Faddeev fixed center approximation to $\pi\bar{K}K^*$ system and the $\pi_1(1600)$

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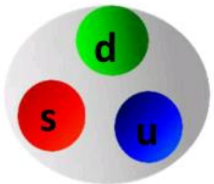
第十八届全国中高能核物理会议

OUTLINE

- Introduction
- Our model
- Numerical results
- Summary

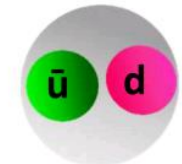
QCD Exotic States



Baryons are red-blue-green triplets

ordinary matter

Mesons are color-anticolor pairs



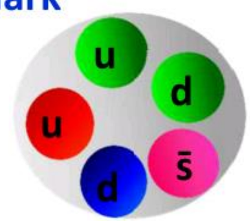
Λ=usd

π=ūd

Other possible combinations of quarks and gluons :

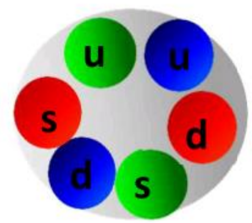
Pentaquark

S= +1
Baryon



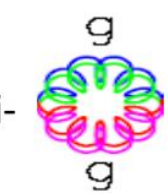
H di-Baryon

Tightly bound
6 quark state



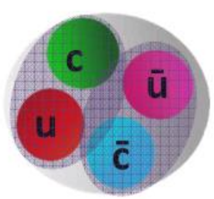
Glueball

Color-singlet multi-gluon bound state



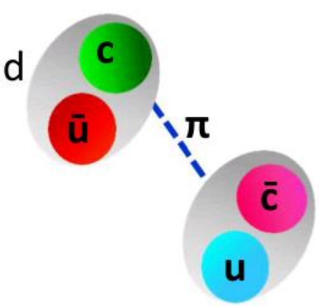
Tetraquark

Tightly bound
diquark &
anti-diquark

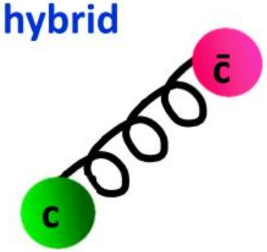


Molecule

loosely bound
meson-
antimeson
“molecule”



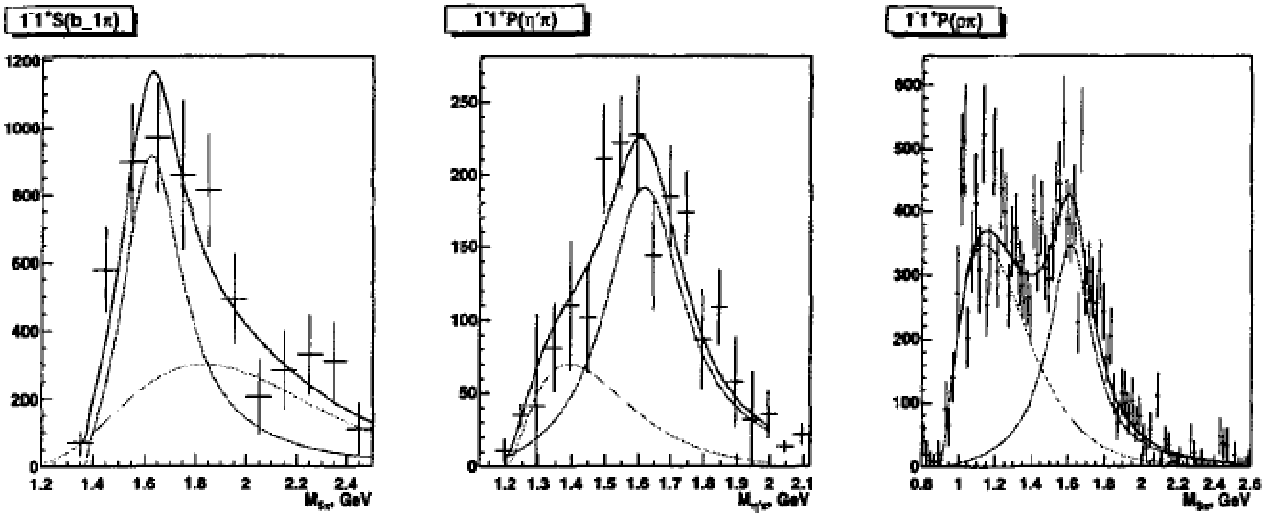
qq̄ -gluon hybrid mesons



by Xiao-Rui Lyu

The observation of $\pi_1(1600)$ from VES collaboration

J^{PC}= 1^{-+} wave in $b_1\pi$, $\eta'\pi$ and $\rho\pi$ channels



Nuclear Physics A663(2000) 596-599

The signal parameters obtained in the fit are:

$M(\pi_1(1600))$=1.61 $\pm$0.02 GeV
$\Gamma(\pi_1(1600))$=0.29 $\pm$0.03 GeV

$f_1\pi$ channel was also include

Yu. P. Gouz et al. (VES Collaboration), AIP Conf. Proc. **272,**572 (1993).

Comparing the results from VES collaboration and model calculations

Models for hybrid decays predict rates for $\pi_1(1600)$

P. R. Page, E. S. Swanson, and A. P. Szczepaniak, Phys. Rev. D59, 034016 (1999).

$b_1\pi : f_1\pi : \eta'\pi : \rho\pi = 24 : 5 : 2 : 9$

Branching ratios from VES collaboration

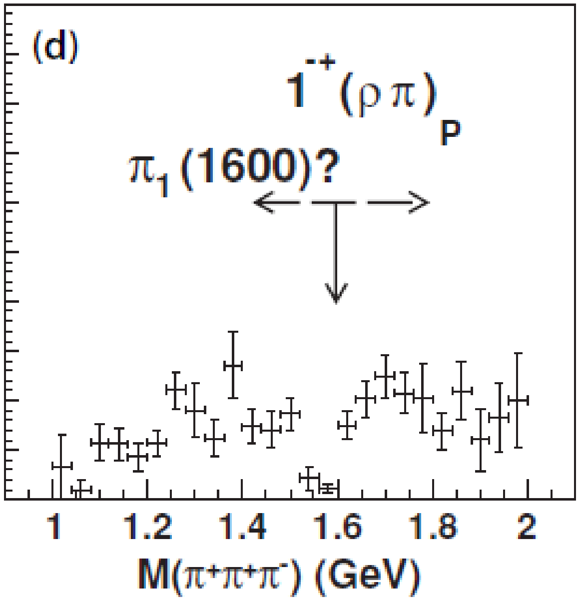
D. V. Amelin et al., Phys. Atom. Nucl. 68, 359 (2005) [Yad. Fiz.68, 388 (2005)].

$b_1\pi : f_1\pi : \eta'\pi : \rho\pi = 1.0 \pm 0.3 : 1.1 \pm 0.3 : < 0.3 : 1.0$

The CLAS experiment result

Phys. Rev. Lett. 102,102002 (2009).

$\gamma p \rightarrow \pi^+\pi^+\pi^-$ (n)



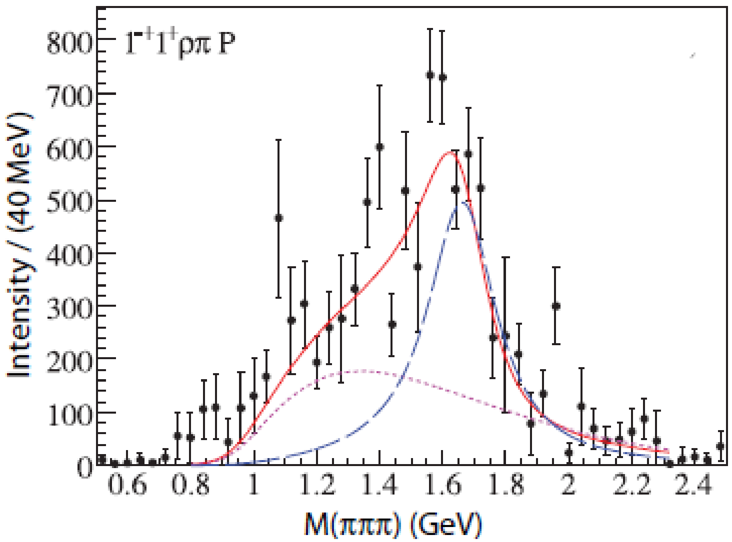
In the 1^{-+} exotic wave , no intensity is observed.

There results imply that the π_1(1600) is not strongly produced in photoproduction, the π_1(1600) does not decay to 3π or both.

The COMPASS experiment result

PRL 104, 241803 (2010)

$\pi^- Pb \rightarrow \pi^-\pi^-\pi^+ X$



$M(\pi_1(1600))$=1.660 ±0.010 GeV
$\Gamma(\pi_1(1600))$=0.269 ±0.021 GeV

Our model: Fixd center approximation (FCA)

We investigate the three-body system of $\pi\bar{K}K^*$ using the FCA approximation to Faddeev equations

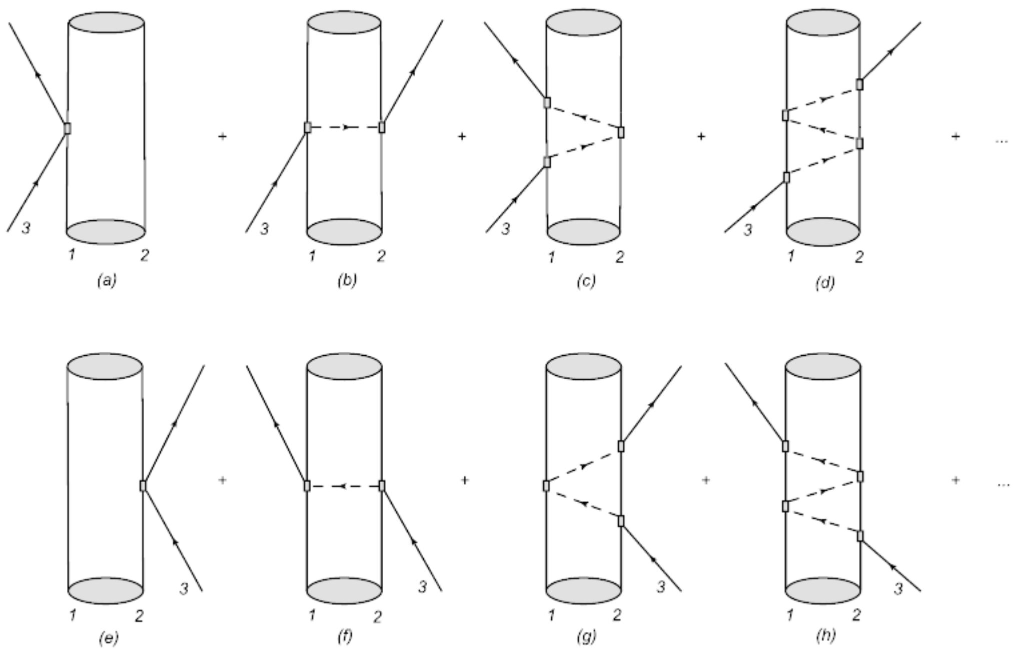


FIG. 1: Diagrammatic representation of the FCA to Faddeev equations.

F.Aceti, Ju-JunXie and E.Oset , Physics Letters B 750(2015) 609-614
Ju-JunXie, E.Oset , Physics Letters B 753(2016) 591-594

We assume $\bar{K}K^*$ forming a cluster as $f_1(1285)$

$$T_1 = t_1 + t_1 G_0 T_2$$
$$T_2 = t_2 + t_2 G_0 T_1$$
$$T = T_1 + T_2$$

t_1 is the combination of the I =1/2 and 3/2 $\pi\bar{K}$ scattering amplitude
t_2 is the combination of the I=1/2 and 3/2 πK^* scattering amplitude.

single-scattering FIG.1(a)，

$$S_1^{(1)} = -it_1(2\pi)^4\delta(k+k_R-k'-k'_R)\frac{1}{V^2}\frac{1}{\sqrt{2w_{p_1}}}\frac{1}{\sqrt{2w_{p'_1}}}\frac{1}{\sqrt{2w_k}}\frac{1}{\sqrt{2w_{k'}}}F_R(\frac{m_{K^*}(\vec{k}-\vec{k}')}{m_K+m_{K^*}})$$

Double-scattering FIG.1(b)

$$S_1^{(2)} = -it_1t_2(2\pi)^4\delta(k+k_R-k'-k'_R)\frac{1}{V^2}\frac{1}{\sqrt{2w_{p_1}}}\frac{1}{\sqrt{2w_{p'_1}}}\frac{1}{\sqrt{2w_k}}\frac{1}{\sqrt{2w_{k'}}}$$

$$\frac{1}{\sqrt{2w_{p_2}}}\frac{1}{\sqrt{2w_{p'_2}}}\int\frac{d^3q}{(2\pi)^3}F_R(q-\frac{m_{K^*}(\vec{k}+\vec{k}')}{m_K+m_{K^*}})\frac{1}{q^{02}-\vec{q}^{\,2}-m_\pi^2+i\varepsilon}$$

To consider states above threshold, we project the form factor into the s-wave

$$F_R(\frac{m_{K^*}(\vec{k}-\vec{k}')}{m_K+m_{K^*}}) \Rightarrow FFS_1(s) = \frac{1}{2}\int_{-1}^{1}F_R(k_1)d(\cos\theta)$$

$$F_R(q-\frac{m_{K^*}(\vec{k}+\vec{k}')}{m_K+m_{K^*}}) = \int dr^3 Exp(-i(q-\frac{m_{K^*}(\vec{k}+\vec{k}')}{m_K+m_{K^*}})\vec{r})\psi(\vec{r})^2$$

we will taken into account that $\vec{k}+\vec{k}'=0$ on average.

Where ψ is an eigenfunction of H, the full Hamiltonian

$$\langle\vec{p}|\psi\rangle = \int d^3k\int d^3k'\langle\vec{p}|\frac{1}{E-H_0}|\vec{k}'\rangle\langle\vec{k}'|V|\vec{k}\rangle\langle\vec{k}|\psi\rangle$$

The expression for the form factor $F_R(q)$

$$F_R(q) = \frac{1}{N}\int_{|\vec{P}|<\Lambda, |\vec{P}-\vec{q}|<\Lambda} \frac{1}{2E_1(\vec{p})}\frac{1}{2E_2(\vec{p})}\frac{1}{M_R - E_1(\vec{p}) - E_2(\vec{p})} \frac{1}{2E_1(\vec{p}-\vec{q})}\frac{1}{2E_2(\vec{p}-\vec{q})}\frac{1}{M_R - E_1(\vec{p}-\vec{q}) - E_2(\vec{p}-\vec{q})}$$

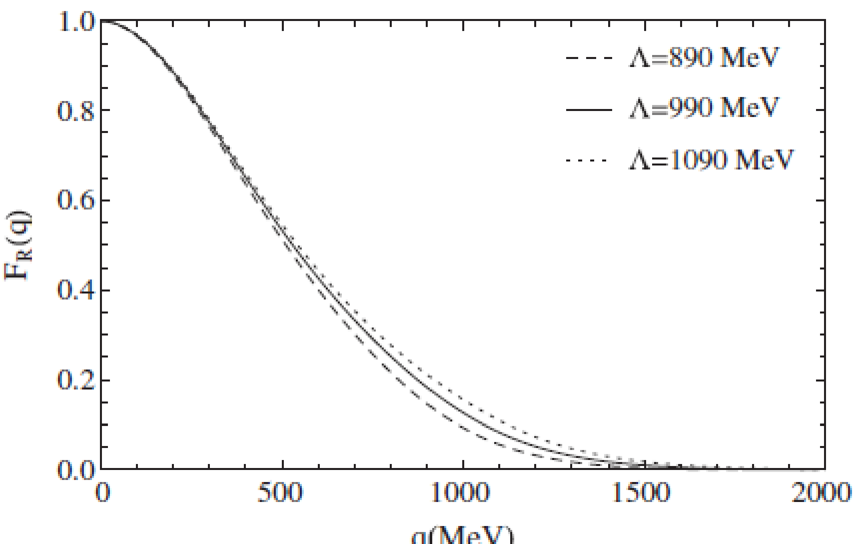
In this work we take Λ = 990 MeV

PHYSICAL REVIEW D 72, 014002 (2005)

The G_0 is the loop function for the π meson propagating inside the cluster

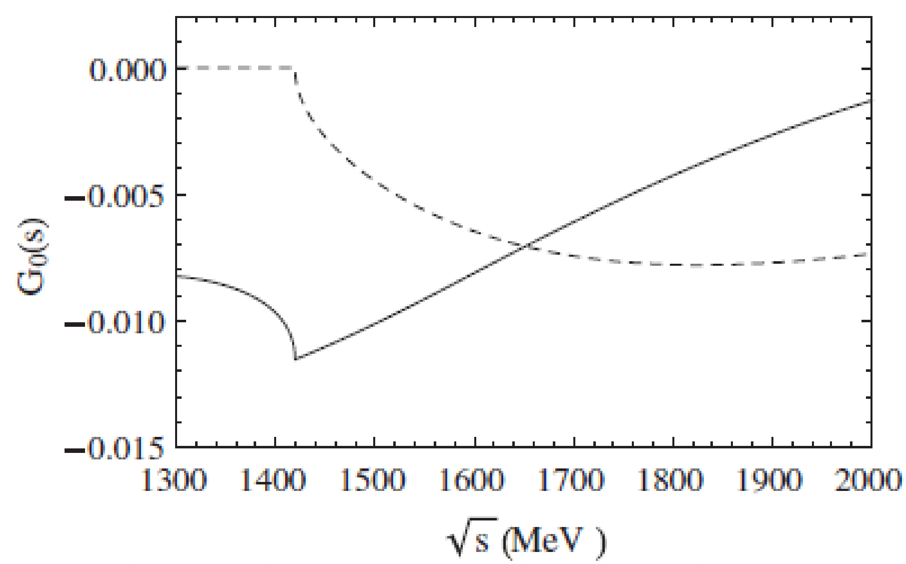
$$G_0 = \frac{1}{2M_R}\int \frac{d^3q}{(2\pi)^3} F_R(q) \frac{1}{q^{0\,2} - \vec{q}^{\,2} - m_3^2 + i\varepsilon}$$

The form factor $F_R(q)$ of $f_1(1285)$ as a $\bar{K}K^*$ bound state



Solid, dashed and dotted line corresponding to different cut-off Λ .

The G_0 as a function of the invariant mass of the $\pi\bar{K}K^*$ system



Real (solid line) and imaginary (dashed line) parts of the G_0 function.

We project the form factor into the s-wave,the only one that we consider. Hence

$$FFS_1(s) = \frac{1}{2}\int_{-1}^{1} F_R(k_1) d(\cos\theta)$$

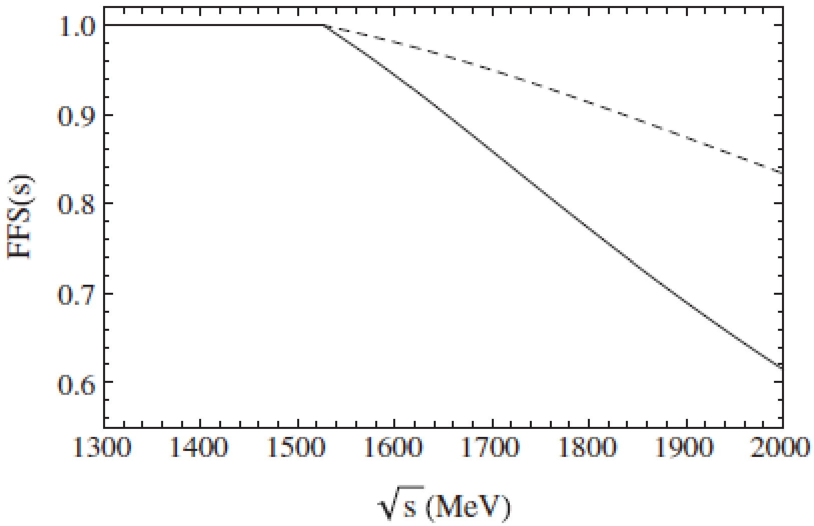
$$FFS_2(s) = \frac{1}{2}\int_{-1}^{1} F_R(k_2) d(\cos\theta)$$

$$k_1 = \frac{m_{K^*}}{m_{\overline{K}} + m_{K^*}} k\sqrt{2(1-\cos\theta)}$$

$$k_2 = \frac{m_{\overline{K}}}{m_{\overline{K}} + m_{K^*}} k\sqrt{2(1-\cos\theta)}$$

$$k = \frac{\sqrt{(s-(m_{\overline{K}} + m_{K^*} + m_\pi)^2)(s-(m_{\overline{K}} + m_{K^*} - m_\pi)^2)}}{2\sqrt{s}}$$

The solid and dashed curves are the results of FFS_1 and FFS_2



The amplitudes for the single-scattering contribution

$$t^{(1,1)}_{\pi\bar{K}K^*} = \left\langle \pi\bar{K}K^* \right| (t_{31}+t_{32}) \left| \pi\bar{K}K^* \right\rangle$$
$$= \{ \langle 11 | \otimes \sqrt{\frac{1}{2}} (\left\langle \frac{1}{2}, -\frac{1}{2} \right| - \left\langle -\frac{1}{2}, \frac{1}{2} \right|) \} (t_{31}+t_{32}) \{ |11\rangle \otimes \sqrt{\frac{1}{2}} (\left| \frac{1}{2}, -\frac{1}{2} \right\rangle - \left| -\frac{1}{2}, \frac{1}{2} \right\rangle) \}$$
$$= (\frac{2}{3} t_{31}^{I=\frac{3}{2}} + \frac{1}{3} t_{31}^{I=\frac{1}{2}}) + (\frac{2}{3} t_{32}^{I=\frac{3}{2}} + \frac{1}{3} t_{32}^{I=\frac{1}{2}})$$

We obtain

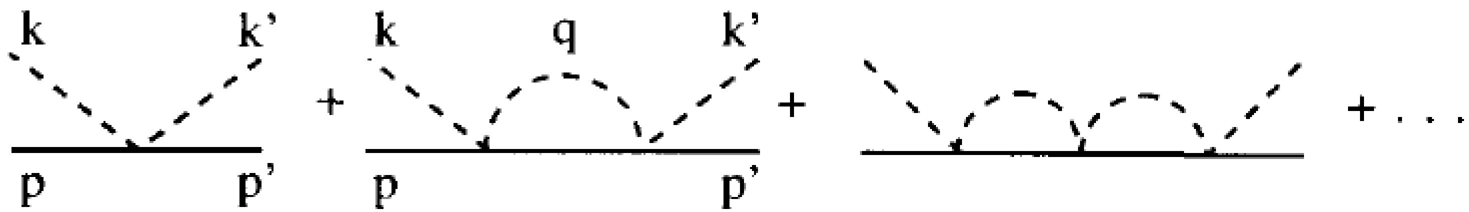
$$t_1 = \frac{2}{3} t_{31}^{I=\frac{3}{2}} + \frac{1}{3} t_{31}^{I=\frac{1}{2}} \qquad t_2 = \frac{2}{3} t_{32}^{I=\frac{3}{2}} + \frac{1}{3} t_{32}^{I=\frac{1}{2}}$$

$\pi\bar{K}K^*$ scattering amplitude (to consider states above threshold)

$$T = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1 \tilde{t}_2 G_0}{1 - \tilde{t}_1 \tilde{t}_2 G_0^2} + \tilde{t}_1 (FFS_1 - 1) + \tilde{t}_2 (FFS_2 - 1)$$

Two-body scattering

The amplitude of two-body scattering can be cast using the BSE



$$T(p_1,k_1;p_2,k_2)=V(p_1,k_1;p_2,k_2)+i\int\frac{d^4q}{(2\pi)^4}\frac{V(p_1,k_1;q,p_1+k_1-q)}{(p_1+k_1-q)^2-m^2+i\varepsilon}\frac{T(q,p_1+k_1-q;p_2,k_2)}{q^2-M^2+i\varepsilon}$$

V can be factorized on shell in the BSEs, and so that the integral equations become algebraic equations

E. Oset, A. Ramos, NPA 635, (1998) 99

$$t=(1-VG)^{-1}V$$

loop propagator

$$G(s)=i\int\frac{d^4q}{(2\pi)^4}\frac{1}{(p_1+k_1-q)^2-m^2+i\varepsilon}\frac{1}{q^2-M^2+i\varepsilon}$$

Leading ordering Lagrangian L_{PPPP} for SU(3) ChPT reads

$$L_{PPPP} = \frac{1}{12 f^2} \mathrm{Tr}([P^{\mu}, \partial^{\nu} P_{\mu}]^2 - MP^4)$$

The pseudoscalar meson mass matrix M is given by

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}$$

The corresponding coupled channels in πK scattering

$$|\pi K\rangle_{I=\frac{1}{2}, I=-\frac{1}{2}} = \sqrt{\frac{1}{3}}|\pi^0 K^0\rangle - \sqrt{\frac{2}{3}}|\pi^- K^+\rangle \qquad |\eta K\rangle_{I=\frac{1}{2}, I=-\frac{1}{2}} = |\eta K^0\rangle$$

$$|\eta' K\rangle_{I=\frac{1}{2}, I=-\frac{1}{2}} = |\eta' K^0\rangle$$

$$|\pi K\rangle_{I=\frac{3}{2}, I=-\frac{1}{2}} = \sqrt{\frac{2}{3}}|\pi^0 K^0\rangle + \sqrt{\frac{1}{3}}|\pi^- K^+\rangle$$

The tree level on-shell and s-wave πK, ηK and η'K amplitude is

$$V_{11}^{1/2} = -\frac{1}{4f^2}(4s + 3t - 4m_\pi^2 - 4m_K^2)$$

$$V_{12}^{1/2} = -\frac{\sqrt{2}}{6f^2}(\text{-}3t + 2m_K^2\, 4m_\eta^2)$$

$$V_{13}^{1/2} = -\frac{1}{12f^2}(-3t + 3m_\pi^2 + 8m_K^2 + m_{\eta'}^2)$$

$$V_{22}^{1/2} = -\frac{2}{9f^2}(3t - m_K^2 - 2m_\eta^2)$$

$$V_{23}^{1/2} = \frac{\sqrt{2}}{18f^2}(3t - 3m_\pi^2 + 2m_K^2 - m_\eta^2 - m_{\eta'}^2)$$

$$V_{33}^{1/2} = -\frac{1}{36f^2}(3t - 6m_\pi^2 + 32m_K^2 - 2m_{\eta'}^2)$$

$$V_{11}^{3/2} = \frac{1}{2f^2}(s - m_\pi^2 - m_K^2)$$

Leading ordering Lagrangian L_{VVPP} for SU(3) ChPT reads

$$L_{VVPP} = \frac{1}{4f}\mathrm{Tr}([V^{\mu},\partial^{\nu}V_{\mu}][P,\partial^{\nu}P])$$

P and V are the SU(3) matrices containing the octet of pseudoscalar and the nonet of vector mesons respectively:

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \qquad V_{\mu} = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \overline{K}^{*0} & \phi \end{pmatrix}_{\mu}$$

The tree level on-shell and s-wave amplitude is

$$V_{ij} = -\frac{1}{8f^2}C_{ij}[3s-(M^2+m^2+M'^2+m'^2)-\frac{1}{s}(M^2-m^2)(M'^2-m'^2)]$$

C_{ij} coefficients in isospin base for I=1/2

PHYSICAL REVIEW D 72, 014002 (2005)

	ϕK	ωK	ρK	$K^*\eta$	$K^*\pi$
ϕK	0	0	0	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{3}{2}}$
ωK	0	0	0	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$
ρK	0	0	-2	$-\frac{3}{2}$	$\frac{1}{2}$
$K^*\eta$	$-\sqrt{\frac{3}{2}}$	$\frac{\sqrt{3}}{2}$	$-\frac{3}{2}$	0	0
$K^*\pi$	$-\sqrt{\frac{3}{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	-2

For I=3/2, there are two channels πK^* and $K\rho$

$C_{11}=1$ $C_{12}=1$ $C_{22}=1$

In the dimensional regularization scheme the loop function gives

$$G_l(\sqrt{s}) = \frac{1}{16\pi^2}\{a(\mu) + \ln\frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s}\ln\frac{m_l^2}{M_l^2} + \frac{q_l}{\sqrt{s}}[\ln(s - (M_l^2 - m_l^2) + 2q_l\sqrt{s})$$
$$+ \ln(s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) - \ln(-s + (M_l^2 - m_l^2) + 2q_l\sqrt{s}) - \ln(-s - (M_l^2 - m_l^2) + 2q_l\sqrt{s})]\}$$

Where μ is the scale of dimensional regularization, a(μ) the subtraction constant

$\pi\bar{K}$ scattering

for $I = 1/2$ $\quad a(\mu) = -1.383 \pm 0.006 \qquad \mu = m_K$

$I = 3/2$ $\quad a(\mu) = -4.643 \pm 0.083. \qquad \mu = m_K$

F.-K. Guo, R.-G. Ping, P.-N. Shen, H.-C. Chiang and B.-S. Zou ,
Nuclear Physics A 773 (2006) 78–94

πK* scattering

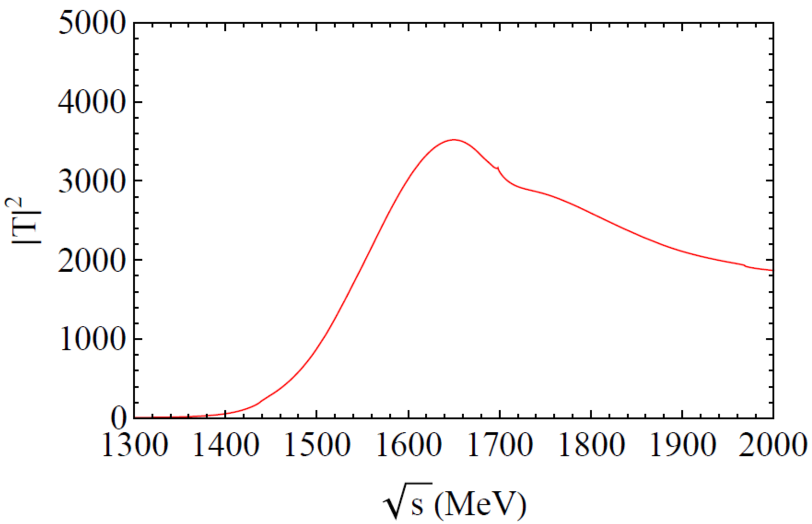
for $I = 1/2$ $\quad a(\mu) = -1.85 \qquad \mu = 900$

$I = 3/2$ $\quad a(\mu) = -1.85 \qquad \mu = 900$

L. Roca, E. Oset, and J. Singh,
PHYSICAL REVIEW D 72, 014002 (2005)

Numerical results

$\pi\bar{K}K^*$ scattering amplitude



The resonant structure around 1650 MeV shows up in the modulus squared

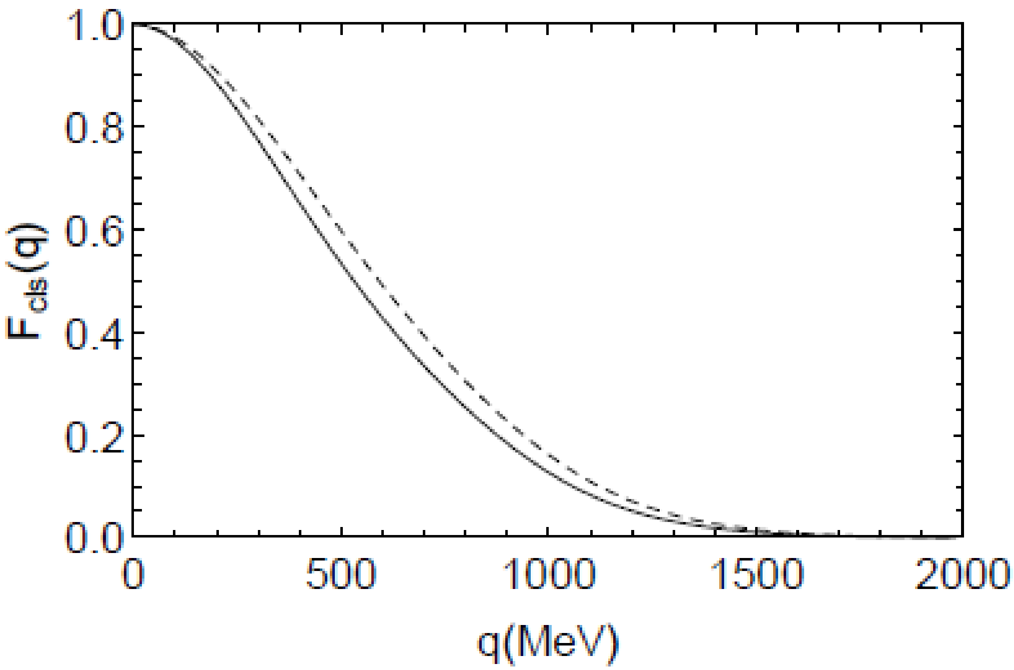
We suggest that this is the origin of the present $\pi_1(1600)$

Xu Zhang, Ju-Jun Xie and Xurong Chen,

Phys. Rev. D 95, 056014 (2017)

$\eta\bar{K}K^*$ system

We assume $\bar{K}K^*$ forming a cluster as $f_1(1285)$ and ηK^* forming a cluster as $K_1(1270)$

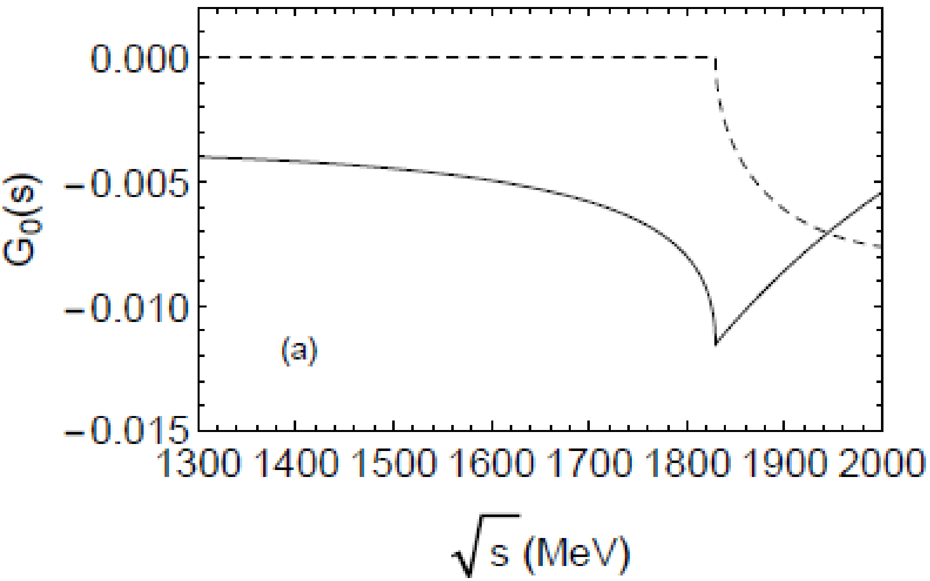


Solid, dashed and dotted line corresponding to $f_1(1285)$ and $K_1(1270)$ respectively.

In this work we take Λ = 990 MeV Λ = 1000 MeV for $f_1(1285)$ and $K_1(1270)$ respectively.

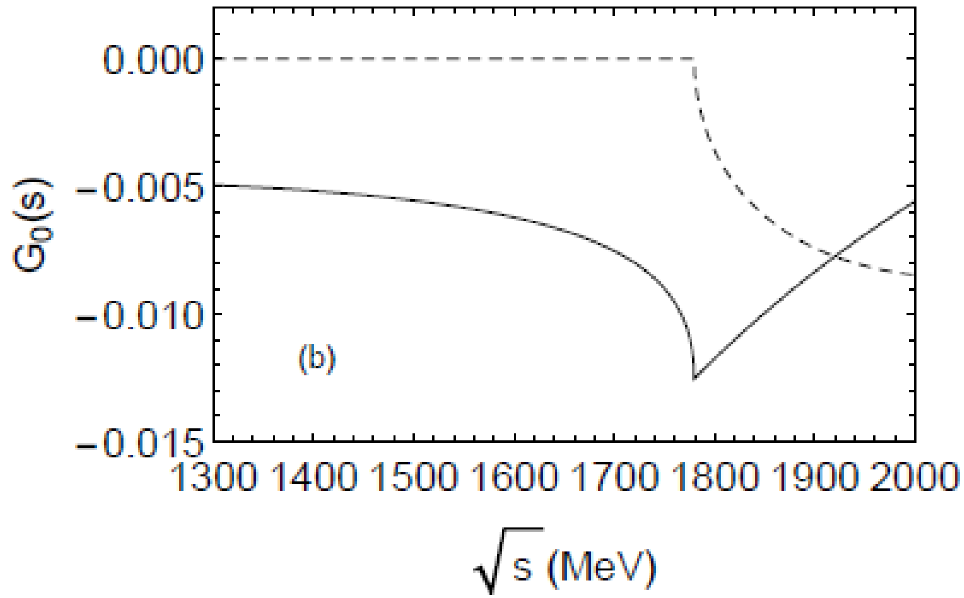
Xu Zhang, Ju-Jun Xie,
arXiv: 1906.07340

The G_0 as a function of the invariant mass of the $\eta(\bar{K}K^*)_{f_1(1285)}$ system



Real (solid line) and imaginary (dashed line) parts of the G_0 function.

The G_0 as a function of the invariant mass of the $\bar{K}(\eta K^*)_{K_1(1270)}$ system

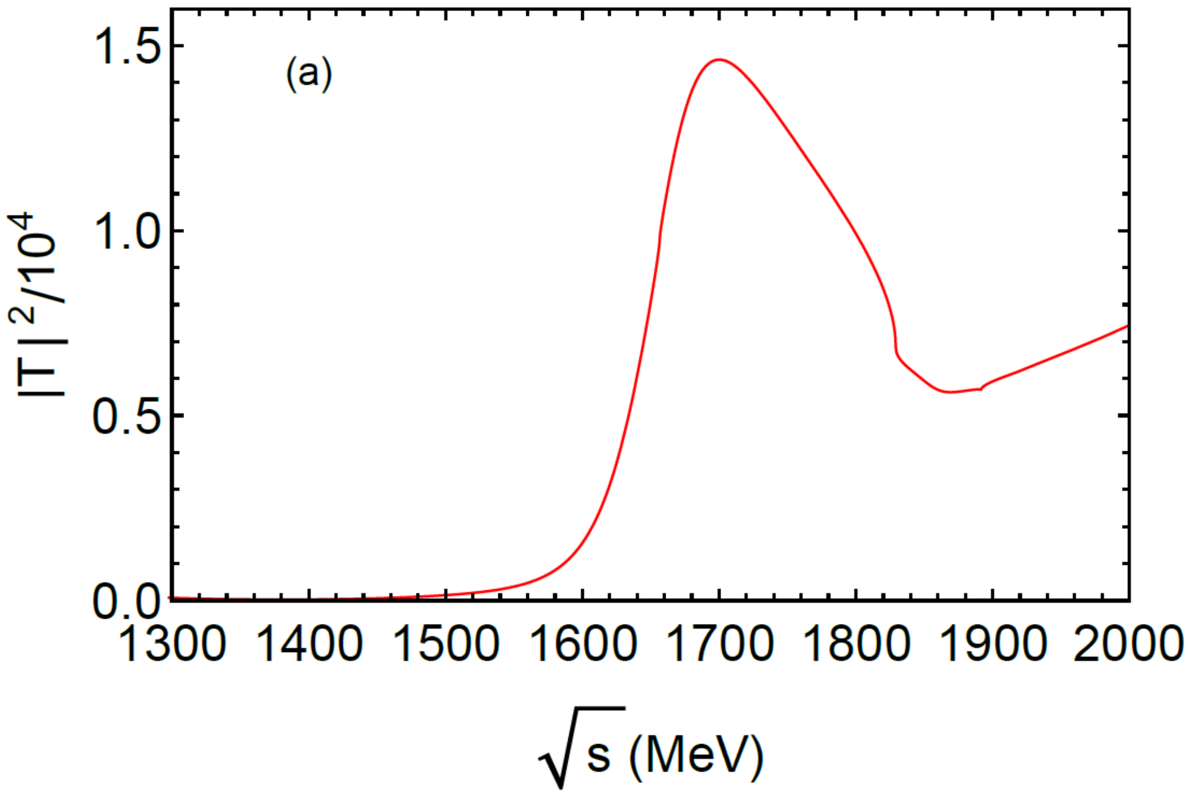


Real (solid line) and imaginary (dashed line) parts of the G_0 function.

Xu Zhang, Ju-Jun Xie, arXiv: 1906.07340

Numerical results

$\eta(\bar{K}K^*)_{f_1(1285)}$ scattering amplitude

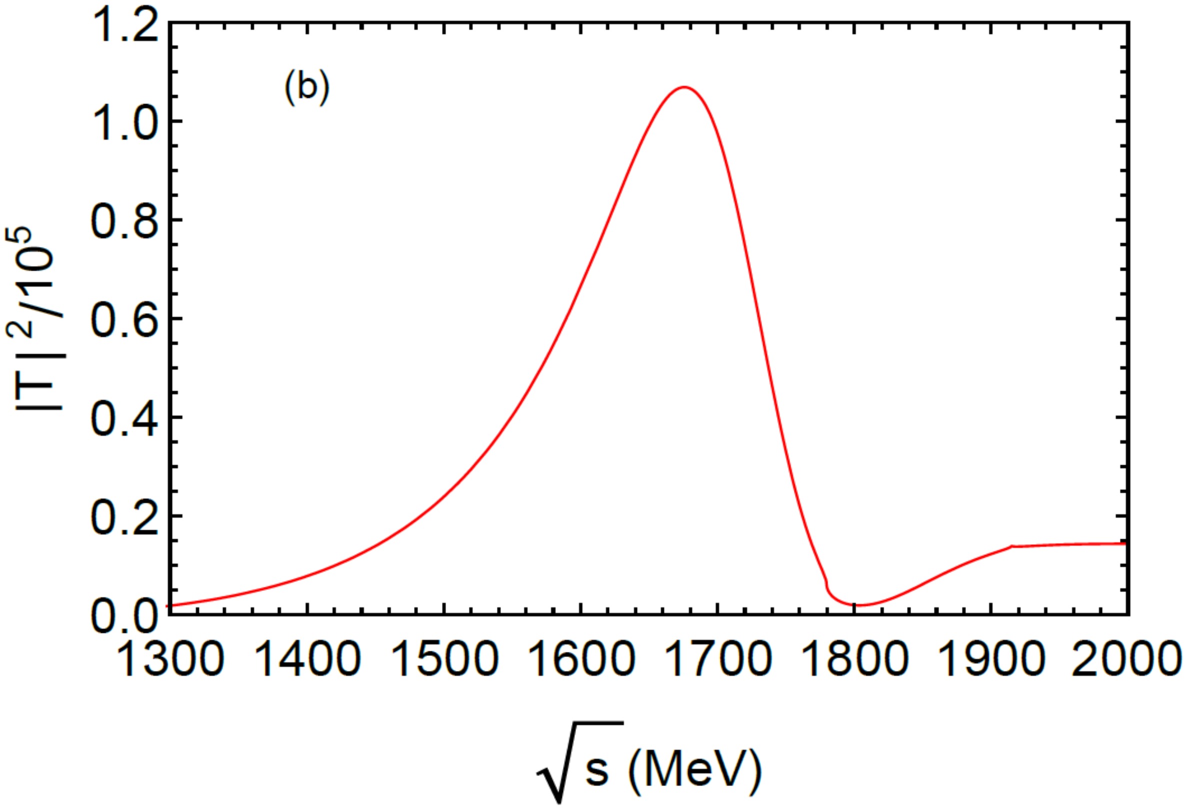


We find evidence of a bound state $I^G(J^{PC}) = 0^+(1^{-+})$ below the $\eta(\bar{K}K^*)_{f_1(1285)}$ threshold with mass around 1700 MeV and width about 180 MeV

Xu Zhang, Ju-Jun Xie,
arXiv: 1906.07340

Numerical results

$\bar{K}(\eta K^*)_{K_1(1270)}$ scattering amplitude



We obtain a bound state $I(J^P) = 0(1^-)$ below the $\bar{K}(\eta K^*)_{K_1(1270)}$ threshold with mass around 1680 MeV and width about 160 MeV

Xu Zhang, Ju-Jun Xie,

arXiv: 1906.07340

Summary

We study the three body systems of $\pi\bar{K}K^*$ by using the fixed center approximation to the Faddeev equations.
There is a resonantstructure around 1650 MeV in the module squared, with quantum numbers $I^G(J^{PC}) = 1^-(1^{-+})$. We associated this resonance to the exotic state $\pi_1(1600)$ with mass 1660 MeV and large uncertainties for the width.

We also study the three body systems of $\eta\bar{K}K^*$. We find evidence of a bound state $I^G(J^{PC}) = 0^+(1^{-+})$ below the $\eta(\bar{K}K^*)_{f_1(1285)}$ threshold with mass around 1700 MeV and width about 180 MeV.
And also we obtain a bound state $I(J^P) = 0(1^-)$ below the $\bar{K}(\eta K^*)_{K_1(1270)}$ threshold with mass around 1680 MeV and width about 160 MeV.

Thank you very much !