Gluon emission from heavy quark in dense nuclear matter

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Outline

1. Introduction

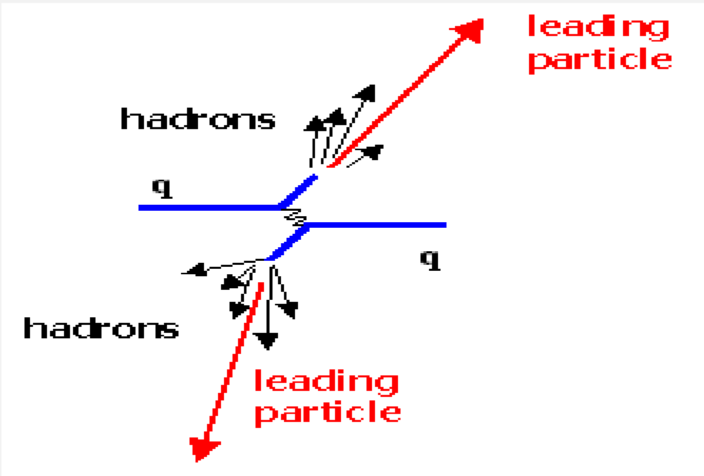
2. General formula for medium-induced gluon emission spectrum

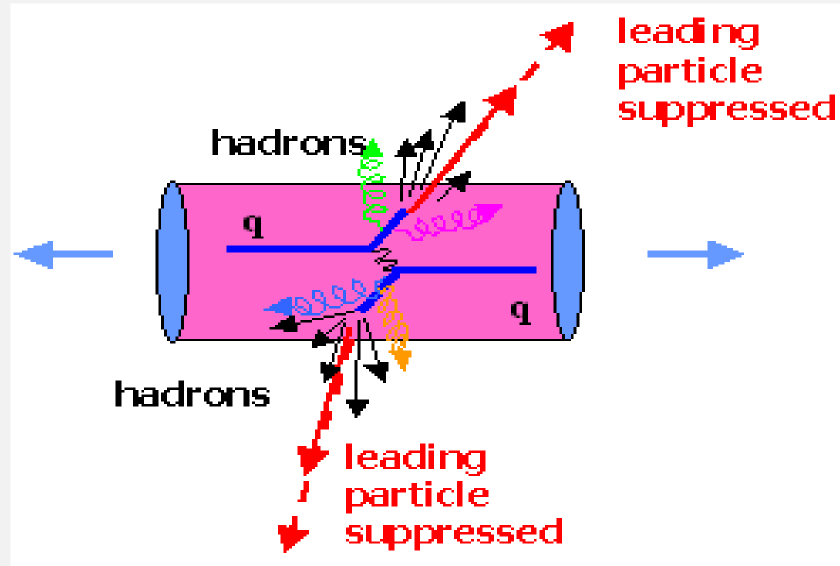
3. Gluon emission with static scattering centers

4. Gluon emission with dynamic scattering centers

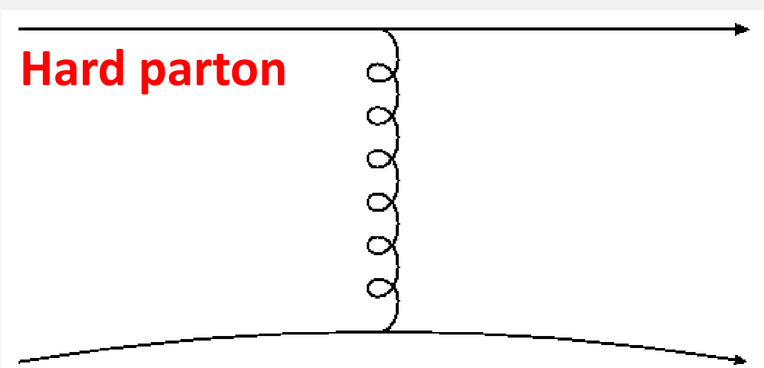
5. Summary

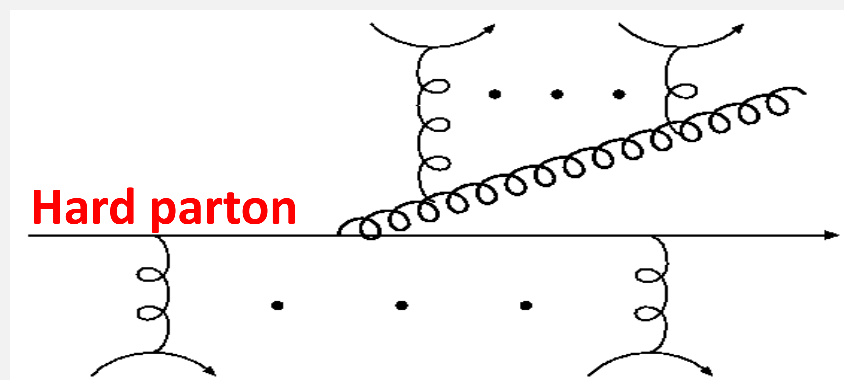
Jets as hard probes of QGP





Jet quenching

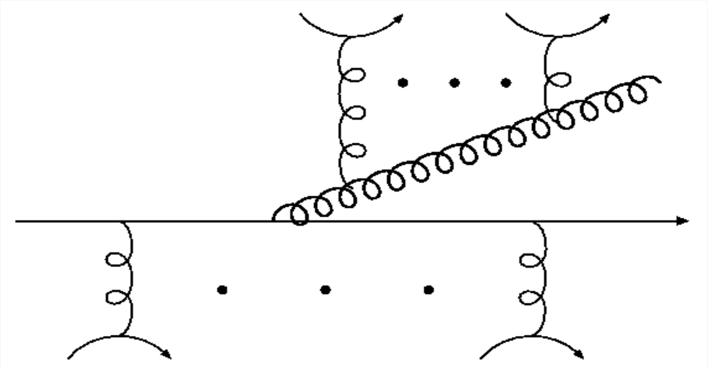




Elastic (collisional)

Inelastic (radiative)

Medium-induced radiative process(1)



Single gluon emission

BDMPS-Z: Baier-Dokshitzer-Mueller-Peigne-Schiff-Zakharov
ASW: Amesto-Salgado-Wiedemann
AMY: Arnold-Moore-Yaffe
GLV: Gyulassy-Levai-Vitev
DGLV: Djordjevic-Gyulassy-Levai-Vitev
HT: Wang-Guo-Majumder

Collinear rescattering expansion: **HT , BDMPS-Z**

Soft gluon emission approximation: **GLV , DGLV, ASW**

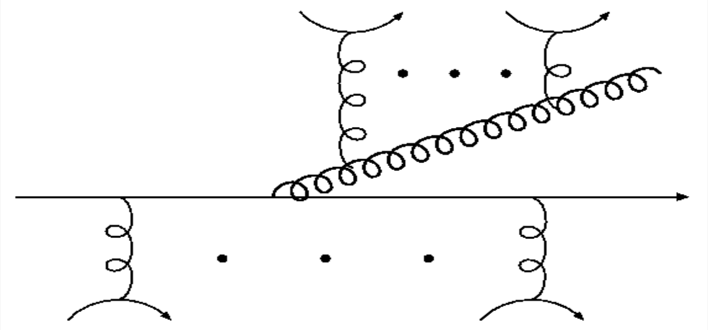
Beyond soft approximation for GLV:

B.Blagojevic et al, PRC(2019); M. D. Sievert , I. Vitev, PRD(2018).

Beyond collinear expansion and soft approximation in DIS framework:

L. Zhang, D.-F. Hou, G. -Y. Qin, PRC(2018) and arXiv:1812.11048v1 ;
Y. -Y. Zhang, G.-Y. Qin, X.-N. Wang, arXiv:1905.12699.

Medium-induced radiative process(2)



Single gluon emission

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Only transverse scattering:

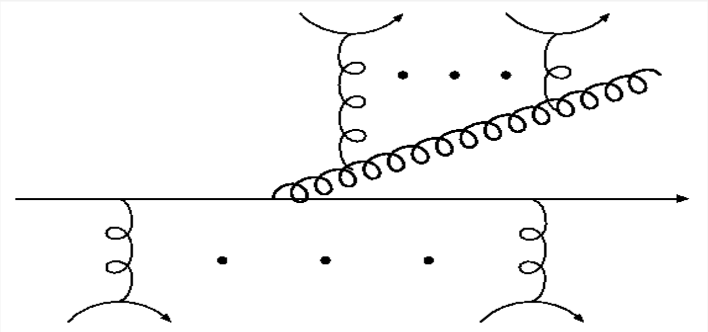
HT: $$\frac{dN_g}{dx dk_\perp^2\, dt} = \frac{2\,\alpha_s}{\pi}\, P(x)\, \frac{\hat{q}}{k_\perp^4}\ \sin^2\!\left(\frac{t-t_i}{2\,\tau_f}\right)$$

GLV: $$\frac{dN_g}{dx} \sim \int d\mathbf{q}_\perp \int d\mathbf{k}_\perp \frac{1}{(q_\perp^2+\mu^2)^2}\, \frac{2(\mathbf{q}_\perp-\mathbf{k}_\perp)}{(\mathbf{q}_\perp-\mathbf{k}_\perp)^2+\chi^2} \left(\frac{\mathbf{k}_\perp}{\mathbf{k}_\perp^2+\chi^2} - \frac{\mathbf{q}_\perp-\mathbf{k}_\perp}{(\mathbf{q}_\perp-\mathbf{k}_\perp)^2+\chi^2}\right) \left(1-\cos\left[\frac{(\mathbf{q}_\perp-\mathbf{k}_\perp)^2+\chi^2}{2xE}\Delta z\right)\right]$$

Include longitudinal scattering in gluon radiation process

L. Zhang, D.-F. Hou, G. -Y. Qin, PRC(2018) and arXiv:1812.11048v1

Medium-induced radiative process(3)



Single gluon emission

BDMPS-Z: Baier-Dokshitzer-Mueller-Peigne-Schiff-Zakharov
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DGLV: Djordjevic-Gyulassy-Levai-Vitev
HT: Wang-Guo-Majumder

Static scattering senter : BDMPS-Z, GLV , ASW

Dynamic scattering senter : DGLV

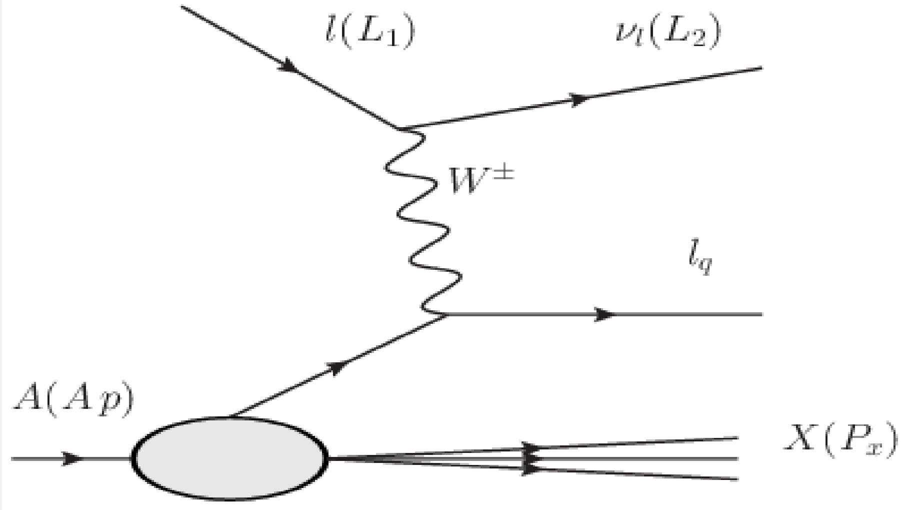
Our work:

- **Beyond collinear rescattering expansion**
- **Beyond soft gluon emission approximation**
- **Include transverse and longitudinal scatterings**
- **For general medium in DIS framework :**
  - **Static: arXiv:1812.11048v1**
  - **Dynamic: in preparation**

Frame work: Deep inelastic scattering (DIS)

The framework of DIS:

$$l(L_1) + A(Ap) \to \nu_l(L_2) + q(l_q) + X(P_X)$$



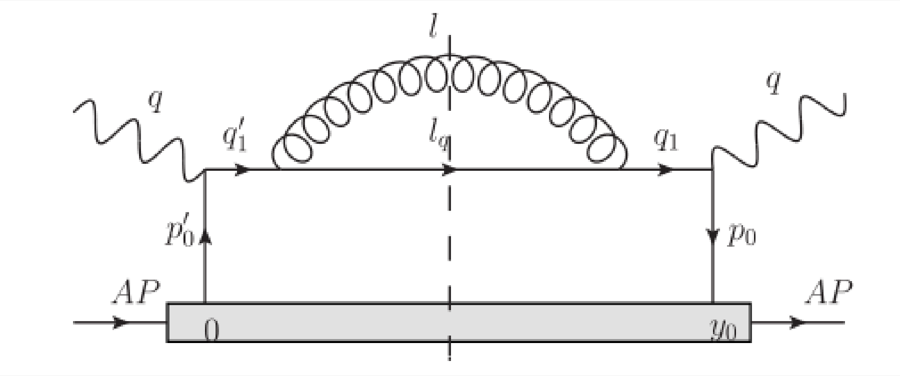
Differential cross section for DIS:

$$E_{L_2}\frac{d\sigma_{DIS}}{d^3\mathbf{L}_2} = \frac{G_F^2}{(4\pi)^3 s} L_{\mu\nu} W^{\mu\nu}$$

$$L_{\mu\nu} = \frac{1}{2}\mathrm{Tr}[\not{L}_1\,\gamma_\mu\,(1-\gamma_5)\,\not{L}_2\,(1+\gamma_5)\,\gamma_\nu]$$

$$W^{\mu\nu} = \frac{1}{2}\sum_X (2\pi)^4\delta^4(q + P_A - P_X - l_q) \times \langle A|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|A\rangle$$

Gluon emission in vacuum

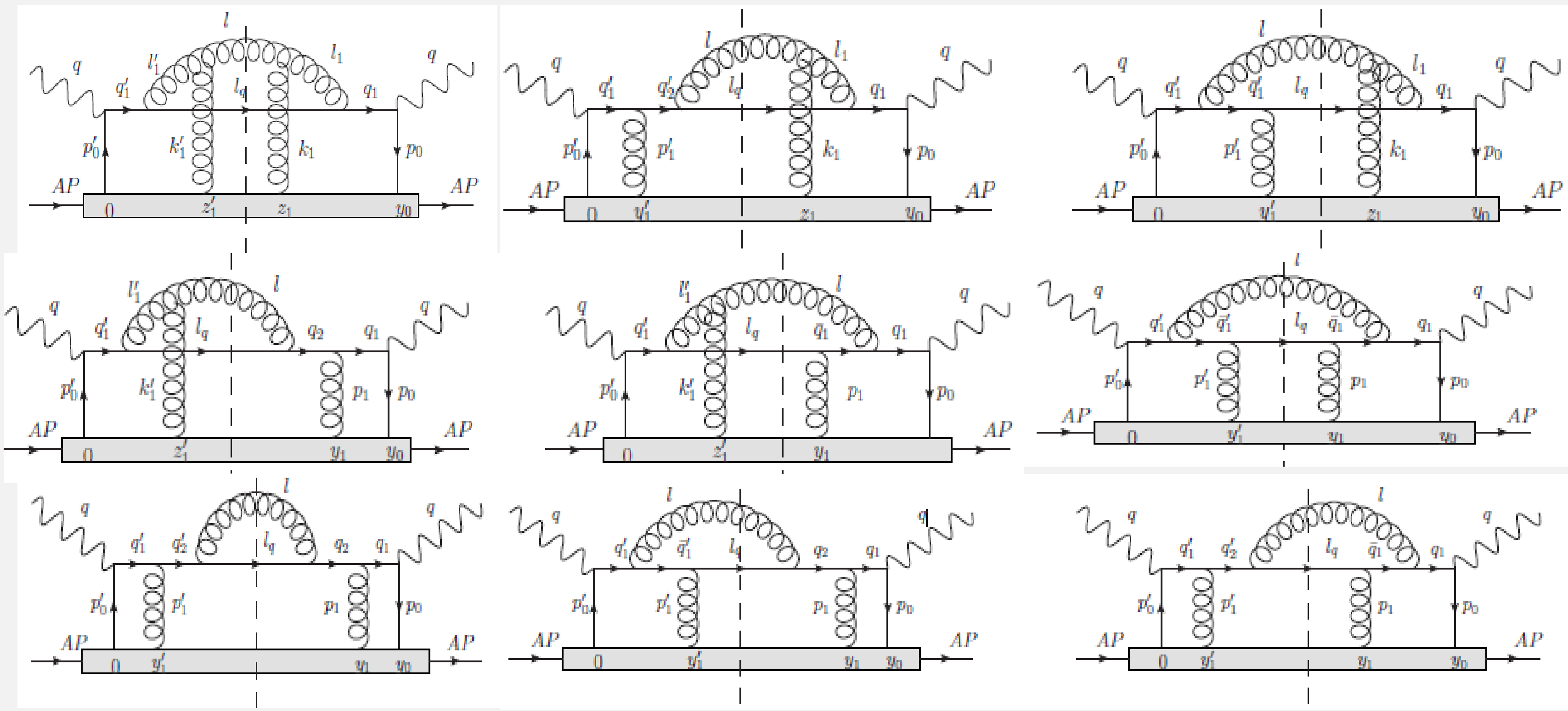


The gluon emission spectrum from heavy quark in vacuum:

$$\frac{dN_g^{vac}}{dl_\perp^2\, dy} = C_F \frac{\alpha_s}{2\pi} P(y) \frac{\left(l_\perp^2 + \frac{y^4}{1+(1-y)^2} M^2\right)}{(l_\perp^2 + y^2 M^2)^2}.$$

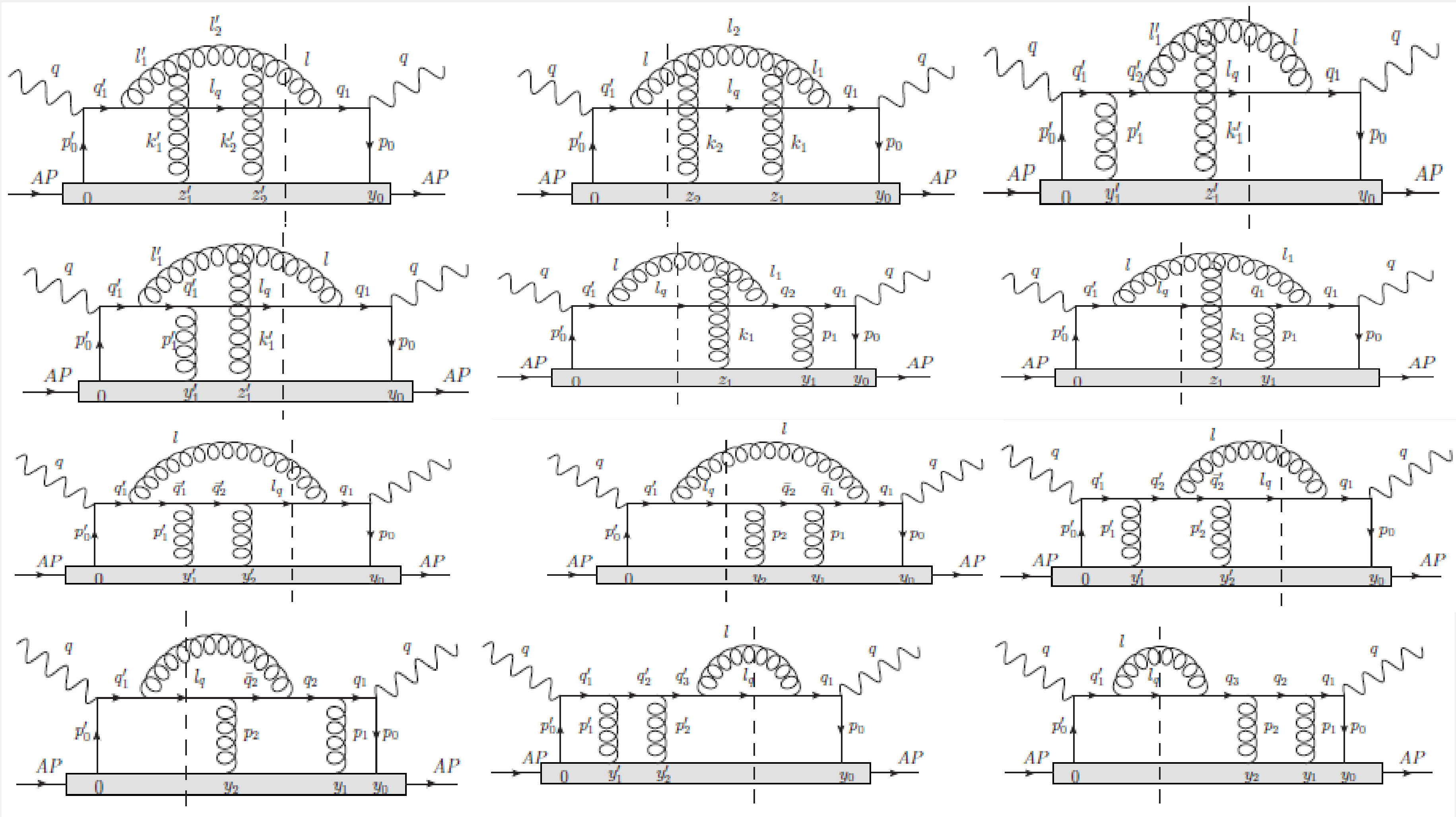
$$y = \frac{l^-}{q^-}, \quad P(y) = \frac{1+(1-y)^2}{y}.$$

Central-cut diagrams with single scattering



X. F. Guo, X. N. Wang; PRL,(2000)
B. W. Zhang, X. N. Wang; Nucl.Phys.A,2003

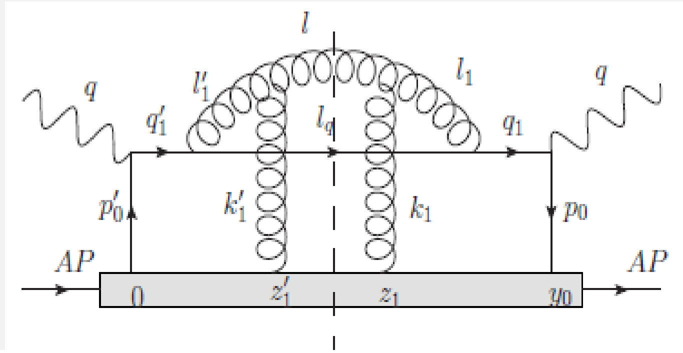
Gluon emission in dense nuclear matter



There are 21 diagrams

X. F. Guo, X. N. Wang; PRL,(2000)
B. W. Zhang, X. N. Wang; Nucl.Phys.A,2003

Look at one diagram



$$\frac{dW^{A\mu\nu}_{(1)}}{dy\,d^2\mathbf{l}_\perp} = \sum_i A C_p^A \frac{1}{4p^+q^-}|V_{ij}|^2 \mathrm{Tr}[\not{p}\gamma^\mu(1-\gamma^5)\{\not{q}+(x_B+x_M)\not{p}\}(1+\gamma^5)\gamma^\nu](2\pi) f_q(x_B+x_M+\tilde{x}_L)$$

$$\times C_A \frac{\alpha_s}{2\pi^2} P(y) \int dZ_1^- \int \frac{d^3\mathbf{k}_1}{(2\pi)^3} \int d\delta z_1^- \int d^3\delta\mathbf{z}_1\, e^{-i\mathbf{k}_1\cdot\delta\mathbf{z}_1} \left(g^2\frac{C_F C_2(R)}{N_c^2-1}\right) \langle A|A^+(\delta z_1^-,\delta\mathbf{z}_1)A^+(0)|A\rangle$$

$$\times\left[2-2\cos\left(\frac{y(1-y)}{(y-\lambda_1^-)(1+\lambda_1^- -y)}\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2}{l_\perp^2+y^2M^2}\frac{Z_1^-}{\tilde{\tau}^-_{\mathrm{form}}}\right)\right]$$

$$\times \frac{1+(1+\lambda_1^- -y)^2}{1+(1-y)^2}\left(\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-}\right)^2 \frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+\frac{(y-\lambda_1^-)^4M^2}{1+(1+\lambda_1^- -y)^2}}{[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2]^2}$$

Scattered gluon field

Hard matrix element

Phase factor

$$Z_1=\frac{z_1+z_1'}{2},\qquad \delta z_1=z_1-z_1'.$$

$$\lambda_1^-=\frac{k_1^-}{q^-},\qquad \tilde{x}_L=\frac{l_\perp^2+y^2M^2}{2p^+q^-y(1-y)}.$$

Look at one diagram

Define the distribution function $\mathcal{D}(\,k_1^-,\,\mathbf{k}_{1\,\perp})$

$$\mathcal{D}(\,k_1^-,\,\mathbf{k}_{1\,\perp}) = \int d\delta z_1^- \int d^3\delta\mathbf{z}_1 e^{-i\mathbf{k}_1\cdot\delta\mathbf{z}_1}\left(g^2\frac{C_F C_2(R)}{N_c^2-1}\right)\langle A|A^+(\delta z_1^-,\delta\mathbf{z}_1)A^+(0)|A\rangle$$

The medium induced gluon emission spectrum for this diagram:

$$\frac{dN_{g,\,(1)}^{med}}{dy d^2\mathbf{l}_\perp} = C_A\frac{\alpha_s}{2\pi^2}P(y)\int dZ_1^- \int \frac{d^3\mathbf{k}_1}{(2\pi)^3}\mathcal{D}(\,k_1^-,\,\mathbf{k}_{1\,\perp})$$

$$\times\left[2-2\cos\left(\frac{y(1-y)}{(y-\lambda_1^-)(1+\lambda_1^- - y)}\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2}{l_\perp^2+y^2M^2}\frac{Z_1^-}{\tilde{\tau}_{\text{form}}^-}\right)\right]$$

$$\times\left[\frac{1+(1+\lambda_1^- - y)^2}{1+(1-y)^2}\left(\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-}\right)^2\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+\frac{(y-\lambda_1^-)^4M^2}{1+(1+\lambda_1^- - y)^2}}{\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+(y-\lambda_1^-)^2M^2\right]^2}\right].$$

$$\tilde{\tau}_{\text{form}}^- = \frac{1}{\tilde{x}_L p^+}$$

Sum over all diagrams

General formula for gluon emission spectrum :

$$\mathcal{D}(k_1^-,\mathbf{k}_{1\perp}) = (2\pi)^3 \frac{dP_{el}}{d^3k_1 dZ_1^-}$$

$$
\begin{aligned}
\frac{dN_g^{med}}{dy\,d^2\mathbf{l}_\perp} = {} & \frac{\alpha_s}{2\pi^2} P(y) \int dZ_1^- \int \frac{d^3\mathbf{k}_1}{(2\pi)^3} \mathcal{D}(k_1^-, \mathbf{k}_{1\perp}) \\
& \times \Bigg\{ \left[2 - 2\cos\left(\frac{y(1-y)}{(y-\lambda_1^-)(1+\lambda_1^- - y)} \frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2}{l_\perp^2 + y^2 M^2} \frac{Z_1^-}{\tilde{\tau}_{\mathrm{form}}^-} \right) \right] \\
& C_A \left[\frac{1+(1+\lambda_1^- - y)^2}{1+(1-y)^2} \left(\frac{y - \frac{\lambda_1^-}{2}}{y-\lambda_1^-} \right)^2 \frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + \frac{(y-\lambda_1^-)^4 M^2}{1+(1+\lambda_1^- - y)^2}}{\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2 \right]^2} \right. \\
& - \frac{1+(1+\lambda_1^- - y)(1-y)}{2[1+(1-y)^2]} \left(\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-} \right) \frac{\mathbf{l}_\perp \cdot (\mathbf{l}_\perp - \mathbf{k}_{1\perp}) + \frac{y^2(y-\lambda_1^-)^2}{1+(1+\lambda_1^- - y)(1-y)} M^2}{[l_\perp^2 + y^2 M^2]\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2 \right]} \\
& \left. - \frac{1+(1+\lambda_1^- - y)(1-\frac{y}{1+\lambda_1^-})}{2[1+(1-y)^2]} \left(\frac{y-\frac{\lambda_1^-}{2}}{y-\lambda_1^-} \right) \frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp}) \cdot \left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp} \right) + \frac{\left(\frac{y}{1+\lambda_1^-} \right)^2 (y-\lambda_1^-)^2}{1+(1+\lambda_1^- - y)(1-\frac{y}{1+\lambda_1^-})} M^2}{\left[\left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp} \right)^2 + (\frac{y}{1+\lambda_1^-})^2 M^2 \right] \left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2 \right]} \right] \\
& + \left[2 - 2\cos\left(\frac{Z_1^-}{\tilde{\tau}_{\mathrm{form}}^-} \right) \right] \left[C_F \frac{\mathbf{l}_\perp^2 + \frac{y^4}{1+(1-y)^2} M^2}{[\mathbf{l}_\perp^2 + y^2 M^2]^2} - \frac{C_A}{2} \frac{\left(y - \frac{\lambda_1^-}{2} \right)^2}{y(y-\lambda_1^-)} \frac{\mathbf{l}_\perp^2 + \frac{y^4}{1+(1-y)^2} M^2}{[\mathbf{l}_\perp^2 + y^2 M^2]^2} \right. \\
& \left. + \left(\frac{C_A}{2} - C_F \right) \frac{1+(1-y)\left(1 - \frac{y}{1+\lambda_1^-} \right)}{1+(1-y)^2} \frac{\mathbf{l}_\perp \cdot \left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp} \right) + \frac{y^2 \left(\frac{y}{1+\lambda_1^-} \right)^2 M^2}{1+(1-y)\left(1 - \frac{y}{1+\lambda_1^-} \right)}}{[l_\perp^2 + y^2 M^2] \left[\left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp} \right)^2 + \left(\frac{y}{1+\lambda_1^-} \right)^2 M^2 \right]} \right] \\
& + C_F \left[\frac{1+\left(1 - \frac{y}{1+\lambda_1^-} \right)^2}{1+(1-y)^2} \frac{\left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp} \right)^2 + \frac{\left(\frac{y}{1+\lambda_1^-} \right)^4}{1+\left(1 - \frac{y}{1+\lambda_1^-} \right)^2} M^2}{\left[\left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp} \right)^2 + \left(\frac{y}{1+\lambda_1^-} \right)^2 M^2 \right]^2} - \frac{\mathbf{l}_\perp^2 + \frac{y^4}{1+(1-y)^2} M^2}{[\mathbf{l}_\perp^2 + y^2 M^2]^2} \right] \Bigg\}.
\end{aligned}
$$

L. Zhang, D.-F. Hou, G. -Y. Qin, arXiv:1812.11048v1

Static screening potential(1)

Static screening potential :

$$A^{\mu}(\mathbf{p}) = g^{\mu -}(2\pi)\delta(p^{-})\frac{-g}{\mathbf{p}_{\perp}^{2}+\mu^{2}}$$

The gluon field correlation function :

$$\langle A|A^{\mu}(\delta\mathbf{z}_1)A^{\nu}(0)|A\rangle = \delta_{+}^{\mu}\delta_{+}^{\nu}\rho^{-}(\delta\mathbf{z}_1)\delta(\delta z_1^{-})\int\frac{d^2\mathbf{p}_{\perp}}{(2\pi)^2}e^{i\mathbf{p}_{\perp}\cdot\delta\mathbf{z}_{1\perp}}\frac{g^2}{(\mathbf{p}_{\perp}^{2}+\mu^{2})^2}$$

Distribution function

$$\mathcal{D}(k_1^{-},\mathbf{k}_{1\perp}) = (2\pi)\delta(k_1^{-})(2\pi)^2\rho^{-}\frac{d\sigma_{\mathrm{el}}}{d^2\mathbf{k}_{1\perp}} = (2\pi)\delta(k_1^{-})\mathcal{D}_{\perp}(\mathbf{k}_{1\perp})$$

$$\mathcal{D}_{\perp}(\mathbf{k}_{1\perp}) = (2\pi)^2\rho^{-}\frac{d\sigma_{\mathrm{el}}}{d^2\mathbf{k}_{1\perp}} = (2\pi)^2\frac{dP_{\mathrm{el}}}{d^2\mathbf{k}_{1\perp}dZ_1^{-}}$$

Static screening potential(2)

$$\frac{dN_g^{\rm med}}{dyd^2l_\perp} = \frac{\alpha_s}{2\pi^2}P(y)\int dZ_1^- \int d^2\mathbf{k}_{1\perp}\frac{dP_{\rm el}}{d^2\mathbf{k}_{1\perp}dZ_1^-}$$

$$\times\left\{C_A\left[2-2\cos\left(\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2}{l_\perp^2+y^2M^2}\frac{Z_1^-}{\tilde{\tau}^-_{\rm form}}\right)\right]\times\left[\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]^2}\right.\right.$$

$$\left.-\frac{1}{2}\frac{(\mathbf{l}_\perp-\mathbf{k}_{1\perp})\cdot(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]}-\frac{1}{2}\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{\left[l_\perp^2+y^2M^2\right]\left[(\mathbf{l}_\perp-\mathbf{k}_{1\perp})^2+y^2M^2\right]}\right]$$

$$+\left(\frac{C_A}{2}-C_F\right)\left[2-2\cos\left(\frac{Z_1^-}{\tilde{\tau}^-_{\rm form}}\right)\right]\left[\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})+\frac{y^4}{1+(1-y)^2}M^2}{\left[l_\perp^2+y^2M^2\right]\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]}-\frac{\mathbf{l}_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{\left[\mathbf{l}_\perp^2+y^2M^2\right]^2}\right]$$

$$\left.+C_F\left[\frac{(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+\frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp-y\mathbf{k}_{1\perp})^2+y^2M^2\right]^2}-\frac{\mathbf{l}_\perp^2+\frac{y^4}{1+(1-y)^2}M^2}{\left[\mathbf{l}_\perp^2+y^2M^2\right]^2}\right]\right\}$$

Beyond collinear rescattering expansion

Beyond soft gluon emission approximation

Soft gluon emission limit

$$\frac{dN_g^{\text{med}}}{dy\,d^2\mathbf{l}_\perp} = \frac{\alpha_s}{2\pi^2} P(y) \int dZ_1^- \int d^2\mathbf{k}_{1\perp} \frac{dP_{\text{el}}}{d^2\mathbf{k}_{1\perp} dZ_1^-} \times C_A \left[2 - 2\cos\left(\frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2 M^2}{l_\perp^2 + y^2 M^2} \frac{Z_1^-}{\tilde{\tau}_{\text{form}}^-} \right) \right]$$

$$\times \left[\frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2}{\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2 M^2\right]^2} - \frac{\mathbf{l}_\perp \cdot (\mathbf{l}_\perp - \mathbf{k}_{1\perp})}{\left[l_\perp^2 + y^2 M^2\right]\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2 M^2\right]} \right]$$

It is consistent with GLV formula (with zero effective mass for radiated gluon) from heavy quarks with static scattering centers .

$$y^2 M \ll y\, M \sim l_\perp \sim k_{1\perp}$$

M.Gyulassy, P.Levai, I. Vitev; PRL,(2000)
M.Gyulassy, P.Levai, I. Vitev; Nucl.Phys.A,(2001)
M.Djordjevic and M.Gyulassy,, Nucl. Phys. A(2004)

Dynamic Scattering Center(1)

The gluon field correlation:

$$\frac{C_2(R)}{N_c^2-1}\langle A|A^\mu(\delta z_1)A^\nu(0)|A\rangle = \int \frac{d^4p}{(2\pi)^4} D_>^{\mu\nu}(p)e^{ip\cdot\delta z_1}$$

$D_>^{\mu\nu}(p)$ **is the effective Hard-Thermal Loop gluon propagator**

$$D_>^{\mu\nu}(p) = -[1+f(p_0)]\,[P^{\mu\nu}(p)\rho_T(p) + Q^{\mu\nu}(p)\rho_L(p)] \qquad (p_0 \leq |(p)|)$$

$$= \theta(1-\frac{p_0^2}{\mathbf{p}^2})\,[1+f(p_0)]\,2\,\mathrm{Im}\left(\frac{P_{\mu\nu}(p)}{p^2-\Pi_T(x)} + \frac{Q_{\mu\nu}(p)}{p^2-\Pi_L(x)}\right)$$

$$x = \frac{p_0}{|\mathbf{p}|}$$

Distribution function

$$\mathcal{D}(k_1^-,\mathbf{k}_{1\perp}) = 2\pi\alpha_s C_F\left[1+f\left(\frac{\sqrt{2}}{2}k_1^-\right)\right]\frac{\mathbf{k}_{1\perp}^2}{k_1^{-2}/2+\mathbf{k}_{1\perp}^2}$$

$$\times\left(\frac{2\,\mathrm{Im}\,\Pi_L(x)}{(\mathbf{k}_{1\perp}^2+\mathrm{Re}\,\Pi_L(x))^2+(\mathrm{Im}\,\Pi_L(x))^2} - \frac{2\,\mathrm{Im}\,\Pi_T(x)}{(\mathbf{k}_{1\perp}^2+\mathrm{Re}\,\Pi_T(x))^2+(\mathrm{Im}\,\Pi_T(x))^2}\right)$$

Dynamic Scattering Center(2)

$$
\begin{aligned}
\frac{dN_g^{med}}{dy\,d^2\mathbf{l}_\perp} = {} & C_F \frac{\alpha_s^2}{\pi} P(y) \int dZ_1^- \int \frac{d^2\mathbf{k}_{1\perp}}{(2\pi)^2} \int \frac{dx}{2\pi} \left[1 + f\left(\frac{x}{\sqrt{1-x^2}}|\mathbf{k}_{1\perp}|\right)\right] \frac{\sqrt{2}\,|\mathbf{k}_{1\perp}|}{\sqrt{1-x^2}} \\
& \times \left(\frac{2\,\mathrm{Im}\,\Pi_L(x)}{(\mathbf{k}_{1\perp}^2 + \mathrm{Re}\,\Pi_L(x))^2 + (\mathrm{Im}\,\Pi_L(x))^2} - \frac{2\,\mathrm{Im}\,\Pi_T(x)}{(\mathbf{k}_{1\perp}^2 + \mathrm{Re}\,\Pi_T(x))^2 + (\mathrm{Im}\,\Pi_T(x))^2} \right) \\
& \times \Bigg\{ \left[2 - 2\cos\left(\frac{y(1-y)}{(y-\lambda_1^-)\,(1+\lambda_1^- - y)} \frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2}{l_\perp^2 + y^2 M^2} \frac{Z_1^-}{\bar{\tau}_{\mathrm{form}}^-} \right) \right] \\
& \times C_A \Bigg[\frac{1 + (1+\lambda_1^- - y)^2}{1+(1-y)^2} \left(\frac{y - \frac{\lambda_1^-}{2}}{y - \lambda_1^-} \right)^2 \frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + \frac{(y-\lambda_1^-)^4 M^2}{1+(1+\lambda_1^- - y)^2}}{\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2\right]^2} \\
& - \frac{1 + (1+\lambda_1^- - y)(1-y)}{2[1+(1-y)^2]} \frac{y - \frac{\lambda_1^-}{2}}{y-\lambda_1^-} \frac{\mathbf{l}_\perp \cdot (\mathbf{l}_\perp - \mathbf{k}_{1\perp}) + \frac{y^2 (y-\lambda_1^-)^2}{1+(1+\lambda_1^- - y)(1-y)} M^2}{[l_\perp^2 + y^2 M^2]\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2\right]} \\
& - \frac{1 + (1+\lambda_1^- - y)\left(1 - \frac{y}{1+\lambda_1^-}\right)}{2[1+(1-y)^2]} \frac{y - \frac{\lambda_1^-}{2}}{y-\lambda_1^-} \frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp}) \cdot \left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right) + \frac{\left(\frac{y}{1+\lambda_1^-}\right)^2 (y-\lambda_1^-)^2}{1+(1+\lambda_1^- - y)\left(1 - \frac{y}{1+\lambda_1^-}\right)} M^2}{\left[\left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)^2 + \left(\frac{y}{1+\lambda_1^-}\right)^2 M^2\right]\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + (y-\lambda_1^-)^2 M^2\right]} \Bigg] \\
& + \left[2 - 2\cos\left(\frac{Z_1^-}{\bar{\tau}_{\mathrm{form}}^-} \right) \right] \Bigg[C_F \frac{l_\perp^2 + \frac{y^4}{1+(1-y)^2} M^2}{[l_\perp^2 + y^2 M^2]^2} - \frac{C_A}{2} \frac{\left(y - \frac{\lambda_1^-}{2}\right)^2}{y(y-\lambda_1^-)} \frac{l_\perp^2 + \frac{y^4}{1+(1-y)^2} M^2}{[l_\perp^2 + y^2 M^2]^2} \\
& + \left(\frac{C_A}{2} - C_F \right) \frac{1 + (1-y)\left(1 - \frac{y}{1+\lambda_1^-}\right)}{1+(1-y)^2} \frac{\mathbf{l}_\perp \cdot \left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right) + \frac{y^2 \left(\frac{y}{1+\lambda_1^-}\right)^2 M^2}{1+(1-y)\left(1 - \frac{y}{1+\lambda_1^-}\right)}}{[l_\perp^2 + y^2 M^2]\left[\left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)^2 + \left(\frac{y}{1+\lambda_1^-}\right)^2 M^2\right]} \Bigg] \\
& + C_F \left[\frac{1 + \left(1 - \frac{y}{1+\lambda_1^-}\right)^2}{1+(1-y)^2} \frac{\left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)^2 + \frac{\left(\frac{y}{1+\lambda_1^-}\right)^4}{1+\left(1 - \frac{y}{1+\lambda_1^-}\right)^2} M^2}{\left[\left(\mathbf{l}_\perp - \frac{y}{1+\lambda_1^-}\mathbf{k}_{1\perp}\right)^2 + \left(\frac{y}{1+\lambda_1^-}\right)^2 M^2\right]^2} - \frac{l_\perp^2 + \frac{y^4}{1+(1-y)^2} M^2}{[l_\perp^2 + y^2 M^2]^2} \right] \Bigg\} .
\end{aligned}
$$

$$k_1^- = \frac{\sqrt{2}\,|\mathbf{k}_{1\perp}|\,x}{\sqrt{1-x^2}}$$

$$\lambda_1^- = \frac{k_1^-}{q^-}$$

In the high energy limit

$k_1^- \ll T, \quad k_1^- \ll q^-$

$$\frac{dN_g^{med}}{dy\,d^2\mathbf{l}_\perp} = C_F \frac{\alpha_s P(y)}{2\pi^3} \int \frac{dZ_1^-}{\lambda_{\rm dyn}^-} \int d^2\mathbf{k}_{1\perp} \frac{\mu^2}{\mathbf{k}_{1\perp}^2(\mathbf{k}_{1\perp}^2+\mu^2)}$$
$$\times \left\{ \left[2 - 2\cos\left(\frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2M^2}{l_\perp^2 + y^2M^2}\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right] \times \left[\frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + \frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2M^2\right]^2}\right.\right.$$
$$-\frac{1}{2}\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp - \mathbf{k}_{1\perp}) + \frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2 + y^2M^2]\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2M^2\right]} - \frac{1}{2}\left.\frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})\cdot(\mathbf{l}_\perp - y\mathbf{k}_{1\perp}) + \frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp - y\mathbf{k}_{1\perp})^2 + y^2M^2\right]\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2M^2\right]}\right]$$
$$+\left(\frac{1}{2} - \frac{C_F}{C_A}\right)\left[2 - 2\cos\left(\frac{Z_1^-}{\tilde{\tau}_{\rm form}^-}\right)\right]\left[\frac{\mathbf{l}_\perp\cdot(\mathbf{l}_\perp - y\mathbf{k}_{1\perp}) + \frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2 + y^2M^2]\left[(\mathbf{l}_\perp - y\mathbf{k}_{1\perp})^2 + y^2M^2\right]} - \frac{l_\perp^2 + \frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2 + y^2M^2]^2}\right]$$
$$\left.+\frac{C_F}{C_A}\left[\frac{(\mathbf{l}_\perp - y\mathbf{k}_{1\perp})^2 + \frac{y^4}{1+(1-y)^2}M^2}{\left[(\mathbf{l}_\perp - y\mathbf{k}_{1\perp})^2 + y^2M^2\right]^2} - \frac{l_\perp^2 + \frac{y^4}{1+(1-y)^2}M^2}{[l_\perp^2 + y^2M^2]^2}\right]\right\}.$$

Beyond collinear rescattering expansion
Beyond soft gluon emission approximation

Soft gluon emission limit

$$\frac{dN_g^{\mathrm{med}}}{dy\,d^2\mathbf{l}_\perp} = C_F \frac{\alpha_s P(y)}{2\pi^3} \int \frac{dZ_1^-}{\lambda_{\mathrm{dyn}}^-} \int d^2\mathbf{k}_{1\perp} \frac{\mu^2}{\mathbf{k}_{1\perp}^2(\mathbf{k}_{1\perp}^2+\mu^2)} \times \left[2 - 2\cos\left(\frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2M^2}{l_\perp^2 + y^2M^2}\,\frac{Z_1^-}{\tilde{\tau}_{\mathrm{form}}^-}\right)\right]$$
$$\times \left[\frac{(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2}{\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2M^2\right]^2} - \frac{\mathbf{l}_\perp \cdot (\mathbf{l}_\perp - \mathbf{k}_{1\perp})}{[l_\perp^2 + y^2M^2]\left[(\mathbf{l}_\perp - \mathbf{k}_{1\perp})^2 + y^2M^2\right]}\right].$$

It is consistent with DGLV formula in dynamic medium

$$y^2 M \ll y\, M \sim l_\perp \sim k_{1\perp}$$

M.Djordjevic and U. Heinz, PRL (2008)

Summary

1. We derive a closed formula for medium-induced single gluon emission via transverse and longitudinal scatterings.

2. Our study is a generalization of both HT and DGLV one-rescattering-one-emission formula:
   —— beyond collinear rescattering expansion used in HT;
   —— beyond soft gluon emission limit used in DGLV.

3. For general medium in DIS framework
   ——both static and dynamic scattering centers.

Outlook

Phenomenological studies of parton energy loss and jet quenching in relativistic heavy-ion collisions.

Thank you !