Spectrum of the fully-heavy tetraquark state $QQ\bar{Q}'\bar{Q}'$

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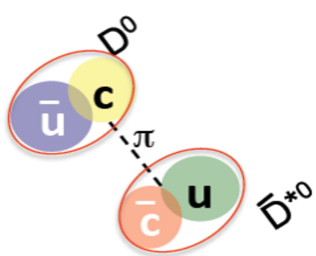
Background

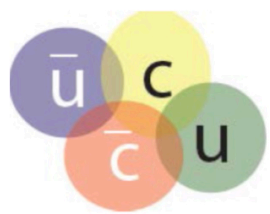
- Since the discovery of X(3872) in 2003, numerous exotic structures “XYZ” have been observed in experiments.

C. Z. Yuan, Int.J.Mod.Phys. A33,1830018

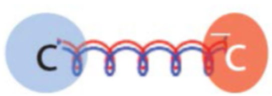
- The theoretical interpretations of “XYZ” include the loosely bound molecular states, the compact tetraquark states, and the hybirds, etc.



molecule



tetraquark



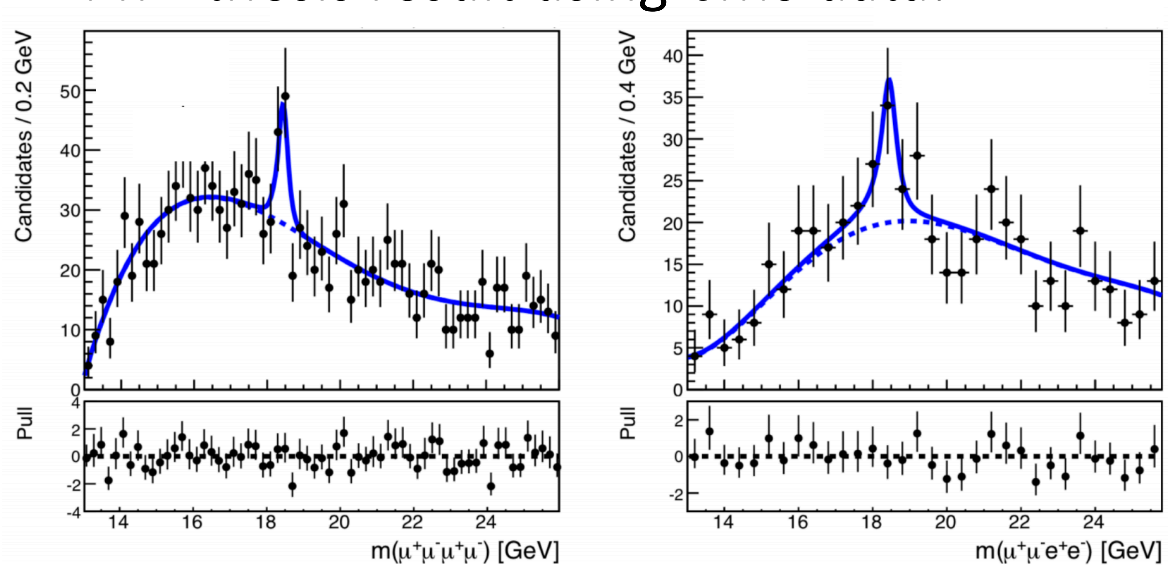
hybird

…

H.X. Chen *et al.*, Phys. Rept. 639, 1
F. K. Guo *et al.*, Rev. Mod. Phys., 015004
Y. R. Liu *et al.*, Prog.Part.Nucl.Phys. 107 237-320

Motivation

- PhD thesis result using CMS data:



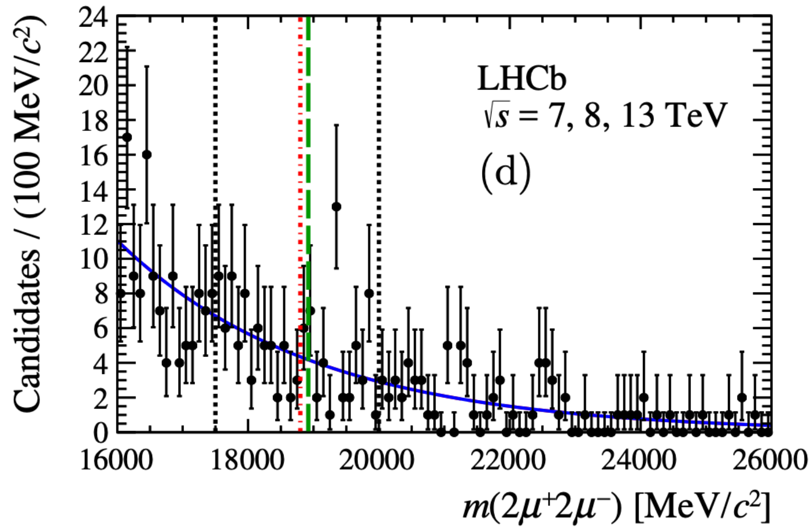
✓ Best mass : 18.4 ± 0.1 (stat.) ± 0.2 (syst.) GeV

✓ $\mathrm{M}(bb\bar{b}\bar{b}) < 2M(\Upsilon(1S))$

✓ Global significance was 3.6σ

JHEP 1705, 013 (2017)
S. Durgut (CMS), Search for Exotic Mesons at CMS (2018), http://meetings.aps.org/Meeting/APR18/Session/U09.6

- LHCb:



✓ No significant excess is found for $X_{bb\bar{b}\bar{b}}$ in the mass range (17.5-20.0) GeV.

JHEP 1810, 086 (2018)

Motivation

- Theoretical works: existence of the stable fully-heavy tetraquark state

✓ Positive: $bb\bar{b}\bar{b} \sim 18-20$ GeV, $cc\bar{c}\bar{c} \sim 5-7$ GeV:

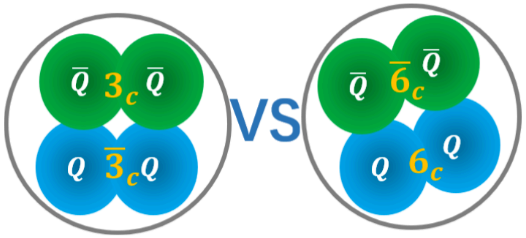
arXiv:1612.00012, Eur. Phys. J. C 78, 647, EPJ Web Conf. 182, 02028,
Phys. Lett. B 718, 545, Phys. Rev. D 70, 014009 ...

✓ Negative: no stable $QQ\bar{Q}\bar{Q}$ states exist.

Phys. Rev. D 97, 094015, Phys. Rev. D.97.054505, arXiv:1901.02564 ...

- A fully-heavy tetra-quark state:

✓ Two color configurations: $\bar{3}_c \otimes 3_c = 1_c$ and $6_c \otimes \bar{6}_c = 1_c$.
✓ Short-range one-gluon-exchange (OGE) potential dominates.
✓ A good candidate for compact tetraquark state.
✓ Nonrelativistic quark model.



Quark model

- Model I

$$V_{ij}(r_{ij}) = \frac{\lambda_i}{2}\frac{\lambda_j}{2}\left(V_{\text{coul}} + V_{\text{conf}} + V_{\text{hyp}} + V_{\text{cons}}\right)$$

$$= \frac{\lambda_i}{2}\frac{\lambda_j}{2}\left(\frac{\alpha_s}{r_{ij}} - \frac{3b}{4}r_{ij} - \frac{8\pi\alpha_s}{3m_i m_j}\mathbf{s}_i \cdot \mathbf{s}_j e^{-\tau^2 r^2}\frac{\tau^3}{\pi^{3/2}} + V_{\text{cons}}\right)$$

The running coupling constant

C. Y. Wong *et.al.,* Phys. Rev. C 65, 014903

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln(A + Q^2/B^2)}$$

- Model II

For antiquark $-\overrightarrow{\lambda}^*$

$$V_{ij}(r_{ij}) = -\frac{3}{16}\sum_{i<j}\lambda_i\lambda_j\left(-\frac{\kappa(1 - \exp(-r_{ij}/r_c))}{r_{ij}} + \lambda r_{ij}^p\right.$$

$$\left.-\Lambda + \boxed{\frac{8\pi}{3m_i m_j}\kappa'(1 - \exp(-r_{ij}/r_c))\frac{\exp(-r_{ij}^2/r_0^2)}{\pi^{3/2}r_0^3}\mathbf{s}_i \cdot \mathbf{s}_j}\right),$$

All the mass information

B. Silvestre-Brac, Few Body Syst. 20, 1

Quark model

- The parameters ⟵ Mass spectra of the mesons

TABLE I. The values of parameters in quark model I and model II.

Model I			m_c[GeV]	m_b[GeV]	b[GeV^2]	τ[GeV]	$V_{\rm cons}$[GeV]	A	B[GeV]	
			1.776	5.102	0.18	0.897	0.62	10	0.31	
Model II	p	r_c	m_c[GeV]	m_b[GeV]	κ	κ'	λ[GeV^2]	Λ[GeV]	A[GeV^{B-1}]	B
	1	0	1.836	5.227	0.5069	1.8609	0.1653	0.8321	1.6553	0.2204

TABLE II. The mass spectra of the heavy quarkonia in units of MeV. The $M_{\rm ex}$, M^{I}_{th}, and M^{II}_{th} refer to the mass spectra of mesons from PDG, in model I , and in model II, respectively.

	$M_{\rm ex}$	M^{I}_{th}	M^{II}_{th}		$M_{\rm ex}$	M^{I}_{th}	M^{II}_{th}
B_c	6274.9	6319.4	6293.5				
η_c	2983.9	3056.5	3006.6	η_b	9399.0	9497.8	9427.9
$\eta_c(2S)$	3637.6	3637.6	3621.2	$\Upsilon(1S)$	9460.30	9503.6	9470.4
J/ψ	3096.9	3085.1	3102.1	$\Upsilon(2S)$	10023.26	9949.7	10017.8
$\psi(2S)$	3686.1	3652.4	3657.8	$\Upsilon(3S)$	10355.2	10389.8	10440.6

Phys.Rev.D 98, 030001

Wave function

- Wave function of tetraquark state: No. of basis $N^3 = 2^3$.

$$\psi_{JJ_z} = \sum \left[\varphi_{n_a J_a}(\mathbf{r}_{12}, \beta_a) \otimes \varphi_{n_b J_b}(\mathbf{r}_{34}, \beta_b) \otimes \phi_{NL_{ab}}(\mathbf{r}, \beta)\right]_{JJ_z},$$

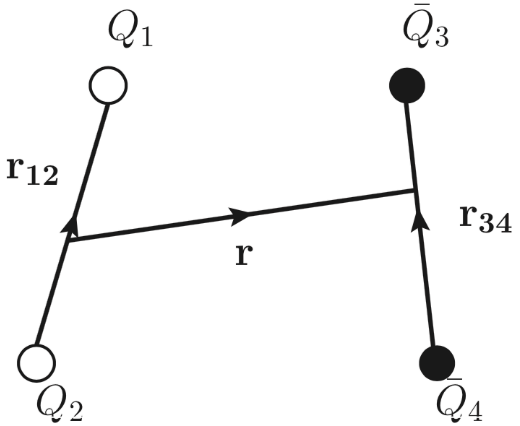
- Basic wave function of each Jacobi coordinate

$$\varphi_{n_a J_a M_a} = \left[\phi_{n_a l_a}(\mathbf{r}_{12}, \beta_a)\chi_{s_a}\right]^{J_a}_{M_a} \chi_f \chi_{c_a}$$

$\chi_{s,f,c}$: the wave function in the spin, flavor, and color space.

- Gaussian function:

$$\phi_{n_a l_a m_a}(\mathbf{r}_{12}, \beta_a) = i^{l_a} r_{12}^{l_a} \sqrt{\frac{4\pi}{(2l_a+1)!!}} \left(\frac{n_a \beta_a^2}{\pi}\right)^{3/4} \times (2n_a\beta_a^2)^{l_a/2} e^{-r^2\beta_a^2 n_a/2} Y_{l_a m_a}(\mathbf{\Omega}_{12})$$



C. Y. Wong, Phys.Rev.C69,055202; Emiko Hiyama *et.al.,* Prog.Part.Nucl.Phys. 51 223-307

S-wave tetraquark state

- S-wave tetraquark states: $\boldsymbol{J}=\boldsymbol{S}$

✓ The ground S-wave state : $l_a = l_b = L_{ab} = 0$
✓ The couple with higher orbital excitations is neglected.

- Color-flavor-spin configuration of $QQ\bar{Q}'\bar{Q}'$: ⇐ Fermi statistics

- $J^{PC}=0^{++}$

$$\chi_1 = \left[[QQ]^1_{\bar{3}_c}[\bar{Q}\bar{Q}]^1_{3_c}\right]^0_{1_c},$$
$$\chi_2 = \left[[QQ]^0_{6_c}[\bar{Q}\bar{Q}]^0_{\bar{6}_c}\right]^0_{1_c}.$$

- $J^{PC}=1^{+-}$

$$\chi_1 = \left[[QQ]^1_{\bar{3}_c}[\bar{Q}\bar{Q}]^1_{3_c}\right]^1_{1_c}.$$

- $J^{PC}=2^{++}$

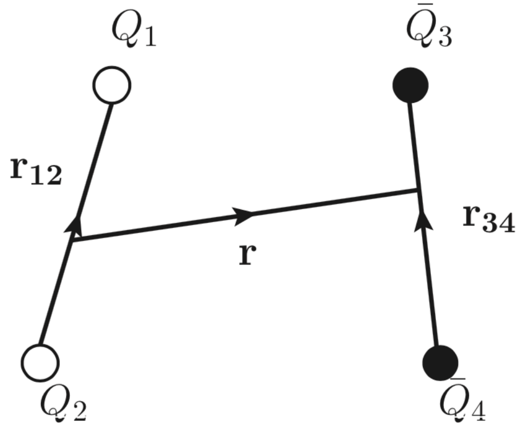
$$\chi_1 = \left[[QQ]^1_{\bar{3}_c}[\bar{Q}\bar{Q}]^1_{3_c}\right]^2_{1_c}.$$

Hamiltonian

$$H = \sum_{i=1}^{4} \frac{p_j^2}{2m_j} + \sum_i m_i + \sum_{i<j} V_{ij} = \frac{p^2}{2u} + V_I + h_{12} + h_{34},$$

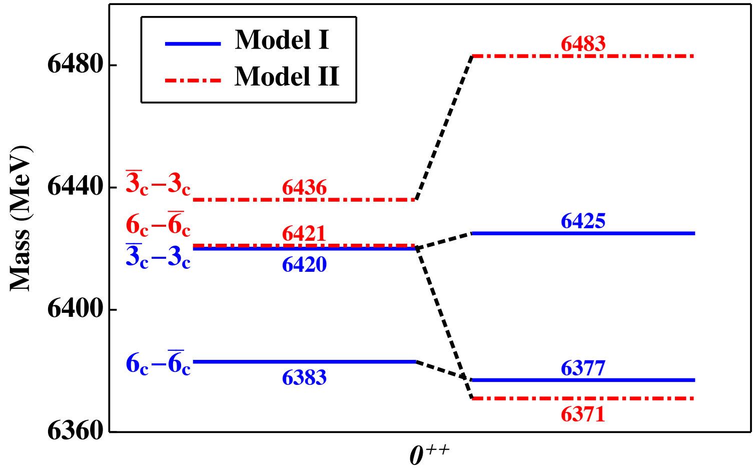
With

$$V_I = V_{13} + V_{14} + V_{23} + V_{24},$$
$$h_{ij} = \frac{p_{ij}^2}{2u_{ij}} + V_{ij} + m_i + m_j,$$

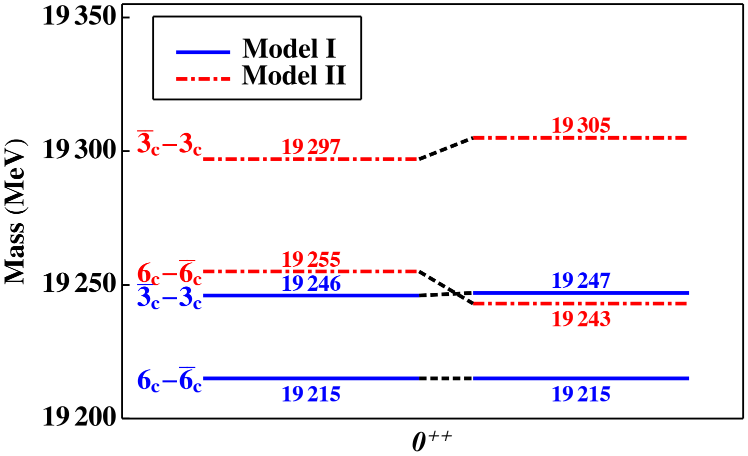


- h_{12}/h_{34} : diagonal in the color-flavor-spin space.
- V_I :the mixing between different color-spin-flavor configurations.
- Solving the Schrödinger equation by variational method.

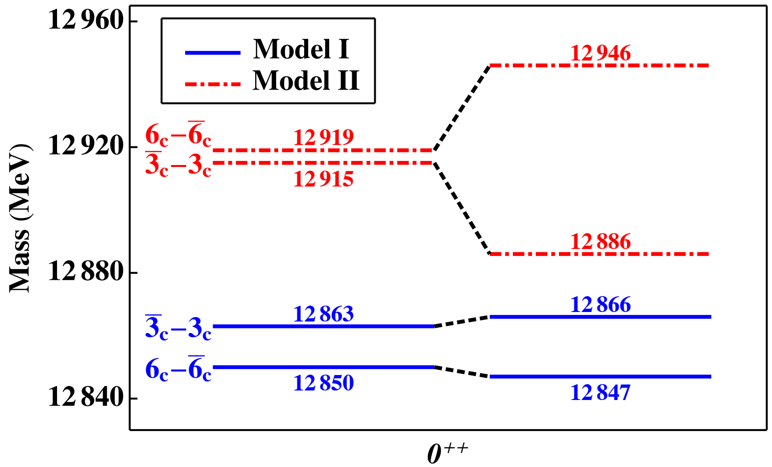
$J^{PC} = 0^{++}$



$cc\bar{c}\bar{c}$



$bb\bar{b}\bar{b}$



$bb\bar{c}\bar{c}$

- 0^{++} state: an admixture of $\bar{3}_c \otimes 3_c$ and $6_c \otimes \bar{6}_c$ configurations.

- The two quark models lead to similar mass spectra up to tens of MeV.

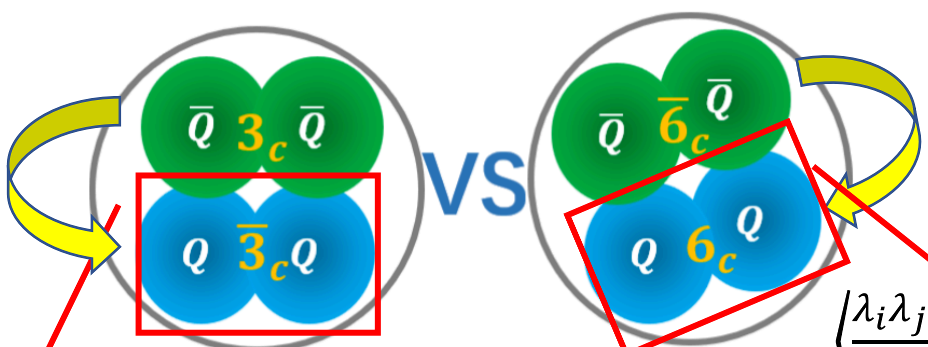
The left (right) half: without (with) mixing between $\bar{3}_c \otimes 3_c$ and $6_c \otimes \bar{6}_c$

$$J^{PC} = 0^{++}$$

- $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$: M($\bar{3}_c \otimes 3_c$)> M($6_c \otimes \bar{6}_c$) in two quark models.

- $bb\bar{c}\bar{c}$: M($\bar{3}_c \otimes 3_c$)> M($6_c \otimes \bar{6}_c$) in model I;
  M($\bar{3}_c \otimes 3_c$)< M($6_c \otimes \bar{6}_c$) in model II;

$-\frac{1}{3}$, attractive



$-\frac{5}{6}$, attractive

Compete

$\left\langle \frac{\lambda_i \lambda_j}{2} \right\rangle = \frac{1}{3}$, repulsive

$\left\langle \frac{\lambda_i \lambda_j}{2} \right\rangle = -\frac{2}{3}$, attractive

$J^{PC} = 0^{++}$

- The mixture:

$J^{PC}=0^{++}$	Model I	M [GeV]	$\bar{3}_c \otimes 3_c$	$6_c \otimes \bar{6}_c$	Model II	M [GeV]	$\bar{3}_c \otimes 3_c$	$6_c \otimes \bar{6}_c$
$cc\bar{c}\bar{c}$	$\beta_a=\beta_b=0.4,\ \beta=0.6$	6.377	11%	89%	$\beta_a=\beta_b=0.5,\ \beta=0.7$	6.371	43%	57%
	$\gamma_a=\gamma_b=0.4,\ \gamma=0.7$	6.425	89%	11%	$\gamma_a=\gamma_b=0.5,\ \gamma=0.8$	6.483	57%	43%
$bb\bar{b}\bar{b}$	$\beta_a=\beta_b=0.7,\ \beta=0.9$	19.215	1%	99%	$\beta_a=\beta_b=0.9,\ \beta=1.1$	19.243	17%	83%
	$\gamma_a=\gamma_b=0.7,\ \gamma=0.9$	19.247	99%	1%	$\gamma_a=\gamma_b=0.8,\ \gamma=1.2$	19.305	83%	17%
$bb\bar{c}\bar{c}$	$\beta_a=0.6, \beta_b=0.5, \beta=0.7$	12.847	14%	86%	$\beta_a=0.7, \beta_b=0.5, \beta=0.8$	12.886	53%	47%
	$\gamma_a=0.6, \gamma_b=0.4, \gamma=0.9$	12.866	86%	14%	$\gamma_a=0.7, \gamma_b=0.5, \gamma=0.9$	12.946	47%	53%

- $6_c \otimes \bar{6}_c$ is important even dominant in the ground state.

- The proportions in the two models are quite different. The mixing is more stronger in model II.

- S-wave tetraquark state $Q_1Q_2\bar{Q}\bar{Q}$:

✓ Orthogonality of χ_s ⇒ Color interactions do not contribute to the mixing.

✓ Only the hyperfine interaction contributes to the couple-channel effects.

Mixture

- A tetraquark state is an admixture of different color configurations.

- For a $Q_1 Q_2 \bar{Q}_3 \bar{Q}_4$ $(Q_1 \neq Q_2$ & $Q_3 \neq Q_4)$:

$$(\sum_n^4 \lambda_n)^2 |\chi_{i,j}\rangle = 0$$

$$\langle\chi_i|(\lambda_1+\lambda_2)^2|\chi_j\rangle = 0$$

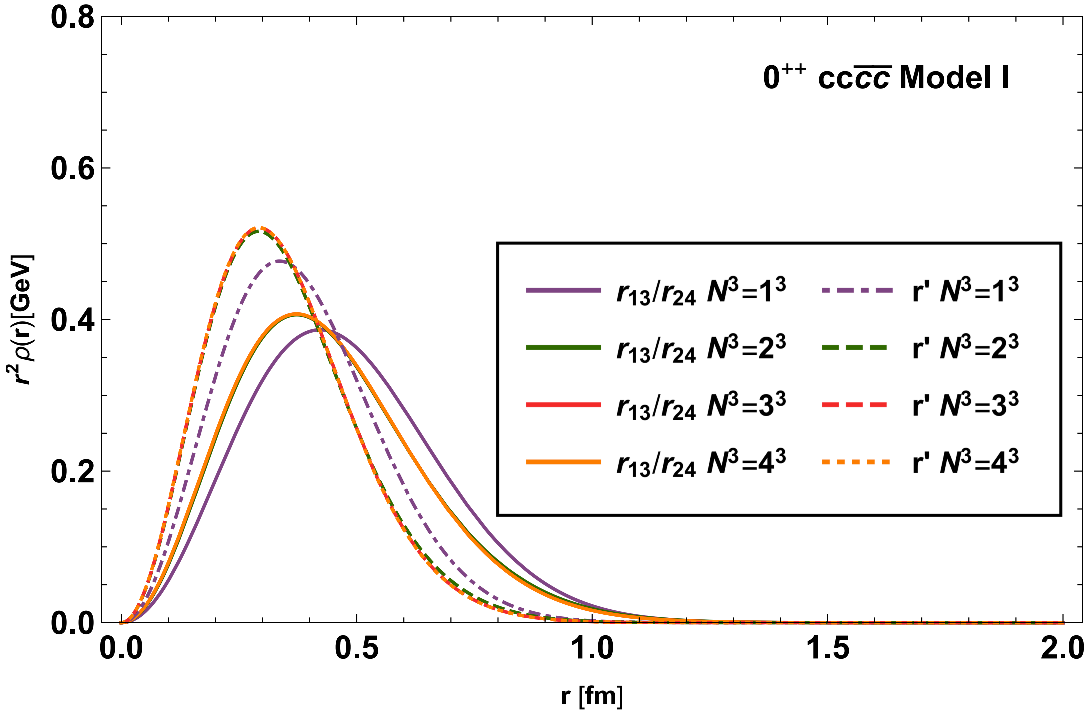
$$\langle\chi_i|(\lambda_3+\lambda_4)^2|\chi_j\rangle = 0$$

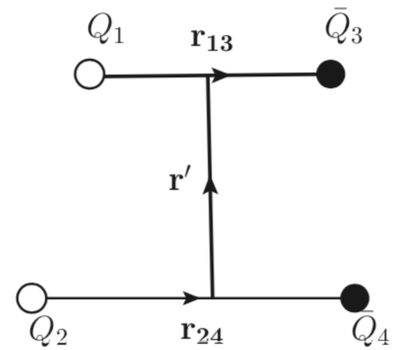
$$\Longrightarrow \quad \langle\chi_i|(\lambda_1+\lambda_2)(\lambda_3+\lambda_4)|\chi_j\rangle = 0$$

	Model I	Model II
OGE Coulomb	$\alpha_s(m_i, m_j)$ ✓	constant κ ✕
Linear confinement	constant b ✕	constant λ ✕
Hyperfine	$\alpha_s(m_i, m_j)$ ✓	constant κ' ✓

Scattering state vs tetraquark state

$J^{PC}=0^{++}$	Model I									
$N^3=2^3$	After mixing	$\bar{3}_c\otimes 3_c$	$6_c\otimes\bar{6}_c$	$1_c\otimes 1_c$	$8_c\otimes 8_c$	$\sqrt{\langle r_{12}^2\rangle}$ fm	$\sqrt{\langle r_{34}^2\rangle}$ fm	$\sqrt{\langle r^2\rangle}$ fm	$\sqrt{\langle r_{13}^2\rangle}$ fm	$\sqrt{\langle r_{24}^2\rangle}$ fm	$\sqrt{\langle r'^2\rangle}$ fm
$cc\bar{c}\bar{c}$	6.377	11%	89%	90%	10%	0.54		0.30	0.49		0.38
$bb\bar{b}\bar{b}$	19.215	1%	99%	75%	25%	0.35		0.19	0.31		0.25
$bb\bar{c}\bar{c}$	12.847	14%	86%	92%	8%	0.39	0.50	0.26	0.41		0.32





- Considerable $8_c\otimes 8_c$
- Small r_{rms}
- Stable spatial extension

Compact tetraquark states

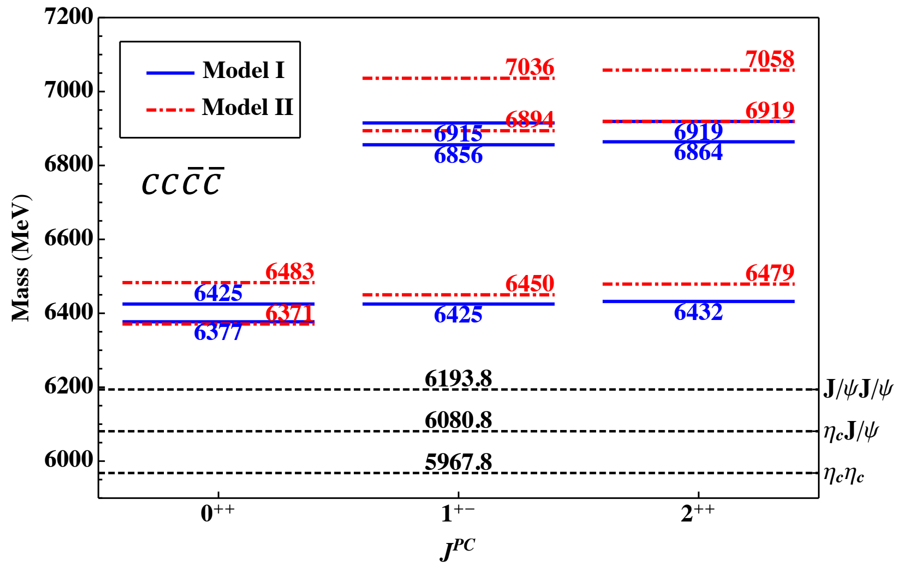
$J^{PC} = 1^{+-}$ and 2^{++}

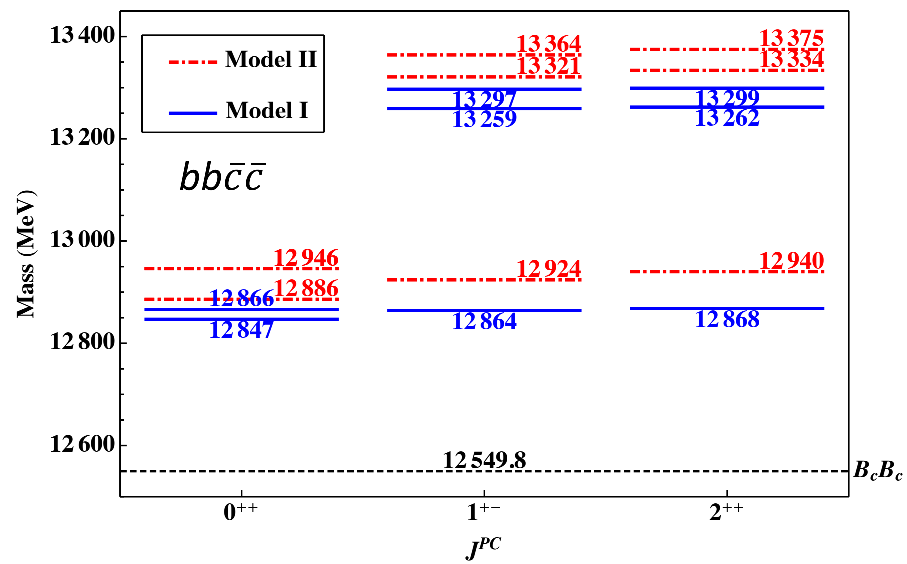
- Only one color configuration: $\bar{3}_c \otimes 3_c$
- The mass spectra of $cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$ and $bb\bar{c}\bar{c}$ states

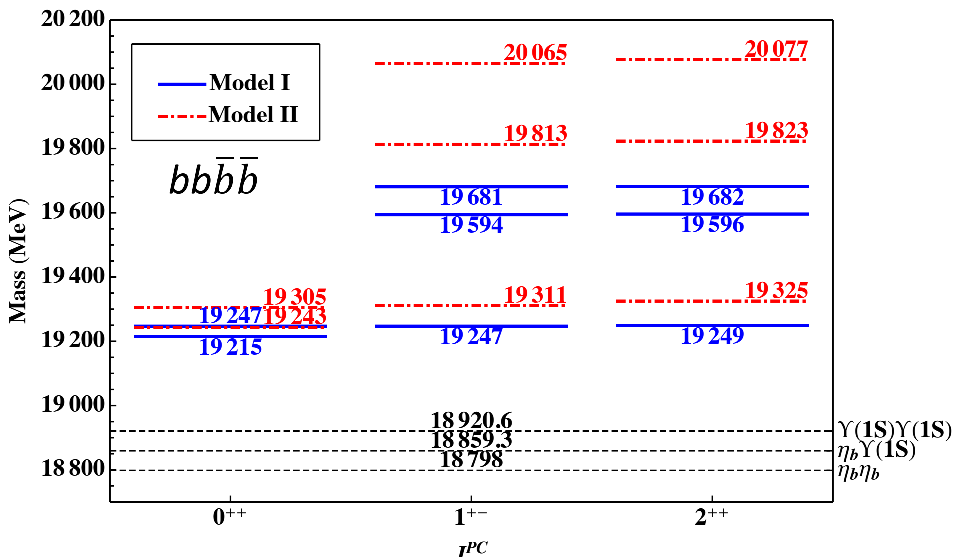
	Model I	nS	$J^{PC}=1^{+-}$	$J^{PC}=2^{++}$	Model II	nS	$J^{PC}=1^{+-}$	$J^{PC}=2^{++}$
$cc\bar{c}\bar{c}$	$\beta_a=0.4$	$1S$	6.425	6.432	$\beta_a=0.5$	$1S$	6.450	6.479
	$\beta_b=0.4$	$2S$	6.856	6.864	$\beta_b=0.5$	$2S$	6.894	6.919
	$\beta=0.6$	$3S$	6.915	6.919	$\beta=0.6$	$3S$	7.036	7.058
$bb\bar{b}\bar{b}$	$\beta_a=0.7$	$1S$	19.247	19.249	$\beta_a=1.0$	$1S$	19.311	19.325
	$\beta_b=0.7$	$2S$	19.594	19.596	$\beta_b=1.0$	$2S$	19.813	19.823
	$\beta=0.9$	$3S$	19.681	19.682	$\beta=1.1$	$3S$	20.065	20.077
$bb\bar{c}\bar{c}$	$\beta_a=0.7$	$1S$	12.864	12.868	$\beta_a=0.7$	$1S$	12.924	12.940
	$\beta_b=0.5$	$2S$	13.259	13.262	$\beta_b=0.5$	$2S$	13.321	13.334
	$\beta=0.7$	$3S$	13.297	13.299	$\beta=0.7$	$3S$	13.364	13.375

- Results in two quark models are similar.
- The mass difference of two states arises from the hyperfine potential.

Numerical results





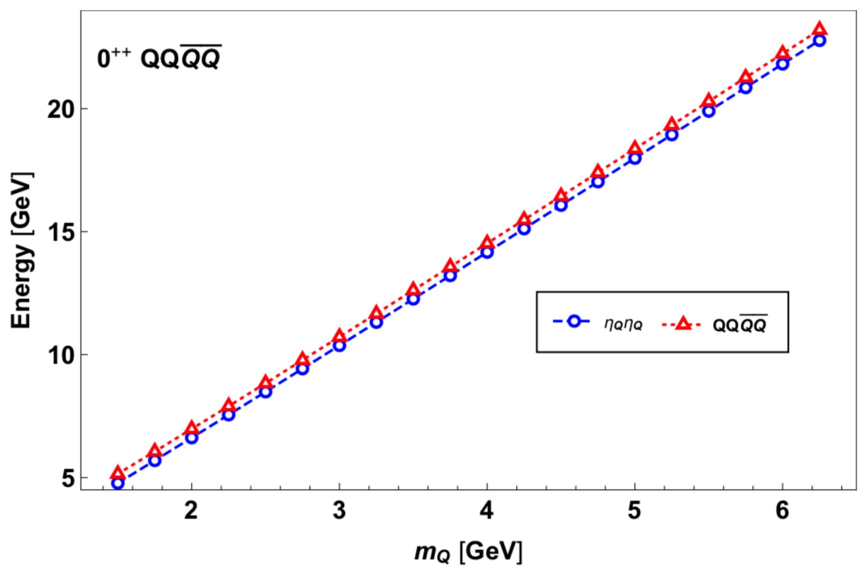


✓ The lowest 0^{++} states are located about 300 ~ 450 MeV above the lowest scattering state.

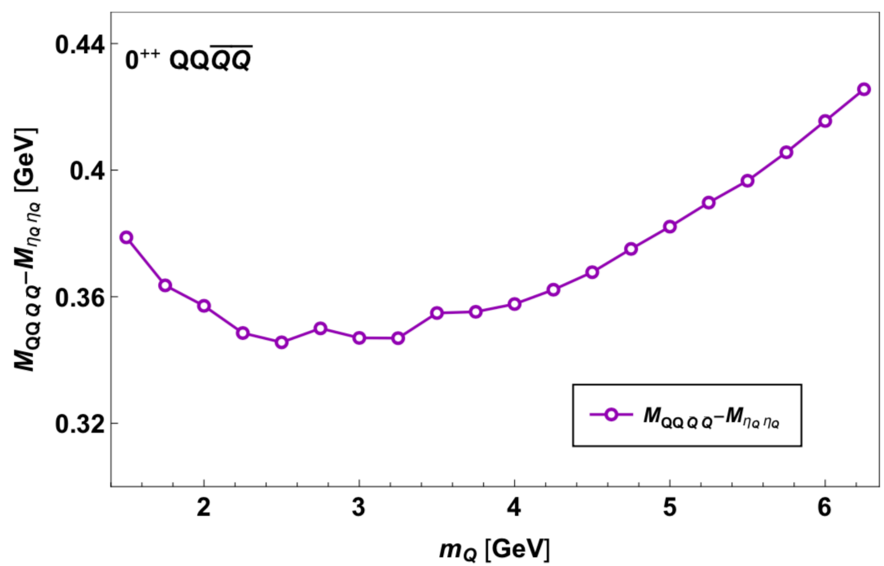
✓ No bound states exist in the two quark models

m_q-denpendence

- The constituent quark mass dependence of the tetraquark spectra



(a)The mass spectra of the tetraquark states $QQ\bar{Q}\bar{Q}$ with $J^{PC} = 0^{++}$.



(b)The mass difference between the tetraquark states and the mass threshold of $\eta_Q\eta_Q$.

- The $\mathrm{M}(QQ\bar{Q}\bar{Q}) > M(\eta_Q\bar{\eta}_Q)$: no bound tetraquark states exist.

Summary and Outlook

➢ The mass spectra of tetraquark states $QQ\bar{Q}'\bar{Q}'$ in two quark models,

• $6_c \otimes \bar{6}_c$ is important even dominant in the ground state.

• Only the hyperfine potential contributes to the mixing between different color configurations.

• No bound $cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$, and $bb\bar{c}\bar{c}$ (or cc$\bar{b}\bar{b}$) states exist in the two quark models.

➢ The extension to the $Q_1Q_2\bar{Q}_3\bar{Q}_4$ state.

➢ The confinement mechanism for multi-quark system need more investigation.

➢ The existence of the tetraquark resonances with narrow decay width.

Thank you for your attention!

Backup slides

Wave function

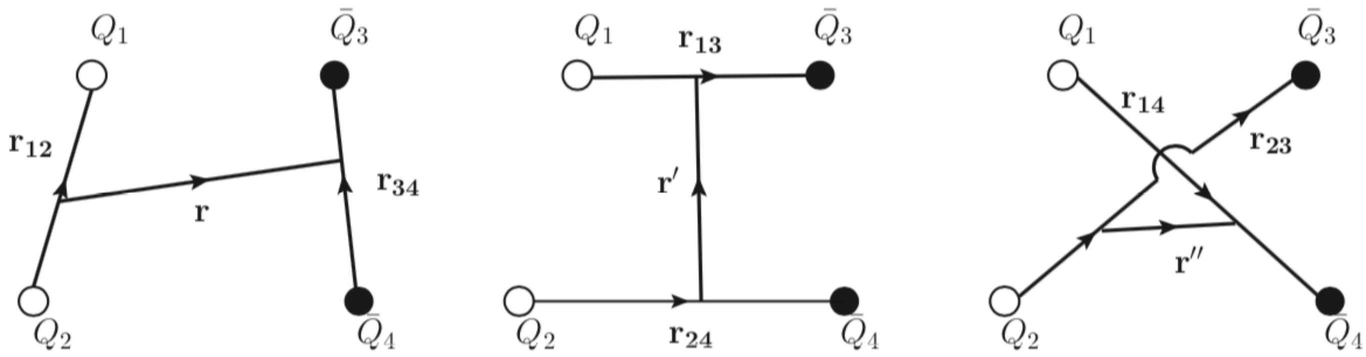


FIG. 1. The Jacobi coordinates in the tetraquark state.

- The Jacobi coordinates transfer as

$$\mathbf{r}_{jk} = \mathbf{r}_j - \mathbf{r}_k = \mathbf{r} + c^a_{jk}\mathbf{r}_{12} + c^b_{jk}\mathbf{r}_{34},$$

$$\mathbf{r} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2} - \frac{m_3\mathbf{r}_3 + m_4\mathbf{r}_4}{m_3 + m_4},$$

$$\mathbf{r}' = \frac{(m_1m_3 - m_2m_4)\mathbf{r} + M_T u_{12}\mathbf{r}_{12} - M_T u_{34}\mathbf{r}_{34}}{(m_1 + m_4)(m_2 + m_3)},$$

$$\mathbf{r}'' = \frac{(m_1m_4 - m_2m_3)\mathbf{r} + M_T u_{12}\mathbf{r}_{12} - M_T u_{34}\mathbf{r}_{34}}{(m_1 + m_3)(m_2 + m_4)},$$

c_{jk}: transform coefficients

- Use the first coordinate configuration.

Numerical results

- The dependence of the mass spectra on the number of the expanding base.

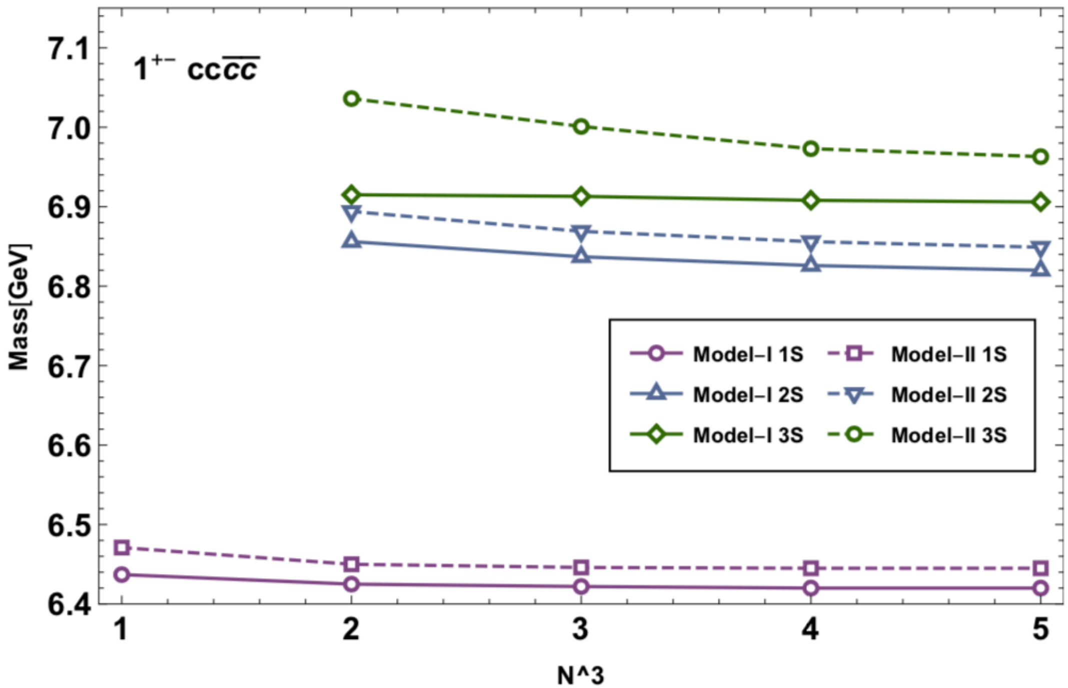


FIG. 2. The dependence of the mass spectrum on the number of Gaussian basis N^3. The line and dashed line represent the numerical results in model I and model II, respectively.

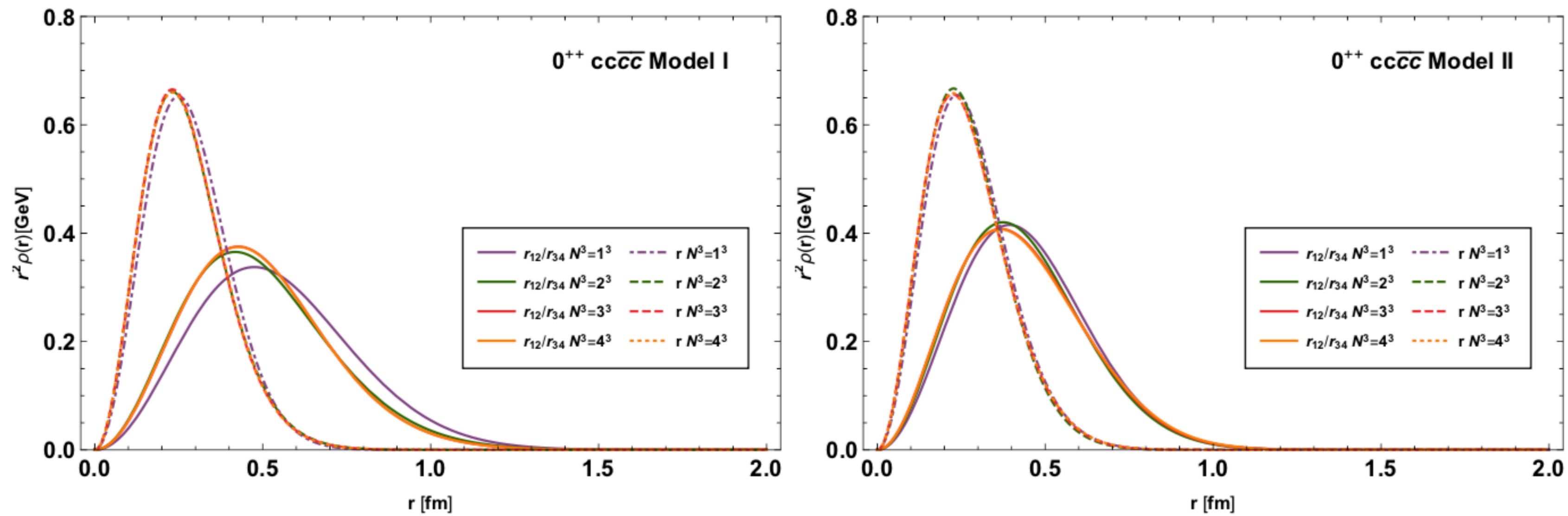
✓ $N^3 = 2^3$

TABLE III. The coefficient c_{ij}.

c^a_{14}	c^a_{13}	c^a_{23}	c^a_{24}	c^b_{14}	c^b_{13}	c^b_{23}	c^b_{24}
$\frac{m_2}{m_1+m_2}$	$\frac{m_2}{m_1+m_2}$	$-\frac{m_1}{m_1+m_2}$	$-\frac{m_1}{m_1+m_2}$	$\frac{m_3}{m_3+m_4}$	$-\frac{m_4}{m_3+m_4}$	$-\frac{m_4}{m_3+m_4}$	$\frac{m_3}{m_3+m_4}$

TABLE IV. The configurations of the diquark (antiquark) constrained by Pauli principle. “S” and “A” represent symmetry and antisymmetry.

$J^P = 1^+$	QQ	$J^P = 0^+$	QQ
S-wave(L=0)	S	S-wave(L=0)	S
Flavor	S	Flavor	S
Spin(S=1)	S	Spin(S=0)	A
Color($\bar{3}_c$)	A	Color(6_c)	S



(a) Model I. (b) Model II.

FIG. 1: The dependence of the root mean square radius $\sqrt{\langle r_{12}\rangle}$ ($\sqrt{\langle r_{34}\rangle}$) and $\sqrt{\langle r\rangle}$ on the extension of the wave function.

$$\rho(r) = \int |\psi(r_{12},\, r_{34},\, r)|^2 d\vec{r}_{12} d\vec{r}_{34} d\hat{\vec{r}}$$

$$\rho(r_{12}) = \int |\psi(r_{12},\, r_{34},\, r)|^2 d\vec{r} d\vec{r}_{34} d\hat{\vec{r}}_{12}$$

TABLE VIII. The comparison of the mass spectra of 0^{++} $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ from Ref. [48] and our results using the same quark model. In the right table, we remove the constrains on the wave functions used in Ref. [48].

	Ref. [48]				without constrains			
$J^{PC}=0^{++}$	$w=0.325$	M [GeV]	$\bar{3}_c\otimes 3_c$	$6_c\otimes\bar{6}_c$		M [GeV]	$\bar{3}_c\otimes 3_c$	$6_c\otimes\bar{6}_c$
$cc\bar{c}\bar{c}$	$\beta_a=\beta_b=0.49,\ \beta=0.69$	6470	66%	34%	$\beta_a=\beta_b=0.4,\ \beta=0.6$	6417	33%	67%
	$\gamma_a=\gamma_b=0.49,\ \gamma=0.69$	6559	34%	66%	$\gamma_a=\gamma_b=0.4,\ \gamma=0.7$	6509	67%	33%
$bb\bar{b}\bar{b}$	$\beta_a=\beta_b=0.88,\ \beta=1.24$	19268	66%	34%	$\beta_a=\beta_b=0.7,\ \beta=0.9$	19226	18%	82%
	$\gamma_a=\gamma_b=0.88,\ \gamma=1.24$	19306	34%	66%	$\gamma_a=\gamma_b=0.7,\ \gamma=0.9$	19268	82%	18%

[48] *M.S. Liu et.al.,* PhysRevD.100.016006.