On decays of $X(3872)$ to $\chi_{cJ}\pi^0$ and $J/\psi\pi^+\pi^-$

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arXiv:1904.07509, with Zhiguang Xiao and Meng-ting Yu

Outline

1. Motivation
2. Gamow state and mathematical backgroud
3. the Friedrichs model
4. The Extended Friedrichs Scheme
5. $X(3872) \to \chi_{cJ}\pi^0, J/\psi\pi^+\pi^-, J/\psi\pi^+\pi^-\pi^0$

Motivation

Uniqueness of $X(3872)$, right on the $D^0\bar{D}^{0*}$ threshold
$M_{X(3872)} = 3871.69 \pm 0.17$ MeV, $\Gamma_{X(3872)} < 1.2$ MeV
Understandings:

- Molecular state
- Tetraquark state
- Mixing state of charmonium and continua
- $\cdots\cdots$

see Reviews:

H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Phys. Rept. 639, 1 (2016)

F.-K. Guo, C. Hanhart, U.-G. Meiner, Q. Wang, Q. Zhao, and B.-S. Zou, Rev. Mod. Phys. 90, 015004 (2018)

Experiment efforts

BES $e^+e^- \to \gamma X(3872)$ with $X(3872) \to \chi_{cJ}\pi^0$, J=0,1,2

M. Ablikim et al., Phys.Rev.Lett. 122 (2019) no.20, 202001

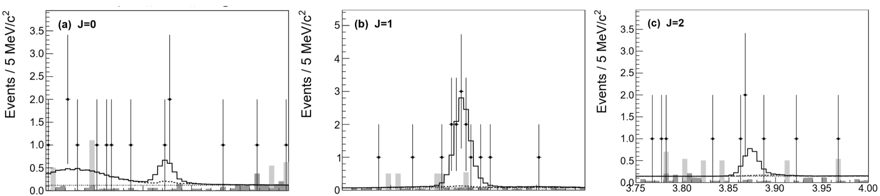


Figure: Distributions of $\pi^0\chi_{cJ}$ mass, $M_{\pi^0\chi_{cJ}}$, from the process $e^+e^- \to \gamma\pi^0\chi_{cJ}$ for (a) J = 0, (b) J = 1, and (c) J = 2. The dashed line is the total background in the fit and includes contributions from events with interchanged γ_1 and γ_2 and cross-feed among the search channels.

$$\frac{\mathcal{B}(X(3872) \to \chi_{c1}\pi^0)}{\mathcal{B}(X(3872) \to J/\psi\pi^+\pi^-)} = 0.88^{+0.33}_{-0.27} \pm 0.10. \quad (1)$$

Experiment efforts

Belle $B^+ \to \chi_{c1}\pi^0 K^+$ V. Bhardwaj et al. Phys. Rev. D 99,(2019) 111101

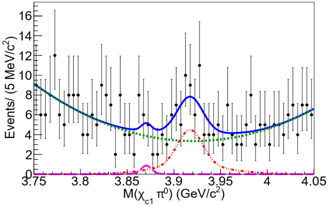


FIG. 2: 1D UML fit to the $M_{\chi_{c1}\pi^0}$ distribution in the -30 MeV $< \Delta E < 20$ MeV signal region for the $B^+ \to (\chi_{c1}\pi^0)K^+$ decay mode. The curves show the $B^+ \to X(3872)(\to \chi_{c1}\pi^0)K^+$ signal (magenta dashed), $B^+ \to X(3915)(\to \chi_{c1}\pi^0)K^+$ signal (red double dotted-dashed), and the background component (green dotted for combinatorial) as well as the overall fit (blue solid). Points with error bar represent the data.

No significant signal of $X(3872) \to \chi_{c1}\pi^0$.

$$\frac{\mathcal{B}(X(3872) \to \chi_{c1}\pi^0)}{\mathcal{B}(X(3872) \to J/\psi\pi^+\pi^-)} < 0.97 \tag{2}$$

at 90% confidence level.

Motivation

S. Dubynskiy and M. B. Voloshin, Phys. Rev., D77, 014013 (2008)
$\Gamma_{\chi_{c0}\pi} : \Gamma_{\chi_{c1}\pi} : \Gamma_{\chi_{c2}\pi} = 0 : 2.7 : 1$ when assuming the $X(3872)$ as a traditional charmonium state
$\Gamma_{\chi_{c0}\pi} : \Gamma_{\chi_{c1}\pi} : \Gamma_{\chi_{c2}\pi} = 2.88 : 0.97 : 1$ as a four-quark state.

Other calculations through EFT method

S. Fleming and T. Mehen, Phys. Rev., D78, 094019 (2008)

S. Fleming and T. Mehen, Phys. Rev., D85, 014016 (2012)

T. Mehen, Phys. Rev., D92, 034019 (2015)

Theoretical calculation of the ratios of $\chi_{cJ}\pi^0$, $J/\psi\pi^+\pi^-$, $J/\psi\pi^+\pi^-\pi^0$ is absent.
Hadron level calculation is not found in the literature.

How to describe $X(3872)$

Uniqueness of $X(3872)$, right on the $D^0\bar{D}^{0*}$ threshold
$M_{X(3872)} = 3871.69 \pm 0.17$ MeV, $\Gamma_{X(3872)} < 1.2$ MeV

We use the extended Friedrichs scheme.

Gamow state and mathematical background

Gamow state, a state with a complex eigenvalue, introduced by Gamow to describe nuclear α decay. G.Gamow , Z.Phys. 51 (1928) 204-212

In conventional QM, due to Hermitian of the Hamiltonian, its eigenvalue is real.

Rigged Hilbert Space (RHS) scheme, developed by Bohm and Gadella, provides a solid mathematical foundation for the unstable state.

A.Bohm, M.Gadella, Dirac kets, Gamow vectors, and Gelfund Triplets, Springer Lectures Notes in Physics Vol.348, Springer, Berlin

One of the properties of Gamow states:

$$\begin{aligned} H|z_R\rangle &= z_R|z_R\rangle \\ \langle z_R|H &= z_R^*\langle z_R| \\ \langle z_R|z_R\rangle &= 0 \end{aligned} \tag{3}$$

$|z_R\rangle$ is not a state in Hilbert space but in RHS.

The Friedrichs Model

[Friedrichs, Commun. Pure Appl. Math.,1(1948),361]

A free Hamiltonian H_0 with a simple continuous spectrum, $\mathbb{R}^+ \equiv [0, \infty)$, plus a discrete eigenvalue ω_0 ($\omega_0 > 0$). An interaction V between the continuous and discrete parts is produced so that the discrete state of H_0 is dissolved in the continuous spectrum and a resonance is produced.

$$H_0|1\rangle = \omega_0|1\rangle, H_0|\omega\rangle = \omega|\omega\rangle. \tag{4}$$

The free Hamiltonian is then

$$H_0 = \omega_0|1\rangle\langle 1| + \int_0^\infty \omega|\omega\rangle\langle\omega|\mathrm{d}\omega, \tag{5}$$

and the interaction V is written as

$$V = \lambda \int_0^\infty [f(\omega)|\omega\rangle\langle 1| + f(\omega)^*|1\rangle\langle\omega|]\mathrm{d}\omega\,. \tag{6}$$

The eigenvalue problem of $H = H_0 + V$ is exactly solvable.

The Friedrichs Model

[Friedrichs, Commun. Pure Appl. Math.,1(1948),361] solve the eigenstate $|\Psi(x)\rangle$ of $H = H_0 + V$ with eigenvalue x,

$$H\Psi(x) = x|\Psi(x)\rangle. \tag{7}$$

Since $|1\rangle$ and $|\omega\rangle$ form a complete set, the eigenstate $|\Psi(x)\rangle$ can be expressed in terms of $|1\rangle$ and $|\omega\rangle$,

$$|\Psi(x)\rangle = \alpha(x)|1\rangle + \int_0^\infty \psi(x,\omega)|\omega\rangle \mathrm{d}\omega. \tag{8}$$

Note that here $|\Psi\rangle$ is a vector in $\Phi^\times$, and it only make senses as an anti-linear functional on vector $|\phi\rangle \in \Phi$, $\langle\phi|\Psi\rangle$. So, $\psi(x,\omega)$ should be treated as a distribution. Substituting (8) into Eq.(7), one can obtain the following relations:

$$\begin{aligned}(\omega_0 - x)\alpha(x) + \lambda\int_0^\infty f(\omega)\psi(x,\omega)\mathrm{d}\omega &= 0,\\ (\omega - x)\psi(x,\omega) + \lambda f(\omega)\alpha(x) &= 0.\end{aligned} \tag{9}$$

[Friedrichs, Commun. Pure Appl. Math.,1(1948),361]

Then, for real $x > 0$, we have

$$\psi_\pm(x,\omega) = -\frac{\lambda\alpha(x)f(\omega)}{\omega - x \pm i\epsilon} + \gamma(\omega)\delta(\omega - x)\,,$$
$$(\omega_0 - x)\alpha_\pm(x) + \lambda f(x)\gamma(x) - \alpha_\pm(x)\lambda^2 \int_0^\infty \frac{|f(\omega)|^2}{\omega - x \pm i\epsilon}\mathrm{d}\omega = 0. \quad (10)$$

one can define

$$\eta^\pm(x) = x - \omega_0 - \lambda^2 \int_0^\infty \frac{|f(\omega)|^2}{x - \omega \pm i\epsilon}\mathrm{d}\omega\,, \quad (11)$$

and analytically continue $\eta^\pm$ to the complex plane $\eta(x)$, and η^+ and η^- are the boundary functions of $\eta(x)$ on the upper rim and lower rim of the cut on the positive axis, respectively.

Resonance state

If $\eta(x) = 0$ has a pair of complex conjugate solutions $z_R \in \mathbb{C}_-$ and $z_R^* \in \mathbb{C}_+$ on the second sheet, the right eigenstates for eigenvalue z_R and z_R^* can be expressed as

$$
\begin{aligned}
|z_R\rangle &= N_R\Big(|1\rangle + \lambda \int_0^\infty \mathrm{d}\omega \frac{f(\omega)}{[z_R - \omega]_+}|\omega\rangle\Big),\\
|z_R^*\rangle &= N_R^*\Big(|1\rangle + \lambda \int_0^\infty \mathrm{d}\omega \frac{f(\omega)}{[z_R^* - \omega]_-}|\omega\rangle\Big),
\end{aligned}
\tag{12}
$$

which are the Gamow states satisfying $H|z_R\rangle = z_R|z_R\rangle$ and $H|z_R^*\rangle = z_R^*|z_R^*\rangle$.

Bound state

If $\eta(x) = 0$ have a solution on the negative real axis on physical Riemann sheet, it represents a bound state. The bound state with eigenvalue z_B can then be represented as

$$|z_B\rangle = N_B\Big(|1\rangle + \lambda \int_0^\infty \frac{f(\omega)}{z_B - \omega}|\omega\rangle \mathrm{d}\omega\Big) \tag{13}$$

where $N_B = (\eta'(z_B))^{-1/2} = (1 + \lambda^2 \int d\omega \frac{|f(\omega)|^2}{(z_B-\omega)^2})^{-1/2}$.

Virtual state

If $\eta(x) = 0$ has a solution on the negative real axis of the second Riemann sheet, it corresponds to a virtual state.
For the simple virtual poles, similar to resonant states, there are two kinds of states, $|z_v^+\rangle$ by analytical continuation from the upper rim and $|z_v^-\rangle$ lower rim of the cut to the second sheet

$$|z_v^{\pm}\rangle = N_v^{\pm}\Big(|1\rangle + \lambda \int_0^{\infty} \frac{f(\omega)}{[z_v - \omega]_{\pm}} |\omega\rangle \mathrm{d}\omega\Big), \quad \langle \tilde{z}_v^{\pm}| = \langle z_v^{\mp}|\,, \tag{14}$$

where $N_v^- = N_v^{+*} = (\eta'^{+}(z_v))^{-1/2} = (1 + \lambda^2 \int d\omega \frac{|f(\omega)|^2}{[(z_v-\omega)_+]^2})^{-1/2}$.

The Extended Friedrichs Scheme

Z. Xiao and ZYZ, J. Math. Phys. 58, 072102 (2017)

Z. Xiao and ZYZ, J. Math. Phys. 58, 062110 (2017)

For a specific total angular momentum, the interaction between a discrete state $|0\rangle$ having a bare energy eigenvalue m_0, and some continuum two-particle states $|E; n, SL\rangle$, where E is the bare energy eigenvalue in the center of mass system (c.m.s) and n, S, L denote the species, total spin and total orbital angular momentum, respectively, can be expressed in the form of extended Friedrichs scheme

$$\begin{aligned} H =& m_0|0\rangle\langle 0| + \sum_{n,S,L}\int_{E_{th,n}}^{\infty} \mathrm{d}E\, E|E; n, SL\rangle\langle E; n, SL| \\ &+ \sum_{n,S,L}\int_{E_{th,n}}^{\infty} \mathrm{d}E\ f^n_{SL}(E)|0\rangle\langle E; n, SL| + h.c. \end{aligned} \quad (15)$$

The coupling form factor between $|A\rangle$ and $|BC\rangle$ in the extended Friedrichs scheme by $f_{SL}(E)$ could be calculated by theoretical models as the QPC model.

The Extended Friedrichs Scheme

Scattering amplitudes could be expressed as

$$S_{fi}(E, E') = \delta(E - E')\Big(\delta_{fi} - 2\pi i \frac{f_i(E) f_f{}^*(E)}{\eta^+(E)}\Big). \tag{16}$$

where the resolvent $\eta^{\pm}(x)$ is defined as

$$\eta^{\pm}(x) = x - m_0 - \sum_{n,S,L} \int_{E_{n,th}}^{\infty} \frac{|f^n_{SL}(E)|^2}{x - E \pm i0} \mathrm{d}E\,. \tag{17}$$

The bound states, virtual states, or resonance state correspond to the zero points of $\eta(z)$ on the different Riemann sheets.

$X(3872)$

ZYZ and Z. Xiao, Phys. Rev. D96, 054031 (2017)

Table: Comparison of the experimental masses and the total widths (in MeV).

$n^{2s+1}L_J$	M_{expt}	Γ_{expt}	M_{BW}	Γ_{BW}	pole	GI
2^3P_2	3927.2 ± 2.6	24 ± 6	3920	10	3920-4i	3979
2^3P_1	3942 ± 9	37^{+27}_{-17}			3934-40i	3953
	3871.69 ± 0.17	< 1.2	3871	0	3871-0i	
2^3P_0	3862^{+66}_{-45}	201^{+242}_{-149}	3878	11	3878-5i	3917
2^1P_1			3895	37	3902-27i	3956

For $X(3872)$, the "elementariness" and "compositeness"

$Z_{c\bar{c}} : X_{\bar{D}^0 D^{0*}} : X_{D^+ D^{-*}} : X_{\bar{D}^* D^*} = 1 : (3.1 \sim 9.3) : (0.45 \sim 0.46) : 0.04$

Only one free parameter $\gamma = 4.0$, so does the following calculation.

$X(3872)$ wave function

Wave function of the $X(3872)$ is expressed explicitly as

$$
\begin{aligned}
|X(3872)\rangle =& N_B\Big(|c\bar{c}\rangle + \int_{M_{00V}}^{\infty} \mathrm{d}E \sum_{S,L} \frac{f_{SL}^{00V}(E)}{z_X - E}\left(|E; D^0\bar{D}^{0*}, SL\rangle + |E; D^{0*}\bar{D}^0, SL\rangle\right) \\
&+ \int_{M_{+-V}}^{\infty} \mathrm{d}E \sum_{S,L} \frac{f_{SL}^{+-V}(E)}{z_X - E}\left(|E; D^+ D^{-*}, SL\rangle + |E; D^{+*} D^-, SL\rangle\right) \\
&+ \int_{M_{0V0V}}^{\infty} \mathrm{d}E \sum_{S,L} \frac{f_{SL}^{0V0V}(E)}{z_X - E}|E; D^{0*}\bar{D}^{0*}, SL\rangle \\
&+ \int_{M_{+V-V}}^{\infty} \mathrm{d}E \sum_{S,L} \frac{f_{SL}^{+V-V}(E)}{z_X - E}|E; D^{+*} D^{-*}, SL\rangle\Big),
\end{aligned}
\tag{18}
$$

where $N_B = \eta'(z_X)^{-1/2}$ is the normalization factor and z_X the mass of $X(3872)$.

$X(3872) \to \chi_{cJ}\pi^0, J/\psi\pi^+\pi^-, J/\psi\pi^+\pi^-\pi^0$

$d\Gamma(\alpha \to \beta) = 2\pi |M_{\beta\alpha}|^2 \delta^4(p_{\beta_1} + p_{\beta_2} - p_\alpha) d^3\vec{p}_{\beta_1} d^3\vec{p}_{\beta_2}$ where $M_{\beta\alpha}$ is the transition amplitude.

$$\Gamma(\alpha \to \beta) = \sum_{l's'} 2\pi |M_{l's'}|^2 \mu' k' = \sum_{l's'} 2\pi |F_{l's'}|^2 \tag{19}$$

$$\begin{aligned}
F_{l's'} &= {}_{l's'}\langle \chi_{cJ}\pi^0 | H_I | X(3872) \rangle = N_B \Big({}^{\chi_{cJ}\pi^0}_{l's'} \langle E' | H_I | c\bar{c} \rangle \\
&+ \int_{M_{00}}^{\infty} \mathrm{d}E \sum_{l,s} \frac{f_{ls}^{00}(E)}{z_X - E} \big({}^{\chi_{cJ}\pi^0}_{l's'} \langle E' | H_I | E \rangle_{ls}^{D^0\bar{D}^{0*}} + C.C. \big) \\
&+ \int_{M_{+-}}^{\infty} \mathrm{d}E \sum_{l,s} \frac{f_{ls}^{+-}(E)}{z_X - E} \big({}^{\chi_{cJ}\pi^0}_{l's'} \langle E' | H_I | E \rangle_{ls}^{D^+D^{-*}} + C.C. \big) \\
&+ \cdots \Big)
\end{aligned} \tag{20}$$

$X(3872) \to \chi_{cJ}\pi^0, J/\psi\pi^+\pi^-, J/\psi\pi^+\pi^-\pi^0$

The Barnes-Swanson model

T. Barnes and E. S. Swanson, Phys. Rev. D 46, 131 (1992)

T. Barnes, N. Black, D. J. Dean, and E. S. Swanson, Phys. Rev. C60, 045202 (1999)

T. Barnes, N. Black, and E. S. Swanson, Phys. Rev. C63, 025204 (2001)

The interaction Hamiltonian of the Barnes-Swanson model in the momentum space is

$$T_{fi} = \begin{cases} -\frac{8\pi\alpha_s}{3m_1m_2}[\vec{S}_1 \cdot \vec{S}_2] & \text{Spin} - \text{spin} \\ \frac{4\pi\alpha_s}{q^2} I & \text{Coulomb} \\ \frac{6\pi b}{q^4} I & \text{Linear} \\ \frac{4i\pi\alpha_s}{q^2}\{\vec{S}_1 \cdot [\vec{q} \times (\frac{\vec{p}_1}{2m_1^2} - \frac{\vec{p}_2}{m_1m_2})] + \vec{S}_2 \cdot [\vec{q} \times (\frac{\vec{p}_1}{m_1m_2} - \frac{\vec{p}_2}{2m_2^2})]\} & \text{OGE spin} - \text{orbit} \\ -\frac{3i\pi b}{q^4}[\frac{1}{m_1^2}\vec{S}_1 \cdot (\vec{q} \times \vec{p}_1) - \frac{1}{m_2^2}\vec{S}_2 \cdot (\vec{q} \times \vec{p}_2)] & \text{Linear spin} - \text{orbit} \\ \frac{4\pi\alpha_s}{m_1m_2q^2}[\vec{S}_1 \cdot \vec{q}\vec{S}_2 \cdot \vec{q} - \frac{1}{3}q^2\vec{S}_1 \cdot \vec{S}_2] & \text{OGE tensor} \end{cases}$$

$X(3872) \to \chi_{cJ}\pi^0, J/\psi\pi^+\pi^-, J/\psi\pi^+\pi^-\pi^0$

The Barnes-Swanson Model
Four kinds of diagrams are considered, among which the quark-antiquark interactions are denoted as $Capture_1$, $Capture_2$, and the quark-quark(antiquark-antiquark) interactions are denoted as $Transfer_1$, and $Transfer_2$.

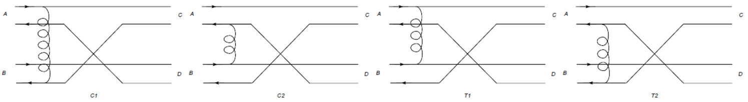


FIG. 2. The four quark rearrangement diagrams of $AB \to CD$ meson-meson scatterings. The arrows represent the quark line directions.

To reduce the so-called "prior-poster" ambiguity, the four "poster" diagrams are considered similarly and averaged to obtain the final result.

$X(3872) \to \chi_{cJ}\pi^0, J/\psi\pi^+\pi^-, J/\psi\pi^+\pi^-\pi^0$

$$F_{l's'} = {}_{l's'}\langle \chi_{cJ}\pi^0|H_I|X(3872)\rangle = N_B\Big({}^{\chi_{cJ}\pi^0}_{l's'}\langle E'|H_I|c\bar{c}\rangle$$

$$+ \int_{M_{00}}^{\infty} \mathrm{d}E \sum_{l,s} \frac{f^{00}_{ls}(E)}{z_X - E} \big({}^{\chi_{cJ}\pi^0}_{l's'}\langle E'|H_I|E\rangle^{D^0\bar{D}^{0*}}_{ls} + C.C.\big)$$

$$+ \int_{M_{+-}}^{\infty} \mathrm{d}E \sum_{l,s} \frac{f^{+-}_{ls}(E)}{z_X - E} \big({}^{\chi_{cJ}\pi^0}_{l's'}\langle E'|H_I|E\rangle^{D^+D^{-*}}_{ls} + C.C.\big) + \cdots \Big)$$

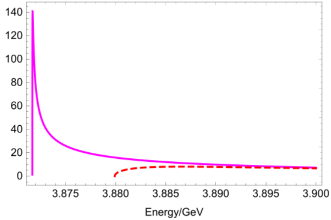


Figure: Comparison of the integrands $\frac{f_{ls}\mathcal{M}_{l's',ls}}{(z_X - E)}$ for $D^0\bar{D}^{0*} \to \chi_{c1}\pi^0$ (solid) and $D^+\bar{D}^{-*} \to \chi_{c1}\pi^0$ (dashed).

$X(3872) \to J/\psi\rho, \omega$

With the Barnes-Swanson model, one could obtain

$D\bar{D}^* \to \chi_{c0}\pi^0, \chi_{c1}\pi^0, \chi_{c2}\pi^0, J/\psi\rho, J/\psi\omega$ ($\rho \to \pi\pi$ and $\omega \to \pi\pi\pi$)

$$\Gamma_{J/\psi\pi\pi} = \int_{2m_\pi}^{m_X - m_{J/\psi}} \sum_{l,s} \frac{|F_{l,s}(X \to J/\psi\rho)|^2 \Gamma_\rho}{(E - m_\rho)^2 + \Gamma_\rho^2/4} \mathrm{d}E,$$

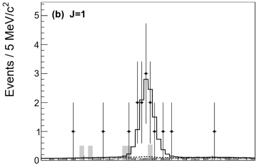
$$\Gamma_{J/\psi\pi\pi\pi} = \int_{3m_\pi}^{m_X - m_{J/\psi}} \sum_{l,s} \frac{|F_{l,s}(X \to J/\psi\omega)|^2 \Gamma_\omega}{(E - m_\omega)^2 + \Gamma_\omega^2/4} \mathrm{d}E,$$

the branch fraction will be

$$\Gamma_{\chi_{c0}\pi} : \Gamma_{\chi_{c1}\pi} : \Gamma_{\chi_{c2}\pi} : \Gamma_{J/\psi 2\pi} : \Gamma_{J/\psi 3\pi} = 1.5 : 1.3 : 1.0 : 16 : 26. \quad (22)$$

This numerical results of $\Gamma_{J/\psi 2\pi} : \Gamma_{J/\psi 3\pi}$ are consistent with the measured ratio $1.0 \pm 0.4 \pm 0.3$ by Belle, 0.8 ± 0.3 by *BABAR*, and $1.6^{+0.4}_{-0.3} \pm 0.2$ by *BESIII*. hep-ex/0505037, PRD82,011101,PRL122, 232002

Comparisons with experiments



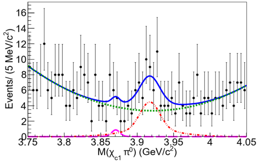


FIG. 2: 1D UML fit to the $M_{\chi_{c1}\pi^0}$ distribution in the -30 MeV $< \Delta E < 20$ MeV signal region for the $B^+ \to (\chi_{c1}\pi^0)K^+$ decay mode. The curves show the $B^+ \to X(3872)(\to \chi_{c1}\pi^0)K^+$ signal (magenta dashed), $B^+ \to X(3915)(\to \chi_{c1}\pi^0)K^+$ signal (red double dotted-dashed), and the background component (green dotted for combinatorial) as well as the overall fit (blue solid). Points with error bar represent the data.

BES

$$\frac{\mathcal{B}(X(3872) \to \chi_{c1}\pi^0)}{\mathcal{B}(X(3872) \to J/\psi\pi^+\pi^-)} = 0.88^{+0.33}_{-0.27} \pm 0.10. \tag{23}$$

Belle

No significant signal of $X(3872) \to \chi_{c1}\pi^0$.

Still an open question.

Summary

- The Extended Friedrichs Scheme might shed more light on hadron physics.
- With only one free parameter, the properties of $X(3872)$ could be understood.
- The isospin breaking effect of $X(3872)$ is understood easily.
- A rough prediction of $X(3872)$ decaying to P-wave $\chi_{cJ}\pi^0, J = 0, 1, 2$, is about one order smaller than its decay to S-wave $J/\psi\pi\pi$.

Thanks for your patience!

Extra slides

Extra slides