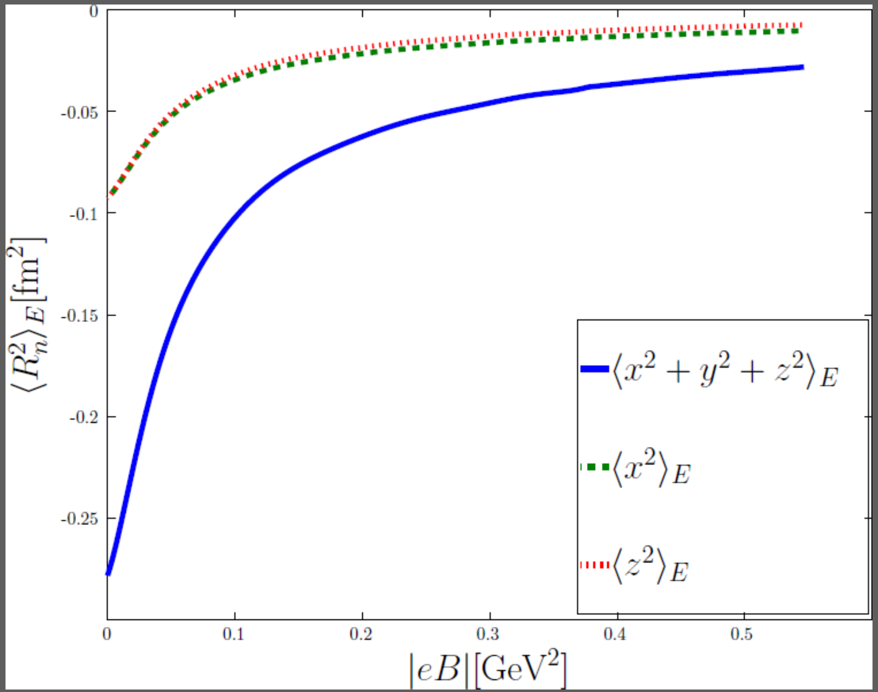
Physically:

- when |eB| is weak, the proton mass decreases, which causes the proton size to increase
- when |eB| is strong, the freedom of the charged meson ($\pi^{+,-}$) is restricted in the x-y plane, which causes the proton size to decrease

The anisotropy of neutron electric charge radius

The neutron mean square(MS) electric charge radii:

$$\langle R_n^2 \rangle_E \equiv \langle n| \int dV\, R^2 \rho_E |n\rangle$$

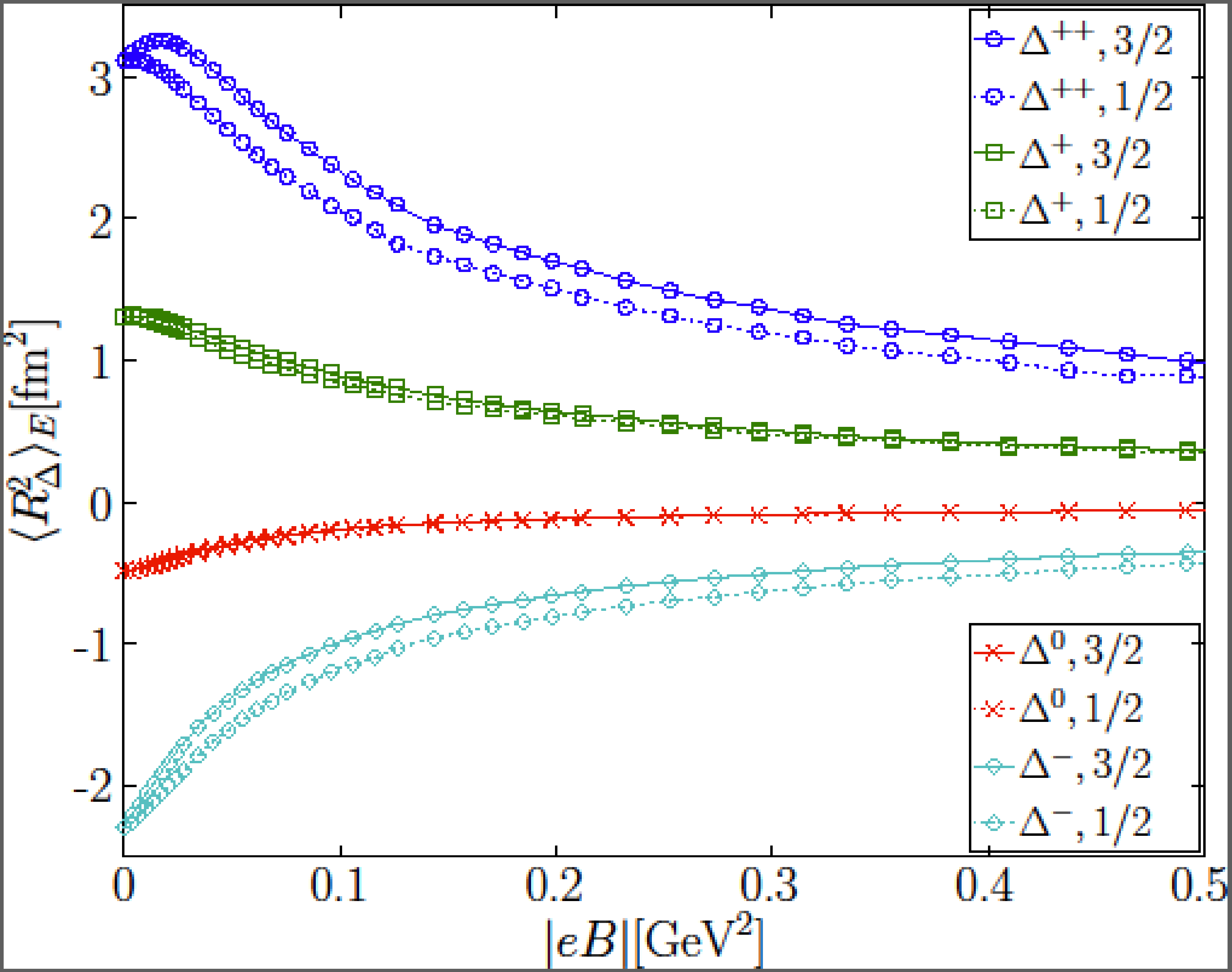


The electric charge radius of neutron is changed

Physically:

- the total electric charge of neutron is neutral but have both positive and negative electric charge distribution
- the negative distribution is more apart from central point which cause MS radii have a minus sign
- the neutron mass always increases, and the freedom of the charged meson ($\pi^{+,-}$) is restricted in the x-y plane, which causes the neutron size to decrease

The Delta isobars mean square electric charge radius



Outline

- Introduction
- Skyrme model
- Nucleon and Delta in the magnetic background
- ***Summary***

Summary:

- In zero density, weak magnetic field region
  - The baryon number is always conserved
  - The electric charge of baryon is always conserved
  - The magnetic field twist the shape of baryon
  - the magnetic moment of Delta isobars can be rewritten by the magnetic moment of proton and neutron

- In zero density, strong magnetic field region
  - The mass of proton and $\Delta^{++,+,0}$ first decreases, and then increases, consequently, the size of them first increases and then decreases.
  - The mass of neutron and Δ^{-} always increases, and consequently, the size of them always decreases
  - In the core part of magnetar, the proton density decrease 3.4% and the neutron density increase 15.3% compared to that in vacuum.

	$J_3 = 3/2$	$J_3 = 1/2$
$\rho_{\Delta^{++}}$	-8.5%	-0.1%
$\rho_{\Delta^{+}}$	-0.9%	$+2.4\%$
$\rho_{\Delta^{0}}$	-7.4%	-5.6%
$\rho_{\Delta^{-}}$	$+19.7\%$	$+9.8\%$

Thank you for your attention!