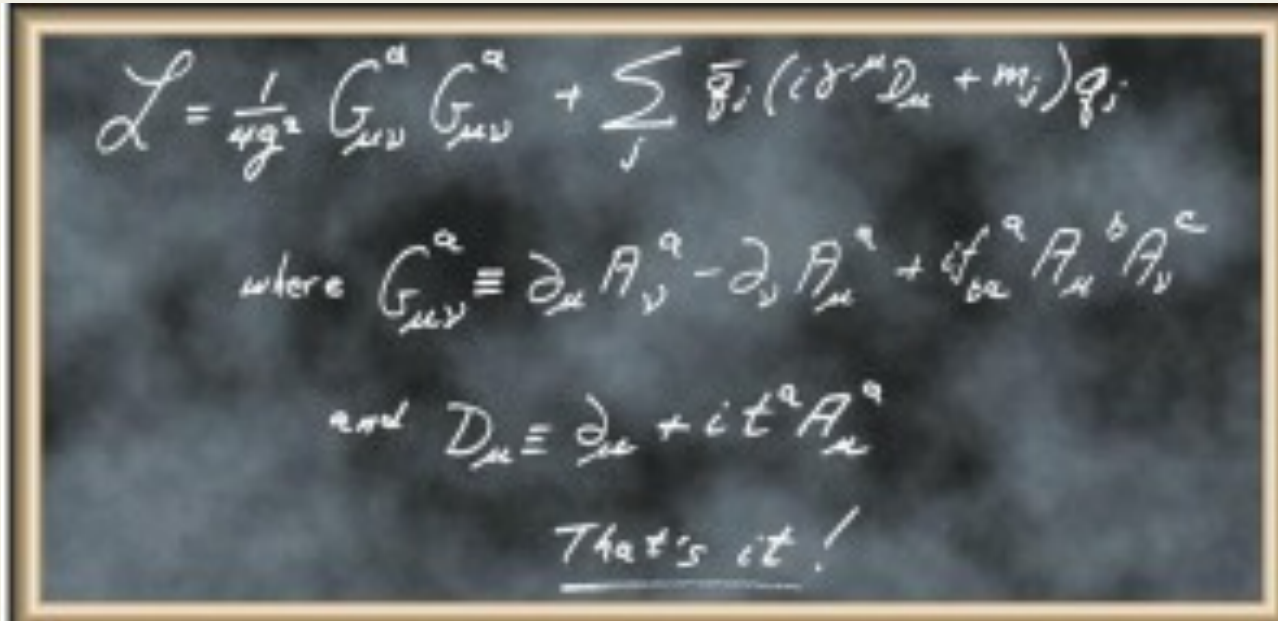


# DSEs meet IQCD

**Lei Chang(常雷)**

leichang@nankai.edu.cn

**Nankai University**



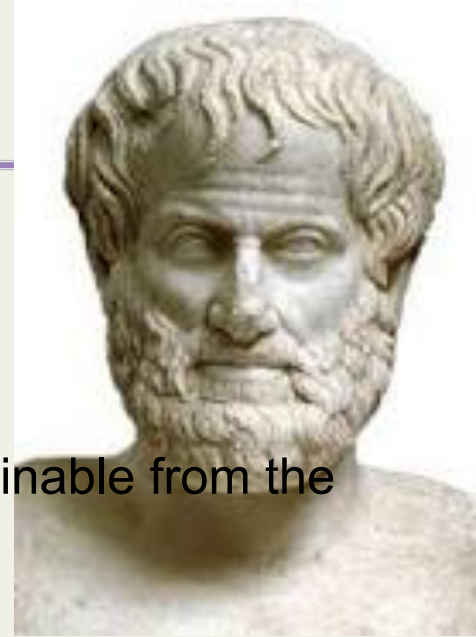
The image shows a chalkboard with handwritten mathematical equations. The first equation is the Lagrangian density for Quantum Chromodynamics (QCD):
$$\mathcal{L} = \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a + \sum_j \bar{q}_j (i\gamma^\mu D_\mu + m_j) q_j$$
The second equation defines the gluon field strength tensor:
$$\text{where } G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + if_{abc} A_\mu^b A_\nu^c$$
The third equation defines the covariant derivative:
$$\text{and } D_\mu \equiv \partial_\mu + it^a A_\mu^a$$
The final phrase written on the board is "That's it!".

- Quark current mass...Higgs boson.
  - **Hadron physics mass-scale – 1 GeV**
    - is an **emergent** feature of the Standard Model
- No amount of staring at  $L_{\text{QCD}}$  can reveal that scale

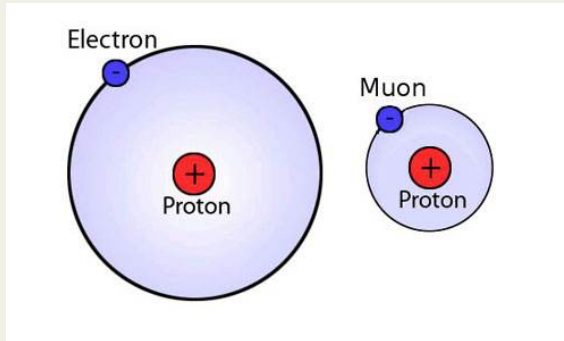
## Holism=Emergentism

---

- Aristotle(亚里士多德): 383BC?-322 BC
- Holism:
  - the idea that items can have properties, (emergent properties), as a whole that are not explainable from the sum of their parts.
- Summarized concisely (Aristotle):
  - *The whole is more than the sum of its parts*
- Hegel(黑格尔) (Stuttgart 1770 – Berlin 1831):
  - Das Wahre ist das Ganze (The true is the whole)*
- Anderson (1972)
  - More is different



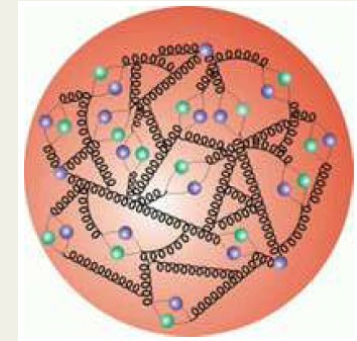
# Bound state and quantum field theory



QED

## Trace anomaly

- All renormalisable four-dimensional theories possess a trace anomaly;
- The size of the trace anomaly in QED must be great deal smaller than that in QCD.



QCD

Field theory Successful:

- Nonrelativistic quantum mechanics to handle bound state;
- Perturbation theory to handle relativistic effects

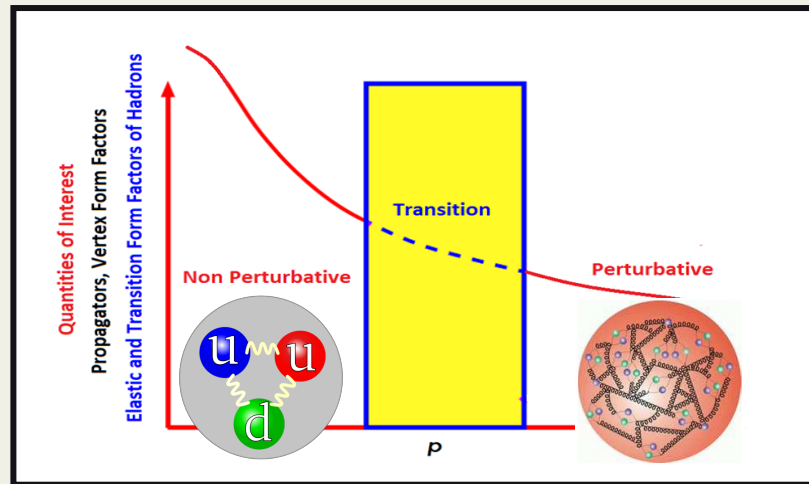
Field theory not Successful yet:

- Growth of the running coupling constant in the infrared region;
- **Confinement**;
- **Dynamical Chiral Symmetry Breaking**;
- Possible nontrivial vacuum structure in hadron



Constituent quark model  $\rightarrow$  intuitive understanding of many low energy observables.

Minimum number of constituents required



Feynman's parton model  $\rightarrow$  intuitive understanding of high-energy phenomena.

Constituent picture;  
Probabilistic interpretation of distribution functions

QCD vacuum in the hadron is very complicated medium  
Individual quarks and gluons are lost in the sea

Both the constituent quark model and the parton model  
are put in peril by QCD with a possible complicated  
vacuum structure.

## Emergent phenomena, 方法论

- **Confinement and DCSB** are emergent phenomena  
Not revealed by any amount of staring at Lagrangian for quantum chromodynamics;  
They determine the character of the QCD's spectrum, the structure and interactions of bound states
- Can one understand confinement and DCB in terms of properties of the degrees-of-freedom used to formulate QCD?  
E.g., is it pointless to attempt to predict the nucleon's form factor on a domain that is not yet accessible?

If YES:

Must rely on the vast array of effective field theories, developed for different systems, in order, to express and understand the consequences of confinement and DCSB, without identifying their source

If NO:

Must develop nonperturbative calculational methods to define and tackle QCD

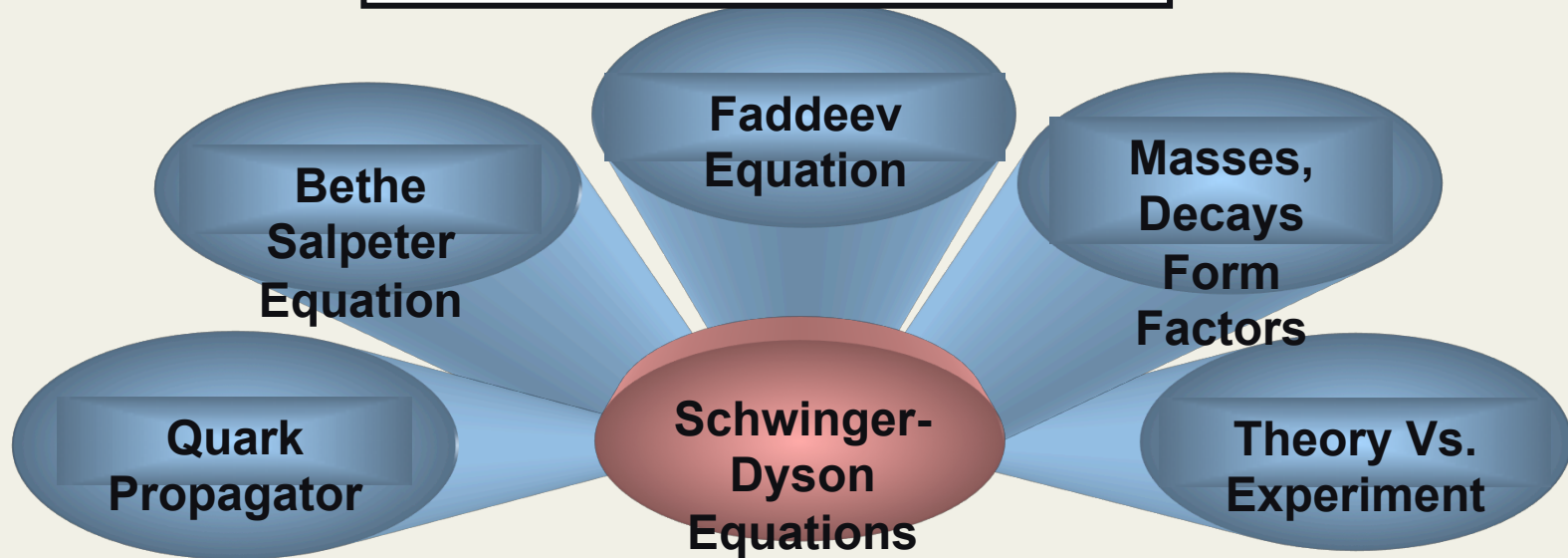
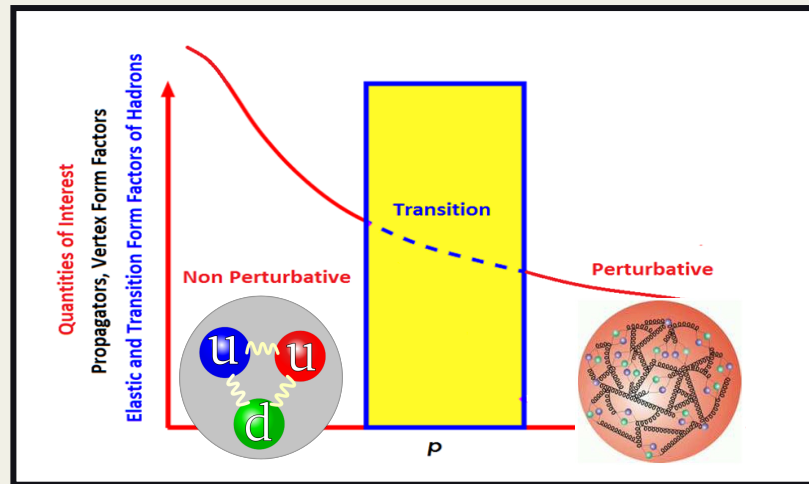
- 1) Lattice-regularized QCD
- 2) Continuum methods in quantum field theory
- 3) Combination of all the above

Currently, each approach has strengths and weaknesses  
So 3) is probably the best:

Combine all available methods to fullest extent reasonably possible.

# Dyson-Schwinger Equation scope

## Study bound state problem within an continuum field theory



- i) A Pattern for the Flavor Dependence of the Quark-Gluon Interaction,  
[arXiv: 1903.07808](#), [Muyang Chen](#) and [Lei Chang](#);
- ii) Excited Bc States via Continuum QCD,  
[arXiv: 1904.00399](#), [Lei Chang](#), [Muyang Chen](#) and [Yuxin Liu](#).

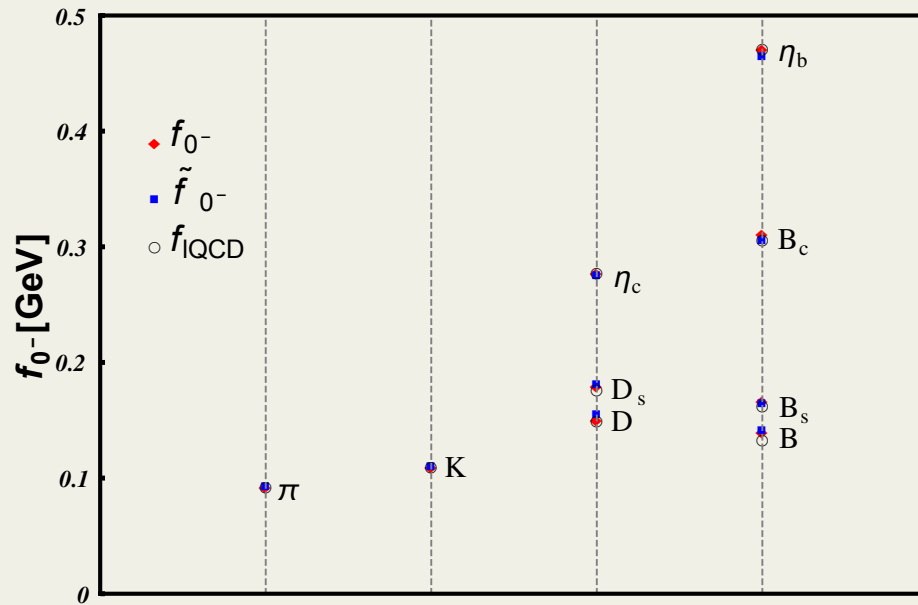


TABLE II. Masses of the first radial excited states of charm-beauty system (in GeV). The experimental data for  $M_{\eta_c(2S)}$ ,  $M_{\psi(2S)}$ ,  $M_{\eta_b(2S)}$  and  $M_{\Upsilon(2S)}$  are taken from Ref.[25],  $M_{B_c^+(2S)}$  and  $M_{B_c^+(2S)} - M_{B_c^{*+}(2S)}^{\text{rec}}$  from Ref.[2]. The mass splitting,  $M_{B_c^{*+}(1S)} - M_{B_c^+(1S)}$ , is quoted from Ref.[22]. The uncertainties of our results correspond to the varying of the parameters in Table I.

	$M_{\eta_c(2S)}$	$M_{\psi(2S)}$	$M_{\psi(2S)} - M_{\eta_c(2S)}$
here	3.606(18)	3.645(18)	0.039
expt.	3.638(1)	3.686(1)	0.048
	$M_{B_c^+(2S)}$	$M_{B_c^{*+}(2S)}$	$M_{B_c^+(2S)} - M_{B_c^{*+}(2S)}^{\text{rec}}$
here	6.813(16)	6.841(18)	0.039
expt.	6.872(2)	—	0.031
	$M_{\eta_b(2S)}$	$M_{\Upsilon(2S)}$	$M_{\Upsilon(2S)} - M_{\eta_b(2S)}$
here	9.915(15)	9.941(15)	0.026
expt.	9.999(4)	10.023(1)	0.024

- They deviate from each other by no more than 3% for all the pseudoscalar mesons;
- We conclude that the av-WTI is perfectly preserved in our approach.

# QCDs Dyson-Schwinger Equations



Quark propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---}$$

Ghost propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---}$$

Ghost-gluon vertex:

$$\text{---}\bigcirc\text{---} = \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

Quark-gluon vertex:

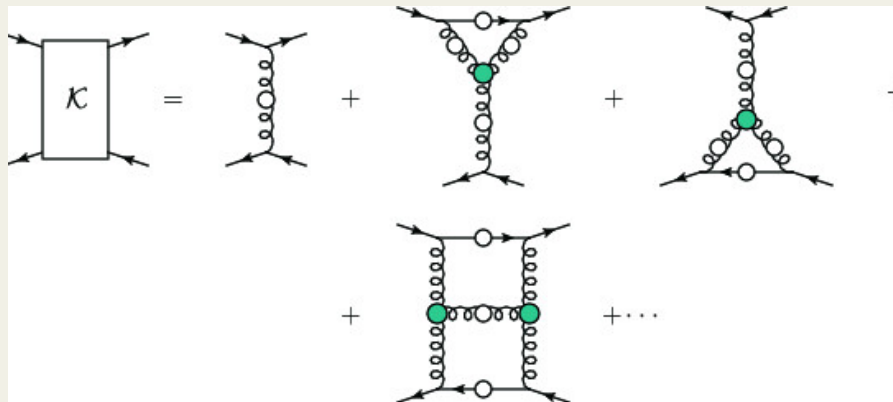
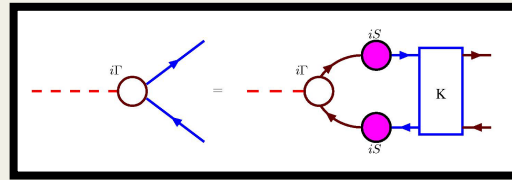
$$\text{---}\bigcirc\text{---} = \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

Gluon propagator:

$$\text{---}\bigcirc\text{---}^{-1} = \text{---}^{-1} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---} + \text{---}\bigcirc\text{---}$$

Image courtesy of Gernot Eichmann

## Bethe-Salpeter Equations



渐进两夸克  
说几个夸克态构成束缚态是难以定义的

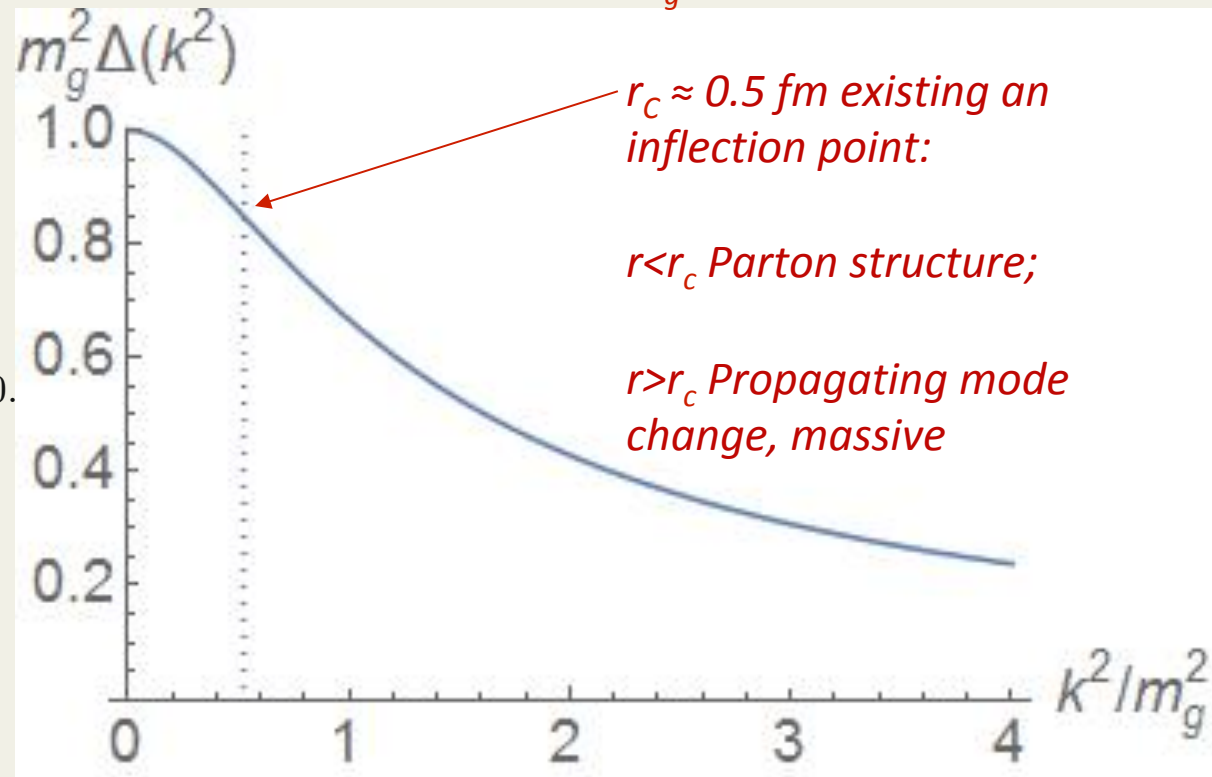
## Spectral representation

$$\Delta(q^2) = \int_0^\infty d\sigma \frac{\rho(\sigma)}{q^2 + \sigma},$$

if

$$\Delta''(q_\star^2) = 2 \int_0^\infty d\sigma \frac{\rho(\sigma)}{(q_\star^2 + \sigma)^3} = 0.$$

Non positive definite

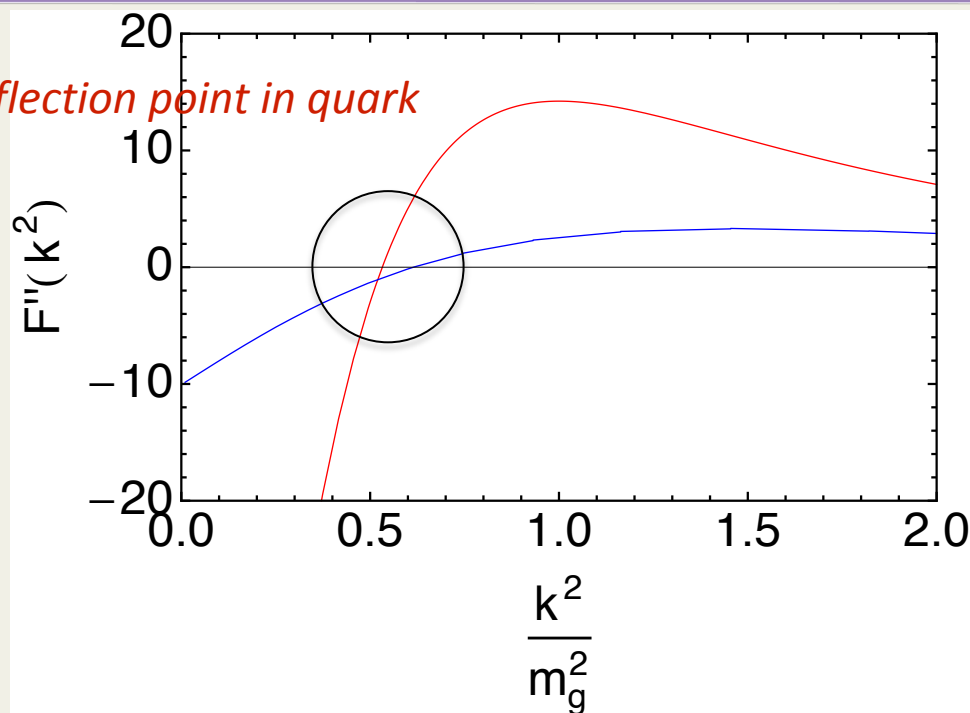
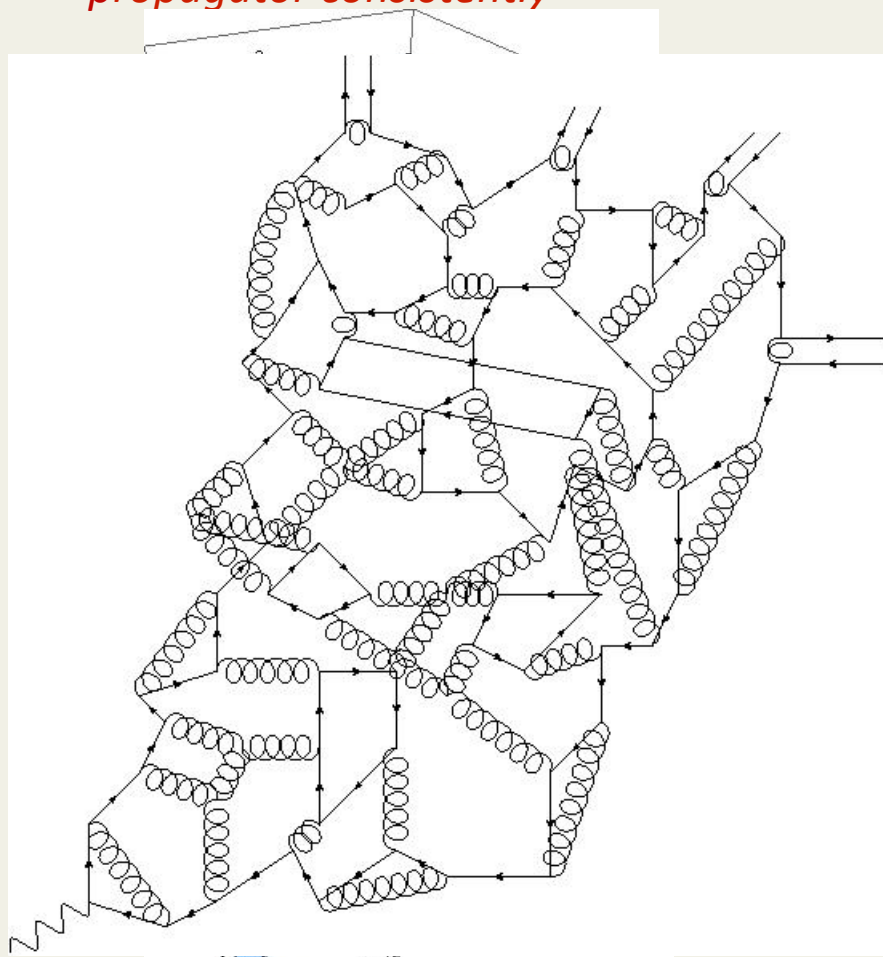


胶子=汉尼拔



# Quark Propagator $S(k) = -i\gamma \cdot k \sigma_V(k^2) + \sigma_S(k^2)$

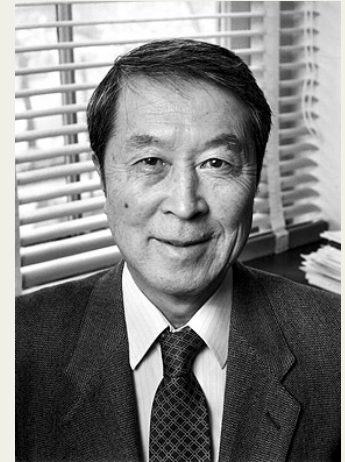
- at around  $k^2 \approx 0.5 \text{ mg}^2$  there exists an inflection point in quark propagator consistently



- Inflection points
- Red line: running gluon propagator
- Blue line: vector part of propagator



- Pion is Massless...



- In October 1934, **Hideki Yukawa** predicated the existence of a “heavy quantum” meson, exchanging nuclear force between neutrons and protons.
- It was discovered by **Cecil Powell** in 1949 in cosmic ray tracks in a photographic emulsion.
- Pion was nicely accommodated in the Eight Fold way of **Murray Gell-Mann** in 1961.
- **Yoichiro Nambu** associated it with CSB in 1960.

# Pion's dichotomy

## Goldstone boson and Bound State

Maris, Roberts and Tandy, Phys. Lett. **B420**(1998) 267-273

### ➤ Pion's Bethe-Salpeter amplitude

#### Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k; P) = \tau^{\pi^j} \gamma_5 \left[ iE_{\pi}(k; P) + \gamma \cdot P F_{\pi}(k; P) \right. \\ \left. + \gamma \cdot k k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} H_{\pi}(k; P) \right]$$

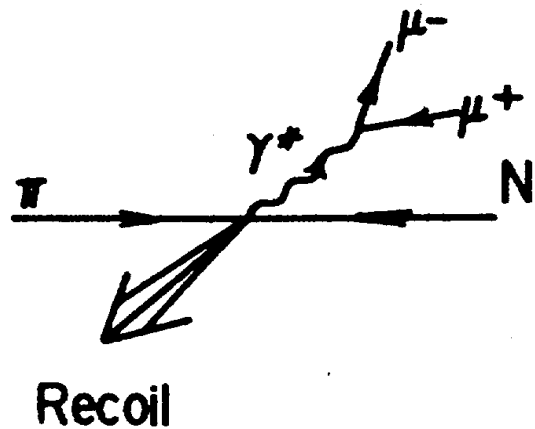
### ➤ Dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

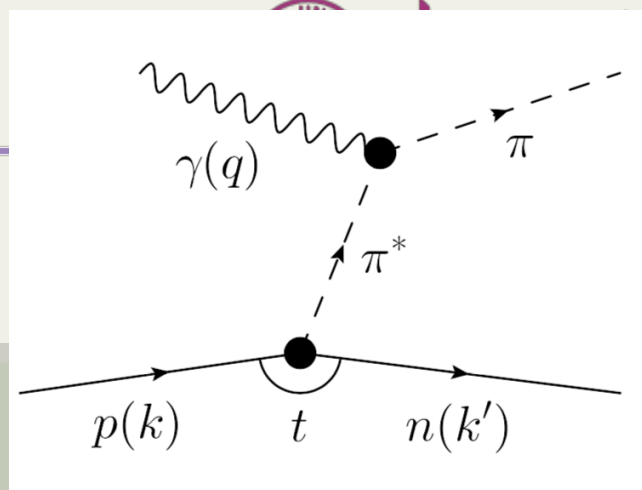
### ➤ Axial-vector Ward-Takahashi identity entails(chiral limit)

$$f_{\pi} E(k; P | P^2 = 0) = B(k^2) + (k \cdot P)^2 \frac{d^2 B(k^2)}{d^2 k^2} + \dots$$

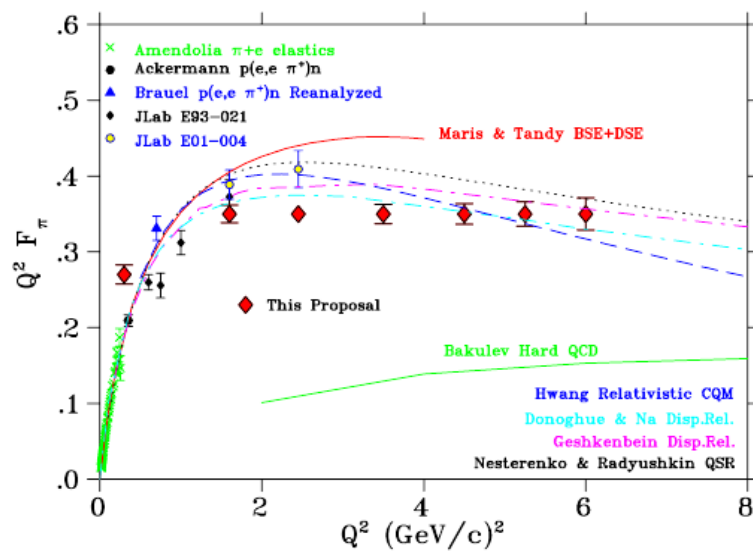
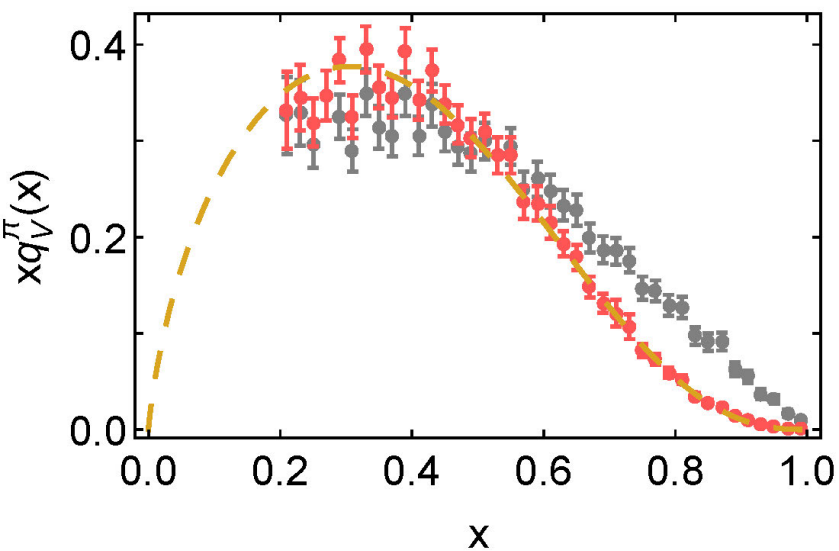
- Given the dichotomy of pion the fine-tuning should not play any role in an explanation of pion properties;
- Descriptions of pion within frameworks that cannot faithfully express symmetries and their breaking patterns(such as constituent-quark models) are unreliable;
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.



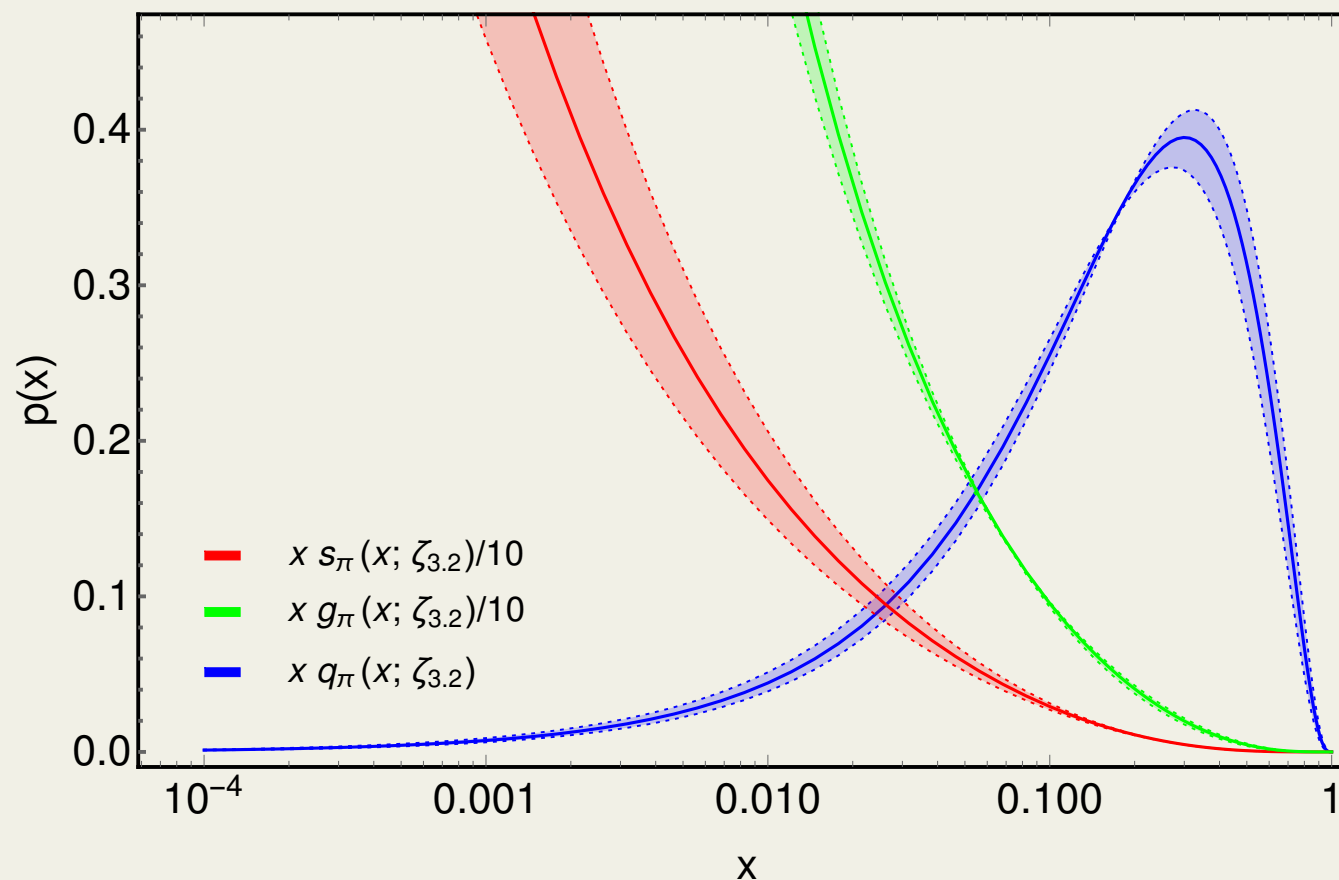
$$\pi N \rightarrow \mu^+ \mu^- X$$



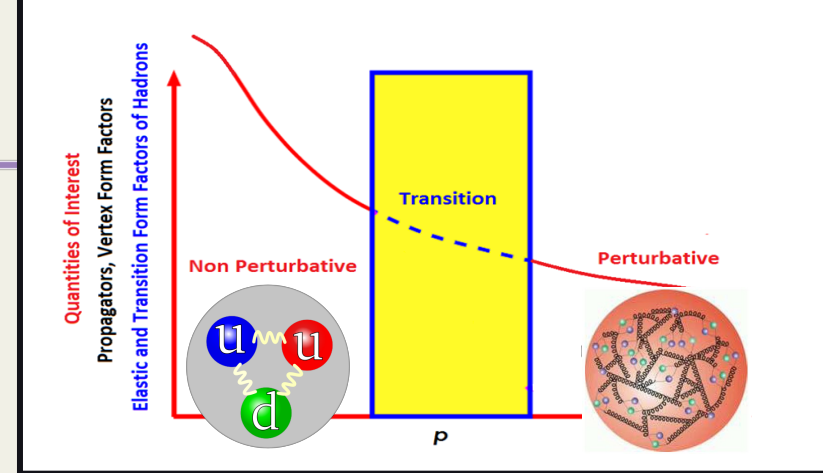
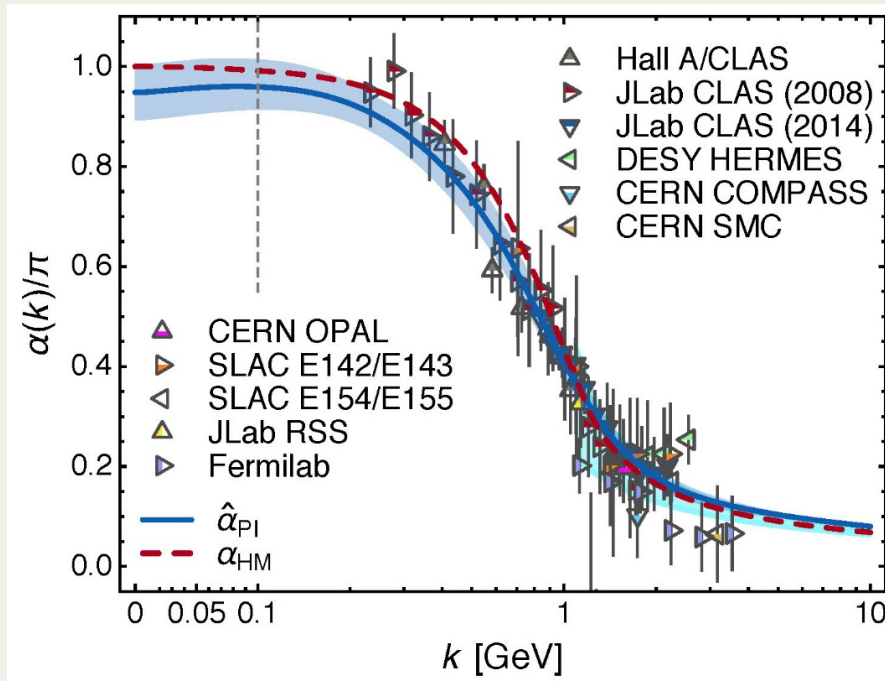
$$p(e, e' \pi^+) n$$



# PDF



# Hadronic Scale

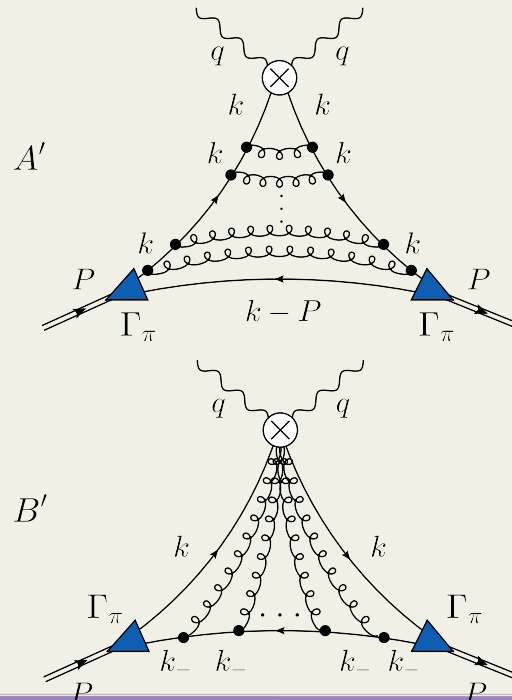
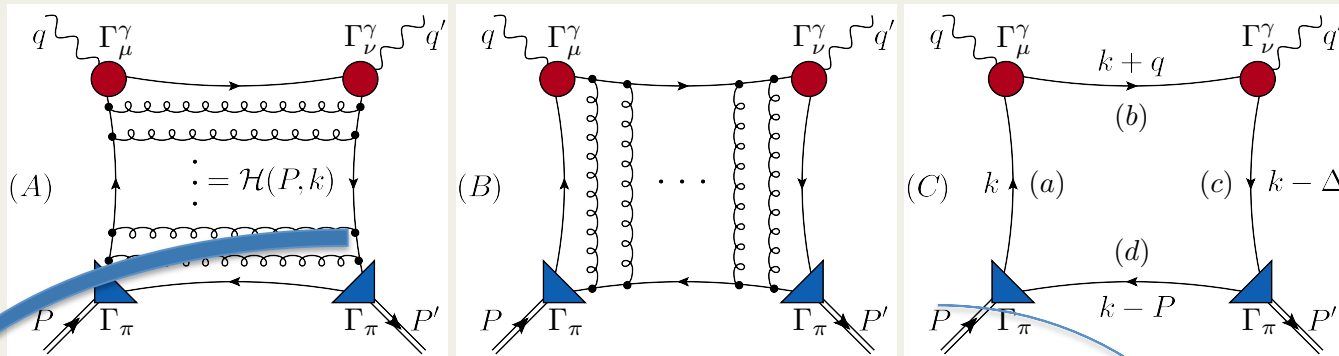


$$\alpha_{PI}(k^2) = \frac{\pi \gamma_m}{\ln[(m_\alpha^2 + k^2)/\Lambda_{QCD}^2]},$$

$$m_\alpha = 0.30 \text{ GeV} \gtrsim \Lambda_{QCD}.$$

- Rainbow-Ladder truncation
- Renormalize our DSEs at the hadronic scale  $\zeta=m_\alpha$
- Pure valences

# Pion Compton Scattering(RL symmetry)



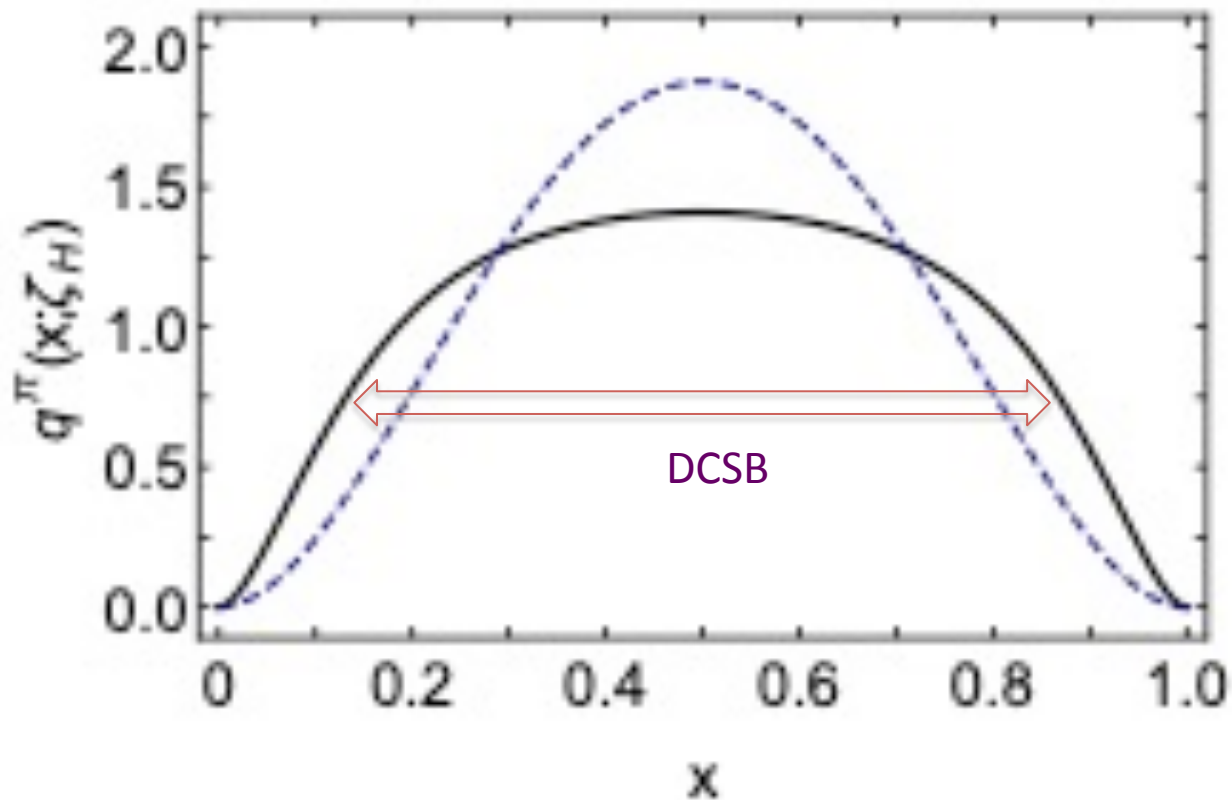
Employing the optical theorem

$$q^\pi(x; \zeta_H) = q^\pi(1-x; \zeta_H);$$

## Hadronic Scale

- Numerical  $m=0,1,2,3,4,5$
- SPM extrapolation to  $m=11$
- PDF extraction

$$q^\pi(x; \zeta_H) = N_c \text{tr} \int_{dk} \delta_n^x(k_\eta) \\ \times n \cdot \partial_{k_\eta} [\Gamma_\pi(k_\eta, -P) S(k_\eta)] \Gamma_\pi(k_{\bar{\eta}}, P) S(k_{\bar{\eta}}),$$





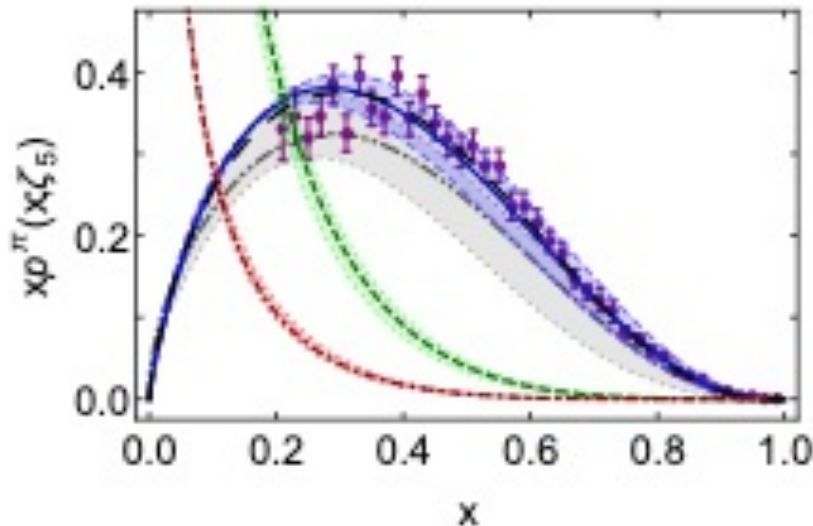
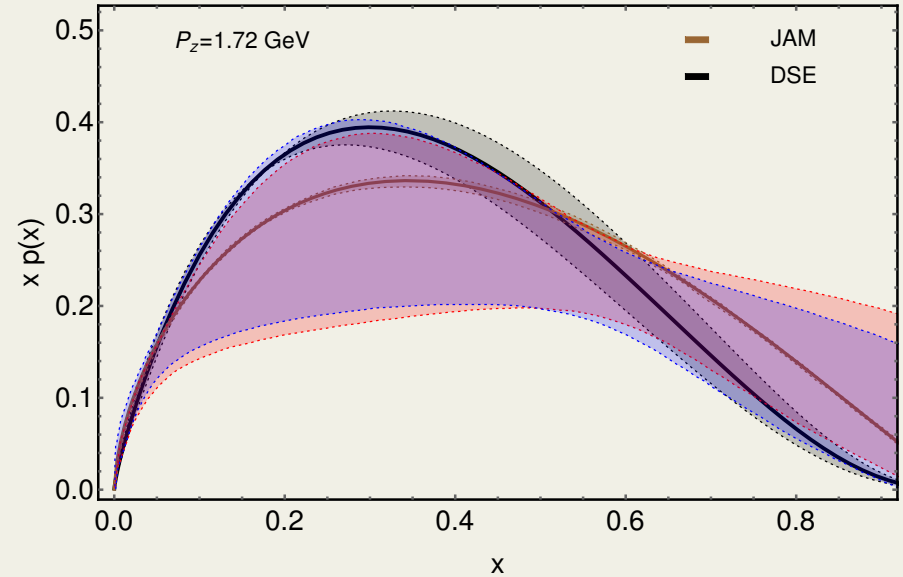
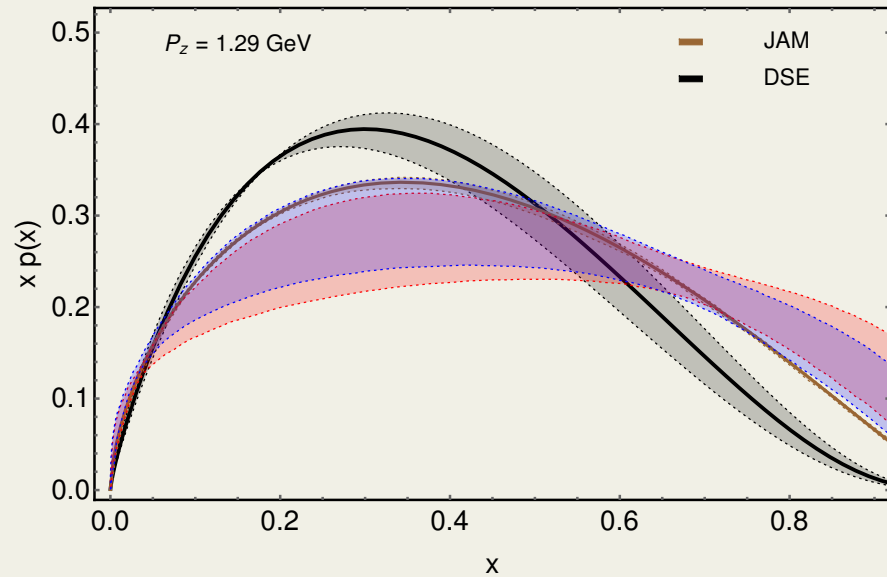


FIG. 7. Pion valence-quark momentum distribution function,  $xq^\pi(x; \zeta)$ , evolved  $\zeta_H \rightarrow \zeta_5 = 5.2 \text{ GeV}$  – solid (blue) curve embedded in shaded band; and long-dashed (black) curve –  $\zeta_5$  result from Ref. [12]. Gluon momentum distribution in pion,  $xg^\pi(x; \zeta_2)$  – dashed (green) curve within shaded band; and sea-quark momentum distribution,  $xS^\pi(x; \zeta_2)$  – dot-dashed (red) curve within shaded band. See Eqs. (39), (40). In all the above cases, the shaded band indicates the effect of  $\zeta_H \rightarrow \zeta_H(1 \pm 0.1)$ . Dot-dot-dashed (grey) curve within shaded band – IQCD result [31]. Data (purple) from Ref. [9], rescaled according to the analysis in Ref. [14].

- Solid Blue=nonsinglet evolution for valence-quark  
 $\langle x_{\text{valence}} \rangle = 0.48(3)$ ,
- glue& sea quarks distribution are generated by singlet evolution  
 $\langle x_{\text{glue}} \rangle = 0.41(2)$ ,  
 $\langle x_{\text{sea}} \rangle = 0.11(2)$ ,
- dot-dot-dashed(grey)=IQCD result for the pion valence-quark distribution
- Pointwise form of the IQCD prediction agrees with our result(with errors).
- Significant: two disparate treatments of the pion have arrived at the same prediction.

IQCD result, R. S. Sufian *et al*, PRD99(2019)074507





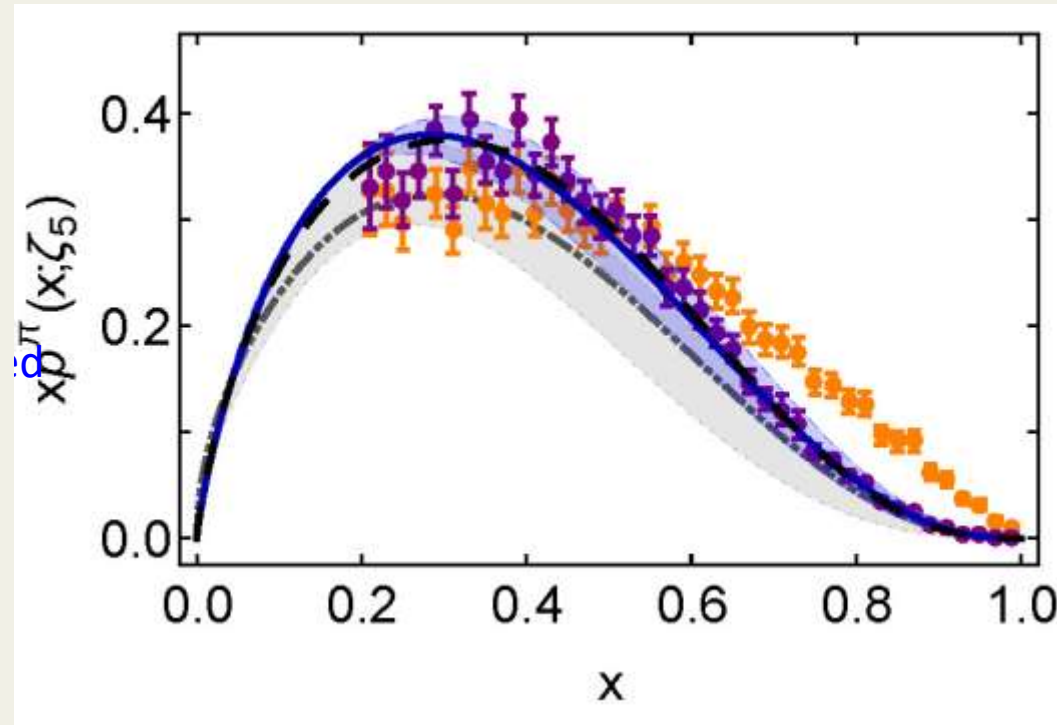
IQCD result, T. Izubuchi *et al*, arXiv:1905.06349.

Valence parton distribution function of pion from fine lattice

还不是时候

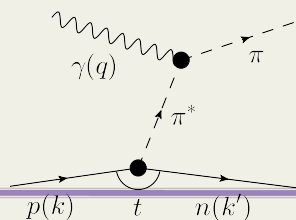
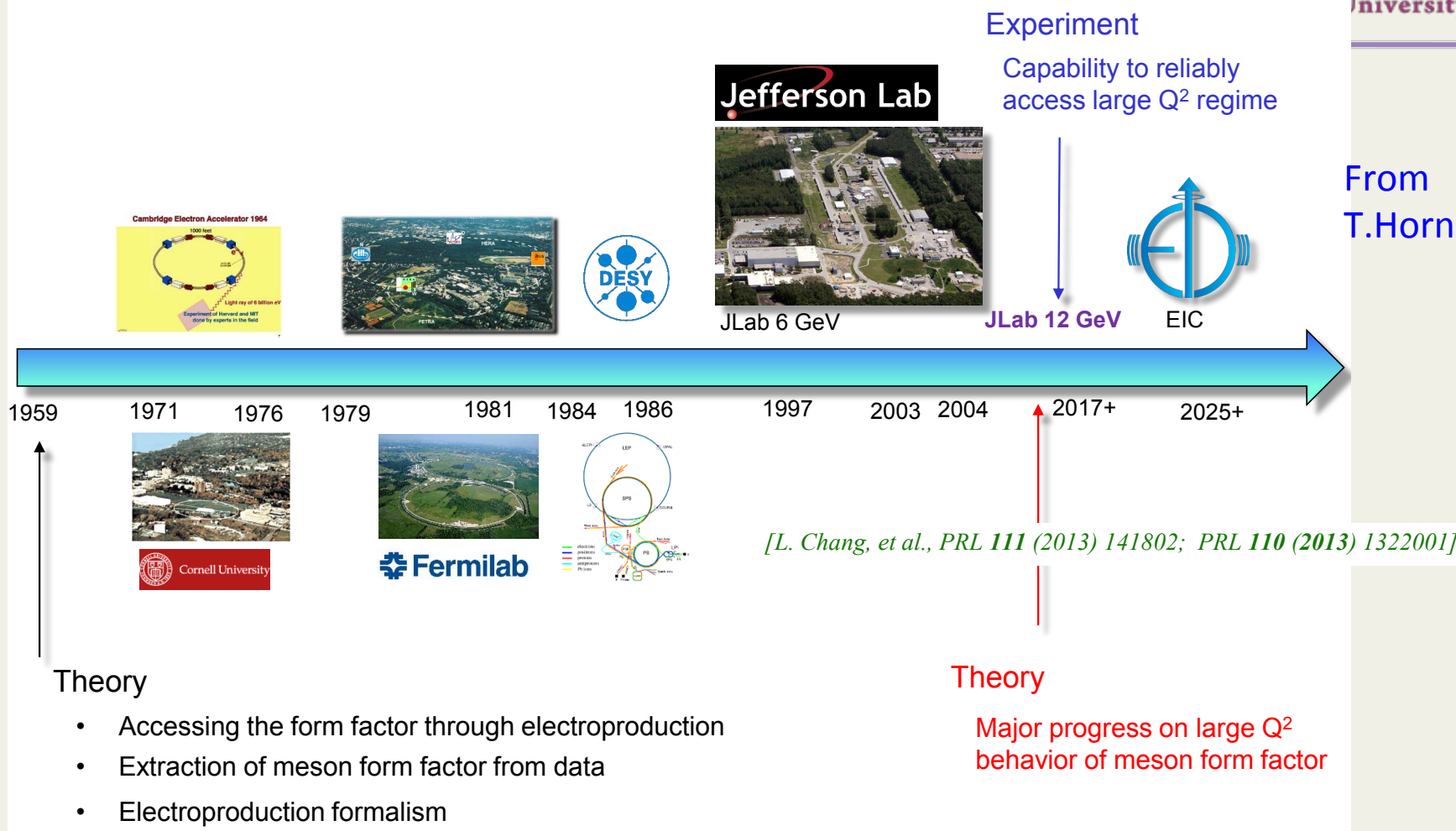
# Pion PDF 20年

- 1989...Conway et al. Phys. Rev.D 39 (1989) 92  
Leading-order analysis of Drell-Yan data
- 2000...Hecht et al. Phys. Rev.C 63 (2001) 025213  
QCD connected model calculation
- 2010...Aicher et al. Phys. Rev. Lett. 105 (2010) 252003  
Consistent next-to-leading order analysis
- 2019/04...Ding, et al.  
Continuum QCD prediction
- 2019/01...Sufian, et al.  
1st exploratory lattice-QCD calculation  
Using lattice-calculated matrix element obtained through spatially separated current-current correlations in coordinate space



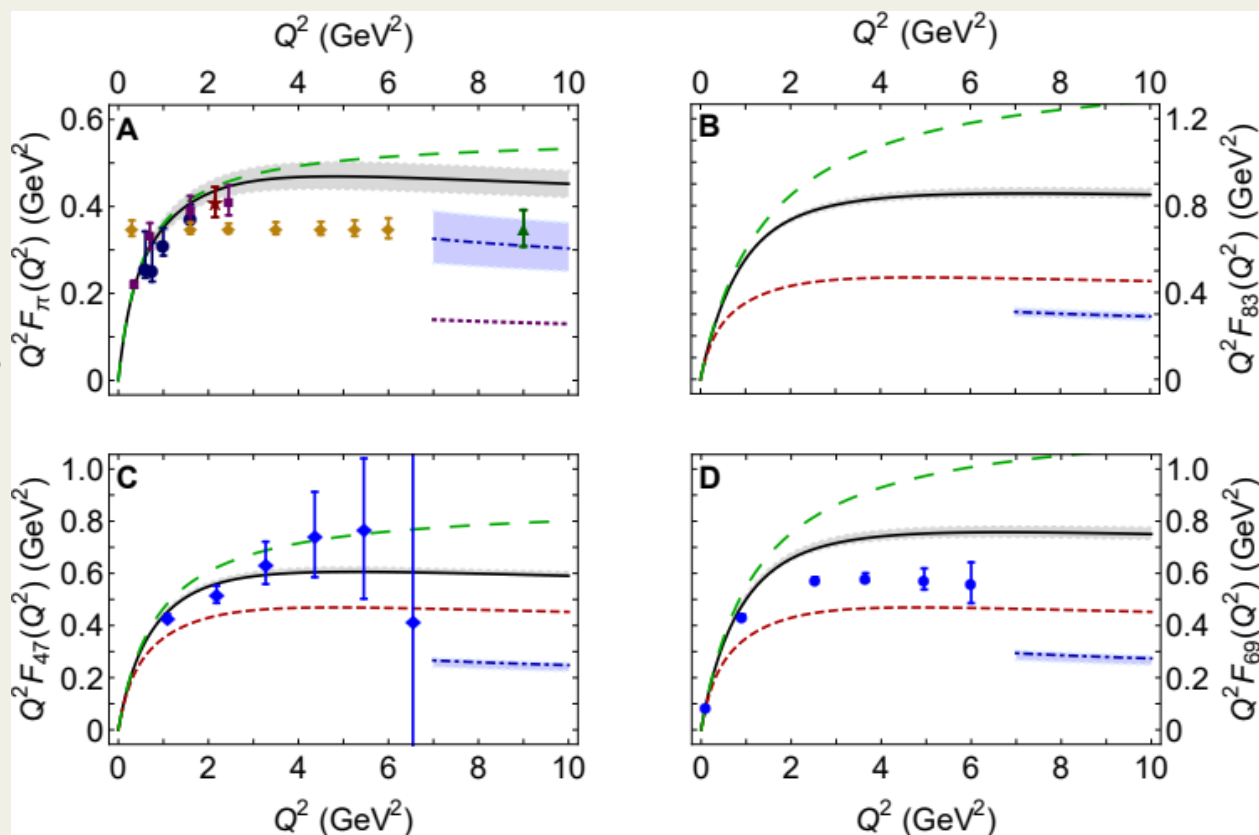
**Sufian**...extending lattice calculation on three other ensembles and the preliminary result gives an indication MORE COINCIDE.

# Meson Form Factor Data Evolution



- 如何理解不在壳的pion?
- 如何理解基于第一性原理计算的格点QCD结果

- ✓ 彩虹梯子近似
- ✓ 赝标介子质量分别  
 $m_{0-}/\text{GeV}=0.14, 0.47, 0.69, 0.83$
- ✓ 电磁形状因子  $Q^2 \lesssim 10 \text{ GeV}^2$ .
- ✓ 与格点结果比较



- IQCD(QCDSF/UKQCD/CSSM Collaborations) in panel C: PRD96(2017)114509
- IQCD(HPQCD Collaboration) in panel D: PRD96(2017)054501

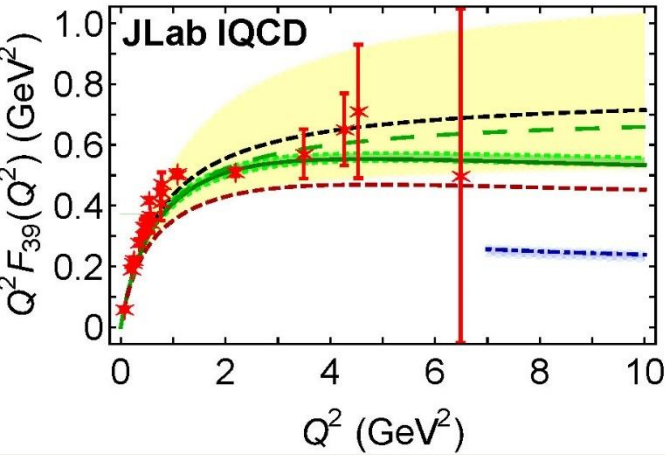


Figure 2. Elastic form factor of a pion-like pseudoscalar meson with mass  $m_0 = 0.39$  GeV. Red asterisks -- JLab lattice results drawn from the ECT\* presentation by D.G. Richards. Curves: Solid green curve within green bands -- our prediction, obtained using the methods described in our submission; long-dashed green curve -- single-pole vector meson dominance result obtained with vector meson mass,  $m_1 = 0.86$  GeV, computed consistent with the form factor prediction; short-dashed black curve within yellow bands -- range of uncertainty in single-monopole fit to IQCD results following from the JLab-quoted error on the meson's radius; and dot-dashed blue curve within blue bands -- result from QCD hard-scattering formula, Eq. (1) in our submission, computed with the consistent meson decay constant and PDA, as predicted by the our analysis. For comparison, the dashed red curve is the DSE prediction for the physical-pion, taken from Fig. 1A in our submission.

The lattice results in Figure 2 were drawn from a presentation by D. G. Richards at the ECT\* Workshop on Mapping Parton Distribution Amplitudes and Functions. That presentation is available here

[https://indico.ectstar.eu/event/22/contributions/503/attachments/387/532/pion\\_pdf\\_richards.pdf](https://indico.ectstar.eu/event/22/contributions/503/attachments/387/532/pion_pdf_richards.pdf)

Notably, in common with the Adelaide lattice-QCD results (Fig. 1C in our submission), the JLab results are consistent with our predictions. The Glasgow results (Fig. 1D in our submission) alone appear to be inconsistent with physics expectations.

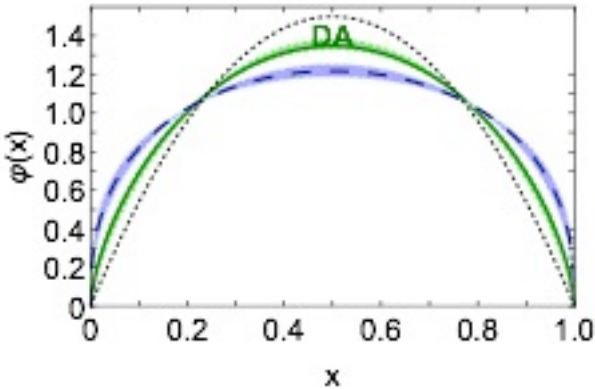


FIG. 2. Dressed-valence-quark distribution amplitude of pion-like pseudoscalar mesons. Solid green curve within green bands -- prediction for  $m_{0-} = 0.39$  GeV; long-dashed blue curve within blue bands -- prediction from Ref. [1] for the physical pion; and dotted black curve -- asymptotic profile,  $\varphi_\infty(x) = 6x(1-x)$ .

	$r_{0-}/\text{fm}$	$m_{1-}/\text{GeV}$	$f_{0-}/\text{GeV}$
DSE	0.58(1)	$0.86^{+0.04}_{-0.02}$	0.109(1)
IQCD	0.55(10)	$0.88^{+0.19}_{-0.13}$	?

# 电磁半径

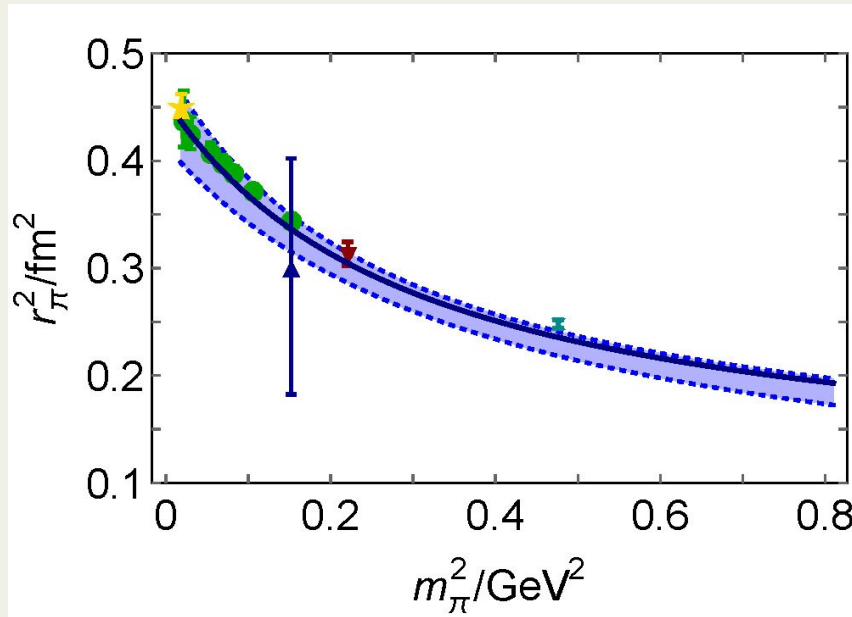
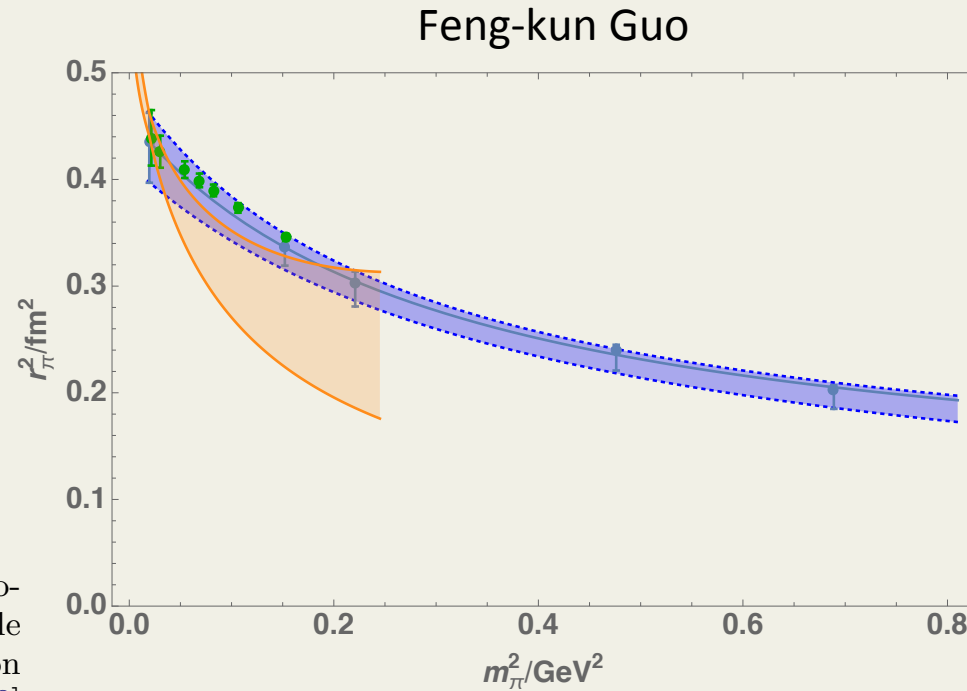
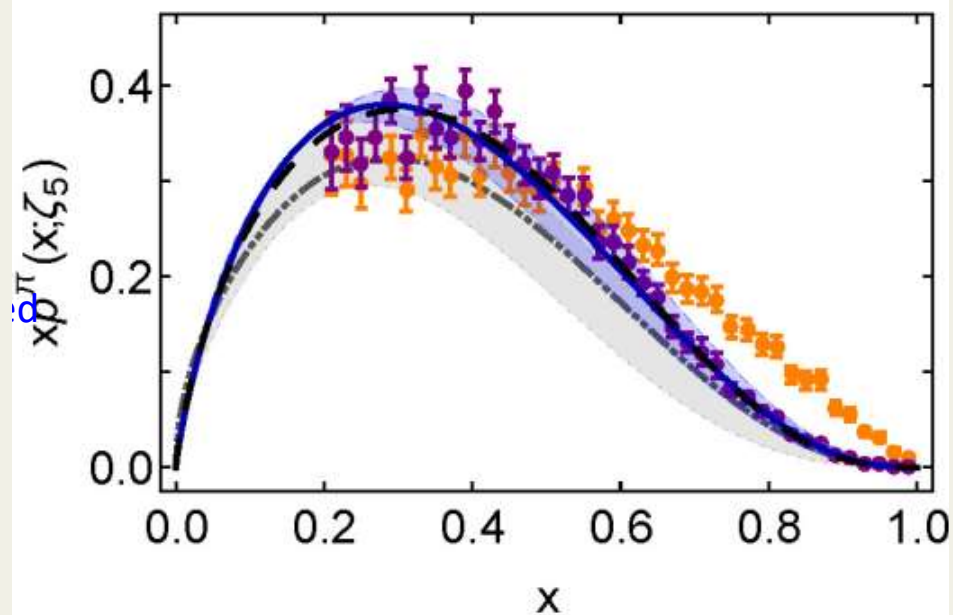
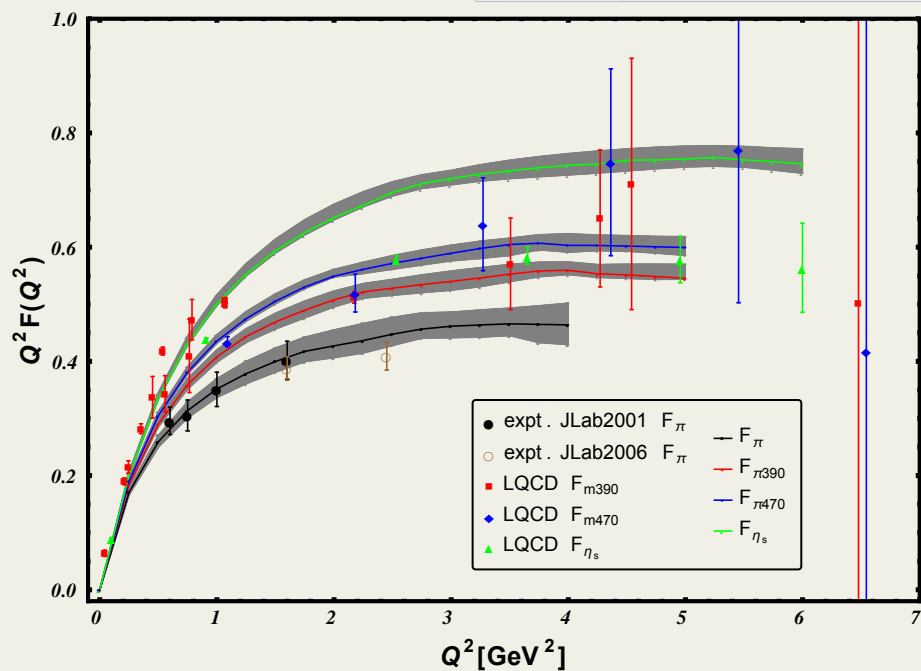


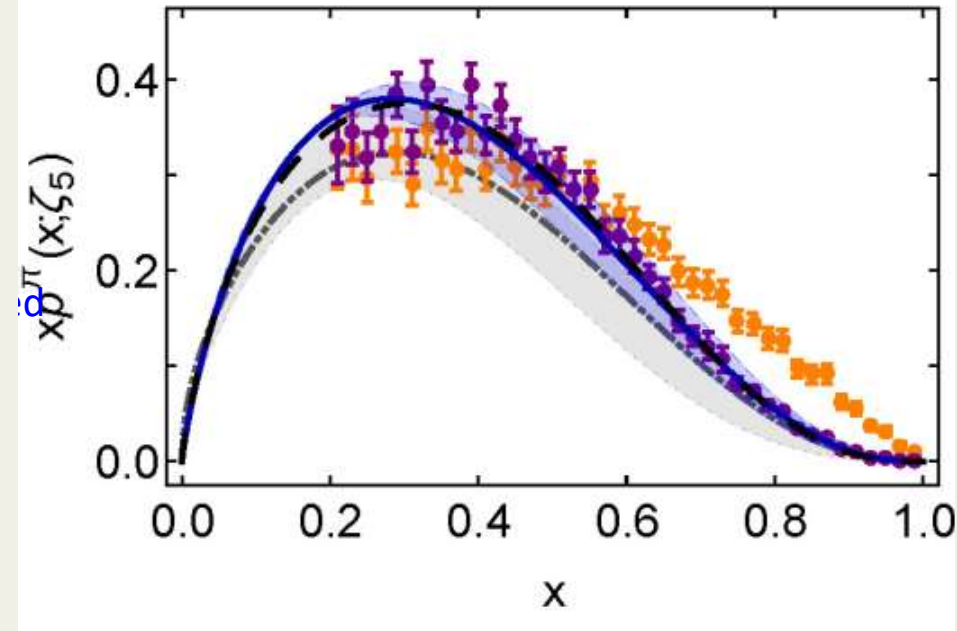
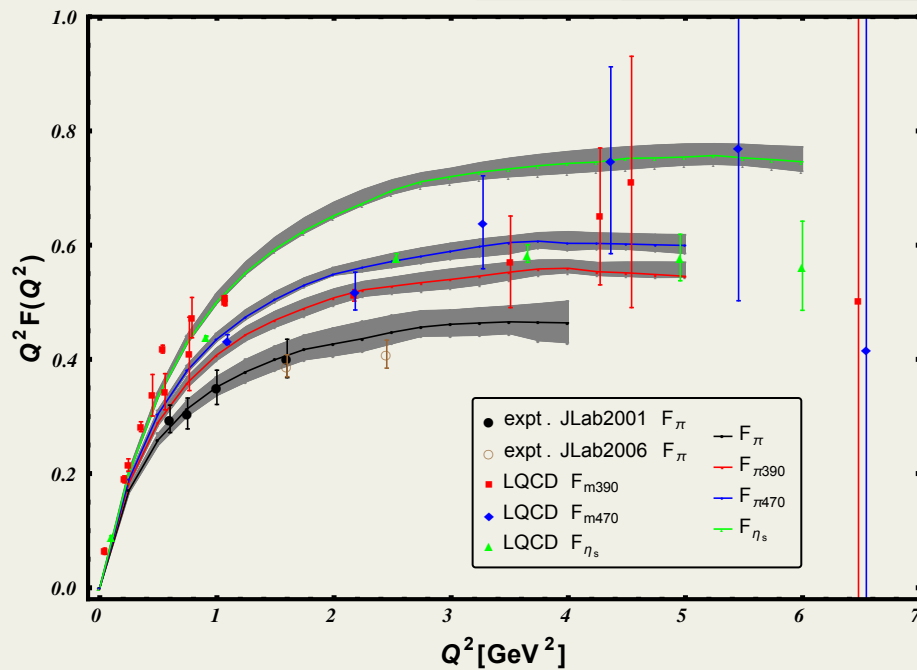
FIG. 2: Lattice-QCD computations of the pion's electromagnetic charge radius (green circles [34], blue up-triangle [35], red down-triangle [36], cyan cross [37]) as a function of  $m_\pi^2$ , compared with a continuum theory prediction [38] (blue curve within bands, which indicate response to reasonable parameter variation). The continuum analysis establishes  $f_\pi r_\pi \approx \text{constant}$ , from which it follows that the size of a Nambu-Goldstone mode decreases in inverse proportion to the active strength of the dominant mass generating mechanism. The empirical value of  $r_\pi$  is marked by the gold star.



- [34] G. Wang, J. Liang, T. Draper, K.-F. Liu and Y.-B. Yang, PoS **LATTICE2018**, 127 (2018).
- [35] B. Chakraborty and D. G. Richards, (2018), *private communication*.
- [36] A. J. Chambers *et al.*, Phys. Rev. D **96**, 114509 (2017).
- [37] J. Koponen, A. C. Zimmermann-Santos, C. T. H. Davies, G. P. Lepage and A. T. Lytle, Phys. Rev. D **96**, 054501 (2017).







Thanks for your attention





# Particle Data Group



南开大学  
Nankai University

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update

$g$   
or gluon

$$I(J^P) = 0(1^-)$$

SU(3) color octet

Mass  $m = 0$ .

Theoretical value. A mass as large as a few MeV may not be precluded, see YNDURAIN 95.

VALUE	DOCUMENT ID	TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •			
	ABREU 92E	DLPH	Spin 1, not 0
	ALEXANDER 91H	OPAL	Spin 1, not 0
	BEHREND 82D	CELL	Spin 1, not 0
	BERGER 80D	PLUT	Spin 1, not 0
	BRANDELIK 80C	TASS	Spin 1, not 0

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YNDURAIN 95	PL B345 524	F.J. Yndurain	(MADU)
ABREU 92E	PL B274 498	P. Abreu <i>et al.</i>	(DELPHI Collab.)
ALEXANDER 91H	ZPHY C52 543	G. Alexander <i>et al.</i>	(OPAL Collab.)
BEHREND 82D	PL B110 329	H.J. Behrend <i>et al.</i>	(CELLO Collab.)
BERGER 80D	PL B97 459	C. Berger <i>et al.</i>	(PLUTO Collab.)
BRANDELIK 80C	PL B97 453	R. Brandelik <i>et al.</i>	(TASSO Collab.)



# In QCD: Gluons become massive!



南开大学  
Nankai University

PHYSICAL REVIEW

VOLUME 125, NUMBER 1

JANUARY 1, 1962

## Gauge Invariance and Mass

JULIAN SCHWINGER

*Harvard University, Cambridge, Massachusetts, and University of California, Los Angeles, California*

(Received July 20, 1961)

It is argued that the gauge invariance of a vector field does not necessarily imply zero mass for an associated particle if the current vector coupling is sufficiently strong. This situation may permit a deeper understanding of nucleonic charge conservation as a manifestation of a gauge invariance, without the obvious conflict with experience that a massless particle entails.

- Schwinger  
1962

PHYSICAL REVIEW D

VOLUME 26, NUMBER 6

15 SEPTEMBER 1982

## Dynamical mass generation in continuum quantum chromodynamics

John M. Cornwall

*Department of Physics, University of California, Los Angeles, California 90024*

(Received 30 April 1982)

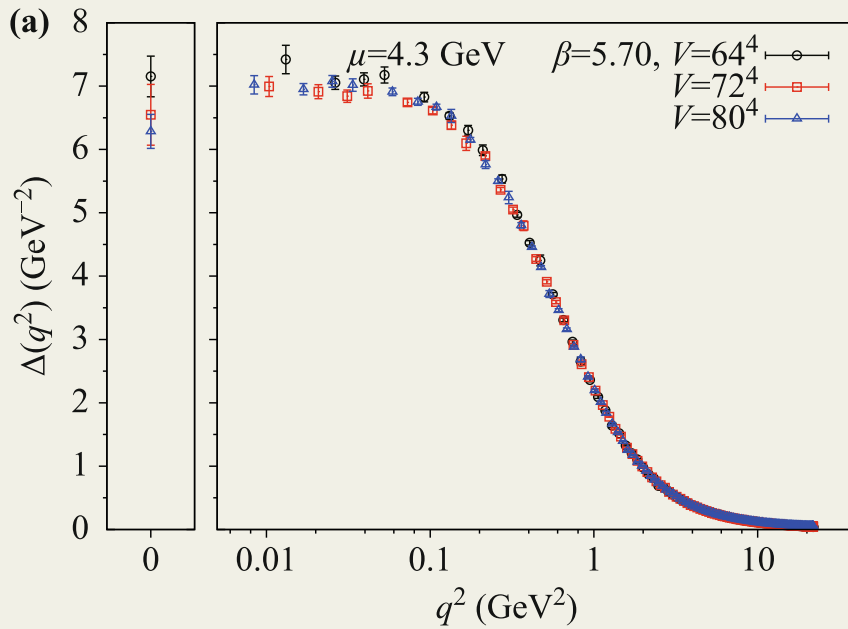
- Cornwall  
1982

$$\Delta_{\mu\nu}^{-1}(q) = \text{diagram (a)} + \text{diagram (b)} + \text{diagram (c)} + \text{diagram (d)} + \text{diagram (e)}$$

$\Pi_{\mu\nu}(q)$ 
 $\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$ 
 $P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$

- Binosi & Papavassiliou  
Phys. Rept. 479  
(2009)1-152  
Pinch Technique: Theory  
and Applications

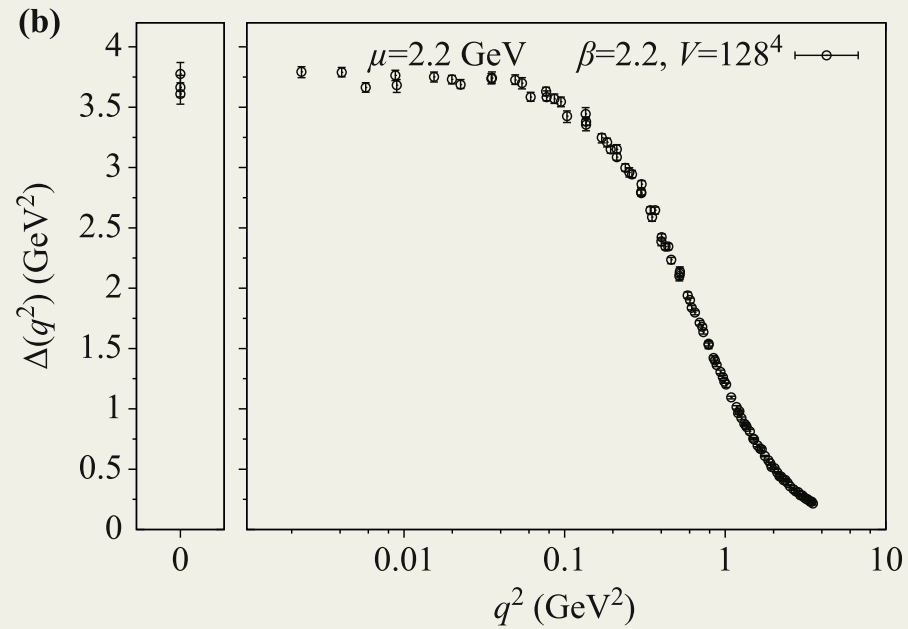
a) SU(3)



I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, Lattice gluodynamics computation of Landau gauge Green's functions in the deep infrared, *Phys. Lett. B* 676, 69 (2009), arXiv: 0901.0736 [hep-lat]

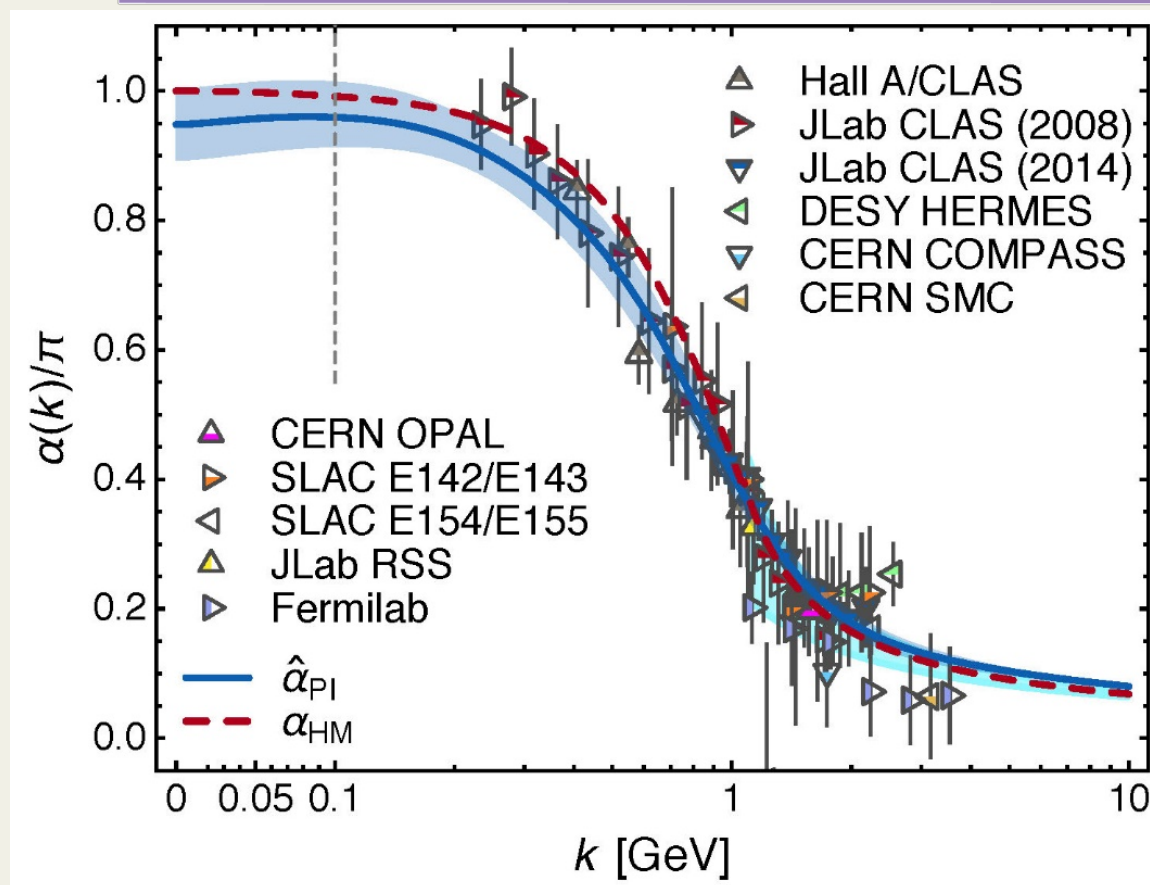
I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, The Landau gauge gluon and ghost propagators in 4D SU(3) gluodynamics in large lattice volumes, arXiv: 0710.1968 [hep-lat]

b) SU(2)

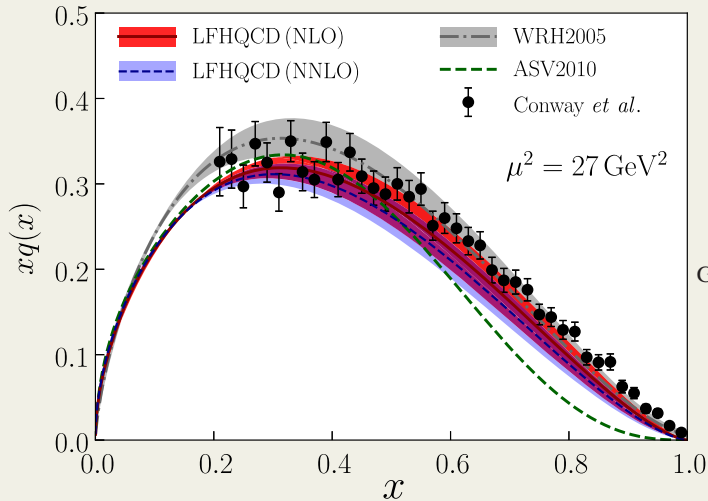


A. Cucchieri and T. Mendes, Numerical test of the Gribov–Zwanziger scenario in Landau gauge, *PoS QCD-TNT 09*, 026 (2009), arXiv: 1001.2584 [hep-lat]

# Process-independent effective-charge in QCD



# Modellers still insist on...

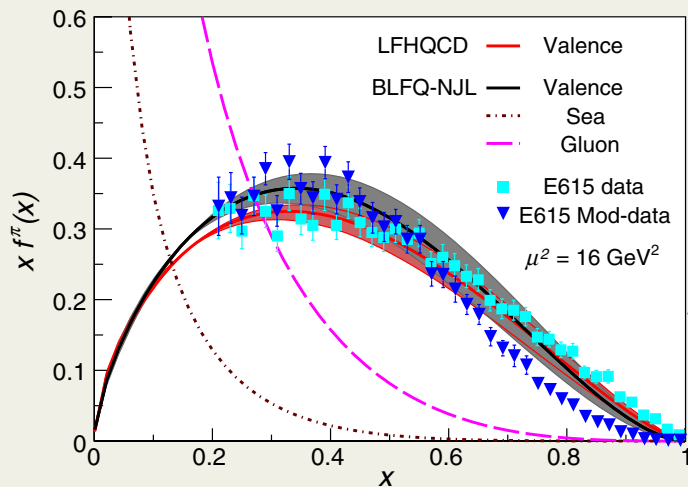


PHYSICAL REVIEW LETTERS **120**, 182001 (2018)

## Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond,<sup>1</sup> Tianbo Liu,<sup>2,3</sup> Raza Sabbir Sufian,<sup>2</sup> Hans Günter Dosch,<sup>4</sup> Stanley J. Brodsky,<sup>5</sup> and Alexandre Deur<sup>2</sup>

(HLFHS Collaboration)



PHYSICAL REVIEW LETTERS **122**, 172001 (2019)

## Parton Distribution Functions from a Light Front Hamiltonian and QCD Evolution for Light Mesons

Jiangshan Lan,<sup>1,2,†</sup> Chandan Mondal,<sup>1,\*</sup> Shaoyang Jia,<sup>3,‡</sup> Xingbo Zhao,<sup>1,2,§</sup> and James P. Vary<sup>3,||</sup>

(BLFQ Collaboration)