北航物理学院

SCHOOL OF PHYSICS,BUAA

Gaussian Expansion Method and its application to DK, DDK, DDDK molecules

Tian-Wei Wu (吴天伟)

Collaborator：Ming-Zhu Liu, Li-Sheng Geng, Emiko Hiyama, Manuel Pavon Valderrama

长沙 20190624

OUTLINE

- Introduction to Gaussian Expansion Method
- Application to DK, DDK, DDDK molecules
- Results and Summary

Introduction to Gaussian Expansion Method

1. What's GEM
2. Advantages of GEM

Precise Few-body Method-GEM

GEM is an ab initio basis expansion method to solve Schödinger Equation of Few-body systems by variational principle.

- **Gaussian basis**

$$\phi^G_{nlm}(r) = \phi^G_{nl}(r) Y_{lm}(\hat{r}),$$

$$\phi^G_{nl}(r) = N_{nl} r^l e^{-\nu_n r^2},$$

$$N_{nl} = \sqrt{\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}(2l+1)!!}} \quad (n = 1 - n_{max}).$$

- **GEM parameters**

$$\nu_n = \frac{1}{r_n^2},$$
$$r_n = r_1 a^{n-1} \quad (n = 1 - n_{max}).$$

- **Schödinger Equation and wave function**

$$(H - E)\Psi_{JM} = 0$$

$$\Psi_{JM} = \sum_{n=1}^{n_{max}} C_n^{(J)} \Phi_{JM,n}.$$

- **Eigen equation and matrix elements**

$$\sum_{n'=1}^{n_{max}} (H^{(J)}_{nn'} - E N^{(J)}_{nn'}) C^{(J)}_{n'} = 0,$$

$$H^{(J)}_{nn'} = \langle \Phi_{JM,n} | H | \Phi_{JM,n'} \rangle,$$
$$N^{(J)}_{nn'} = \langle \Phi_{JM,n} | 1 | \Phi_{JM,n'} \rangle.$$

High precision

- **The First ten Hydrogen energy levels**

Table 1: The first ten Energies of H

E_n	GEM/eV	Analytical/eV	Relative error
1	-13.60566399	-13.60566439	-2.9E-8
2	-3.40141599	-3.40141609	-2.9E-8
3	-1.51174039	-1.51174048	-6.4E-8
4	-0.85035538	-0.85035402	-1.5E-7
5	-0.54422639	-0.54422657	-4.3E-7
6	-0.3779342	-0.3779351	-2.23E-6
7	-0.2776640	-0.2776666	-9.08E-6
8	-0.2125854	-0.2125885	-1.46E-5
9	-0.1679680	-0.1679712	-1.88E-5
10	-0.1360548	-0.1360566	-1.34E-5

Obtain ten energy levels in one calculation with high precision.

- **Hydrogen wave functions**

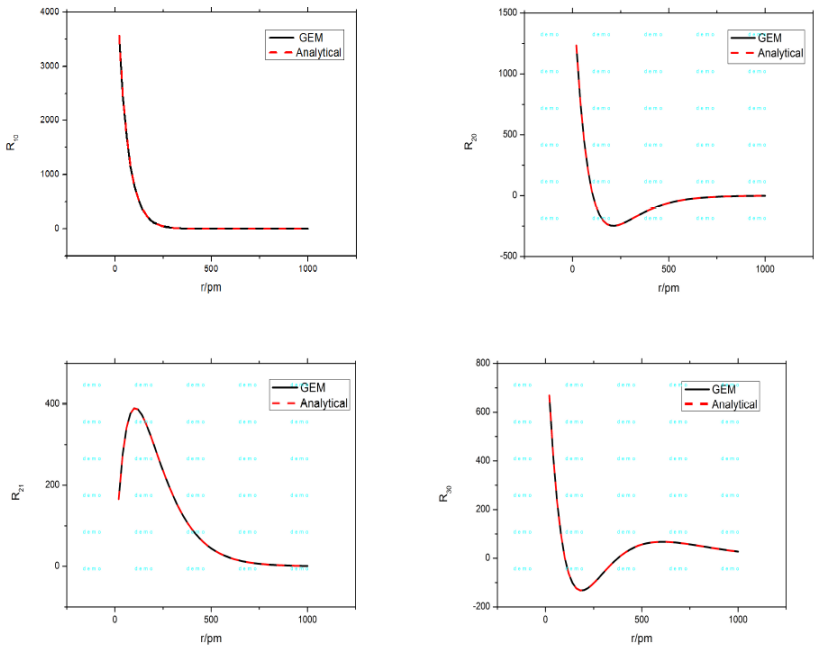
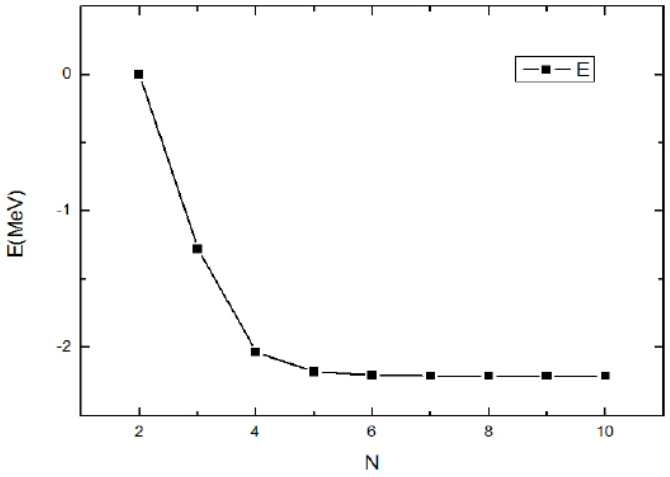


Figure 1: Radial wave functions of H

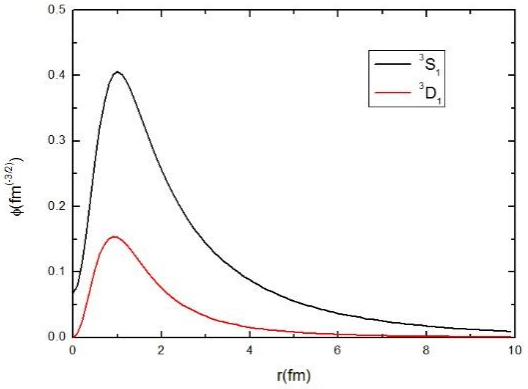
Rapid convergence

- **Deuteron binding energy with Gaussian basis numbers**



Convergent at a very small basis number！ Useful and important ！

- **Deuteron：S-D mixing state**



- **Calculated properties of Deuteron**

Properties	AV8	Reid93
Binding Energy	2.223855MeV	2.224575MeV
Distribution	$93.91\%^3S_1, 6.09\%^3D_1$	$94.30\%^3S_1, 5.70\%^3D_1$
RMS Radius	3.960fm	3.938fm
Magnetic Moment	$0.845\mu_N$	$0.847\mu_N$
Quadrupole Moment	2.852×10^{-3}bar	2.703×10^{-3}bar

Application to DK, DDK, DDDK molecules

1. DK, DDK, DDDK MOLECULES
2. DDD_{S0}* SYSTEM
3. RESULTS AND CONCLUSION

Observation of Ds0*(2317)

In 2003, the BABAR collaboration observed a particle at the invariant mass 2320MeV.

- $D_s^+ \to K^+K^-\pi^+$ E=(2316.8 ± 0.4) MeV Width=(8.6 ± 0.4) MeV
- $D_s^+ \to K^+K^-\pi^+\pi^0$ E=(2317.6 ± 1.3) MeV Width=(8.8 ± 1.1) MeV

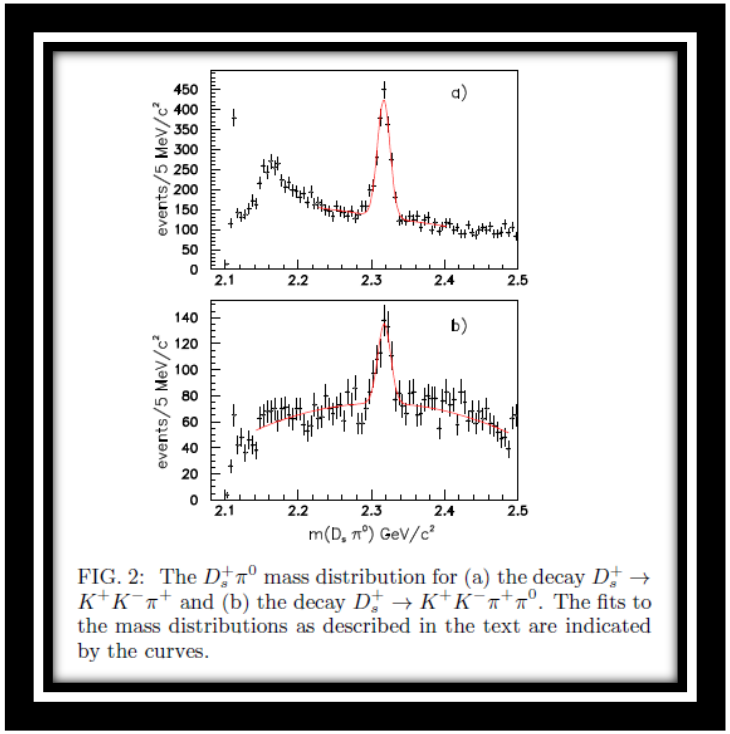


FIG. 2: The $D_s^+\pi^0$ mass distribution for (a) the decay $D_s^+ \to K^+K^-\pi^+$ and (b) the decay $D_s^+ \to K^+K^-\pi^+\pi^0$. The fits to the mass distributions as described in the text are indicated by the curves.

$c\bar{s}$ or DK molecule?

The naïve quark model predicted mass as a $c\bar{s}$ state is about 160 MeV higher than the Exp.

Fitting of Ds0*(2317)

- **DK molecule picture：**

  Binding energy 45MeV, <3.8MeV width.

  Strong Chiral LO attractive interaction and repulsive NLO interaction.

- **DK potential (S-wave, spin=0, isospin=0)**

$$V_{DK}(\vec{r};R_c) = C_S \frac{e^{-(r/R_S)^2}}{\pi^{3/2}R_S^3} + C(R_C)\frac{e^{-(r/R_c)^2}}{\pi^{3/2}R_c^3},$$

  NLO repulsive core and LO attractive part

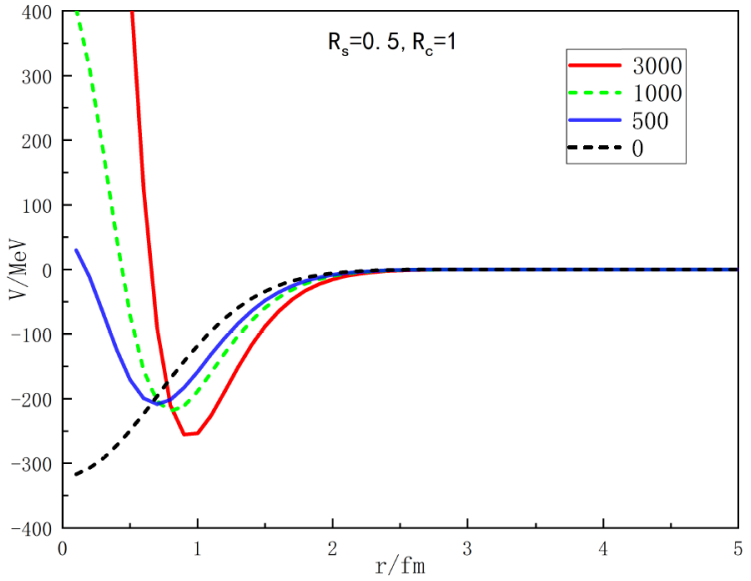
- **LO Weinberg-Tomozawa (WT) DK interaction**

$$V_{DK}(\vec{q}) = -\frac{C_W(I)}{2f_\pi^2},$$

$$C_W(0) = 2 \quad \text{and} \quad C_W(1) = 0,$$

$$V_{DK}(\vec{r}) = -\frac{C_W(I)}{2f_\pi^2}\delta^{(3)}(\vec{r}),$$

$$V_{DK}(r;R_c) = -\frac{C_W(I)}{2f_\pi^2}\frac{e^{-(r/R_c)^2}}{\pi^{3/2}R_c^3},$$



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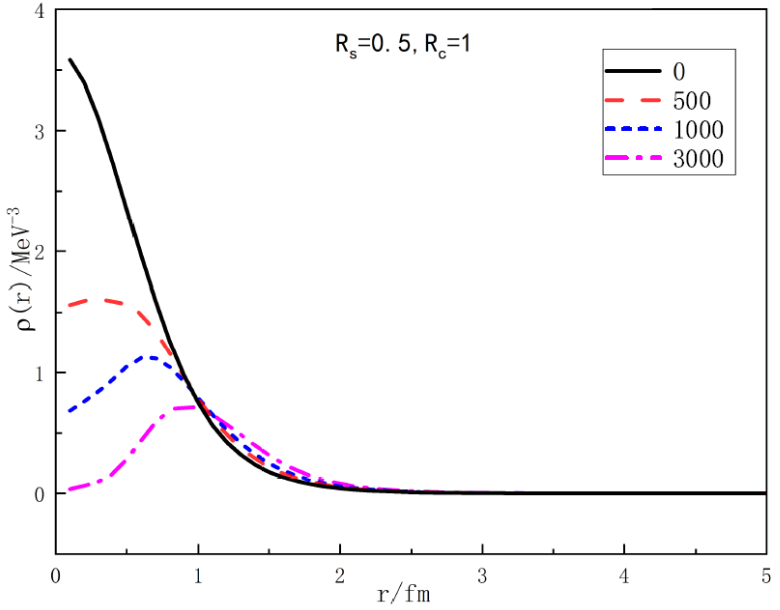


Can we build up **multi-component** molecular states?

- **Experiments, theory, and lattice QCD** all show that DK interaction is strong enough to form the Ds0*(2317)

- A natural question is: if we add one more D, **can they form molecules of three hadrons or more**?

- We study DDK and DDDK systems to explore this straightforward and naive question

Jacobian coordinates and wave functions of DDK and DDDK systems

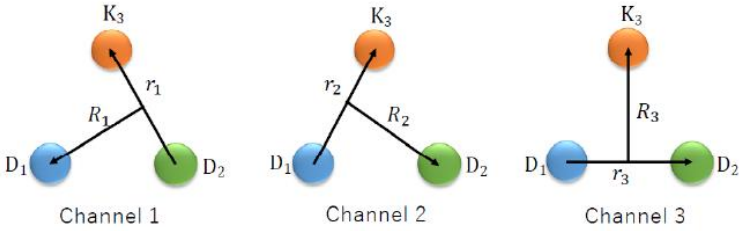
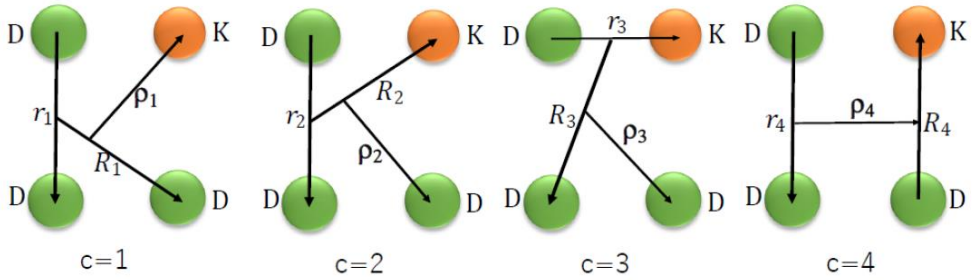


FIG. 2. Three Jacobian coordinates for *DDK* system



$$\Psi^{total}_{JM} = \sum_{c,\alpha} C_{c,\alpha} \Psi^{c}_{JM,\alpha}(\mathbf{r}_c, \mathbf{R}_c)$$

$$\Psi^{c}_{JM,\alpha}(\mathbf{r}_c, \mathbf{R}_c) = H^{c}_{T,t} \otimes [\Phi^{c}_{lL,\Lambda}]_{JM}$$

$$\Phi^{c}_{lL,\Lambda}(\mathbf{r}_c, \mathbf{R}_c) = [\phi^{G}_{n_c l_c}(\mathbf{r}_c)\psi^{G}_{N_c L_c}(\mathbf{R}_c)]_{\Lambda},$$

$$\phi^{G}_{nlm}(\mathbf{r}_c) = N_{nl} r_c^{l} e^{-\nu_n r_c^2} Y_{lm}(\hat{r}_c),$$

$$\psi^{G}_{NLM}(\mathbf{R}_c) = N_{NL} R_c^{L} e^{-\lambda_n R_c^2} Y_{LM}(\hat{R}_c).$$

$$\Psi^{total}_{I(J^P)} = \sum_{c,\alpha} A_{c,\alpha} \Psi^{c}_{\alpha}(\boldsymbol{r}_c, \boldsymbol{R}_c, \boldsymbol{\rho}_c), \qquad c = 1 - 18$$

$$\Psi^{c}_{\alpha}(\boldsymbol{r}_c, \boldsymbol{R}_c, \boldsymbol{\rho}_c) = H^{c}_{t,T,I} \otimes \Phi^{c,JP}_{lL\lambda,\sigma\Lambda}.$$

$$\Phi^{c}_{lL\lambda,\sigma\Lambda} = [\phi^{G}_{n_c l_c}(\boldsymbol{r}_c)\psi^{G}_{N_c L_c}(\boldsymbol{R}_c)]_{\sigma_c}\varphi^{G}_{\nu_c \lambda_c}(\boldsymbol{\rho}_c)]_{\Lambda},$$

$$\phi^{G}_{nlm}(\boldsymbol{r}_c) = N_{nl} r_c^{l} e^{-\nu_n r_c^2} Y_{lm}(\hat{r}_c),$$

$$\psi^{G}_{NLM}(\boldsymbol{R}_c) = N_{NL} R_c^{L} e^{-\lambda_N R_c^2} Y_{LM}(\hat{R}_c),$$

$$\varphi^{G}_{\nu\lambda\mu}(\boldsymbol{R}_c) = N_{\nu\lambda} \rho_c^{\lambda} e^{-\omega_\nu \rho_c^2} Y_{\lambda\mu}(\hat{\rho}_c).$$

DD interaction

- **DD OBE potential**

$$V_{DD}(r;\Lambda) = V_\rho(r;\Lambda) + V_\omega(r;\Lambda) + V_\sigma(r;\Lambda)$$

$$V_\sigma(r;\Lambda) = -g_\sigma^2\, m_\sigma\, W_C(m_\sigma r, \frac{\Lambda}{m_\sigma})\,,$$

$$V_\rho(r;\Lambda) = +\vec{\tau}_1\cdot\vec{\tau}_2\, g_\rho^2\, m_\rho\, W_C(m_\rho r, \frac{\Lambda}{m_\rho})\,,$$

$$V_\omega(r;\Lambda) = +g_\omega^2\, m_\omega\, W_C(m_\omega r, \frac{\Lambda}{m_\omega})\,,$$

$$W_C(x,\lambda) = \frac{e^{-x}}{4\pi x} - \lambda\,\frac{e^{-\lambda x}}{4\pi\lambda x} - \frac{(\lambda^2-1)}{2\lambda}\,\frac{e^{-\lambda x}}{4\pi}.$$

$$m_\rho = 0.770\text{GeV},\ m_\omega = 0.780\ \text{GeV},\ m_\sigma = 0.6\ \text{GeV},$$

$$g_\rho = g_\omega = 2.6,\ g_\sigma = 3.4.$$

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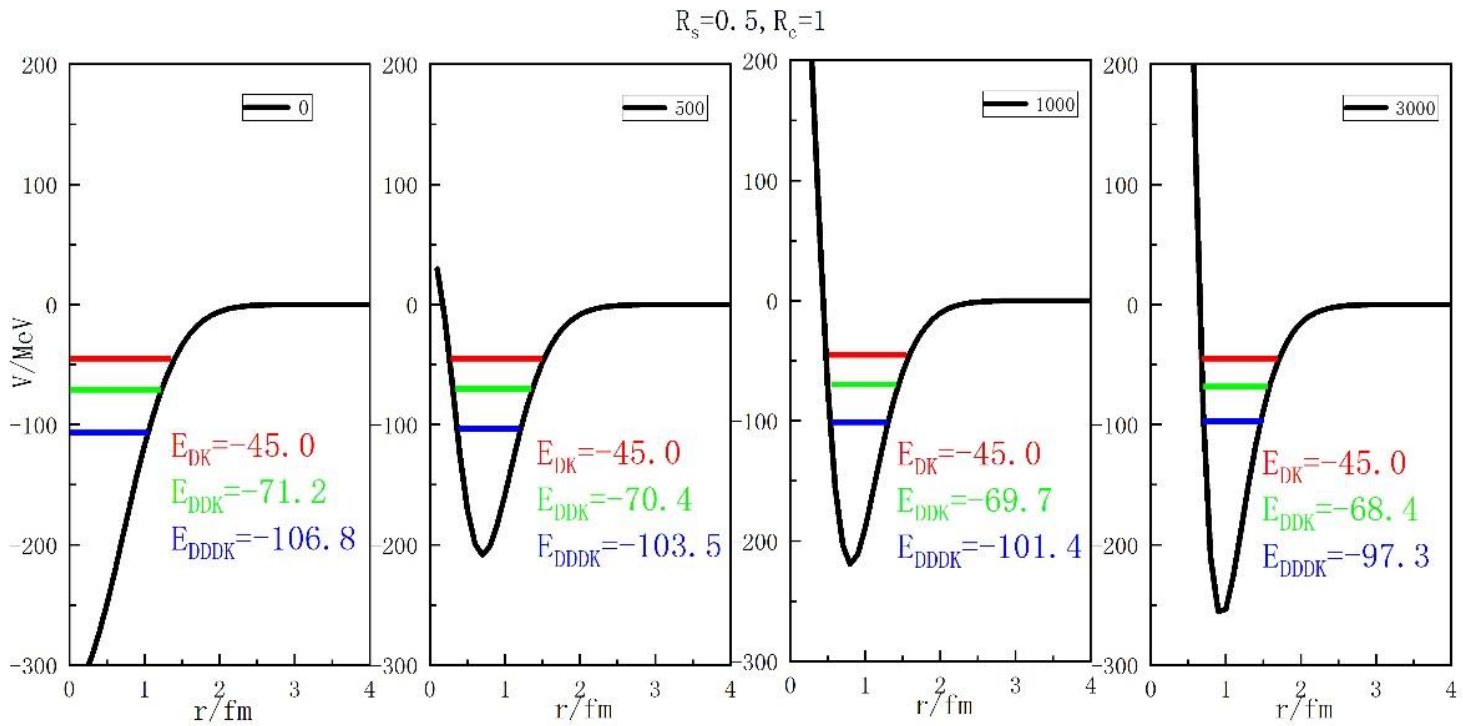
The cutoff Λ is set by reproducing the X(3872) pole, yielding 1.01GeV, here for simplicity we set it to 1.0GeV.

DDK and DDDK Binding energy



$$V_{DK}(\vec{r};R_c) = C_S \frac{e^{-(r/R_S)^2}}{\pi^{3/2}R_S^3} + C(R_C)\frac{e^{-(r/R_c)^2}}{\pi^{3/2}R_c^3},$$

DK, DDK, DDDK binding energy with the repulsive core parameter Cs ranging from 0-3000MeV.

DDK and DDDK Binding energies with different sets of DK potential

TABLE IV. Binding energies (in units of MeV) of DDK and $DDDK$ systems with and withou the DD interaction.

$\frac{C_S}{\pi R_S^3}$	$\frac{C(R_c)}{\pi R_c^3}$	E_2	E_3(only V_{DK})	$E_3(V_{DK}+V_{DD})$	E_4(onlyV_{DK})	$E_4(V_{DK}+V_{DD})$
		$R_S = 0.5$fm		$R_c = 1$fm		
0	-320.1	-45.0	-65.8	-71.2	-89.4	-106.8
500	-455.4	-45.0	-65.8	-70.4	-89.2	-103.5
1000	-562.6	-45.0	-65.7	-69.7	-88.8	-101.4
3000	-838.7	-45.0	-65.0	-68.4	-87.0	-97.3
		$R_S = 0.5$fm		$R_c = 2$fm		
0	-149.1	-45.0	-66.0	$-68.8, -45.1$	$-88.7, -66.3$	$-97.6, -70.7$
500	-178.4	-45.0	-65.9	$-68.2, -45.5$	$-88.5, -66.7$	$-95.5, -70.9$
1000	-195.0	-45.0	$-65.8, -45.2$	$-67.9, -45.8$	$-88.2, -66.9$	$-94.5, -71.2$
3000	-225.9	-45.0	$-65.3, -45.6$	$-67.2, -46.6$	$-87.0, -67.0$	$-92.6, -71.7$
		$R_S = 0.5$fm		$R_c = 3$fm		
0	-107.0	-45.0	$-66.2, -47.3$	$-68.0, -48.3$	$-88.8, -70.2$	$-94.4, -74.3$
500	-119.4	-45.0	$-66.2, -48.2$	$-67.7, -49.3$	$-88.7, -71.0$	$-93.2, -74.8$
1000	-125.6	-45.0	$-66.1, -48.7$	$-67.5, -49.8$	$-88.4, -71.3$	$-92.5, -75.2$
3000	-136.2	-45.0	$-65.8, -49.4$	$-67.1, -50.7$	$-87.6, -71.7$	$-91.4, -75.7$

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DD interaction makes the DDK and DDDK systems more bound, but only by a few MeV.

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As the range of the attraction becomes larger, two bound state solutions appear instead of one.

RMS radius and expectation values of the kinetic and potential terms of DK and DDK systems

$\frac{C_S}{\pi R_S^3}$	$\frac{C(R_c)}{\pi R_c^3}$	$r_2(DK)$	$r_3(DK)$	$r_3(DD)$	$< T >$	$< V_{DK} >$	$< V_{DD} >$
			$R_S = 0.5$fm $R_c = 1$fm				
0	−320.1	1.28	1.32	1.36	124.37	−189.61	−5.98
500	−455.4	1.39	1.44	1.47	99.51	−164.83	−5.03
1000	−562.6	1.46	1.53	1.54	91.43	−156.67	−4.51
3000	−838.7	1.61	1.69	1.68	93.24	−157.80	−3.82
			$R_S = 0.5$fm $R_c = 2$fm				
0	−149.1	1.74	1.80	1.80	60.20	−125.74	−3.23
500	−178.4	1.91	1.98	1.96	51.00	−116.59	−2.64
1000	−195.0	1.99	2.07	2.04	50.63	−116.12	−2.43
3000	−225.9	2.13	2.22	2.15	53.61	−118.59	−2.24
			$R_S = 0.5$fm $R_c = 3$fm				
0	−107.0	2.13	2.19	2.17	39.49	−105.35	−2.13
500	−119.4	2.31	2.38	2.34	34.80	−100.73	−1.77
1000	−125.6	2.37	2.47	2.42	34.90	−100.77	−1.65
3000	−136.2	2.53	2.61	2.53	36.66	−102.24	−1.54

RMS radius and expectation values of the kinetic and potential terms of DK and DDK systems

$\frac{C_S}{\pi R_S^3}$	$\frac{C(R_c)}{\pi R_c^3}$	$r_2(DK)$	$r_3(DK)$	$r_3(DD)$	$< T >$	$< V_{DK} >$	$< V_{DD} >$
$R_S = 0.5$fm $R_c = 1$fm							
0	−320.1	1.28	1.32	1.36	124.37	−189.61	−5.98
500	−455.4	1.39	1.44	1.47	99.51	−164.83	−5.03
1000	−562.6	1.46	1.53	1.54	91.43	−156.67	−4.51
3000	−838.7	1.61	1.69	1.68	93.24	−157.80	−3.82
$R_S = 0.5$fm $R_c = 2$fm							
0	−149.1	1.74	1.80	1.80	60.20	−125.74	−3.23
500	−178.4	1.91	1.98	1.96	51.00	−116.59	−2.64
1000	−195.0	1.99	2.07	2.04	50.63	−116.12	−2.43
3000	−225.9	2.13	2.22	2.15	53.61	−118.59	−2.24
$R_S = 0.5$fm $R_c = 3$fm							
0	−107.0	2.13	2.19	2.17	39.49	−105.35	−2.13
500	−119.4	2.31	2.38	2.34	34.80	−100.73	−1.77
1000	−125.6	2.37	2.47	2.42	34.90	−100.77	−1.65
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- The RMS radius of the Ds0(2317), which ranges from 1.2 to 2.6 fm, increases with the cutoff Rc

RMS radius and expectation values of the kinetic and potential terms of DK and DDK systems

$\frac{C_S}{\pi R_S^3}$	$\frac{C(R_c)}{\pi R_c^3}$	$r_2(DK)$	$r_3(DK)$	$r_3(DD)$	$< T >$	$< V_{DK} >$	$< V_{DD} >$
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- The RMS radius of the Ds0(2317), which ranges from 1.2 to 2.6 fm, increases with the cutoff Rc

- The geometry of the DDK system is more or less of a equilateral triangle.

RMS radius and expectation values of the kinetic and potential terms of DK and DDK systems

$\frac{C_S}{\pi R_S^3}$	$\frac{C(R_c)}{\pi R_c^3}$	$r_2(DK)$	$r_3(DK)$	$r_3(DD)$	$< T >$	$< V_{DK} >$	$< V_{DD} >$
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0	−320.1	1.28	1.32	1.36	124.37	−189.61	−5.98
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- The RMS radius of the Ds0(2317), which ranges from 1.2 to 2.6 fm, increases with the cutoff Rc
- The geometry of the DDK system is more or less of a equilateral triangle.
- The DD interaction is weakly attractive, accounting for only a few MeV of the total potential energy

DDs0* and DDDs0* results

The DDDs0* system, which is equivalent to DDDK 4-body system by regarding the DK as a Ds0*(2317).

- **DDs0* OKE Potential**

$$V_{OKE}(\vec{q}) = -h^2 \frac{\omega_K^2}{f_\pi^2} \frac{1}{\mu_K^2 + \vec{q}^2},$$

$$\mu_K = \sqrt{m_K^2 - \omega_K^2}.$$

- **DDs0* Potential in coordinate space**

$$V_{DD^*_{s0}}(r) = -h^2 \frac{\omega_K^2}{f_\pi^2} \left(\frac{e^{-\mu_K r}}{4\pi r} - \frac{e^{-\Lambda' r}}{4\pi r} - \frac{(\Lambda'^2 - \mu_K^2) e^{-\Lambda' r}}{8\pi \Lambda'} \right)$$

$$h = 0.7 \text{ and } f_\pi = 130 \text{ MeV}.$$

Λ'	$B_{DD^*_{s0}}$	$B_{DDD^*_{s0}}$(only $V_{DD^*_{s0}}$)	$B_{DDD^*_{s0}}$($V_{DD} + V_{DD^*_{s0}}$)
0.8	−5.1	−11.5	−13.9
1.0	−8.5	−18.9	−22.5
1.2	−11.7	−25.8	−30.3
1.4	−14.5	−31.9	−37.2
1.6	−17.0	−37.2	−43.3

DDs0* binding energy is 50-62MeV, DDDs0* binding energy is 59-88MeV, with respect to DDK and DDDK thresholds. which is consistent with DDK and DDDK results.

Summary

- We have addressed the question of whether one can build up multi-component molecular states. The answer is yes, where we find a bound DDK trimer and DDDK tetramer.

- We predict the DDK trimer will bind by about 70MeV and the DDDK tetramer by about 100MeV, with variations of a few MeV at most stemming from the uncertainties in the DK and DD potentials.

- In addition, even if one treats the Ds0(2317) as a genuine $c\bar{s}$ state, we still predict DDs0 and DDDs0 bound states with the same quantum numbers as the DDK trimer and DDDK tetramer.

- To those of D*K, B$\bar{K}$ and B*$\bar{K}$, we naively expect the existence of the heavy quark symmetry partners of the DDK and DDDK states .

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Thanks for your attention !

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