Radiative transitions and magnetic moments of the charmed and bottom vector mesons in chiral perturbation theory

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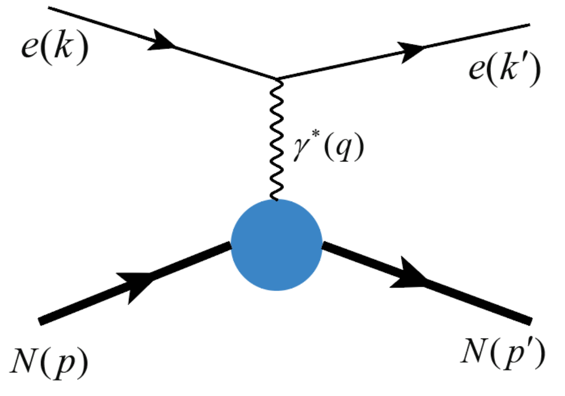
Outline

- Introduction
- Electromagnetic form factors and magnetic moments
- Effective Lagrangians
- Radiative transitions
- Magnetic moments
- Summary

Introduction

- Nucleon electromagnetic form factors are fundamental quantities related to the charge and magnetization distributions inside the nucleon.



$$T_{fi} = -i \int j_\mu (\frac{-1}{q^2}) J^\mu d^4x,$$

$$j^\mu = -e\bar{u}(k')\gamma^\mu u(k) e^{i(k'-k)\cdot x}$$

$$J^\mu = e\bar{u}(p') \left[F_1(q^2)\gamma^\mu + \frac{\kappa}{2M} F_2(q^2) i\sigma^{\mu\nu} q_\nu \right] u(p) e^{i(p'-p)\cdot x},$$

$$G_E = F_1 + \frac{\kappa q^2}{4M^2} F_2,$$

$$G_M = F_1 + \kappa F_2.$$

Sachs' form factors

$F_1(q^2)$ and $F_2(q^2)$ are Dirac and Pauli form factors, respectively.

Int. J. Mod. Phys. E **12**, 1 (2003); J. Phys. G **34**, S23 (2007); Phys. Rept. **550-551**, 1 (2015); Rev. Mod. Phys. **91**, 015001 (2019).

- Probing the shape and inner structure of hadrons still remain an intriguing and challenging topic.

Introduction

- Unlike proton and neutron, vast majority of hadronic states are unstable against strong interactions
- Magnetic moments can be related to the form factors by extrapolating the form factor $G_M(q^2)$ to zero moment transfer.
- The radiative transition is a very effective way to help us catch a glimpse of quark dynamics in the hadrons.
- We report the study on the radiative transitions and magnetic moments of the charmed and bottom vector mesons.

$\boldsymbol{\bar{D}^{*0}/\bar{D}^{0}}$	$\boldsymbol{D^{*-}/D^{-}}$	$\boldsymbol{D_s^{*-}/D_s^{-}}$
$\boldsymbol{B^{*+}/B^{+}}$	$\boldsymbol{B^{*0}/B^{0}}$	$\boldsymbol{B_s^{*0}/B_s^{0}}$

Introduction

- As a consequence of heavy quark spin symmetry, the mass shifts between these spin triplets and singlets are generally small.
- $m_{D^*} - m_D \sim m_\pi$, $m_{B^*} - m_B \ll m_\pi$.

$D^*(2007)^0$ DECAY MODES

$\overline{D}^*(2007)^0$ modes are charge conjugates of modes below.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D^0\pi^0$	(64.7 ± 0.9) %
Γ_2	$D^0\gamma$	(35.3 ± 0.9) %

$D^*(2010)^\pm$ DECAY MODES

$D^*(2010)^-$ modes are charge conjugates of the modes below.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D^0\pi^+$	(67.7 ± 0.5) %
Γ_2	$D^+\pi^0$	(30.7 ± 0.5) %
Γ_3	$D^+\gamma$	$(\ 1.6\pm0.4)$ %

Particle Data Group, Phys. Rev. D **98**, 030001

D_s^{*+} DECAY MODES

D_s^{*-} modes are charge conjugates of the modes below.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$D_s^+\gamma$	(93.5 ± 0.7) %
Γ_2	$D_s^+\pi^0$	$(\ 5.8\pm0.7)$ %
Γ_3	$D_s^+e^+e^-$	$(\ 6.7\pm1.6)\times10^{-3}$

B^* DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$B\gamma$	dominant

?

B_s^* DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$B_s\gamma$	dominant

Introduction

- **Before our study:**

1. Various quark models: Phys. Rev. D **36**, 2074 (1987); Phys. Lett. B **284**, 421 (1992); Z. Phys. C **67**, 633 (1995); Phys. Rev. D **53**, 1349 (1996).
2. Heavy quark effective theory and vector meson dominance model: Phys. Lett. B **316**, 555 (1993).
3. Quark-potential models: Phys. Rev. D **32**, 189 (1985); Phys. Rev. D **64**, 094007 (2001); Phys.Lett. B **537**, 241 (2002).
4. QCD sum rules: Phys. Lett. B **334**, 169 (1994); Phys. Lett. B **368**, 163 (1996); Mod. Phys. Lett. A **12**, 3027 (1997).
5. Lattice QCD simulations: Eur. Phys. J. C **71**, 1734 (2011).
6. Constituent quark-meson model: Phys. Rev. D **58**, 034004 (1998).
7. Chiral effective field theory: Phys. Rev. D **47**, 1030 (1993); Phys. Lett. B **296**, 415 (1992); Phys. Rept. **281**, 145 (1997).
8. Extended Nambu-Jona-Lasinio model: Chin. Phys. C **38**, 013103 (2014); Chin. Phys. C **39**, 113103 (2015).

- **The SU(3) chiral perturbation theory (χPT) is used in our work**.

Electromagnetic form factors and magnetic moments

- $V \to P\gamma$

$$\langle P(p')|J^{\mu}_{em}(q^2)|V(p,\varepsilon_V)\rangle = e\mu'(q^2)\epsilon^{\mu\nu\alpha\beta}p_\nu q_\alpha \varepsilon_{V\beta},$$

$$H_{\text{int}} = \int d^3x e A_\mu J^{\mu}_{em},$$

- In heavy quark limit, $|\mathcal{H}(p)\rangle = \sqrt{m_H}\left[|\mathcal{H}(v)\rangle + \mathcal{O}(1/m_H)\right].$

$$\langle P(p')|J^{\mu}_{em}|V(p,\varepsilon_V)\rangle = e\sqrt{m_V m_P}\mu'(q^2)\epsilon^{\mu\nu\alpha\beta}v_\nu q_\alpha \varepsilon_{V\beta},$$

Phys. Rept. **281**, 145 (1997)

- The radiative decay rate

$$\Gamma[V \to P\gamma] = \frac{1}{3}\int d\Omega_{\hat{q}} \frac{1}{32\pi^2}\frac{|\boldsymbol{q}|}{m_V^2}\sum|\mathcal{M}|^2, \Longrightarrow \Gamma[V \to P\gamma] = \frac{\alpha}{3}\frac{m_P}{m_V}|\mu'(0)|^2|\boldsymbol{q}|^3,$$

- The transition magnetic moment $\mu_{V\to P\gamma} = \frac{e}{2}\mu'(0).$

Electromagnetic form factors and magnetic moments

- $V \to V\gamma$
- The magnetic moment of a vector meson,

$$\begin{aligned}\mathcal{G}^\mu(q^2) &= \langle V(p',\varepsilon'^*)|J^\mu_{em}(q^2)|V(p,\varepsilon)\rangle \\ &= -\mathcal{G}_1(q^2)(\varepsilon\cdot\varepsilon'^*)(p+p')^\mu \\ &\quad +\mathcal{G}_2(q^2)\left[(\varepsilon\cdot q)\varepsilon'^{*\mu}-(\varepsilon'^*\cdot q)\varepsilon^\mu\right] \\ &\quad +\mathcal{G}_3(q^2)\frac{(\varepsilon\cdot q)(\varepsilon'^*\cdot q)}{2m_V^2}(p+p')^\mu.\end{aligned}$$

Breit frame

$$q^\mu=(p-p')^\mu=(0,\boldsymbol{Q}),\quad \boldsymbol{Q}=Q\hat{z},\quad p^\mu=(p^0,\tfrac{1}{2}\boldsymbol{Q}),$$
$$p'^\mu=(p^0,-\tfrac{1}{2}\boldsymbol{Q}),\quad -q^2=Q^2\geq 0,\quad p^0=\sqrt{m_V^2+\tfrac{1}{4}Q^2}.$$

Charge

Quadrupole

$$\begin{aligned}\mathcal{G}^0(Q^2) &= 2p^0\Big\{\mathcal{G}_C(Q^2)(\varepsilon\cdot\varepsilon'^*)+\frac{\mathcal{G}_Q(Q^2)}{2m_V^2}\Big[(\varepsilon\cdot\boldsymbol{Q})(\varepsilon'^*\cdot\boldsymbol{Q}) \\ &\quad -\frac{1}{3}(\varepsilon\cdot\varepsilon'^*)Q^2\Big]\Big\},\end{aligned} \tag{9}$$

Dipole

$$\begin{aligned}\boldsymbol{\mathcal{G}}(Q^2) &= \mathcal{G}_2(Q^2)\left[(\varepsilon'^*\cdot\boldsymbol{Q})\varepsilon-(\varepsilon\cdot\boldsymbol{Q})\varepsilon'^*\right] \\ &= 2p^0\frac{\mathcal{G}_M(Q^2)}{2m_V}\left[(\varepsilon'^*\cdot\boldsymbol{Q})\varepsilon-(\varepsilon\cdot\boldsymbol{Q})\varepsilon'^*\right],\end{aligned}$$

Effective Lagrangians

- **The leading order chiral Lagrangians**
- The Lagrangian of Goldstone bosons and photon,

$$\Gamma_\mu \equiv \frac{1}{2}\left[u^\dagger\left(\partial_\mu - ir_\mu\right)u + u\left(\partial_\mu - il_\mu\right)u^\dagger\right],$$
$$u_\mu \equiv \frac{i}{2}\left[u^\dagger\left(\partial_\mu - ir_\mu\right)u - u\left(\partial_\mu - il_\mu\right)u^\dagger\right],$$

$$\phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}.$$

$$u^2 = U = \exp\left(\frac{i\phi}{f_\phi}\right), \qquad r_\mu = l_\mu = -eQA_\mu,$$

$$Q = Q_l = \mathrm{diag}(2/3, -1/3, -1/3)$$

$$\mathcal{L}_{\phi\gamma}^{(2)} = \frac{f_\phi^2}{4}\,\mathrm{Tr}\left[\nabla_\mu U\,(\nabla^\mu U)^\dagger\right],$$

$$\nabla_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu.$$

Phys. Rev. D **96**, 076011 (2017);
Phys. Lett. B **777**, 169 (2018);
Eur. Phys. J. C **77**, 869 (2017).

Effective Lagrangians

- The Lagrangians for heavy matter field

$$\mathcal{L}^{(1)}_{H\phi} = -i\langle \bar{\mathcal{H}} v\cdot \mathcal{D}\mathcal{H}\rangle - \frac{1}{8}\Delta\langle \bar{\mathcal{H}}\sigma^{\mu\nu}\mathcal{H}\sigma_{\mu\nu}\rangle + g\langle \bar{\mathcal{H}}\not{u}\gamma_5\mathcal{H}\rangle,$$

$$\begin{aligned}\mathcal{H} &= \left(P^*_\mu\gamma^\mu + iP\gamma_5\right)\frac{1-\not{v}}{2},\\ \bar{\mathcal{H}} &= \gamma^0\mathcal{H}^\dagger\gamma^0 = \frac{1-\not{v}}{2}\left(P^{*\dagger}_\mu\gamma^\mu + iP^\dagger\gamma_5\right),\end{aligned}$$

- The Lagrangians for describing the (transition) magnetic moments at the tree level

$$\mathcal{L}^{(2)}_{H\gamma} = \tilde{a}\langle \bar{\mathcal{H}}\sigma^{\mu\nu}\tilde{f}^{+}_{\mu\nu}\mathcal{H}\rangle + a\langle \mathcal{H}\sigma^{\mu\nu}\bar{\mathcal{H}}\rangle\,\mathrm{Tr}(f^{+}_{\mu\nu}),$$

$$\begin{aligned} f^R_{\mu\nu} &= f^L_{\mu\nu} = -eQ\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right),\\ f^{\pm}_{\mu\nu} &= u^\dagger f^R_{\mu\nu}u \pm u f^L_{\mu\nu}u^\dagger,\\ \tilde{f}^{\pm}_{\mu\nu} &= f^{\pm}_{\mu\nu} - \frac{1}{3}\mathrm{Tr}(f^{\pm}_{\mu\nu}),\end{aligned}$$

- The Lagrangians for the interactions of matter fields with two Goldstone fields

$$\mathcal{L}^{(2)}_{H\phi\phi} = ib\langle \bar{\mathcal{H}}\sigma^{\mu\nu}[u_\mu, u_\nu]\mathcal{H}\rangle.$$

Effective Lagrangians

- **The higher order chiral Lagrangians**
- The electromagnetic chiral Lagrangians at $O(p^3)$ read

$$\mathcal{L}_{H\gamma}^{(3)} = -i\tilde{c}\langle\bar{\mathcal{H}}\sigma^{\mu\nu}v\cdot\nabla\tilde{f}_{\mu\nu}^{+}\mathcal{H}\rangle - ic\langle\mathcal{H}\sigma^{\mu\nu}\bar{\mathcal{H}}\rangle v\cdot\nabla\mathrm{Tr}(f_{\mu\nu}^{+}).$$

$$\tilde{a} \rightarrowtail \tilde{a} + \tilde{c}v\cdot q, \qquad a \rightarrowtail a + cv\cdot q.$$

- At $O(p^4)$

$$\begin{aligned}\mathcal{L}_{H\gamma}^{(4)} &= \tilde{d}\langle\mathcal{H}\sigma^{\mu\nu}\tilde{\chi}_{+}\bar{\mathcal{H}}\rangle\mathrm{Tr}(f_{\mu\nu}^{+}) + \bar{d}\langle\bar{\mathcal{H}}\sigma^{\mu\nu}\mathcal{H}\rangle\mathrm{Tr}(\tilde{f}_{\mu\nu}^{+}\tilde{\chi}_{+})\\ &+d\langle\bar{\mathcal{H}}\sigma^{\mu\nu}\{\tilde{\chi}_{+},\tilde{f}_{\mu\nu}^{+}\}\mathcal{H}\rangle,\end{aligned}$$

$$\begin{aligned}\chi &= 2B_0\mathrm{diag}(m_u,m_d,m_s) = \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2),\\ \chi_\pm &= u^\dagger\chi u^\dagger \pm u\chi^\dagger u.\end{aligned}$$

$$\begin{aligned}\chi_+ &= \mathrm{diag}(2m_\pi^2, 2m_\pi^2, 4m_K^2 - 2m_\pi^2),\\ \tilde{\chi}_+ &= \chi_+ - \frac{1}{3}\mathrm{Tr}(\chi_+).\end{aligned}$$

- The contributions of $O(p^4)$ tree diagrams are added into our numerical results as errors.

Radiative transitions

- The standard power counting scheme gives the chiral order of a Feynman diagram as

$$\mathcal{O} = 4N_L - 2I_M - I_H + \sum_n nN_n,$$

- The order of the (transition) magnetic moment is $\mathcal{O}_\mu = \mathcal{O} - 1.$
- The transition form factors of $V \to P\gamma$ can be expressed as follows,

$$\mu'_{V\to P\gamma} = \left[\mu'^{(1)}_{\text{tree}}\right] + \left[\mu'^{(2)}_{\text{loop}}\right] + \left[\mu'^{(3)}_{\text{tree}} + \mu'^{(3)}_{\text{loop}}\right],$$

Tree Level

$\boxed{O(p^2)}$ (a) $\boxed{O(p^4)}$ (b)

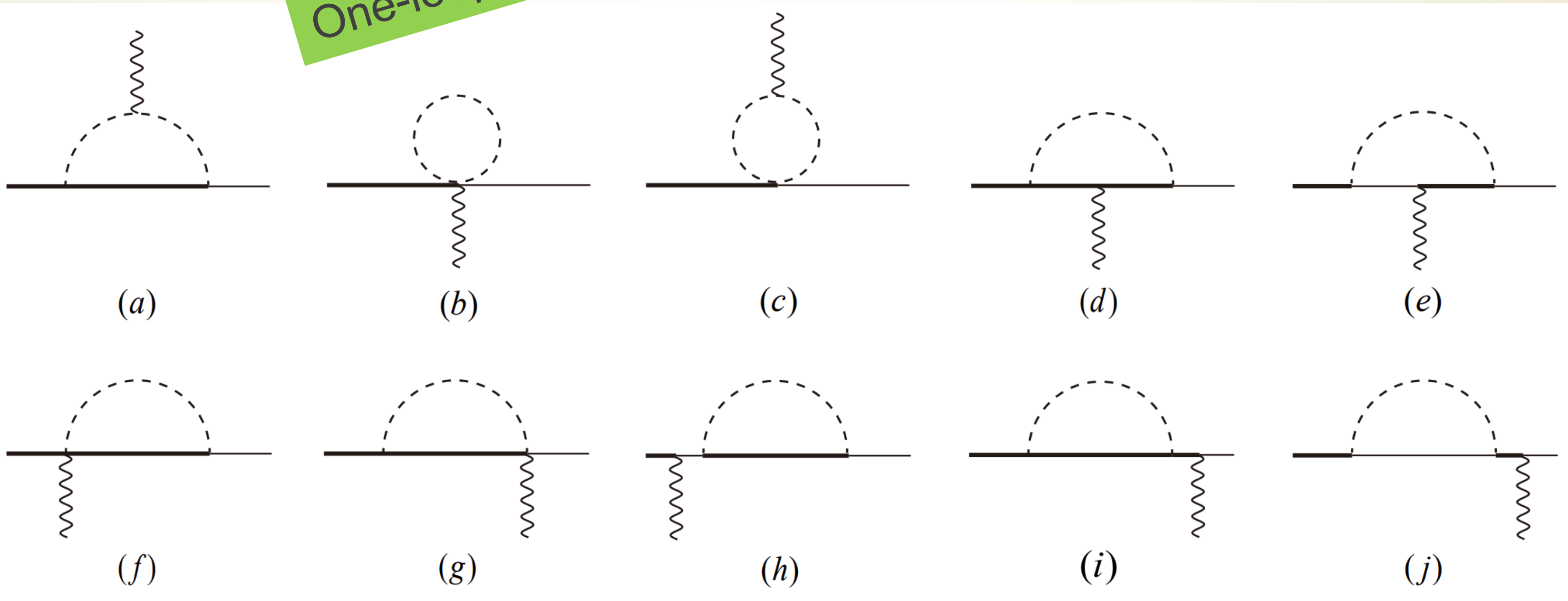
Transition form factors at $O(p^2)$

$$\mu'^{(a)}_{\bar{D}^{*0}\to\bar{D}^0\gamma} = \frac{16}{3}(\tilde{a} - 3a),$$
$$\mu'^{(a)}_{D^{*-}\to D^-\gamma} = -\frac{8}{3}(\tilde{a} + 6a),$$
$$\mu'^{(a)}_{D_s^{*-}\to D_s^-\gamma} = -\frac{8}{3}(\tilde{a} + 6a),$$

Radiative transitions

- The one-loop diagrams

One-loop Level



- Except for figure (a), the leading order low-energy-constants (LECs) $a, \tilde{a}, b$ also appear in above loop diagrams.

Radiative transitions

- **Estimation of the leading order LECs**
- Estimate the values of $\boldsymbol{a}$ and $\tilde{\boldsymbol{a}}$ with constituent quark model

$$\langle P|\mathcal{L}_{\rm em}|V\rangle = 2\sqrt{m_V m_P}\langle P|\sum_i \frac{e_i}{2m_i}\boldsymbol{\sigma}|V\rangle\cdot\mathbf{B},$$

$$|V\rangle = \frac{1}{\sqrt{2}}|\bar{Q}\uparrow q\downarrow + \bar{Q}\downarrow q\uparrow\rangle,$$

$$|P\rangle = \frac{1}{\sqrt{2}}|\bar{Q}\uparrow q\downarrow - \bar{Q}\downarrow q\uparrow\rangle.$$

Phys. Rev. D **47**, 1030 (1993).

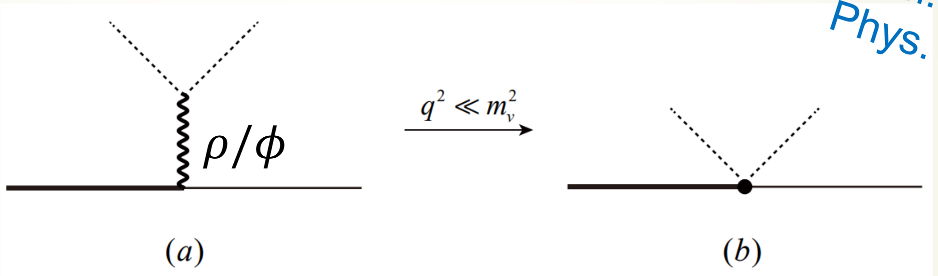
$$\langle P|\mathcal{L}_{\rm em}|V\rangle = 2\sqrt{m_V m_P}(\mu_{\bar{Q}} - \mu_q), \quad \mu_i = e_i/(2m_i)$$

$$\tilde{a} = -\frac{1}{8m_q}, \qquad a = \frac{1}{24m_{\bar{Q}}},$$

Radiative transitions

- Estimate the value of b with resonance saturation model

Phys. Rev. C **65**, 044001 (2002);
Nucl. Phys. B **321**, 311 (1989);
Phys. Rept. **281**, 145 (1997).



- $VP\rho$ Lagrangians

$$\mathcal{L}_{H\rho} = i\beta\langle\bar{\mathcal{H}}v^\mu(\mathcal{V}_\mu - \rho_\mu)\mathcal{H}\rangle + i\lambda\langle\bar{\mathcal{H}}\sigma^{\mu\nu}F_{\mu\nu}(\rho)\mathcal{H}\rangle,$$

$$F_{\mu\nu}(\rho) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu + [\rho_\mu, \rho_\nu], \quad \rho_\mu = i\frac{g_v}{\sqrt{2}}\hat{\rho}_\mu,$$

- $\rho\pi\pi$ (ϕKK) Lagrangians

$$\hat{\rho}^\mu = \begin{pmatrix} \frac{\rho_0+\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho_0+\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}^\mu .$$

$$\mathcal{L}_{v\phi} = f_\phi^2 a \mathrm{Tr}(\Gamma_\mu^{(0)}\rho^\mu + \rho^\mu\Gamma_\mu^{(0)}), \quad a = 2,$$

$$b = -\frac{2\lambda g_v^2 f_\phi^2}{m_v^2},$$

Radiative transitions

- **The numerical results**

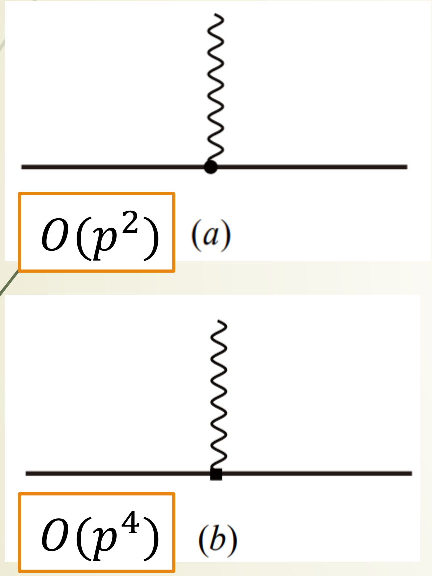
TABLE IV: The radiative decay widths for $V \to P\gamma$ (in units of keV). $\mathrm{Br}_{\mathrm{expt}}$ and Γ_{expt} denote the branching ratio and decay width measured in experiments. $\Gamma_{1,\ldots,4}$ are the model predictions.

Decay modes	SU(2)		SU(3)		Experimental data and model predictions				
	$\Delta = 0$	$\Delta \neq 0$	$\Delta = 0$	$\Delta \neq 0$	$\mathrm{Br}_{\mathrm{expt}} \vert \Gamma_{\mathrm{expt}}$ [9]	Γ_1 [14]	Γ_2 [15]	Γ_3 [19]	Γ_4 [25]
$\bar{D}^{*0} \to \bar{D}^0\gamma$	$30.0^{+7.3}_{-6.6}$	$23.9^{+5.0}_{-6.3}$	$22.9^{+8.2}_{-7.0}$	$16.2^{+6.5}_{-6.0}$	$(38.1 \pm 2.9)\% \vert \cdots$	20.0 ± 0.3	26.5	11.5	12.9 ± 2
$D^{*-} \to D^-\gamma$	$1.0^{+0.9}_{-0.6}$	$0.5^{+0.5}_{-0.4}$	$1.8^{+1.3}_{-0.9}$	$0.73^{+0.7}_{-0.3}$	$(1.6 \pm 0.4)\% \vert 1.33 \pm 0.33$	0.9 ± 0.02	0.93	1.04	0.23 ± 0.1
$D_s^{*-} \to D_s^-\gamma$	$\cdots$	$\cdots$	$0.15^{+0.5}_{-0.1}$	$0.32^{+0.3}_{-0.3}$	$(94.2 \pm 0.7)\% \vert \cdots$	0.18 ± 0.01	0.21	0.19	0.13 ± 0.05
$B^{*+} \to B^+\gamma$	$0.75^{+0.2}_{-0.2}$	$0.71^{+0.2}_{-0.2}$	$0.63^{+0.2}_{-0.2}$	$0.58^{+0.2}_{-0.2}$	$\cdots \vert \cdots$	0.4 ± 0.03	0.58	0.19	0.13 ± 0.03
$B^{*0} \to B^0\gamma$	$0.19^{+0.05}_{-0.05}$	$0.18^{+0.05}_{-0.05}$	$0.25^{+0.06}_{-0.06}$	$0.23^{+0.06}_{-0.06}$	$\cdots \vert \cdots$	0.13 ± 0.01	0.18	0.07	0.38 ± 0.06
$B_s^{*0} \to B_s^0\gamma$	$\cdots$	$\cdots$	$0.05^{+0.03}_{-0.03}$	$0.04^{+0.03}_{-0.03}$	$\cdots \vert \cdots$	0.068 ± 0.017	0.12	0.05	0.22 ± 0.04

- $\boldsymbol{\Gamma_1}$: light-front quark model; $\boldsymbol{\Gamma_2}$: relativistic independent quark model; $\boldsymbol{\Gamma_3}$: relativistic quark model; $\boldsymbol{\Gamma_4}$: QCD sum rule.

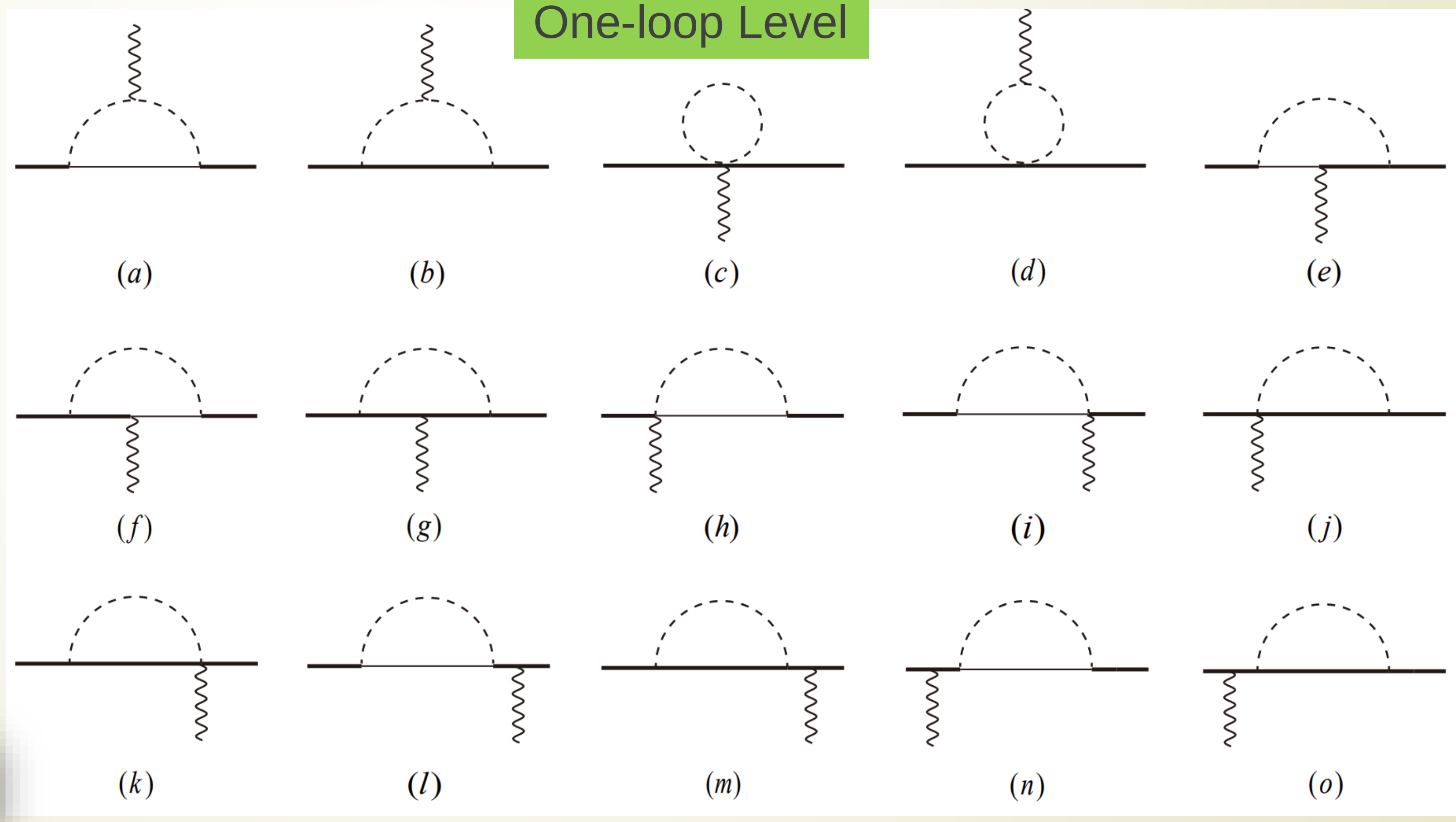
Magnetic moments

- **The Feynman diagrams that contribute to the magnetic moments**



Tree Level

One-loop Level



$$\mu_V = \left[\mu^{(1)}_{\text{tree}}\right] + \left[\mu^{(2)}_{\text{loop}}\right] + \left[\mu^{(3)}_{\text{tree}} + \mu^{(3)}_{\text{loop}}\right],$$

Magnetic moments

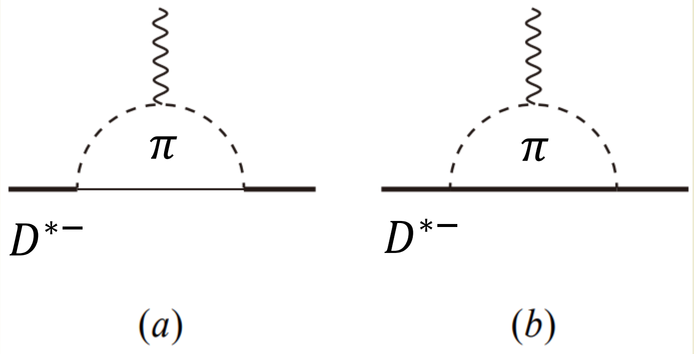
- The anomalous magnetic moments of nucleons reveal that the proton and neutron are not elementary particles and they have internal substructures. As in the case of nucleons, the magnetic moments of D^* and B^* also encode important information of their underlying substructures.

TABLE VIII: The transition magnetic moments and magnetic moments of the charmed and bottom vector mesons calculated in the SU(2) case order by order (in units of μ_N).

Physical quantity	$\Delta = 0$				$\Delta \neq 0$			
	$\mathcal{O}_\mu(p^1)$ Tree	$\mathcal{O}_\mu(p^2)$ Loop	$\mathcal{O}_\mu(p^3)$ Loop	Total	$\mathcal{O}_\mu(p^1)$ Tree	$\mathcal{O}_\mu(p^2)$ Loop	$\mathcal{O}_\mu(p^3)$ Loop	Total
$\mu_{\bar{D}^{*0}\to\bar{D}^0\gamma}$	−2.24	0.21	−0.10	−2.13	−2.24	0.29	0.04	−1.91
$\mu_{D^{*-}\to D^-\gamma}$	0.55	−0.21	0.05	0.39	0.55	−0.29	0.02	0.28
$\mu_{B^{*+}\to B^+\gamma}$	−1.80	0.16	−0.09	−1.73	−1.80	0.19	−0.07	−1.68
$\mu_{B^{*0}\to B^0\gamma}$	0.99	−0.16	0.046	0.88	0.99	−0.19	0.04	0.84
$\mu_{\bar{D}^{*0}}$	1.48	−0.21	0.11	1.38	1.48	0.07	0.05	1.60
$\mu_{D^{*-}}$	−1.31	0.21	−0.05	−1.14	−1.31	−0.07	−0.007	−1.39
$\mu_{B^{*+}}$	1.93	−0.16	0.09	1.86	1.93	−0.13	0.09	1.90
$\mu_{B^{*0}}$	−0.86	0.16	−0.05	−0.75	−0.86	0.13	−0.05	−0.78

SU(2) case

Magnetic moments



- **The results in SU(3) case**

TABLE IX: The transition magnetic moments and magnetic moments of charmed and bottom vector mesons calculated in the SU(3) case order by order (in units of μ_N).

Physical quantity	$\Delta = 0$				$\Delta \neq 0$			
	$\mathcal{O}_\mu(p^1)$ Tree	$\mathcal{O}_\mu(p^2)$ Loop	$\mathcal{O}_\mu(p^3)$ Loop	Total	$\mathcal{O}_\mu(p^1)$ Tree	$\mathcal{O}_\mu(p^2)$ Loop	$\mathcal{O}_\mu(p^3)$ Loop	Total
$\mu_{\bar{D}^{*0}\to\bar{D}^0\gamma}$	−2.24	0.71	−0.34	−1.86	−2.24	0.81	−0.13	−1.57
$\mu_{D^{*-}\to D^-\gamma}$	0.55	−0.21	0.19	0.54	0.55	−0.29	0.08	0.34
$\mu_{D_s^{*-}\to D_s^-\gamma}$	0.20	−0.50	0.15	−0.15	0.20	−0.51	0.10	−0.21
$\mu_{B^{*+}\to B^+\gamma}$	−1.80	0.55	−0.34	−1.58	−1.80	0.58	−0.30	−1.52
$\mu_{B^{*0}\to B^0\gamma}$	0.99	−0.16	0.17	1.0	0.99	−0.19	0.14	0.95
$\mu_{B_s^{*0}\to B_s^0\gamma}$	0.65	−0.39	0.13	0.38	0.65	−0.39	0.11	0.36
$\mu_{\bar{D}^{*0}}$	1.48	−0.71	0.40	1.18	1.48	−0.40	0.40	1.48
$\mu_{D^{*-}}$	−1.31	0.21	−0.21	−1.31	−1.31	−0.07	−0.24	−1.62
$\mu_{D_s^{*-}}$	−0.96	0.50	−0.16	−0.62	−0.96	0.47	−0.21	−0.69
$\mu_{B^{*+}}$	1.93	−0.55	0.34	1.71	1.93	−0.52	0.36	1.77
$\mu_{B^{*0}}$	−0.86	0.16	−0.17	−0.87	−0.86	0.13	−0.19	−0.92
$\mu_{B_s^{*0}}$	−0.51	0.39	−0.13	−0.25	−0.51	0.38	−0.14	−0.27

SU(3) case

Magnetic moments

- **A comparison with other models**

TABLE VII: The magnetic moments of the charmed and bottom vector mesons (in units of nucleon magnetons μ_N), and a comparison with the Bag model (Bag), extended Nambu-Jona-Lasinio model (NJL) and extended Bag model (Extended Bag) predictions.

States	SU(2)		SU(3)		The results from other theoretical works		
	$\Delta = 0$	$\Delta \neq 0$	$\Delta = 0$	$\Delta \neq 0$	Bag [20]	NJL [32]	Extended Bag [22]
$\bar{D}^{*0}$	$1.38^{+0.25}_{-0.25}$	$1.60^{+0.25}_{-0.25}$	$1.18^{+0.25}_{-0.25}$	$1.48^{+0.22}_{-0.38}$	0.89	$\cdots$	1.28
D^{*-}	$-1.14^{+0.15}_{-0.15}$	$-1.39^{+0.15}_{-0.15}$	$-1.31^{+0.20}_{-0.15}$	$-1.62^{+0.24}_{-0.08}$	−1.17	−1.16	−1.13
D_s^{*-}	$\cdots$	$\cdots$	$-0.62^{+0.15}_{-0.15}$	$-0.69^{+0.22}_{-0.10}$	−1.03	−0.98	−0.93
B^{*+}	$1.86^{+0.25}_{-0.25}$	$1.90^{+0.20}_{-0.20}$	$1.71^{+0.25}_{-0.25}$	$1.77^{+0.25}_{-0.30}$	1.54	1.47	1.56
B^{*0}	$-0.75^{+0.11}_{-0.11}$	$-0.78^{+0.11}_{-0.11}$	$-0.87^{+0.13}_{-0.11}$	$-0.92^{+0.15}_{-0.11}$	−0.64	$\cdots$	−0.69
B_s^{*0}	$\cdots$	$\cdots$	$-0.25^{+0.11}_{-0.11}$	$-0.27^{+0.13}_{-0.10}$	−0.47	$\cdots$	−0.51

- [20] Phys. Rev. D **22**, 773 (1980); [22] arXiv:1803.01809 [hep-ph]; [32] Chin. Phys. C **39**, 113103 (2015).

Summary

- We have systematically studied the radiative transitions and magnetic moments of charmed and bottom vector mesons with xPT up to $O(p^4)$.
- The LECs $\tilde{a}$, a, and b in the $O(p^4)$ Lagrangians are estimated with the quark model and resonance saturation model, respectively.
- Our result for $D^{*-} \to D^- \gamma$ is in accordance with the experimental measurement. We also investigate the convergence of the chiral expansion of the transition magnetic moments in the SU(2) and SU(3) cases with the mass splitting kept and unkept.
- As a byproduct, the full widths of $\bar{D}^{*0}$ and D_s^{*-} are estimated to be $77.7^{+26.7}_{-20.5}$ keV and $0.62^{+0.45}_{-0.50}$ keV, respectively.
- We also calculate the magnetic moments of the D^* and B^* mesons.

Thank you