A Bridge Between Euclidean Space and Minkowski Space Physics

—Combine the DSE approach with MIT bag model

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Based on Phys. Rev. D 99, 074013 and the work in preparation.

Together with Lei Chang, Yuxin Liu.

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- There is one way that can solve this gap partly, that is the quasi parton distribution functions. They can exact the quasi parton distribution functions from the LQCD calculations. But the procedure is quite complicated.
- Here we give out a much simpler way to study the dress effect (under the rainbow ladder approximation and beyond rainbow ladder approximation) of parton distribution functions partly.

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Dyson-Schwinger Equation

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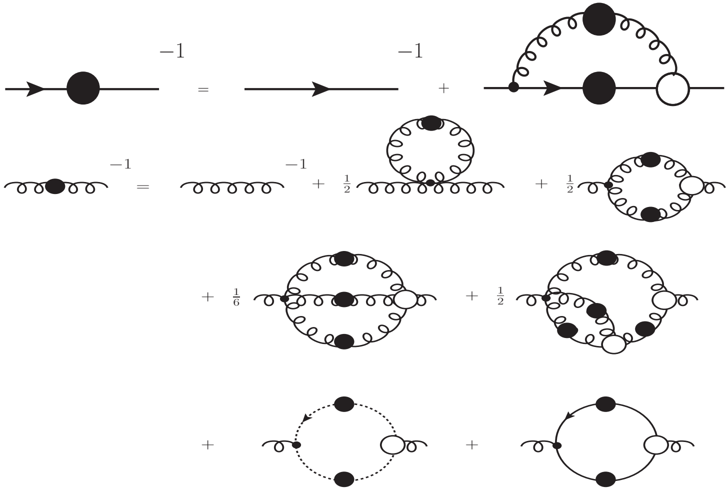
$$\frac{1}{Z[J]} < (\partial^2 + \mu^2)\phi(x) > |_J = J(x) \tag{2}$$

Do the successive derivative with respect to the external field $J(x)$, we can get the general expression

$$\begin{aligned} &< \frac{\delta}{\delta\phi(x)} \left[\int d^4x' \mathcal{L}\left(\phi(x')\right) \right] \phi(x_1) \cdots \phi(x_n) > \\ &= \sum_{i=1}^{n} < T\{\phi(x_1) \cdots \left(-i\delta^4(x - x_i)\right) \cdots \phi(x_n)\} > \end{aligned} \tag{3}$$

Dyson-Schwinger Equation

The schematic representation of DSEs in QCD:



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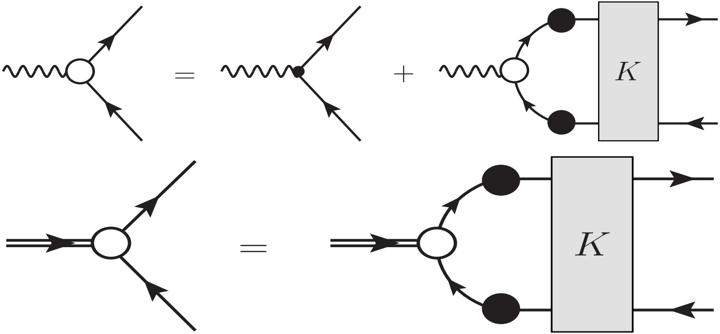
Bethe-Salpeter Equation

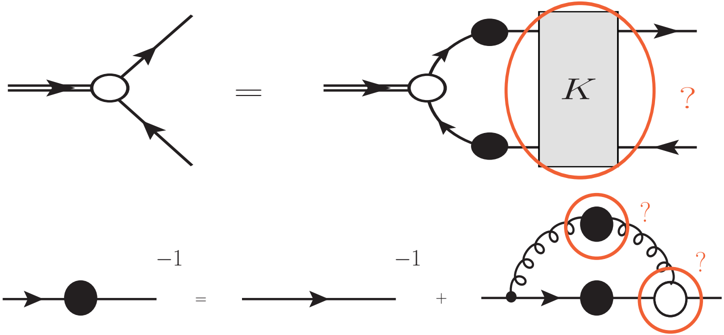
In the special case of two body systems, they are describe by the Bethe-Salpeter equations.



By solving these equations, we can get the wave function of two body bound states, such as mesons, diquarks etc. and give out the predictions for their properties.

DSE Approach

Since DSEs are infinitely coupled, to make them functional in real calculations, we need to make a truncation.



$$S^{-1}(p) = Z_2 S_0^{-1}(p) + Z_2 g^2 \int_{dq}^{\Lambda} D_{\alpha\beta}(k) t^a \gamma_\alpha S(q) t^a \Gamma_\beta(q,p) \qquad (4)$$

$$[\Gamma(k;0)]_{EF} = Z_v [\Gamma_0(k;0)]_{EF} + \int_{dq}^{\Lambda} [K(k,q;0)]_{EF}^{GH} [\chi(q;0)]_{GH} \qquad (5)$$

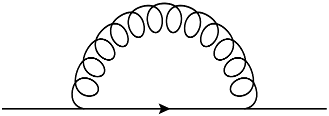
Rainbow ladder approximation

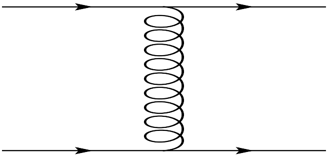
The most successful truncation is the rainbow ladder approximation.

$$\Gamma^{\mu}(p+k,p) = \gamma^{\mu} \tag{6}$$

For the consistency between DSE and BSE [PhysRevD.52.4736 Munczek 1995]

$$K(p,q;0) = -\frac{\delta\Sigma(p)}{\delta S(q)} \tag{7}$$





The gluon propagator

Usually, one would model the gluon propagator as
$g^2 D_{\alpha\beta}(k) = \mathcal{G}(k^2)(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2})$

$$\mathcal{G}(k^2) = \mathcal{G}_{ir}(k^2) + \mathcal{G}_{uv}(k^2) \tag{8}$$

$$\mathcal{G}_{ir}(k^2) = \frac{8\pi^2}{\omega^5} m_g^3 e^{-k^2/\omega^2} \tag{9}$$

$$\mathcal{G}_{uv}(k^2) = \frac{8\pi^2 \gamma_m}{\ln[\tau + (1 + k^2/\Lambda_{QCD}^2)^2]} \frac{1 - e^{-k^2/4m_t^2}}{k^2} . \tag{10}$$

A quenched gluon propagator!
The rainbow-ladder approximation had showed successes in light quark ground state mesons.
Then what about beyond rainbow ladder approximation?

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Truncation Beyond Rainbow Ladder

We use the longitudinal part of Munczek vertex: [PhysRevD.52.4736 Munczek 1995]

$$i\Gamma_\nu(p+k,p) = \frac{\partial}{\partial p^\nu} \int_0^1 d\alpha S^{-1}(p+\alpha k) \tag{11}$$

It satisfies the Ward-Takahashi identity:

$$ik_\nu \Gamma_\nu(p+k,p) = S^{-1}(p+k) - S^{-1}(p) \tag{12}$$

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- drawback: we can't represent it with Feynman diagram.
- advantage: It's an analytical expression. then we may derive the scattering kernel analytically.

Scattering Kernel

The scattering kernel for the Munczek quark gluon vertex can be calculated out as

$$[K(k,q;0)]_{EF}^{GH} = -\frac{\delta[\Sigma(k)]_{EF}}{\delta[S(q)]_{GH}} = ① + ② \,, \tag{13}$$

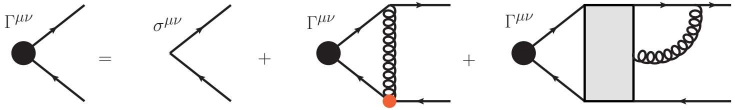
$$① = -Z_2 g^2 D_{\mu\nu}(q-k) t^a \, [\gamma_\mu]_{EG} \, t^a \, [\Gamma_\nu(q,k)]_{HF} \tag{14}$$

$$\begin{aligned} ② = {} & Z_2 g^2 \int_{dl}^{\Lambda} D_\mu \nu(l-k) t^a \, [\gamma_\mu]_{EM} \, [S(l)]_{MN} \, t^a \\ & \times \frac{\partial}{i\partial k^\nu} \int_0^1 d\alpha \left[S^{-1}(k+\alpha(l-k)) \right]_{NG} \\ & \times \delta^{(4)} \left(k + \alpha(l-k) - q \right) \left[S^{-1} \left(k + \alpha(l-k) \right) \right]_{HF} \,. \end{aligned} \tag{15}$$

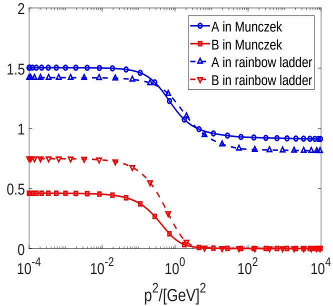
BSE with Munczek's Vertex

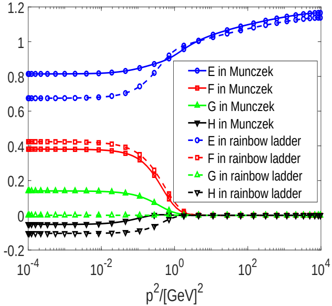
The BSE with Munczek's vertex takes as

$$
\begin{aligned}
[\Gamma(k;0)]_{EF} = Z_v[\Gamma_0(k;0)]_{EF} \\
& - Z_2 g^2 \int_d q^\Lambda D_{\mu\nu}(q-k) t^a [\gamma_\mu]_{EG} [\chi(q;0)]_{GH} t^a [\Gamma_\nu(q,k)]_{HF} \\
& + Z_2 g^2 \int_{dq}^\Lambda D_{\mu\nu}(q-k) t^a [\gamma_\mu]_{EM} [S(q)]_{MN} t^a [\Lambda_\nu(q,k;0)]_{NF}
\end{aligned}
\tag{16}
$$



Results of Munczek vertex





$$S^{-1}(p) = i\gamma \cdot pA(p^2) + B(p^2)$$

$$\begin{aligned}\Gamma_{\mu\nu}(p;0) &= \sigma_{\mu\nu}E(p^2)\\ &+\Big((i\gamma\cdot p)\sigma_{\mu\nu} + \sigma_{\mu\nu}(i\gamma\cdot p)\Big)F(p^2)\\ &+\Big((i\gamma\cdot p)\sigma_{\mu\nu} - \sigma_{\mu\nu}(i\gamma\cdot p)\Big)G(p^2)\\ &+(i\gamma\cdot p)\sigma_{\mu\nu}(i\gamma\cdot p)H(p^2)\end{aligned}$$

	$m_g/[GeV]$	$\omega/[GeV]$
RL	0.82	0.5
Mun	0.436	0.355

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Quark field in the bag

The wave function of a fermion in the bag is the solution of free massless Dirac equation:

$$\varphi_m(\boldsymbol{x}, t) = N \begin{bmatrix} j_0\left(\frac{\omega_{n,\kappa} r}{R}\right) U_m \\ i\boldsymbol{\sigma} \cdot \hat{\boldsymbol{x}} j_1\left(\frac{\omega_{n,\kappa} r}{R}\right) U_m \end{bmatrix} e^{-i\frac{\omega_{n\kappa} t}{R}} , \tag{17}$$

$\omega = \omega_{1,-1} = 2.04$.
Second quantization, a quark field in the bag center at $\boldsymbol{a}$:

$$\psi(\boldsymbol{x}, t) = \sum_{m=\uparrow,\downarrow} a_{q_m}(\boldsymbol{a}) \varphi_m(\boldsymbol{x} - \boldsymbol{a}, t) + \cdots \tag{18}$$

The annihilation and creation operator satisfy

$$\left\{ a_i(\boldsymbol{a}), a_j^\dagger(\boldsymbol{b}) \right\} = \delta_{ij} \int d^3x \varphi_j^\dagger(\boldsymbol{x} - \boldsymbol{b}) \varphi_i(\boldsymbol{x} - \boldsymbol{a}) \, . \tag{19}$$

Proton field in the bag

In the constituent quark model, a spin-up proton

$$
\begin{aligned}
|P\uparrow\rangle = \frac{1}{\sqrt{18}}\,(&2u_\uparrow u_\uparrow d_\downarrow - u_\uparrow u_\downarrow d_\uparrow - u_\downarrow u_\uparrow d_\uparrow \\
&+2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow d_\uparrow u_\downarrow - u_\downarrow d_\uparrow u_\uparrow \\
&+2d_\downarrow u_\uparrow u_\uparrow - d_\uparrow u_\uparrow u_\downarrow - d_\uparrow u_\downarrow u_\uparrow)\ .
\end{aligned} \tag{20}
$$

So the proton field in the bag

$$
\begin{aligned}
&|P\uparrow, \boldsymbol{r} = \boldsymbol{a}\rangle \\
&= \frac{1}{\sqrt{18}}\Big(2a^\dagger_{u_\uparrow}(\boldsymbol{a})a^\dagger_{u_\uparrow}(\boldsymbol{a})a^\dagger_{d_\downarrow}(\boldsymbol{a}) \\
&\qquad -a^\dagger_{u_\uparrow}(\boldsymbol{a})a^\dagger_{u_\downarrow}(\boldsymbol{a})a^\dagger_{d_\uparrow}(\boldsymbol{a}) \\
&\qquad -a^\dagger_{u_\downarrow}(\boldsymbol{a})a^\dagger_{u_\uparrow}(\boldsymbol{a})a^\dagger_{d_\uparrow}(\boldsymbol{a}) + \cdots\Big)\,|0,\ \boldsymbol{r} = \boldsymbol{a}\rangle\ ,
\end{aligned} \tag{21}
$$

$|0,\ \boldsymbol{r} = \boldsymbol{a}\rangle$ is the empty bag center at $\boldsymbol{r} = \boldsymbol{a}$

Peierls-Yoccoz (PY) projection method

The static hadron bag state $H_B(\boldsymbol{x})$ can be decomposed in terms of the plane wave in the momentum space

$$|H_B(\boldsymbol{x})\rangle = \int \frac{d^3p}{(2\pi)^3} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \left[\frac{\phi(\boldsymbol{p})}{W_H(p)}\right] |H(\boldsymbol{p})\rangle \,. \tag{22}$$

$$|H(\boldsymbol{p})\rangle = \left[\frac{W_H(p)}{\phi(\boldsymbol{p})}\right] \int d^3x e^{-i\boldsymbol{x}\cdot\boldsymbol{p}} |H_B(\boldsymbol{x})\rangle \,, \tag{23}$$

the normalization relation:

$$\langle H(\boldsymbol{p})|H(\boldsymbol{p}')\rangle = (2\pi)^3 \delta^{(3)}(\boldsymbol{p}-\boldsymbol{p}')\, W_H(p) \,. \tag{24}$$

we can get

$$|\phi(\boldsymbol{p})|^2 = \int d^3r e^{-i\boldsymbol{r}\cdot\boldsymbol{p}} \langle H_B(\boldsymbol{0})|H_B(\boldsymbol{r})\rangle = |\phi_3(\boldsymbol{p})|^2 \,. \tag{25}$$

where

$$|\phi_n(\boldsymbol{p})|^2 = \int d^3a e^{-i\boldsymbol{p}\cdot\boldsymbol{a}} \left[\int d^3x \varphi^\dagger(\boldsymbol{x}-\boldsymbol{a})\varphi(\boldsymbol{x})\right]^n \,. \tag{26}$$

Quark distribution in the bag

The definition of quark distribution function in a bag:

$$q_i(x) = 2M \int \frac{d\xi^-}{4\pi} e^{iq^+\xi^-} \times \langle N; \boldsymbol{p} = 0 | \bar{\psi}_i(\xi)\gamma^+\psi_i(0) | N; \boldsymbol{p} = 0 \rangle \Big|_{\xi^+, \xi_\perp = 0} , \tag{27}$$

Define

$$\mathcal{M} = \langle N; \boldsymbol{p} = 0 | \bar{\psi}_i(\xi)\gamma^+\psi_i(0) | N; \boldsymbol{p} = 0 \rangle . \tag{28}$$

The result is

$$\mathcal{M} = \sum_m \langle N; \mathbf{0} | P_{f,m} | N; \mathbf{0} \rangle \int \frac{d^3k_1}{(2\pi)^3} e^{i(\omega\xi^0/R - \boldsymbol{k}_1 \cdot \boldsymbol{\xi})} \times \bar{\varphi}(\boldsymbol{k}_1)\gamma^+\varphi(\boldsymbol{k}_1) \frac{|\phi_2(\boldsymbol{k}_1)|^2}{|\phi_3(\mathbf{0})|^2} . \tag{29}$$

Quark distribution in the bag

$$
\begin{aligned}
q_i(x) = &\left(\sum_m \langle N; \mathbf{0}|P_{f,m}|N; \mathbf{0}\rangle\right) \frac{M}{2\pi} \int d\xi^- e^{i(q^+ + \tilde{k}_1^+)\xi^-} \int \frac{d^3 k_1}{(2\pi)^3} \\
&\bar{\varphi}(\boldsymbol{k}_1)\gamma^+ \varphi(\boldsymbol{k}_1) \frac{\phi_2(|\boldsymbol{k}_1)|^2}{|\phi_3(\mathbf{0})|^2}\,, \\
= &\sqrt{2}M \left(\sum_m \langle N; \mathbf{0}|P_{f,m}|N; \mathbf{0}\rangle\right) \int_{k_{min}}^{\infty} \frac{kdk}{(2\pi)^2} \\
&\bar{\varphi}(\boldsymbol{k})\gamma^+ \varphi(\boldsymbol{k}) \frac{|\phi_2(\boldsymbol{k})|^2}{|\phi_3(\mathbf{0})|^2}\,,
\end{aligned} \tag{30}
$$

where $k_{min} = |\omega/R - Mx|$.

The time component has been integrated out.

Quark distribution in the bag

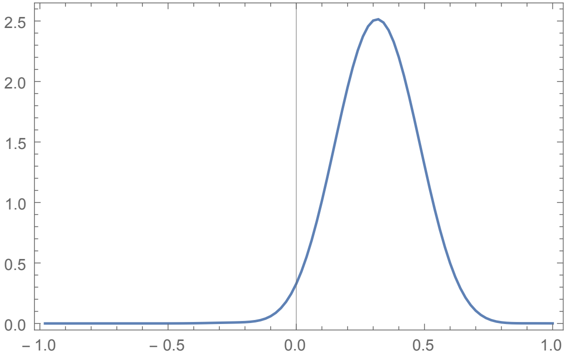
Calculate the inner product directly,

$$\bar{\varphi}(\boldsymbol{k})\gamma^{+}\varphi(\boldsymbol{k}) = \frac{4\pi R^3\omega^4}{\sqrt{2}(\omega^2-\sin^2\omega)}\left[t_{00}^2(k)+t_{11}^2(k)+2\hat{k}_z t_{00}(k)t_{11}(k)\right], \tag{31}$$

we finally get

$$q_i(x) = \frac{4\pi M R^3\omega^4}{(\omega^2-\sin^2\omega)}\left(\sum_m \langle N;\mathbf{0}|P_{f,m}|N;\mathbf{0}\rangle\right)\times \int_{k_{min}}^{\infty}\frac{kdk}{(2\pi)^2}\left[t_{00}^2(k)+t_{11}^2(k)+2\hat{k}_z t_{00}(k)t_{11}(k)\right]\frac{|\phi_2(\boldsymbol{k})|^2}{|\phi_3(\mathbf{0})|^2}. \tag{32}$$

Quark distribution in the bag



B	$(0.113\text{GeV})^4$
R	1.46 fm
M	1.10 GeV
ω	2.04

$$\int_{-1}^{1} dx\ q(x) = 0.99994\,. \tag{33}$$

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Take the transversity distribution function as a example.

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Definition

$$h(x) = 2M \int \frac{d\xi^-}{4\pi} e^{iq^+\xi^-} \times \\ \langle N; \boldsymbol{p} = 0; S | \bar{\psi}_i(\xi) i\sigma^{1+}\gamma_5 \psi_i(0) | N; \boldsymbol{p} = 0; S \rangle \Big|_{\xi^+, \xi_\perp = 0} . \tag{34}$$

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$$\bar{\phi}(\boldsymbol{k}) i\sigma^{1+}\gamma_5 \phi(\boldsymbol{k}) \\ = \frac{4\pi R^3 \omega^4}{\sqrt{2}(\omega^2 - \sin^2\omega)} \left[t_{00}^2(k) + \hat{k}_z^2 t_{11}^2(k) + 2\hat{k}_z t_{00}(k) t_{11}(k) \right] , \quad (35)$$

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The result is

$$h(x) = \frac{4\pi M R^3\omega^4}{(\omega^2 - \sin^2\omega)} \left(\langle N; \mathbf{0}; S|P_{f,m}|N; \mathbf{0}; S\rangle \right) \times \qquad (36)$$
$$\int_{k_{min}}^{\infty} \frac{k dk}{(2\pi)^2} \left[t_{00}^2(k) + \hat{k}_z^2 t_{11}^2(k) + 2\hat{k}_z t_{00}(k) t_{11}(k) \right] \frac{|\phi_2(\boldsymbol{k})|^2}{|\phi_3(\mathbf{0})|^2}.$$

Dress effects on the transversity distribution

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But usually we calculate the dressed vertex in Euclidean space (LQCD, DSE, FRG, ...) and the distribution function are define in the Minkowski.
Fortunately, we can see in the bag model, we have integrated out the time during the calculation of distribution functions. It is OK for us to use the dressed quark vertex in Euclidean space to look insight into the dress effects of the distribution functions.

Dress effects on the transversity distribution

Dressed transversity distribution function

$$h(x) = 2M \int \frac{d\xi^-}{4\pi} e^{iq^+\xi^-} \times \\ \langle N; \boldsymbol{p}=0; S|\bar{\psi}_i(\xi)\, i\Gamma_5^{1+}\psi_i(0)|N; \boldsymbol{p}=0; S\rangle\Big|_{\xi^+,\xi_\perp=0} . \tag{37}$$

In Euclidean space

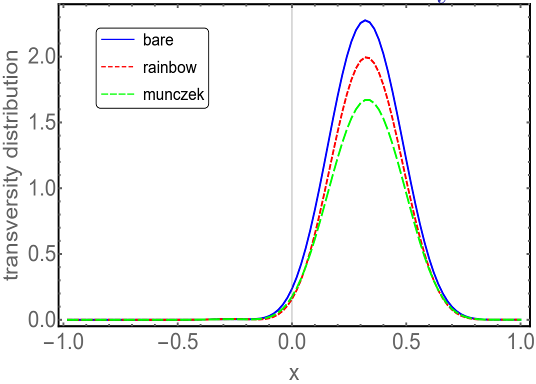
$$\Gamma_5^{\mu\nu}(k;0) = \sigma^{\mu\nu}\gamma_5 * E(k^2) + \{i\not{k}, \sigma^{\mu\nu}\gamma_5\} * F(k^2) \\ + [i\not{k}, \sigma^{\mu\nu}\gamma_5] * G(k^2) + i\not{k}\sigma^{\mu\nu}\gamma_5 i\not{k} * H(k^2) , \tag{38}$$

We can get the result of dressed transversity distribution is

Dress effects on the transversity distribution

$$
\begin{aligned}
h(x) = {} & \frac{4\pi M R^3 \omega^4}{(\omega^2 - \sin^2\omega)} \left(\langle N; \mathbf{0} | P_{f,m} | N; \mathbf{0}\rangle\right) \int_{k_{min}}^{\infty} \frac{kdk}{(2\pi)^2} \frac{|\phi_2(\boldsymbol{k})|^2}{|\phi_3(\mathbf{0})|^2} \times \\
& \Big\{ \left[t_{00}^2(k) + \hat{k}_z^2 t_{11}^2(k) + 2\hat{k}_z t_{00}(k) t_{11}(k) \right] * E(k^2) \\
& + \left[2Mx t_{00}^2(k) - 2\left(\frac{\omega}{R}\hat{k}_3^2 + k_3\right) t_{11}^2(k) - 2k(1-\hat{k}_3^2) t_{00}(k) t_{11}(k) \right] * F(k^2) \\
& + \Bigg[(Mx)^2 t_{00}^2(k) + \left((Mx)^2 \hat{k}_3^2 + \frac{k^2}{2}(1-\hat{k}_3^2)^2 + 2k(1-\hat{k}_3^2)(Mx)\hat{k}_3 \right) \\
& \qquad t_{11}^2(k) - 2\left(k(1-\hat{k}_3^2)(Mx) + (Mx)^2 \hat{k}_3 \right) t_{00}(k) t_{11}(k) \Bigg] * H(k^2) \Big\} ,
\end{aligned}
\tag{39}
$$

Dress effects on the transversity distribution



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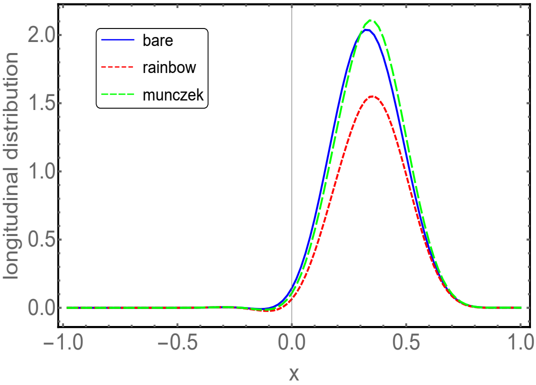
R	1.46 fm
M	1.10 GeV
ω	2.04

Approximation	bare	RL	Munczek
tensor charge δq	0.783	0.710	0.649
up quark δu	1.184	1.008	0.888
down quark δd	-0.296	-0.251	-0.222

$$\delta q = \int_{-1}^{1} dx h(x)\,,$$

Dress effects on the longitudinal distribution



B	$(0.113\text{GeV})^4$
R	1.46 fm
M	1.10 GeV
ω	2.04

Approximation	bare	RL	Munczek
axial charge $\triangle q$	0.776	0.584	0.784
axial coupling g_A	1.29	0.972	1.306

$$\triangle q = \int_{-1}^{1} dx g(x) \,,$$

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Summary and outlook

We have done

- We go beyond the rainbow ladder approximation by utilizing the Munczek's quark gluon vertex and derive the four particle scattering kernel analytically.
- Here we combine the MIT bag model with DSE approach to give a quite simple way to explore the dress effects of distribution functions.

We will do

- Fix the gluon parameters by solving the BSEs for mesons.
- Compare the results with the experimental results.