



Mesonic dynamic and QCD phase transition

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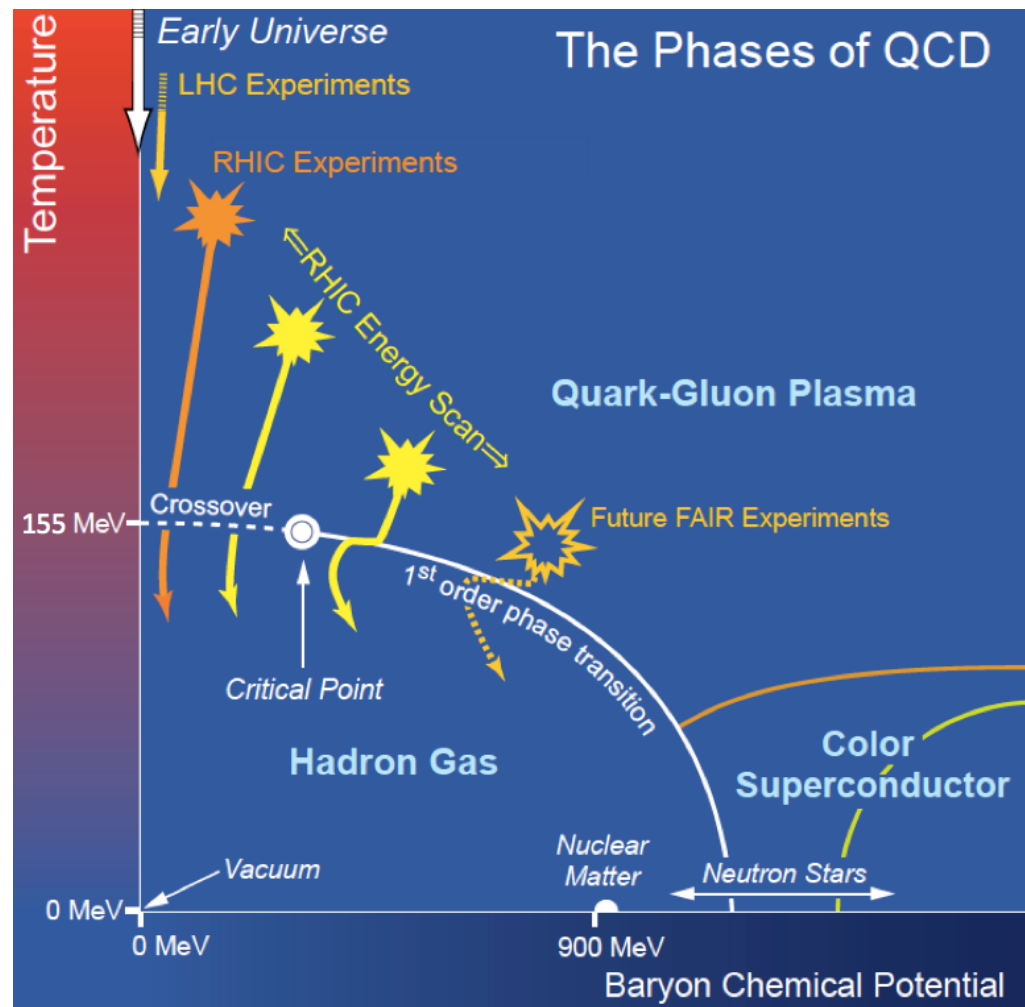
This work is done in cooperation with Rui Wen(温睿) and Wei-jie Fu(付伟杰)

Outline

- Introduction
- Thermodynamic quantities
- Baryon number fluctuation
- Effective potential and Yukawa coupling
- Summary

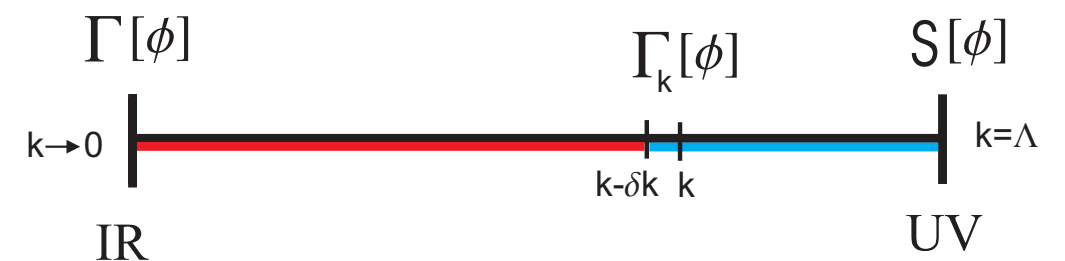
Introduction

QCD Phase diagram



The Hot QCD White Paper (2015)

Functional renormalization group



effective action of the PQM: $N_f = 2$ **2 flavor**

$$\Gamma_k = \int_x \left\{ Z_{q,k} \bar{q} (\gamma_\mu \partial_\mu - \gamma_0 (\mu + ig A_0)) q + \frac{1}{2} Z_{\phi,k} (\partial_\mu \phi)^2 \right. \\ \left. + h_k \bar{q} (T^0 \sigma + i \gamma_5 \vec{T} \cdot \vec{\pi}) q + V_k(\rho) - c \sigma \right\} + \dots$$

flow equation:

$$\partial_t \Gamma_k = \frac{1}{2} \left(\text{Diagram 1} - \text{Diagram 2} - \text{Diagram 3} + \frac{1}{2} \text{Diagram 4} \right)$$

Quark-Meson model

Gluon

Glue potential:

$$V_{glue}(L, \bar{L}) = -\frac{a(T)}{2} \bar{L}L + b(T) \ln M_H(L, \bar{L}) + \frac{c(T)}{2} (L^3 + \bar{L}^3) + d(T) (\bar{L}L)^2$$

$$M_H(L, \bar{L}) = 1 - 6\bar{L}L + 4(L^3 + \bar{L}^3) - 3(\bar{L}L)^2$$

**Distribution
function**

$$n_F(x, T) = \frac{1}{\exp(\frac{x}{T}) + 1}$$

Replace



$$n_F(x, T, L, \bar{L}) = \frac{1 + 2\bar{L}e^{x/T} + Le^{2x/T}}{1 + 3\bar{L}e^{x/T} + 3Le^{2x/T} + e^{3x/T}}$$

$$\partial_t \Gamma_k = \frac{1}{2} \left(\text{diagram 1} - \text{diagram 2} - \text{diagram 3} + \frac{1}{2} \text{diagram 4} \right)$$

The diagrams represent Feynman diagrams for the time derivative of the effective action. Diagram 1 is a loop with a wavy line and a cross. Diagram 2 is a loop with a dashed line and a cross. Diagram 3 is a loop with a solid line and a cross. Diagram 4 is a loop with a dashed line and a cross. The first three diagrams are subtracted, and the fourth is added with a factor of 1/2. A red horizontal line is drawn under the first three diagrams.

$$L(\vec{x}) = \frac{1}{N_c} \langle \text{Tr} \mathcal{P}(\vec{x}) \rangle$$

$$\bar{L}(\vec{x}) = \frac{1}{N_c} \langle \text{Tr} \mathcal{P}^\dagger(\vec{x}) \rangle$$

$$\mathcal{P}(\vec{x}) = \mathcal{P} \exp \left(ig \int_0^\beta d\tau A_0(\vec{x}, \tau) \right)$$

Break of the $O(4)$ symmetry

In finite temperature field theory

$O(4)$ -symmetry replaced by $\mathbb{Z}_2 \otimes O(3)$

momentum: $p = (\omega_n, \vec{p}) = (2\pi nT, \vec{p})$

Here we investigate the influence of the meson wave function renormalization

$$\Gamma_k = \int_x \{ Z_{q,k} \bar{q} (\gamma_\mu \partial_\mu - \gamma_0 (\mu + igA_0)) q + \frac{1}{2} Z_{\phi,k} (\partial_\mu \phi)^2 + h_k \bar{q} (T^0 \sigma + i\gamma_5 \vec{T} \cdot \vec{\pi}) q + V_k(\rho) - c\sigma \} + \dots$$

Propagator:

$$G_k^\pi = \frac{1}{\underline{Z}_{\phi,k} (q_0^2 + \vec{q}^2 (1+r_B) + \bar{m}_\pi^2)}$$



$$G_k^\pi = \frac{1}{\underline{Z}_{\phi,k}^\parallel q_0^2 + \underline{Z}_{\phi,k}^\perp \vec{q}^2 (1+r_B) + m_\pi^2}$$

$$\Gamma_k = \int_x \{ Z_{q,k} \bar{q} (\gamma_\mu \partial_\mu - \gamma_0 (\mu + igA_0)) q + \frac{1}{2} Z_{\phi,k}^\parallel (\partial_0 \phi)^2 + \frac{1}{2} Z_{\phi,k}^\perp (\partial_i \phi)^2 + h_k \bar{q} (T^0 \sigma + i\gamma_5 \vec{T} \cdot \vec{\pi}) q + V_k(\rho) - c\sigma \} + \dots$$

Flow equation

The flow of the wave function renormalization:

(anomalous dimension)

$$\eta_{\phi,k}^{\perp} = -\frac{\partial_t Z_{\phi,k}^{\perp}}{Z_{\phi,k}^{\perp}}$$

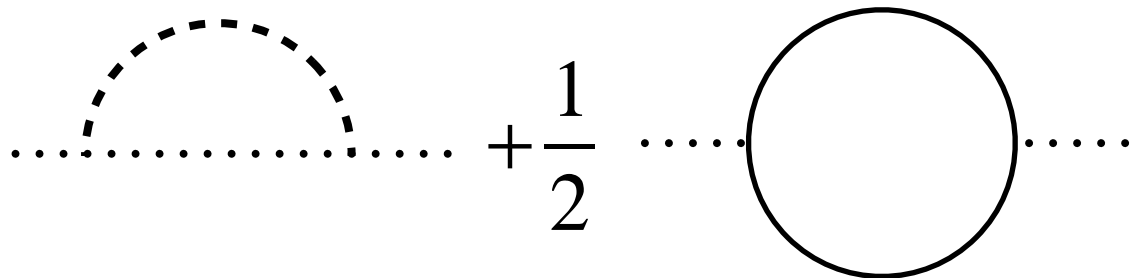
$$\eta_{\phi,k}^{\parallel} = -\frac{\partial_t Z_{\phi,k}^{\parallel}}{Z_{\phi,k}^{\parallel}}$$

$$\partial_t = k \frac{\partial}{\partial k}$$

Wetterich equation:

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left\{ \tilde{\partial}_t \ln \left(\Gamma_k^{(2)} + R_k \right) \right\}$$

$$\partial_t Z_{\phi,k} \sim -\frac{1}{2} \tilde{\partial}_t (G_k^{\sigma} F_k^{\sigma\pi} G_k^{\pi} F_k^{\sigma\pi}) + \frac{1}{2} \tilde{\partial}_t (G_k^{q\bar{q}} F_k^{\bar{q}q} G_k^{q\bar{q}} F_k^{\bar{q}q})$$

$$-\frac{1}{2} \cdots \cdots \cdots + \frac{1}{2} \cdots \cdots \cdots$$


Thermodynamic quantities

Renormalized dimensionless meson mass and quark mass:

$$\bar{m}_{\pi,k}^2 = \frac{V'_k(\rho)}{k^2 Z_{\phi,k}^\perp}$$

$$\bar{m}_{\sigma,k}^2 = \frac{V'_k(\rho) + 2\rho V''_k(\rho)}{k^2 Z_{\phi,k}^\perp}$$

$$\bar{m}_{q,k}^2 = \frac{h_k^2 \rho}{2k^2 Z_{q,k}^2}$$

Fluctuation:

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n} \frac{p}{T^4}$$

Kurtosis:

$$\kappa\sigma^2 = \frac{\chi_4^B}{\chi_2^B}$$

Trace anomaly:

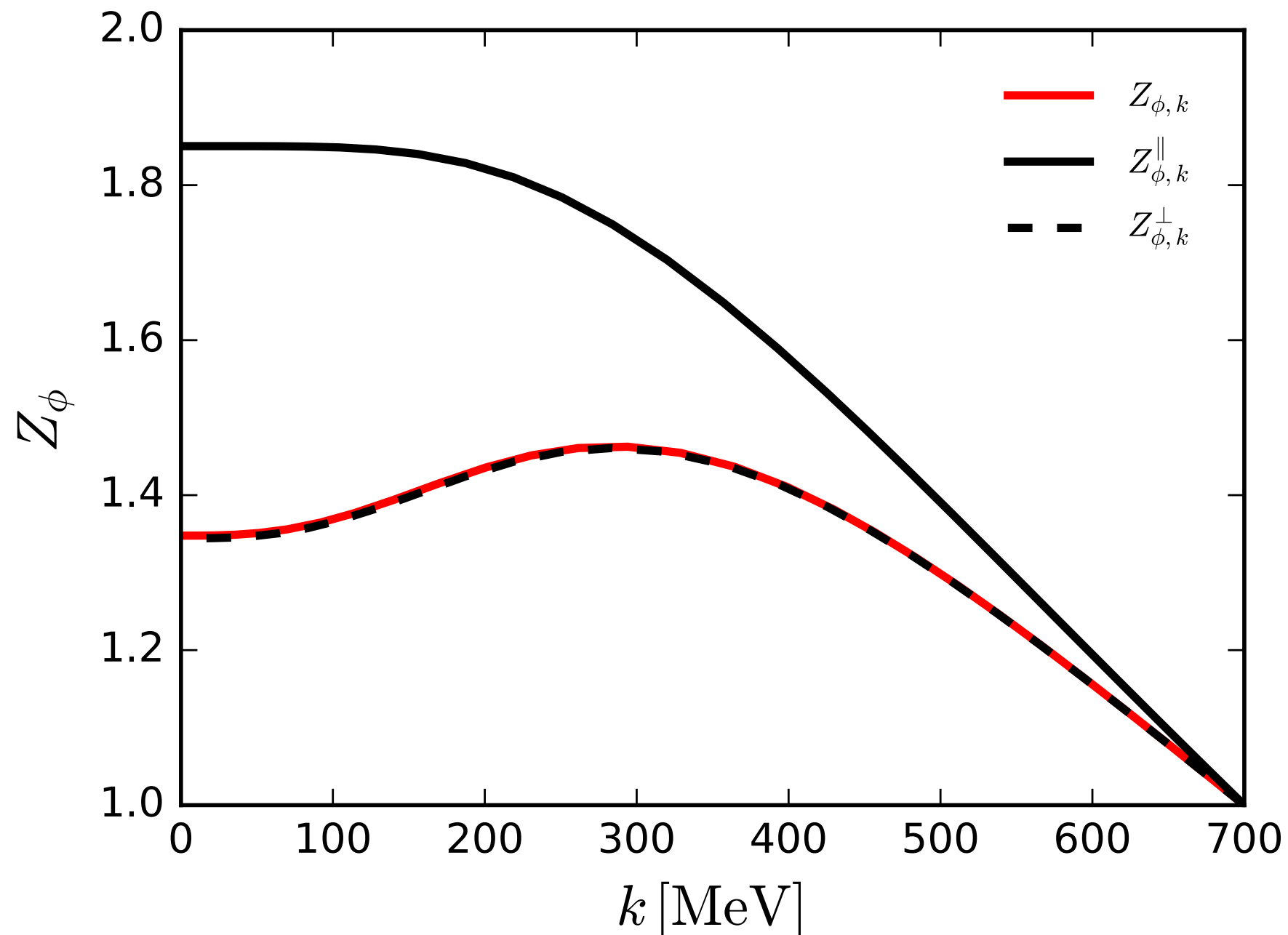
$$\frac{\varepsilon - 3p}{T^4}$$

Pion decay constant:

$$f_\pi = \langle \sigma \rangle$$

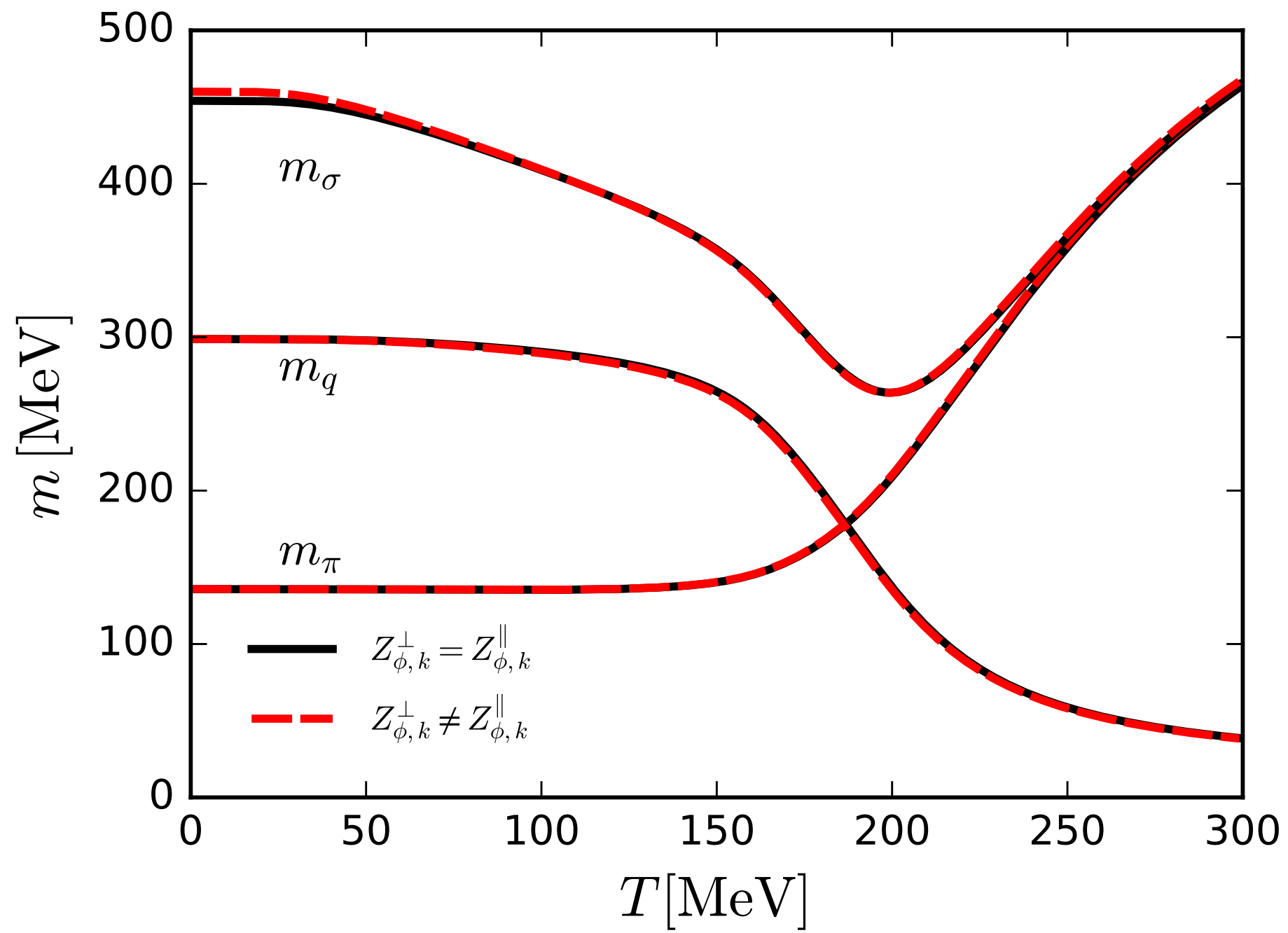
Thermodynamic quantities

The meson wave function renormalization as a function of k at $T=0$



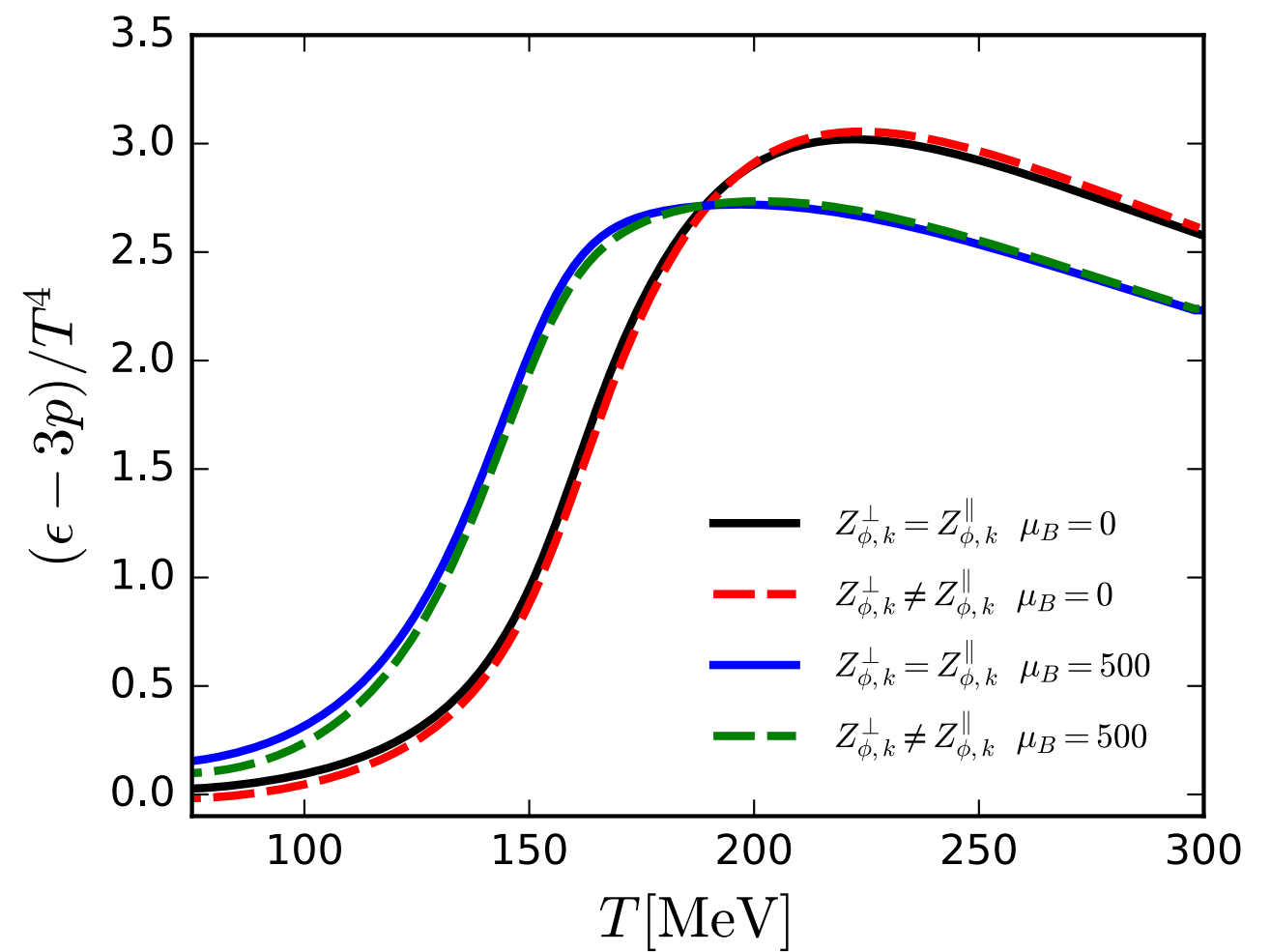
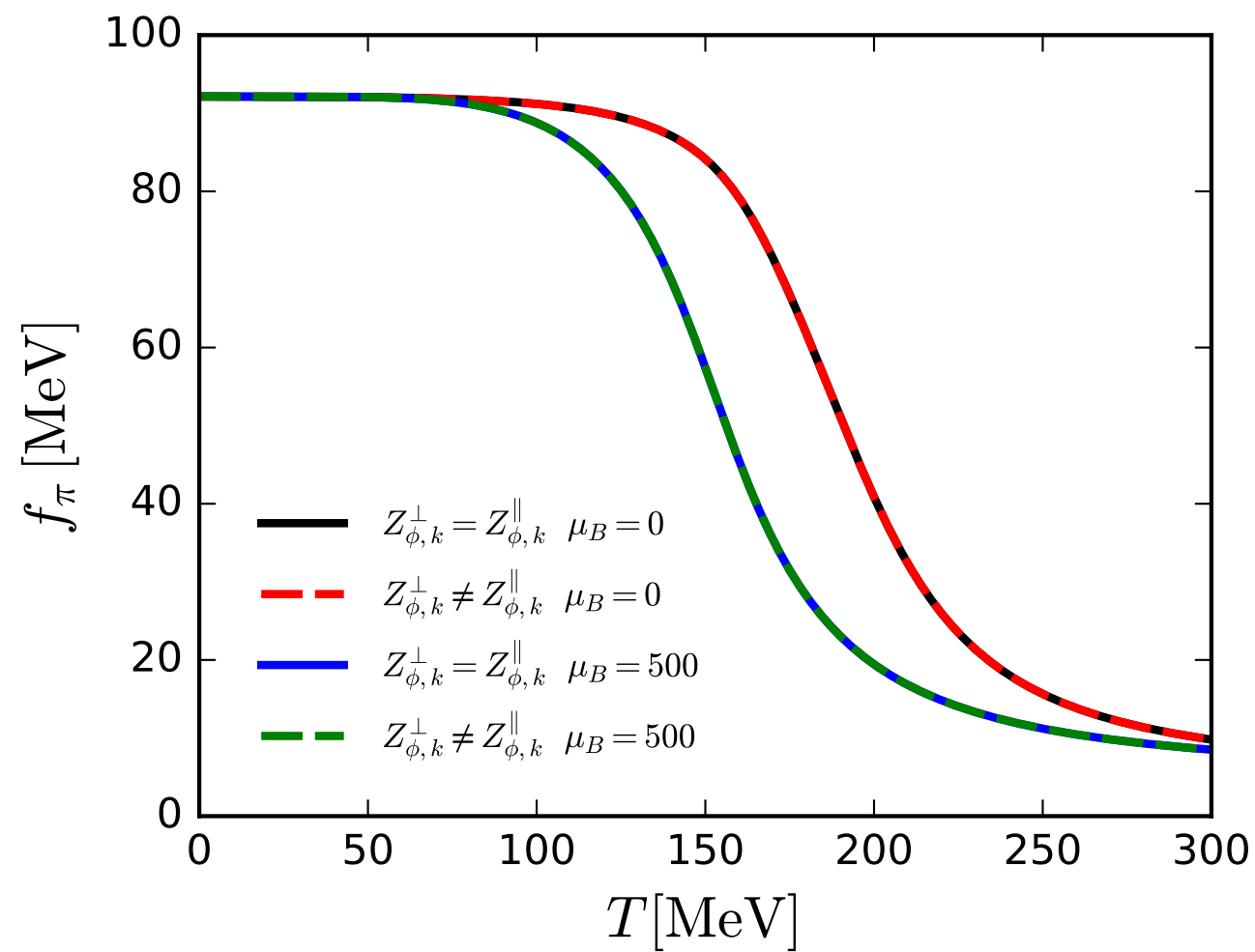
Thermodynamic quantities

The pion mass and quark mass as a function of temperature

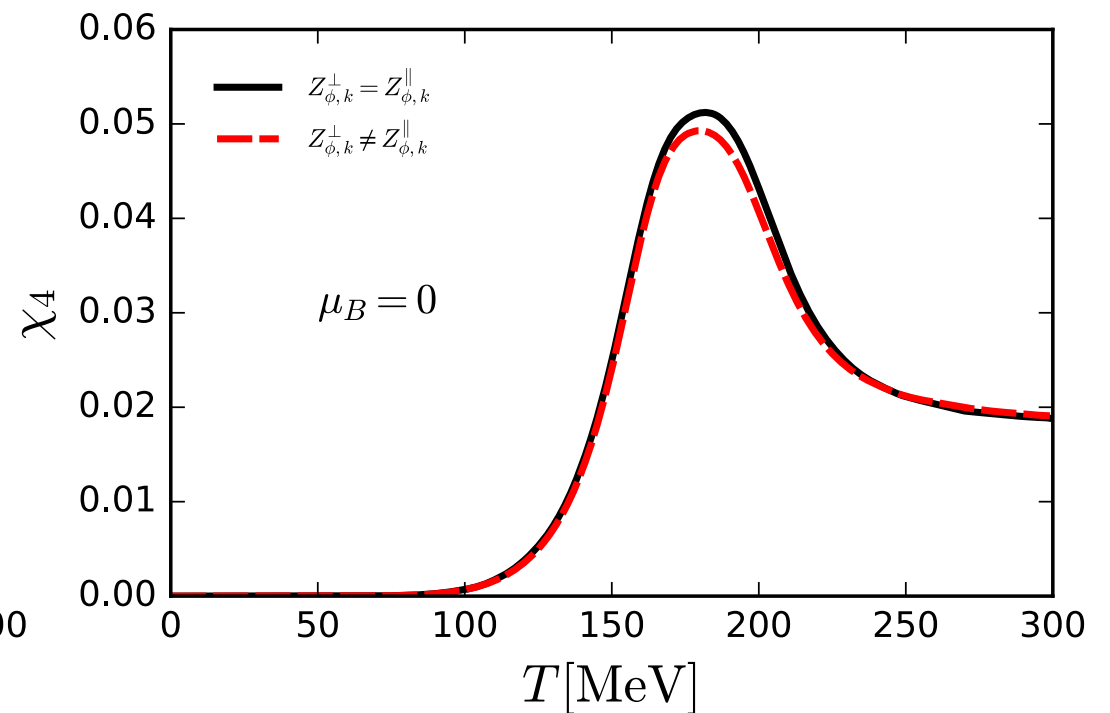
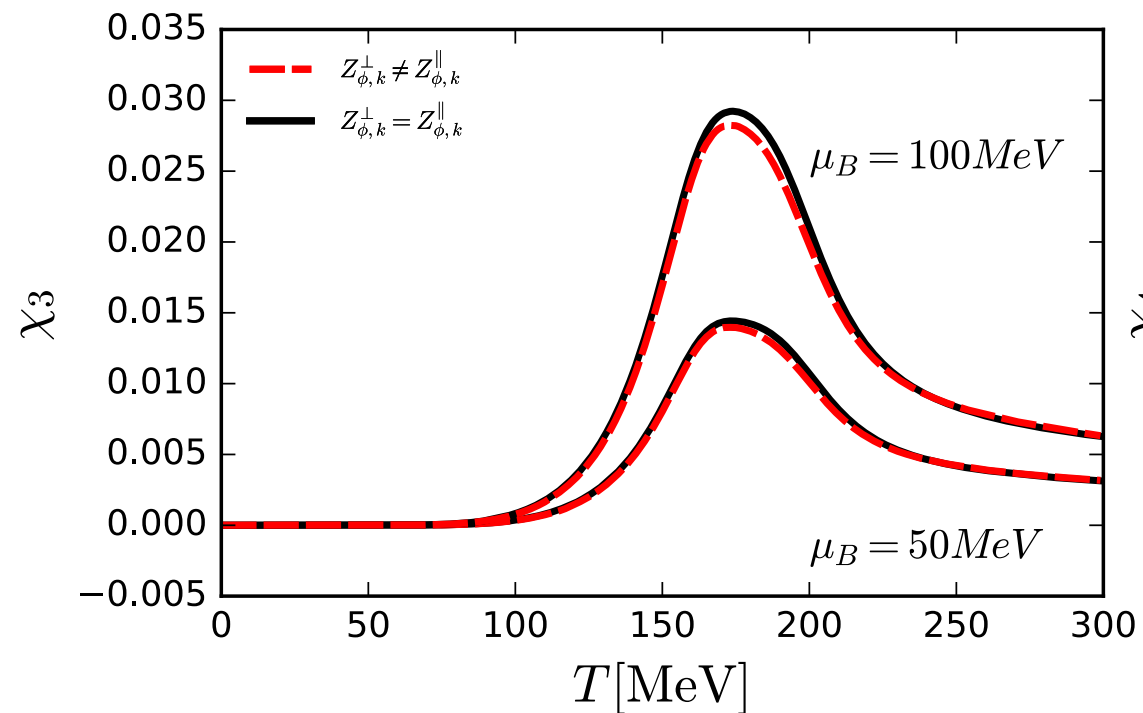
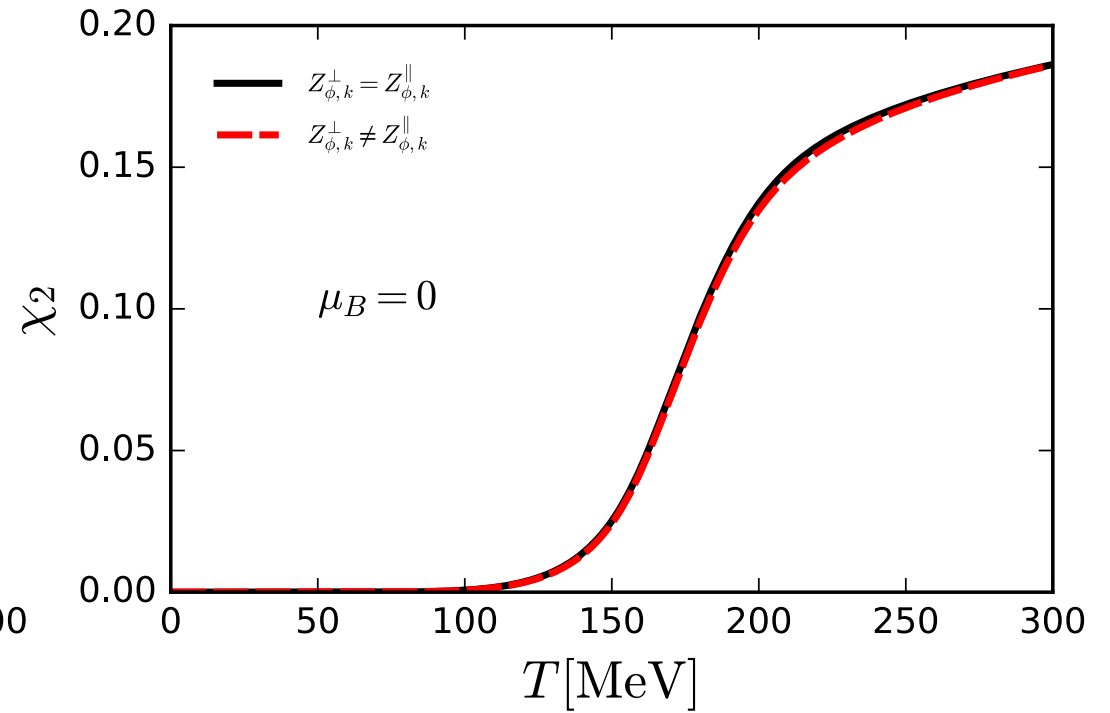
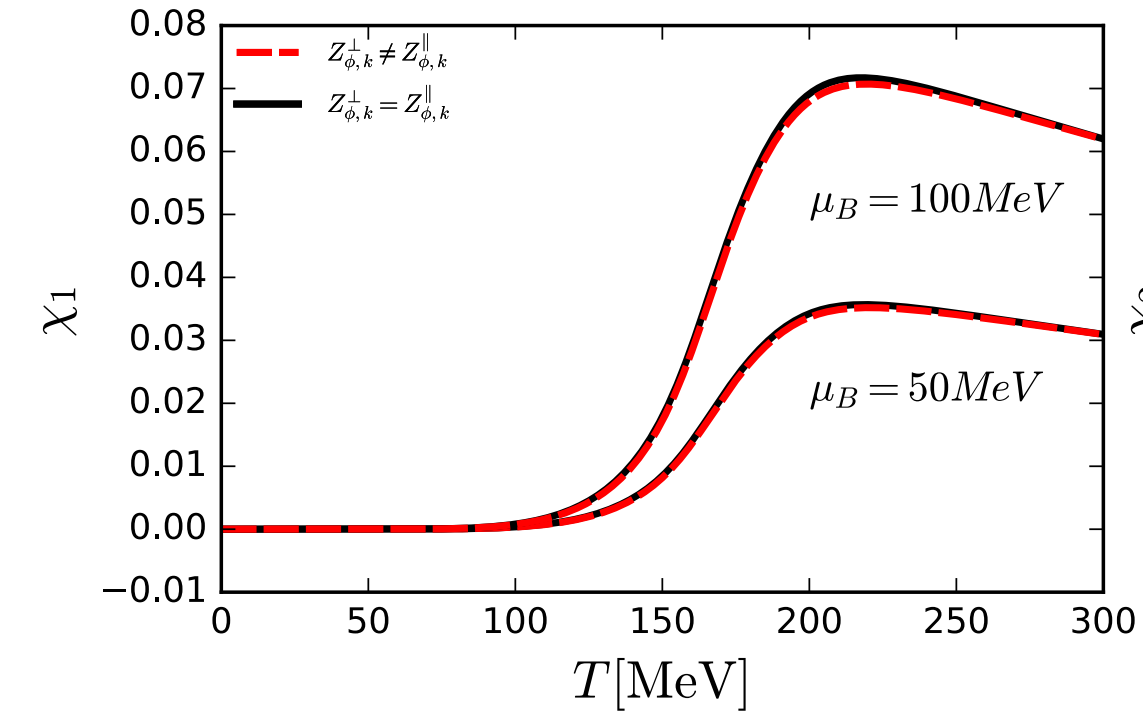


Thermodynamic quantities

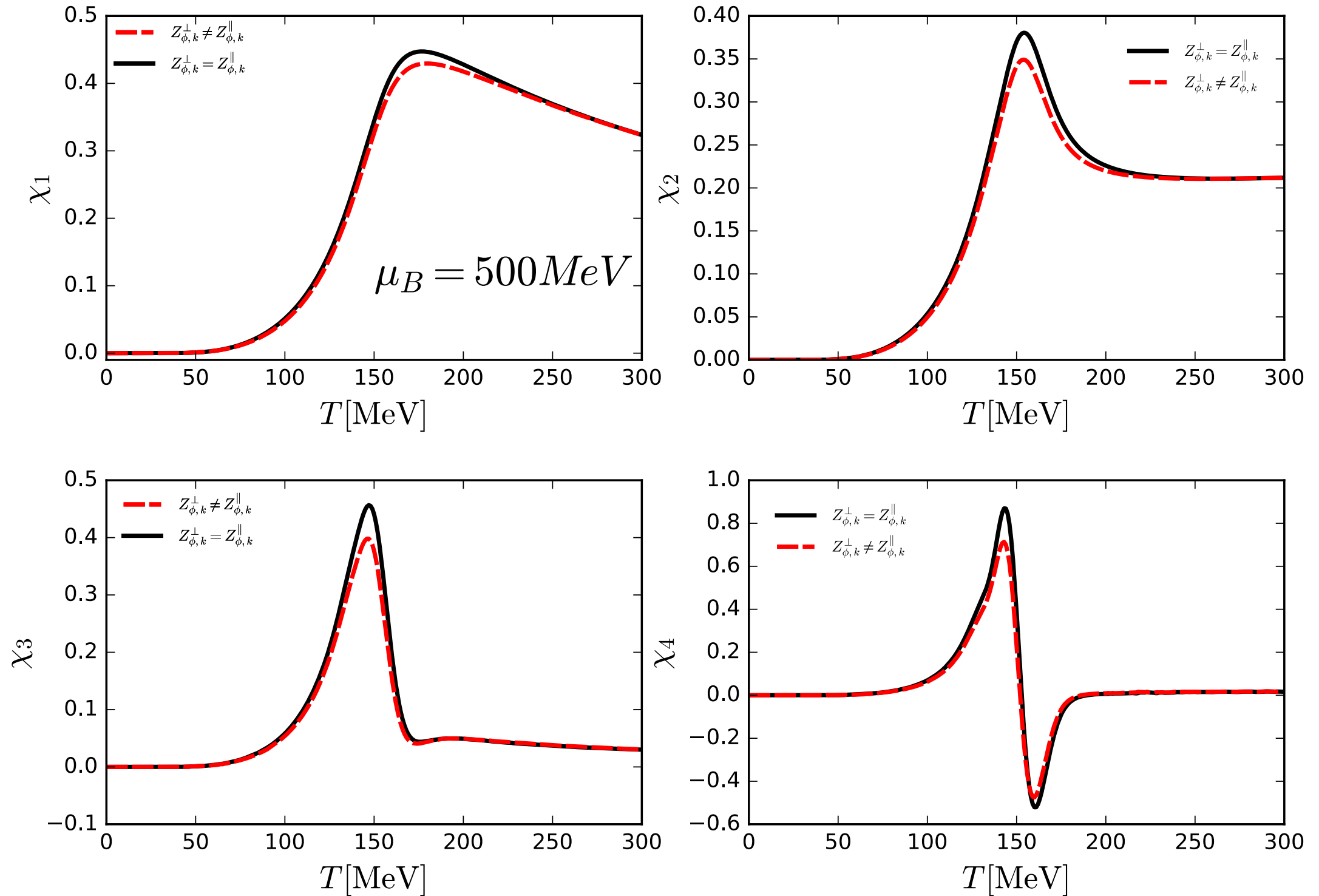
Trace-anomaly and Pion decay constant as a function of temperature



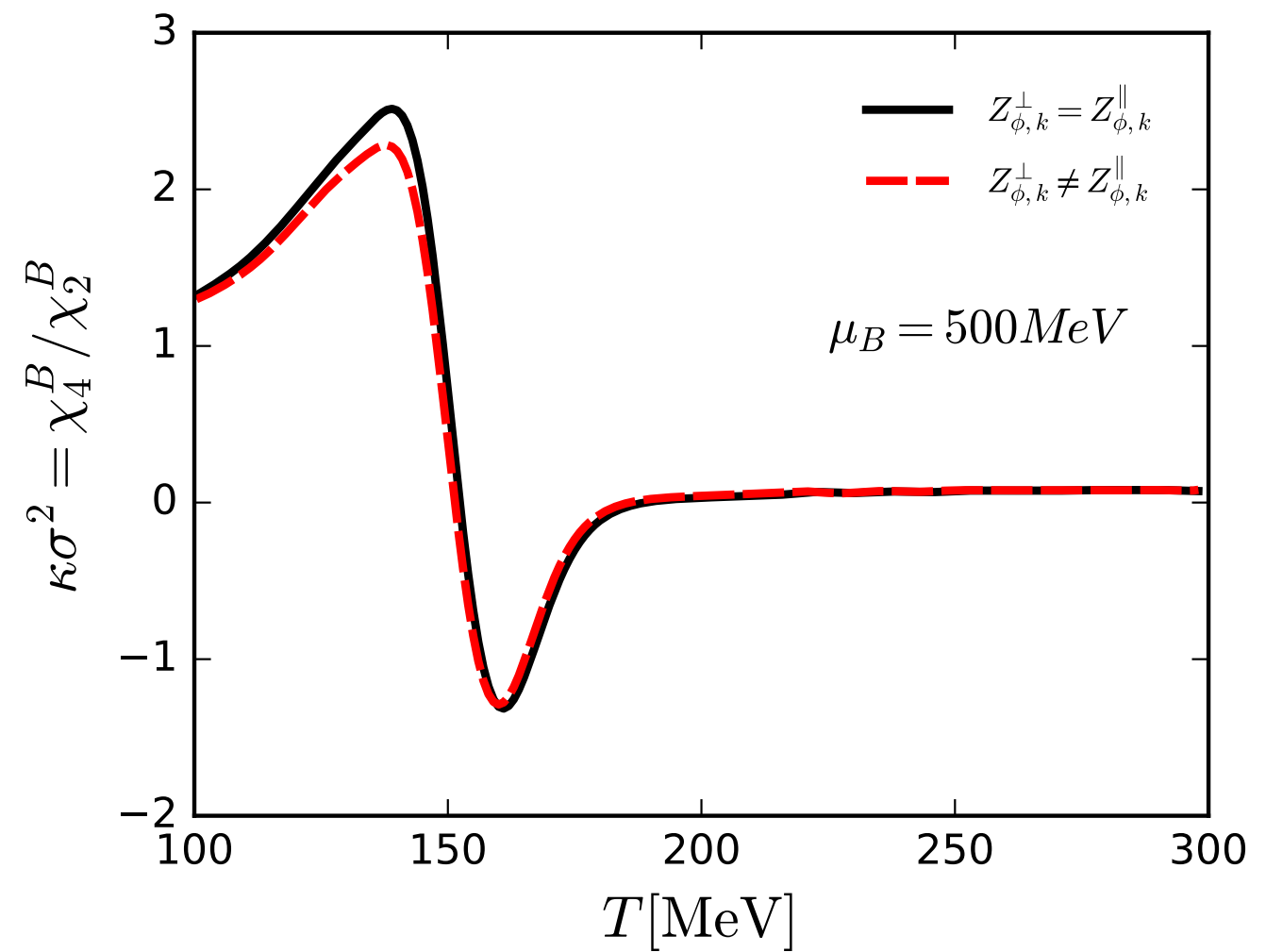
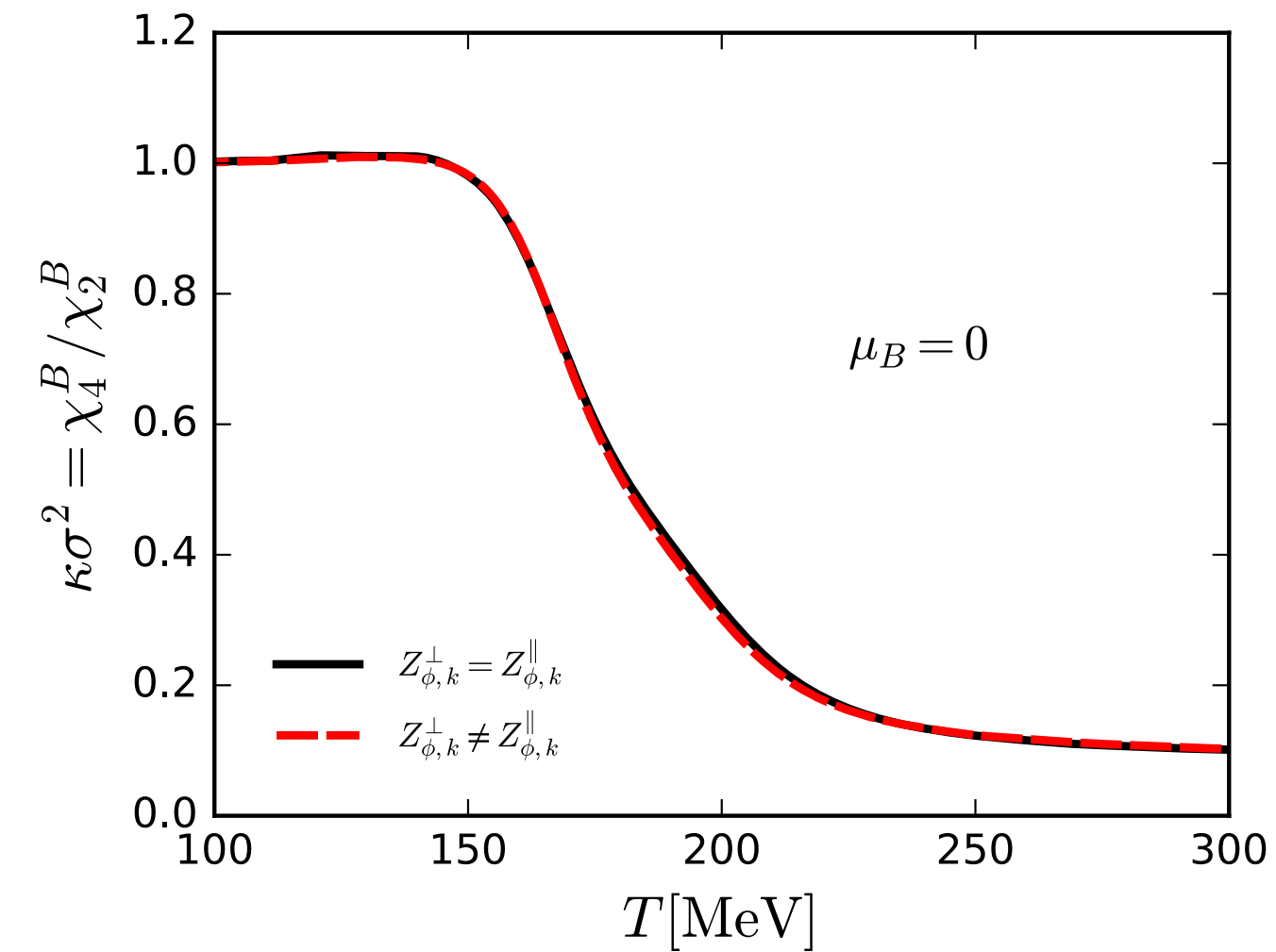
Baryon number fluctuation



Baryon number fluctuation



Baryon number fluctuation



Effective potential and Yukawa coupling

Effective potential:

$$\bar{V}_k(\bar{\rho}) = \sum_{n=0}^N \frac{\bar{\lambda}_{n,k}}{n!} (\bar{\rho} - \bar{K}_k)^n$$

$$n_V = 5$$

Yukawa coupling:

$$\bar{h}_k(\bar{\rho}) = \sum_{n=0}^N \frac{\bar{h}_{n,k}}{n!} (\bar{\rho} - \bar{K}_k)^n$$

$$n_h = 5$$

Fix point:

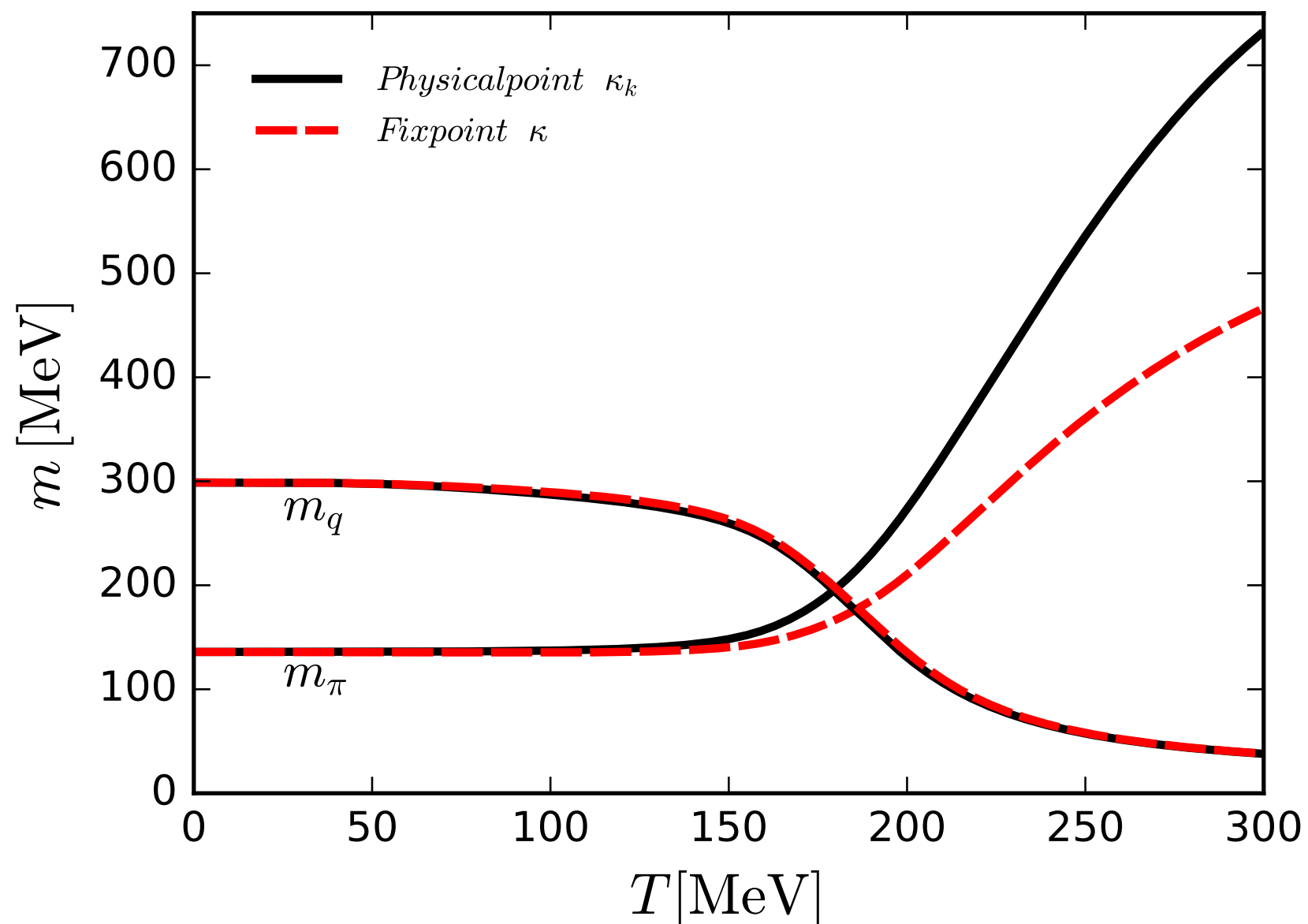
$$\bar{K}_k = Z_{\phi,k}^{\perp} K$$

Physical point:

$$\bar{K}_k = Z_{\phi,k}^{\perp} K_k$$

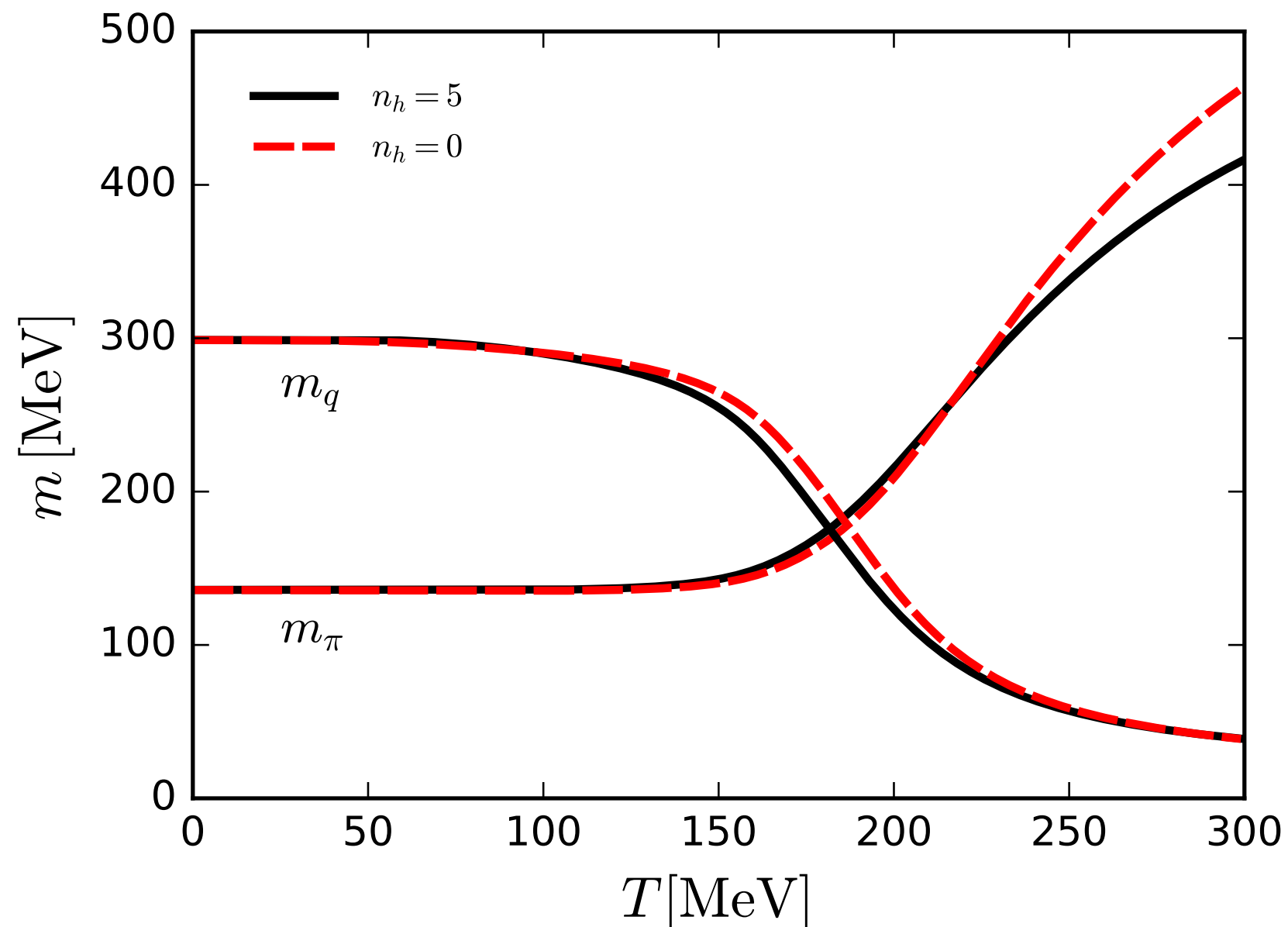
Effective potential and Yukawa coupling

The fix point and physical point for effective potential:



Effective potential and Yukawa coupling

The $n=0$ for Yukawa coupling and $n=5$ for Yukawa coupling:



Summary

- ★ The meson wave function renormalization has a little affect on the low order thermodynamic quantities and will little suppress the higher order fluctuations at high chemical potential.
- ★ The physical point expansion of the effective potential will accelerate the de-couple of the meson mass as the function of temperature. The expansion of the Yukawa coupling will slow down the de-couple of the meson mass as the function of temperature

Thanks for your attention