Chiral Solitons and Vacuum Polarization

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Outline

I. Introduction

II. Different Chiral soliton models and their relation

III. Chiral solitons & vacuum polarization

IV. Summary

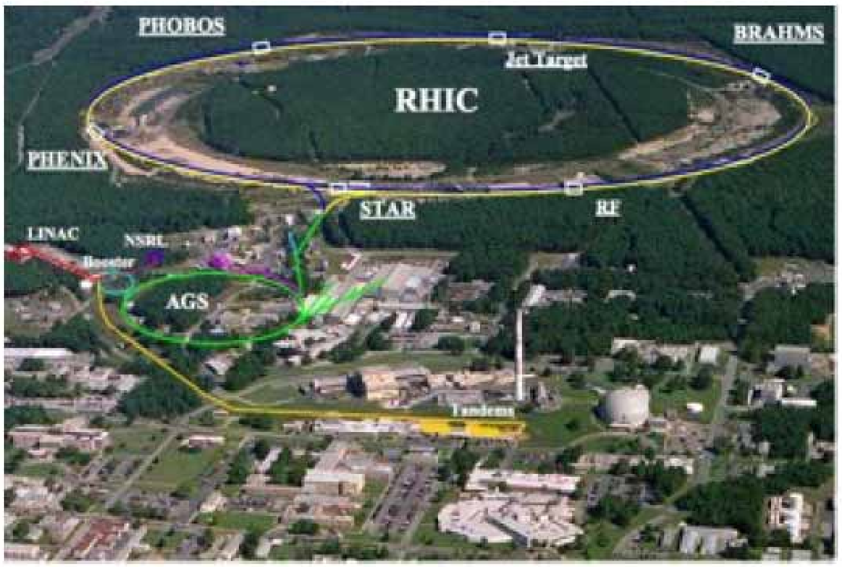
I. Introduction

QCD and the vacuum

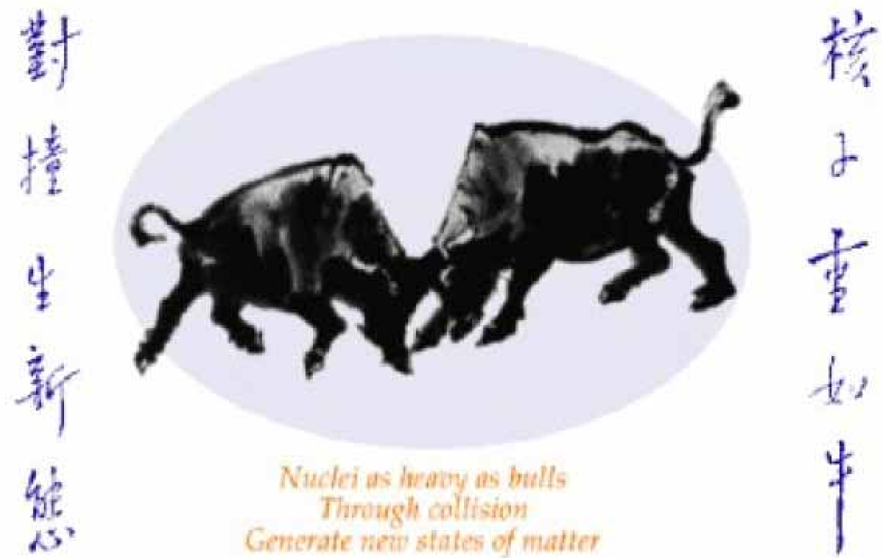
- ✓ Quantum chromodynamics (QCD) is the fundamental theory of strong interaction.
- ✓ QCD vacuum is a very nontrivial issue
  - ➢ Chiral symmetry breaking, color confinement
  - ➢ T. D. Lee, vacuum engineering
    high energy collisions of heavy nuclei,
    nuclear matter at extreme condition

Lee T D and Wick G C 1974 Phys. Rev. D 9 2291

Vacuum engineering: RHIC



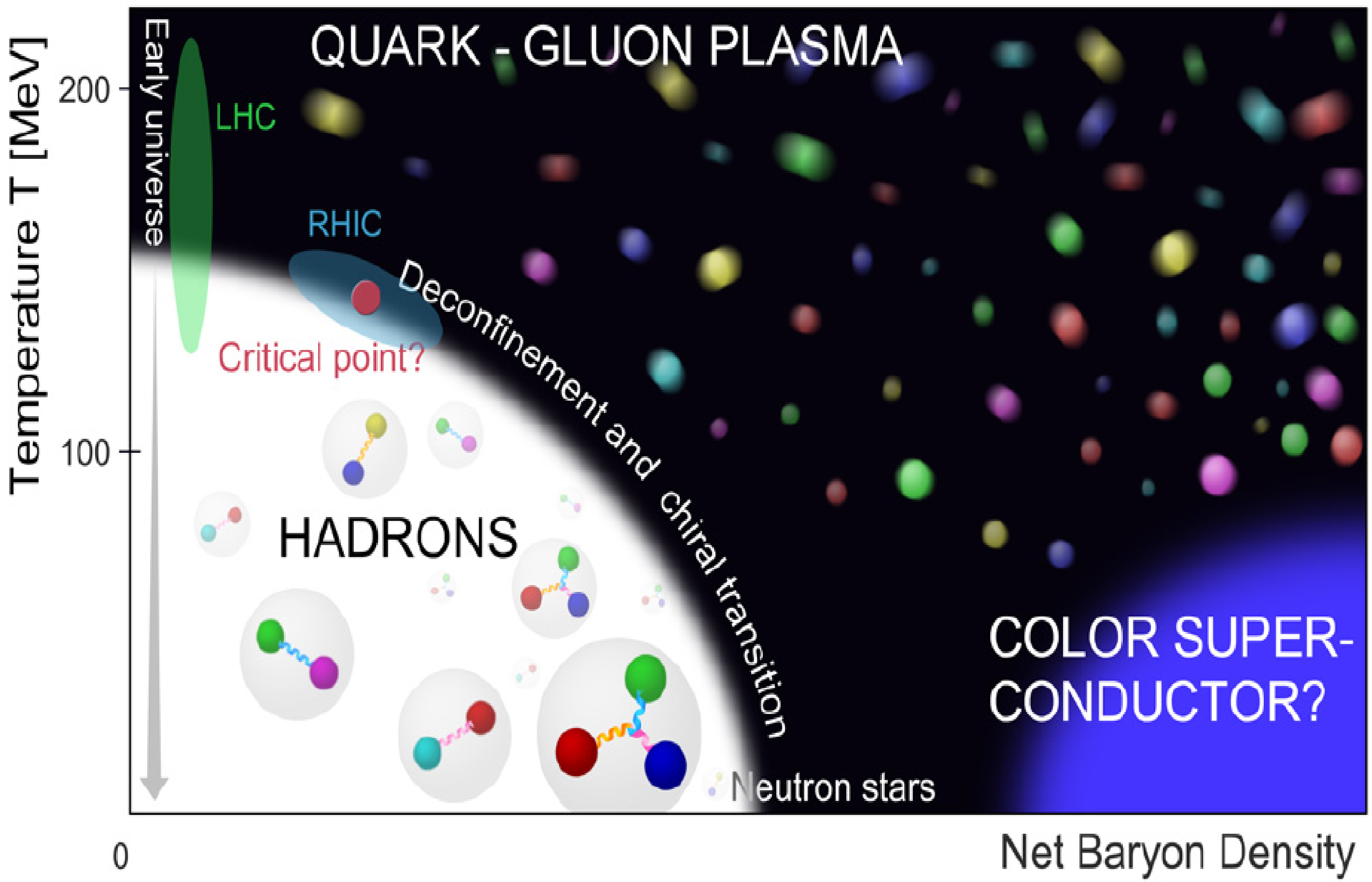
A



B

Figure : (A) The Relativistic Heavy Ion Accelerator (RHIC) at Brookhaven National Laboratory (BNL). (B) Heavy ions colliding: One of the artworks associated with BNL.

QCD phase diagram



Emergent quarks

- Quark emergent properties in different vacuums

At low T ➡ Confinement , Hadron state
Chiral symmetry breaking

At high T ➡ Deconfinement , sQGP, liquid state
Chial symmety restoring

•The QCD vacuum is nontrivial and nonperturpative.

"…Try to understand the ground state first.
If it is understood then properties of the excitations
will follow naturally. …"

from 《The QCD Vacuum,Hadrons and Superdense Matter》 by E.V. Shuryak

- High temperature vacuum
  - ✓ QCD phase transitions
    - ➢Deconfinement phase transition
    - ➢Chiral phase transition
  - ✓ Topological structure of thermal QCD vacuum topological solitons in QCD, instantons and dyons

Shuryak E and Sulejmanpasic T 2013 Phys. Lett. B 726 257
Liu Y, Shuryak E and Zahed I 2015 Phys. Rev. D 92 085006
Larsen R and Shuryak E 2016 Phys. Rev. D 93 054029
D. Diakonov, Nucl. Phys. Proc. Suppl. 195, 5-45,2009

- Low temperature vacuuum

  ✓ Bag models, e.g. MIT, SLAC, FL, CDM ...

  ——confinement

  ✓ Chiral models, e.g. NJL, LσM, Skyrme...

  ——chiral symmetry breaking

R. Friedberg, T.D. Lee, 1977 Phys. Rev. D 16 1096
Birse M.C. 1990 Prog. Part. Nucl. Phys. 25 1
D.I. Diakonov, V.Yu. Petrov and P.V. Pobylitsa, 1988 Nuclear Physics B 306 809
T. Schafer, E. V. Shuryak and J. J. M. Verbaarschot, 1994 Nucl. Phys. B 412 143
Zahed, I. and G. E. Brown, 1986 Phys.Rep. 142 1

Solitons

QCD vacuum → Nucleons

Effective models

- Nontopological solitons

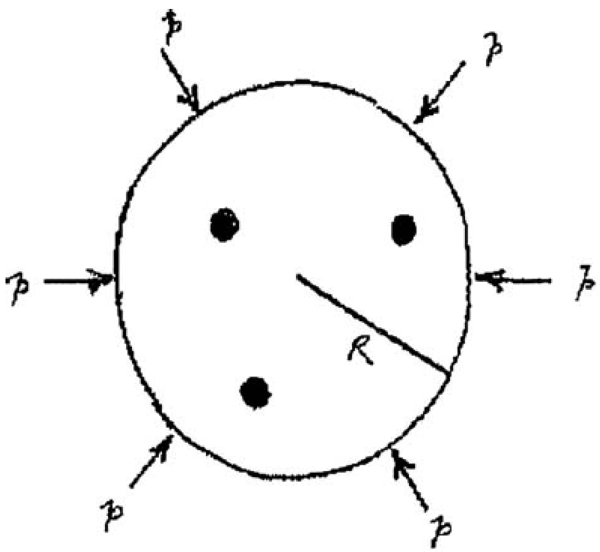
  Soliton models for hadrons,
  e.g. Friedberg-Lee model, color dielectric model, NJL model, Chiral quark solitons, ...

- Topological solitons

  Skyrme solitons,
  Monopoles, instantons and dyons from QCD.

II. Different chiral soliton models and their relation

The soliton bag

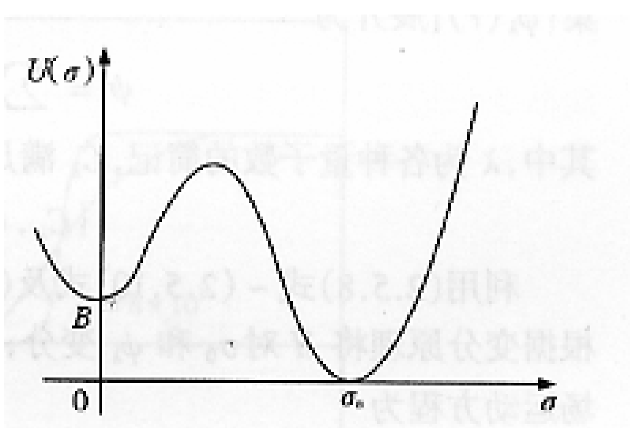


Soliton energy:

$$\frac{4}{3}\pi R^3 p + 4\pi R^2 s + \frac{N\xi}{R}$$

Soliton size R is determined by following equation:

$$4\pi R^3(Rp + 2s) = N\xi$$



Confinement: color dielectric function $\kappa(\sigma) = (1 - (\sigma / \sigma_v)^2)^2$

R. Friedberg, T.D. Lee, 1977 Phys. Rev. D 16 1096

The Skyrme soliton model

In large N_c limit, baryons could be regarded as solitons in a bosonic (or meson) field theory.

$$\mathcal{L}_{\mathrm{Skyr}} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] + \frac{1}{32e^2}\mathrm{Tr}\left\{\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]\left[U^\dagger\partial^\mu U, U^\dagger\partial^\nu U\right]\right\}$$

Effective chiral lagrangian

where, $U(\vec{r}) = \exp[i\vec{\tau}\cdot\vec{\phi}(\vec{r})]$

hedgehog link, $\vec{\phi}(\vec{r}) = \hat{r}\phi(r), \quad \hat{r} = \frac{\vec{r}}{r}$

$$M = \sigma + i\vec{\tau}\cdot\vec{\pi} = f_\pi\left[\cos\phi(r) + i\vec{\tau}\cdot\hat{r}\sin\phi(r)\right] = f_\pi \exp[i\vec{\tau}\cdot\hat{r}\phi(r)]$$

T.H.R. Skyrme, Nucl. Phys. 31, 556 (1962)

Non-trivial topology

Static soliton configurations U(r) represent mappings U :

$$\mathbb{R}^3 \rightarrow SU(2)$$

which requires the boundary condition

$$U(r) \xrightarrow{r \to \infty} 1$$

Compactify the three–dimensional space to a sphere S^3, then

$$U : S^3 \rightarrow S^3, \qquad \pi_3(S^3) = Z$$

E. Witten, Nucl. Phys. B 223, 422 (1983)
J. Goldstone and F. Wilczek, Phys. Rev. Lett. 47 (1981) 986

The linear sigma model

$$H = \int \mathrm{d}\boldsymbol{r}\, \psi^{+}\{\boldsymbol{\alpha}\cdot\boldsymbol{p}c + g\beta(\sigma + i\gamma_5 \boldsymbol{\pi}\cdot\boldsymbol{\tau})\}\psi$$

$$+\frac{1}{\hbar c}\int \mathrm{d}\boldsymbol{r}\,\{\tfrac{1}{2}(\nabla\sigma)^2 + \tfrac{1}{2}(\nabla\boldsymbol{\pi})^2 + \lambda(\sigma^2 + \boldsymbol{\pi}^2 - \sigma_0^2)^2\}$$

$$\varphi = \sigma/\sigma_0\,, \qquad \boldsymbol{\chi} = \boldsymbol{\pi}/\sigma_0\,, \qquad \boldsymbol{x} = g\sigma_0\boldsymbol{r}\,,$$

$$\sigma^2(r) + \pi^2(r) = \sigma_0^2 \qquad \chi_\alpha(\boldsymbol{x}) = \hat{\boldsymbol{x}}_\alpha \chi(x)\,, \quad \hat{\boldsymbol{x}} = \boldsymbol{x}/x.$$

$$\phi(x) = \cos\theta(x)\,, \qquad \chi(x) = \sin\theta(x)$$

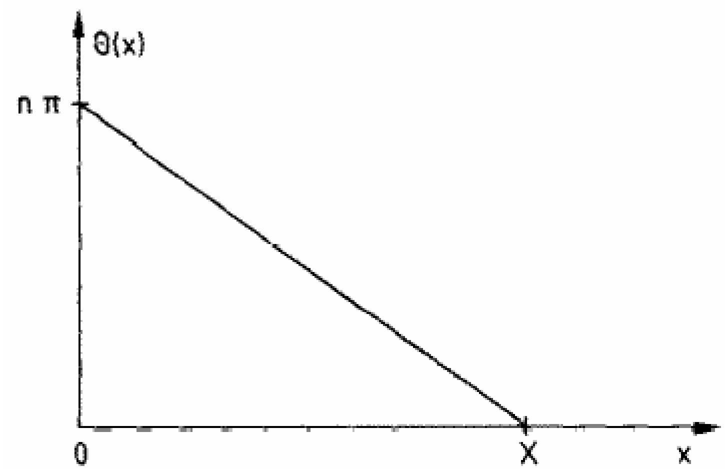
Kahana, S. and G. Ripka (1984). Nucl.Phys., A429, 462.

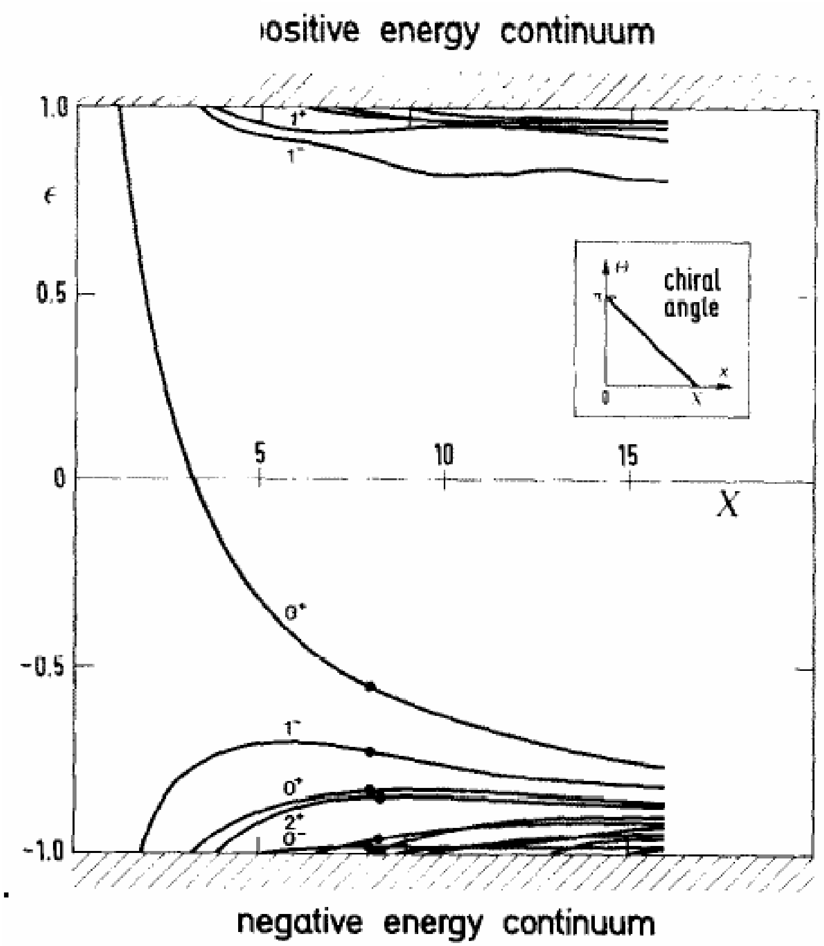
Kahana, S., G. Ripka and V. Soni (1984). Nucl.Phys., A415, 351.

Polarized Dirac sea

$$\left[\frac{\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}}{i}+\beta(\varphi+i\gamma_5\hat{\mathbf{x}}\cdot\boldsymbol{\tau}\chi)\right]|\lambda\rangle=\varepsilon_\lambda|\lambda\rangle$$

$$-\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)+\sin 2\theta+\frac{Ng^2}{2\pi}\frac{\mathrm{d}\varepsilon_\lambda}{\mathrm{d}\theta}=0$$





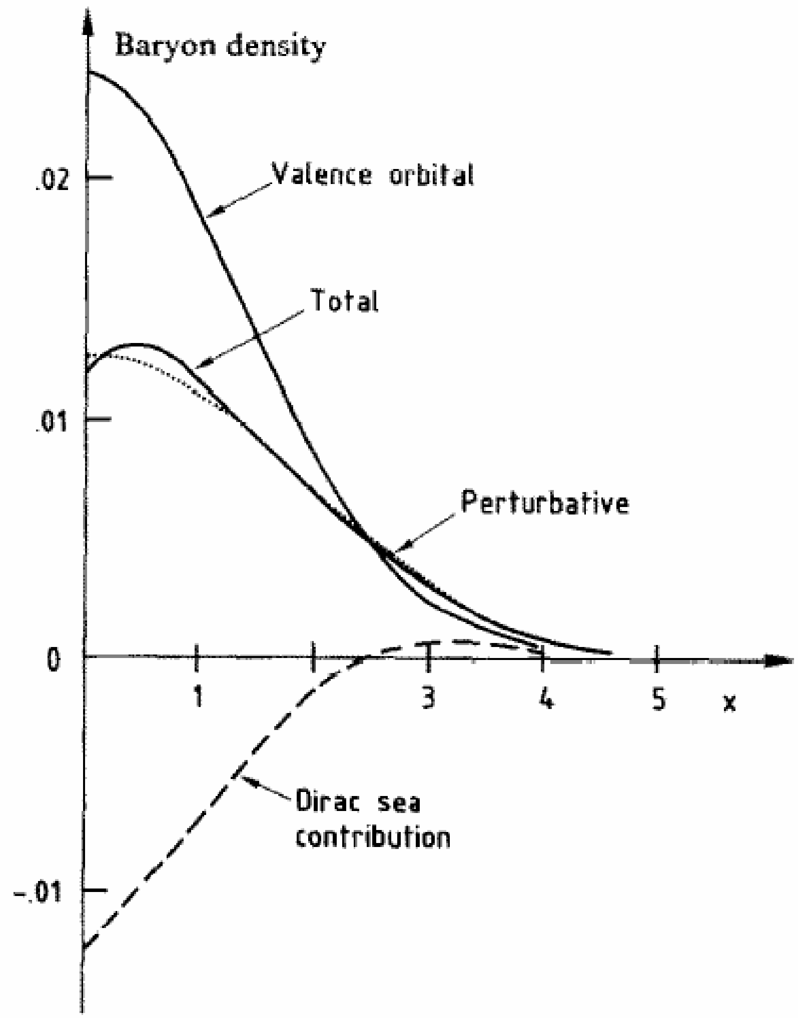
Kahana, S. and G. Ripka (1984). Nucl.Phys., A429, 462.

Polarized Dirac sea

The contributions to baryon density:

The valence quark and of the remaining Dirac sea quark.

The dotted line is that from the Skyrme model for comparison.



Kahana, S. and G. Ripka (1984). Nucl.Phys., A429, 462.

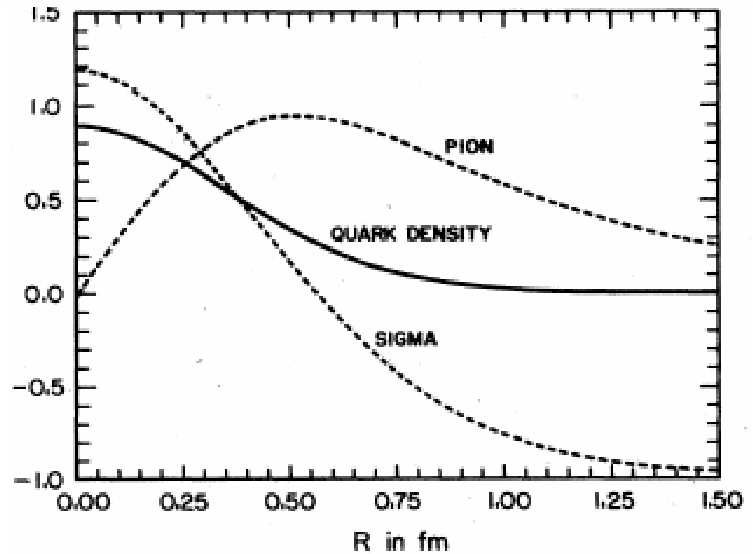
Non-topological soliton

$$\mathscr{L}(x)=i\bar{\psi}(x)\partial\!\!\!/\psi(x)+\tfrac{1}{2}\partial_\mu\hat{\sigma}(x)\partial^\mu\hat{\sigma}(x)+\tfrac{1}{2}\partial_\mu\vec{\hat{\phi}}(x)\partial^\mu\vec{\hat{\phi}}(x)+g\bar{\psi}(x)[\hat{\sigma}(x)+i\vec{\tau}\cdot\vec{\hat{\phi}}(x)\gamma_5]\psi(x)$$
$$-\frac{\lambda^2}{4}[\hat{\sigma}(x)^2+\vec{\hat{\phi}}(x)^2-\nu^2]^2-F_\pi m_\pi{}^2\hat{\sigma}(x)\ .$$

$$\frac{1}{r}\frac{d^2}{d^2r}r\sigma(r)=\frac{\partial U}{\partial\sigma}-\frac{3g}{4\pi}[G^2(r)-F^2(r)]\ ,$$

$$\frac{1}{r}\left[\frac{d^2}{d^2r}-\frac{2}{r^2}\right]rh(r)=\frac{\partial U}{\partial h}+\frac{3g}{2\pi}G(r)F(r)\ ,$$

$$\begin{bmatrix}\omega+g\sigma(r) & \frac{d}{dr}+\frac{2}{r}-gh(r)\\ \frac{d}{dr}+gh(r) & -\omega+g\sigma(r)\end{bmatrix}\begin{bmatrix}G(r)\\F(r)\end{bmatrix}=\begin{bmatrix}G(r)\\F(r)\end{bmatrix}$$



Birse, M. C. and M. K. Banerjee (1984). Phys.Lett., B136, 284.

Birse, M. C. and M. K. Banerjee (1985). Phys.Rev., D31, 118.

Chiral quark soliton model

From QCD instanton vacuum

$$\mathcal{Z} = \int D\pi^A \int D\psi^\dagger D\psi \;\exp \int d^4x\; \psi^\dagger(x) \left[i\partial\!\!\!/ + iMU^{\gamma_5}(x) \right] \psi(x)$$

$$\mathcal{L}' = \bar{\Psi}(i\gamma^\mu \partial_\mu - m_0 - MU^{\gamma_5})\Psi$$

$$\sigma^2 + \vec{\pi}^2 = M^2 \qquad MU^{\gamma_5} = \sigma + i\gamma_5 \vec{\pi}\vec{\tau}$$

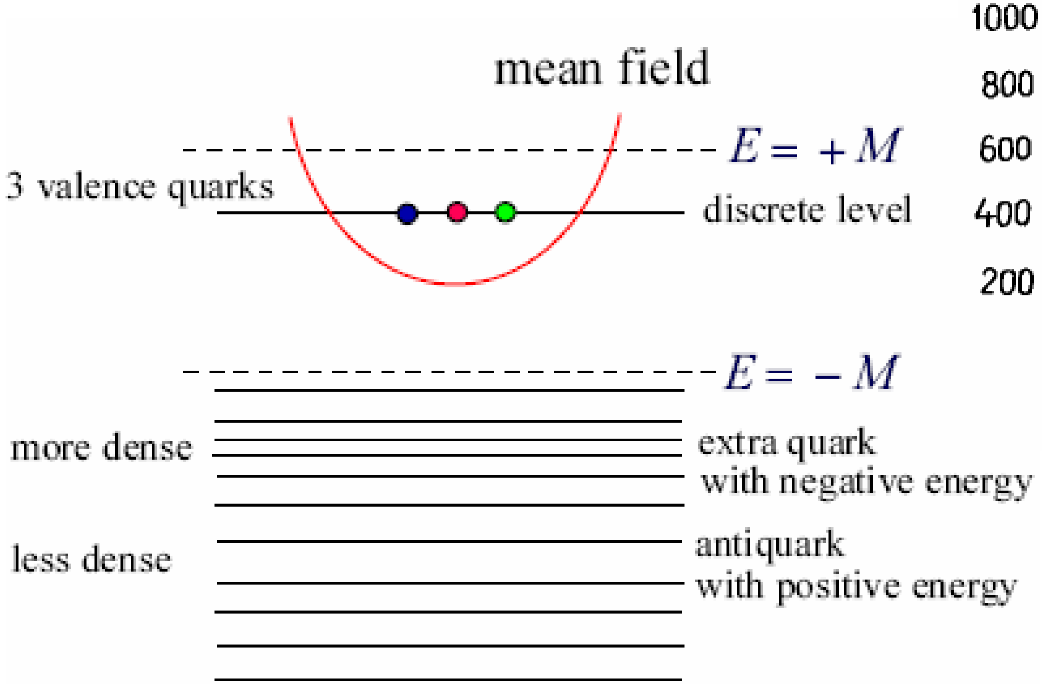
$$\left[\gamma_4 \gamma_k \partial_k + M \gamma_4 e^{i\gamma_5 (\tau n) P(r)} \right] \psi = E\psi$$

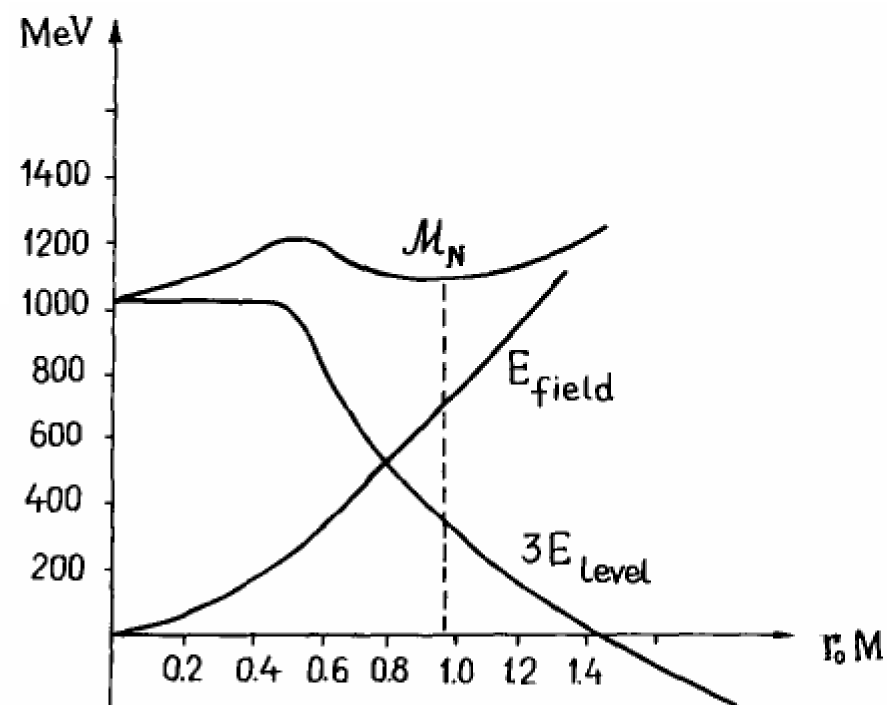
$$\pi^A(\mathbf{x}) = n^A P(r), \qquad n^A = \frac{x^A}{r}, \qquad r = |\mathbf{x}|,$$

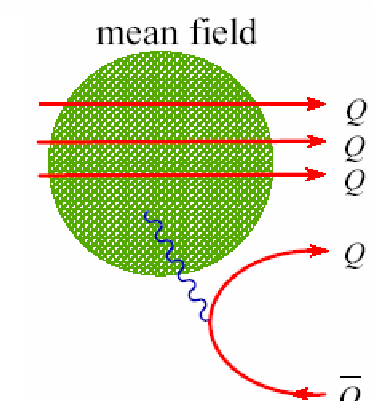
Diakonov, D., V. Petrov and P. Pobylitsa (1988). Nucl.Phys., B306, 809.

Polarized Dirac sea

$$M_N = 3\left(E_{\rm val}[\pi(x)] + E_{\rm sea}[\pi(x)]\right)$$







Dmitri Diakonov, Eur Phys J A (2005) 24, s1, 3–8

The NJL model

$$\mathcal{L} = \bar{\Psi}(x)\left[i\partial\!\!\!/ - \hat{m}\right]\Psi(x) + \frac{G}{2}\left[\left(\bar{\Psi}(x)\Psi(x)\right)^2 + \left(\bar{\Psi}(x)i\gamma_5\vec{\tau}\Psi(x)\right)^2\right]$$

$$\mathcal{L}' = \bar{\Psi}(i\partial\!\!\!/ - \sigma - i\vec{\pi}\cdot\vec{\tau}\gamma_5)\Psi - \frac{1}{2G}(\sigma^2 + \vec{\pi}^2) + \frac{m_0}{G}\sigma$$

$$\sigma(r) = M\cos\Theta(r) \quad \text{and} \quad \pi(r) = M\sin\Theta(r)$$

$$\delta_\Theta\left[N_c\epsilon_{val}(\Theta) + E_{sea}(\Theta)\right]\Big|_{\Theta=\Theta_c} = 0$$

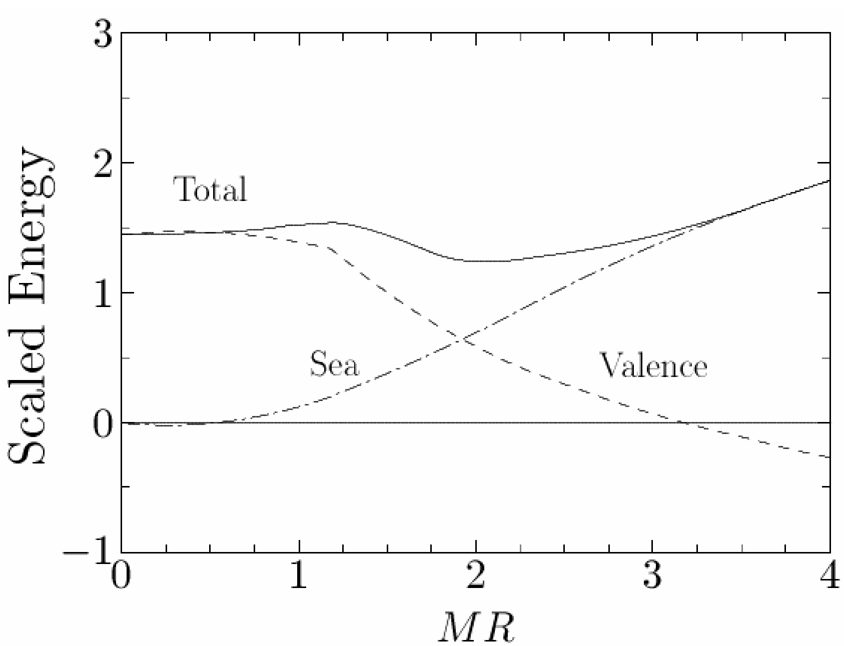
$$\sin\Theta(r)S(\vec{r}) - \cos\Theta(r)P(\vec{r}) = 0$$

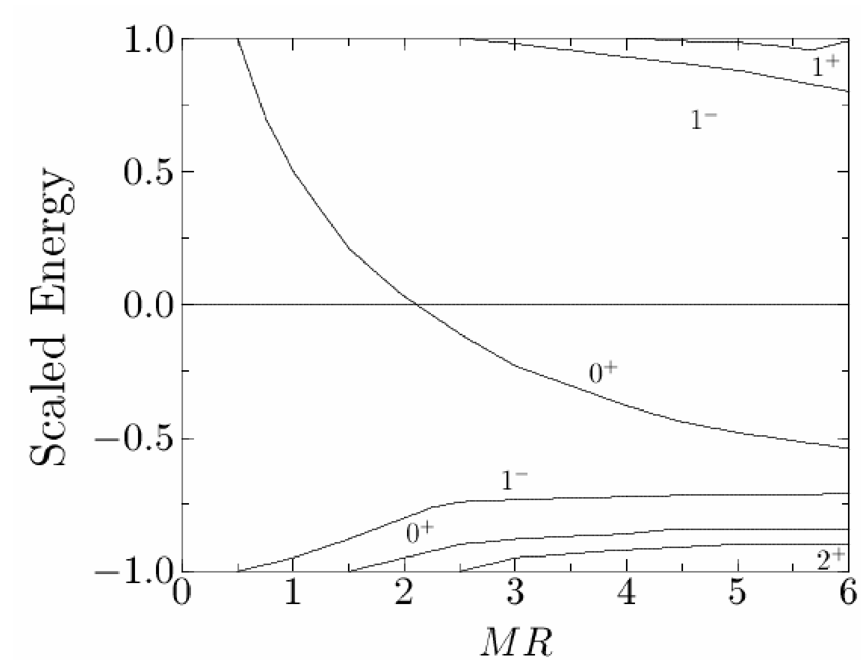
$$S(\vec{r}) = N_cM\left[\sum_n R_2^\Lambda(\epsilon_n)\bar{\Phi}_n(\vec{r})\Phi_n(\vec{r}) + \theta(\epsilon_{val})\bar{\Phi}_{val}(\vec{r})\Phi_{val}(\vec{r})\right]$$

$$P(\vec{r}) = N_cM\left[\sum_n R_2^\Lambda(\epsilon_n)\bar{\Phi}_n(\vec{r})i\gamma_5(\tau^a\hat{n}^a)\Phi_n(\vec{r}) + \theta(\epsilon_{val})\bar{\Phi}_{val}(\vec{r})i\gamma_5(\tau^a\hat{n}^a)\Phi_{val}(\vec{r})\right]$$

C. Christov et al., Prog. Part. Nucl. Phys. 37 (1996) 91
Alkofer, R., H. Reinhardt and H. Weigel (1994). Phys.Rep., 265 139

Polarized Dirac sea





$$M_{cl} = N_c \epsilon_{val} \theta(\epsilon_{val}) + E^{\Lambda}_{sea}$$

$$h \Phi_n(\vec{r}) = \epsilon_n \Phi_n(\vec{r})$$

$$h(\sigma, \vec{\pi}) = -i\gamma^0 \gamma^k \nabla_k + \gamma^0(\sigma + i\vec{\pi} \cdot \vec{\tau}\gamma_5)$$

C. Christov et al., Prog. Part. Nucl. Phys. 37 (1996) 91
Alkofer, R., H. Reinhardt and H. Weigel (1994). Phys.Rep., 265 139

III. Chiral solitons & vacuum polarization

Linear sigma model

$$\mathcal{L} = \bar{\psi}[i\gamma^{\mu}\partial_{\mu} + g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})]\psi + \frac{1}{2}(\partial_{\mu}\sigma\partial^{\mu}\sigma + \partial_{\mu}\vec{\pi}\partial^{\mu}\vec{\pi}) - U(\sigma,\vec{\pi})$$

ψ represents the light quark field $\psi=(u,d)$. The scalar field σ and the pion field $\vec{\pi}=(\pi_1,\pi_2,\pi_3)$.

where, $$U(\sigma,\vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - \nu^2)^2 + H\sigma$$

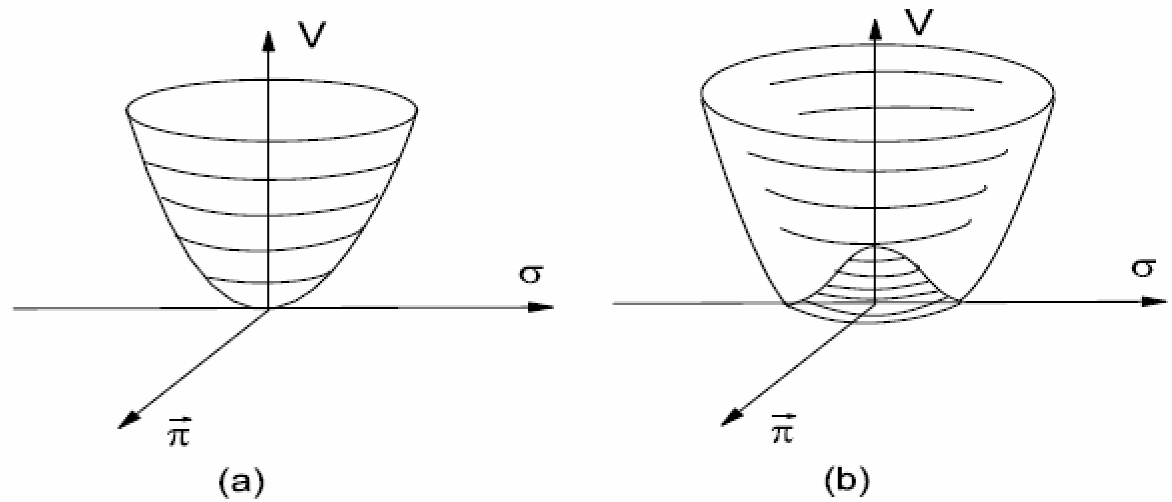
Parameters: $\lambda = 82.1, \quad g = 5.38, \quad \nu = 91.7MeV$

$$H = f_{\pi}m_{\pi}^2, \quad f_{\pi} = 93MeV, \quad m_{\pi} = 138MeV, \; m_{\sigma} = 1200MeV$$

M.C. Birse and M.K. Banerjee, Phys.Rev.D31,118(1985).

Chiral symmetry breaking

$$U(\sigma,\vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - \nu^2)^2 + H\sigma$$



Non-Chiral limit: $H \neq 0$ → Explicit chiral symmetry breaking

Chiral limit: $H = 0$ → Spontaneous chiral symmetry breaking of the vacuum

MFA and "hedgehog" ansatz

$$\sigma(\boldsymbol{r},t) = \sigma(r), \quad \psi(\boldsymbol{r},t) = e^{-i\varepsilon t}\phi_0(r)$$

$$\phi_0(r) = \begin{pmatrix} u(r) \\ i\vec{\sigma}\cdot\hat{\boldsymbol{r}}v(r) \end{pmatrix}\chi$$

$$\vec{\pi}(\boldsymbol{r},t) = \hat{\boldsymbol{r}}\pi(r)$$

Soliton equations (Non-chiral limit)

$$\frac{du(r)}{dr} = -g\pi(r)u(r) - (\varepsilon - g\sigma(r))v(r)$$

$$\frac{dv(r)}{dr} = -\frac{2v(r)}{r} + g\pi(r)v(r) + (\varepsilon + g\sigma(r))u(r)$$

$$\frac{d^2\sigma(r)}{dr^2} + \frac{2}{r}\frac{d\sigma(r)}{dr} = \frac{\partial U}{\partial \sigma} - Ng(u^2(r) - v^2(r))$$

$$\frac{d^2\pi(r)}{dr^2} + \frac{2}{r}\frac{d\pi(r)}{dr} - \frac{2\pi(r)}{r^2} = \frac{\partial U}{\partial \pi} - 2Ngu(r)v(r)$$

N=3 for the nucleon and $\vec{\pi}(\boldsymbol{r}) = \hat{\boldsymbol{r}}\pi(r)$

Soliton equations (Chiral limit)

$$\frac{du(r)}{dr} = -g\pi u(r) - (\varepsilon - g\sigma(r))v(r)$$

$$\frac{dv(r)}{dr} = -\frac{2v(r)}{r} + g\pi(r)v(r) + (\varepsilon + g\sigma(r))u(r)$$

$$r^2 \frac{d^2\theta(r)}{dr^2} + 2r\frac{d\theta(r)}{dr} + \frac{Ngr^2}{\sigma_v}\left[(u^2(r) - v^2(r))\sin\theta(r) + 2u(r)v(r)\cos\theta(r)\right] = 0$$

where, $\pi^2(r) + \sigma^2(r) = \sigma_v^2$

$$\sigma(r) = \sigma_v \cos\theta(r), \quad \pi(r) = \sigma_v \sin\theta(r), \quad \vec{\pi}(\boldsymbol{r}) = \hat{\boldsymbol{r}}\pi(r)$$

Normalization condition

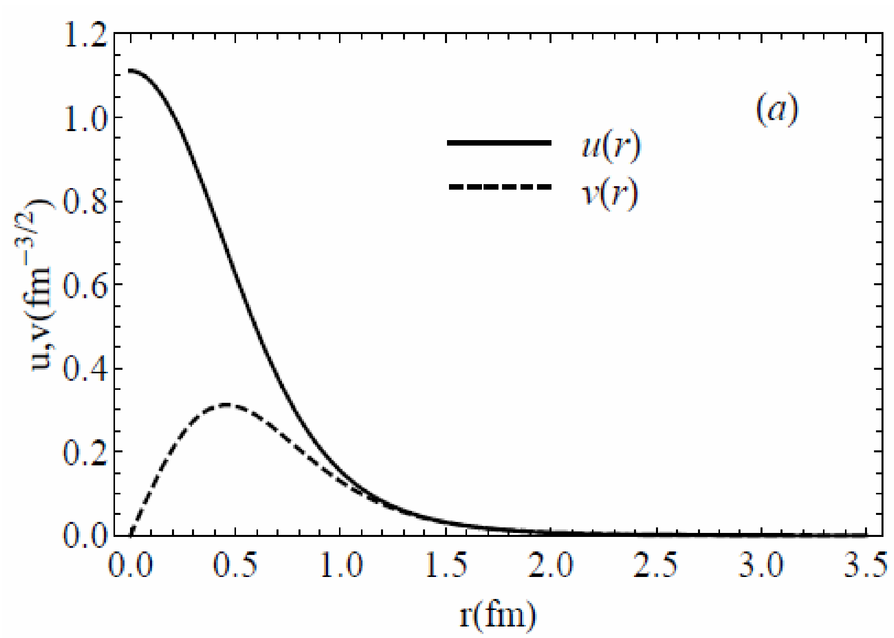
$$4\pi \int r^2 (u^2(r) + v^2(r)) dr = 1$$

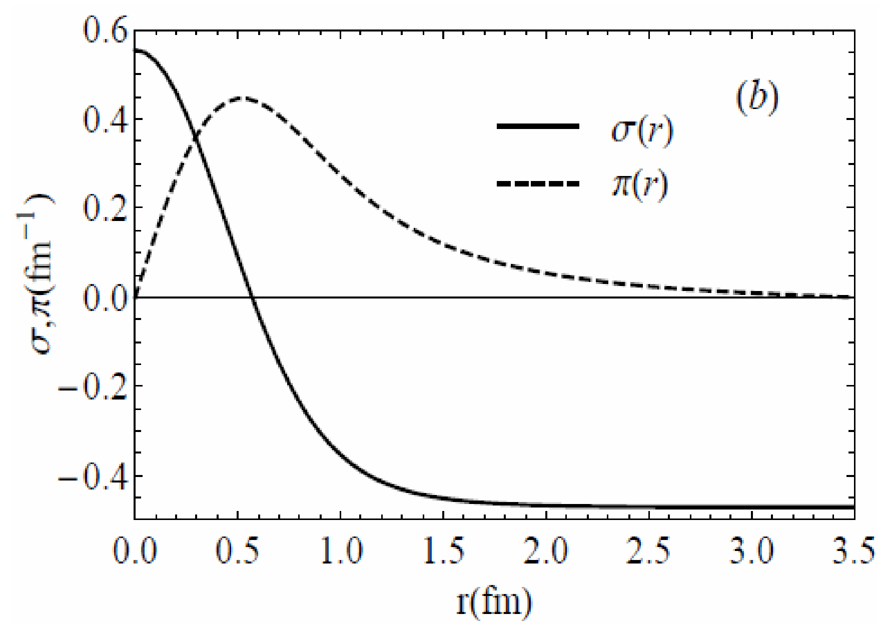
Boundary conditions

$$v(0) = 0, \ \frac{d\sigma(0)}{dr} = 0, \ \pi(0) = 0$$

$$u(\infty) = 0, \ \sigma(\infty) = -f_\pi, \ \pi(\infty) = 0$$

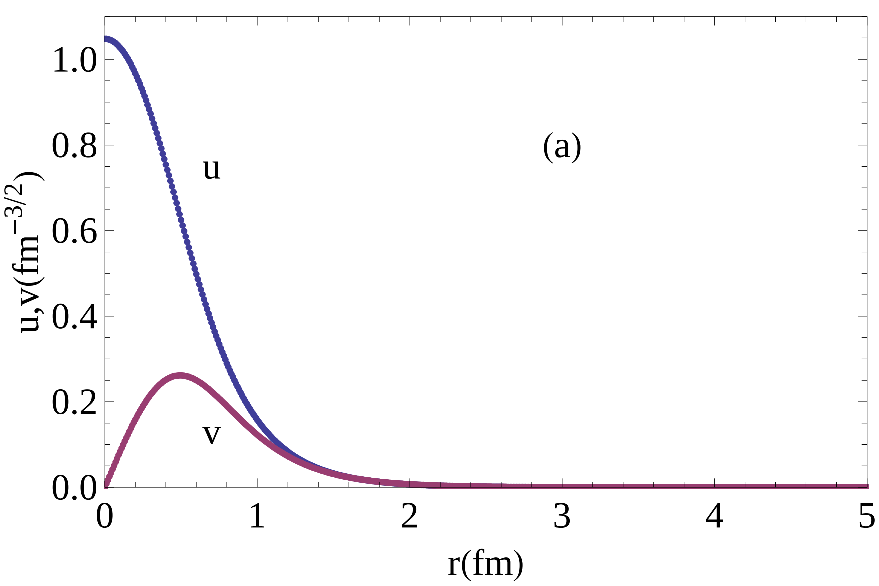
Soliton solutions (Non-chiral limit)

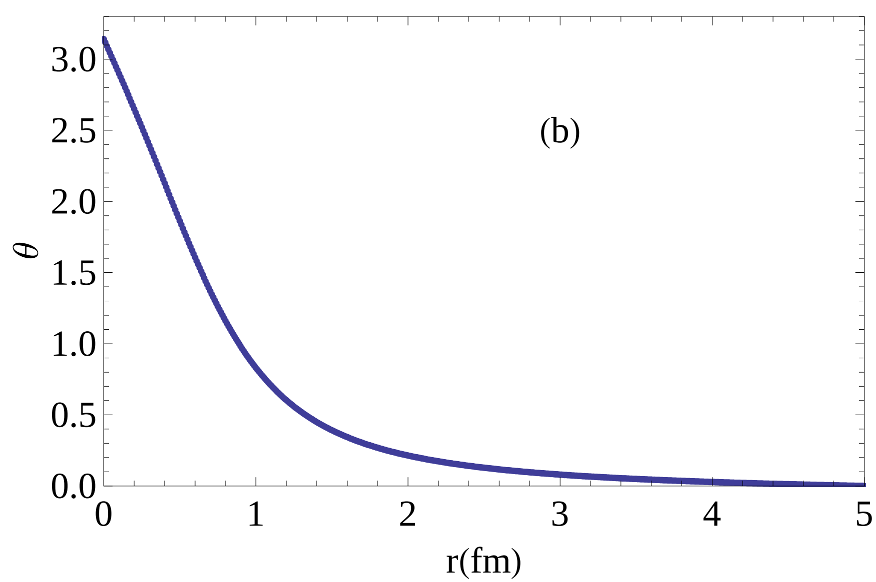




Soliton solutions (Chiral limit)

$$\pi^2(r)+\sigma^2(r)=\sigma_v^2$$





$$\sigma(r)=\sigma_v\cos\theta(r),\quad \pi(r)=\sigma_v\sin\theta(r),\quad \vec{\pi}(\boldsymbol{r})=\hat{\boldsymbol{r}}\pi(r)$$

Soliton solutions could be interpreted as nucleons.

Nucleon energy or mass

$$E = 3\varepsilon + 4\pi \int drr^2 \left[\frac{1}{2}\left(\frac{d\sigma(r)}{dr}\right)^2 + \frac{1}{2}\left(\frac{d\pi(r)}{dr}\right)^2 + \frac{\pi(r)^2}{r^2} + U \right]$$

Proton charge radius

$$r_p^2 = 4\pi \int drr^4 (u^2(r) + v^2(r))$$

The nucleon properties

$$E = M = N\varepsilon + 4\pi\int_0^\infty r^2\left[\frac{1}{2}\left(\frac{d\sigma}{dr}\right)^2 + U\left(\sigma\right)\right]$$

$$R^2 = 4\pi\int_0^\infty r^4\left[u^2(r) + v^2(r)\right]dr$$

$$\mu_p = \frac{8\pi}{3}\int_0^\infty r^3 u(r) v(r) dr$$

$$\frac{g_A}{g_V} = \frac{20\pi}{3}\int_0^\infty r^2\left[u^2(r) - \frac{1}{3}v^2(r)\right]dr$$

Soliton solutions (Non-chiral limit)

(MeV)	$M_q = 450$ $g = 4.83$	$M_q = 500$ $g = 5.38$	$M_q = 550$ $g = 5.91$	$M_q = 600$ $g = 6.45$	$M_q = 900$ $g = 9.67$
ε(MeV)	95	32	-30	-94	-506
E(MeV)	1223	1119	999	867	-104
r_p(fm)	0.72	0.69	0.66	0.65	0.56

Soliton solutions (Chiral limit)

(MeV)	$M_q = 450$ $g = 4.83$	$M_q = 500$ $g = 5.38$	$M_q = 550$ $g = 5.91$	$M_q = 600$ $g = 6.45$	$M_q = 900$ $g = 9.67$
ε(MeV)	32	-18	-68	-118	-416
E(MeV)	1161	1048	928	802	-5
r_p(fm)	0.73	0.71	0.69	0.68	0.64

NJL model

$$\mathcal{L} = \bar{\Psi}(x)\left[i\partial\!\!\!/ - m_0\right]\Psi(x) + \frac{G}{2}\left[\left(\bar{\Psi}(x)\Psi(x)\right)^2 + \left(\bar{\Psi}(x)i\gamma_5\vec{\tau}\Psi(x)\right)^2\right]$$

After bosonization,

$$\mathcal{L}' = \bar{\Psi}(i\partial\!\!\!/ - \sigma - i\vec{\pi}\cdot\vec{\tau}\gamma_5)\Psi - \frac{1}{2G}(\sigma^2 + \vec{\pi}^2) + \frac{m_0}{G}\sigma$$

The effective action is

$$S(\sigma,\vec{\pi}) = -N_c \mathrm{Tr}\,\log\,D(\sigma,\vec{\pi}) + \frac{1}{2G}\int \mathrm{d}^4x(\sigma^2 + \vec{\pi}^2) - \frac{m_0}{G}\int d^4x\sigma$$

The effective potential & gap equation

After proper time regularization,

$$V_{eff} = \frac{N_c}{4\pi^2}\int_{\Lambda^{-2}}^{\infty}\frac{du}{u^3}\left(e^{-uM^2} - e^{-um_0^2}\right) + \frac{1}{2G}\left(\sigma^2 + \vec{\pi}^2\right) - \frac{m_0}{G}\sigma$$

The gap equation,is

$$M = m_0 - G\langle\bar{\psi}\psi\rangle$$

where,

$$\langle\bar{\psi}\psi\rangle = -\frac{N_c M}{2\pi^2}\int_{\Lambda^{-2}}^{\infty}\frac{du}{u^2}e^{-uM^2}$$

Parameters set in NJL model

The GMOR relation is $m_\pi^2 f_\pi^2 = -m_0 \langle \bar{\psi}\psi \rangle$

where the pion decay constant,

$$f_\pi^2 = M^2 \frac{N_c}{4\pi^2} \int_{\Lambda^{-2}}^{\infty} \frac{du}{u} e^{-uM^2}$$

The independent parameters here we choose are

Λ and M.

Soliton equations in NJL model

$$\frac{du(r)}{dr} = -\pi u(r) - (\varepsilon - \sigma(r))v(r)$$

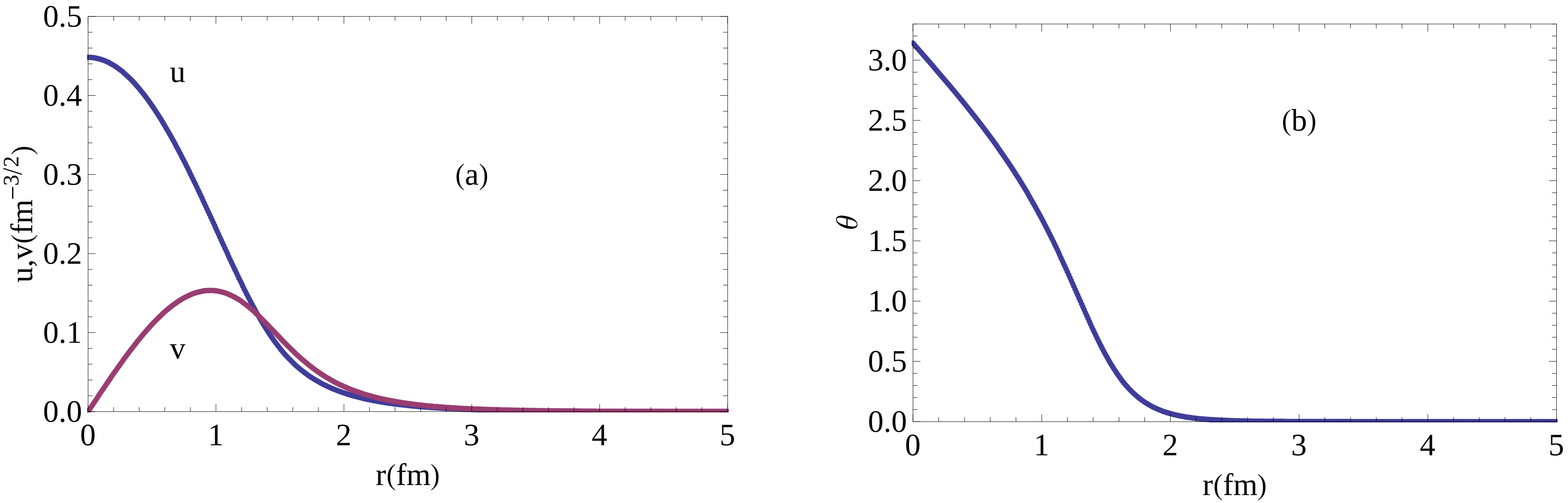
$$\frac{dv(r)}{dr} = -\frac{2v(r)}{r} + \pi(r)v(r) + (\varepsilon + \sigma(r))u(r)$$

$$\frac{m_0}{G}\sin\theta(r) - N\left[(u^2(r) - v^2(r))\sin\theta(r) + 2u(r)v(r)\cos\theta(r)\right] = 0$$

where, $\pi^2(r) + \sigma^2(r) = M^2$

$$\sigma(r) = M\cos\theta(r), \quad \pi(r) = M\sin\theta(r), \quad \vec{\pi}(\boldsymbol{r}) = \hat{\boldsymbol{r}}\pi(r)$$

Soliton solutions in NJL model



NJL Soliton solutions

(MeV) (MeV)	$M_q = 246$ $\Lambda = 800$	$M_q = 291$ $\Lambda = 700$	$M_q = 348$ $\Lambda = 650$	$M_q = 378$ $\Lambda = 640$	$M_q = 517$ $\Lambda = 650$
ε(MeV)	68	27	-25	-53	-187
E(MeV)	452	347	203	123	-262
r_p(fm)	1.22	1.15	1.14	1.09	1.06

IV. Summary

- QCD vacuum is very nontrivial which properties could be changed, which leads to the different emergent phenomena of quarks.

- Through chiral soliton models we show that the vacuum is polarized which has a deep meaning for the study of nucleon structure.

Outlook

- The quantum corrections from the polarized vacuum should be further calculated...
- The QCD vacuum of nontrivial topology and its implication for nucleon structure should be further studied…
- How to incorporate confinement into chiral soliton models from a nontrivial QCD vacuum deserves a deep exploration ...

Thank you!