手征有效场论与QCD高能行为的匹配：标量流部分

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outline

1. Introduction
2. ChEFT v.s. QCD
3. Phenomenology
4. Summary

1 introduction

- QCD works above Λ_{QCD}。
- Chiral EFT (ChEFT) is an unrenormalizable theory. The unknown coupling constants grow in number as we go to higher orders to improve the accuracy of calculations in high energy region.
- To get information about the couplings: experimental determination, matching with QCD in high energy region.

RChT

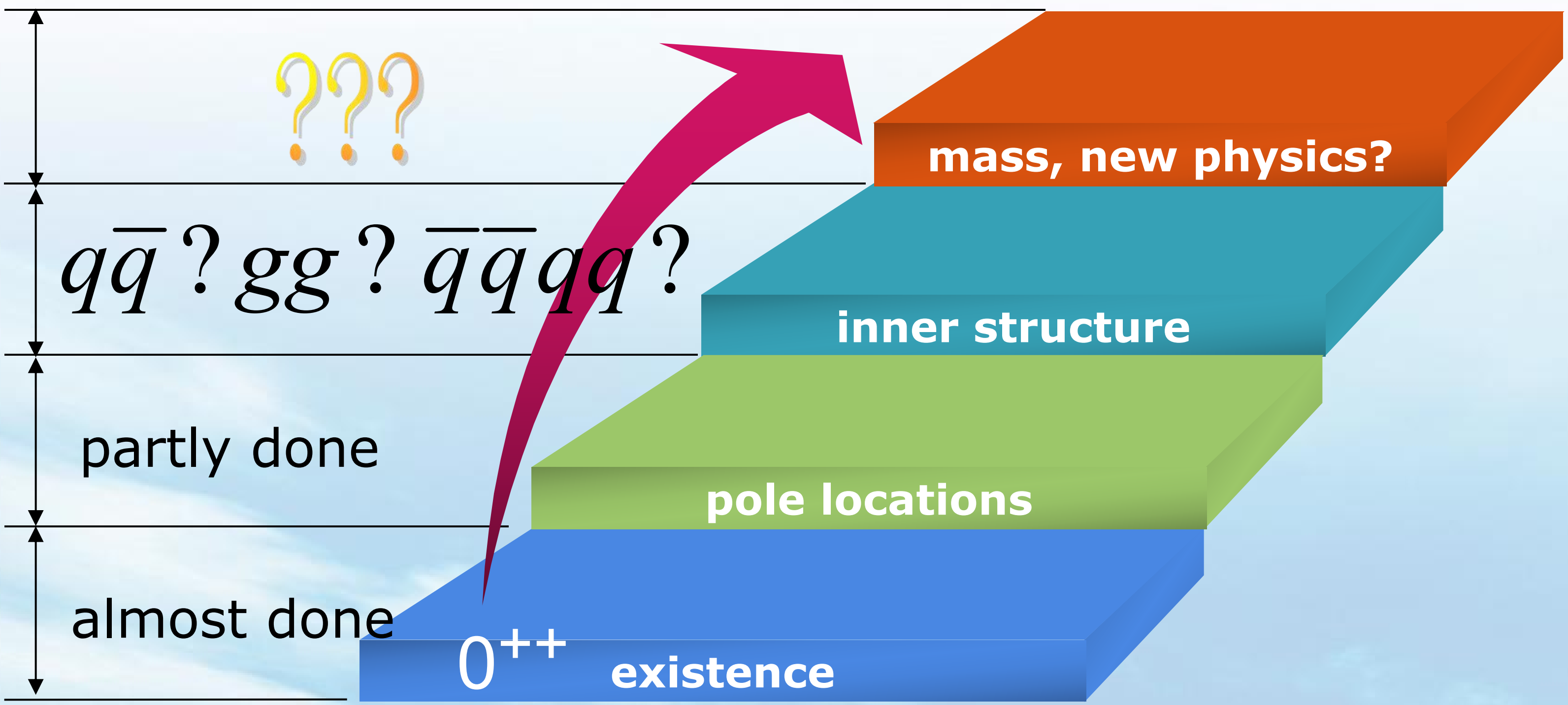
- In 0.5-2GeV, there are lots of unflavored mesons.
- RChT: by Ecker, Gasser, Pich, Rafael.
- It introduces heavier resonances as new degrees of freedom.
- It is the 'full theory' of ChPT

GF in high energy region

- Unkown couplings in RChT? Experiment and theory constraints.
- Matching their Green functions (GFs).
- RChT should give the same high energy behavior as that of QCD.
- Extend to the unphysical region of LQCD?

Scalars

- Scalars: the same quantum number as that of QCD



2.Matching:SVV,SAA

- Matching GF between QCD and ChEFT in the high energy region, using large Nc and OPE.

$$\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} = i^2 \int d^4x\, d^4y\, e^{i(p_1\cdot x+p_2\cdot y)} \langle 0|T\left\{S^i(0)A^j_\mu(x)A^k_\nu(y)\right\}|0\rangle$$

$$\left(\Pi^{ijk}_{SVV}\right)_{\mu\nu} = i^2 \int d^4x\, d^4y\, e^{i(p_1\cdot x+p_2\cdot y)} \langle 0|T\left\{S^i(0)V^j_\mu(x)V^k_\nu(y)\right\}|0\rangle$$

$$S^i(x) = \left(\bar{q}\lambda^i q\right)(x) \qquad V^i_\mu(x) = \left(\bar{q}\gamma_\mu\frac{\lambda^i}{2}q\right)(x) \qquad A^i_\mu(x) = \left(\bar{q}\gamma_\mu\gamma_5\frac{\lambda^i}{2}q\right)(x)$$

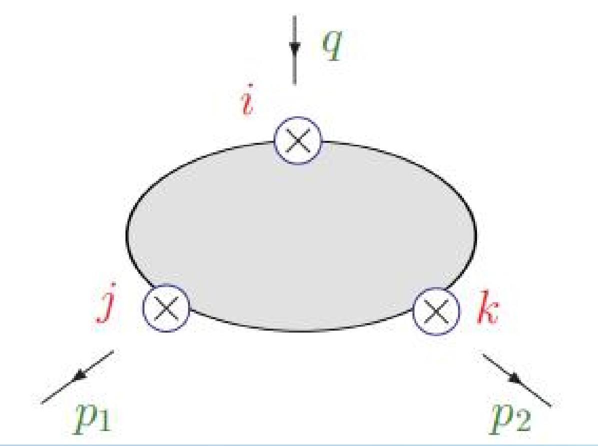
- Ward identity

$$p_1^\mu\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} = -2\,d^{ijk}B_0F^2\frac{(p_2)_\nu}{p_2^2}$$

$$p_2^\nu\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} = -2\,d^{ijk}B_0F^2\frac{(p_1)_\mu}{p_1^2}$$

$$p_1^\mu\left(\Pi^{ijk}_{SVV}\right)_{\mu\nu} = 0$$

$$p_2^\nu\left(\Pi^{ijk}_{SVV}\right)_{\mu\nu} = 0$$



SAA， QCD

- P and Q are the Lorentz structure of momentum, they vanish by timing $p_{1\mu}$ and $p_{2\nu}$.

$$\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu} = d^{ijk} B_0 \left[-2\,F^2 \frac{(p_1)_\mu (p_2)_\nu}{p_1^2 p_2^2} + \mathcal{F}_A\left(p_1^2, p_2^2, q^2\right)\, P_{\mu\nu} + \mathcal{G}_A\left(p_1^2, p_2^2, q^2\right)\, Q_{\mu\nu}\right]$$

$$P_{\mu\nu} = (p_2)_\mu\,(p_1)_\nu - p_1\cdot p_2\, g_{\mu\nu}\,,$$

$$Q_{\mu\nu} = p_1^2\,(p_2)_\mu\,(p_2)_\nu + p_2^2\,(p_1)_\mu\,(p_1)_\nu - p_1\cdot p_2\,(p_1)_\mu\,(p_2)_\nu - p_1^2\,p_2^2\, g_{\mu\nu}$$

$$\lim_{\lambda\to\infty}\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu}(\lambda p_1, \lambda p_2) = -2\,d^{ijk}\,B_0 F^2 \frac{1}{\lambda^2}\frac{1}{p_1^2\,p_2^2\,q^2}\left[q^2\,(p_1)_\mu\,(p_2)_\nu + Q_{\mu\nu} - p_1\cdot p_2\,P_{\mu\nu}\right] + \mathcal{O}\left(\frac{1}{\lambda^3}\right)$$

$$\lim_{\lambda\to\infty}\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu}(\lambda p_1, p_2) = -2\,d^{ijk}\,B_0 F^2 \frac{1}{\lambda}\frac{(p_1)_\mu\,(p_2)_\nu}{p_1^2 p_2^2} + \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

$$\lim_{\lambda\to\infty}\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu}(p_1, \lambda p_2) = -2\,d^{ijk}\,B_0 F^2 \frac{1}{\lambda}\frac{(p_1)_\mu\,(p_2)_\nu}{p_1^2 p_2^2} + \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

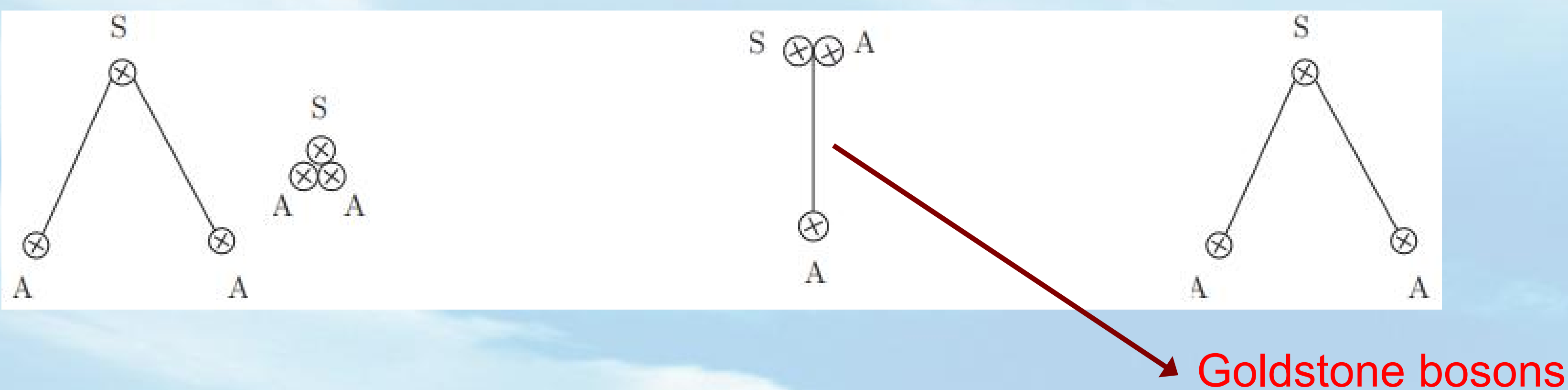
$$\lim_{\lambda\to\infty}\left(\Pi^{ijk}_{SAA}\right)_{\mu\nu}(\lambda p_1, q - \lambda p_1) = \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

SAA, ChEFT

- Lagrangians of ChPT and RChT.

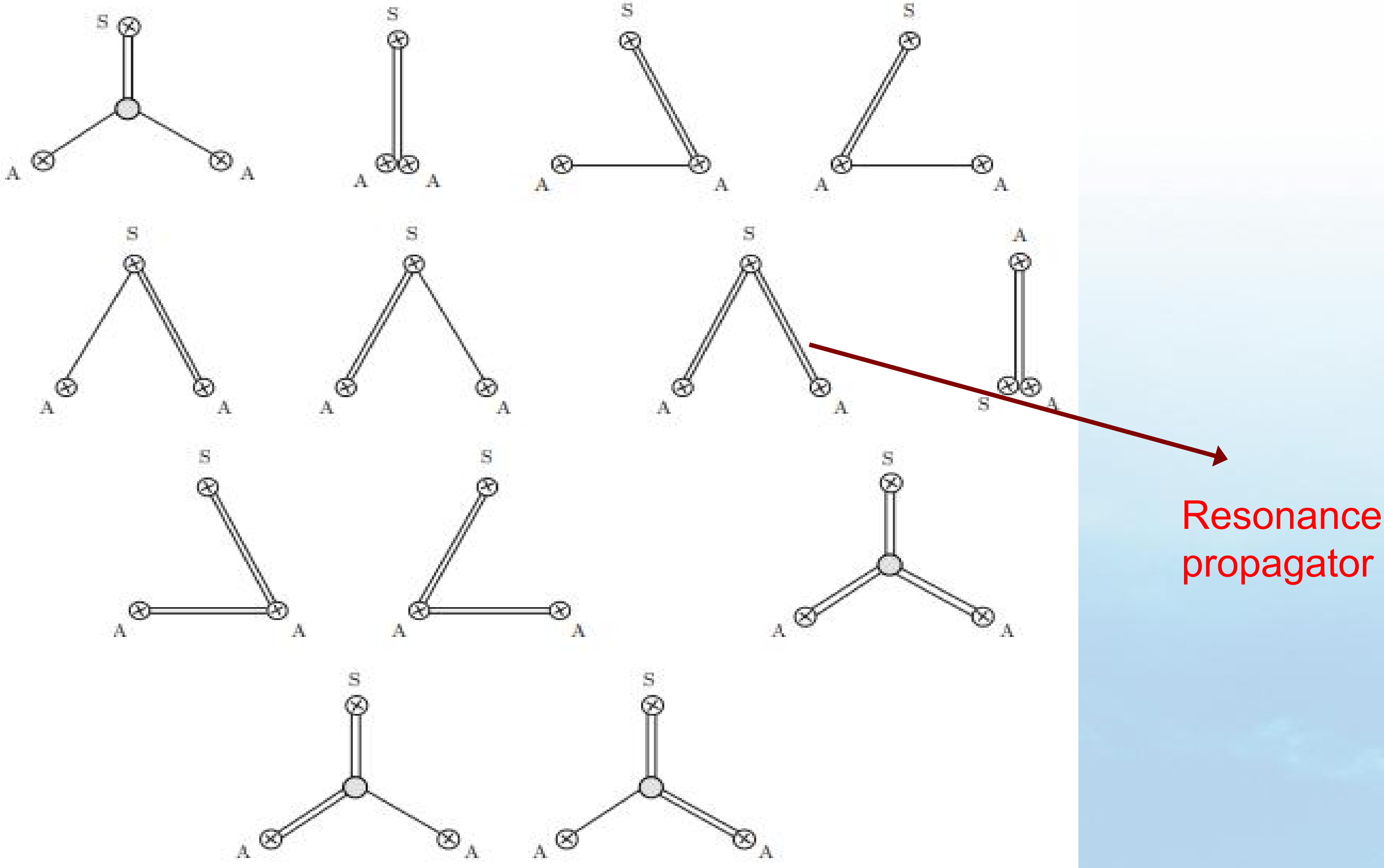
Coupling	Operator	Coupling	Operator	Coupling	Operator
$F^2/4$	$\langle u_\mu u^\mu + \chi_+ \rangle$	λ_{12}^S	$\langle S \{ \nabla_\alpha f_-^{\mu\alpha}, u_\mu \} \rangle$	λ_1^{SA}	$\langle \{ \nabla_\mu S, A^{\mu\nu} \} u_\nu \rangle$
$\tilde{L}_5$	$\langle u_\mu u^\mu \chi_+ \rangle$	λ_{16}^S	$\langle S f_{-\mu\nu} f_-^{\mu\nu} \rangle$	λ_2^{SA}	$\langle \{ S, A_{\mu\nu} \} f_-^{\mu\nu} \rangle$
$\tilde{C}_{12}$	$\langle h_{\mu\nu} h^{\mu\nu} \chi_+ \rangle$	λ_{17}^S	$\langle S \nabla_\alpha \nabla^\alpha (u_\mu u^\mu) \rangle$	λ_6^{AA}	$\langle A_{\mu\nu} A^{\mu\nu} \chi_+ \rangle$
$\tilde{C}_{80}$	$\langle f_{-\mu\nu} f_-^{\mu\nu} \chi_+ \rangle$	λ_{18}^S	$\langle S \nabla_\mu \nabla^\mu \chi_+ \rangle$	λ^{SAA}	$\langle S A_{\mu\nu} A^{\mu\nu} \rangle$
$\tilde{C}_{85}$	$\langle f_{-\mu\nu} \{ \chi_+^\mu, u^\nu \} \rangle$	λ_6^A	$\langle A_{\mu\nu} [u^\mu, \nabla^\nu \chi_+] \rangle$		
		λ_{16}^A	$\langle A_{\mu\nu} \{ f_-^{\mu\nu}, \chi_+ \} \rangle$		
		λ_{17}^A	$\langle A_{\mu\nu} \nabla_\alpha \nabla^\alpha f_-^{\mu\nu} \rangle$		

- ChPT's contribution to SAA GFs.



SAA

- Contribution of RChT



SAA

- GFs of RChT

$$
\begin{aligned}
\mathcal{F}_A(p_1^2,p_2^2,q^2) \;=\;& 32\left(\hat{C}_{12}-\hat{C}_{80}-\hat{C}_{85}\right)-32\,\lambda_{16}^{S}P_S-16\,\lambda_6^{AA}P_A(p_1^2)\,P_A(p_2^2)\\
&+8\sqrt{2}\,\left(2\,\lambda_{16}^{A}-\lambda_6^{A}+(\,\lambda_1^{SA}+2\,\lambda_2^{SA}\,)\,P_S\,\right)\,\left(P_A(p_1^2)+P_A(p_2^2)\right)\\
&-16\,\lambda^{SAA}P_S\,P_A(p_1^2)\,P_A(p_2^2)\,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{G}_A(p_1^2,p_2^2,q^2) \;=\;& \frac{8}{p_1^2p_2^2}\Big(2\,\hat{L}_5+4\,\hat{C}_{12}(p_1^2+p_2^2-q^2)-2\,\hat{C}_{85}\,(p_1^2+p_2^2)+2\,c_d\,P_S\\
&-2\,\lambda_{12}^{S}\,(p_1^2+p_2^2\,)P_S-2\,\lambda_{17}^{S}\,q^2\,P_S\\
&-\sqrt{2}\,(\lambda_6^{A}-\lambda_1^{SA}P_S)\,\left(p_1^2\,P_A(p_1^2)+p_2^2\,P_A(p_2^2)\right)\Big)\,,
\end{aligned}
$$

$$P_S=\frac{c_m-\lambda_{18}^{S}\,q^2}{M_S^2-q^2}\qquad\qquad P_A(p^2)=\frac{F_A-2\sqrt{2}\,\lambda_{17}^{A}\,p^2}{M_A^2-p^2}$$

- Matching them with those of QCD, we obtain the constraints about couplings. Since p_1 and p_2 go to infinity arbitrarily, one can require each formalism of momentum is matched.

SAA matching

- constrains

$$
\begin{aligned}
\hat{L}_5 &= \hat{C}_{12} = \hat{C}_{80} = \hat{C}_{85} = 0, \\
\lambda_6^A &= \lambda_{16}^A = \lambda_{12}^S = \lambda_{16}^S = 0, \\
\lambda_6^{AA} &= -\frac{F^2}{16 F_A^2}, \\
\lambda_1^{SA} &= \frac{1}{\sqrt{2}\, F_A}\left(c_d - \frac{F^2}{8 c_m}\right), \\
\lambda_2^{SA} &= -\frac{c_d}{2\sqrt{2}\, F_A}.
\end{aligned}
$$

- 15 couplings, 4 of them remain λ_{17}^A λ_{17}^S λ_{18}^S λ^{SAA}
- also from $\Pi_{SS-PP}^{ij}(t)$ $F_S^{ij}(t)$, one can knows three more couplings, only 1 remain

$$
\begin{aligned}
\lambda_{17}^S &= \lambda_{18}^S = 0, \\
\lambda_{17}^A &= 0,
\end{aligned}
$$

V. Cirigliano, et.al., NPB753 (2006) 179
G. Ecker, PLB223 (1989) 425

SVV， QCD

- The GF contain only forms of P and Q, and it satisfies Ward identity.

$$\left(\Pi_{SVV}^{ijk}\right)_{\mu\nu} = d^{ijk} B_0 \left[\mathcal{F}_V\left(p_1^2, p_2^2, q^2\right) P_{\mu\nu} + \boxed{\mathcal{G}_V\left(p_1^2, p_2^2, q^2\right)} Q_{\mu\nu} \right]$$

- QCD part.

$$\lim_{\lambda\to\infty} \left(\Pi_{SVV}^{ijk}\right)_{\mu\nu} (\lambda p_1, \lambda p_2) = -d^{ijk} B_0 F^2 \frac{1}{\lambda^2} \frac{1}{p_1^2 p_2^2 q^2} \left[\boxed{2\, Q_{\mu\nu}} + \left(p_1^2 + p_2^2 + q^2\right) P_{\mu\nu} \right] + \mathcal{O}\left(\frac{1}{\lambda^3}\right)$$

$$\lim_{\lambda\to\infty} \left(\Pi_{SVV}^{ijk}\right)_{\mu\nu} (\lambda p_1, p_2) = -2\, d^{ijk} \frac{1}{\lambda} \frac{\Pi_{VT}(p_2^2)}{p_1^2} P_{\mu\nu} + \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

$$\lim_{\lambda\to\infty} \left(\Pi_{SVV}^{ijk}\right)_{\mu\nu} (p_1, \lambda p_2) = -2\, d^{ijk} \frac{1}{\lambda} \frac{\Pi_{VT}(p_1^2)}{p_2^2} P_{\mu\nu} + \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

$$\lim_{\lambda\to\infty} \left(\Pi_{SVV}^{ijk}\right)_{\mu\nu} (\lambda p_1, q - \lambda p_1) = \mathcal{O}\left(\frac{1}{\lambda^2}\right)$$

SVV

- $O(p^6)$ and higher order lagrangians.

Coupling	Operator	Contributes to	Coupling	Operator	Contributes to
λ^{SVV}	$\langle S V_{\mu\nu} V^{\mu\nu} \rangle$	$\mathcal{F}_{SVV}$	λ_{15}^{S}	$\langle S f_{+\mu\nu} f_+^{\mu\nu} \rangle$	$\mathcal{F}_{SVV}$
λ_3^{SV}	$\langle \{S, V_{\mu\nu}\} f_+^{\mu\nu} \rangle$	$\mathcal{F}_{SVV}$	λ_6^{V}	$\langle V_{\mu\nu} \{f_+^{\mu\nu}, \chi_+\} \rangle$	$\mathcal{F}_{SVV}$
λ_6^{VV}	$\langle V_{\mu\nu} V^{\mu\nu} \chi_+ \rangle$	$\mathcal{F}_{SVV}$	$\tilde{C}_{61}$	$\langle f_{+\mu\nu} f_+^{\mu\nu} \chi_+ \rangle$	$\mathcal{F}_{SVV}$

- RChT does not contribute to the GFs with Q.

$$\left(\Pi_{SVV}^{ijk}\right)_{\mu\nu} = d^{ijk} B_0 \left[\mathcal{F}_V\left(p_1^2, p_2^2, q^2\right) P_{\mu\nu} + \boxed{\mathcal{G}_V\left(p_1^2, p_2^2, q^2\right)} Q_{\mu\nu} \right]$$

$$\lim_{\lambda\to\infty} \left(\Pi_{SVV}^{ijk}\right)_{\mu\nu} (\lambda p_1, \lambda p_2) = -d^{ijk} B_0 F^2 \frac{1}{\lambda^2} \frac{1}{p_1^2 p_2^2 q^2} \left[\boxed{2 Q_{\mu\nu}} + \left(p_1^2 + p_2^2 + q^2\right) P_{\mu\nu} \right] + \mathcal{O}\left(\frac{1}{\lambda^3}\right),$$

SVV

- We need to include higher order ones to finish the match.

Coupling	Operator	Contributes to	Coupling	Operator	Contributes to
κ_1^{SVV}	$\langle \nabla^\mu V_{\mu\nu} \nabla_\alpha V^{\alpha\nu} S' \rangle$	$\mathcal{G}_{SVV}$	λ^{SVV}	$\langle S V_{\mu\nu} V^{\mu\nu} \rangle$	$\mathcal{F}_{SVV}$
κ_2^{SVV}	$\langle \{\nabla^\mu V_{\mu\nu}, V^{\alpha\nu}\} \nabla_\alpha S \rangle$	$\mathcal{F}_{SVV}, \mathcal{G}_{SVV}$	λ_{15}^{S}	$\langle S f_{+\mu\nu} f_+^{\mu\nu} \rangle$	$\mathcal{F}_{SVV}$
κ_3^{SVV}	$\langle \nabla^\alpha V_{\mu\nu} \nabla_\alpha V^{\mu\nu} S \rangle$	$\mathcal{F}_{SVV}$	λ_3^{SV}	$\langle \{S, V_{\mu\nu}\} f_+^{\mu\nu} \rangle$	$\mathcal{F}_{SVV}$
κ_4^{SVV}	$\langle \{\nabla^\alpha V_{\mu\nu}, V^{\mu\nu}\} \nabla_\alpha S \rangle$	$\mathcal{F}_{SVV}$	c_m	$\langle S \chi_+ \rangle$	-
κ_5^{SVV}	$\langle \{\nabla_\alpha V_{\mu\nu}, V^{\alpha\mu}\} \nabla^\nu S \rangle$	$\mathcal{F}_{SVV}$	κ_6^{SVV}	$\langle \nabla_\alpha V_{\mu\nu} \nabla^\mu V^{\alpha\nu} S \rangle$	$\mathcal{F}_{SVV}$

- From power-counting, we also need to consider such terms

$$\langle R_a \, \chi(p^4) \rangle, \; \langle R_a \, R_b \, \chi(p^2) \rangle$$

SVV

- The higher order lagrangians, corresponding to $O(p^8)$ and higher terms.

Coupling	Operator	Coupling	Operator	Coupling	Operator
κ_1^{SVV}	$\langle \nabla^\mu V_{\mu\nu} \nabla_\alpha V^{\alpha\nu} S \rangle$	κ_1^{SV}	$\langle \{ V_{\alpha\nu}, \nabla_\mu f_+^{\mu\nu} \} \nabla^\alpha S \rangle$	κ_1^{V}	$\langle \{ \nabla^\alpha V_{\alpha\nu}, f_+^{\mu\nu} \} \nabla_\mu \chi_+ \rangle$
κ_2^{SVV}	$\langle \{ \nabla^\mu V_{\mu\nu}, V^{\alpha\nu} \} \nabla_\alpha S \rangle$	κ_2^{SV}	$\langle \{ \nabla^\alpha V_{\alpha\nu}, \nabla_\mu f_+^{\mu\nu} \} S \rangle$	κ_2^{V}	$\langle \{ \nabla^\alpha V_{\alpha\nu}, \nabla_\mu f_+^{\mu\nu} \} \chi_+ \rangle$
κ_3^{SVV}	$\langle \nabla_\alpha V_{\mu\nu} \nabla^\alpha V^{\mu\nu} S \rangle$	κ_3^{SV}	$\langle \{ \nabla^\alpha V_{\mu\nu}, f_+^{\mu\nu} \} \nabla_\alpha S \rangle$	κ_3^{V}	$\langle \{ \nabla^\alpha V_{\mu\nu}, f_+^{\mu\nu} \} \nabla_\alpha \chi_+ \rangle$
κ_4^{SVV}	$\langle \{ \nabla^\alpha V_{\mu\nu}, V^{\mu\nu} \} \nabla_\alpha S \rangle$	κ_4^{SV}	$\langle \{ \nabla_\mu V_{\alpha\nu}, \nabla^\alpha f_+^{\mu\nu} \} S \rangle$	κ_4^{V}	$\langle \{ \nabla_\mu V_{\alpha\nu}, \nabla^\alpha f_+^{\mu\nu} \} \chi_+ \rangle$
κ_5^{SVV}	$\langle \{ \nabla_\alpha V_{\mu\nu}, V^{\alpha\mu} \} \nabla^\nu S \rangle$	κ_5^{SV}	$\langle \{ V^{\alpha\nu}, \nabla_\alpha f_{+\,\mu\nu} \} \nabla^\mu S \rangle$	κ_5^{V}	$\langle \{ \nabla^\mu V^{\alpha\nu}, f_{+\,\mu\nu} \} \nabla_\alpha \chi_+ \rangle$
κ_6^{SVV}	$\langle \nabla_\alpha V_{\mu\nu} \nabla^\mu V^{\alpha\nu} S \rangle$	κ_1^{S}	$\langle \nabla^\alpha f_+^{\mu\nu} \nabla_\mu f_{+\,\alpha\nu} S \rangle$	κ_1^{VV}	$\langle \nabla^\alpha V^{\mu\nu} \nabla_\mu V_{\alpha\nu} \chi_+ \rangle$
		κ_2^{S}	$\langle \{ f_+^{\mu\nu}, \nabla^\alpha f_{+\,\alpha\nu} \} \nabla_\mu S \rangle$	κ_2^{VV}	$\langle \{ V^{\mu\nu}, \nabla^\alpha V_{\alpha\nu} \} \nabla_\mu \chi_+ \rangle$
		κ_3^{S}	$\langle \nabla^\alpha f_+^{\mu\nu} \nabla_\alpha f_{+\,\mu\nu} S \rangle$	κ_3^{VV}	$\langle \nabla^\alpha V^{\mu\nu} \nabla_\alpha V_{\mu\nu} \chi_+ \rangle$

- In EFT: when a resonance is integrated out , it contributes $O(p^2)$ interaction of light pseudoscalar mesons.

SVV

- GFs of SVV, from RChT

$$
\begin{aligned}
\mathcal{F}_V(p_1^2,p_2^2,q^2) \;=\;& -32\,\hat{C}_{61} - 32\,\lambda_{15}^S\,P_S + 16\sqrt{2}\,(\lambda_6^V + \lambda_3^{SV}\,P_S)\,(P_V(p_1^2) + P_V(p_2^2)) \\
& -16\,(\lambda_6^{VV} + \lambda^{SVV}\,P_S)\,P_V(p_1^2)\,P_V(p_2^2) \\
& -4\Big(\,(2\,\kappa_2^{SVV} + 2\,\kappa_3^{SVV} + \kappa_6^{SVV})\,(p_1^2 + p_2^2) \\
& \qquad -(2\,\kappa_3^{SVV} - 4\,\kappa_4^{SVV} + 2\,\kappa_5^{SVV} + \kappa_6^{SVV})\,q^2\Big)\,P_S\,P_V(p_1^2)\,P_V(p_2^2) \\
& +4\sqrt{2}\,(2\kappa_1^{SV} - 2\kappa_3^{SV} + \kappa_4^{SV} + \kappa_5^{SV})\,P_S\,(p_1^2\,P_V(p_2^2) + p_2^2\,P_V(p_1^2)) \\
& +4\sqrt{2}\,(2\kappa_3^{SV} - \kappa_4^{SV} + \kappa_5^{SV})\,q^2\,P_S\,(P_V(p_1^2) + P_V(p_2^2)) \\
& +4\sqrt{2}\,(2\kappa_3^{SV} + \kappa_4^{SV} - \kappa_5^{SV})\,P_S\,(p_1^2\,P_V(p_1^2) + p_2^2\,P_V(p_2^2)) \\
& +8\,(\kappa_1^S + 2\kappa_3^S)\,q^2\,P_S - 8\,(\kappa_1^S + 2\,\kappa_2^S + 2\,\kappa_3^S)\,(p_1^2 + p_2^2)\,P_S \\
& -4\Big(\,(\kappa_1^{VV} + 2\kappa_2^{VV} + 2\kappa_3^{VV})\,(p_1^2 + p_2^2) - (\kappa_1^{VV} + 2\kappa_3^{VV})\,q^2\Big)\,P_V(p_1^2)\,P_V(p_2^2) \\
& +4\sqrt{2}\,(2\kappa_1^V + 2\kappa_3^V + \kappa_4^V + \kappa_5^V)\,(p_1^2\,P_V(p_1^2) + p_2^2\,P_V(p_2^2)) \\
& +4\sqrt{2}\,(2\kappa_3^V - \kappa_4^V + \kappa_5^V)\,q^2\,(P_V(p_1^2) + P_V(p_2^2)) \\
& -4\sqrt{2}\,(2\kappa_3^V - \kappa_4^V + \kappa_5^V)\,(p_1^2\,P_V(p_2^2) + p_2^2\,P_V(p_1^2))
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{G}_V(p_1^2,p_2^2,q^2) \;=\;& 8\,(\kappa_1^{SVV} - 2\,\kappa_2^{SVV})\,P_S\,P_V(p_1^2)\,P_V(p_2^2) - 32\,\kappa_2^S\,P_S \\
& +8\sqrt{2}\,(\kappa_1^{SV} - \kappa_2^{SV})\,P_S\,(P_V(p_1^2) + P_V(p_2^2)) \\
& +8\sqrt{2}\,(\kappa_1^V - \kappa_2^V)\,(P_V(p_1^2) + P_V(p_2^2)) - 16\,\kappa_2^{VV}\,P_V(p_1^2)\,P_V(p_2^2)
\end{aligned}
$$

$$
P_V(p^2) = \frac{F_V - 2\sqrt{2}\,\lambda_{22}^V\,p^2}{M_V^2 - p^2}
$$

SVV

- Matching results

$$
\begin{aligned}
\kappa_2^S = \kappa_2^{VV} &= 0\,,\\
\kappa_1^S + 2\,\kappa_3^S &= 0\,,\\
\kappa_1^{VV} + 2\,\kappa_3^{VV} &= 0\,,\\
\kappa_1^{SV} - \kappa_2^{SV} &= 0\,,\\
2\,\kappa_3^{SV} + \kappa_4^{SV} - \kappa_5^{SV} &= -\frac{2\sqrt{2}\,\lambda_{15}^S}{F_V}\,,\\
2\,\kappa_1^{SV} - 2\,\kappa_3^{SV} + \kappa_4^{SV} + \kappa_5^{SV} &= 0\,,\\
2\,\kappa_3^{SV} - \kappa_4^{SV} + \kappa_5^{SV} &= \frac{4\,\lambda_6^V}{c_m} + \frac{M_V^2}{c_m}(2\kappa_1^V + 2\kappa_3^V + \kappa_4^V + \kappa_5^V)\,,\\
\kappa_1^V - \kappa_2^V &= 0\,,\\
2\,\kappa_3^V - \kappa_4^V + \kappa_5^V &= 0\,,\\
\kappa_1^V + 2\,\kappa_3^V + \kappa_5^V &= -\frac{\sqrt{2}\,\hat{C}_{61}}{F_V}\,,\\
\kappa_1^{SVV} - 2\,\kappa_2^{SVV} &= \frac{F^2}{4\,c_m\,F_V^2}\\
2\,\kappa_3^{SVV} - 4\,\kappa_4^{SVV} + 2\,\kappa_5^{SVV} + \kappa_6^{SVV} &= -\frac{4\,\lambda_6^{VV}}{c_m} + \frac{F^2}{4\,c_m\,F_V^2}\,,\\
2\,\kappa_2^{SVV} + 2\kappa_3^{SVV} + \kappa_6^{SVV} &= -\frac{F^2}{4\,c_m\,F_V^2} - \frac{4\sqrt{2}\lambda_3^{SV}}{F_V} - \frac{\sqrt{2}\,M_S^2\,(2\,\kappa_3^{SV} - \kappa_4^{SV} + \kappa_5^{SV})}{F_V}\\
&\quad - \frac{\sqrt{2}\,M_V^2\,(2\,\kappa_3^{SV} + \kappa_4^{SV} - \kappa_5^{SV})}{F_V}
\end{aligned}
$$

29 parameters， 14+1 conctraints, and one is from two point GFs and forward dispersion relation. $\lambda_{22}^V = 0$

G. Ecker, PLB223 (1989) 425

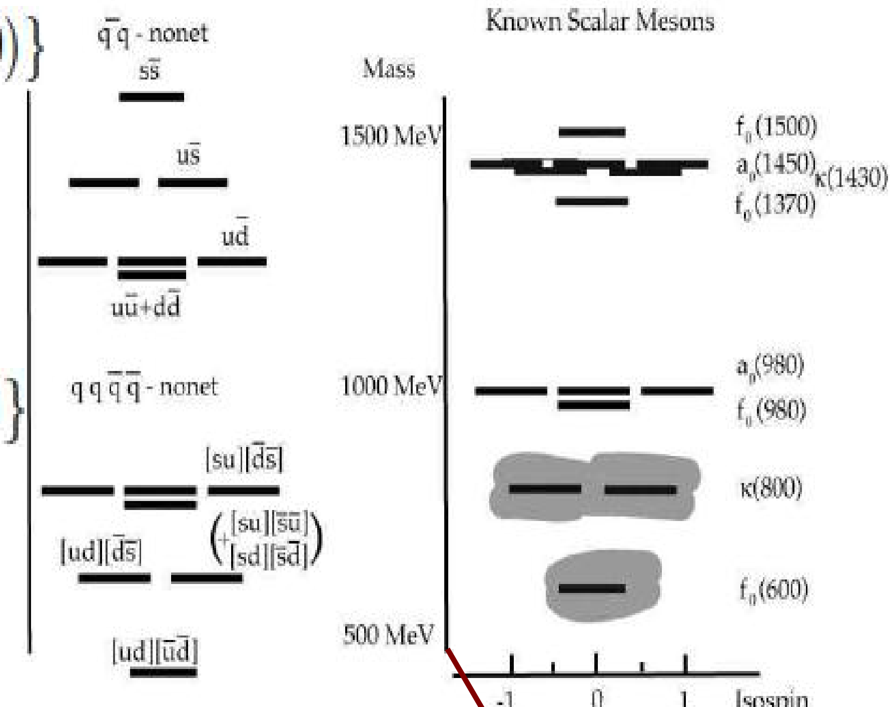
3. Phenomenology of SAA

- Firstly we need to know which resonances should be filled in the current.

$$S_H = \{f_0(1370), K_0^*(1430), a_0(1450), f_0(1500)\}$$

$$S_L = \{f_0(500), K_0^*(700), a_0(980), f_0(980)\}$$

$M_a > M_\kappa$, tetraquark?



Jaffe
Phys. Rept. 409 (2005) 1

$$S = \begin{pmatrix} \frac{1}{\sqrt{2}}a^0 + \frac{1}{\sqrt{3}}S_0 + \frac{1}{\sqrt{6}}S_8 & a^+ & \kappa^+ \\ a^- & -\frac{1}{\sqrt{2}}a^0 + \frac{1}{\sqrt{3}}S_0 + \frac{1}{\sqrt{6}}S_8 & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & \frac{1}{\sqrt{3}}S_0 - \sqrt{\frac{2}{3}}S_8 \end{pmatrix}$$

Mixing mechanism

- The light and heavy mesons of I=1(1/2) could have mixing

$$\begin{pmatrix} a_{0,L} \\ a_{0,H} \end{pmatrix} = \begin{pmatrix} \cos\varphi_a & \sin\varphi_a \\ -\sin\varphi_a & \cos\varphi_a \end{pmatrix} \begin{pmatrix} a_0(980) \\ a_0(1450) \end{pmatrix}$$

$$\begin{pmatrix} K^*_{0,L} \\ K^*_{0,H} \end{pmatrix} = \begin{pmatrix} \cos\varphi_k & \sin\varphi_k \\ -\sin\varphi_k & \cos\varphi_k \end{pmatrix} \begin{pmatrix} K^*_0(700) \\ K^*_0(1430) \end{pmatrix}$$

- The lagrangian of these mesons

$$\begin{aligned} \mathcal{L}_{I=1,1/2} \; = \; & c_d^L \langle S_L u_\mu u^\mu \rangle + \alpha_L \langle S_L u_\mu \rangle \langle u^\mu \rangle + c_m^L \langle S_L \chi_+ \rangle \\ & + c_d^H \langle S_H u_\mu u^\mu \rangle + \alpha_H \langle S_H u_\mu \rangle \langle u^\mu \rangle + c_m^H \langle S_H \chi_+ \rangle \end{aligned}$$

f_0 mixing

- I=0 mesons are more complicated. We only consider the mixing between the heavier ones

$$\begin{pmatrix} f_0(1370) \\ f_0(1510) \\ f_0(1710) \end{pmatrix} = A \begin{pmatrix} S_8 \\ S_0 \\ S_1 \end{pmatrix}$$

$$A = \begin{pmatrix} \cos\gamma\cos\beta\cos\alpha - \sin\gamma\sin\alpha & \cos\gamma\cos\beta\sin\alpha + \sin\gamma\cos\alpha & -\cos\gamma\sin\beta \\ -\sin\gamma\cos\beta\cos\alpha - \cos\gamma\sin\alpha & -\sin\gamma\cos\beta\sin\alpha + \cos\gamma\cos\alpha & \sin\gamma\sin\beta \\ \sin\beta\cos\alpha & \sin\beta\sin\alpha & \cos\beta \end{pmatrix}$$

- $f_0(1710)$-$f_0(1370)$-$f_0(1500)$ mixing

Large Nc contribution at NLO

- Large Nc is not a perfect counting for scalars, so we include the NLO ones.
- Constraints from two-point GFs may be violated in the phenomenology of scalars.

$$F_V\, G_V = F^2\,, \qquad F_V^2 - F_A^2 = F^2\,, \qquad F_V^2\, M_V^2 = F_A^2\, M_A^2\,,$$
$$4\, c_d\, c_m = F^2\,, \qquad c_d = c_m\,,$$

$$c_d = c_m = \frac{F}{2} = 46.2\,\mathrm{MeV} \neq \begin{array}{l} 13\,\mathrm{MeV} \lesssim c_d \lesssim 40\,\mathrm{MeV} \\ 30\,\mathrm{MeV} \lesssim c_m \lesssim 100\,\mathrm{MeV} \end{array}$$

- LO+NLO contributions.

$$\begin{aligned}\mathcal{L}_{I=0,S_1} = \;& c_d^H \langle S_H\, u_\mu u^\mu \rangle + c_m^H \langle S_H\, \chi_+ \rangle + \alpha_H \langle S_H\, u_\mu \rangle \langle u^\mu \rangle + \beta_H \langle S_H \rangle \langle u_\mu u^\mu \rangle \\ & + \gamma_H \langle S_H \rangle \langle u_\mu \rangle \langle u^\mu \rangle + c_d' S_1 \langle u_\mu u^\mu \rangle + c_m' S_1 \langle \chi_+ \rangle + \gamma' S_1 \langle u_\mu \rangle \langle u^\mu \rangle\end{aligned}$$

Refinement of experimental data

- For scalars, the datas are not complete and some of them are even contradicted with each other. We choose the more 'reliable' ones by analysis.

Final State	$\mathrm{Br}[f_0(1370)]$	$\mathrm{Br}[f_0(1500)]$	$\mathrm{Br}[f_0(1710)]$
$\boxed{4\pi}$	$\boxed{0.83 \pm 0.18}$	0.495 ± 0.033 [20]	-
$a_1\pi$	0.050 ± 0.017	0.059 ± 0.025 [21]	-
$\pi\pi$	0.076 ± 0.039	0.349 ± 0.023 [20]	0.148 ± 0.071 [26, 25]
$\eta\eta$	0.023 ± 0.010	0.051 ± 0.009 [20]	0.173 ± 0.079 [20, 25]
$\eta\eta'$	-	0.019 ± 0.008 [20]	-
$K\overline{K}$	0.069 ± 0.039	0.086 ± 0.010 [20]	0.36 ± 0.12 [25]
$K\overline{K}/\pi\pi$	0.91 ± 0.20 [24]	0.246 ± 0.026 [20]	-
$\eta\eta/\pi\pi$	0.31	0.145 ± 0.027 [20]	-
$\eta\eta'/\pi\pi$	-	0.055 ± 0.024 [20]	-
$a_1\pi/\pi\pi$	0.66	0.170 ± 0.086 [20, 21]	-
$\pi\pi/K\overline{K}$	-	-	$0.41^{+0.11}_{-0.17}$ [26]
$\eta\eta/K\overline{K}$	-	-	0.48 ± 0.15 [20]
$\eta\eta'/K\overline{K}$	-	-	-
$a_1\pi/K\overline{K}$	-	-	-

FSI

- Final state interactions between the light pseudoscalars can not be ignored.

$$\begin{pmatrix} \mathcal{M}_{f_i\to\pi\pi} \\ \mathcal{M}_{f_i\to K\overline{K}} \end{pmatrix}^{\mathrm{FSI}} = \sqrt{S}\begin{pmatrix} \mathcal{M}_{f_i\to\pi\pi} \\ \mathcal{M}_{f_i\to K\overline{K}} \end{pmatrix}^{\mathrm{bare}}$$

$$\sqrt{S} = \mathcal{O}^T\sqrt{S_{\mathrm{diag}}}\,\mathcal{O}$$

$$S_{\mathrm{diag}} = \begin{pmatrix} e^{2i\delta^{I=0}_{\pi\pi}} & 0 \\ 0 & e^{2i\delta^{I=0}_{K\overline{K}}} \end{pmatrix}$$

$$\mathcal{O} = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix}$$

Energy (GeV)	$\delta^{I=0}_{\pi\pi}$ (Deg)	$\delta^{I=0}_{K\overline{K}}$ (Deg)
1.395	308.05	−71.46
1.504	340.18	−78.92
1.720	373.59	−107.20

K-Matrix, extend to 1.8GeV, L.Y.Dai&M.R.Pennington, PLB736(2014)11; PRD90 (2014) 036004;

Fit to the widths

Width	Our fit (MeV)	Exp. (MeV)
$\Gamma_{f_0(1370)\to\pi\pi}$	11.7 ± 5.7	20.8 ± 10.7 [55, 56]
$\Gamma_{f_0(1370)\to K\bar{K}}$	10.7 ± 3.2	19.0 ± 10.6 [55, 56]
$\Gamma_{f_0(1370)\to\eta\eta}$	10.4 ± 4.3	6.41 ± 2.88 [57, 58]
$\Gamma_{f_0(1500)\to\pi\pi}$	38.1 ± 5.6	38.0 ± 2.5 [59, 60]
$\Gamma_{f_0(1500)\to K\bar{K}}$	9.39 ± 2.2	9.37 ± 1.09 [59, 60]
$\Gamma_{f_0(1500)\to\eta\eta}$	5.50 ± 4.1	5.56 ± 0.98 [59, 60]
$\Gamma_{f_0(1500)\to\eta\eta'}$	0.0	2.07 ± 0.87
$\Gamma_{f_0(1710)\to\pi\pi}$	20.5 ± 6.6	20.5 ± 9.9
$\Gamma_{f_0(1710)\to K\bar{K}}$	50.0 ± 15.3	50.0 ± 16.7
$\Gamma_{f_0(1710)\to\eta\eta}$	23.8 ± 9.8	24.0 ± 11.0
$\Gamma_{f_0(1710)\to\eta\eta'}$	30.9 ± 20.2	—
$\Gamma_{a_0^+(1450)\to\pi^+\eta}$	24.4 ± 12.0	24.7 ± 5.3
$\Gamma_{a_0(1450)\to\pi^0\eta}$	24.5 ± 12.0	24.7 ± 5.3
$\Gamma_{a_0^+(1450)\to\pi^+\eta'}$	9.14 ± 7.6	8.7 ± 4.5
$\Gamma_{a_0(1450)\to\pi^0\eta'}$	9.18 ± 7.7	8.7 ± 4.5
$\Gamma_{a_0^+(1450)\to K^+\overline{K}^0}$	21.0 ± 7.3	21.7 ± 7.4
$\Gamma_{a_0^0(1450)\to K^+K^-}$	10.6 ± 3.7	—
$\Gamma_{a_0^0(1450)\to K^0\overline{K}^0}$	10.4 ± 3.6	—

$\Gamma_{K_0^{*+}(1430)\to\pi^0K^+}$	80.5 ± 12.8	—
$\Gamma_{K_0^{*+}(1430)\to\pi^+K^0}$	159.7 ± 25.5	—
$\Gamma_{K_0^{*0}(1430)\to\pi^0K^0}$	80.0 ± 12.8	—
$\Gamma_{K_0^{*0}(1430)\to\pi^-K^+}$	160.6 ± 25.6	—
$\Gamma_{K_0^{*+}(1430)\to\eta K^+}$	20.7 ± 14.3	—
$\Gamma_{K_0^{*0}(1430)\to\eta K^0}$	20.5 ± 14.2	—
$\Gamma_{K_0^{*+}(1430)\to\pi K}$	240.1 ± 38.3	251.1 ± 27.0
$\Gamma_{a_0^+(980)\to\pi^+\eta}$	81.2 ± 16.9	—
$\Gamma_{a_0(980)\to\pi^0\eta}$	81.7 ± 17.0	—
$\Gamma_{a_0^+(980)\to K^+\overline{K}^0}$	14.4 ± 5.5	14.2 ± 1.8
$\Gamma_{a_0^0(980)\to K^+K^-}$	7.66 ± 2.8	—
$\Gamma_{a_0^0(980)\to K^0\overline{K}^0}$	6.68 ± 2.7	—
$\Gamma_{K_0^{*+}(700)\to\pi^0K^+}$	1.56 ± 1.9	—
$\Gamma_{K_0^{*+}(700)\to\pi^+K^0}$	3.04 ± 3.6	—
$\Gamma_{K_0^{*0}(700)\to\pi^0K^0}$	1.53 ± 1.8	—
$\Gamma_{K_0^{*0}(700)\to\pi^-K^+}$	3.09 ± 3.7	—
$\Gamma_{K_0^{*}(700)\to\pi K}$	4.59 ± 5.5	478 ± 127

Our couplins

- $c_d=c_m$: it is satisfied in light scalars , they are larger for light ones than that of heavy ones
- $a_0(980)$ has a small mixing angle with $a_0(1450)$, while that of κ and $K_0^*(1430)$ is almost 90 degree.

glueball component is mainly from $f_0(1710)$, its NLO contribution from large Nc could be ignored.

Parameter	Our fit	Mixing angle	Our fit
$M_{a_0(980)}$	1023.8 ± 22.6		
c_d^L	15.6 ± 1.9	α	-98.8 ± 41.9
c_d^H	3.07 ± 1.00	β	-39.8 ± 13.7
c_d'	0.0	γ	-27.8 ± 44.4
c_m^L	13.3 ± 6.8	ω	53.6 ± 4.7
c_m^H	9.21 ± 3.21	φ_a	4.78 ± 3.75
c_m'	0.0	φ_k	90.3 ± 22.5
α_L	17.9 ± 3.2		
α_H	0.88 ± 1.50		
β_H	-3.42 ± 0.53		
γ_H	-6.45 ± 1.19		
γ'	1.43 ± 3.26		
$\chi^2_{d.o.f}$	0.40		

SAA

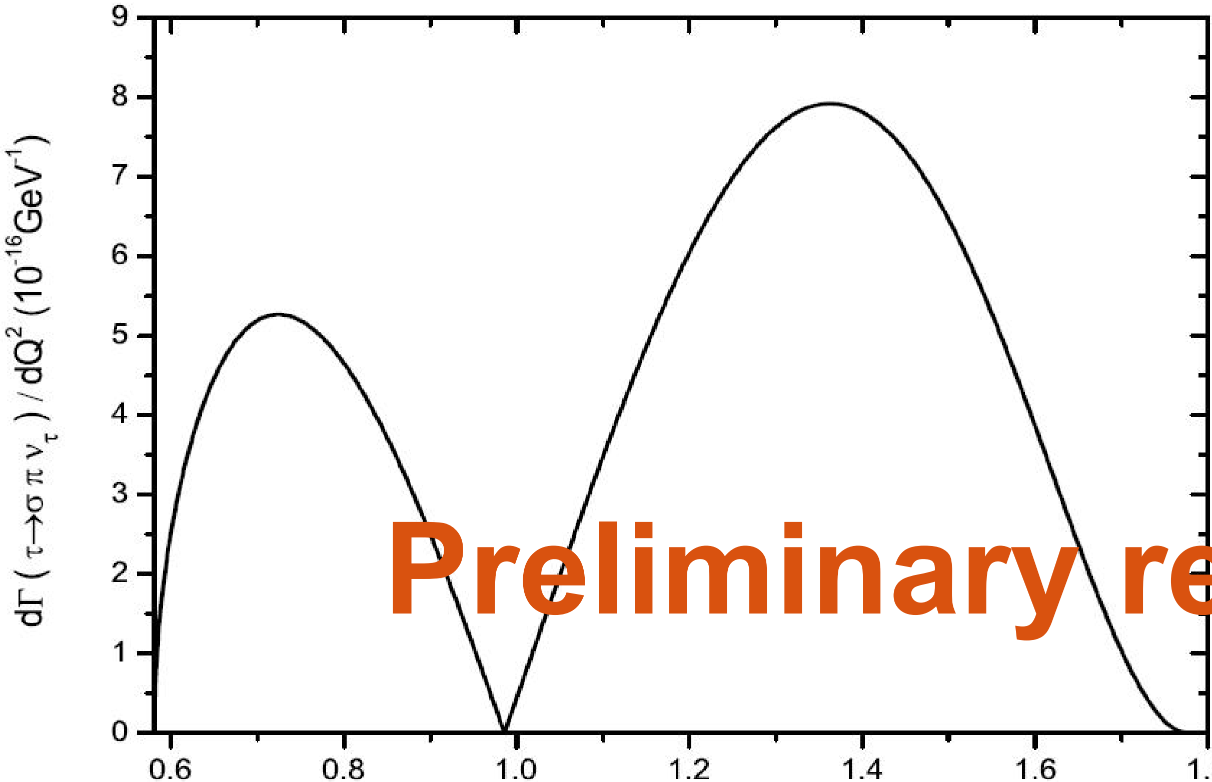
Decaying particle	Ratio	Our fit	Exp.
$f_0(1370)$	$\mathrm{Br}[K\overline{K}/\pi\pi]$	0.912 ± 0.374	0.91 ± 0.20 [55]
	$\mathrm{Br}[\eta\eta/\pi\pi]$	0.889 ± 0.771	0.31 ± 0.80 [56, 57]
$f_0(1500)$	$\mathrm{Br}[K\overline{K}/\pi\pi]$	0.246 ± 0.006	0.246 ± 0.026
	$\mathrm{Br}[\eta\eta/\pi\pi]$	0.144 ± 0.002	0.145 ± 0.027
	$\mathrm{Br}[\eta'\eta/\pi\pi]$	0.0	0.055 ± 0.024
$f_0(1710)$	$\mathrm{Br}[\pi\pi/K\overline{K}]$	0.410 ± 0.037	0.41 ± 0.14 [58, 59]
	$\mathrm{Br}[\eta\eta/K\overline{K}]$	0.476 ± 0.282	0.48 ± 0.15
$a_0(1450)$	$\mathrm{Br}[\pi\eta'/\pi\eta]$	0.375 ± 0.163	0.35 ± 0.16
	$\mathrm{Br}[K\overline{K}/\pi\eta]$	0.859 ± 0.269	0.88 ± 0.23 [56]
$K_0^*(1430)$	$\mathrm{Br}[\eta K/\pi K]$	0.086 ± 0.074	0.092 ± 0.031 [60]
$a_0(980)$	$\mathrm{Br}[K\overline{K}/\pi\eta]$	0.175 ± 0.057	0.183 ± 0.024

- $f_0(1710)$ has a large glueball component， f1370 is composed of u,d mainly, and f1500 is composed of s mainly.

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} -0.82 \pm 0.22 & 0.12 \pm 0.49 & 0.57 \pm 0.16 \\ 0.07 \pm 0.48 & -0.95 \pm 0.24 & 0.30 \pm 0.25 \\ 0.57 \pm 0.14 & 0.29 \pm 0.23 & 0.77 \pm 0.09 \end{pmatrix} \begin{pmatrix} N \\ S \\ G \end{pmatrix}$$

$$|S\rangle \equiv |\bar{s}s\rangle$$
$$|N\rangle \equiv \frac{1}{\sqrt{2}}|\bar{u}u + \bar{d}d\rangle$$

Other phenomenology



Preliminary results

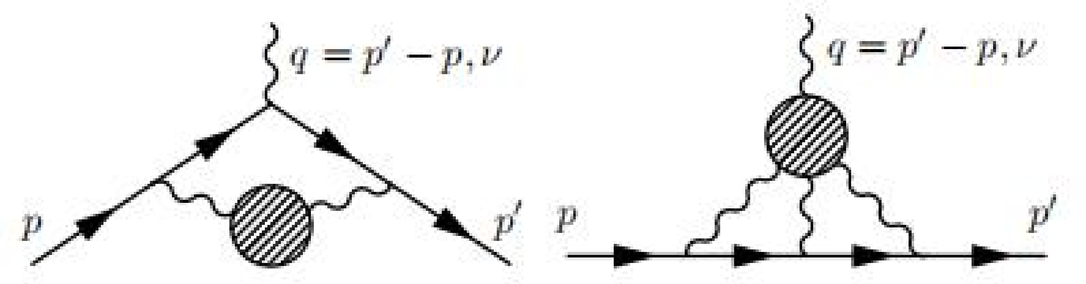
widths	fit.I	data (eV)
$\Gamma_{\sigma\to\gamma\gamma}$	2027	2050 ± 210
$\Gamma_{f_0(980)\to\gamma\gamma}$	254	320 ± 50
$\Gamma_{a_0\to\gamma\gamma}$	424	400
$\Gamma_{\phi\to\sigma\gamma}$	1.1	-
$\Gamma_{\phi\to f_0(980)\gamma}$	65.9	1370 ± 81
$\Gamma_{\phi\to a_0(980)\gamma}$	1.8	324 ± 26

Prospects

- VPP？ Tensors？

$$\lim_{p^2\to 0} p^2 \prod_{\mu,PPV}^{abc}(p,q,r) = \frac{-4iB^2F^2f^{abc}q_\mu}{q^2}$$

- g-2 constraint？



4. Summary

Matching

We match the three point GFs of SAA and SVV between QCD and ChEFT, and obtain the constaints for couplings.

Phenomenology

We study the decay widths of sclars into two pseudoscalars. We found that: $f_0(1370)$,$f_0(1500)$, $f_0(1710)$ are mainly composed of uu(dd), ss, glueball, respectively.

Prospects

More phenomenology, measurements, are needed to check the theory.

Thanks for your attention！