Chiral and Spin effects from Wigner function approach

高建华

山东大学（威海）

ArXiv:1902.06510 JHG, Z.T. Liang

ArXiv:1802.06216 JHG, Z.T. Liang, Q. Wang, X.N. Wang

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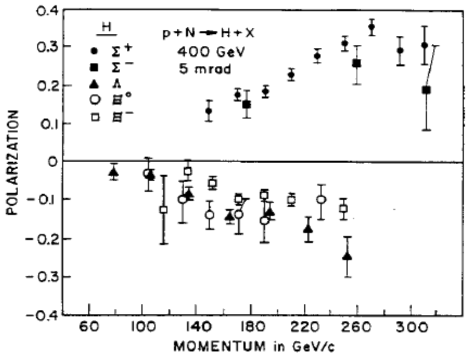
Outline

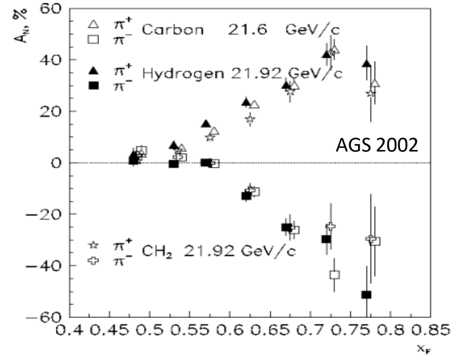
- **Introduction**
- **Chiral effects for massless Fermions**
- **Spin effects for massive Fermions**
- **Summary and outlook**

Spin Physics at High Energy

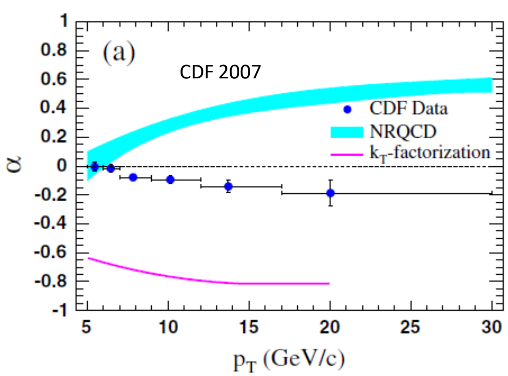
Hyperon Polarization



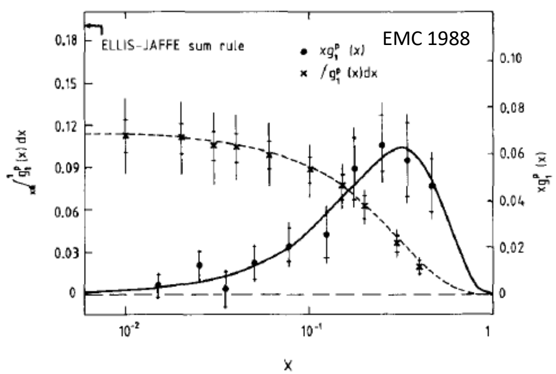
Single Spin Asymmetry



J/Ψ Polarization

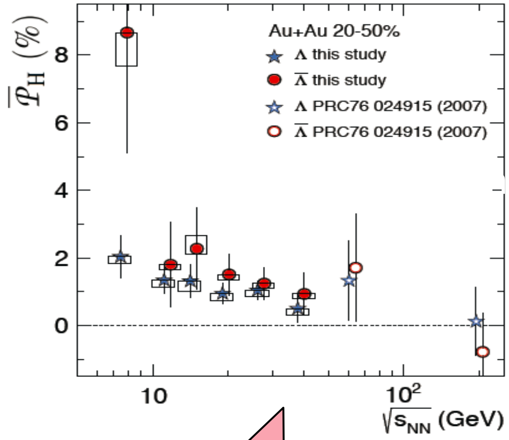


Proton Spin Crisis



Chirality/Spin in Relativistic HIC

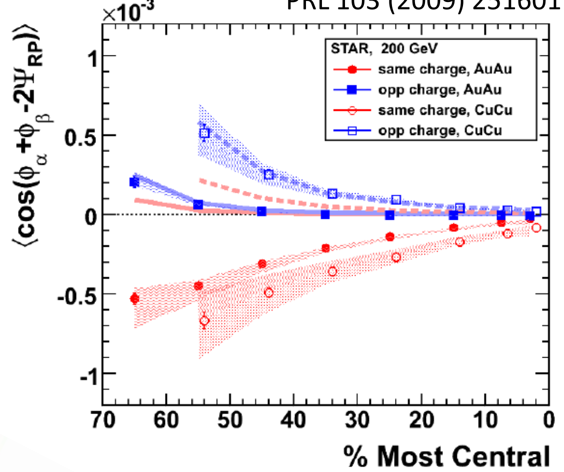
STAR Nature 548 (2017) 62-65

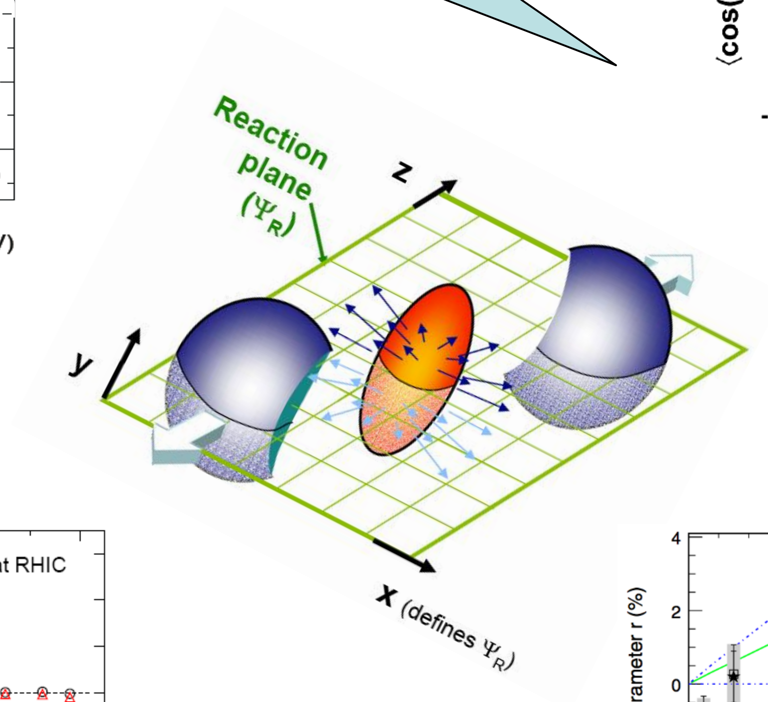


Global Polarization Effect !

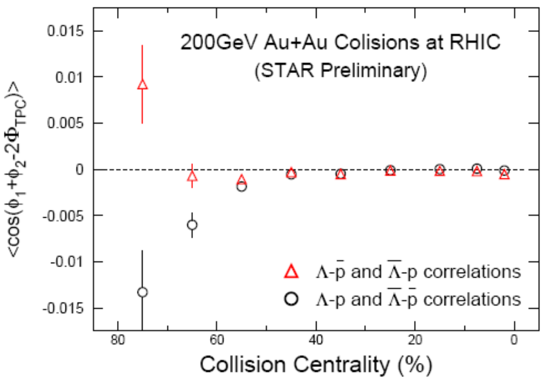
Chiral Magnetic Effect ?

PRL 103 (2009) 251601



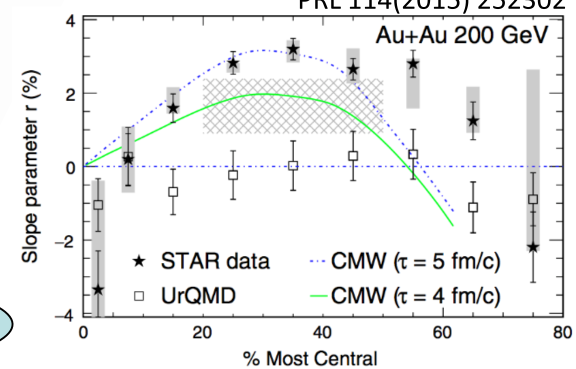


Chiral Magnetic Wave ?

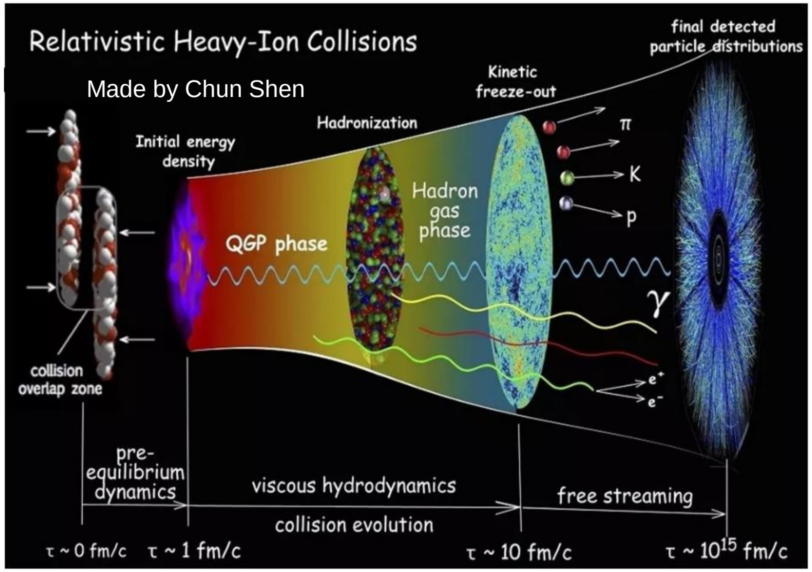


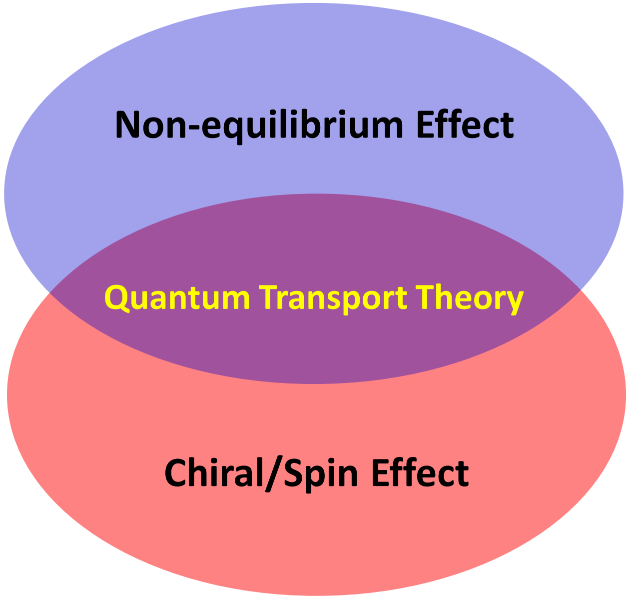
Chiral Vortical Effect ?

PRL 114(2015) 252302

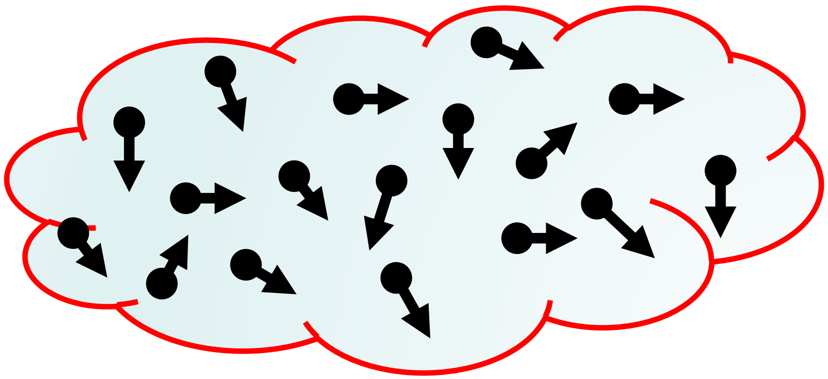


Quantum Transport Theory





Classical Transport Theory



Particle distribution function:

$$f(t, \vec{x}, \vec{p})$$

Conserved current:

$$j^\mu(x) \equiv \int \frac{d^3p}{2E_p} p^\mu f(t, \vec{x}, \vec{p})$$

Energy-momentum tensor:

$$T^{\mu\nu}(x) \equiv \int \frac{d^3p}{2E_p} p^\mu p^\nu f(t, \vec{x}, \vec{p})$$

The classical Boltzmann equation with the background EM fields

$$p^\mu \left(\partial_\mu - F_{\mu\nu}\partial_p^\nu\right) f(t, \vec{x}, \vec{p}) = \mathcal{C}[f]$$

→ collision term

One distribution function + One transport equation

Wigner Functions and Equations

Wigner matrix elements for spin-1/2 fermion in Abelian gauge field:

$$W_{\alpha\beta}(x,p) = \left\langle : \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}_\beta \left(x+\frac{y}{2}\right) U\left(x+\frac{y}{2}, x-\frac{y}{2}\right) \psi_\alpha \left(x-\frac{y}{2}\right) : \right\rangle$$

$$\begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} \\ W_{21} & W_{22} & W_{23} & W_{24} \\ W_{31} & W_{32} & W_{33} & W_{34} \\ W_{41} & W_{42} & W_{43} & W_{44} \end{pmatrix}$$

gauge link

16 independent Wigner functions:

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5 \mathscr{P} + \gamma^\mu \mathscr{V}_\mu + \gamma^5\gamma^\mu \mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

scalar, pseudo, vector, axial, tensor

32 Wigner equations in background field :

Vasak, Gyulassy, Elze, Annals Phys. 1987

$$\begin{aligned} \Pi^\mu \mathscr{V}_\mu = m\mathscr{F}, \quad G^\mu \mathscr{A}_\mu &= -2m\mathscr{P}, \\ \Pi_\mu \mathscr{F} + \frac{1}{2} G^\nu \mathscr{S}_{\mu\nu} &= m\mathscr{V}_\mu, \\ G_\mu \mathscr{P} - \epsilon_{\mu\nu\rho\sigma}\Pi^\nu \mathscr{S}^{\rho\sigma} &= 2m\mathscr{A}_\mu, \\ \frac{1}{2}(G_\mu \mathscr{V}_\nu - G_\nu \mathscr{V}_\mu) - \epsilon_{\mu\nu\rho\sigma}\Pi^\rho \mathscr{A}^\sigma &= m\mathscr{S}_{\mu\nu}. \end{aligned}$$

$$\begin{aligned} G^\mu \mathscr{V}_\mu = 0, \quad \Pi^\mu \mathscr{A}_\mu &= 0, \\ \frac{1}{2} G_\mu \mathscr{F} - \Pi^\nu \mathscr{S}_{\mu\nu} &= 0, \\ \Pi_\mu \mathscr{P} + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma} G^\nu \mathscr{S}^{\rho\sigma} &= 0, \\ (\Pi_\mu \mathscr{V}_\nu - \Pi_\nu \mathscr{V}_\mu) + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{A}^\sigma &= 0. \end{aligned}$$

$$\Pi^\mu \equiv p^\mu - \frac{1}{2} j_1\left(\frac{1}{2}\Delta\right) F^{\mu\nu}\partial_\nu^p \qquad G^\mu \equiv \partial_x^\mu - j_0\left(\frac{1}{2}\Delta\right) F^{\mu\nu}\partial_\nu^p \qquad \Delta \equiv \partial^p \cdot \partial_x$$

Massless Fermions

Decoupled Wigner equations for massless Fermions:

$$\begin{aligned} \Pi^\mu \mathscr{V}_\mu &= 0, \quad \Pi^\mu \mathscr{A}_\mu = 0, \\ G^\mu \mathscr{V}_\mu &= 0, \quad G^\mu \mathscr{A}_\mu = 0, \\ \epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{A}^\sigma &= -2\left(\Pi_\mu \mathscr{V}_\nu - \Pi_\nu \mathscr{V}_\mu\right), \\ \epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{V}^\sigma &= -2\left(\Pi_\mu \mathscr{A}_\nu - \Pi_\nu \mathscr{A}_\mu\right). \end{aligned}$$

8 functions 16 equations

$$\begin{aligned} \Pi_\mu \mathscr{F} + \frac{1}{2} G^\nu \mathscr{S}_{\mu\nu} &= 0, \\ \frac{1}{2} G_\mu \mathscr{F} - \Pi^\nu \mathscr{S}_{\mu\nu} &= 0, \\ -G_\mu \mathscr{P} + \epsilon_{\mu\nu\rho\sigma} \Pi^\nu \mathscr{S}^{\rho\sigma} &= 0, \\ \Pi_\mu \mathscr{P} + \frac{1}{4} \epsilon_{\mu\nu\rho\sigma} G^\nu \mathscr{S}^{\rho\sigma} &= 0. \end{aligned}$$

8 functions 16 equations

Right/Left ($s = \pm 1$)

$$\mathscr{J}_s^\mu \equiv \frac{1}{2}\left(\mathscr{V}_\mu + s\mathscr{A}_\mu\right)$$

$$j^\mu = \int d^4p \mathscr{V}^\mu, \quad j_5^\mu = \int d^4p \mathscr{A}^\mu, \quad T^{\mu\nu} = \int d^4p p^\mu \mathscr{V}^\nu$$

$$\Pi_\mu \mathscr{J}_s^\mu = 0, \quad G_\mu \mathscr{J}_s^\mu = 0, \quad \epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{J}_s^\sigma = -2s\left(\Pi_\mu \mathscr{J}_{s\nu} - \Pi_\nu \mathscr{J}_{s\mu}\right)$$

4 independent functions + 8 coupled equations

S.Ochs, U. Heinz Annals Phys. 266 (1998), JHG, Q. Wang Phys.Lett. B749 (2015)

$\hbar$ Expansion in Wigner functions

Wigner functions, G^μ, Π^μ operators expand in $\hbar$:

$$\mathscr{J}^\mu = \sum_{k=0}^{\infty} \hbar^k \mathscr{J}^\mu_{(k)} \quad G^\mu = \sum_{k=0}^{\infty} \hbar^k G^\mu_{(k)} \quad \Pi^\mu = \sum_{k=0}^{\infty} \hbar^k \Pi^\mu_{(k)}$$

$$\mathscr{J}^\mu = (\mathscr{J}^0, \vec{\mathscr{J}})$$

Express $\vec{\mathscr{J}}$ as $\mathscr{J}^0$:

$$\vec{\mathscr{J}}^{(n)} = \frac{\vec{p}}{p_0} \mathscr{J}_0^{(n)} + \vec{C}\left[\mathscr{J}_0^{(n-1)}, \mathscr{J}_0^{(n-2)}, \cdots, \mathscr{J}_0^{(0)}\right]$$

Evolution equation for $\mathscr{J}_0^{(n)}$:

$$\sum_{k=0}^{n}\left[G_0^{(k)} \mathscr{J}_0^{(n-k)} + \vec{G}^{(k)} \cdot \vec{\mathscr{J}}^{(n-k)}\right] = 0$$

Constraint equation for $\mathscr{J}_0^{(n)}$:

$$\sum_{k=0}^{n}\left[\Pi_0^{(k)} \mathscr{J}_0^{(n-k)} - \vec{\Pi}^{(k)} \cdot \vec{\mathscr{J}}^{(n-k)}\right] = 0$$

1 independent functions + 2 Wigner equations

JHG, Z.T. Liang, Q. Wang, X.N. Wang, arXiv:1802.06216

Wigner Functions Disentangling Theorem

The constraint equation gives on-shell condition:

$$\sum_{k=0}^{n}\left[\Pi_0^{(k)}\mathscr{J}_0^{(n-k)}-\vec{\Pi}^{(k)}\cdot\vec{\mathscr{J}}^{(n-k)}\right]=0$$

$O(\hbar^0)$:

$$\mathscr{J}_0^{(0)} = p_0 f_p^{(0)}\delta\left(p^2\right),$$

$O(\hbar^1)$:

$$\mathscr{J}_0^{(1)} = p_0 f_p^{(1)}\delta\left(p^2\right)+s\left(\vec{B}\cdot\vec{p}\right)f_p^{(0)}\delta'\left(p^2\right),$$

$O(\hbar^2)$: ...

$O(\hbar^0)+O(\hbar^1)$

$$\mathscr{J}_0 = p_0 f_p\delta\left(p^2+\frac{s\hbar\vec{B}\cdot\vec{p}}{p_0}\right)$$

$$p_0=|\vec{p}|\left(1-\hbar\vec{B}\cdot\vec{\Omega}_p\right)$$

Particle distribution function

$$f_p = f_p^{(0)}+\hbar f_p^{(1)}$$

Transport equation

$$\left[p^\mu G_\mu^{(0)}+\frac{s\hbar}{2}\vec{G}^{(0)}\cdot\left(\frac{1}{p_0}\vec{G}^{(0)}\times\vec{p}\right)\right]\left[f_p\delta\left(p^2+\frac{s\hbar\vec{B}\cdot\vec{p}}{p_0}\right)\right] = 0$$

1 independent functions + 1 transport equation at any $O(\hbar^n)$

JHG, Z.T. Liang, Q. Wang, X.N. Wang, arXiv:1802.06216

Chiral kinetic equations

Particle: integrate p_0 from 0 to $+\infty$ $\overline{\text{Particle}}$: integrate p_0 from $-\infty$ to 0

Chiral kinetic equation in 3D ($\vec{p}$)

$$\vec{v} = \left(1+2\hbar\vec{B}\cdot\vec{\Omega}_p\right)\hat{\vec{p}} - \hbar\left(\hat{\vec{p}}\cdot\vec{\Omega}_p\right)\vec{B}$$

$$\partial_t f_p + \frac{1}{\left(1+\hbar\vec{B}\cdot\vec{\Omega}_p\right)}\left[\vec{v}+\hbar\vec{E}\times\vec{\Omega}_p+\hbar(\hat{\vec{p}}\cdot\vec{\Omega}_p)\vec{B}\right]\cdot\vec{\nabla}_x f_p + \frac{1}{\left(1+\hbar\vec{B}\cdot\vec{\Omega}_p\right)}\left[\vec{E}+\vec{v}\times\vec{B}+\hbar|\vec{p}|\vec{\nabla}_x(\vec{B}\cdot\vec{\Omega}_p)+\hbar(\vec{E}\cdot\vec{B})\vec{\Omega}_p\right]\cdot\vec{\nabla}_p f_p = 0$$

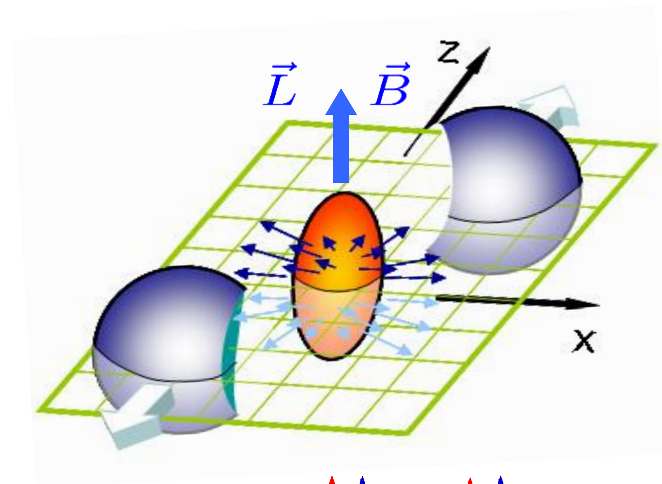
Stephanov & Yin PRL 109,(2012)162001, Son & Yamamoto PRD 87 (2013) 8, 085016

Berry curvature: $\vec{\Omega}_p = \frac{s\hat{\vec{p}}}{2\vec{p}^2}$ Berry monopole: $\vec{\nabla}_p\cdot\vec{\Omega}_p = 2\pi s\delta^3(\vec{p})$

Various Chiral Effects

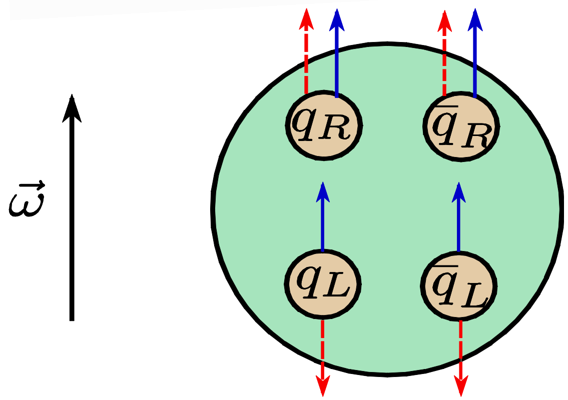
$$j^\mu = \xi_B B^\mu + \xi \omega^\mu$$

CME: $\xi_B = \frac{\mu_5}{2\pi^2}$ CVE: $\xi = \frac{\mu\mu_5}{\pi^2}$



$$j_5^\mu = \xi_{B5} B^\mu + \xi_5 \omega^\mu$$

CSE: $\xi_{B5} = \frac{\mu}{2\pi^2}$, LPE: $\xi_5 = \frac{T^2}{6} + \frac{\mu^2 + \mu_5^2}{2\pi^2}$



$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} j_\mu \qquad \partial_\mu j^\mu = 0, \qquad \partial_\mu j_5^\mu = C E \cdot B.$$

Massive Fermions

Wigner equations in background field at $O(\hbar)$: $\nabla^\mu \equiv \partial_x^\mu - F^{\mu\nu}\partial_\nu^p$

16 Wigner functions

32 Wigner equations

$$
\begin{aligned}
\nabla^\mu \mathscr{V}_\mu &= 0 & m\mathscr{F} &= p^\mu \mathscr{V}_\mu \\
p^\mu \mathscr{A}_\mu &= 0 & m\mathscr{P} &= -\frac{1}{2}\nabla^\mu \mathscr{A}_\mu \\
\frac{1}{2}\nabla_\mu \mathscr{F} - p^\nu \mathscr{S}_{\mu\nu} &= 0 & m\mathscr{V}_\mu &= p_\mu \mathscr{F} + \frac{1}{2}\nabla^\nu \mathscr{S}_{\mu\nu} \\
p_\mu \mathscr{P} + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\nabla^\nu \mathscr{S}^{\rho\sigma} &= 0 & m\mathscr{A}_\mu &= \frac{1}{2}\nabla_\mu \mathscr{P} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}p^\nu \mathscr{S}^{\rho\sigma} \\
(p_\mu \mathscr{V}_\nu - p_\nu \mathscr{V}_\mu) + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\nabla^\rho \mathscr{A}^\sigma &= 0 & m\mathscr{S}_{\mu\nu} &= \frac{1}{2}(\nabla_\mu \mathscr{V}_\nu - \nabla_\nu \mathscr{V}_\mu) - \epsilon_{\mu\nu\rho\sigma}p^\rho \mathscr{A}^\sigma
\end{aligned}
$$

Choose $\mathscr{F}$ and $\mathscr{A}^\mu$ as the independent fundamental components

Weickgenannt, Sheng, Speranza & Wang 1902.06513; Hattori, Hidaka & Yang 1903.01653; Wang, Guo, Shi & Zhuang 1903.03461； Li, Yee 1905.10463

Eleven of 32 provide the expressions of other components:

$$
\mathscr{P} = -\frac{\hbar}{2m}\nabla^\mu \mathscr{A}_\mu, \quad \mathscr{V}_\mu = \frac{1}{m}p_\mu \mathscr{F} - \frac{\hbar}{2m^2}\epsilon_{\mu\nu\rho\sigma}\nabla^\nu p^\rho \mathscr{A}^\sigma
$$

$$
\mathscr{S}_{\mu\nu} = -\frac{1}{m}\epsilon_{\mu\nu\rho\sigma}p^\rho \mathscr{A}^\sigma + \frac{\hbar}{2m^2}(\nabla_\mu p_\nu - \nabla_\nu p_\mu)\mathscr{F}
$$

Vasak, Gyulassy, Elze Annals Phys. 1987； ArXiv:1902.06510 JHG and Liang

Transport or Constraint Equations

Five of 32 lead to coupled transport equation for $\mathscr{F}$ and $\mathscr{A}^\mu$:

$$p\cdot\nabla\mathscr{F} = \frac{\hbar}{2m}p^\mu(\partial_\lambda^x\tilde{F}_{\mu\nu})\partial_p^\lambda\mathscr{A}^\nu$$

$$p\cdot\nabla\mathscr{A}_\mu = F_{\mu\nu}\mathscr{A}^\nu + \frac{\hbar}{2m}p^\nu(\partial_\lambda^x\tilde{F}_{\mu\nu})\partial_p^\lambda\mathscr{F}$$

Five of 32 modify on-shell conditions :

$$\left(p^2 - m^2\right)\mathscr{F} = -\frac{\hbar}{m}p^\mu\tilde{F}_{\mu\nu}\mathscr{A}^\nu$$

$$\left(p^2 - m^2\right)\mathscr{A}_\mu = -\frac{\hbar}{m}p^\nu\tilde{F}_{\mu\nu}\mathscr{F}$$

One of 32 provide a subsidiary condition:

$$p_\mu\mathscr{A}^\mu = 0$$

All the rest **10** of the 32 Wigner equations are satisfied automatically !

4 independent Wigner functions,
1 is $\mathscr{F}$, **3** are from 4-vector $\mathscr{A}^\mu$

satisfy

4 on-shell conditions
4 transport equations

Vasak, Gyulassy, Elze Annals Phys. 1987； ArXiv:1902.06510 JHG and Liang

Solve the constraint equations

Solve the modified on-shell conditions :

$$\left(p^2-m^2\right)\mathscr{F} = -\frac{\hbar}{m}p^\mu\tilde{F}_{\mu\nu}\mathscr{A}^\nu,$$
$$\left(p^2-m^2\right)\mathscr{A}_\mu = -\frac{\hbar}{m}p^\nu\tilde{F}_{\mu\nu}\mathscr{F},$$

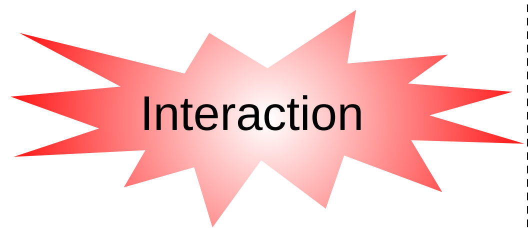
$\Longrightarrow$

$$\mathscr{F} = \delta\left(p^2-m^2\right)\mathcal{F}+\frac{\hbar}{m}\tilde{F}_{\mu\nu}p^\mu\mathcal{A}^\nu\delta'\left(p^2-m^2\right)$$
$$\mathscr{A}_\mu = \delta\left(p^2-m^2\right)\mathcal{A}_\mu+\frac{\hbar}{m}p^\nu\tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)$$

Introduce $\mathcal{F}$ and $\mathcal{A}^\mu$ as new independent Wigner functions.

Convenient to deal with transient EM field:

$$\mathscr{F}^i = \delta\left(p^2-m^2\right)\mathcal{F}^i$$
$$\mathscr{A}^i_\mu = \delta\left(p^2-m^2\right)\mathcal{A}^i_\mu$$



$$\mathscr{F}^f = \delta\left(p^2-m^2\right)\mathcal{F}^f$$
$$\mathscr{A}^f_\mu = \delta\left(p^2-m^2\right)\mathcal{A}^f_\mu$$

$t < t_i, \quad F^{\mu\nu} = 0$

$t_i < t < t_f, \quad F^{\mu\nu} \neq 0$

$t > t_f, \quad F^{\mu\nu} = 0$

$t = t_i$

$t = t_f$

Unintegrated Kinetic Equations

Substitute the solution into the transport equations

$$\mathscr{F} = \delta\left(p^2 - m^2\right)\mathcal{F} + \frac{\hbar}{m}\tilde{F}_{\mu\nu}p^\mu \mathcal{A}^\nu \delta'\left(p^2 - m^2\right)$$

$$\mathscr{A}_\mu = \delta\left(p^2 - m^2\right)\mathcal{A}_\mu + \frac{\hbar}{m}p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2 - m^2\right)$$

$$p\cdot\nabla\mathscr{F} = \frac{\hbar}{2m}p^\mu(\partial^x_\lambda \tilde{F}_{\mu\nu})\partial^\lambda_p \mathscr{A}^\nu,$$

$$p\cdot\nabla\mathscr{A}_\mu = F_{\mu\nu}\mathscr{A}^\nu + \frac{\hbar}{2m}p^\nu(\partial^x_\lambda \tilde{F}_{\mu\nu})\partial^\lambda_p \mathscr{F}.$$

$$p\cdot\nabla\left[\mathcal{F}\delta\left(p^2 - m^2\right) + \frac{\hbar}{m}\tilde{F}_{\mu\nu}p^\mu\mathcal{A}^\nu\delta'\left(p^2 - m^2\right)\right] = \frac{\hbar}{2m}(\partial^x_\lambda\tilde{F}_{\mu\nu})\partial^\lambda_p\left[p^\mu\mathcal{A}^\nu\delta\left(p^2 - m^2\right)\right],$$

$$p\cdot\nabla\left[\mathcal{A}_\mu\delta\left(p^2 - m^2\right) + \frac{\hbar}{m}p^\nu\tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2 - m^2\right)\right] = F_{\mu\nu}\left[\mathcal{A}^\nu\delta\left(p^2 - m^2\right) + \frac{\hbar}{m}p_\lambda\tilde{F}^{\nu\lambda}\mathcal{F}\delta'\left(p^2 - m^2\right)\right] + \frac{\hbar}{2m}(\partial^x_\lambda\tilde{F}_{\mu\nu})\partial^\lambda_p\left[p^\nu\mathcal{F}\delta\left(p^2 - m^2\right)\right].$$

$$p_\mu\mathcal{A}^\mu\,\delta\left(p^2 - m^2\right) = 0$$

Unintegrated kinetic equations:

Manifest Lorentz Covariance !

Singular Dirac delta function !

Integrated Kinetic Equations

Integrated kinetic equations in **4**-vector form:

$$
\begin{aligned}
p\cdot\nabla\mathcal{F} &= -\frac{\hbar\, p^\mu}{2mE_p^2}\left[\tilde{F}_{\mu\nu}\bar{p}^\lambda\nabla_\lambda - E_p^2(\bar{\partial}_x^\lambda\tilde{F}_{\mu\nu})\bar{\partial}_\lambda^p\right]\mathcal{A}^\nu \\
p\cdot\nabla\mathcal{A}_\mu &= F_{\mu\nu}\mathcal{A}^\nu - \frac{\hbar\, p^\nu}{2mE_p^2}\left[\tilde{F}_{\mu\nu}\bar{p}^\lambda\nabla_\lambda - E_p^2(\bar{\partial}_x^\lambda\tilde{F}_{\mu\nu})\bar{\partial}_\lambda^p\right]\mathcal{F} \\
p^\mu\cdot\mathcal{A}_\mu &= 0
\end{aligned}
$$

$$
\begin{aligned}
p &= (E_p,\vec{p}) \qquad \bar{p} = (0,\vec{p}) \\
\bar{\partial}_\mu^x &= (0,\vec{\nabla}_x) \qquad \bar{\partial}_\mu^p = (0,\vec{\nabla}_p) \\
\nabla^\mu &= \nabla_x^\mu - F^{\mu\nu}\bar{\partial}_\nu^p \\
E_p &= \sqrt{\vec{p}^2+m^2}
\end{aligned}
$$

Integrated kinetic equations in **3**-vector form:

$$
\begin{aligned}
\left(\nabla_t+\vec{v}\cdot\vec{\nabla}\right)\mathcal{F} &= -\frac{\hbar}{2mE_p}\left[(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p) - (\vec{B}\cdot\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\vec{v}\right]\cdot\vec{\mathcal{A}} \\
\left(\nabla_t+\vec{v}\cdot\vec{\nabla}\right)\vec{\mathcal{A}} &= \vec{B}\times\vec{\mathcal{A}} - \vec{E}(\vec{v}\cdot\vec{\mathcal{A}}) - \frac{\hbar}{2mE_p}(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla}+E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\mathcal{F}
\end{aligned}
$$

$$
\vec{v}=\vec{p}/E_p, \quad \nabla_t=\partial_t+\vec{E}\cdot\vec{\nabla}_p, \quad \vec{\nabla}=\vec{\nabla}_x+\vec{B}\times\vec{\nabla}_p,
$$

$$
\mathcal{A}^0 = \vec{v}\cdot\vec{\mathcal{A}}
$$

Simplified Version

Vector current and energy-momentum tensor at $O(\hbar)$:

$$j^\mu = \int d^4p\, \mathscr{V}^\mu, \qquad T^{\mu\nu} = \int d^4p\, p^\nu \mathscr{V}^\mu$$

$$\mathscr{V}_\mu = \frac{1}{m} p_\mu \mathscr{F} - \frac{\hbar}{2m^2} \epsilon_{\mu\nu\rho\sigma} \nabla^\nu p^\rho \mathscr{A}^\sigma,$$

Covariant unintegrated kinetic equations for $\mathcal{A}^\mu$ at $O(1)$:

$$p \cdot \nabla \left[\mathcal{A}_\mu \delta\left(p^2 - m^2\right) + \frac{\hbar}{m} \cancel{p^\nu \tilde{F}_{\mu\nu} \mathcal{F} \delta'\left(p^2 - m^2\right)} \right] = F_{\mu\nu} \left[\mathcal{A}^\nu \delta\left(p^2 - m^2\right) + \frac{\hbar}{m} \cancel{p_\lambda \tilde{F}^{\nu\lambda} \mathcal{F} \delta'\left(p^2 - m^2\right)} \right] + \frac{\hbar}{2m} \cancel{(\partial_\lambda^x \tilde{F}_{\mu\nu}) \partial_p^\lambda \left[p^\nu \mathcal{F} \delta\left(p^2 - m^2\right) \right]}.$$

⇩

$$p \cdot \nabla \left[\mathcal{A}_\mu \delta\left(p^2 - m^2\right) \right] = F_{\mu\nu} \mathcal{A}^\nu \delta\left(p^2 - m^2\right) \quad + \quad p_\mu \mathcal{A}^\mu \delta\left(p^2 - m^2\right) = 0$$

Inserting the solved $\mathcal{A}^\mu$ into the transport equation for $\mathcal{F}$

$$p \cdot \nabla \left[\mathcal{F} \delta\left(p^2 - m^2\right) + \frac{\hbar}{m} \tilde{F}_{\mu\nu} p^\mu \mathcal{A}^\nu \delta'\left(p^2 - m^2\right) \right] = \frac{\hbar}{2m} (\partial_\lambda^x \tilde{F}_{\mu\nu}) \partial_p^\lambda \left[p^\mu \mathcal{A}^\nu \delta\left(p^2 - m^2\right) \right]$$

Simplified Version

Define:

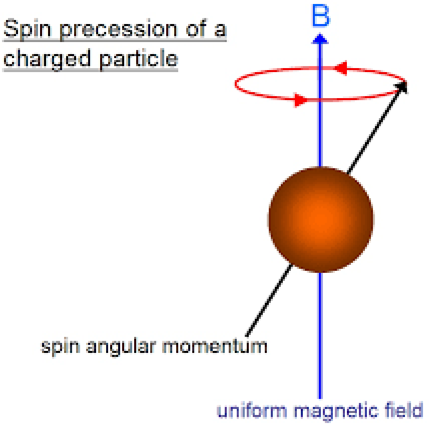
$$\frac{\mathcal{A}_\mu}{\mathcal{F}} = P s_\mu \qquad s^2 = -1, \quad p \cdot s = 0$$

P : Spin polarization magnitude

s^μ : Spin polarization direction

Decoupled equations for P and s_μ :

$$p \cdot \nabla \left[P \delta\left(p^2 - m^2\right)\right] = 0,$$
$$p \cdot \nabla \left[s_\mu \delta\left(p^2 - m^2\right)\right] = F_{\mu\nu} s^\nu \delta\left(p^2 - m^2\right).$$



Rewrite the transport equations for $\mathcal{F}$ as:

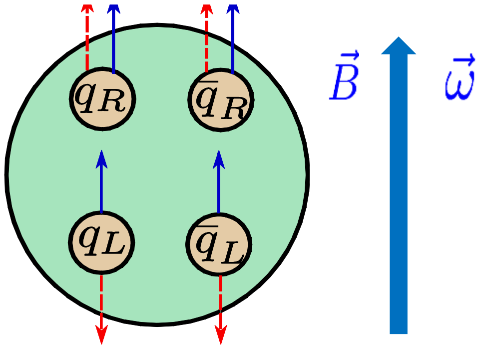
$$p \cdot \nabla \left[\mathcal{F} \delta\left(p^2 - m^2 - 2E_p \Delta E\right)\right] = \frac{\hbar}{2m} (\partial^x_\lambda \tilde{F}^{\rho\sigma}) \partial^\lambda_p \left[p_\rho s_\sigma P \mathcal{F} \delta\left(p^2 - m^2 - 2E_p \Delta E\right)\right]$$

The effective interaction energy:

$$\Delta E = -\frac{\hbar P}{2mE_p} \tilde{F}^{\rho\sigma} p_\rho s_\sigma$$

CSE with mass correction

Chiral separation effect :



$$j_5^\mu = \frac{\mu}{2\pi^2}B^\mu + \frac{1}{2\pi^2}\left(\frac{\pi^2 T^2}{3} + \mu^2 + \mu_5^2\right)\omega^\mu$$

Global equilibrium solution with constant $\Omega_{\mu\nu}$ & $F_{\mu\nu}$

$$\mathcal{A}_\mu = 0, \quad \mathcal{F} = \frac{m}{2\pi^3}\left[\frac{\theta(u\cdot p)}{e^{(u\cdot p-\mu)/T}+1} + \frac{\theta(-u\cdot p)}{e^{-(u\cdot p-\mu)/T}+1}\right]$$

$\beta_\mu = u_\mu/T$

$\Omega_{\mu\nu} = \partial_\mu\beta_\nu - \partial_\nu\beta_\mu$

$$\mathscr{A}_\mu = \frac{\hbar}{m}p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2 - m^2\right) \Rightarrow j_5^\mu = \int d^4p\mathscr{A}^\mu = \sigma B^\mu$$

$$\Rightarrow \sigma = \frac{\hbar}{2\pi^2}\int_0^\infty dp\left(n_+ - n_-\right), \quad n_\pm = \frac{1}{e^{(E_p\mp\mu)/T}+1}$$

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Chiral limit: $\sigma|_{m=0} = \frac{\hbar\mu}{2\pi^2}$ Zero temperature limit: $\sigma|_{T\to 0} = \frac{\hbar\sqrt{\mu^2 - m^2}}{2\pi^2}$

Quantum magnetization effect

Wigner function associated to spin magnetic moment density:

$$\mathscr{S}_{\mu\nu} = -\frac{1}{m}\epsilon_{\mu\nu\rho\sigma}p^{\rho}\mathscr{A}^{\sigma} + \frac{\hbar}{2m^2}\left(\nabla_{\mu}p_{\nu} - \nabla_{\nu}p_{\mu}\right)\mathscr{F}$$

Spin magnetic moment vector:

$$M_{\mu} = \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}u^{\nu}\int d^4p\,\mathscr{S}^{\alpha\beta}$$

Global equilibrium solution with constant $\Omega_{\mu\nu}$ & $F_{\mu\nu}$:

$$\mathcal{A}_{\mu} = 0 \qquad \mathcal{F} = \frac{m}{2\pi^3}\left[\frac{\theta(u\cdot p)}{e^{(u\cdot p-\mu)/T}+1} + \frac{\theta(-u\cdot p)}{e^{-(u\cdot p-\mu)/T}+1}\right]$$

Quantum magnetization effect

$$M_{\mu} = \hbar\kappa B_{\mu} - \frac{\hbar\rho}{m}\omega_{\mu}$$

Susceptibility:

$$\kappa = \frac{m}{2\pi^2}\int\frac{dp}{E_p}\left(n_+ + n_-\right)$$

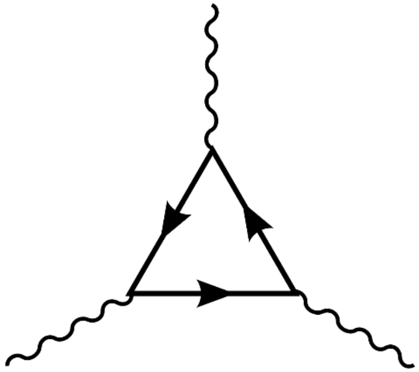
Charge density:

$$\rho = \frac{1}{\pi^2}\int dp\,p^2\left(n_+ - n_-\right)$$

Chiral Anomaly

Chiral anomaly:

$$\partial_\mu j_5^\mu = -2mj_5 - \frac{e^2}{2\pi^2} E \cdot B$$



Pseudo scalar Wigner function:

$$\mathscr{P} = -\frac{\hbar}{2m} \nabla^\mu \mathscr{A}_\mu$$

$$\mathscr{A}_\mu = \delta\left(p^2 - m^2\right) \mathcal{A}_\mu + \frac{\hbar}{m} p^\nu \tilde{F}_{\mu\nu} \mathcal{F} \delta'\left(p^2 - m^2\right)$$

Integrate over momentum:

$$j_5^\mu = \int d^4 p \mathscr{A}^\mu \qquad j_5 = \int d^4 p \mathscr{P}$$

$$\partial_\mu j_5^\mu = -\frac{2m}{\hbar} j_5 + \hbar C E \cdot B$$

$$C = \frac{1}{2m} \int d^4 p \, \partial^\lambda \left[\mathcal{F} \partial_\lambda \delta(p^2 - m^2)\right]$$

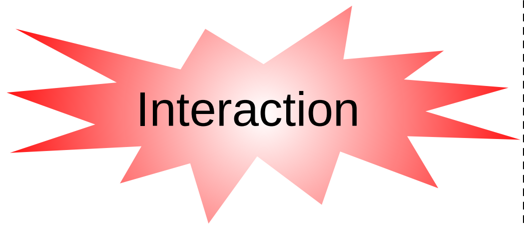
Global Polarization Generation

Transient EM field process:

$$\mathscr{A}_\mu = \delta\left(p^2-m^2\right)\mathcal{A}_\mu + \frac{\hbar}{m}p^\nu \tilde{F}_{\mu\nu}\mathcal{F}\delta'\left(p^2-m^2\right)$$

$$\mathscr{F}^i = \delta\left(p^2-m^2\right)\mathcal{F}^i \neq 0$$

$$\mathscr{A}^i_\mu = \delta\left(p^2-m^2\right)\mathcal{A}^i_\mu = 0$$



$$\mathscr{A}^f_\mu = \delta\left(p^2-m^2\right)\mathcal{A}^f_\mu = ?$$

$t < t_i, \quad F^{\mu\nu} = 0$

$t_i < t < t_f, \quad F^{\mu\nu} \neq 0$

$t > t_f, \quad F^{\mu\nu} = 0$

$t = t_i$

$t = t_f$

Evolution equation for spin polarization vector up to $O(1)$:

$$\left(\nabla_t + \vec{v}\cdot\vec{\nabla}\right)\vec{A} = \vec{B}\times\vec{A} - \vec{E}(\vec{v}\cdot\vec{A})$$

near t_i

$\vec{A}(t_i) = 0$

$$\frac{\partial \vec{A}}{\partial t} = 0$$

No way to generate the polarization from a zero initial value !

Global Polarization Generation

Evolution equation for spin vector up to $O(\hbar)$:

$$\left(\nabla_t + \vec{v}\cdot\vec{\nabla}\right)\vec{A} \;=\; \vec{B}\times\vec{A} - \vec{E}(\vec{v}\cdot\vec{A}) - \frac{\hbar}{2mE_p}(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla} + E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\mathcal{F}$$

near t_i ⇩ $\vec{A}(t_i)=0$

$$\frac{\partial\vec{A}}{\partial t} \;=\; -\frac{\hbar}{2mE_p}(\vec{B}+\vec{E}\times\vec{v})(\vec{v}\cdot\vec{\nabla} + E_p\overleftarrow{\nabla}_x\cdot\vec{\nabla}_p)\mathcal{F}(t_i)$$

Polarization seed: EM field + inhomogeneous $\mathcal{F}(t_i)$

Self-consistent background EM field: Vasak, Gyulassy, Elze, Annals Phys. 1987

$$\partial_\mu F^{\mu\nu} = j^\nu \;\Rightarrow\; \partial_\lambda\partial^\lambda F_{\mu\nu} = (\partial_\mu j_\nu - \partial_\nu j_\mu)$$

Global polarization: **Vorticity → EM field → polarization**

Particle scattering generation: arxiv:1904.09152, J.J. Zhang, R.H. Fang, Q. Wang, X.N. Wang

Summary and Outlook

- Wigner function approach can demonstrate:

  Chiral systems in a background EM field can be described sufficiently by one distribution function and one equation up to any order of $\hbar$.

- Wigner function approach can describe:

  Chiral anomaly, chiral magnetic effect, chiral vortical effect, chiral separate effect, quantum magnetization effect and global polarization.

- How can we introduce quantum gauge fields instead of background fields systematically and consistently?

Thanks for your attention!