

Three body open flavor decays of higher vector charmonium and bottomonium

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Outline

1 Introduction

2 3P_0 Model

3 Numerical Result

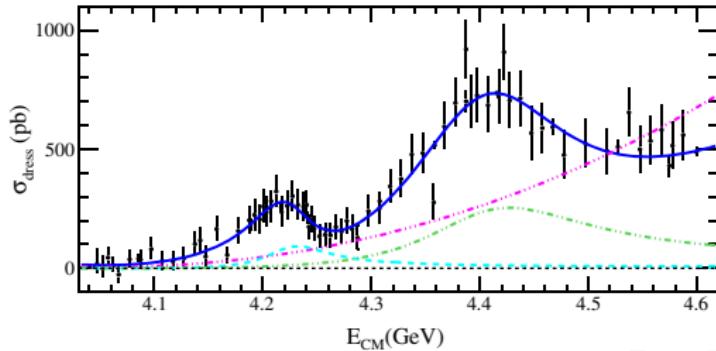
4 Summary

Introduction

- 2008, Belle, $e^+e^- \rightarrow D^0D^-\pi^+$ [PRL 100, 062001 \(2008\)](#)

$$\frac{\mathcal{B}[\psi(4415) \rightarrow D^0 D^- \pi_{\text{nonresonant}}^+]}{\mathcal{B}[\psi(4415) \rightarrow D \bar{D}_2^*(2460) \rightarrow D^0 D^- \pi^+]} < 0.22 \quad (1)$$

- 2009, Belle, $e^+e^- \rightarrow D^0 D^{*-} \pi^+ + \text{c.c.}$: found no evidence of $\psi(4260)$, $\psi(4360)$, $\psi(4415)$, $\psi(4630)$ or $\psi(4660)$ with limited statistics [PRD 80, 091101 \(2009\)](#)
- 2018, BESIII, $e^+e^- \rightarrow D^0 D^{*-} \pi^+$: $Y(4220)$, $\psi(4415)$ [PRL 122, 102002 \(2019\)](#)



Introduction

- 2010, Belle, $\Upsilon(5S) \rightarrow B^{0,+} + \text{others}$: PRD 81, 112003 (2010)

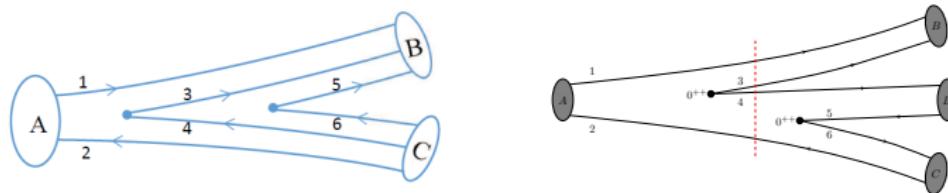
$$f(B\bar{B}\pi) = (0.0 \pm 1.1 \pm 0.3)\%, \quad (2)$$

$$f(B\bar{B}^*\pi + B^*\bar{B}\pi) = (7.3^{+2.3}_{-2.1} \pm 0.8)\%, \quad (3)$$

$$f(B^*\bar{B}^*\pi) = (1.0^{+1.4}_{-1.3} \pm 0.4)\%. \quad (4)$$

- 3P_0 model, $X(4660)[c\bar{c}] \rightarrow \Lambda_c \bar{\Lambda}_c$: Xiao et al. EPJC 78, 605 (2018)

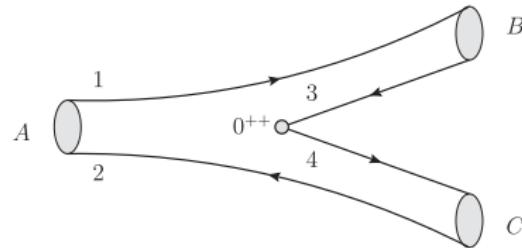
- $\Gamma_{\psi(4S,5S,6S)} \sim \text{MeV}$
- $\Gamma_{\psi(3D,4D,5D)} \sim 0.1 \text{ MeV}$



3P_0 Model

- In the 3P_0 model, a light $q\bar{q}$ pair is created with the vacuum quantum number $J^{PC} = 0^{++}$, and then rearranged with the quarks within the initial meson to produce two final mesons.

$$J^{PC} = 0^{++} \Rightarrow L = 1, S = 1 \Rightarrow {}^{2S+1}L_J = {}^3P_0 \quad (5)$$



L. Micu, Nucl. Phys. B 10, 521 (1969).

Transition operator

- Interaction Hamiltonian PRD 54, 6811 (1996)

$$H_{\text{int}} = g_s \int d^3x \bar{\psi} \psi \quad (6)$$

where

$$\gamma = g_s / 2m_q \quad (7)$$

- Decay amplitude

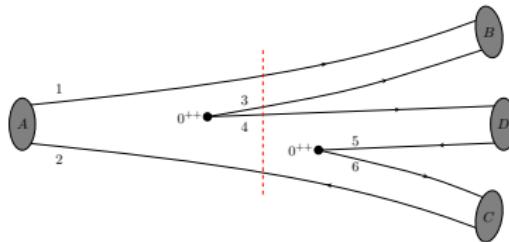
$$\delta^{(3)}(\mathbf{P}_f - \mathbf{P}_i) \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}} = \langle f | H_{\text{int}} | i \rangle \quad (8)$$

- In the nonrelativistic limit, the transition operator reads

$$T = -3\gamma \sum_m \langle 1m; 1-m | 00 \rangle \int d^3k_4 d^3k_5 \delta^3(k_4 + k_5) \mathcal{Y}_1^m \left(\frac{k_4 - k_5}{2} \right) \chi_{1,-m}^{45} \varphi_0^{45} \omega_0^{45} b_{4i}^\dagger(\mathbf{k}_4) d_{5j}^\dagger(\mathbf{k}_5) \quad (9)$$

Transition operator

- Two pairs of $q\bar{q}$ are created in the three body open flavor decay processes



- At the 2nd order EPJC 78, 605 (2018)

$$\delta^{(3)}(\mathbf{P}_f - \mathbf{P}_i) \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}} = \sum_k \frac{\langle f | H_{\text{int}} | k \rangle \langle k | H_{\text{int}} | i \rangle}{E_k - E_i} \approx \frac{\langle f | H_{\text{int}} H_{\text{int}} | i \rangle}{2m_q} \quad (10)$$

where we take $E_k - E_i$ as a constant: $E_k - E_i \approx 2m_q$ (closure approximation). PRD 44, 799 (1991); PRL 67, 1066 (1991)

Closure approximation: $E_k - E_i \approx 2m_q$

- (focus on 1^{--} charmonium)
- quark level
 - intermediate state – initial state $\sim q\bar{q}$
- hadron level
 - Intermediate states: $E_k \sim (4.0 - 4.1)$ GeV:

$$D\bar{D}_1, D^*\bar{D}_1, D^*\bar{D}_0, D^*\bar{D}_2, J/\psi f_0(500), \\ h_c(1P)\pi, h_c(1P)\eta, \chi_{c0}(1P)\omega, \chi_{c2}(1P)\omega, \dots$$

- Higher mass states [$\psi(4660)$, $\psi(4415)$, $\psi(4360)$]: ✓
- Lower mass states [$\psi(4040)$, $\psi(4160)$]: ✗

Transition operator

- Transition operator $T_2 \sim T_1^2/2m_q$

$$\begin{aligned} T = & \frac{9\gamma^2}{2m_q} \sum_{mm'} \langle 1m; 1-m | 00 \rangle \langle 1m'; 1-m' | 00 \rangle \int d^3k_3 d^3k_4 d^3k_5 d^3k_6 \\ & \times \delta^3(\mathbf{k}_3 + \mathbf{k}_4) \delta^3(\mathbf{k}_5 + \mathbf{k}_6) \mathcal{Y}_1^m \left(\frac{\mathbf{k}_3 - \mathbf{k}_4}{2} \right) \mathcal{Y}_1^{m'} \left(\frac{\mathbf{k}_5 - \mathbf{k}_6}{2} \right) \\ & \times \chi_{1,-m}^{34} \varphi_0^{34} \omega_0^{34} a_{3i}^\dagger(\mathbf{k}_3) b_{4j}^\dagger(\mathbf{k}_4) \chi_{1,-m'}^{56} \varphi_0^{56} \omega_0^{56} c_{5i'}^\dagger(\mathbf{k}_5) d_{6j'}^\dagger(\mathbf{k}_6) \end{aligned}$$

Mock state

- Meson

$$\begin{aligned}
 & \left\langle B \left(n_B^{2S_B+1} L_B J_B M_{J_B} \right) (\mathbf{P}_B) \right\rangle \\
 = & \sqrt{2E_B} \sum_{M_{L_B} M_{S_B}} \langle L_B M_{L_B} S_B M_{S_B} | J_B M_{J_B} \rangle \\
 & \times \int d^3 \mathbf{k}_a d^3 \mathbf{k}_b \delta^3(\mathbf{k}_a + \mathbf{k}_b - \mathbf{P}_B) \\
 & \times \psi_{n_B L_B M_{L_B}}(\mathbf{k}_a, \mathbf{k}_b) \chi_{S_B M_{S_B}}^{ab} \varphi_B^{ab} \omega_B^{ab} |q_a(\mathbf{k}_a) \bar{q}_b(\mathbf{k}_b)\rangle
 \end{aligned}$$

- Normalization

$$\langle B(\mathbf{P}_B) | B(\mathbf{P}'_B) \rangle = 2E_B \delta^3(\mathbf{P}_B - \mathbf{P}'_B) \quad (11)$$

- Spatial wave function → simple harmonic oscillator (SHO).

C. Hayne and N. Isgur, Phys. Rev. D 25, 1944 (1982).

Decay width

- Helicity amplitude

$$\begin{aligned} \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C} M_{J_D}}(A \rightarrow BCD) = & \frac{\gamma^2}{2m_q} \sqrt{16E_A E_B E_C E_D} \times \sum_{mm'} \sum_{M_{L_{A,B,C,D}}, M_{S_{A,B,C,D}}} \langle 1m; 1-m|00\rangle \langle 1m'; 1-m'|00\rangle \\ & \times \langle L_A M_{L_A} S_A M_{S_A} | J_A M_{J_A} \rangle \langle L_B M_{L_B} S_B M_{S_B} | J_B M_{J_B} \rangle \langle L_C M_{L_C} S_C M_{S_C} | J_C M_{J_C} \rangle \\ & \times \langle L_D M_{L_D} S_D M_{S_D} | J_D M_{J_D} \rangle \langle \chi_{S_B M_{S_B}}^{13} \chi_{S_C M_{S_C}}^{26} \chi_{S_D M_{S_D}}^{45} | \chi_{S_A M_{S_A}}^{12} \chi_{1,-m}^{34} \chi_{1,-m'}^{56} \rangle \\ & \times \langle \varphi_B^{13} \varphi_C^{26} \varphi_D^{45} | \varphi_A^{12} \varphi_0^{34} \varphi_0^{56} \rangle \times I_{M_{L_B} M_{L_C} M_{L_D}}^{M_{L_A} mm'}(\mathbf{p}), \end{aligned}$$

- Decay width

$$\Gamma = \int_0^\infty dE_B dE_C \frac{\pi^3}{M_A} \frac{1}{2J_A + 1} \sum_{M_{J_{A,B,C,D}}} \left| \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C} M_{J_D}} \right|^2. \quad (12)$$

Parameter

- $\gamma_{c\bar{c}} = 6.95, \gamma_{b\bar{b}} = 10.42$ PRD 72, 054026 (2005); PRD 92, 054034 (2015)
- $m_u = m_d = 330$ MeV, $m_s = 419$ MeV,
 $m_c = 1628$ MeV, $m_b = 4977$ MeV PRD 32, 189 (1985)
- SHO strength β PRD 69, 054008 (2004); 72, 054026 (2005); 92, 054034 (2015); 93, 034035 (2016)

$$\int d^3\mathbf{p} \left| \psi_{nlm}^{\text{SHO}}(\mathbf{p}) \right|^2 p^2 = \int d^3\mathbf{p} |\Phi(\mathbf{p})|^2 p^2$$

TABLE II. Masses and harmonic oscillator strength β 's of final state mesons used in the decays (in units of MeV).

- $\beta_{c\bar{c}} = 500 \pm 50$ MeV
- $\beta_{b\bar{b}} = 600 \pm 50$ MeV
- $\alpha_{q\bar{q}} = 400$ MeV

Meson	State	Mass [55]	β [28,51]
π	1S_0	138.0	400
ρ	3S_1	775.3	400
ω	3S_1	782.6	400
η	1S_0	547.9	400
D	1S_0	1867.2	600
D^*	3S_1	2008.6	520
D_s	1S_0	1968.3	650
D_s^*	3S_1	2112.2	560
B	1S_0	5279.5	580
B^*	3S_1	5324.6	540
B_s	1S_0	5366.9	640
B_s^*	3S_1	5415.4	600

Decay width: $\psi(4360)$

TABLE III. The partial decay widths (in MeV) of the vector charmonium with a mass of 4368 MeV.

State	$\psi(4^3S_1)$	$\psi(3^3D_1)$
$\Gamma_{DD\pi}$	0.27	0.14
$\Gamma_{DD^*\pi}$	1.40	1.21
$\Gamma_{D^*D^*\pi}$	0.60	0.25
$\Gamma_{DD\eta}$	0.6 keV	0.3 keV

- Possible assignment: $\psi(3D)$, $\psi(4S)$ [PRD 79, 094004 \(2009\); Int. J. Mod. Phys. E 22, 1330026 \(2013\).](#)
- S - and D -waves are of same order.
- $D\bar{D}^*\pi$ mode is dominant.

$$\mathcal{B}[\psi(4^3S_1) \rightarrow D\bar{D}^*\pi] \sim 1.5\%, \quad \mathcal{B}[\psi(3^3D_1) \rightarrow D\bar{D}^*\pi] \sim 1.3\%.$$

Decay width: $\psi(4415)$

TABLE IV. The partial decay widths (in MeV) of the vector charmonium with a mass of 4421 MeV.

State	$\psi(4^3S_1)$	$\psi(5^3S_1)$	$\psi(3^3D_1)$
$\Gamma_{DD\pi}$	0.38	0.11	0.21
$\Gamma_{DD^*\pi}$	2.01	0.96	1.84
$\Gamma_{D^*D^*\pi}$	1.07	0.59	0.52
$\Gamma_{DD\eta}$	5.4 keV	1.7 keV	2.9 keV

- Assignment: $\psi(4S)$ [PLB 72, 57 \(1977\); PRD 72, 054026 \(2005\)](#);
- $\mathcal{B}[\psi(4S) \rightarrow DD^*\pi] \sim 3.2\%$ — PDG upper limit (11%) [PRD 98, 030001 \(2018\)](#)
- $\mathcal{B}[\psi(4S) \rightarrow DD\pi] \sim 0.6\%$ — Belle upper limit (2.2%) [PRL 100, 062001 \(2008\)](#)
- Other assignments: $\psi(5S)$, $\psi(3D)$ [PRD 79, 094004 \(2009\); Int. J. Mod. Phys. E 22, 1330026 \(2013\)](#)

Decay width: $\psi(4660)$

TABLE I: The possible assignments of the $\psi(4660)$ with the predicted masses (MeV) from various models.

State	QM [46]	QM [47]	QM [48]	SSE/EA[49]	NR/GI [50]	SP [10]	LP/SP [51]
$\psi(4^3S_1)$	4625	4450	4389	4398/4426	4406/4450	4273	4412/4281
$\psi(5^3S_1)$	4641	4642/4672	...	4463	4711/4472
$\psi(6^3S_1)$	4804/4828	...	4608	...
$\psi(3^3D_1)$...	4520	4426	4464/4477	...	4317	4478/4336
$\psi(4^3D_1)$	4641	4690/4707
$\psi(5^3D_1)$	4804/4855

TABLE V. The partial decay width (MeV) of the vector charmonium with a mass of 4643 MeV.

State	$\psi(4^3S_1)$	$\psi(5^3S_1)$	$\psi(6^3S_1)$	$\psi(3^3D_1)$	$\psi(4^3D_1)$	$\psi(5^3D_1)$
$\Gamma_{DD\pi}$	1.14	0.31	0.09	0.63	0.17	0.05
$\Gamma_{DD^*\pi}$	6.65	2.83	1.10	6.99	2.99	1.16
$\Gamma_{D^*D^*\pi}$	5.97	2.68	1.13	4.12	2.11	0.96
$\Gamma_{DD\rho}$	0.85	0.41	0.16	1.86	0.64	0.22
$\Gamma_{DD\omega}$	0.24	0.12	0.05	0.59	0.20	0.07
$\Gamma_{DD\eta}$	53.2 keV	15.3 keV	4.2 keV	29.1 keV	8.2 keV	2.2 keV
$\Gamma_{DD^*\eta}$	0.25	0.12	0.05	0.20	0.11	0.05
$\Gamma_{D^*D^*\eta}$	58.2 keV	38.7 keV	19.1 keV	8.5 keV	9.9 keV	7.5 keV
$\Gamma_{D_sD_s\eta}$	3.0 keV	0.8 keV	0.2 keV	1.6 keV	0.4 keV	0.1 keV
$\Gamma_{D_sD_s^*\eta}$	1.9 keV	1.3 keV	0.6 keV	12 eV	11 eV	7 eV

- Possible assignment: $\psi(4S, 5S, 6S)$, $\psi(3D, 4D, 5D)$.
- $D^{(*)}\bar{D}^{(*)}\pi$ modes dominant.
- $D\bar{D}\rho$, $D\bar{D}\omega$ and $D\bar{D}^*\eta$ modes are also important.

Decay width

- The variation of the $\Gamma_{D^+ D^- \pi^0}$ with the mass of the charmonium

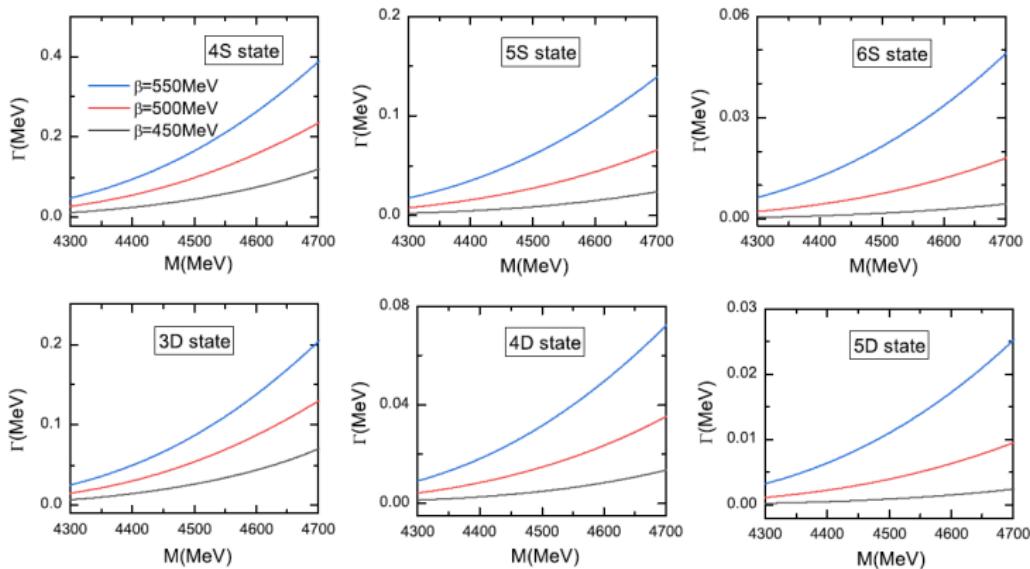


FIG. 2. The variation of the $D^+ D^- \pi^0$ partial decay width with the mass of the D -wave vector charmonium. Note that $\Gamma_{D^+ D^- \pi^0} = \frac{1}{6} \Gamma_{DD\pi}$ since we have ignored the isospin breaking. The blue, red, and black lines correspond to the predictions with different values of the harmonic oscillator strength $\beta = 450, 500$, and 550 MeV, respectively.

Decay width

- The variation of the $\Gamma_{D^+ D^{*-} \pi^0}$ with the mass of the charmonium

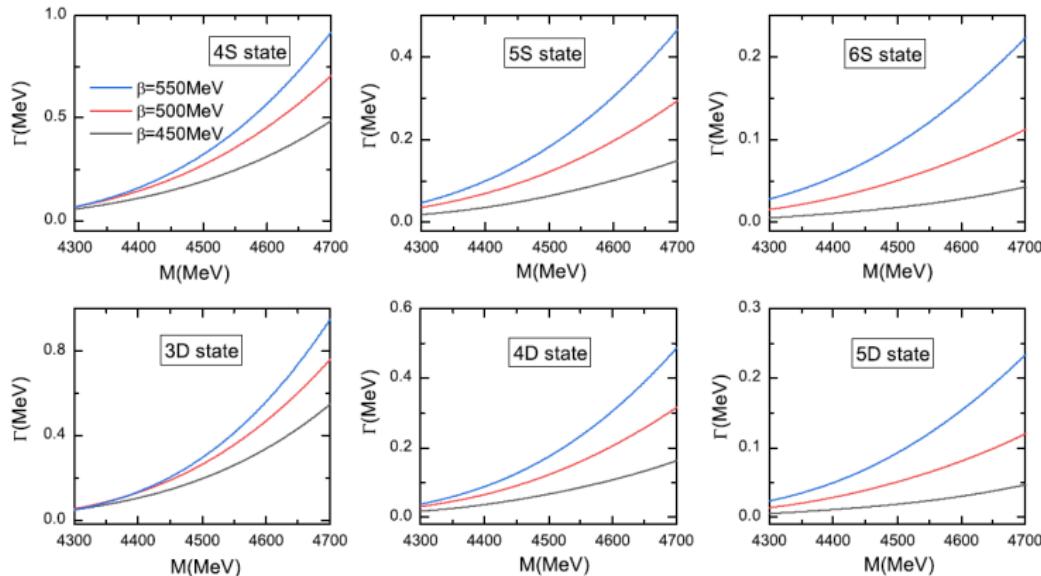


FIG. 3. The variation of the $D^+ D^{*-} \pi^0$ partial decay width with the mass of the D -wave vector charmonium. Note that $\Gamma_{D^+ D^{*-} \pi^0} = \frac{1}{12} \Gamma_{DD^* \pi}$ since we have ignored the isospin breaking. The blue, red, and black lines correspond to the predictions with different values of the harmonic oscillator strength $\beta = 450, 500$, and 550 MeV, respectively.

Decay width

- The variation of the $\Gamma_{D^{*+}D^{*-}\pi^0}$ with the mass of the charmonium

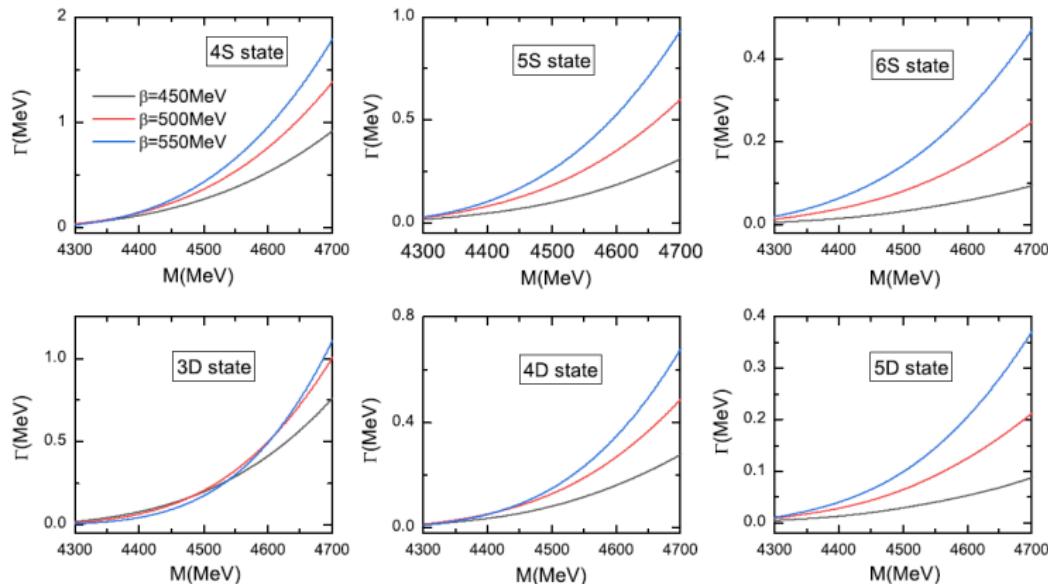


FIG. 4. The variation of the $D^{*+}D^{*-}\pi^0$ partial decay width with the mass of the D -wave vector charmonium. Note that $\Gamma_{D^{*+}D^{*-}\pi^0} = \frac{1}{6}\Gamma_{D^*D^*\pi}$ since we have ignored the isospin breaking. The blue, red, and black lines correspond to the predictions with different values of the harmonic oscillator strength $\beta = 450, 500$, and 550 MeV, respectively.

Decay width

- The partial widths are sensitive to mass.
- The partial widths increased as the mass increase.
- partial widths are not sensitive to β .

$\Upsilon(10860)$, $\Upsilon(11020)$

TABLE VII. The $B^{(*)}\bar{B}^{(*)}\pi$ partial decay widths of the vector bottomonium (in units of MeV). \mathcal{B}_{exp} represents the branching ratio for each corresponding channel.

Meson	State	Mode	$\beta = 550 \text{ MeV}$		$\beta = 600 \text{ MeV}$		$\beta = 650 \text{ MeV}$		[36]	$\mathcal{B}_{\text{exp}} [55]$	
			Γ_{th}	\mathcal{B}_i	Γ_{th}	\mathcal{B}_i	Γ_{th}	\mathcal{B}_i			
$\Upsilon(10860)$	5^3S_1	$BB\pi$	0.12	0.2%	0.20	0.4%	0.28	0.5%	(23–30) keV		
		$BB^*\pi$	1.36	2.7%	1.22	2.4%	0.94	1.8%	(5–6.6) keV		
		$B^*B^*\pi$	0.68	1.3%	0.61	1.2%	0.47	0.9%	(0.0 \pm 1.2)%		
$\Upsilon(11020)$	6^3S_1	$BB\pi$	0.17	0.3%	0.34	0.7%	0.55	1.1%	(7.3 \pm 2.3)%		
		$BB^*\pi$	2.50	5.1%	3.17	6.5%	3.41	7.0%	(1.0 \pm 1.4)%		
		$B^*B^*\pi$	2.12	4.3%	2.69	5.5%	2.97	6.1%			

- $\Upsilon(10860)$: $\mathcal{B}_{BB\pi}$ and $\mathcal{B}_{B^*B^*\pi}$ are consistent with the exp. data, while $\mathcal{B}_{BB^*\pi}$ is smaller but very close to the Belle's measurement.
- $\Upsilon(11020)$: $\mathcal{B}_{BB^*\pi}$ and $\mathcal{B}_{B^*B^*\pi}$ are quite large.

PLB 671, 55 (2009)

Summary

- Extended the 3P_0 model to study the three body open flavor decays of heavy quarkonium
- Charmonium
 - $\Gamma[\psi(4360) \rightarrow DD^*\pi]$ can reach up to 1 MeV.
 - $\mathcal{B}[\psi(4415) \rightarrow DD^{(*)}\pi]$ are within the exp. upper limits.
 - $\psi(4660)$ decays dominantly into $D^{(*)}D^{(*)}\pi$.
- Bottomonium
 - $\mathcal{B}[\Upsilon(5S) \rightarrow BB\pi/B^*B^*\pi]$ are consistent with the exp.
 - $\mathcal{B}[\Upsilon(5S) \rightarrow BB^*\pi]$ is smaller but close to the Belle measurement.
 - $\mathcal{B}[\Upsilon(6S) \rightarrow BB^*\pi/B^*B^*\pi]$ can reach up to several MeV.

Thank You!