In-medium effect on the thermodynamics and transport coefficients in van der Waals hadron resonance gas

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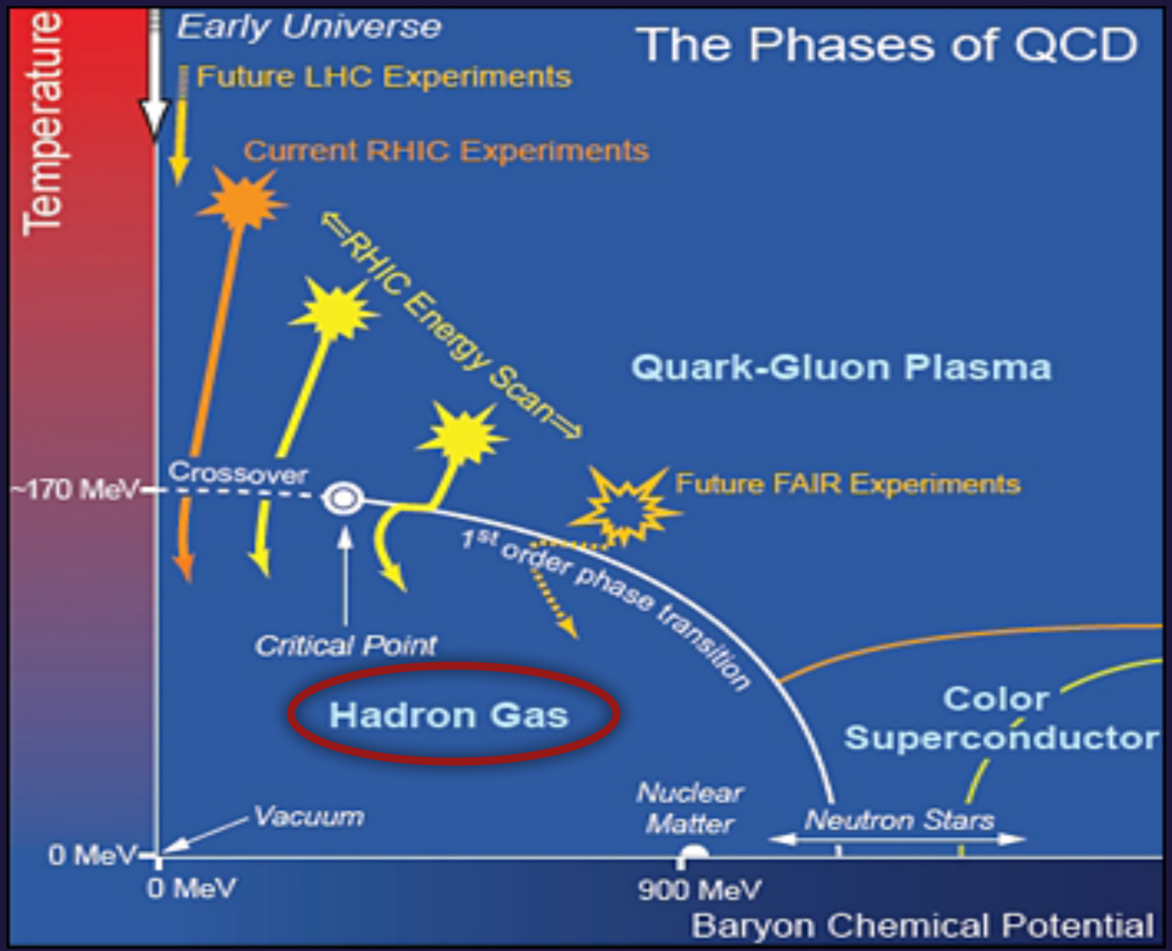
Based on: arXiv:1905.08146

In collaboration with Jin-Wen Kang, Ben-Wei Zhang

"第十八届全国中高能核物理大会暨第十二届全国中高能核物理"研讨会，

湖南长沙，21–25 June 2019

Phase diagram of strongly interacting matter



Crossover region:140-190 MeV

Our goals: in hadronic matter

1: thermal evolution of hadron masses .

2: The bulk thermodynamics and the equation of state (EoS)

3: Transport coefficients

Experiment: heavy-ion collisions

Theory: quantum chromodynamics (QCD)

Hadronic phase of strongly interacting matter

Methods:

Lattice QCD simulation

Base on first-principle

Is difficult at finite chemical potential due to sign problem .

Effective QCD models:

NJL model, PQM model/PLSM

Successful in description of chiral symmetry broken and restoration and de/confinement.

Is applicable at finite chemical potential

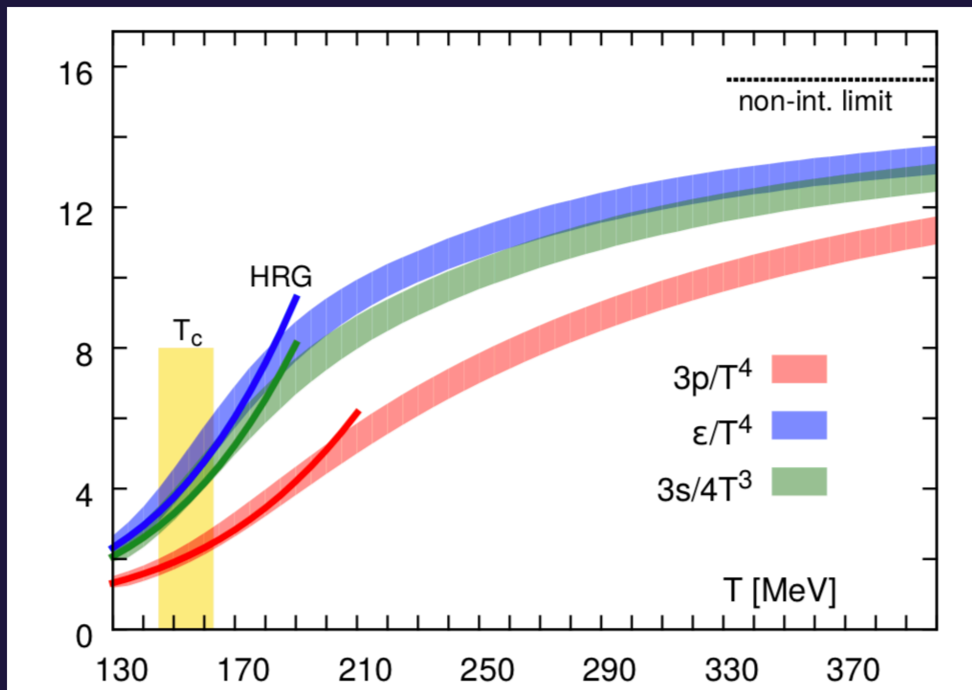
HRG model:

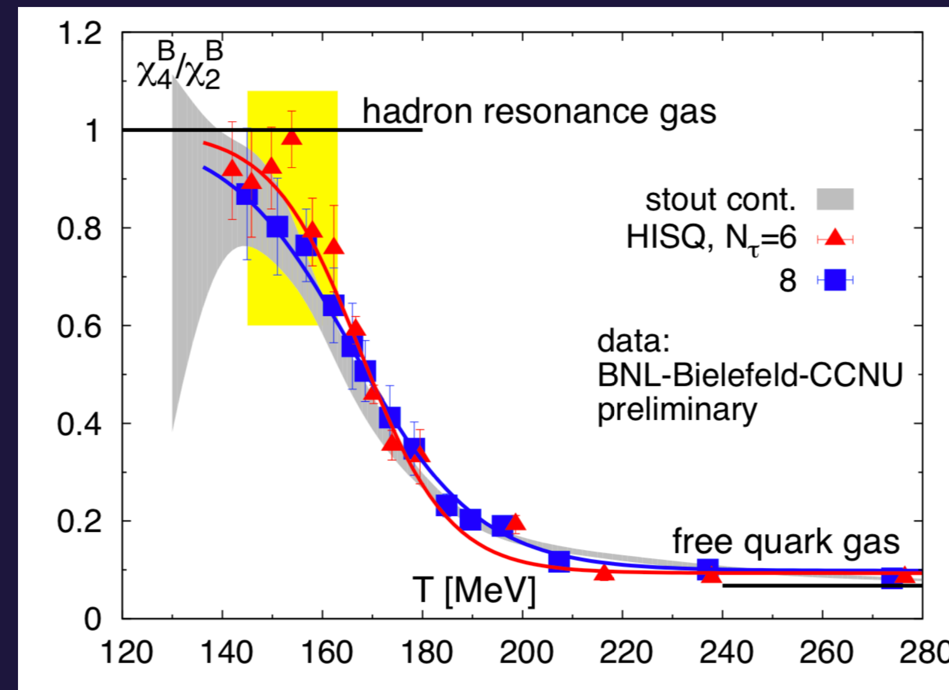
most simple statistical model

……

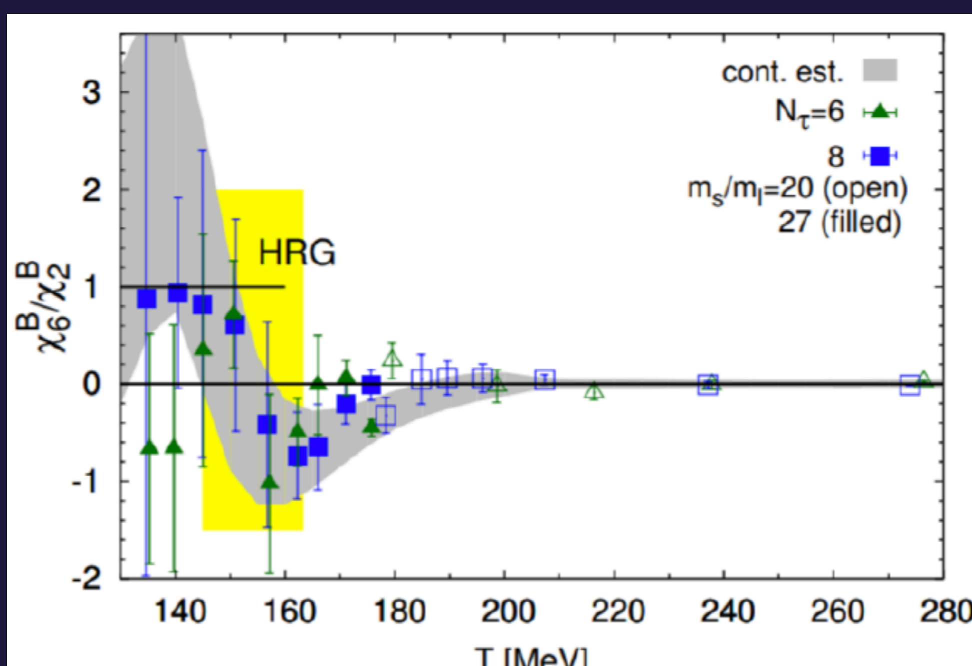
Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz, and K. K. Szabo, Nature 443,675 (2006).

EoS in HRG model and lattice QCD at $\mu_B = 0$





$T_c^{\mu=0} = 155 \sim 160$ MeV



$$\chi_{ijk}^{BQS} = \frac{\partial^{i+j+k}(P/T^4)}{\partial(\mu_B/T)^i\,\partial(\mu_Q/T)^j\,\partial(\mu_S/T)^k}$$

Observables in HRG model are consistent with lQCD at T<150 MeV.

However HRG model breaks down at T>Tc especially for description of baryon number fluctuations.

Bazavov et al., PRD 90, 094503(2014); *Borsanyi et al., PLB* 730, 99 (2014)

V. Vovchenko, M. I. Gorenstein and H. Stoecker, Phys. Rev. Lett. **118**, *no.*18, 182301(2017).

Hadron resonance gas model

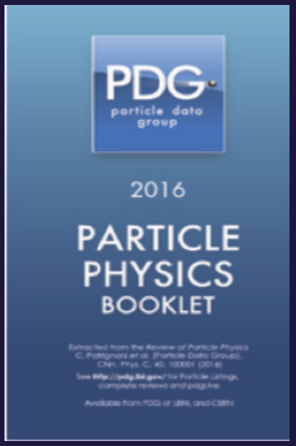
Dashen-Ma-Bernstein:

"If interactions mediated by narrow resonances, properties of interacting hadron gas are those of noninteracting hadron-resonance gas."

HRG model: the assumption of thermal equilibrium of a system composed by free hadrons and resonances.

$$\mu_i = B_i\mu_B + Q_i\mu_Q + S_i\mu_S$$

$$\ln Z^{id} = \sum_i \ln Z_i^{id} = \sum_i \pm \frac{Vg_i}{(2\pi)^3}\int d^3p \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$



$E_i = \sqrt{\vec{p}^{\,2} + m_i^2}$ where m usually takes as vacuum mass from PDG.

$$P^{id} = T\frac{\partial \ln Z^{id}}{\partial V} = \sum_i g_i \int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3E_i} f_i^{id}(T,\mu_i)$$

$$n^{id} = \frac{T}{V}\left(\frac{\partial \ln Z^{id}}{\partial \mu_i}\right)_{V,T} = \sum_i g_i \int \frac{d^3p}{(2\pi)^3} f_i^{id}(T,\mu_i)$$

$$\epsilon^{id} = -\frac{1}{V}\left(\frac{\partial \ln Z^{id}}{\partial \frac{1}{T}}\right)_{\frac{\mu}{T}} = \sum_i g_i \int \frac{d^3p}{(2\pi)^3} E_i f_i^{id}(T,\mu_i)$$

But hadron mass will be changed with the increase of temperature and density actually!

R. F. Dashen and R. Rajaraman, Phys. Rev. D **10**, 694(1974).

A. Andronic, et al, Phys. Lett. B **718**, 80(2012).

Interactions in hadron resonance gas: *attractive interaction* and *repulsive interaction* .

Classical Van der Waals equation

$$P(T,V,N) = \frac{NT}{V - bN} - a\frac{N^2}{V^2}$$

Nobel Prize in 1910

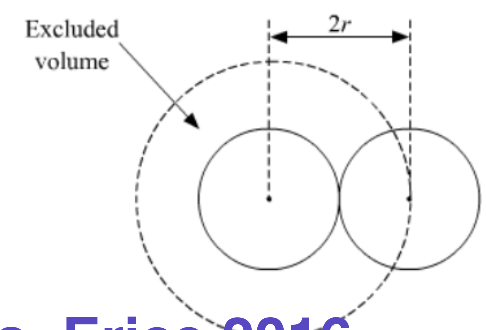
Two ingredients:

1) Short-range repulsion: particles are hard spheres,

$$V \to V - bN, \qquad b = 4\frac{4\pi r^3}{3}$$

2) Attractive interactions in mean-field approximation,

$$P \to P - a\,n^2$$



Volodymyr Vovchenko, Erice 2016

VDWHRG model

1). VDW interactions only exist between baryon-baryon and antibaryon-antibaryon pairs.

2). VDW parameters (a and b) extracted from ground of state of nuclear matter.

$(E/A = -16\ \text{MeV},\ n_0 = 0.16\ \text{fm}^{-3})$

V. Vovchenko, et al, Phys. Rev. C **91**, *no.*6, 064314(2015).

V. Vovchenko, M. I. Gorenstein and H. Stoecker, Phys. Rev. Lett. **118**, *no.*18, 182301(2017).

V. Vovchenko, Phys. Rev. C **96**, *no.*1, 015206(2017).

Total pressure of system within VDWHRG model in GCE.

$$P(T,\mu) = P_M(T,\mu) + P_B(T,\mu) + P_{\bar{B}}(T,\mu) \qquad \mu(\mu_B, \mu_Q, \mu_S)$$

$$p_M(T,\mu) = \sum_{i \in M} p_i^{id}(T,\mu_i) \qquad P_{B(\bar{B})}(T,\mu) = \sum_{i \in B(\bar{B})} P^{id}(T, \mu_i^{B(\bar{B})*}) - a n_{B(\bar{B})}^2$$

Solving transcendental equations numerically

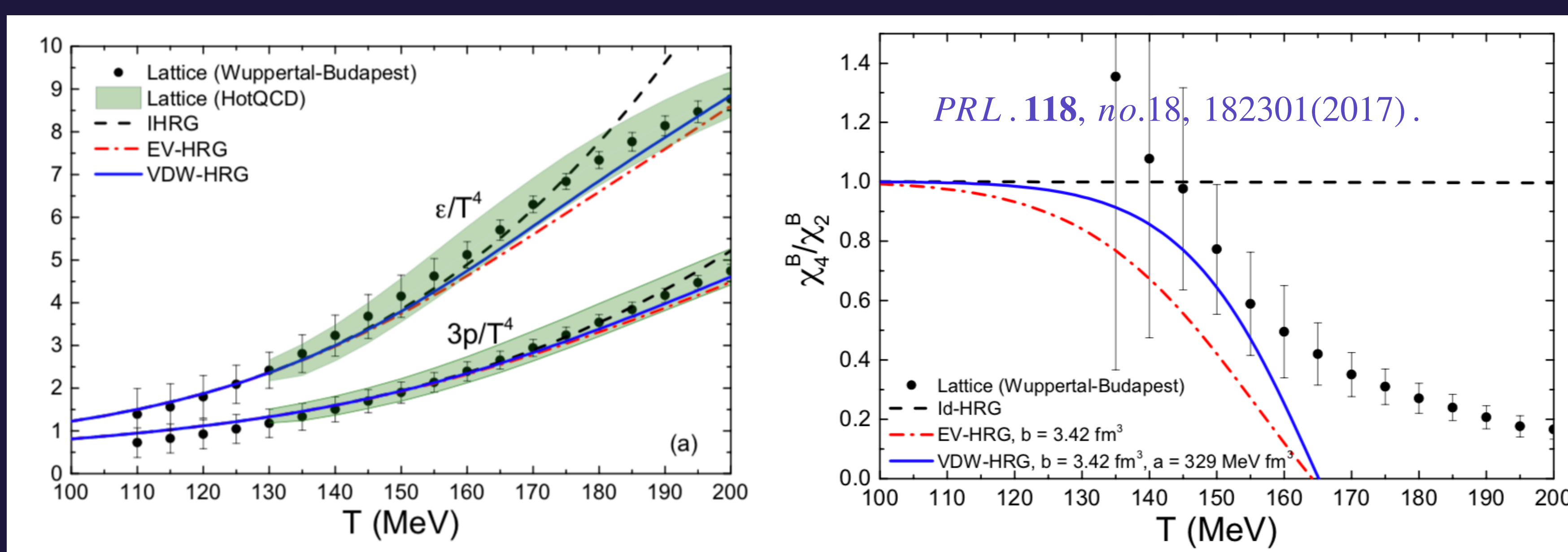
$$\mu_i^{B*} - \mu_i = -bP_B - abn_B^2 + 2an_B$$

$$n_B = \left(\frac{\partial P_B}{\partial \mu}\right)_T = \frac{\sum_{i \in B} n_i^{id}(T,\mu_i^{B*})}{1 + b\sum_{i \in B} n_i^{id}(T,\mu_i^{B*})}$$

$\Rightarrow$

$$s_B = \frac{\sum_{i \in B} s_i^{id}(T,\mu_i^{B*})}{1 + b\sum_{i \in B} s_i^{id}(T,\mu_i^{B*})}$$

$$\epsilon_B = \frac{\sum_{i \in B} \epsilon_i^{id}(T,\mu_i^{B*})}{1 + b\sum_{i \in B} \epsilon_i^{id}(T,\mu_i^{B*})} - an_B^2(T,\mu)$$



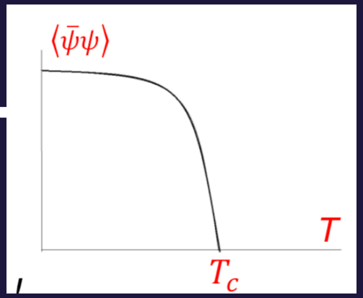
The effect of thermal hadron mass not be considered.

$b = 3.42\ \mathrm{fm}^3,\ a = 329\ \mathrm{MeV\ fm}^3$

$at\ \mu_B = 0$

V. Vovchenko, M. I. Gorenstein and H. Stoecker, Phys. Rev. Lett. **118**, no.18, 182301(2017).

Mass sensitivity of hadrons at finite T and μ_B



Spontaneous chiral symmetry breaking ⟶ hadron mass $\xrightarrow{T\uparrow\ \mu_B\uparrow}$?

Effective models: Polyakov-Nambu-Jona-Lasinio(PNJL) model, Polyakov Linear Sigma model (PLSM).

- **SU(3) Polyakov-loop Linear Sigma Model**

  $(\pi, \eta, \eta', K, \sigma, a_0, f_0, \kappa)$

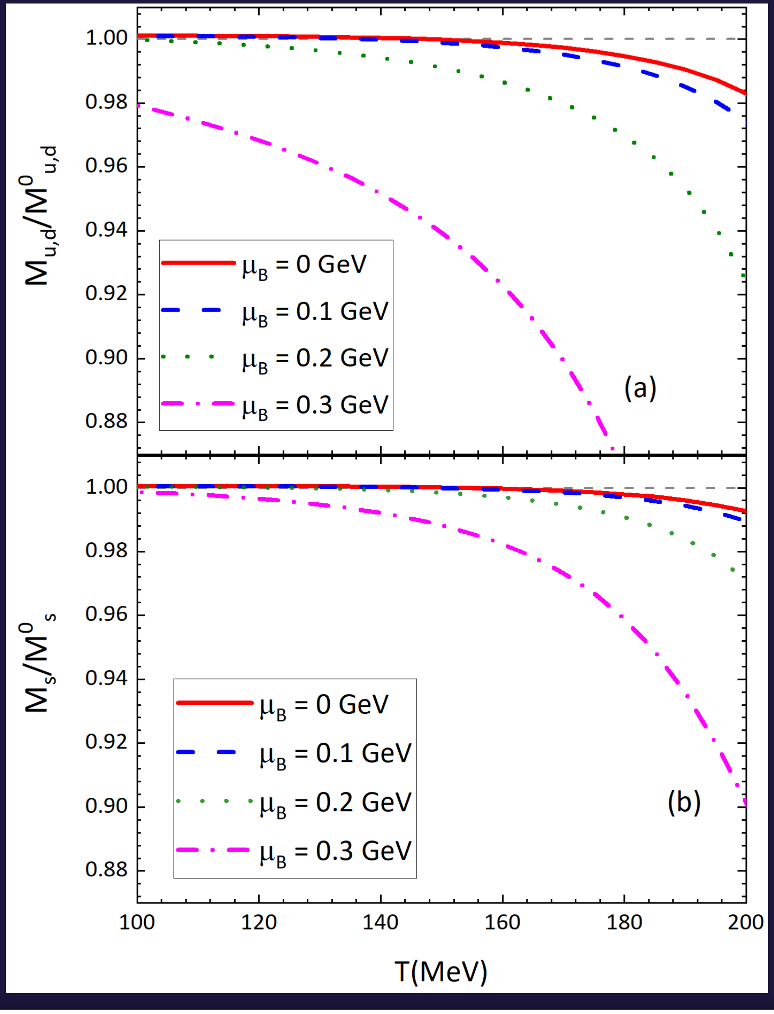
- **Generalized mass scaling rule of hadrons**

$$M_{B/M}(T,\mu) = M_{B/M}(0,0) + (N_q - N_s)\delta M_q(T,\mu) + N_s \delta M_s(T,\mu)$$

$N_{q/s}$ the number of light quark/strange in a given hadron .

$M_{q/s}$ light /strange constituent quark mass.

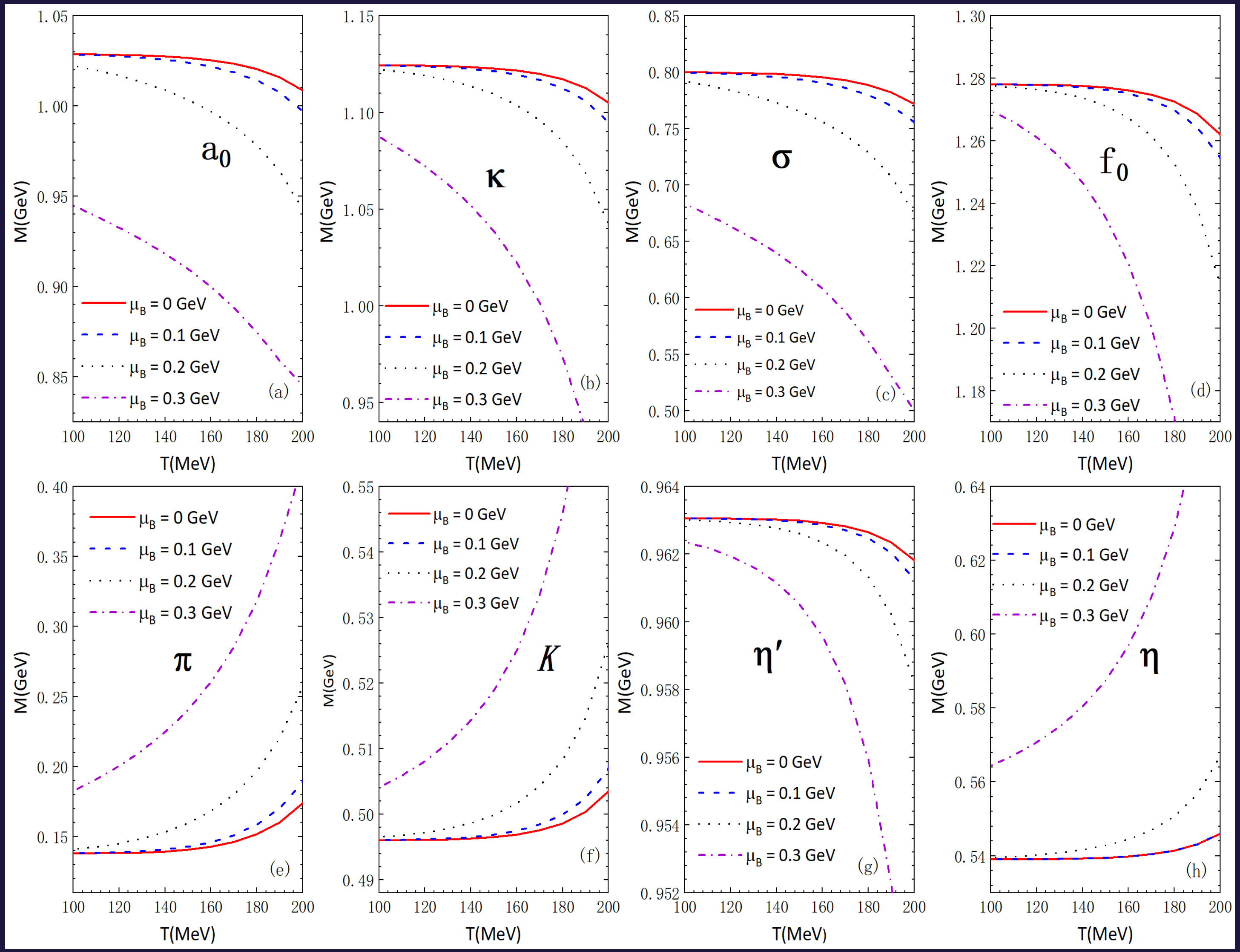
$M_{B/M}(0,0)$ vacuum mass of baryon/meson.



The T behavior of $M_{u,d/s}$ shows a smooth crossover .
The starting temperature at which $M_{u,d/s}$ begins to melt is about $T^{\mu=0} \sim 160/180$ MeV

A . N . Tawfik and A . M . Diab, Phys . Rev . C **91**, no.1, 015204(2015) .
B . J . Schaefer and M . Wagner, Phys . Rev . D **79**, 014018(2008) .
G . P . Kadam, H . Mishra, Phys . Rev . C **93**, 025205(2016) .
J . Jankowski, D . Blaschke and M . Spalinski, Phys . Rev . D **87**, no.10, 105018(2013)

Various meson states in hot and dense hadronic matter



Sector	symbol	PDG	PLSM
scalar mesons	a_0	$a_0(980^{\pm 20})$	1028
$J^{PC} = 0^{++}$	κ	$K_0^*(1425^{\pm 50})$	1124
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Pseudoscalar mesons	π	$\pi^0(134.97^{\pm 6.9})$	138
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	η'	$\eta'(957.78^{\pm 0.06})$	963

π, K, η states increases as temperature increases.

$\mathrm{a}_0, \kappa, \sigma, \mathrm{f}_0, \eta'$ states decrease as temperature increases.

All considered HRG models in our work:

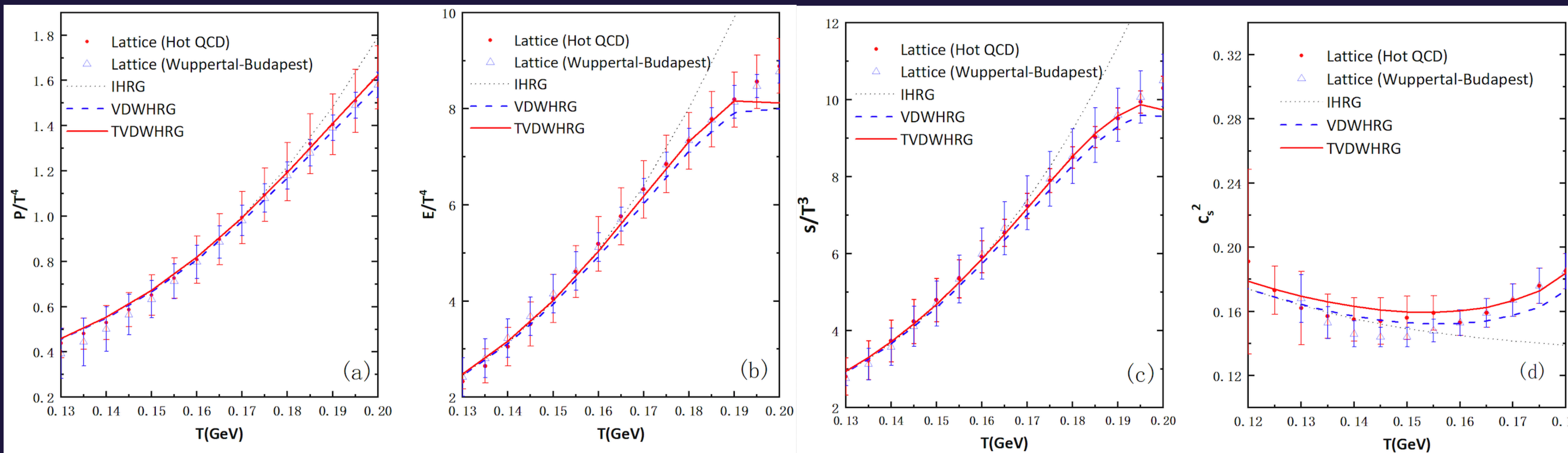
1. **IHRG model**
2. **THRG model**

(thermal hadron mass effect +IHRG model)

3. **VDWHRG model**
4. **TVDWHRG model**

(thermal hadron mass effect+VDWHRG model)

Thermodynamics in TVDWHRG model *vs* Lattice data



$a = 239$ MeV fm^3, $b = 3.42$ fm^3 *at* $\mu_B = 0$

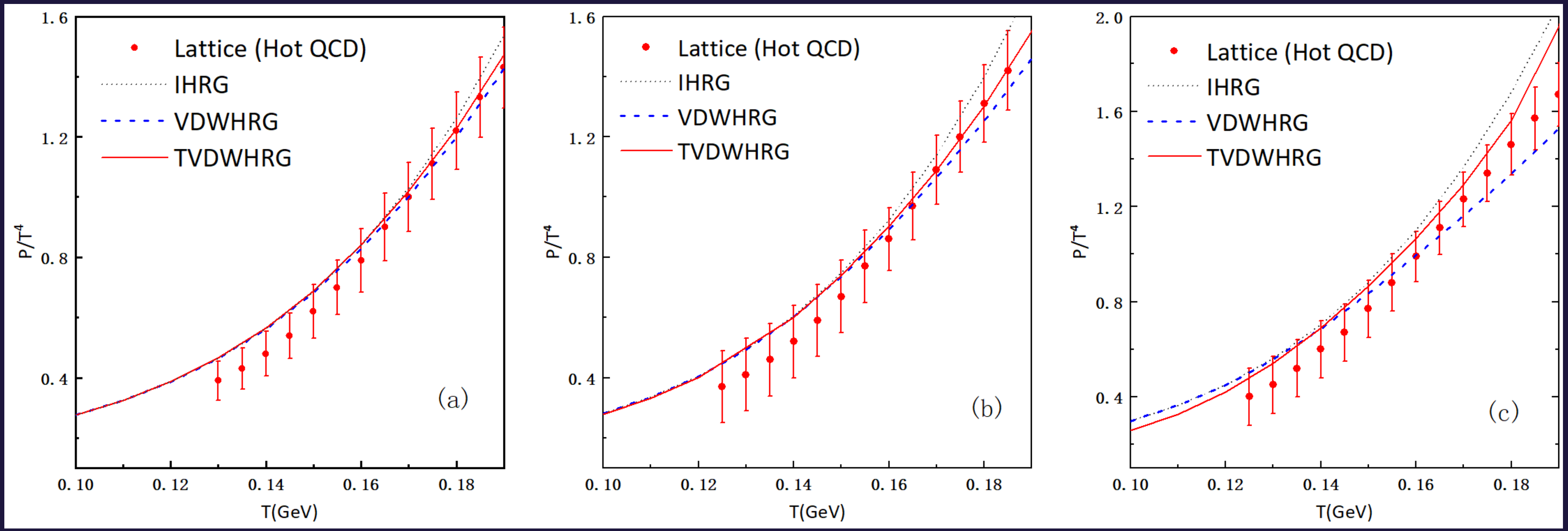
H.X.Zhang, J.W.Kang, and B.W.Zhang, arxiv : 1905.08146.

1. Vacuum hadron masses up to 2 GeV are taken from PDG2014.
2. $P/T^4, E/T^4, s/T^3$ within TVDWHRG model have an improved agreement with lattice data than within VDWHRG model at crossover region 0.16 GeV $< T <$ 0.19 GeV.
3. The speed of sound square, $c_s^2 = \partial P/\partial \epsilon$, is closer to lattice data at $0.165 < T < 0.18$ GeV within TVDWHRG model.

S.Borsanyi, Z.Fodor, C.Hoelbling, S.D.Katz, S.KriegandK.K.Szabo, Phys.Lett.B 730, 99(2014).

Bazavov et al., Phys.Rev.D **90**, 094503(2014).

Pressure in TVDWHRG model vs Lattice data



$\mu_B = 0.1\text{GeV}$ $\mu_B = 0.2\text{GeV}$ $\mu_B = 0.3\text{GeV}$

P/T^4 have better agreement to lattice data at $\mu_B = 0.1,\ 0.2$ GeV while for $\mu_B > 0.2$ GeV all considered models fail to simulate lattice data.

Reasons for failure:

(I) The parameters a and b may vary with μ_B.

(II) Lattice data we used here only up to μ_B^2.

Bazavov et al., Phys. Rev. D **90**, 094503(2014).

N. Sarkar and P. Ghosh, Phys. Rev. C **98**, 014907(2018).

Transport coefficients

Shear viscosity (η) , thermal conductivity (λ) and electrical conductivity (σ_{el}) in quasi-particle kinetic theory under the relaxation time approximation

$$\eta = \frac{1}{15T}\sum_a g_a \int \frac{d^3p}{(2\pi)^3}\frac{p^4}{E_a^2}\tau_a f_a(1 \pm f_a)$$

$$\lambda = \left(\frac{w}{n_B T}\right)^2 \sum_a g_a \int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3E_a^2}\tau_a\left(B_a - \frac{n_B E_a}{w}\right)^2 f_a(1 \pm f_a)$$

$$w = \epsilon + P$$

$$\sigma_{el} = \frac{1}{3T}\frac{4\pi}{137}\sum_a g_a e_a^2 \int \frac{d^3p}{(2\pi)^3}\frac{p^2}{E_a}\tau_a f_a(1 \pm f_a)$$

τ_a : the relaxation time

Only elastic scattering process $a(p_a) + b(p_b) \rightarrow a(p_c) + b(p_d)$

$$\tau_a^{-1} \equiv \sum_b n_b \langle \sigma_{ab} v_{ab} \rangle = \sum_b \frac{\beta \int_{S_0}^{\infty} \sigma_{ab\to ab}\gamma(S) K_1(\beta\sqrt{S})\frac{1}{2\sqrt{S}} dS}{4m_a^2 m_b^2 K_2(\beta m_a) K_2(\beta m_b)} n_b$$

$$\sum_b n_b = n_B + n_{\bar{B}} + n_M$$

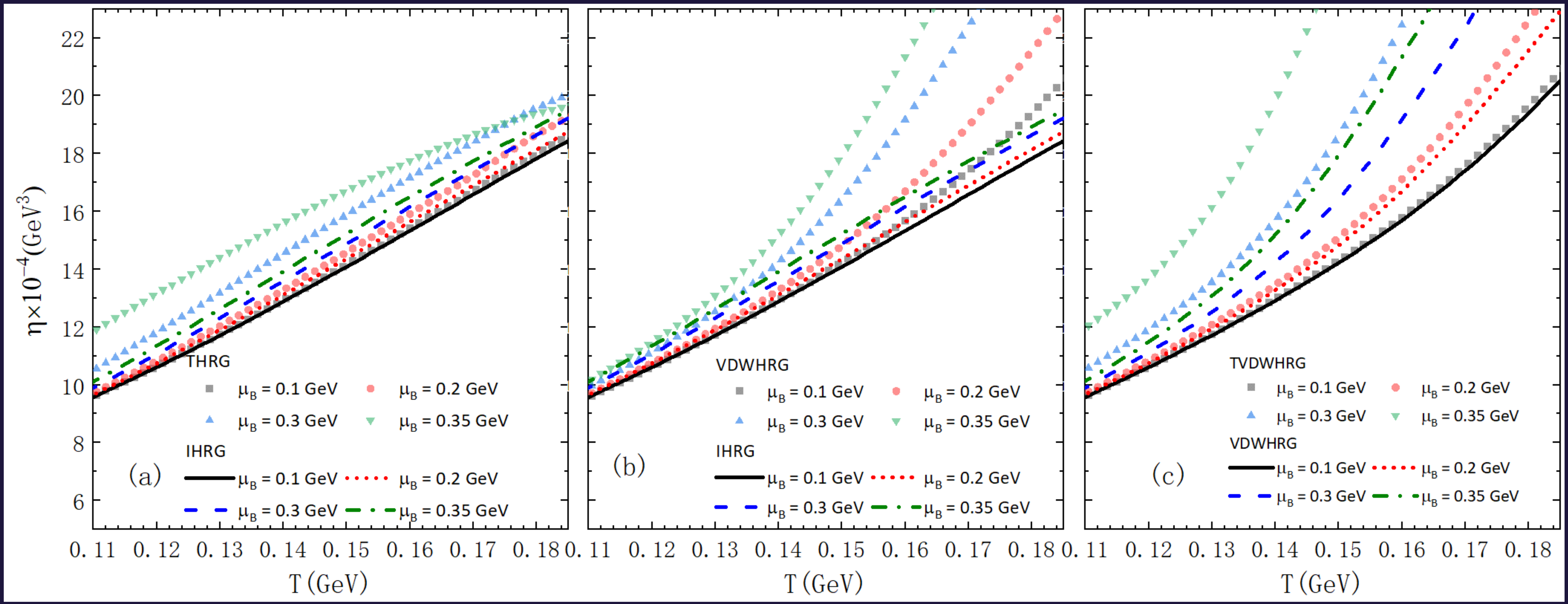
$n_B + n_{\bar{B}}$ are different in the cases of IHRG and VDWHRG

P. Chakraborty and J. I. Kapusta, Phys. Rev. C 83, 014906(2011).

A. Abhishek, H. Mishra and S. Ghosh, Phys. Rev. D 97 (2018) *no.*1 014005.

η

Shear viscosity



(a) η increases monotonically with the increase of T and μ_B.

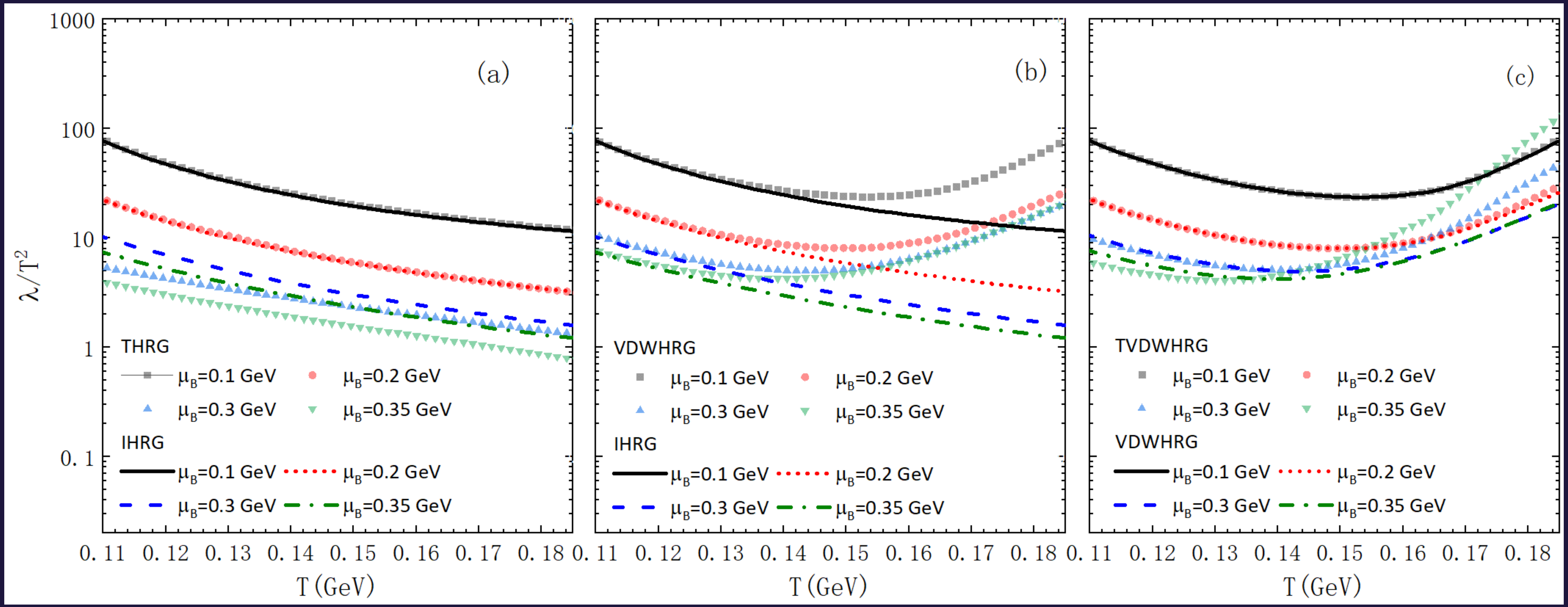
(a) Considering thermal hadron mass effect strengthen η increase.

(b) The increase of η can be improved further with the inclusion of VDW interactions.

(c) The effect of VDW interactions can be enhanced by including thermal hadron masses.

λ/T^2

Scaled thermal conductivity



(a) λ/T^2 decreases with the increase of T and μ_B .

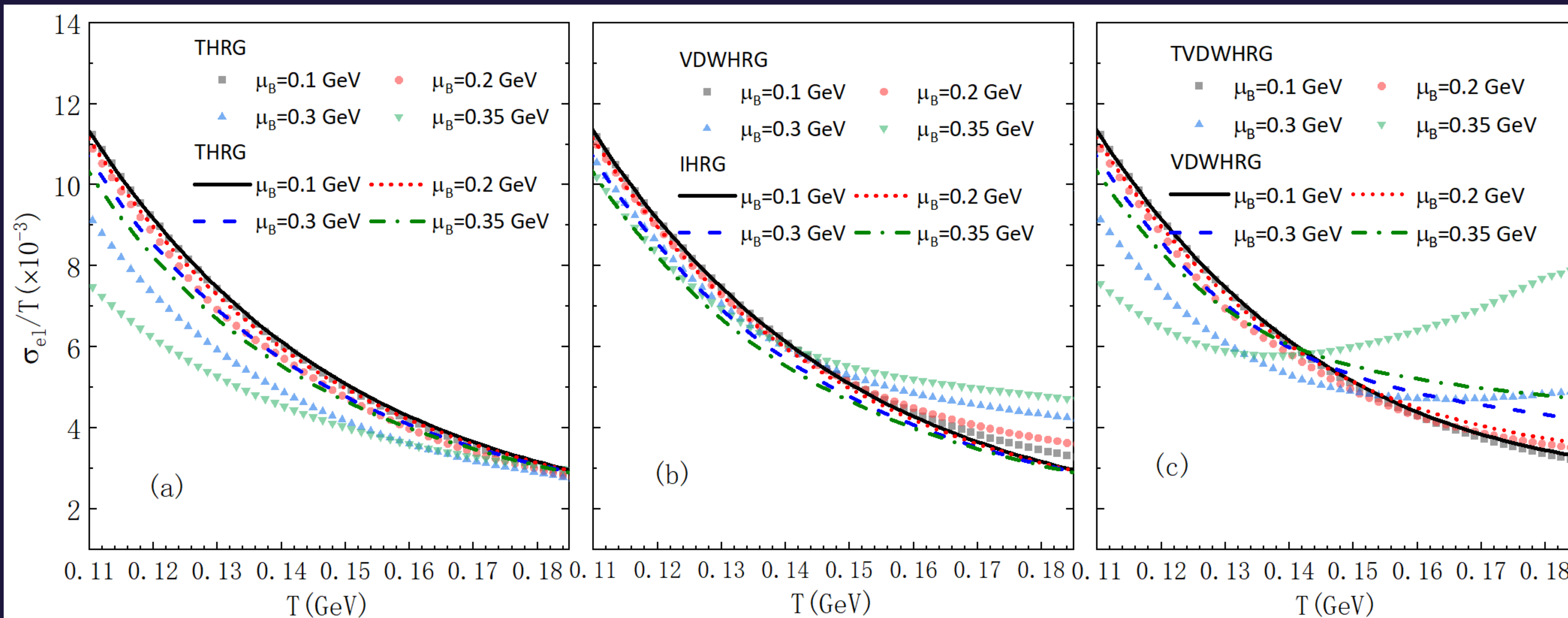
(a) The effect of thermal hadron mass strengthen λ/T^2 decreases especially at $\mu_B \geqslant 0.3$ GeV while the effect of thermal hadron mass on λ/T^2 is negligible at $\mu_B \leqslant 0.2$ GeV .

(b) Considering VDW effect on λ/T^2 can lead to an improvement of λ/T^2 with temperature at higher T ($T > 0.16$ GeV) .

(c) The effect of VDW interactions will be enhannced for $\mu_B > 0.2$ GeV at high T by including thermal mass effect .

σ_{el}/T

Scaled electrical conductivity



(a) σ_{el}/T decreases with the increase of T and μ_B.

(a) Considering thermal hadron mass effect strengthen σ_{el}/T decrease.

(b) σ_{el}/T including VDW interactions increases as μ_B increases at higher T.

(c) The effect of VDW interactions will be enhannced for $\mu_B > 0.2$ GeV at high T. by including thermal mass effect.

Summary&Outlook

(1) Thermodynamical observables in TVDWHRG model have an improved agreement with the lattice data at $\mu=0$ than within VDWHRG model, though the general behavior of thermodynamics don't change much.

(2) Considering thermal hadron mass effect can strengthen shear viscosity increases while weaken thermal and electrical conductivities .

(3) The transport coefficients have a significant enhancement at T>0.16 GeV by including the VDW interactions between hadrons even change the behavior of transport coefficients.

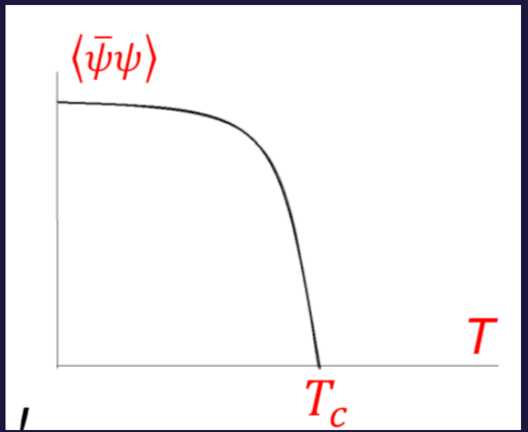
(4) The effect of VDW interactions on thermal and electrical conductivities can be enhanced further at high T by taking into account thermal hadron masses for $\mu>0.2$ GeV .

(5) Choosing appropriate effective model and VDW parameters to better consistent with lattice result.

谢谢大家！

Backup

Hadrons in PLSM at finite T and μ_B



Spontaneous chiral symmetry breaking ⟶ hadron mass $\xrightarrow{T\uparrow\ \mu_B\uparrow}$ **?**

Effective models:Polyakov-Nambu-Jona-Lasinio(PNJL) model, Polyakov Linear Sigma model (PLSM).

The effective Lagrangian of PLSM: $\mathscr{L} = \mathscr{L}_{quark} + \mathscr{L}_{meson} - \mathscr{U}(\phi, \phi^*, T)$

Quark part : $\mathscr{L}_{quark} = \sum_f \bar{q}_f(i\gamma^\mu D_\mu - gT_a(\sigma_a + i\gamma_5\pi_a))q_f$ g is Yukawa coupling constant.

Mesonic part:

$$\mathscr{L}_{meson} = Tr(\partial_\mu M^\dagger \partial^\mu M) - m^2 Tr(M^\dagger M) - \lambda_1[Tr(M^\dagger M]^2 - \lambda_2[Tr(M^\dagger M]^2$$

$$+c[\det(M) + \det(M^\dagger)] \quad +Tr[H(M + M^\dagger)]$$

$[U(1)_A$ symmetry breaking] [explicit symmetry breaking]

chiral fields $M = \sum_{a=0}^{8} T_a(\sigma_a + i\pi_a$ is scalar and pseudoscalar nonet), $H = \sum_{a=0}^{8} h_a T_a$, $T_a = \frac{\lambda_a}{2}$ generator operator

Polyakov-loop potential: $\mathscr{U}(\phi, \phi^*, T)$ mimics de/confinement transition

ϕ, ϕ^* are order parameters for confinement/deconfinement.

Different parameterizations of Polyakov potential: **Polynomial form, logarithmic form,Fukushima form**

A. N. Tawfik and A. M. Diab, Phys. Rev. C **91**, *no.*1, 015204(2015).

B. J. Schaefer and M. Wagner, Phys. Rev. D **79**, 014018(2008).

B. J. Schaefer et al, Phys. Rev. D **81**, 074013(2010).

*C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D*73, 014019 (2006).

*S. Roessner, C. Ratti, and W. Weise, Phys. Rev. D*75, 034007 (2007).

*K. Fukushima, Phys. Rev. D*77, 114028 (2008).

The thermodynamic potential of PLSM in mean field approximation

$$\Omega(T,\mu;\sigma_x,\sigma_y,\phi,\phi^*) = \frac{-T\ln\mathcal{Z}}{V} = \Omega_{\bar{q}q} + U(\sigma_x,\sigma_y) + \mathcal{U}(\phi,\phi^*,T)$$

Quark-antiquark potential:

$$\Omega_{\bar{q}q} = -2T\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\Big(\ln[1+3(\phi+\phi^* e^{-(E_f-\mu_f)/T})\times e^{-(E_f-\mu_f)/T}+e^{-3(E_f-\mu_f)/T}]$$
$$\ln[1+3(\phi+\phi^* e^{-(E_f+\mu_f)/T})\times e^{-(E_f+\mu_f)/T}+e^{-3(E_f+\mu_f)/T}]\Big)$$

Mesonic potential part:

$$U(\sigma_x,\sigma_y) = \frac{m^2}{2}(\sigma_x^2+\sigma_y^2) - h_x\sigma_x - h_y\sigma_y - \frac{c}{2\sqrt{2}}\sigma_x^2\sigma_y + \frac{\lambda_1}{2}\sigma_x^2\sigma_y^2 + \frac{1}{8}(2\lambda_1+\lambda_2)\sigma_x^4 + \frac{1}{4}(\lambda_1+\lambda_2)\sigma_y^4$$

Logarithmic Polyakov-loop potential:

$$\frac{\mathcal{U}_{log}(\phi,\phi^*,T)}{T^4} = -\frac{1}{2}a(T)\phi\phi^* + b(T)\ln[1-6\phi\phi^* + 4(\phi^3+\phi^{*3}) - 3(\phi\phi^*)^2]$$

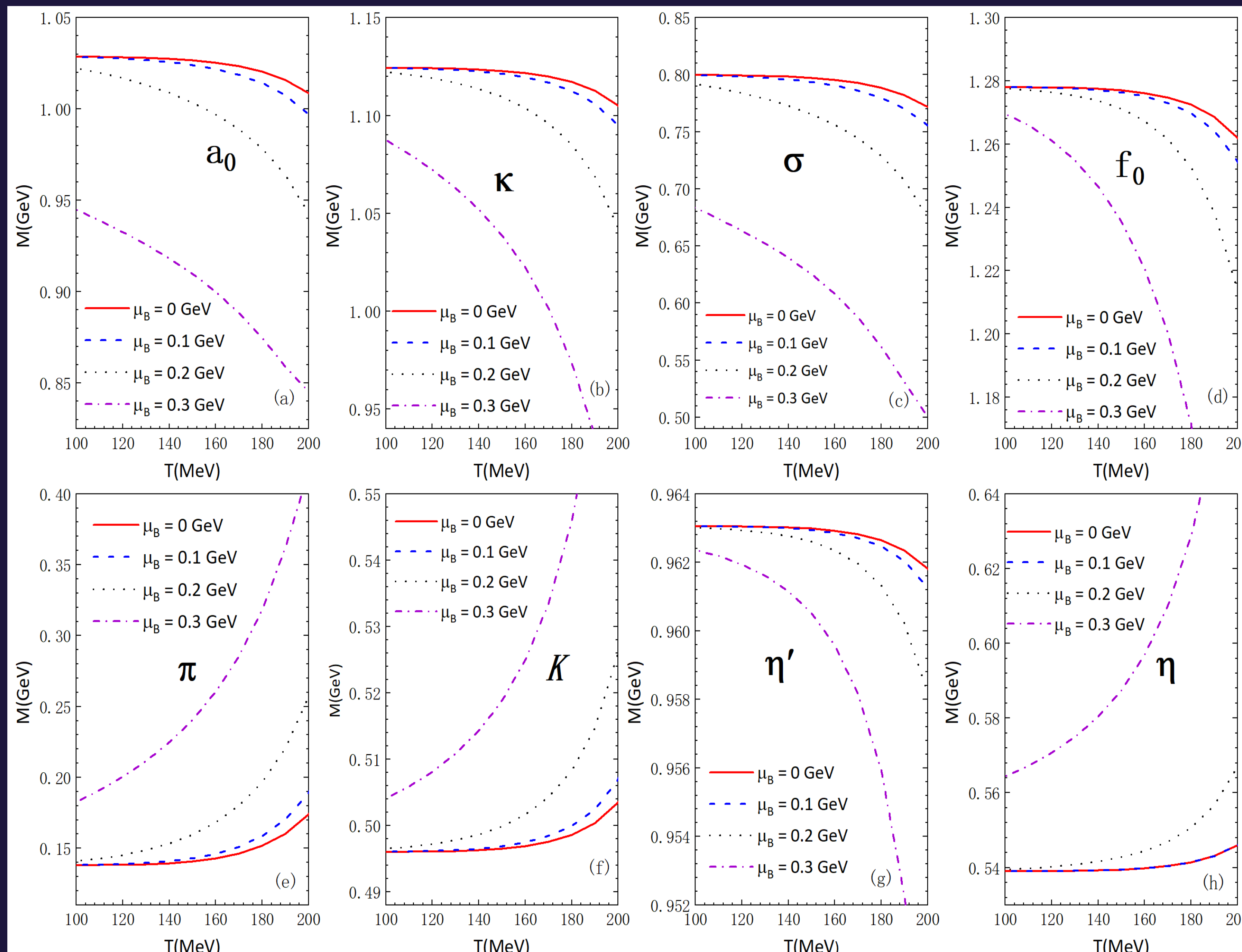
Constituent light and strange quark masses:

$$m_q = g\frac{\sigma_x}{2},\ m_s = g\frac{\sigma_y}{\sqrt{2}}$$ chiral condenstates relates chiral transition : $\sigma_x = \langle\bar{q}q\rangle,\ \sigma_y = \langle\bar{s}s\rangle$.

Minimizing the thermodynamic potential with respect to order parameters (sigma fields and Polyakov loop fields)

$$\frac{\partial\Omega}{\partial\sigma_x} = \frac{\partial\Omega}{\partial\sigma_y} = \frac{\partial\Omega}{\partial\phi} = \frac{\partial\Omega}{\partial\phi^*}\Big|_{min} = 0 \quad\Longrightarrow\quad \sigma_x(T,\mu_B),\ \sigma_y(T,\mu_B),\ \phi(T,\mu_B),\ \phi^*(T,\mu_B)$$

Various meson states in hot and dense hadronic matter



Meson masses at finite T and μ

$$M^2_{i,ab} = \frac{\partial^2 \Omega(T,\mu_f)}{\partial \varphi_{i,a} \partial \varphi_{i,b}} \Big|_{min} = m^2_{i,ab} + \delta_T m^2_{i,ab}$$

$m^2_{\mathrm{i,ab}}$: tree – level mass matrix,

$\delta_T m^2_{i,ab}$: mass modification due to T and μ_B

Sector	symbol	PDG	PLSM
scalar mesons	a_0	$a_0(980^{\pm 20})$	1028
$J^{PC} = 0^{++}$	κ	$K_0^*(1425^{\pm 50})$	1124
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π, K, η states increases as temperature increases .

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Thermodynamic potential density

$$\Omega(T,\mu) = \frac{-T\ln\mathcal{Z}}{V} = U(\sigma_x,\sigma_y) + \mathcal{U}(\phi,\phi*,T) + \Omega_{\bar\psi\psi}$$

Quark-antiquark potential

$$\Omega_{\bar\psi\psi} = -2TN\sum_{f=u,d,s}\int_0^\infty \frac{d^3p}{(2\pi)^3}(\ln[1+3(\phi+\phi* e^{-(E_f-\mu_f)/T})\times e^{-(E_f-\mu_f)/T} + e^{-3(E_f-\mu_f)/T}]$$

$$\ln[1+3(\phi+\phi* e^{-(E_f+\mu_f)/T})\times e^{-(E_f+\mu_f)/T} + e^{-3(E_f+\mu_f)/T}])$$

Mesonic potential

$$U(\sigma_x,\sigma_y) = \frac{m^2}{2}(\sigma_x^2+\sigma_y^2) - h_x\sigma_x - h_y\sigma_y - \frac{c}{2\sqrt{2}}\sigma_x^2\sigma_y + \frac{\lambda_1}{2}\sigma_x^2\sigma_y^2 + \frac{1}{8}(2\lambda_1+\lambda_2)\sigma_x^4 + \frac{1}{4}(\lambda_1+\lambda_2)\sigma_y^4$$

Polyakov-loop potential

$$\frac{\mathcal{U}_{poly}(\phi,\phi*,T)}{T^4} = -\frac{b2(T)}{2}\phi\phi* - \frac{b3}{6}(\phi^3+\phi*^3) + \frac{b4}{4}(\phi\phi*)^2 \qquad b2(T) = a0 + a1(\frac{T_0}{T}) + a2(\frac{T_0}{T})^2 + a3(\frac{T_0}{T})^3$$

$$\frac{\mathcal{U}_{log}(\phi,\phi*,T)}{T^4} = -\frac{1}{2}a(T)\phi\phi* + b(T)\ln[1-6\phi\phi* + 4(\phi^3+\phi*^3) - 3(\phi\phi*)^2]$$

$$a(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2},\ b(T) = b_3\frac{T_0^3}{T^3}$$

$$\frac{\mathcal{U}_{Fuku}(\phi,\phi*,T)}{T^4} = -\frac{b}{T^3}[54e^{-a/T}\phi\phi* + \ln(1-6\phi\phi* - 3(\phi\phi*)^2 + 4(\phi^3+\phi*^3))]$$

m^2, h_x, h_y, λ_1, λ_2 *and* c, *which can be fixed experimentally.*

ϕ *and* ϕ *are deconfinement order parameters.*

B. J. Schaefer et al, Phys. Rev. D **81**, 074013(2010).

σ_x , σ_y , ϕ and ϕ^* can be obtained by minimizing the potential

$$\frac{\partial\Omega}{\sigma_x} = \frac{\partial\Omega}{\sigma_y} = \frac{\Omega}{\phi} = \frac{\Omega}{\phi^*}\Big|_{min} = 0$$

$\sigma_x = \bar{\sigma}_x$, $\sigma_y = \bar{\sigma}_y$, $\phi = \bar{\phi}$ and $\phi^* = \bar{\phi}^*$ are the global minimum.

All parameters

$c = 4807.84$ MeV

$m^2 = -(306.26)^2$ MeV^3

$h_x = (120.73)^3$ MeV^3

$h_y = (336.41)^3$ MeV^3

$\lambda_1 = 13.49$

$\lambda_2 = 46.48$

$g = 6.5$

$m_\sigma = 800$ MeV

$M_s^0 = 433$ MeV

$M_q^0 = 300$ MeV

Polynomial

$T_0 = 270$ MeV

$a_0 = 6.75$

$a_1 = -1.95$

$a_2 = 2.625$

$a_3 = -7.44$

$b_4 = 7.5$

Logarithmic

$a_0 = 3.51$

$a_1 = -2.47$

$a_2 = 15.2$

$b_3 = -1.75$

$T_0 = 270$ MeV

Fukushima

$a = 664$ MeV

$b = (196.2\ \text{MeV})^3$

C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D73, 014019 (2006).

S. Roessner, C. Ratti, and W. Weise, Phys. Rev. D75, 034007 (2007).

K. Fukushima, Phys. Rev. D77, 114028 (2008).

A. N. Tawfik and A. M. Diab, Phys. Rev. C **91**, *no.*1, 015204(2015)