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Light-flavor mesons at low temperatures in Chiral EFT

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Outline:

1. Introduction

2. Chiral perturbation theory and eta-eta' mixing

3. S-wave meson-meson scattering and scalar resonances

4. Thermal behaviors of light pseudoscalar and scalar mesons at low temperatures

5. Summary

Introduction

Lowest QCD scalar resonances

$f_0(500)/\sigma$: precise $\pi\pi$ scattering data + dispersive technique + chiral EFT. Well determined pole positions !

[Xiao,Zheng, NPA01] [Caprini,Colangelo,Leutwyler,PRL06] [Garcia-Martin, et al., PRD11]

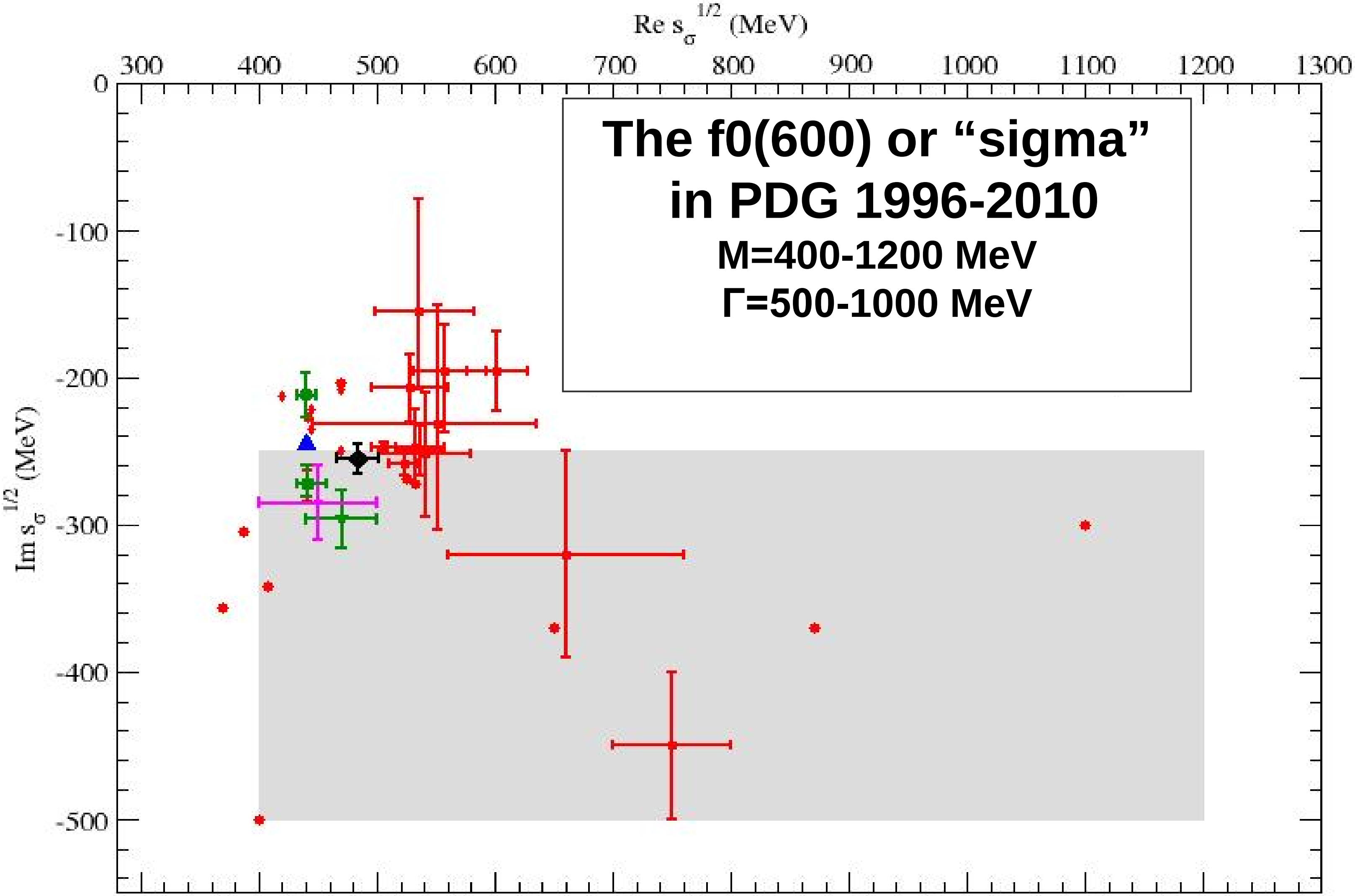
$f_0(980)$: $\pi\pi$ scattering data + (dispersive technique,Unitarized chiral EFT). Confirmed pole in the complex energy plane !
[Garcia-Martin, et al., PRD11] [Oller, Oset, Pelaez, PRD99]

$K^*_0(700)/\kappa$: πK scattering data+ (dispersive technique,Unitarized chiral EFT). Confirmed pole in the complex energy plane !
[Zheng, Zhou, et al.,] [Descotes-Genon, Moussallam, EPJC06] [Pelaez, Rodas, PRD16]

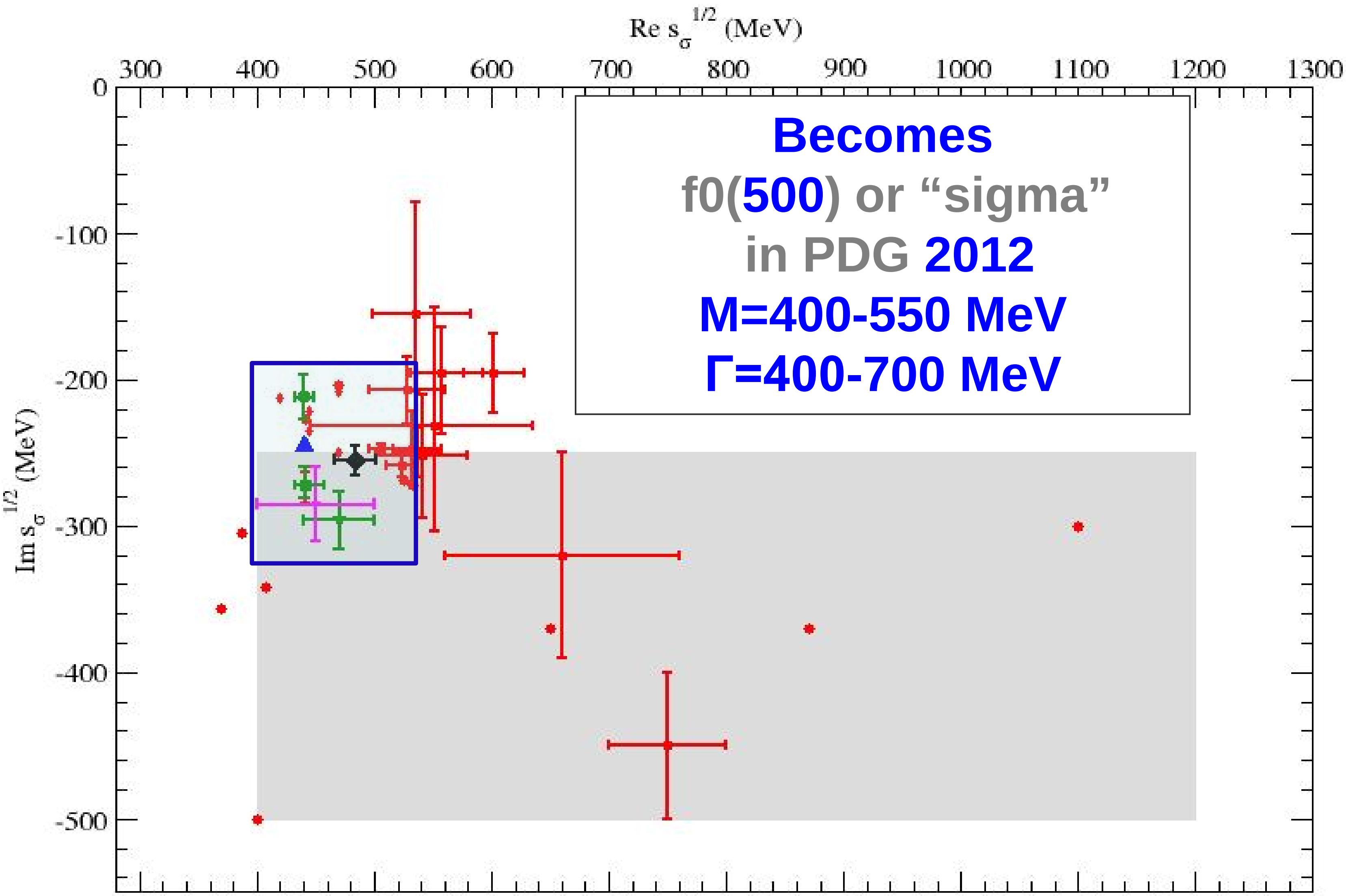
$a_0(980)$: Absence of the $\pi\eta$ scattering data. Lattice can help.

[ZHG, L.Liu, Meissner, Oller, Rusetsky, PRD17]

Lightest QCD scalar resonance: sigma



From J.Pelaez



From J.Pelaez

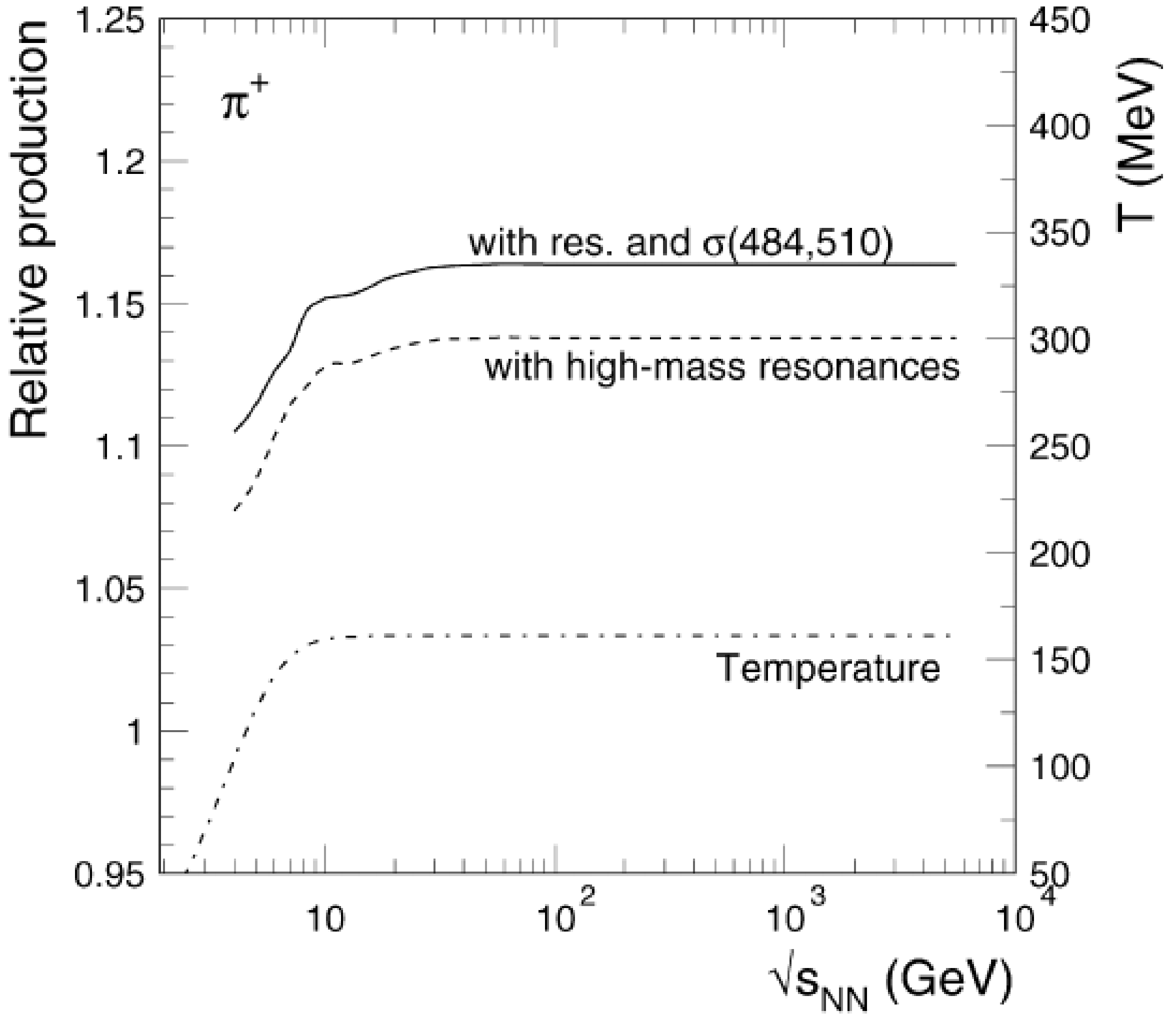
Importance of the broad sigma:

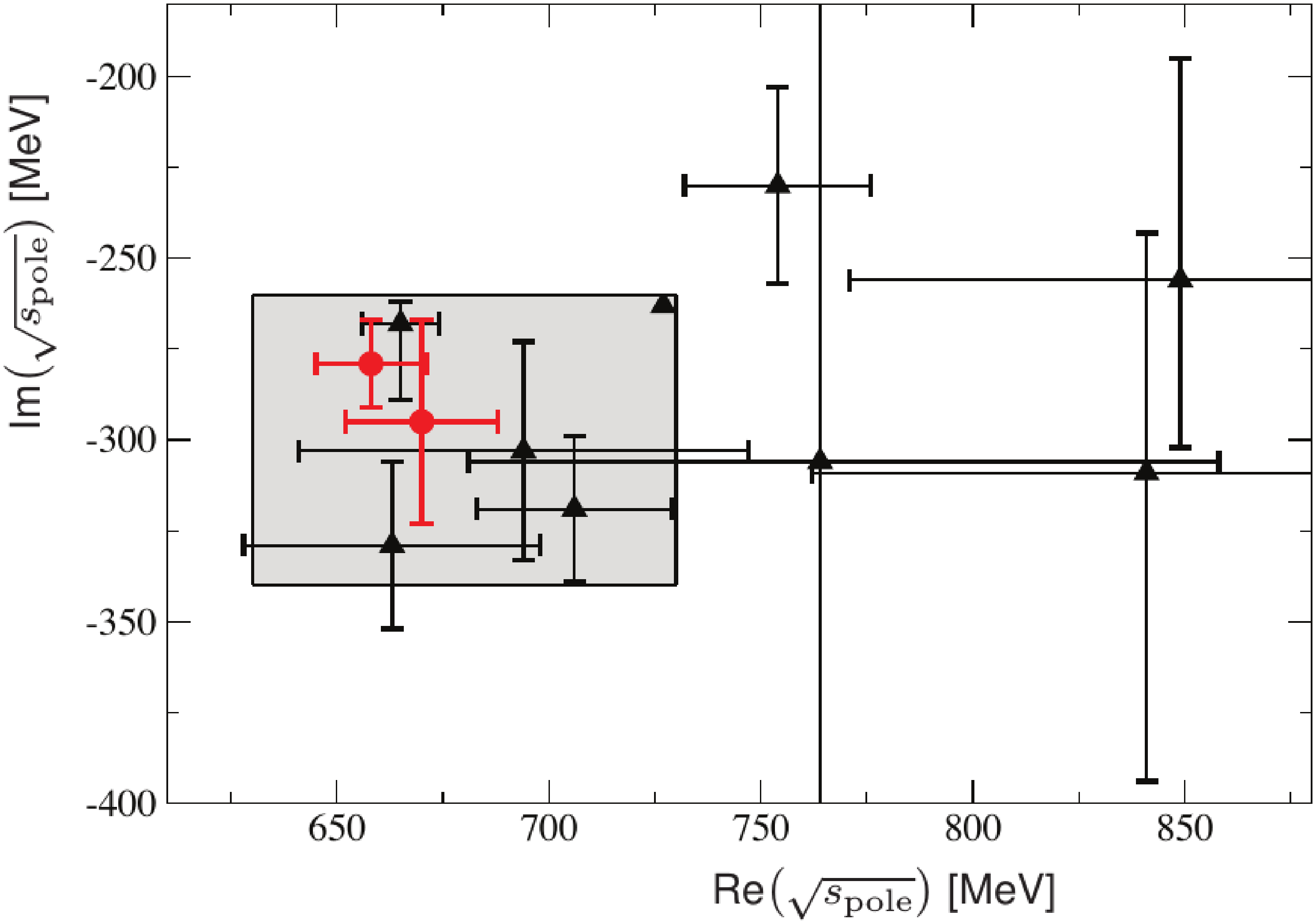
to improve the description of the hadron yields

[Andronic, Braun-Munzinger, Stachel, PLB'09]

Problem:

Breit-Wigner formula unsuitable for such a broad resonance !





Kappa or K*0(700) from PDG

f_0(980) and a_0(980) :

theoretically less clear, due to the nearby Kbar-K threshold.
Coupled-channel analysis needed !
[Garcia-Martin, et al., PRD11] [Oller, Oset, Pelaez, PRD99]

[ZHG, L.Liu, Meissner, Oller, Rusetsky, PRD17] ....

In this talk, we focus on the thermal behaviors of these scalar resonances at finite temperatures.

- **Chiral EFT is used to describe sigma, kappa, f_0(980) and a_0(980) simultaneously.**
- **Scattering and relevant inputs from Exp/Lattice are nicely reproduced in our approach.**
- **The QCD $U_A(1)$ anomaly is tentatively addressed by examining the thermal masses of the eta and eta'.**
- **The thermal behaviors of both light pseudoscalar and scalar mesons are pure predicitons !**

Chiral peturbation theory and eta-eta' mixing

The relevant dynamical d.o.f: π, K, η, η'

$SU_L(3) \otimes SU_R(3) \to SU_V(3)$: Goldstone π, K, η_8 [$SU(3)$ χPT]
[Gasser and Leutwyler, NPB'85]

$N_C \to \infty : M^2_{\eta_0} \sim \mathcal{O}(1/N_C), \therefore\ \eta_0$ becomes Goldstone
[Witten, NPB'79]

$U(3)\,\chi PT : \pi,\ K,\ \eta_8$ and η_0

A consistent power counting scheme in U(3) χPT:

$$\delta \sim p^2/\Lambda_\chi^2 \sim m^2_{\pi,K,\eta}/\Lambda_\chi^2 \sim 1/N_C$$

In this case, it is essential to generalize from SU(3) to U(3) ChPT

$$\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta_8+\frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta_8+\frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta_8+\frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

Leading order in the δ counting:

$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \boxed{\frac{F^2}{3}M_0^2 \ln^2 \det u}$$

Leads to a massive η_0

NLO: $\mathcal{O}(N_C p^4)$ and $\mathcal{O}(N_C^0 p^2)$

$$\mathcal{L}^{(\delta^1)} = L_5\langle u_\mu u^\mu \chi_+\rangle + L_8/2\langle \chi_+\chi_+ + \chi_-\chi_-\rangle$$
$$+ + F^2\tilde{\Lambda}_1\langle u_\mu\rangle\langle u^\mu\rangle + F^2\tilde{\Lambda}_2 \ln(\det U)\langle U^\dagger\chi - \chi^\dagger U\rangle + \ldots$$

NNLO: $\mathcal{O}(N_C^{-2}p^0)$, $\mathcal{O}(N_C^{-1}p^2)$, $\mathcal{O}(N_C^0 p^4)$ and $\mathcal{O}(N_C p^6)$

$$\mathcal{L}^{(\delta^2)} = \tilde{v}_0^{(4)} X^4 + \tilde{v}_1^{(2)} X^2\langle u_\mu u^\mu\rangle + L_4\langle u_\mu u^\mu\rangle\langle \chi_+\rangle$$
$$+ C_1\langle u_\rho u^\rho h_{\mu\nu} h^{\mu\nu}\rangle + \ldots$$

[Herrera-Siklody, Latorre, Pascual, Taron, NPB'97]
[Bijnens, Colangelo, Ecker, JHEP'99], [Jiang, Ge, Wang, '14]

δ counting in U(3) χPT: a systematical expansion to describe eta-eta' mixing

η-η' mixing at LO: $\quad \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle + \frac{F^2}{12} M_0^2 X^2$

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \frac{1}{2}\partial_\mu \eta_8 \partial^\mu \eta_8 + \frac{1}{2}\partial_\mu \eta_1 \partial^\mu \eta_1 \\
& - \frac{4\overline{m}_K^2 - \overline{m}_\pi^2}{3}\eta_8\eta_8 - \frac{2\overline{m}_K^2 + \overline{m}_\pi^2 + 3M_0^2}{3}\eta_1\eta_1 - \frac{4\sqrt{2}(\overline{m}_K^2 - \overline{m}_\pi^2)}{3}\eta_1\eta_8
\end{aligned}
$$

with

$$
\overline{m}_\pi^2 = 2Bm_{ud}\,, \qquad \overline{m}_K^2 = B(m_s + m_{ud})
$$

$$
\Longrightarrow
$$

$$
\begin{aligned}
\eta_8 \quad &= \quad c_\theta \overline{\eta} + s_\theta \overline{\eta}'\,, \quad (c_\theta = \cos\theta \quad s_\theta = \sin\theta) \\
\eta_1 \quad &= \quad -s_\theta \overline{\eta} + c_\theta \overline{\eta}'\,,
\end{aligned}
$$

The leading order calculation leads to the conventional one-mixing angle scheme.

$\overline{\eta}$-$\overline{\eta}'$ **will get mixing again due to higher order effects.**

General parameterizations of $\overline{\eta}$, $\overline{\eta}'$ bilinear terms

$$\begin{aligned}\mathcal{L} \;=\;& \frac{\delta_1}{2}\,\partial_\mu\partial_\nu\overline{\eta}\partial^\mu\partial^\nu\overline{\eta} + \frac{\delta_2}{2}\,\partial_\mu\partial_\nu\overline{\eta}'\partial^\mu\partial^\nu\overline{\eta}' + \delta_3\,\partial_\mu\partial_\nu\overline{\eta}\partial^\mu\partial^\nu\overline{\eta}' \\ & + \frac{1+\delta_{\overline{\eta}}}{2}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta} + \frac{1+\delta_{\overline{\eta}'}}{2}\partial_\mu\overline{\eta}'\partial^\mu\overline{\eta}' + \delta_k\,\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}' \\ & - \frac{m_{\overline{\eta}}^2 + \delta_{m_{\overline{\eta}}^2}}{2}\overline{\eta}\,\overline{\eta} - \frac{m_{\overline{\eta}'}^2 + \delta_{m_{\overline{\eta}'}^2}}{2}\overline{\eta}'\overline{\eta}' - \delta_{m^2}\,\overline{\eta}\,\overline{\eta}'\,.\end{aligned}$$

δ_i's are calculated up to NNLO in δ-counting U(3) ChPT

LO from $\mathcal{L}^{\delta^0}$: $\quad \delta_i = 0\,,$

NLO from $\mathcal{L}^{\delta^1}$:

$$\begin{aligned}\delta_1^{NLO} &= \delta_2^{NLO} = \delta_3^{NLO} = 0\,,\\ \delta_{\overline{\eta}}^{NLO} &= \frac{8L_5}{3F^2}\left[m_\pi^2(-c_\theta^2 - 4\sqrt{2}c_\theta s_\theta + s_\theta^2) + 2m_K^2(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2\right] + s_\theta^2\Lambda_1\,,\\ \delta_{\overline{\eta}'}^{NLO} &= \frac{8L_5}{3F^2}\left[m_\pi^2(c_\theta^2 + 4\sqrt{2}c_\theta s_\theta - s_\theta^2) + 2m_K^2(c_\theta^2 - 2\sqrt{2}c_\theta s_\theta + 2s_\theta^2\right] + c_\theta^2\Lambda_1\,,\\ \delta_k^{NLO} &= -\frac{16L_5}{3F^2}(m_K^2 - m_\pi^2)(\sqrt{2}c_\theta^2 - c_\theta s_\theta - \sqrt{2}s_\theta^2) - c_\theta s_\theta\Lambda_1\,,\end{aligned}$$

……

NNLO from $\mathcal{L}^{\delta^2}$ and chiral loops:

$$\delta_1^{NNLO} = \frac{32C_{12}}{3F^2}\left[m_\pi^2(-c_\theta^2 - 4\sqrt{2}c_\theta s_\theta + s_\theta^2) + 2m_K^2(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2\right],$$

$$\begin{aligned}
&\delta_{\overline{\eta}}^{NNLO} = \\
&\frac{8L_4}{F^2}(2m_K^2 + m_\pi^2) + \cdots \\
&+\frac{128L_5L_8}{3F^4}\left[m_\pi^4(-c_\theta^2 - 4\sqrt{2}c_\theta s_\theta + s_\theta^2) + m_K^4(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)\right] + \cdots \\
&+\frac{16C_{14}}{3F^4}\left[3m_\pi^4 + 4m_K^4(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2) - 4m_K^2m_\pi^2(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)\right] + \cdots \\
&+\frac{c_\theta^2}{F^2}A_0(m_\pi^2) + \cdots, \\
&\cdots\cdots
\end{aligned}$$

$$A_0(m^2) = -\frac{m^2}{16\pi^2}\log\frac{m^2}{\mu^2}$$

[X.K.Guo, ZHG, Oller, Sanz-Cillero, JHEP’15]

- We also give the relation to the popular two-mixing angle scheme:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \frac{1}{F} \begin{pmatrix} F_8 \cos\theta_8 & -F_0 \sin\theta_0 \\ F_8 \sin\theta_8 & F_0 \cos\theta_0 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} .$$

$$F_\eta^8 = \cos\theta_8 F_8 \qquad F_{\eta'}^8 = \sin\theta_8 F_8$$
$$F_\eta^0 = -\sin\theta_0 F_0 \qquad F_{\eta'}^0 = \cos\theta_0 F_0$$

$$\langle 0|A_\mu^a|\eta^{(\prime)}\rangle = i p_\mu F^a_{\eta^{(\prime)}}$$

Now the mixing parameters (F_0 F_8 θ_0 θ_8)are calculated in terms of the chiral low energy constants up to NNLO !

First complete NNLO calculation of eta-eta' mixing !

Eta-eta' mixing in ChPT in literature

Some previous (partial) lower-order calculations:

LO in δ-expansion, Georgi, PRD'94

LO in p^2 and NLO in $1/N_c$, Peris, PLB'94.

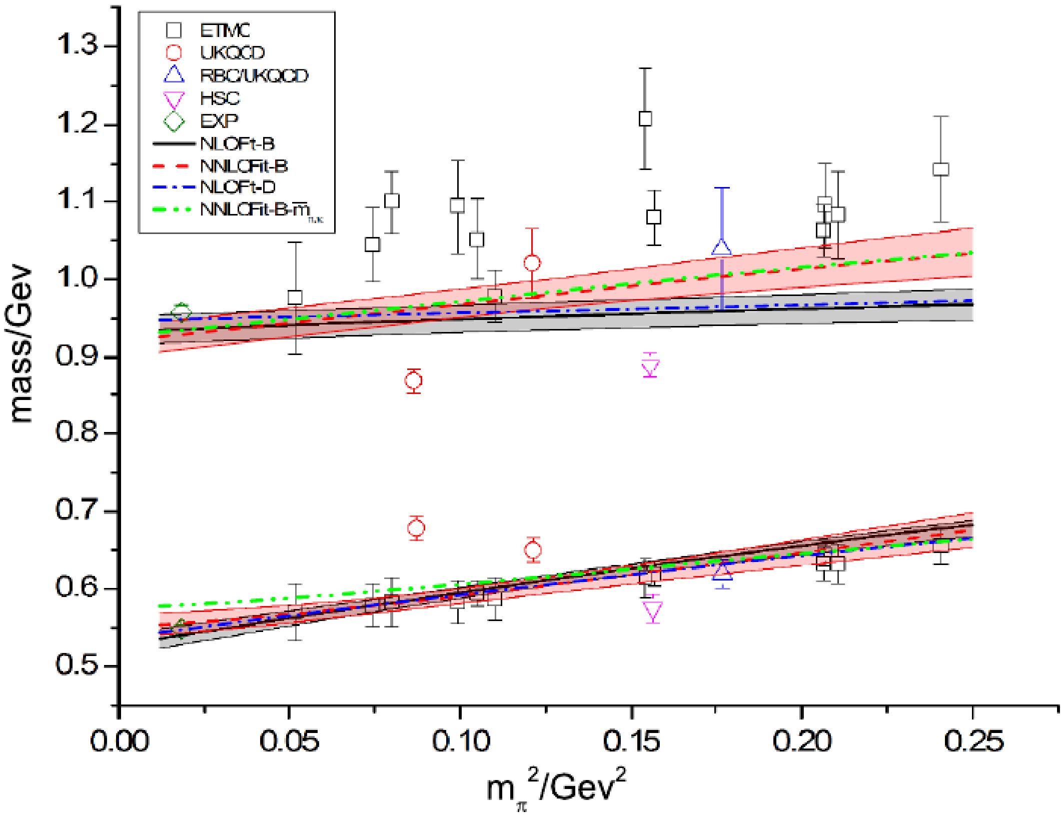
NLO in p^2, LO in $1/N_c$, Gerard, Kou, PLB'05; Degrande, Gerard JHEP'09; Mathieu, Vento PLB'10.

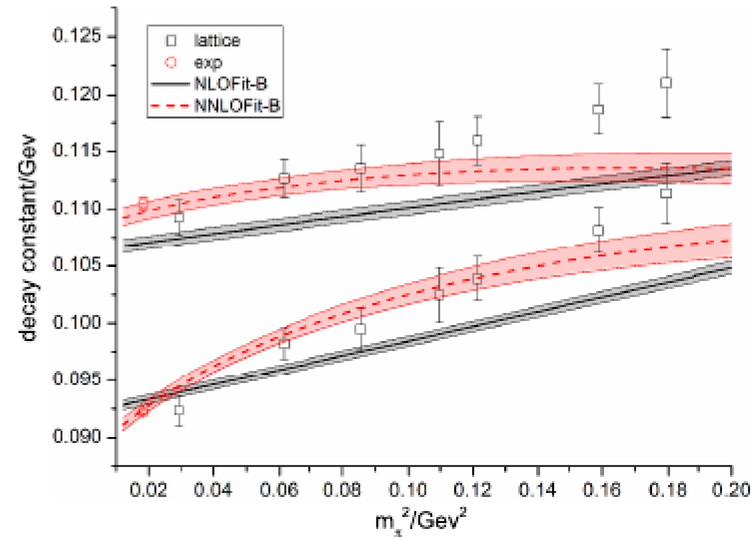
NLO in p^2 and $1/N_c$, Herrera-Siklody *et al.*, PLB'98

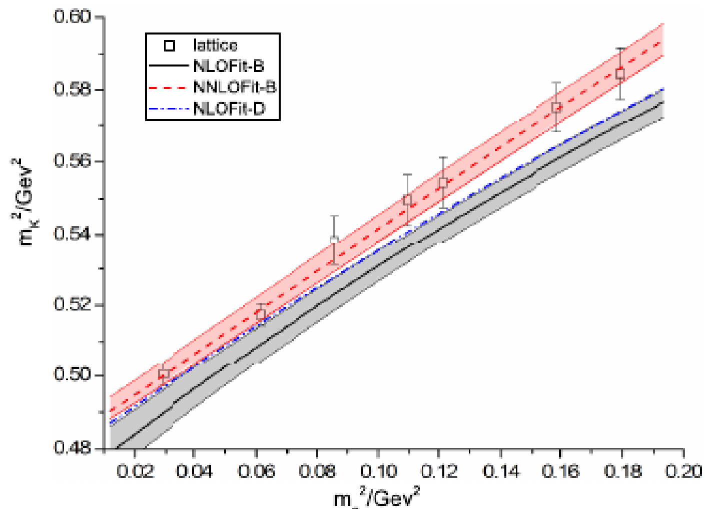
One-loop calculation+resonance exchange (partial NNLO), Z.H.Guo,Oller, PRD'11

FKS formalism, Feldmann,Kroll,Stech,PRD'98

etc







[X.K.Guo, ZHG, Oller, Sanz-Cillero, JHEP'15]

Nice reproduction of the lattice results with reasonable chiral LECs !

S-wave meson-meson scattering and Scalar resonances

ChPT is reliable at threshold energy region.

E.g. Leading order amplitudes of pi-eta, K-Kbar, pi-eta' scattering

$$T_{J=0}^{I=1,\pi\eta\to\pi\eta}(s)^{(2)} = \frac{(c_\theta - \sqrt{2}s_\theta)^2 m_\pi^2}{3F_\pi^2},$$

$$T_{J=0}^{I=1,\pi\eta\to K\bar{K}}(s)^{(2)} = \frac{c_\theta(3m_\eta^2 + 8m_K^2 + m_\pi^2 - 9s) + 2\sqrt{2}s_\theta(2m_K^2 + m_\pi^2)}{6\sqrt{6}F_\pi^2},$$

$$T_{J=0}^{I=1,\pi\eta\to\pi\eta'}(s)^{(2)} = \frac{(\sqrt{2}c_\theta^2 - c_\theta s_\theta - \sqrt{2}s_\theta^2)m_\pi^2}{3F_\pi^2},$$

$$T_{J=0}^{I=1,K\bar{K}\to K\bar{K}}(s)^{(2)} = \frac{s}{4F_\pi^2},$$

$$T_{J=0}^{I=1,K\bar{K}\to\pi\eta'}(s)^{(2)} = \frac{s_\theta(3m_{\eta'}^2 + 8m_K^2 + m_\pi^2 - 9s) - 2\sqrt{2}c_\theta(2m_K^2 + m_\pi^2)}{6\sqrt{6}F_\pi^2},$$

$$T_{J=0}^{I=1,\pi\eta'\to\pi\eta'}(s)^{(2)} = \frac{(\sqrt{2}c_\theta + s_\theta)^2 m_\pi^2}{3F_\pi^2},$$

There is no resonance in such amplitudes !

[ZHG,Oller, PRD’11] [ZHG,Oller,Ruiz de Elvira PRD’12,PLB’12]

[ZHG,Liu,Meissner,Oller,Rusetsky,PRD’17]

Unitarization: Algebraic approximation of N/D (a variant version of K-matrix)

$$T_J(s) = \frac{N(s)}{1 + G(s)\ N(s)}$$

- **The s-channel unitariy is exact. The crossed-channel dyanmics is included in a perturbative manner.**

- **Unitarity condition:** $\mathrm{Im}G(s) = -\rho(s)$

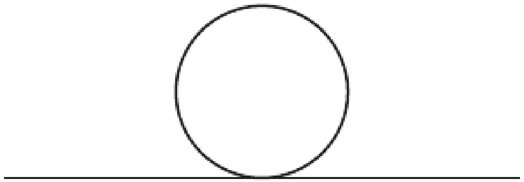
$$G(s) = \boxed{a^{SL}(s_0)} - \frac{s - s_0}{\pi} \int_{4m^2}^{\infty} \frac{\rho(s')}{(s' - s)(s' - s_0)} ds'$$

- ***N*(*s*): given by the partial wave chiral amplitudes**

$$\mathcal{V}^{(S,I)}_{J,D_1\phi_1\to D_2\phi_2}(s) = \frac{1}{2}\int_{-1}^{+1} \mathrm{d}\cos\varphi P_J(\cos\varphi) V^{(S,I)}_{D_1\phi_1\to D_2\phi_2}(s, t(s,\cos\varphi))$$

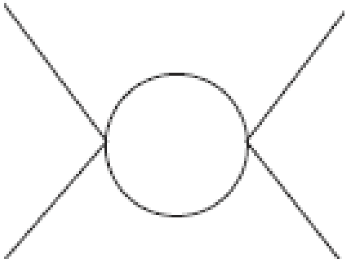
For eta-eta' mixing, temperatures enter via the loop funcionts

$$A_0(m^2) = -m^2 \ln\frac{m^2}{\mu^2} - \int_0^\infty dp\, \frac{8p^2}{E_p}\frac{1}{e^{\frac{E_p}{T}}-1}$$



For meson-meson scattering, temperatures enter via G(s)

$$\begin{aligned} G(s) &= i\int \frac{\mathrm{d}^4 q}{(2\pi)^4}\frac{1}{(q^2-m_1^2)[(P-q)^2-m_2^2]} \\ &= i\int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\int \frac{dq_0}{2\pi}\frac{1}{q_0^2-E_1^2}\frac{1}{(P_0-q_0)^2-E_2^2} \end{aligned}$$



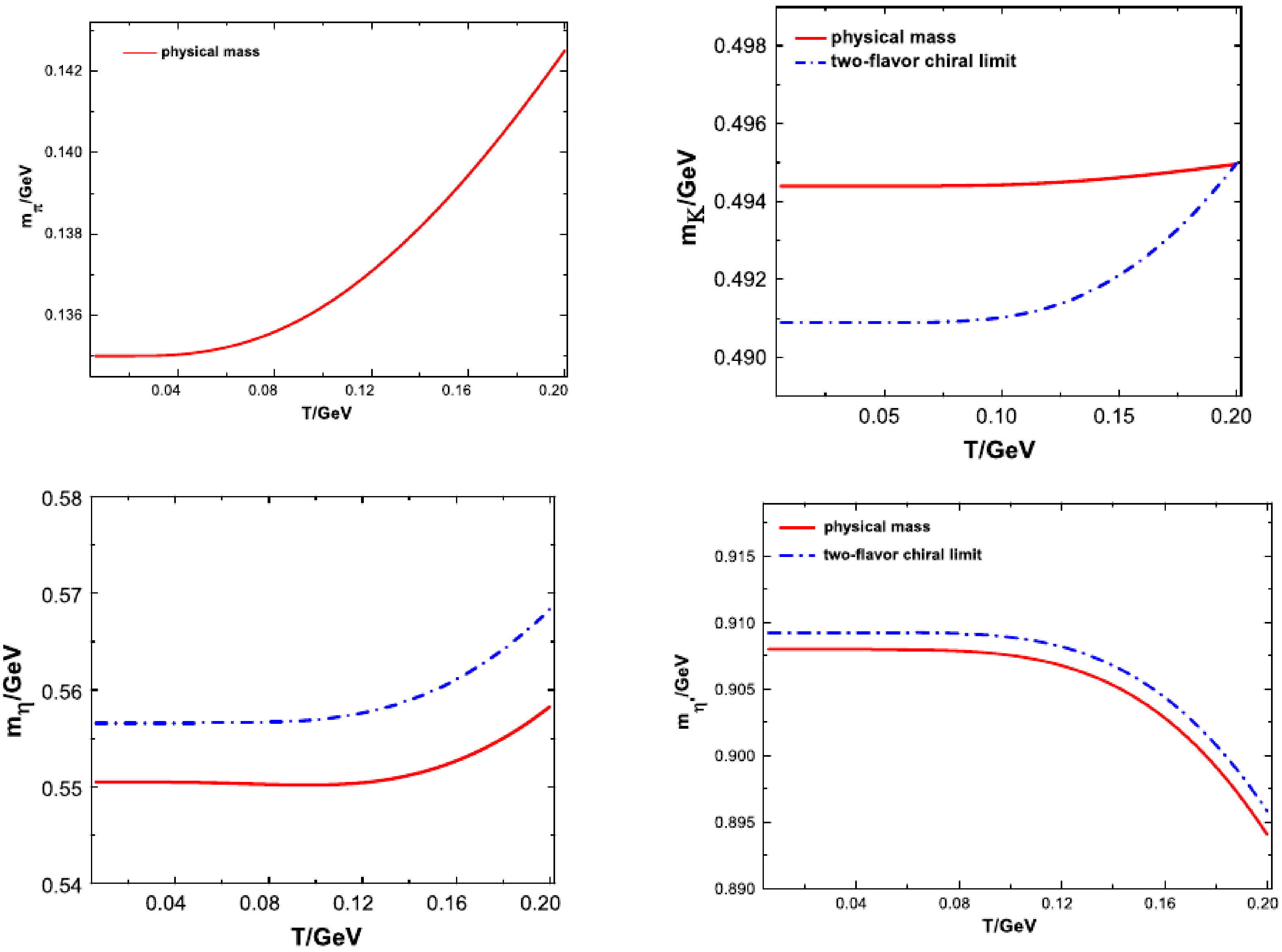
$$q_0 \to i\omega_n = i2\pi nT\,, \quad dq_0 \to i2\pi T$$

$$= \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\sum_{n=-\infty}^{n=+\infty} T\frac{1}{\omega_n^2+E_1^2}\frac{1}{(P_0-i\omega_n)^2-E_2^2}$$

Standard Matsubara techniques to calculate G(s) .

Be careful about the complex extrapolation !

Thermal behaviors of light pseudoscalar mesons and scalar resonances at low temperatures



[Gu, Duan, ZHG, PRD'18]

Relevant channels to study σ, $f_0(980)$, κ, $a_0(980)$

- IJ=10: $\pi\eta$, $K\bar{K}$, $\pi\eta'$

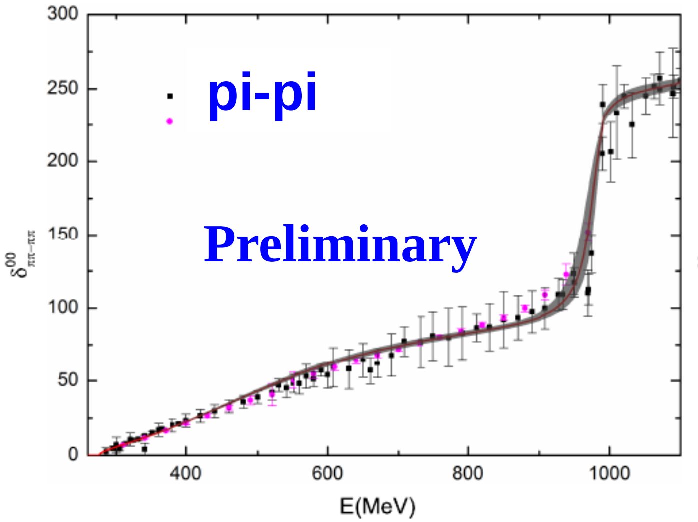
  $a_0(980)$

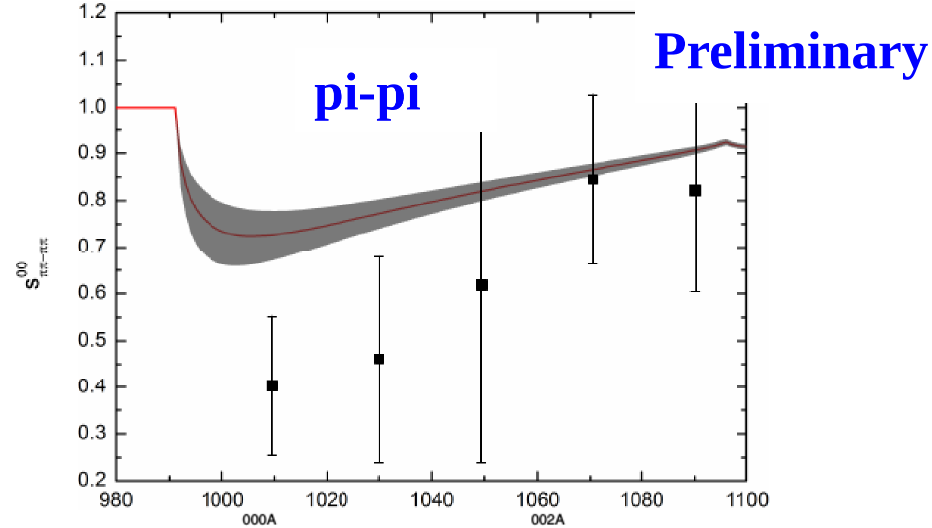
- IJ=$\frac{1}{2}$0 : $K\pi$, $K\eta$, $K\eta'$

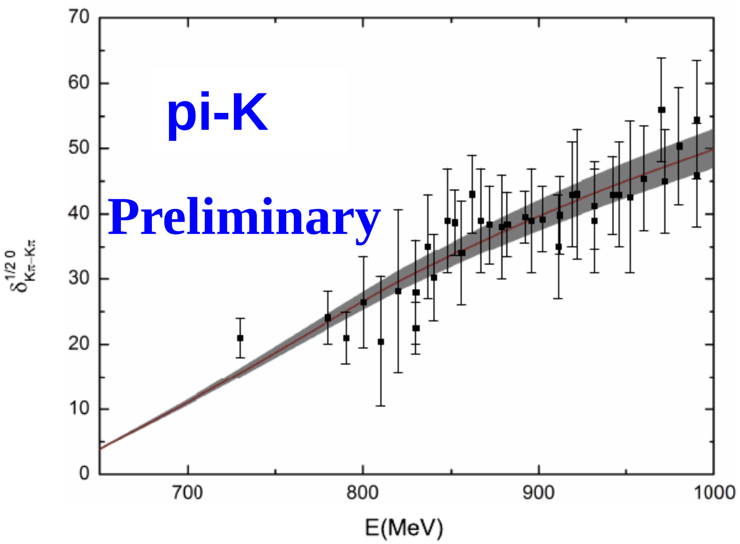
  κ

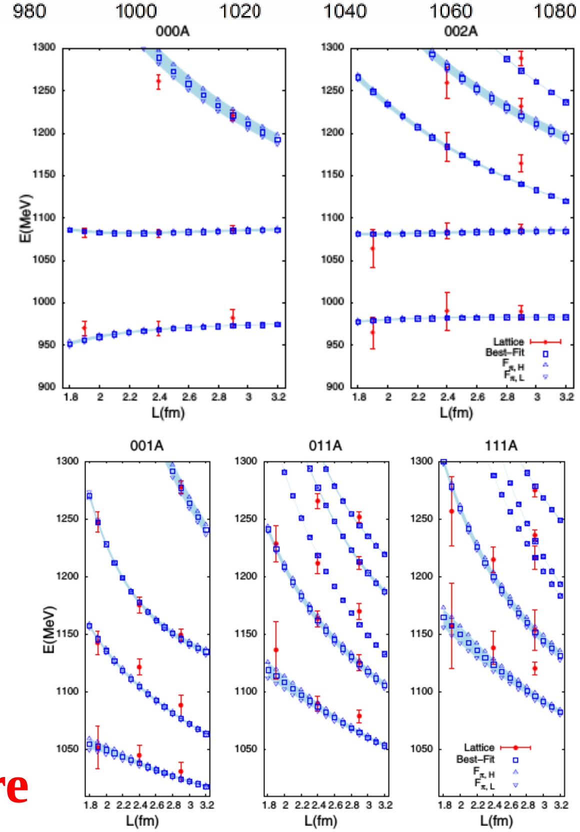
- IJ=00: $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$

  σ, $f_0(980)$









pi-eta

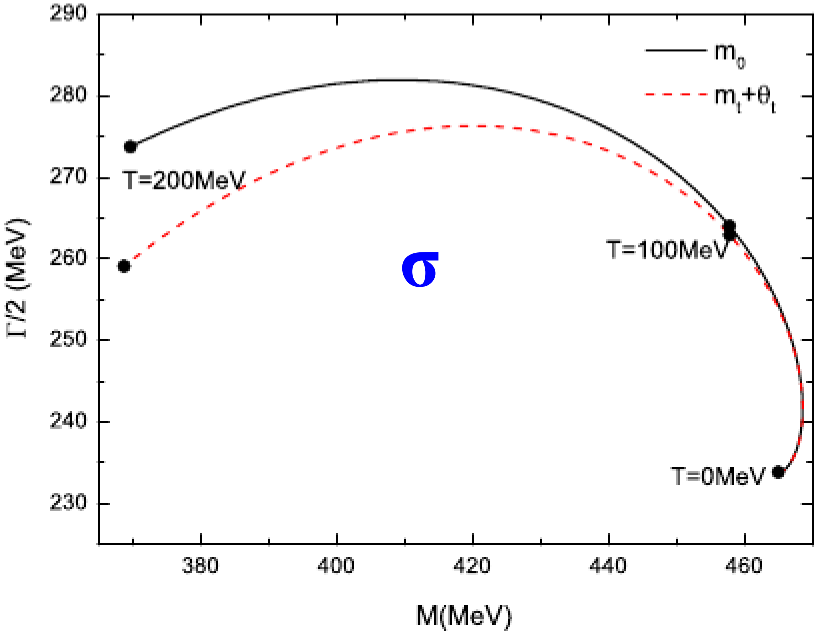
[ZHG, et. al.,PRD' 17]

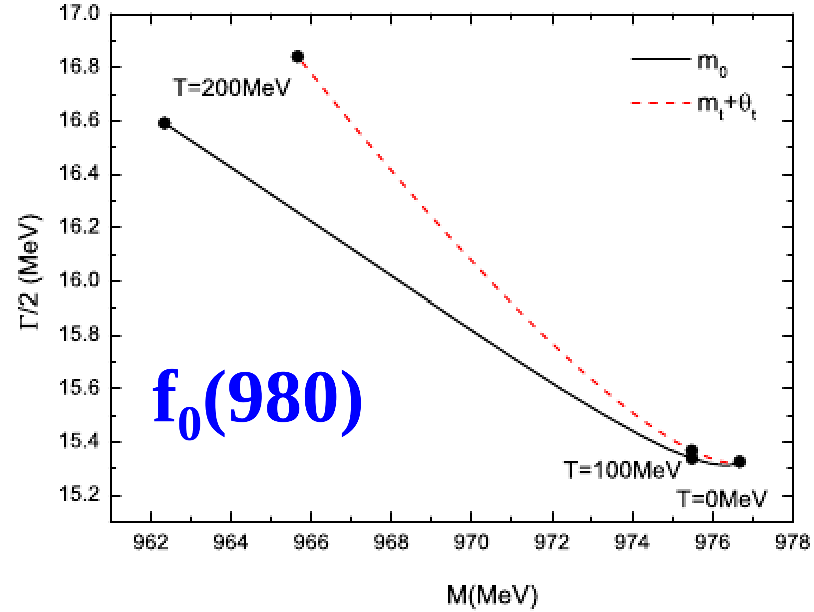
Lattice data

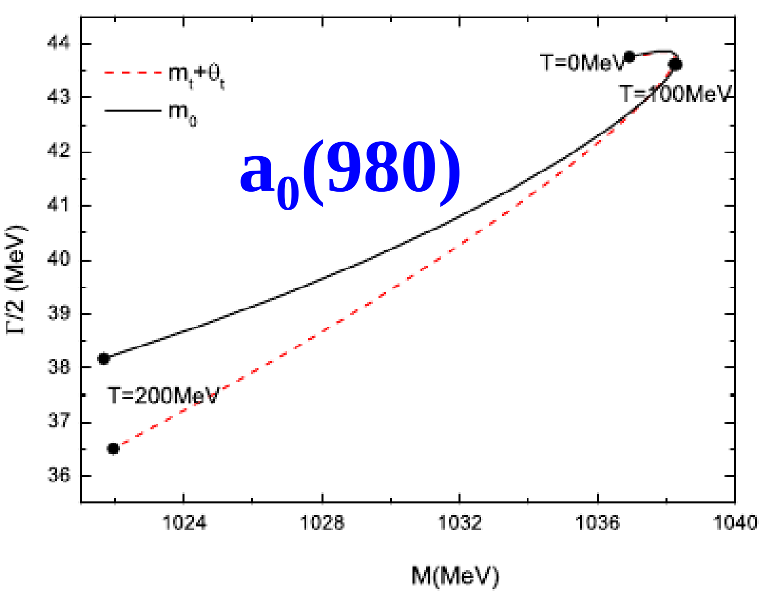
[Dudek, et. al.,PRD'16]

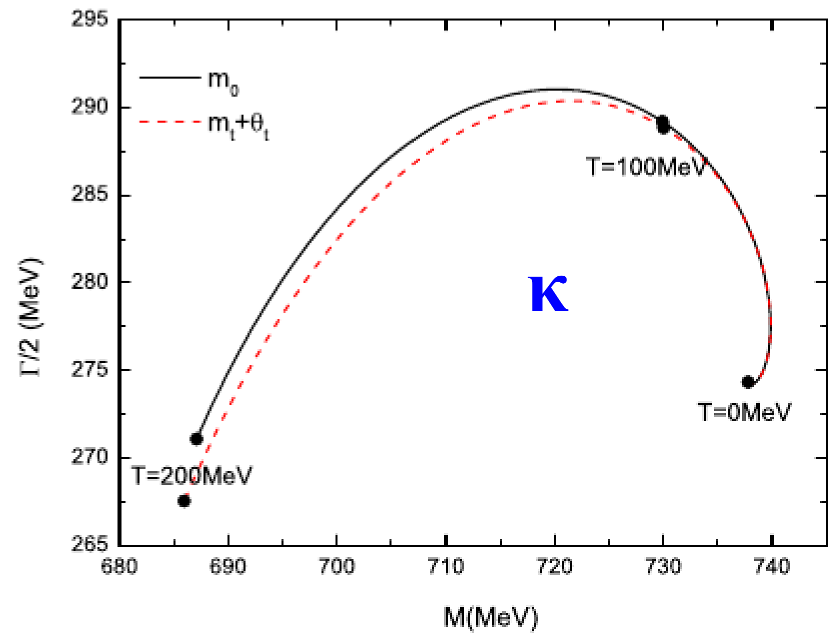
Preliminary fits to Exp and Lattice scattering observables at zero temperature

Preliminary results









Summary

• Chiral perturbation theory provides a reliable framework to study the interactions of pi, K, eta and eta' and also the possible resonances in their scattering at zero temperature.

• The finite-temperature effects are pure predictions in chiral perturbation theory.

• Our preliminary results show interesting thermal behaviors of light scalar resonances of QCD !

• This approach can be straightforwardly extended to other systems and can be a useful tool to study the thermal behaviors of resonances !

谢谢大家！