Three-Dimensional Fragmentation Functions and Semi-Inclusive e^+e^--Annihilation At High Energies

梁作堂 (Liang Zuo-tang)

山东大学(Shandong University)

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Based on: K.B. Chen, S.Y. Wei and ZTL, Front. Phys. (China)10, 101204 (2015) (short review);
X.N. Wang and ZTL, PRD75, 094002 (2007);
S.Y. Wei, Y.K Song and ZTL, PRD89, 014024 (2014);
S.Y. Wei, K.B. Chen, Y.K. Song and ZTL, PRD91, 034015 (2015);
K.B. Chen, W.H. Yang, S.Y. Wei and ZTL, PRD94, 034003 (2016);
K.B. Chen, W.H. Yang, Y.J. Zhou and ZTL, PRD95, 034009 (2017);
W.H. Yang, K.B. Chen and ZTL, PRD96, 054016 (2017).

Introduction: QCD and hadron physics

Two important quantities: parton distribution function (PDF) ⟷ hadron structure

fragmentation function (FF) ⟷ hadronization

hadron structure (PDFs)

hadronization (FFs)

QCD parton model

the measurable quantities (cross section, correlations, azimuthal asymmetries, polarizations, ...)

(1) High precision calculations to establish the connections

(2) High precision measurements & parameterizations

(3) Theoretical calculations (model, LQCD, …….)

Contents

I. Introduction

Collinear expansion, PDFs and FFs defined via quark-quark correlator

II. General kinematic analysis for $e^+e^- \to V\pi X$

- The basic Lorentz tensors for the hadronic tensor
- Cross section in terms of structure functions

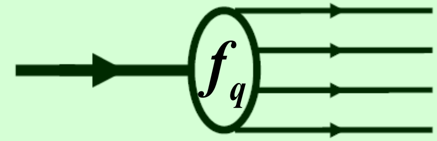
III. Parton model results for $e^+e^- \to V\bar{q}X$ up to twist-4

- Collinear expansion for semi-inclusive $e^+e^- \to h\bar{q}X$
- Structure functions up to twist-4
- Numerical estimation of Lambda polarization and spin alignment of *K**

IV. Summary and outlook

Introduction: Intuitive definition of PDFs

Parton model: A fast moving proton ═ A beam of partons



✪ Consider only longitudinal momentum ⟹ one-dimensional PDFs $f_q(x)$

$f_q(x)$: number density, x=k/p: fractional momentum

✪ Including spin ⟹ spin dependent one-dimensional PDFs:

$$f_1(x,s_q;S) = f_1(x) + \lambda_q \lambda\, g_{1L}(x) + \vec{s}_{Tq} \cdot \vec{S}_T\, h_{1T}(x)$$

helicity distribution transversity

✪ Including transverse momentum ⟹ three-dimensional (or TMD) PDFs:

$$f(x,k_\perp,s_q;p,S) = f_1(x,k_\perp) + \vec{S}_T \cdot \frac{\hat{p}\times\vec{k}_\perp}{M} f_{1T}^\perp(x,k_\perp) + \lambda_q \lambda g_{1L}(x,k_\perp) + \lambda_q \frac{\vec{S}_T\cdot\vec{k}_\perp}{M} g_{1T}^\perp(x,k_\perp)$$

$$+ \vec{s}_{\perp q} \cdot \frac{\hat{p}\times\vec{k}_\perp}{M} h_1^\perp(x,k_\perp) + \vec{s}_{\perp q}\cdot\vec{S}_T h_{1T}(x,k_\perp) + \frac{\vec{s}_{\perp q}\cdot\vec{k}_\perp}{M}\frac{\vec{S}_T\cdot\vec{k}_\perp}{M} h_{1T}^\perp(x,k_\perp) + \frac{\vec{s}_{\perp q}\cdot\vec{k}_\perp}{M}\lambda h_{1L}^\perp(x,k_\perp)$$

Introduction: Leading twist TMD PDFs

The 8 three-dimensional (or TMD) PDFs

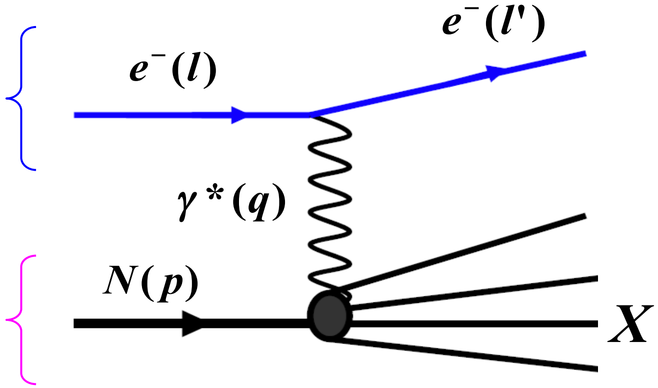
nucleon polarization ↑ / quark polarization →	U	L	T
U	$f_1(x,k_\perp)$ number density		$h_1^\perp(x,k_\perp)$ Boer-Mulders function
L		$g_{1L}(x,k_\perp)$ helicity distribution	$h_{1L}^\perp(x,k_\perp)$ Worm-gear/longi-transversity
T	$f_{1T}^\perp(x,k_\perp)$ Sivers function	$g_{1T}^\perp(x,k_\perp)$ Worm-gear/trans-helicity	$h_{1T}(x,k_\perp)$ transversity distribution $h_{1T}^\perp(x,k_\perp)$ pretzelosity

In quantum field theory, they are defined via Lorentz decomposition of the quark-quark correlator $\hat{\Phi}^{(0)}(k;p,S)=\int d^4z\, e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$

Inclusive deep inelastic scattering (DIS) $e^- + N \to e^- + X$

The differential cross section

$$d\sigma = \frac{\alpha_{em}^2}{sQ^4} L^{\mu\nu}(l,\lambda_l,l',\lambda_l')W_{\mu\nu}(q,p,S)\frac{d^3l'}{2E'}$$



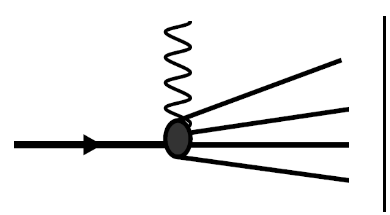
leptonic tensor

hadronic tensor

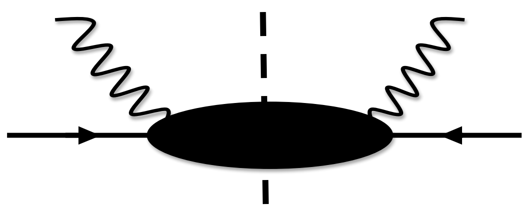
The hadronic tensor:

$$W_{\mu\nu}(q,p,S) = \sum_X \langle p,S|J_\mu(0)|X\rangle\langle X|J_\nu(0)|p,S\rangle(2\pi)^4\delta^4(p+q-p_X)$$

$$W_{\mu\nu}(q,p,S) = \left| \quad \right|^2 =$$

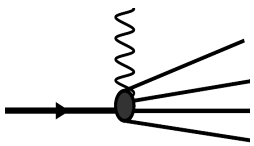
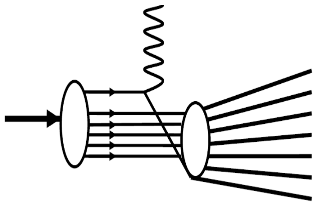
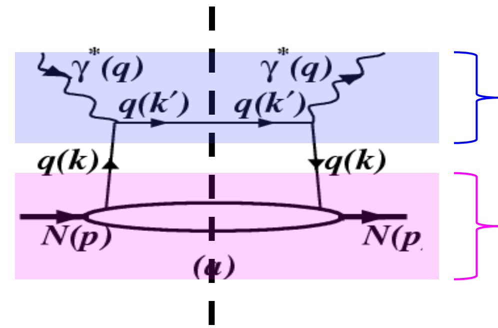


$|\mathcal{M}|^2$



$\mathcal{M}\times\mathcal{M}^*$

Parton model without QCD:

$W_{\mu\nu}(q,p,S) = \Big|$  $\Big|^2 = \Big|$  $\Big|^2 =$ 

$$W_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} Tr\left[\hat{H}_{\mu\nu}(k,q)\hat{\phi}(k,p,S)\right]$$

the calculable hard part $\hat{H}_{\mu\nu}(k,q) = \gamma_\mu(\not{k}+\not{q})\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$

the quark-quark correlator $\hat{\phi}(k,p,S) = \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\psi(z)|p,S\rangle$

Collinear approximation: $k \approx xp$

$$\Rightarrow \quad W_{\mu\nu}(q,p) \approx \left[(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) + \frac{1}{2xq\cdot p}(q+2xp)_\mu(q+2xp)_\nu\right] f_q(x)$$

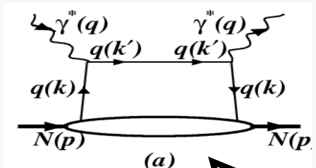
operator expression of the number density : $f_q(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-}\langle p|\bar{\psi}(0)\frac{\gamma^+}{2}\psi(z)|p\rangle$

no local (color) gauge invariance!

Inclusive DIS with "multiple gluon scattering"

To get the gauge invariance, we need to take the "multiple gluon scattering" into account

$W_{\mu\nu}(q,p,S) =$



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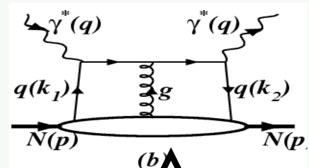

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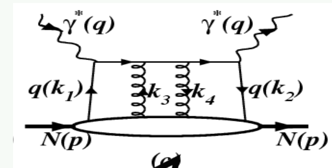


$+ \cdots$

$$W_{\mu\nu}(q,p,S) = W_{\mu\nu}^{(0)}(q,p,S) + W_{\mu\nu}^{(1)}(q,p,S) + W_{\mu\nu}^{(2)}(q,p,S) + \ldots$$

$$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(0)}(k,q)\hat{\phi}^{(0)}(k;p,S)\right]$$

$$W_{\mu\nu}^{(1)}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(1)\rho}(k_1,k_2,q)\hat{\phi}_{\rho}^{(1)}(k_1,k_2;p,S)\right]$$

no gauge invariance!

the quark-gluon-quark correlator $\hat{\phi}_{\rho}^{(1)}(k_1,k_2;p,S) = \int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)A_\rho(y)\psi(z)|p,S\rangle$

Collinear approximation:

- ✪ The hard parts: $\hat{H}_{\mu\nu}^{(0)}(k,q) \approx \hat{H}_{\mu\nu}^{(0)}(x)$, $\hat{H}_{\mu\nu}^{(1)\rho}(k_1,k_2,q) \approx \hat{H}_{\mu\nu}^{(1)\rho}(x_1,x_2)$
- ✪ The gluon field: $A_\rho(y) \approx n\cdot A(y)\dfrac{p_\rho}{n\cdot p}$

Using Ward identities, e.g., $p_\rho \hat{H}_{\mu\nu}^{(1,L)\rho}(x_1,x_2) = \hat{H}_{\mu\nu}^{(0)}(x_1)/(x_2-x_1-i\varepsilon)$

all $\hat{H}_{\mu\nu}^{(j)}(x_i)$'s reduce to $\hat{H}_{\mu\nu}^{(0)}(x)$ ⇨

Inclusive DIS: LO pQCD, leading twist

$$W_{\mu\nu}(q,p,S) \approx \tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}^{(0)}(k;p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

LO & leading twist

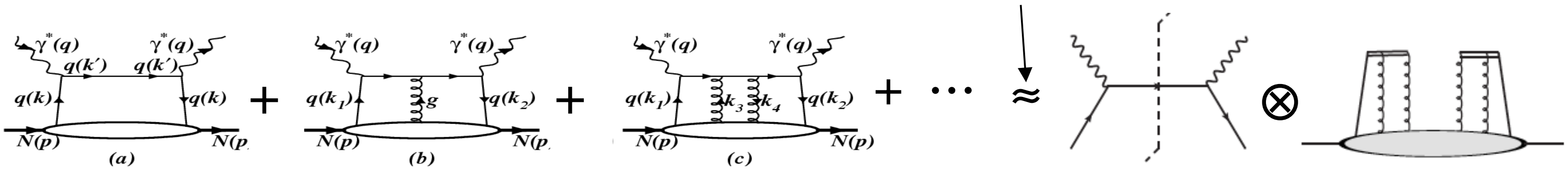
$$\hat{\Phi}^{(0)}(k;p,S) = \int d^4z e^{ikz} \langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

gauge invariant !

$$\mathcal{L}(0,z) = \mathcal{L}^{\dagger}(-\infty,0)\mathcal{L}(-\infty,z) \qquad \mathcal{L}(-\infty,z) = Pe^{ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{z}_\perp)}$$

gauge link

collinear approximation



$$\hat{h}^{(0)}_{\mu\nu} = \gamma_\mu \not{n} \gamma_\nu$$

$$\hat{H}^{(0)}_{\mu\nu}(x) = \hat{h}^{(0)}_{\mu\nu}\delta(x - x_B) \quad \Rightarrow \quad \tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int dx \mathrm{Tr}\left[\hat{\Phi}^{(0)}(x;p,S)\, h^{(0)}_{\mu\nu}\right]\delta(x - x_B)$$

$$\hat{\Phi}^{(0)}(x;p,S) \equiv \int \frac{d^4k}{(2\pi)^4}\delta\left(x - \frac{k^+}{p^+}\right)\hat{\Phi}^{(0)}(k;p,S) = \int \frac{p^+dz^-}{2\pi} e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle$$

⇒ (only) one-dimensional imaging of the nucleon via inclusive DIS

Inclusive DIS: LO pQCD, leading & higher twists

Collinear expansion:

Ellis, Furmanski, Petronzio, (1982,1983)
Qiu, Sterman (1990,1991)

✪ Expanding the hard parts at k =xp:

$$\hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k=xp,q)$$

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^{\rho}} \equiv \left.\frac{\partial \hat{H}^{(0)}_{\mu\nu}(k,q)}{\partial k^{\rho}}\right|_{k=xp}$$

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(x) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^{\rho}} \omega_{\rho}{}^{\rho'} k_{\rho'} + \ldots\ldots$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) = \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2) + \frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_1^{\sigma}} \omega_{\sigma}{}^{\sigma'} k_{1\sigma'} + \ldots\ldots$$

✪ Decomposition of the gluon field:

$$A_{\rho}(y) = n\cdot A(y)\frac{p_{\rho}}{n\cdot p} + \omega_{\rho}{}^{\rho'} A_{\rho'}(y)$$

Using the Ward identities such as,

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^{\rho}} = -\hat{H}^{(1)\rho}_{\mu\nu}(x,x), \qquad p_{\rho}\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$$

to replace the derivatives etc.

$$x = k^{+}/p^{+}$$

$$\omega_{\rho}{}^{\rho'} \equiv g_{\rho}{}^{\rho'} - \bar{n}_{\rho} n^{\rho'}$$

$$\omega_{\rho}{}^{\rho'} k_{\rho'} = (k - xp)_{\rho}$$

$$p^{\pm} = \frac{1}{\sqrt{2}}(p_0 \pm p_3)$$

$$n = (0,1,\vec{0}_{\perp})$$

$$\bar{n} = (1,0,\vec{0}_{\perp})$$

Adding all terms with the same hard part together ⟹

Inclusive DIS: LO pQCD, leading & higher twists

$$W_{\mu\nu}(q,p,S)=\tilde{W}^{(0)}_{\mu\nu}(q,p,S)+\tilde{W}^{(1)}_{\mu\nu}(q,p,S)+\tilde{W}^{(2)}_{\mu\nu}(q,p,S)+\dots$$

---

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

depends on x only !

$$\hat{\Phi}^{(0)}(k,p,S)=\int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

gauge invariant quark-quark correlator

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twist-3, 4 and 5 contributions

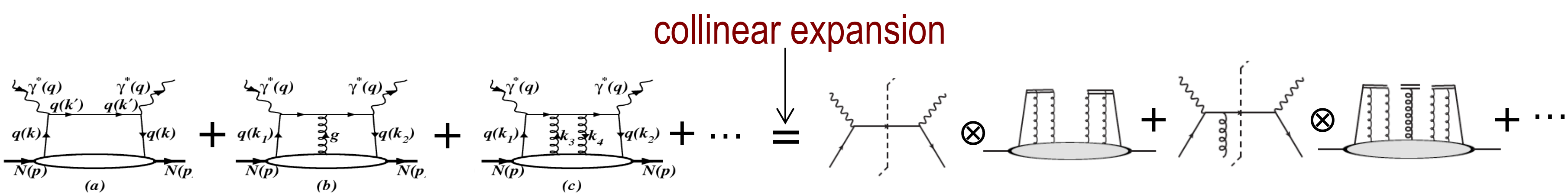
$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S)=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Phi}^{(1)}_{\rho'}(k_1,k_2,p,S)\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}\right]$$

$$\hat{\Phi}^{(1)}_{\rho}(k_1,k_2;p,S)=\int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

$$D_\rho(y)=-i\partial_\rho+gA_\rho(y)$$

gauge invariant quark-gluon-quark correlator

⇒ A consistent framework for inclusive DIS $e^-N\to e^-X$ including leading & higher twists



Inclusive DIS: LO pQCD, leading & higher twists

Simplified expressions for hadronic tensors

The “collinearly expanded hard parts” take very simple forms such as:

$$\hat{H}^{(0)}_{\mu\nu}(x)=\hat{h}^{(0)}_{\mu\nu}\delta(x-x_B),\qquad \hat{h}^{(0)}_{\mu\nu}=\gamma_\mu \not{n}\gamma_\nu$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}=\frac{\pi}{2q\cdot p}\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\delta(x_1-x_B),\qquad \hat{h}^{(1)\rho}_{\mu\nu}=\gamma_\mu\bar{\not{n}}\gamma^\rho\not{n}\gamma_\nu$$

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\mathrm{Tr}\left[\hat{\Phi}^{(0)}(x_B;p,S)h^{(0)}_{\mu\nu}\right]$$

contributes at twist-2, 3 and 4

$$\hat{\Phi}^{(0)}(x;p,S)\equiv\int\frac{d^4k}{(2\pi)^4}\delta(x-\frac{k^+}{p^+})\hat{\Phi}^{(0)}(k;p,S)=\int\frac{p^+dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle$$

one-dimensional gauge invariant quark-quark correlator

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S)=\frac{\pi}{2q\cdot p}\mathrm{Tr}\left[\hat{\varphi}^{(1)}_{\rho'}(x_B;p,S)h^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\right]$$

contributes at twist-3, 4 and 5

$$\hat{\varphi}^{(1)}_{\rho}(x;p,S)\equiv\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta(x-\frac{k_1^+}{p^+})\hat{\Phi}^{(1)}_\rho(k_1,k_2;p,S)=\int\frac{p^+dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle$$

one-dimensional gauge invariant quark-gluon-quark correlator

ONE dimensional, depend only on ONE parton momentum!

PDFs defined via quark-quark correlator

- Expand the quark-quark correlator in terms of the Γ-matrices:

$$\hat{\Phi}^{(0)}(x;p,S)=\frac{1}{2}\Big[\Phi^{(0)}(x;p,S)+i\gamma_5\tilde{\Phi}^{(0)}(x;p,S)+\gamma^\alpha\Phi^{(0)}_\alpha(x;p,S)+\gamma_5\gamma^\alpha\tilde{\Phi}^{(0)}_\alpha(x;p,S)+i\gamma_5{}^\alpha\sigma^{\alpha\beta}\Phi^{(0)}_{\alpha\beta}(x;p,S)\Big]$$

(scalar) (pseudo-scalar) (vector) (axial vector) (tensor)

- Make Lorentz decompositions

$$S=\lambda\frac{p^+}{M}\bar{n}+S_T-\lambda\frac{M^2}{2p^+}n$$

$$\Phi^{(0)}(x;p,S)=Me(x)$$

$$\tilde{\Phi}^{(0)}(x;p,S)=\lambda Me_L(x)$$

$$\Phi^{(0)}_\alpha(x;p,S)=p^+\bar{n}_\alpha f_1(x)+M\varepsilon_{\perp\alpha\rho}S_T^\rho f_T(x)+\frac{M^2}{p^+}n_\alpha f_3(x)$$

$$\tilde{\Phi}^{(0)}_\alpha(x;p,S)=\lambda p^+\bar{n}_\alpha g_{1L}(x)+MS_{T\alpha}g_T(x)+\lambda\frac{M^2}{p^+}n_\alpha g_{3L}(x)$$

$$\Phi^{(0)}_{\rho\alpha}(x;p,S)=p^+\bar{n}_{[\rho}S_{T\alpha]}h_{1T}(x)-M\varepsilon_{T\rho\alpha}h_T(x)+\lambda M\bar{n}_{[\rho}n_{\alpha]}h_L(x)+\frac{M^2}{p^+}n_{[\rho}S_{T\alpha]}h_{3T}(x)$$

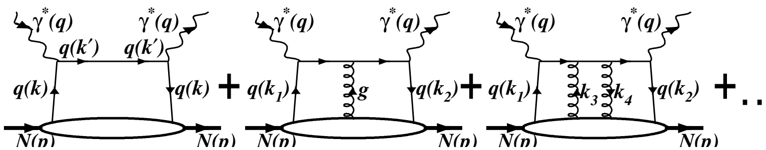
blue: twist-2
black: twist-3, M/Q suppressed
brown: twist-4, $(M/Q)^2$ suppressed

Collinear expansion in high energy reactions

共线展开(collinear expansion)的必要性与重要性：

（1）给出规范链接，保证规范不变性

（2）使高扭度计算大大简化



Before collinear expansion: $W_{\mu\nu}(q,p,S) = W_{\mu\nu}^{(0)}(q,p,S) + W_{\mu\nu}^{(1)}(q,p,S) + W_{\mu\nu}^{(2)}(q,p,S) + ...$

$$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(k,q)\right]$$

$$W_{\mu\nu}^{(1)}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{\phi}_\rho^{(1)}(k_1,k_2,p,S)\hat{H}_{\mu\nu}^{(1)\rho}(k_1,k_2,q)\right]$$

no gauge link, no gauge invariance!

After collinear expansion: $W_{\mu\nu}(q,p,S) = \tilde{W}_{\mu\nu}^{(0)}(q,p,S) + \tilde{W}_{\mu\nu}^{(1)}(q,p,S) + \tilde{W}_{\mu\nu}^{(2)}(q,p,S) + ...$

$$\tilde{W}_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(x)\right]$$

$$\tilde{W}_{\mu\nu}^{(1)}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}_{\rho'}^{(1)}(k_1,k_2,p,S)\hat{H}_{\mu\nu}^{(1)\rho}(x_1,x_2)\omega_\rho^{\ \rho'}\right]$$

with gauge link, gauge invariant!

and they can be simplified to:

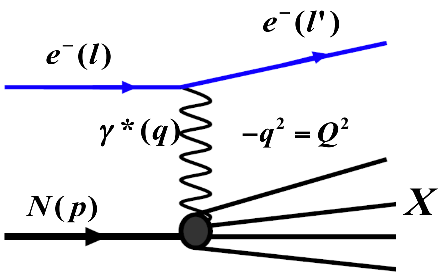
$$\tilde{W}_{\mu\nu}^{(0)}(q,p,S) = \mathrm{Tr}\left[\hat{\Phi}^{(0)}(x_B;p,S)h_{\mu\nu}^{(0)}\right]$$

$$\tilde{W}_{\mu\nu}^{(1)}(q,p,S) = \frac{\pi}{2q\cdot p}\mathrm{Tr}\left[\hat{\varphi}_{\rho'}^{(1)}(x_B;p,S)h_{\mu\nu}^{(1)\rho}\omega_\rho^{\ \rho'}\right]$$

Collinear expansion in high energy reactions

Successfully to all processes where only ONE hadron is explicitly involved.

Inclusive DIS $e^- N \to e^- X$



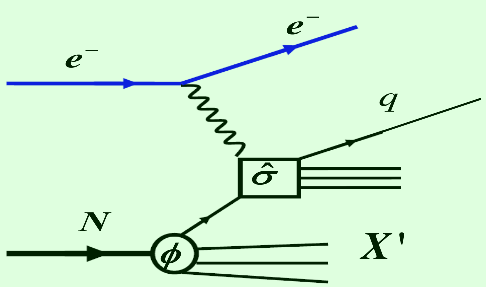
Yes!

where collinear expansion was first formulated.

R. K. Ellis, W. Furmanski and R. Petronzio,
Nucl. Phys. B207,1 (1982); B212, 29 (1983).

Semi-Inclusive DIS

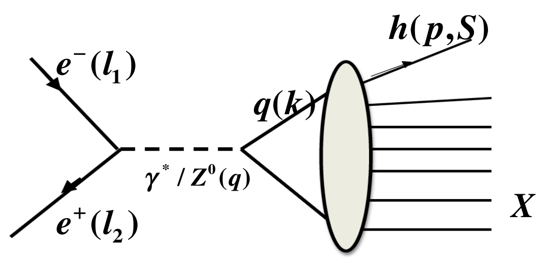
$$e + N \to e + q(jet) + X$$



Yes!
ZTL & X.N. Wang,
PRD (2007);

Inclusive

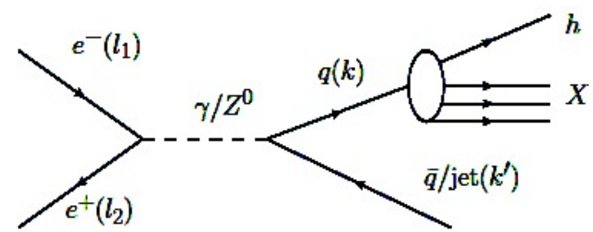
$$e^- + e^+ \to h + X$$



Yes!
S.Y. Wei, Y.K Song,& ZTL,
PRD (2014);

Semi-Inclusive

$$e^- + e^+ \to h + \bar{q}(jet) + X$$



Yes!
S.Y. Wei, K.B. Chen, Y.K Song, &
ZTL, PRD (2015).

Semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$

---

twist-2, 3 and 4 contributions

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\hat{\Phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(x)]\ (2E_{k'})(2\pi)^3\delta^3(\vec{k}' - \vec{k} - \vec{q})$$

$$\hat{\Phi}^{(0)}(k,p,S) = \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

depends on x only !

---

twist-3, 4 and 5 contributions

$$\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \sum_{c=L,R} \mathrm{Tr}[\hat{\Phi}_{\rho'}^{(1)}(k_1,k_2,p,S)\ \hat{H}_{\mu\nu}^{(1,c)\rho}(x_1,x_2)\ \omega_{\rho}^{\ \rho'}]\ (2E_{k'})(2\pi)^3\delta^3(\vec{k}' - \vec{k}_c - \vec{q})$$

$$\hat{\Phi}_{\rho}^{(1)}(k_1,k_2,p,S) = \int d^4z d^4y e^{ik_1y + ik_2(z-y)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

⇨ A consistent framework for $e^-N \to e^- + q(jet) + X$ at LO pQCD including higher twists

ZTL & X.N. Wang, PRD (2007); Y.K. Song, J.H. Gao, ZTL & X.N. Wang, PRD (2011) & PRD (2014).

Semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S;k_\perp) = \mathrm{Tr}\left[\hat{\Phi}^{(0)}(x_B,k_\perp;p,S)\, h_{\mu\nu}^{(0)}\right]$$

$$\hat{\Phi}^{(0)}(x,k'_\perp;p,S) = \int \frac{d^4k}{(2\pi)^4}\delta(x-\frac{k^+}{p^+})\delta^2(k_\perp - k'_\perp)\hat{\Phi}^{(0)}(k;p,S) = \int \frac{p^+dz^-}{2\pi}d^2z_\perp e^{ixp^+z^- - ik_\perp\cdot z_\perp}\langle N|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|N\rangle$$

three-dimensional gauge invariant quark-quark correlator

$$\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S;k_\perp) = \frac{\pi}{2q\cdot p}\mathrm{Tr}\left[\hat{\varphi}_{\rho'}^{(1)}(x_B,k_\perp;p,S)\, h_{\mu\nu}^{(1)\rho}\omega_\rho^{\ \rho'}\right]$$

$$\hat{\varphi}_\rho^{(1)}(x,k_\perp;p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta(x-\frac{k_1^+}{p^+})\delta^2(k_{1\perp} - k_\perp)\hat{\Phi}_\rho^{(1)}(k_1,k_2;p,S)$$

$$= \int \frac{p^+dz^-}{2\pi}d^2z_\perp e^{ixp^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}\langle N|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0,z)\psi(z)|N\rangle$$

three-dimensional gauge invariant quark-gluon-quark correlator

TMD PDFs defined via quark-quark correlator

The quark-quark correlator: $\hat{\Phi}^{(0)}(k;p,S)=\int d^4 z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$

integrate over k^- : $\hat{\Phi}^{(0)}(x,k_\perp;p,S)=\int dz^- d^2 z_\perp e^{i(xp^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$

Expansion in terms of the Γ-matrices

$$
\begin{aligned}
\hat{\Phi}^{(0)}(x,k_\perp;p,S)=\frac{1}{2}\Big[\ &\Phi^{(0)}(x,k_\perp;p,S) && \text{—— scalar}\\
&+i\gamma_5\ \tilde{\Phi}^{(0)}(x,k_\perp;p,S) && \text{—— pseudo-scalar}\\
&+\gamma^\alpha\ \Phi^{(0)}_\alpha(x,k_\perp;p,S) && \text{—— vector}\\
&+\gamma_5\gamma^\alpha\ \tilde{\Phi}^{(0)}_\alpha(x,k_\perp;p,S) && \text{—— axial vector}\\
&+i\gamma_5\sigma^{\alpha\beta}\ \Phi^{(0)}_{\alpha\beta}(x,k_\perp;p,S)\Big] && \text{—— tensor}
\end{aligned}
$$

TMD PDFs defined via quark-quark correlator

The Lorentz decomposition

totally 8(twist 2)+16(twist 3)+8(twist 4) components

$$\Phi_S^{(0)}(x,k_\perp;p,S) = Me(x,k_\perp) + (\tilde{k}_\perp \cdot S_T)e_T^\perp(x,k_\perp)$$ ← twist-3

$$\Phi_\alpha^{(0)}(x,k_\perp;p,S) = p^+\bar{n}_\alpha\left[f_1(x,k_\perp) + \frac{\tilde{k}_\perp \cdot S_T}{M}f_{1T}^\perp(x,k_\perp)\right]$$ ← twist-2

$$+\ k_{\perp\alpha}f^\perp(x,k_\perp) + M\tilde{S}_{T\alpha}f_T(x,k_\perp) + \tilde{k}_{\perp\alpha}\left[\lambda f_L^\perp(x,k_\perp) + \frac{k_\perp \cdot S_T}{M}f_T^\perp(x,k_\perp)\right]$$ ← twist-3

$$+\frac{M^2}{p^+}n_\alpha\left[f_3(x,k_\perp) + \frac{\tilde{k}_\perp \cdot S_T}{M}f_{3T}^\perp(x,k_\perp)\right]$$ ← twist-4

See e.g., K. Goeke, A. Metz, M. Schlegel, PLB 618, 90 (2005);
P. J. Mulders, lectures in 17th Taiwan nuclear physics summer school, August, 2014.

TMD PDFs defined via quark-quark correlator

The Lorentz decomposition

totally 8(twist 2)+16(twist 3)+8(twist 4) components

$$\tilde{\Phi}^{(0)}(x,k_\perp;p,S) = M\left[\lambda e_L(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} e_T(x,k_\perp)\right]$$

twist-3

$$\tilde{\Phi}^{(0)}_\alpha(x,k_\perp;p,S) = p^+\bar{n}_\alpha\left[\lambda g_{1L}(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} g_{1T}^\perp(x,k_\perp)\right]$$

twist-2

$$- MS_{T\alpha} g_T(x,k_\perp) - k_{\perp\alpha}\left[\lambda g_L^\perp(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} g_T^\perp(x,k_\perp)\right] + \varepsilon_{\perp\alpha\beta}k_\perp^\beta g^\perp(x,k_\perp)$$

$$+ \frac{M^2}{p^+} n_\alpha\left[\lambda g_{3L}(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} g_{3T}^\perp(x,k_\perp)\right]$$

twist-4

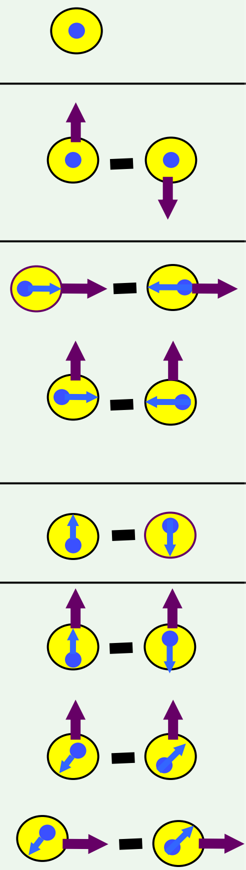
$$\Phi^{(0)}_{\rho\alpha}(x,k_\perp;p,S) = p^+\bar{n}_{[\rho}S_{T\alpha]}h_{1T}(x,k_\perp) + \frac{p^+\bar{n}_{[\rho}k_{\perp\alpha]}}{M}\left[\lambda h_{1L}^\perp(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} h_{1T}^\perp(x,k_\perp)\right] + \frac{p^+\bar{n}_{[\rho}\varepsilon_{\perp\alpha]\beta}k_\perp^\beta}{M} h_1^\perp(x,k_\perp)$$

$$+ S_{T[\rho}k_{\perp\alpha]}h_T^\perp(x,k_\perp) + M\varepsilon_{\perp\rho\alpha}h(x,k_\perp) - \bar{n}_{[\rho}n_{\alpha]}\left[M\lambda h_L(x,k_\perp) - (k_\perp \cdot S_T)h_T(x,k_\perp)\right]$$

$$+ \frac{M^2}{p^+}\left\{n_{[\rho}S_{T\alpha]}h_{3T}(x,k_\perp) + \frac{n_{[\rho}k_{\perp\alpha]}}{M}\left[\lambda h_{3L}^\perp(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} h_{3T}^\perp(x,k_\perp)\right] + \frac{n_{[\rho}\varepsilon_{\perp\alpha]\beta}k_\perp^\beta}{M} h_3^\perp(x,k_\perp)\right\}$$

Twist-2 TMD PDFs defined via quark-quark correlator

Leading twist (twist 2)

quark	polarization nucleon →		TMD PDFs (8)	if no gauge link	integrated over $k_\perp$	name
U (f)	U		$f_1(x,k_\perp)$		$q(x)$	number density
	T		$f_{1T}^\perp(x,k_\perp)$	0	×	Sivers function
L (g)	L		$g_{1L}(x,k_\perp)$		$\Delta q(x)$	helicity distribution
	T		$g_{1T}^\perp(x,k_\perp)$		×	worm gear/trans-helicity
T (h)	U		$h_1^\perp(x,k_\perp)$	0	×	Boer-Mulders function
	$T(//)$		$h_{1T}(x,k_\perp)$		$\delta q(x)$	transversity distribution
	$T(\perp)$		$h_{1T}^\perp(x,k_\perp)$			pretzelocity
	L		$h_{1L}^\perp(x,k_\perp)$		×	worm gear/ longi-transversity

Twist-3 TMD PDFs defined via quark-quark correlator

Next to the leading twist (twist-3)

they are NOT probability distributions but include the quantum interference effects.

quark	polarization nucleon →	TMD PDFs (16)	if no gauge link	integrated over $k_\perp$	name
U (f)	U	$e(x,k_\perp)$, $f^\perp(x,k_\perp)$	0 $\frac{f_1(x,k_\perp)}{x}$	$e(x)$, ×	number density
	L	$f_L^\perp(x,k_\perp)$	0	×	Sivers function
	T	$e_T^\perp(x,k_\perp)$, $f_T(x,k_\perp)$, $f_T^\perp(x,k_\perp)$	0 0 0	× $f_T(x)$	
$\vec{L}$ (g)	U	$g^\perp(x,k_\perp)$	0	×	
	L	$e_L(x,k_\perp)$, $g_L^\perp(x,k_\perp)$	0 $\frac{g_{1L}(x,k_\perp)}{x}$	$e_L(x)$, ×	helicity distribution
	T	$e'^\perp_T(x,k_\perp)$, $g_T(x,k_\perp)$, $g_T^\perp(x,k_\perp)$	0 0 $\frac{g_{1T}(x,k_\perp)}{x}$	× $g'_T(x)$	worm gear/trans-helicity
$\vec{T}$ (h)	U	$h(x,k_\perp)$	0	$h(x)$	Boer-Mulders function
	T(//)	$h_T^\perp(x,k_\perp)$	$\frac{h_{1T}^\perp(x,k_\perp)}{x}$	×	transversity distribution
	$T(\perp)$	$h_T^{\perp'}(x,k_\perp)$	$\frac{k_\perp^2 h_{1T}^\perp(x,k_\perp)}{M^2 x}$	×	pretzelocity
	L	$h_L(x,k_\perp)$	$\frac{k_\perp^2 h_{1L}^\perp(x,k_\perp)}{M^2 x}$	$h_L(x)$	worm gear/ longi-transversity

TMD PDFs defined via quark-quark correlator

quark polarization →

Twist-2 TMD PDFs

nucleon polarization ↑	U	L	T
U	$f_1(x,k_\perp)$ number density		$h_1^\perp(x,k_\perp)$ Boer-Mulders function
L		$g_{1L}(x,k_\perp)$ helicity distribution	$h_{1L}^\perp(x,k_\perp)$ Worm-gear/longi-transversity
T	$f_{1T}^\perp(x,k_\perp)$ Sivers function	$g_{1T}^\perp(x,k_\perp)$ Worm-gear/trans-helicity	$h_{1T}(x,k_\perp)$ transversity distribution $h_{1T}^\perp(x,k_\perp)$ pretzelosity

Twist-3 TMD PDFs

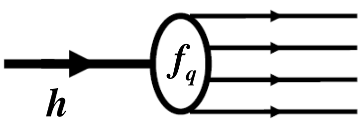
nucleon polarization ↑	U	L	T
U	$e(x,k_\perp), f^\perp(x,k_\perp)$ number density	$g^\perp(x,k_\perp)$	$h(x,k_\perp)$ Boer-Mulders function
L	$f_L^\perp(x,k_\perp)$	$e_L(x,k_\perp), g_L^\perp(x,k_\perp)$ helicity distribution	$h_L(x,k_\perp)$ Worm gear/ longi-transversity
T	$e_T^\perp(x,k_\perp)$, $f_T^{\perp 1}(x,k_\perp), f_T^{\perp 2}(x,k_\perp)$ Sivers function	$e_T(x,k_\perp)$, $g_T(x,k_\perp), g_T^\perp(x,k_\perp)$ Worm gear/ trans-helicity	$h_T^\perp(x,k_\perp)$ transversity distribution $h_T(x,k_\perp)$ pretzelosity

Fragmentation Function v.s. Parton Distribution Function

TMDs = TMD PDFs + TMD FFs

Parton distribution functions (PDFs):

$h \to q + X$



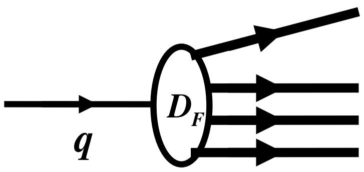
a hadron ⟶ a beam of partons

number density of parton in the beam

$$\hat{\Phi}(k;p,S) = \sum_X \int d^4 z e^{ikz} \times \langle h | \bar{\psi}(0) | X \rangle \langle X | \mathcal{L}(0,z) \psi(z) | h \rangle$$

Fragmentation functions (FFs):

$q \to h + X$



a quark ⟶ a jet of hadrons

number density of hadron in the jet

$$\hat{\Xi}(k_F;p,S) = \sum_X \int d^4 \xi e^{ik_F\xi} \times \langle 0 | \mathcal{L}(0,\xi) \psi(\xi) | hX \rangle \langle hX | \bar{\psi}(0) | 0 \rangle$$

"conjugate" to each other

⇨ Studies on FFs and PDFs should keep pace with each other.

Description of polarization of particles with different spins

Spin 1/2 hadrons:

The spin density matrix: $\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \frac{1}{2}(1 + \vec{S}\cdot\vec{\sigma})$

Vector polarization: $S^\mu = (0, \vec{S}_T, \lambda)$

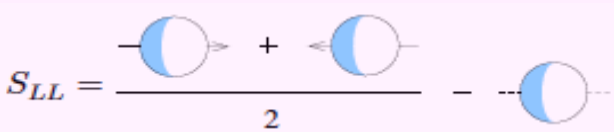
Spin 1 hadrons:

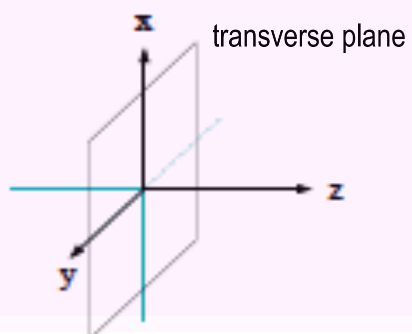
The spin density matrix: $\rho = \begin{pmatrix} \rho_{11} & \rho_{10} & \rho_{1-1} \\ \rho_{01} & \rho_{00} & \rho_{0-1} \\ \rho_{-11} & \rho_{-10} & \rho_{-1-1} \end{pmatrix} = \frac{1}{3}(1 + \frac{3}{2}\vec{S}\cdot\vec{\Sigma} + 3T^{ij}\Sigma^{ij})$

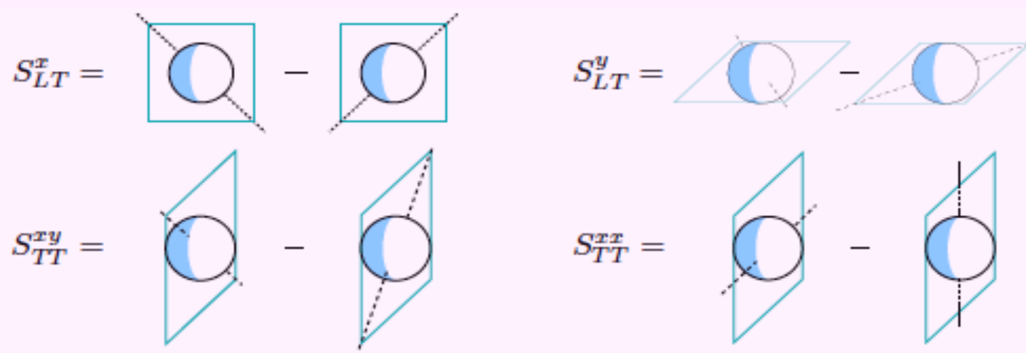
Vector polarization: $S^\mu = (0, \vec{S}_T, \lambda)$ — 3

Tensor polarization: S_{LL}, $S_{LT}^\mu = (0, S_{LT}^x, S_{LT}^y, 0)$, $S_{TT}^{x\mu} = (0, S_{TT}^{xx}, S_{TT}^{xy}, 0)$ — 5

3 + 5 = 8 independent components





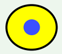
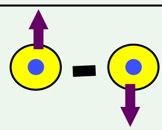
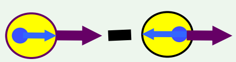
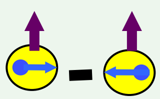
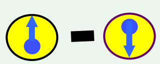
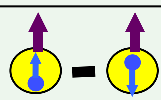
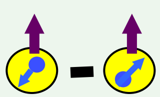
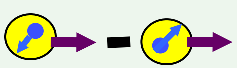


See e.g. A. Bacchetta, & P.J. Mulders, PRD62, 114004 (2000).

Twist-2 TMD FFs defined via quark-quark correlator (spin-1/2)

Leading twist (twist 2)

D, G, H: quark un-, longitudinally, transversely polarized

quark (polarization)	hadron (polarization)	pictorially	TMD FFs (8)	integrated over $k_{F\perp}$	name
U	U		$D_1(z,k_{F\perp})$	$D_1(z)$	number density
	T		$D_{1T}^{\perp}(z,k_{F\perp})$	×	Sivers-type function
L	L		$G_{1L}(z,k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T		$G_{1T}^{\perp}(x,k_{\perp})$	×	
T	U		$H_1^{\perp}(z,k_{F\perp})$	×	Collins function
	$T(//)$		$H_{1T}(z,k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	$T(\perp)$		$H_{1T}^{\perp}(z,k_{F\perp})$		
	L		$H_{1L}^{\perp}(z,k_{F\perp})$	×	

See e.g., K.B. Chen, S.Y. Wei, W.H. Yang, & ZTL, PRD94, 034003 (2016).

Twist-2 TMD FFs defined via quark-quark correlator (spin-1)

Quark pol	Hadron pol	TMD FFs (2+6+10=18)	integrated over $k_{F\perp}$	name
U (D)	U	$D_1(z,k_{F\perp})$	$D_1(z)$	number density
	T	$D_{1T}^{\perp}(z,k_{F\perp})$	×	Sivers-type function
	LL	$D_{1LL}(z,k_{F\perp})$	$D_{1LL}(z)$	spin alignment
	LT	$D_{1LT}^{\perp}(z,k_{F\perp})$	×	
	TT	$D_{1TT}^{\perp}(z,k_{F\perp})$	×	
L (G)	L	$G_{1L}(z,k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T	$G_{1T}^{\perp}(z,k_{F\perp})$	×	
	LT	$G_{1LT}^{\perp}(z,k_{F\perp})$	×	
	TT	$G_{1TT}^{\perp}(z,k_{F\perp})$	×	
T (H)	U	$H_1^{\perp}(z,k_{F\perp})$	×	Collins function
	$T(//)$	$H_{1T}(z,k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	$T(\perp)$	$H_{1T}^{\perp}(z,k_{F\perp})$		
	L	$H_{1L}^{\perp}(z,k_{F\perp})$	×	
	LL	$H_{1LL}^{\perp}(z,k_{F\perp})$	×	
	LT	$H_{1LT}(z,k_{F\perp})$, $H_{1LT}^{\perp}(z,k_{F\perp})$	$H_{1LT}(z)$	
	TT	$H_{1TT}^{\perp}(z,k_{F\perp})$, $H'^{\perp}_{1TT}(z,k_{F\perp})$	×, ×	

See e.g., K.B. Chen, S.Y. Wei, W.H. Yang, & ZTL, PRD94, 034003 (2016).

Twist-3 TMD FFs defined via quark-quark correlator (spin-1)

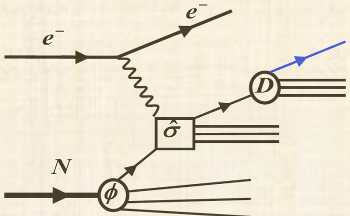
Quark pol	Hadron pol	TMD FFs (4+12+20=36)	integrated over $k_{F\perp}$	name
U (D)	U	$E(z,k_{F\perp}),\ D^{\perp}(z,k_{F\perp})$	$E(z)$, ×	number density
	L	$D_L^{\perp}(z,k_{F\perp})$	×	Sivers-type function
	T	$E_T^{\perp}(z,k_{F\perp}),\ D_T(z,k_{F\perp}),\ D_T^{\perp}(z,k_{F\perp})$	×, $D_T(z)$	
	LL	$E_{LL}(z,k_{F\perp}),\ D_{LL}^{\perp}(z,k_{F\perp})$	$E_{LL}(z)$, ×	spin alignment
	LT	$E_{LT}^{\perp}(z,k_{F\perp}),\ D_{LT}(z,k_{F\perp}),\ D_{LT}^{\perp}(z,k_{F\perp})$	×, $D_{LT}(z)$	
	TT	$E_{TT}^{\perp}(z,k_{F\perp}),\ D_{TT}^{\perp}(z,k_{F\perp}),\ D'^{\perp}_{TT}(z,k_{F\perp})$	×, ×, ×	
L (G)	U	$G^{\perp}(z,k_{F\perp})$	×	
	L	$E_L(z,k_{F\perp}),\ G_L^{\perp}(z,k_{F\perp})$	$E_L(z)$, ×	spin transfer (longitudinal)
	T	$E'^{\perp}_T(z,k_{F\perp}),\ G_T(z,k_{F\perp}),\ G_T^{\perp}(z,k_{F\perp})$	×, $G_T(z)$	
	LL	$G_{LL}^{\perp}(z,k_{F\perp})$	×	
	LT	$E'^{\perp}_{LT}(z,k_{F\perp}),\ G_{LT}(z,k_{F\perp}),\ G_{LT}^{\perp}(z,k_{F\perp})$	×, $G_{LT}(z)$	
	TT	$E'^{\perp}_{TT}(z,k_{F\perp}),\ G_{TT}^{\perp}(z,k_{F\perp}),\ G'^{\perp}_{TT}(z,k_{F\perp})$	×, ×, ×	
T (H)	U	$H(z,k_{F\perp})$	$H(z)$	Collins function
	T(//)	$H_T^{\perp}(z,k_{F\perp})$	×	spin transfer (transverse)
	T(⊥)	$H'^{\perp}_T(z,k_{F\perp})$	×	
	L	$H_L(z,k_{F\perp})$	$H_L(z)$	
	LL	$H_{LL}(z,k_{F\perp})$	$H_{LL}(z)$	
	LT	$H_{LT}^{\perp}(z,k_{F\perp}),\ H'^{\perp}_{LT}(z,k_{F\perp})$	×, ×	
	TT	$H_{TT}^{\perp}(z,k_{F\perp}),\ H'^{\perp}_{TT}(z,k_{F\perp})$	×, ×	

Contents

Access TMDs via semi-inclusive high energy reactions

Semi-inclusive high energy reactions

SiDIS:

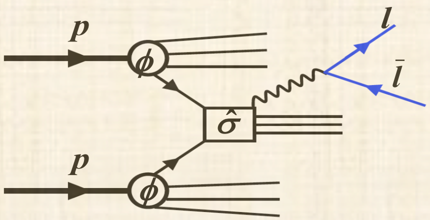


$e+N \to e+h+X$

PDFs: $f_1, f_{1T}^{\perp}, g_{1L}, h_1, h_{1L}^{\perp}, h_{1T}^{\perp} \ldots$

FFs: $D_1, D_{1T}^{\perp}, G_{1L}, G_{1T}^{\perp}, H_1^{\perp}, \ldots$

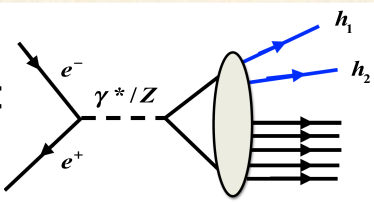
Drell-Yan:



$p+p \to l+\bar{l}+X$

PDFs: $f_1, f_{1T}^{\perp}, g_{1L}, h_1, h_{1L}^{\perp}, h_{1T}^{\perp} \ldots$

e^-e^+ annihilation:

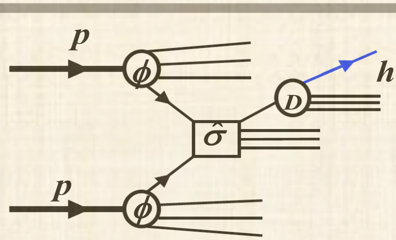


$e^-e^+ \to h_1h_2X$

FFs: $D_1, D_{1T}^{\perp}, G_{1L}, G_{1T}^{\perp}, H_1^{\perp}, \ldots$

Inclusive hadron production in hadron-hadron collisions $h_1+h_2 \to h+X$

$pp \to hX$



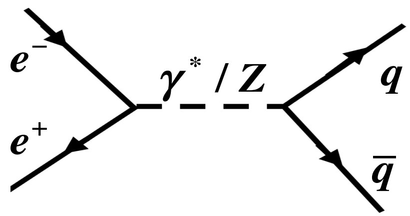
PDFs: $f_1, f_{1T}^{\perp}, g_{1L}, h_1, h_{1L}^{\perp}, h_{1T}^{\perp} \ldots$

FFs: $D_1, D_{1T}^{\perp}, G_{1L}, G_{1T}^{\perp}, H_1^{\perp}, \ldots$

Quark polarization in $e^+e^- \to q\bar{q}$

$e^-e^+ \to Z \to V\pi X$: the best place to study tensor polarization dependent FFs

$e^+e^- \to \gamma^*/Z \to q\bar{q}$

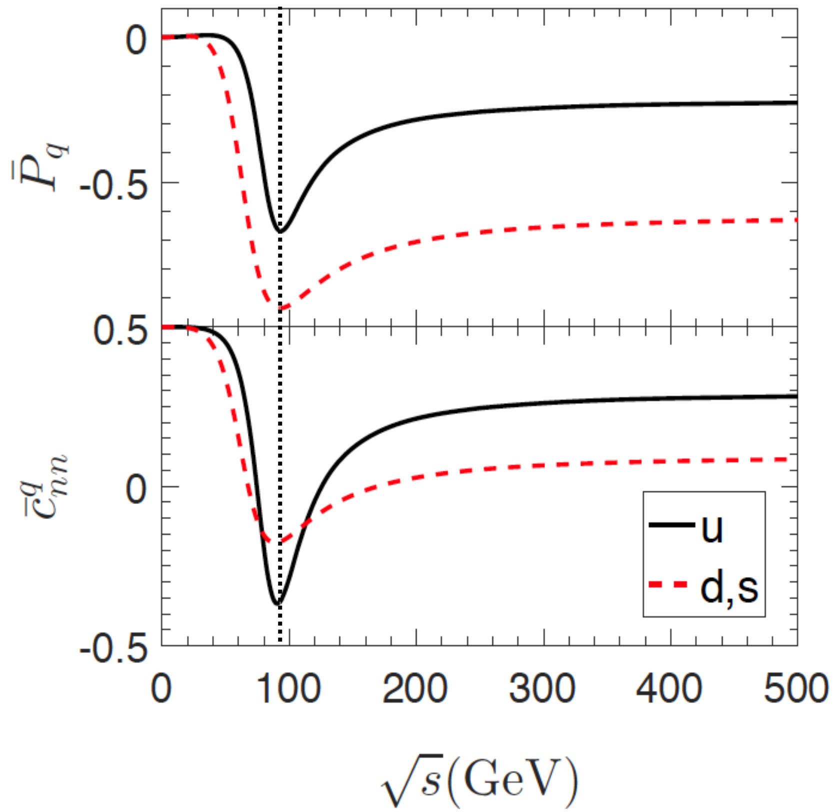


Longitudinal polarization of q or $\bar{q}$:

$$\bar{P}_q = -\frac{\chi c_1^e c_3^q + \chi_{\mathrm{int}}^q c_V^e c_A^q}{e_q^2 + \chi c_1^e c_1^q + \chi_{\mathrm{int}}^q c_V^e c_V^q},$$

Correlation of transverse polarizations of q and $\bar{q}$:

$$\bar{c}_{nn}^q = \frac{e_q^2 + \chi c_1^e c_2^q + \chi_{int}^q c_V^e c_V^q}{2(e_q^2 + \chi c_1^e c_1^q + \chi_{\mathrm{int}}^q c_V^e c_V^q)}$$



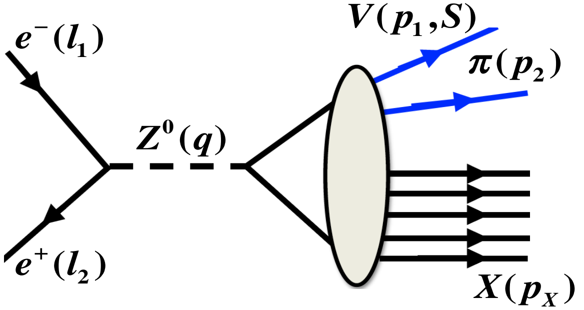
Singly "polarized" process $e^-e^+ \to h_1(\uparrow) + h_2 + X$ ⟹ unpolarized & longitudinally polarized FFs

Doubly "polarized" process $e^-e^+ \to h_1(\uparrow) + h_2(\uparrow) + X$ ⟹ transversely polarized FFs

$e^-e^+ \to Z \to V\pi X$: the best place to study tensor polarization dependent FFs

The general kinematic analysis

$$\frac{2E_1E_2d^6\sigma}{d^3p_1d^3p_2} = \frac{\alpha^2}{sQ^4}\chi L_{\mu\nu}(l_1,l_2)W^{\mu\nu}(q,p_1,S,p_2)$$



The hadronic tensor:

$$W_{\mu\nu}(q,p_1,S,p_2) = W^{S\mu\nu} + iW^{A\mu\nu}$$
$$= \sum_{\sigma,i} W^S_{\sigma i}\, h^{S\mu\nu}_{\sigma i} + \sum_{\sigma,j} \tilde{W}^S_{\sigma j}\, \tilde{h}^{S\mu\nu}_{\sigma j} + i\sum_{\sigma,i} W^A_{\sigma i}\, h^{A\mu\nu}_{\sigma i} + i\sum_{\sigma,j} \tilde{W}^A_{\sigma j}\, \tilde{h}^{A\mu\nu}_{\sigma j}$$

$\sigma = U, V, S_{LL}, S_{LT}, S_{TT}$

polarization

the basic Lorentz tensors (BLTs):

P-even: $\hat{\mathcal{P}} h^{\mu\nu} = h_{\mu\nu}$

P-odd: $\hat{\mathcal{P}} \tilde{h}^{\mu\nu} = -\tilde{h}_{\mu\nu}$

See: K.B. Chen, S.Y. Wei, W.H. Yang, & ZTL, PRD95, 034003 (2016).

General kinematic analysis for $e^+e^- \to V\pi X$

The basic Lorentz tensors (BLTs) for the hadronic tensor

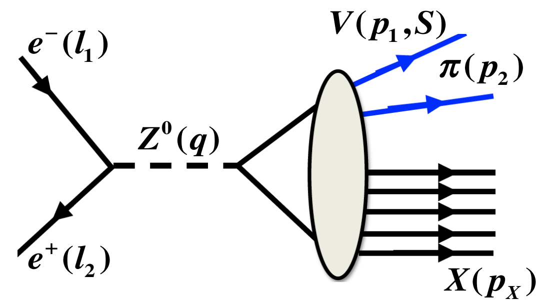
unpolarized:
5+4=9

$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2},\ p_{1q}^\mu p_{1q}^\nu,\ p_{2q}^\mu p_{2q}^\nu,\ p_{1q}^{\{\mu} p_{2q}^{\nu\}} \right\}$$

$$\tilde{h}_{Ui}^{S\mu\nu} = \left\{ \varepsilon^{\{\mu q p_1 p_2}(p_{1q}^{\nu\}},\ p_{2q}^{\nu\}}) \right\}$$

$$h_{U}^{A\mu\nu} = p_{1q}^{[\mu} p_{2q}^{\nu]}$$

$$\tilde{h}_{Ui}^{A\mu\nu} = \left\{ \varepsilon^{\mu\nu q p_1},\ \varepsilon^{\mu\nu q p_2} \right\}$$



(polarization dependent set) = (polarization dependent Lorentz (pseudo)scalar) × (the unpolarized set)

unpolarized

$$\begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix}$$

longitudinal polarization

$$\begin{pmatrix} h_{Li}^{S\mu\nu} \\ \tilde{h}_{Li}^{S\mu\nu} \\ h_{Li}^{A\mu\nu} \\ \tilde{h}_{Li}^{A\mu\nu} \end{pmatrix} = \lambda \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_{U}^{A\mu\nu} \end{pmatrix}$$

transverse polarization

$$\begin{pmatrix} h_{Ti}^{S\mu\nu} \\ \tilde{h}_{Ti}^{S\mu\nu} \\ h_{Ti}^{A\mu\nu} \\ \tilde{h}_{Ti}^{A\mu\nu} \end{pmatrix} = \left\{ (p_2 \cdot S) \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_{U}^{A\mu\nu} \end{pmatrix},\ \varepsilon^{S q p_1 p_2} \begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{U}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix} \right\}$$

General kinematic analysis for $e^+e^- \to V\pi X$

The basic Lorentz tensors (BLTs) for the hadronic tensor (continued)

S_{LL}-dependent part: 5+4=9

$$\begin{pmatrix} h^{S\mu\nu}_{LLi} \\ \tilde{h}^{S\mu\nu}_{LLi} \\ h^{A\mu\nu}_{LLi} \\ \tilde{h}^{A\mu\nu}_{LLi} \end{pmatrix} = S_{LL} \begin{pmatrix} h^{S\mu\nu}_{Ui} \\ \tilde{h}^{S\mu\nu}_{Ui} \\ h^{A\mu\nu}_{U} \\ \tilde{h}^{A\mu\nu}_{Ui} \end{pmatrix}$$

S_{LT}-dependent part: 9+9=18

$$\begin{pmatrix} h^{S\mu\nu}_{LTi} \\ \tilde{h}^{S\mu\nu}_{LTi} \\ h^{A\mu\nu}_{LTi} \\ \tilde{h}^{A\mu\nu}_{LTi} \end{pmatrix} = \left\{ (p_2 \cdot S_{LT}) \begin{pmatrix} h^{S\mu\nu}_{Ui} \\ \tilde{h}^{S\mu\nu}_{Ui} \\ h^{A\mu\nu}_{U} \\ \tilde{h}^{A\mu\nu}_{Ui} \end{pmatrix}, \quad \varepsilon^{S_{LT} q p_1 p_2} \begin{pmatrix} \tilde{h}^{S\mu\nu}_{Ui} \\ h^{S\mu\nu}_{Ui} \\ \tilde{h}^{A\mu\nu}_{Ui} \\ h^{A\mu\nu}_{U} \end{pmatrix} \right\}$$

S_{TT}-dependent part: 9+9=18

$$\begin{pmatrix} h^{S\mu\nu}_{TTi} \\ \tilde{h}^{S\mu\nu}_{TTi} \\ h^{A\mu\nu}_{TTi} \\ \tilde{h}^{A\mu\nu}_{TTi} \end{pmatrix} = \left\{ S^{p_2 p_2}_{TT} \begin{pmatrix} h^{S\mu\nu}_{Ui} \\ \tilde{h}^{S\mu\nu}_{Ui} \\ h^{A\mu\nu}_{U} \\ \tilde{h}^{A\mu\nu}_{Ui} \end{pmatrix}, \quad \varepsilon^{S^{p_2}_{TT} q p_1 p_2} \begin{pmatrix} \tilde{h}^{S\mu\nu}_{Ui} \\ h^{S\mu\nu}_{Ui} \\ \tilde{h}^{A\mu\nu}_{Ui} \\ h^{A\mu\nu}_{U} \end{pmatrix} \right\}$$

See K.B. Chen, S.Y. Wei, W.H. Yang, & ZTL, PRD95, 034003 (2016).

General kinematic analysis for $e^+e^- \to V\pi X$

The cross section in terms of "structure functions" (the Lorentz invariant form)

The unpolarized part:

$$\frac{2E_1E_2 d\sigma^U}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi(\mathcal{F}_U + \tilde{\mathcal{F}}_U)$$

$$y_1 = \frac{2l_1\cdot p_1}{Q^2},\quad y_2 = \frac{2l_1\cdot p_2}{Q^2},\quad \tilde{y} = \frac{\varepsilon^{l_1 q p_1 p_2}}{Q^4}$$

$$\mathcal{F}_U = F_U^0 + y_1 F_U^1 + y_2 F_U^2 + y_1^2 F_U^{11} + y_2^2 F_U^{22} + y_1 y_2 F_U^{12}$$

$$\tilde{\mathcal{F}}_U = \tilde{y}(\tilde{F}_U^0 + y_1 \tilde{F}_U^1 + y_2 \tilde{F}_U^2)$$

All others take the same form, e.g.:

$$\frac{2E_1E_2 d\sigma^V}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi\left[(q\cdot S)(\mathcal{F}_{V1} + \tilde{\mathcal{F}}_{V1}) + (p_2\cdot S)(\mathcal{F}_{V2} + \tilde{\mathcal{F}}_{V2}) + \varepsilon^{Sqp_1p_2}(\mathcal{F}_{V3} + \tilde{\mathcal{F}}_{V3})\right]$$

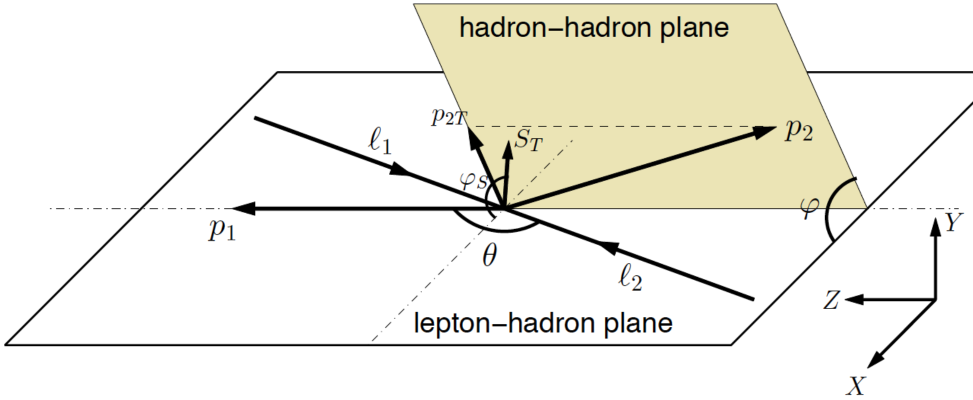
$$\frac{2E_1E_2 d\sigma^{LL}}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi S_{LL}(\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL})$$

$$\mathcal{F}_U \leftrightarrow \mathcal{F}_\sigma \qquad \tilde{\mathcal{F}}_U \leftrightarrow \tilde{\mathcal{F}}_\sigma$$

General kinematic analysis for $e^+e^- \to V\pi X$

The Helicity-Gottfried-Jackson (Helicity-GJ) frame

- c.m. frame of e^+e^-
- p_1 in z-direction
- lepton-hadron plane = oxz plane (e^--V-plane)



$$V:\quad p_1 = (E_1,\ 0,\ 0,\ p_{1z})$$

$$\pi:\quad p_2 = (E_2,\ |\vec{p}_{2T}|\cos\varphi,\ |\vec{p}_{2T}|\sin\varphi,\ p_{2z})$$

$$e^-:\quad l_1 = Q(1,\ \sin\theta,\ 0,\ \cos\theta)/2$$

$$e^+:\quad l_2 = Q(1,\ -\sin\theta,\ 0,\ -\cos\theta)/2$$

$$Z:\quad q = l_1 + l_2 = Q(1,\ 0,\ 0,\ 0)$$

independent variables

$$s = q^2 = Q^2$$

$$\xi_1 = 2q\cdot p_1 / Q^2$$

$$\xi_2 = 2q\cdot p_2 / Q^2$$

$$\theta \quad \text{or} \quad y = 2l_2\cdot p_1 / Q^2$$

$$p_{2T} \equiv |\vec{p}_{2T}|, \quad \varphi$$

General kinematic analysis for $e^+e^- \to V\pi X$

The cross section in Helicity-GJ-frame: unpolarized and longitudinally polarized parts

$$\frac{2E_1E_2 d\sigma^U}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi(\mathcal{F}_U + \tilde{\mathcal{F}}_U)$$

The structure functions: $F_{jxx}^{yy} = F_{jxx}^{yy}(s,\xi_1,\xi_2,p_{2T})$
$\tilde{F}_{jxx}^{yy} = \tilde{F}_{jxx}^{yy}(s,\xi_1,\xi_2,p_{2T})$

$$\mathcal{F}_U = (1+\cos^2\theta)F_{1U} + \sin^2\theta F_{2U} + \cos\theta F_{3U}$$
$$+\cos\varphi[\sin\theta F_{1U}^{\cos\varphi} + \sin 2\theta F_{2U}^{\cos\varphi}]$$
$$+\cos 2\varphi \sin^2\theta F_U^{\cos 2\varphi}$$

1

$\cos\varphi$

$\cos 2\varphi$

parity conserving

$$\tilde{\mathcal{F}}_U = \sin\varphi[\sin\theta \tilde{F}_{1U}^{\sin\varphi} + \sin 2\theta \tilde{F}_{2U}^{\sin\varphi}]$$
$$+\sin 2\varphi \sin^2\theta \tilde{F}_U^{\sin 2\varphi}$$

$\sin\varphi$

$\sin 2\varphi$

parity violating

$\cos\theta, \cos 2\theta, \sin\theta, \sin 2\theta$

$\cos 2\theta, \sin\theta, \sin 2\theta$

General kinematic analysis for $e^+e^- \to V\pi X$

The cross section in Helicity-GJ-frame: unpolarized and longitudinally polarized parts

$$\frac{2E_1E_2 d\sigma^U}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi(\mathcal{F}_U + \tilde{\mathcal{F}}_U)$$

The structure functions: $F^{yy}_{jxx} = F^{yy}_{jxx}(s,\xi_1,\xi_2,p_{2T})$, $\tilde{F}^{yy}_{jxx} = \tilde{F}^{yy}_{jxx}(s,\xi_1,\xi_2,p_{2T})$

$$\mathcal{F}_U = (1+\cos^2\theta)F_{1U} + \sin^2\theta F_{2U} + \cos\theta F_{3U} + \cos\varphi[\sin\theta F^{\cos\varphi}_{1U} + \sin 2\theta F^{\cos\varphi}_{2U}] + \cos 2\varphi \sin^2\theta F^{\cos 2\varphi}_U$$

1, $\cos\varphi$, $\cos 2\varphi$

$$\tilde{\mathcal{F}}_U = \sin\varphi[\sin\theta \tilde{F}^{\sin\varphi}_{1U} + \sin 2\theta \tilde{F}^{\sin\varphi}_{2U}] + \sin 2\varphi \sin^2\theta \tilde{F}^{\sin 2\varphi}_U$$

$\sin\varphi$, $\sin 2\varphi$

$$\frac{2E_1E_2 d\sigma^L}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi\lambda(\mathcal{F}_L + \tilde{\mathcal{F}}_L)$$

$\mathcal{F}_L \leftrightarrow \tilde{\mathcal{F}}_U$, $\tilde{\mathcal{F}}_L \leftrightarrow \mathcal{F}_U$

$F^{xxx}_{jL} \leftrightarrow \tilde{F}^{xxx}_{jU}$, $\tilde{F}^{xxx}_{jL} \leftrightarrow F^{xxx}_{jU}$

$$\mathcal{F}_L = \sin\varphi[\sin\theta F^{\sin\varphi}_{1L} + \sin 2\theta F^{\sin\varphi}_{2L}] + \sin 2\varphi \sin^2\theta F^{\sin 2\varphi}_L$$

$$\tilde{\mathcal{F}}_L = (1+\cos^2\theta)\tilde{F}_{1L} + \sin^2\theta \tilde{F}_{2L} + \cos\theta \tilde{F}_{3L} + \cos\varphi[\sin\theta \tilde{F}^{\cos\varphi}_{1L} + \sin 2\theta \tilde{F}^{\cos\varphi}_{2L}] + \cos 2\varphi \sin^2\theta \tilde{F}^{\cos 2\varphi}_L$$

$$\frac{2E_1E_2 d\sigma^{LL}}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi S_{LL}(\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL})$$

$\mathcal{F}_{LL} \leftrightarrow \mathcal{F}_U$, $\tilde{\mathcal{F}}_{LL} \leftrightarrow \tilde{\mathcal{F}}_U$

$F^{xxx}_{jLL} \leftrightarrow F^{xxx}_{jU}$, $\tilde{F}^{xxx}_{jLL} \leftrightarrow \tilde{F}^{xxx}_{jU}$

$$\mathcal{F}_{LL} = (1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL} + \cos\varphi[\sin\theta F^{\cos\varphi}_{1LL} + \sin 2\theta F^{\cos\varphi}_{2LL}] + \cos 2\varphi \sin^2\theta F^{\cos 2\varphi}_{LL}$$

$$\tilde{\mathcal{F}}_{LL} = \sin\varphi[\sin\theta \tilde{F}^{\sin\varphi}_{1LL} + \sin 2\theta \tilde{F}^{\sin\varphi}_{2LL}] + \sin 2\varphi \sin^2\theta \tilde{F}^{\sin 2\varphi}_{LL}$$

General kinematic analysis for $e^+e^- \to V\pi X$

The cross section in Helicity-GJ-frame: transverse polarization dependent parts

$$\frac{2E_1E_2 d\sigma^T}{d^3p_1 d^3p_2} = \frac{\alpha^2}{s^2}\chi|\vec{S}_T|(\mathcal{F}_T + \tilde{\mathcal{F}}_T)$$

$$\tan\varphi_S = S_T^x / S_T^y$$

$$\begin{aligned}
\mathcal{F}_T = & \sin\varphi_S[\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S}] \\
& + \sin(\varphi_S+\varphi)\sin^2\theta F_T^{\sin(\varphi_S+\varphi)} \\
& + \sin(\varphi_S-\varphi)[(1+\cos^2\theta)F_{1T}^{\sin(\varphi_S-\varphi)} + \sin^2\theta F_{2T}^{\sin(\varphi_S-\varphi)} + \cos\theta F_{3T}^{\sin(\varphi_S-\varphi)}] \\
& + \sin(\varphi_S-2\varphi)[\sin\theta F_{1T}^{\sin(\varphi_S-2\varphi)} + \sin 2\theta F_{2T}^{\sin(\varphi_S-2\varphi)}] \\
& + \sin(\varphi_S-3\varphi)\sin^2\theta F_T^{\sin(\varphi_S-3\varphi)}
\end{aligned}$$

$\sin\varphi_S$
$\sin(\varphi_S+\varphi)$
$\sin(\varphi_S-\varphi)$
$\sin(\varphi_S-2\varphi)$
$\sin(\varphi_S-3\varphi)$

$$\begin{aligned}
\tilde{\mathcal{F}}_T = & \cos\varphi_S[\sin\theta \tilde{F}_{1T}^{\cos\varphi_S} + \sin 2\theta \tilde{F}_{2T}^{\cos\varphi_S}] \\
& + \cos(\varphi_S+\varphi)\sin^2\theta \tilde{F}_T^{\cos(\varphi_S+\varphi)} \\
& + \cos(\varphi_S-\varphi)[(1+\cos^2\theta)\tilde{F}_{1T}^{\cos(\varphi_S-\varphi)} + \sin^2\theta \tilde{F}_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta \tilde{F}_{3T}^{\cos(\varphi_S-\varphi)}] \\
& + \cos(\varphi_S-2\varphi)[\sin\theta \tilde{F}_{1T}^{\cos(\varphi_S-2\varphi)} + \sin 2\theta \tilde{F}_{2T}^{\cos(\varphi_S-2\varphi)}] \\
& + \cos(\varphi_S-3\varphi)\sin^2\theta \tilde{F}_T^{\cos(\varphi_S-3\varphi)}
\end{aligned}$$

$\cos\varphi_S$
$\cos(\varphi_S+\varphi)$
$\cos(\varphi_S-\varphi)$
$\cos(\varphi_S-2\varphi)$
$\cos(\varphi_S-3\varphi)$

General kinematic analysis for $e^+e^- \to V\pi X$

The cross section in Helicity-GJ-frame: transverse polarization dependent parts

$$\frac{2E_1E_2\,d\sigma^{T}}{d^3p_1\,d^3p_2}=\frac{\alpha^2}{s^2}\chi|\vec{S}_T|(\mathcal{F}_T+\tilde{\mathcal{F}}_T)$$

$$\begin{aligned}\mathcal{F}_T &= \sin\varphi_S[\sin\theta F_{1T}^{\sin\varphi_S}+\sin2\theta F_{2T}^{\sin\varphi_S}]\\ &+\sin(\varphi_S+\varphi)\sin^2\theta F_T^{\sin(\varphi_S+\varphi)}\\ &+\sin(\varphi_S-\varphi)[(1+\cos^2\theta)F_{1T}^{\sin(\varphi_S-\varphi)}+\sin^2\theta F_{2T}^{\sin(\varphi_S-\varphi)}+\cos\theta F_{3T}^{\sin(\varphi_S-\varphi)}]\\ &+\sin(\varphi_S-2\varphi)[\sin\theta F_{1T}^{\sin(\varphi_S-2\varphi)}+\sin2\theta F_{2T}^{\sin(\varphi_S-2\varphi)}]\\ &+\sin(\varphi_S-3\varphi)\sin^2\theta F_T^{\sin(\varphi_S-3\varphi)}\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{F}}_T &= \cos\varphi_S[\sin\theta \tilde{F}_{1T}^{\cos\varphi_S}+\sin2\theta \tilde{F}_{2T}^{\cos\varphi_S}]\\ &+\cos(\varphi_S+\varphi)\sin^2\theta \tilde{F}_T^{\cos(\varphi_S+\varphi)}\\ &+\cos(\varphi_S-\varphi)[(1+\cos^2\theta)\tilde{F}_{1T}^{\cos(\varphi_S-\varphi)}+\sin^2\theta \tilde{F}_{2T}^{\cos(\varphi_S-\varphi)}+\cos\theta \tilde{F}_{3T}^{\cos(\varphi_S-\varphi)}]\\ &+\cos(\varphi_S-2\varphi)[\sin\theta \tilde{F}_{1T}^{\cos(\varphi_S-2\varphi)}+\sin2\theta \tilde{F}_{2T}^{\cos(\varphi_S-2\varphi)}]\\ &+\cos(\varphi_S-3\varphi)\sin^2\theta \tilde{F}_T^{\cos(\varphi_S-3\varphi)}\end{aligned}$$

$$\frac{2E_1E_2\,d\sigma^{LT}}{d^3p_1\,d^3p_2}=\frac{\alpha^2}{s^2}\chi|\vec{S}_{LT}|(\mathcal{F}_{LT}+\tilde{\mathcal{F}}_{LT})$$

$$\begin{aligned}\mathcal{F}_{LT} &= \cos\varphi_{LT}[\sin\theta F_{1LT}^{\cos\varphi_{LT}}+\sin2\theta F_{2LT}^{\cos\varphi_{LT}}]\\ &+\cos(\varphi_{LT}+\varphi)\sin^2\theta F_{LT}^{\cos(\varphi_{LT}+\varphi)}\\ &+\cos(\varphi_{LT}-\varphi)[(1+\cos^2\theta)F_{1LT}^{\cos(\varphi_{LT}-\varphi)}+\sin^2\theta F_{2LT}^{\cos(\varphi_{LT}-\varphi)}+\cos\theta F_{3LT}^{\cos(\varphi_{LT}-\varphi)}]\\ &+\cos(\varphi_{LT}-2\varphi)[\sin\theta F_{1LT}^{\cos(\varphi_{LT}-2\varphi)}+\sin2\theta F_{2LT}^{\cos(\varphi_{LT}-2\varphi)}]\\ &+\cos(\varphi_{LT}-3\varphi)\sin^2\theta F_{LT}^{\cos(\varphi_{LT}-3\varphi)}\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{F}}_{LT} &= \sin\varphi_{LT}[\sin\theta \tilde{F}_{1LT}^{\sin\varphi_{LT}}+\sin2\theta \tilde{F}_{2LT}^{\sin\varphi_{LT}}]\\ &+\sin(\varphi_{LT}+\varphi)\sin^2\theta F_{LT}^{\sin(\varphi_{LT}+\varphi)}\\ &+\sin(\varphi_{LT}-\varphi)[(1+\cos^2\theta)\tilde{F}_{1LT}^{\sin(\varphi_{LT}-\varphi)}+\sin^2\theta \tilde{F}_{2LT}^{\sin(\varphi_{LT}-\varphi)}+\cos\theta \tilde{F}_{3LT}^{\sin(\varphi_{LT}-\varphi)}]\\ &+\sin(\varphi_{LT}-2\varphi)[\sin\theta \tilde{F}_{1LT}^{\sin(\varphi_{LT}-2\varphi)}+\sin2\theta \tilde{F}_{2LT}^{\sin(\varphi_{LT}-2\varphi)}]\\ &+\sin(\varphi_{LT}-3\varphi)\sin^2\theta \tilde{F}_{LT}^{\sin(\varphi_{LT}-3\varphi)}\end{aligned}$$

$$\frac{2E_1E_2\,d\sigma^{TT}}{d^3p_1\,d^3p_2}=\frac{\alpha^2}{s^2}\chi|\vec{S}_{TT}|(\mathcal{F}_{TT}+\tilde{\mathcal{F}}_{TT})$$

$$\begin{aligned}\mathcal{F}_{TT} &= \cos2\varphi_{TT}\sin^2\theta F_{TT}^{\cos2\varphi_{TT}}\\ &+\cos(2\varphi_{TT}-\varphi)[\sin\theta F_{1TT}^{\cos(2\varphi_{TT}-\varphi)}+\sin2\theta F_{2TT}^{\cos(2\varphi_{TT}-\varphi)}]\\ &+\cos(2\varphi_{TT}-2\varphi)[(1+\cos^2\theta)F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)}+\sin^2\theta F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)}+\cos\theta F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}]\\ &+\cos(2\varphi_{TT}-3\varphi)[\sin\theta F_{1TT}^{\cos(2\varphi_{TT}-3\varphi)}+\sin2\theta F_{2TT}^{\cos(2\varphi_{TT}-3\varphi)}]\\ &+\cos(2\varphi_{TT}-4\varphi)\sin^2\theta F_{TT}^{\cos(2\varphi_{TT}-4\varphi)}\end{aligned}$$

$$\begin{aligned}\tilde{\mathcal{F}}_{TT} &= \sin2\varphi_{TT}\sin^2\theta \tilde{F}_{TT}^{\sin2\varphi_{TT}}\\ &+\sin(2\varphi_{TT}-\varphi)[\sin\theta \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-\varphi)}+\sin2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-\varphi)}]\\ &+\sin(2\varphi_{TT}-2\varphi)[(1+\cos^2\theta)\tilde{F}_{1TT}^{\sin(2\varphi_{TT}-2\varphi)}+\sin^2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-2\varphi)}+\cos\theta \tilde{F}_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}]\\ &+\sin(2\varphi_{TT}-3\varphi)[\sin\theta \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-3\varphi)}+\sin2\theta \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-3\varphi)}]\\ &+\sin(2\varphi_{TT}-4\varphi)\sin^2\theta \tilde{F}_{TT}^{\sin(2\varphi_{TT}-4\varphi)}\end{aligned}$$

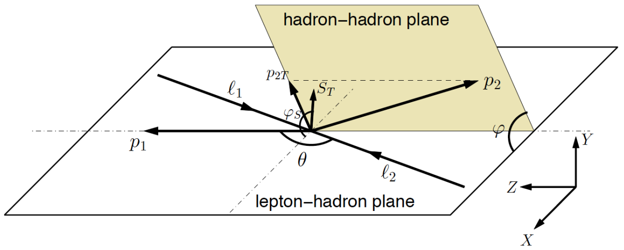
General kinematic analysis for $e^+e^- \to V\pi X$

Hadron polarizations averaged over the azimuthal angle φ

Longitudinal components

$$\langle\lambda\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde{F}_{1L} + \sin^2\theta\tilde{F}_{2L} + \cos\theta\tilde{F}_{3L}]$$

$$\langle S_{LL}\rangle = \frac{1}{2F_{Ut}}[(1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL}]$$



Transversal components

w.r.t. the hadron-hadron plane

$$\langle S_T^n\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)F_{1T}^{\sin(\varphi_S-\varphi)} + \sin^2\theta F_{2T}^{\sin(\varphi_S-\varphi)} + \cos\theta F_{3T}^{\sin(\varphi_S-\varphi)}]$$

$$\langle S_T^t\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde{F}_{1T}^{\cos(\varphi_S-\varphi)} + \sin^2\theta\tilde{F}_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta\tilde{F}_{3T}^{\cos(\varphi_S-\varphi)}]$$

$$\langle S_{LT}^n\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde{F}_{1LT}^{\sin(\varphi_{LT}-\varphi)} + \sin^2\theta\tilde{F}_{2LT}^{\sin(\varphi_{LT}-\varphi)} + \cos\theta\tilde{F}_{3LT}^{\sin(\varphi_{LT}-\varphi)}]$$

$$\langle S_{LT}^t\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)F_{1LT}^{\cos(\varphi_{LT}-\varphi)} + \sin^2\theta F_{2LT}^{\cos(\varphi_{LT}-\varphi)} + \cos\theta F_{3LT}^{\cos(\varphi_{LT}-\varphi)}]$$

$$\langle S_{TT}^{nn}\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)} + \sin^2\theta F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)} + \cos\theta F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}]$$

$$\langle S_{TT}^{nt}\rangle = \frac{2}{3F_{Ut}}[(1+\cos^2\theta)\tilde{F}_{1TT}^{\sin(2\varphi_{TT}-2\varphi)} + \sin^2\theta\tilde{F}_{2TT}^{\sin(2\varphi_{TT}-2\varphi)} + \cos\theta\tilde{F}_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}]$$

w.r.t. the lepton-hadron plane

$$\langle S_T^x\rangle = \frac{2}{3F_{Ut}}[\sin\theta\tilde{F}_{1T}^{\cos\varphi_S} + \sin 2\theta\tilde{F}_{2T}^{\cos\varphi_S}]$$

$$\langle S_T^y\rangle = \frac{2}{3F_{Ut}}[\sin\theta F_{1T}^{\sin\varphi_S} + \sin 2\theta F_{2T}^{\sin\varphi_S}]$$

$$\langle S_{LT}^x\rangle = \frac{2}{3F_{Ut}}[\sin\theta F_{1LT}^{\cos\varphi_{LT}} + \sin 2\theta F_{2LT}^{\cos\varphi_{LT}}]$$

$$\langle S_{LT}^y\rangle = \frac{2}{3F_{Ut}}[\sin\theta\tilde{F}_{1LT}^{\sin\varphi_{LT}} + \sin 2\theta\tilde{F}_{2LT}^{\sin\varphi_{LT}}]$$

$$\langle S_{TT}^{xx}\rangle = \frac{2}{3F_{Ut}}\sin^2\theta F_{1TT}^{\cos 2\varphi_{TT}}$$

$$\langle S_{TT}^{xy}\rangle = \frac{2}{3F_{Ut}}\sin^2\theta\tilde{F}_{1TT}^{\sin 2\varphi_{TT}}$$

Contents

I. Introduction

Collinear expansion, PDFs and FFs defined via quark-quark correlator

II. General kinematic analysis for $e^+e^- \to V\pi X$

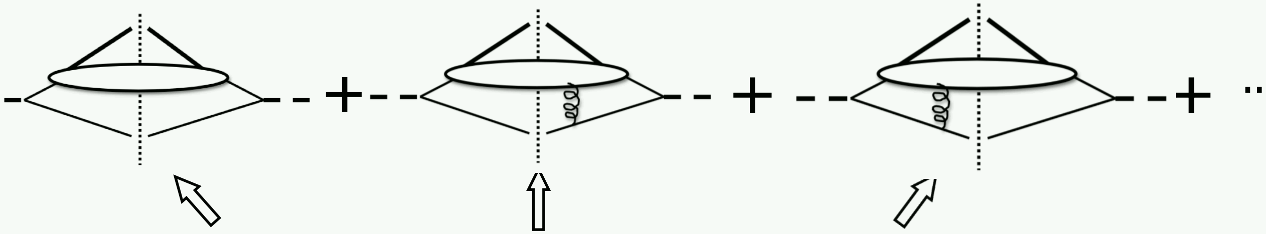
- The basic Lorentz tensors for the hadronic tensor
- Cross section in terms of structure functions

III. Parton model results for $e^+e^- \to V\bar{q}X$ up to twist-4

- Collinear expansion for semi-inclusive $e^+e^- \to h\bar{q}X$
- Structure functions up to twist-4
- Numerical estimation of Lambda polarization and spin alignment of *K**

IV. Summary and outlook

Semi-Inclusive e^+e^--annihilation with "multiple gluon scattering"

$W_{\mu\nu}^{(si)}(q,p,S,k') =$ 

$$W_{\mu\nu}^{(si)}(q,p,S,k') = W_{\mu\nu}^{(0,si)}(q,p,S,k') + W_{\mu\nu}^{(si,1,L)}(q,p,S,k') + W_{\mu\nu}^{(si,1,R)}(q,p,S,k') + \ldots$$

$$W_{\mu\nu}^{(si,0)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(si,0)}(k,k',q)\Pi^{(0)}(k,p,S)\right]$$

$$W_{\mu\nu}^{(si,1,L)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(1,L)\rho}(k_1,k_2,k',q)\Pi_\rho^{(1,L)}(k_1,k_2,p,S)\right]$$

the quark-quark correlator: $\hat{\Pi}^{(0)}(k;p,S) = \sum_X \int d^4z e^{-ikz}\langle 0|\psi(z)|hX\rangle\langle hX|\bar{\psi}(0)|0\rangle$

the quark-gluon-quark correlator:

$$\hat{\Pi}_\rho^{(1,L)}(k_1,k_2;p,S) = \sum_X g\int d^4\xi d^4\eta e^{-ik_1\xi}e^{-i(k_2-k_1)\eta}\langle 0|A_\rho(\eta)\psi(0)|hX\rangle\langle hX|\bar{\psi}(0)|0\rangle$$

the hard parts: $\hat{H}_{\mu\nu}^{(si,0)}(k,k',q) = \Gamma_\mu^q(\not{q}-\not{k})\Gamma_\nu^q(2\pi)^4\delta^4(q-k-k')$ $\qquad \Gamma_\mu^q = \gamma_\mu(c_V^q - c_A^q\gamma_5)$

$$\hat{H}_{\mu\nu}^{(si,1,L)\rho}(k_1,k_2,k',q) = \Gamma_\mu^q(\not{q}-\not{k}_1)\gamma^\rho\frac{\not{k}_2-\not{q}}{(k_2-q)^2-i\varepsilon}\Gamma_\nu^q(2\pi)^4\delta^4(q-k_1-k')$$

Collinear expansion:

See e.g., S.Y. Wei, Y.K. Song and ZTL, PRD89, 014024 (2014).

✪ Expanding the hard part:

$$\hat{H}^{(0)}_{\mu\nu}(z) \equiv \hat{H}^{(0)}_{\mu\nu}(k = \frac{p}{z}, q)$$

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(z) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(z)}{\partial k^\rho} \omega_\rho{}^{\rho'} k_{\rho'} + \ldots\ldots$$

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(z)}{\partial k^\rho} \equiv \left.\frac{\partial \hat{H}^{(0)}_{\mu\nu}(k,q)}{\partial k^\rho}\right|_{k=\frac{p}{z}}$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q) = \hat{H}^{(1,L)\rho}_{\mu\nu}(z_1,z_2) + \frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(z_1,z_2)}{\partial k_1^\sigma} \omega_\sigma{}^{\sigma'} k_{1\sigma'} + \ldots\ldots$$

$$z = p^+ / k^+$$

✪ Decompositing the gluon field: $A_\rho(y) = n \cdot A(y) \frac{p_\rho}{n \cdot p} + \omega_\rho{}^{\rho'} A_{\rho'}(y)$

✪ Using the Ward identities such as,

$$p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(z_1,z_2) = -\frac{z_1 z_2}{z_2 - z_1 - i\varepsilon} \hat{H}^{(0)}_{\mu\nu}(z_1) \qquad p_\rho \hat{H}^{(1,R)\rho}_{\mu\nu}(z_1,z_2) = -\frac{z_1 z_2}{z_2 - z_1 + i\varepsilon} \hat{H}^{(0)}_{\mu\nu}(z_2)$$

to replace the derivatives etc.

✪ Adding all terms with the same hard part together ⟹

Simplified expressions for hadronic tensors

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k_\perp) = \frac{1}{2}\mathrm{Tr}\left[\hat{\Xi}^{(0)}(z_B,k_\perp,p,S)\, h_{\mu\nu}^{(0)}\right]$$

$$\hat{h}_{\mu\nu}^{(0)} = \Gamma_\mu^q \not{n} \Gamma_\nu^q / p^+$$

$$\hat{\Xi}^{(0)}(z,k'_\perp,p,S) = \sum_X \frac{1}{2\pi}\int \frac{p^+ d\xi^-}{2\pi} d^2\xi_\perp e^{ip^+\xi^-/z - ik_\perp\cdot\xi_\perp} \langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle$$

$$\tilde{W}_{\mu\nu}^{(1,L,si)}(q,p,S,k_\perp) = -\frac{\pi}{4q\cdot p}\mathrm{Tr}\left[\hat{\Xi}_{\rho'}^{(1,L)}(z_B,k_\perp,p,S)\, h_{\mu\nu}^{(1)\rho}\omega_\rho^{\ \rho'}\right]$$

$$\hat{h}_{\mu\nu}^{(1)\rho} = \Gamma_\mu^q \bar{\not{n}}\gamma^\rho \not{n}\Gamma_\nu^q$$

$$\hat{\Xi}_\rho^{(1,L)}(z,k_\perp,p,S) = \int \frac{p^+ d\xi^-}{2\pi} d^2\xi_\perp e^{ip^+\xi^-/z - ik_\perp\cdot\xi_\perp} \langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle\langle 0|\mathcal{L}^\dagger(\infty,0)D_\rho(0)\psi(0)|hX\rangle$$

Parton model results for $e^- + e^+ \to Z \to V + \bar{q}(jet) + X$

Structure functions up to twist-4:

$$\kappa_M \equiv \frac{M}{Q} \qquad k_{\perp M} \equiv \frac{|\vec{k}_\perp|}{M}$$

unpolarized

$$zW_{U1} = c_1^e c_1^q (D_1 - 4\kappa_M^2 \,\mathrm{Re}\, D_{-3dd}/z)$$
$$zW_{U3} = 2c_3^e c_3^q (D_1 - 4\kappa_M^2 \,\mathrm{Re}\, D_{-3dd}/z)$$

S_{LL}-dependent:

$$zW_{LL1} = c_1^e c_1^q (D_{1LL} - 4\kappa_M^2 \,\mathrm{Re}\, D_{-3ddLL}/z)$$
$$zW_{LL3} = 2c_3^e c_3^q (D_{1LL} - 4\kappa_M^2 \,\mathrm{Re}\, D_{-3ddLL}/z)$$

S_T-dependent:

$$zW_{T1}^{\sin(\varphi-\varphi_S)} = k_{\perp M} c_1^e c_1^q (D_{1T}^\perp - 4\kappa_M^2 \,\mathrm{Re}\, D_{-3ddT}^\perp/z)$$
$$zW_{T3}^{\sin(\varphi-\varphi_S)} = 2k_{\perp M} c_3^e c_3^q (D_{1T}^\perp - 4\kappa_M^2 \,\mathrm{Re}\, D_{-3ddT}^\perp/z)$$
$$z\tilde{W}_{T1}^{\cos(\varphi-\varphi_S)} = k_{\perp M} c_1^e c_3^q (G_{1T}^\perp - 4\kappa_M^2 \,\mathrm{Re}\, D_{-3ddT}^{\perp 3}/z)$$
$$z\tilde{W}_{T3}^{\cos(\varphi-\varphi_S)} = 2k_{\perp M} c_3^e c_1^q (G_{1T}^\perp - 4\kappa_M^2 \,\mathrm{Re}\, D_{-3ddT}^{\perp 3}/z)$$

totally: 18 non-zeros at twist-2
36 at twsit-3
27 at twist-4

(1) twist-2 & 4 ⟺ even number of φ & φ_S
twist-3 ⟺ odd number of φ & φ_S

(2) Wherever there is twist-2 contribution, there is a twist-4 addendum to it.

Parton model results for $e^- + e^+ \to Z \to V + \bar{q}(jet) + X$

Hadron polarizations (averaged over φ):

Longitudinal components

$$\langle\lambda\rangle^{(0)} = \frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y) G_{1L}}{\sum_q T_0^q(y) D_1}[1+(\alpha_U-\alpha_L)\kappa_M^2]$$

spin transfer, parity violated

$$\langle S_{LL}\rangle = \frac{1}{2}\frac{\sum_q T_0^q(y) D_{1LL}}{\sum_q T_0^q(y) D_1}[1+(\alpha_U-\alpha_{LL})\kappa_M^2]$$

induced polarization, parity conserved

Transverse components w.r.t. hadron-hadron plane

$$\langle S_T^t\rangle = -\frac{2}{3}k_{\perp M}\frac{\sum_q P_q(y) T_0^q(y) G_{1T}^\perp}{\sum_q T_0^q(y) D_1}[1+(\alpha_U-\alpha_T^t)\kappa_M^2]$$

$$\langle S_{LT}^n\rangle = -\frac{2}{3}k_{\perp M}\frac{\sum_q P_q(y) T_0^q(y) G_{1LT}^\perp}{\sum_q T_0^q(y) D_1}[1+(\alpha_U-\alpha_{LT}^n)\kappa_M^2]$$

$$\langle S_{TT}^{nt}\rangle = \frac{2}{3}k_{\perp M}^2\frac{\sum_q P_q(y) T_0^q(y) G_{1TT}^\perp}{\sum_q T_0^q(y) D_1}[1+(\alpha_U-\alpha_{TT}^{nt})\kappa_M^2]$$

"worm-gear" effects, parity violated

$$\langle S_T^n\rangle = \frac{2}{3}k_{\perp M}\frac{\sum_q T_0^q(y) D_{1T}^\perp}{\sum_q T_0^q(y) D_1}[1+(\alpha_U-\alpha_T^n)\kappa_M^2]$$

$$\langle S_{LT}^t\rangle = -\frac{2}{3}k_{\perp M}\frac{\sum_q T_0^q(y) D_{1LT}^\perp}{\sum_q T_0^q(y) D_1}[1+(\alpha_U-\alpha_{LT}^t)\kappa_M^2]$$

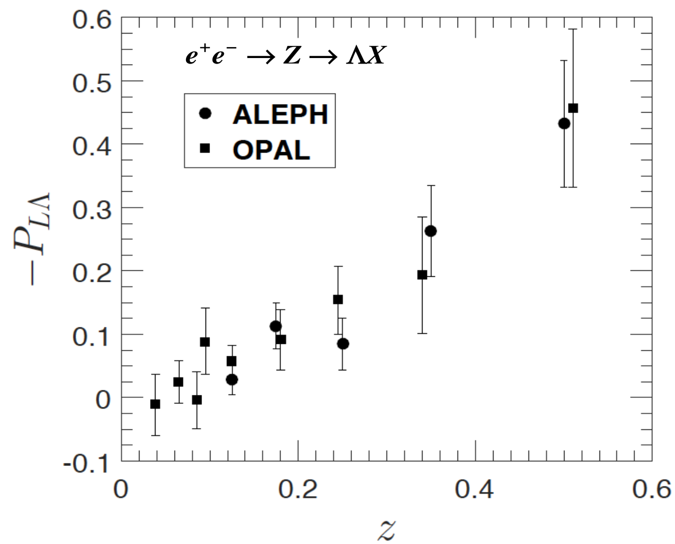
$$\langle S_{TT}^{nn}\rangle = -\frac{2}{3}k_{\perp M}^2\frac{\sum_q T_0^q(y) D_{1TT}^\perp}{\sum_q T_0^q(y) D_1}[1+(\alpha_U-\alpha_{TT}^{nn})\kappa_M^2]$$

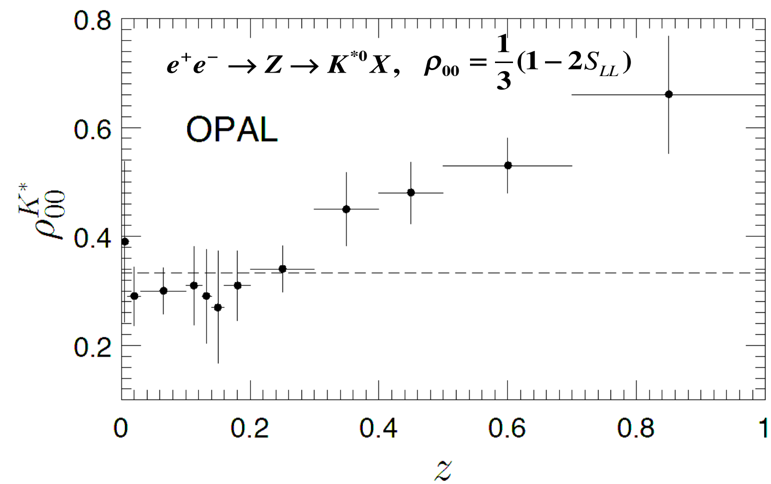
induced polarization, parity conserved

strong energy dependence — very different — weak energy dependence

Numerical estimations for inclusive processes

We have some data from LEP on $e^+e^- \to Z \to hX$





spin transfer, parity violated,
strong energy dependence

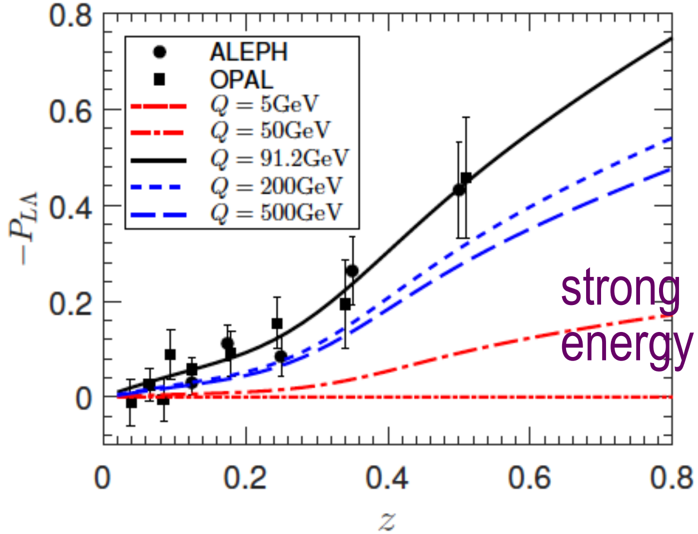
induced polarization, parity conserved
weak energy dependence

⇨ A phenomenological analysis at leading twist with pQCD evolutions of FFs.

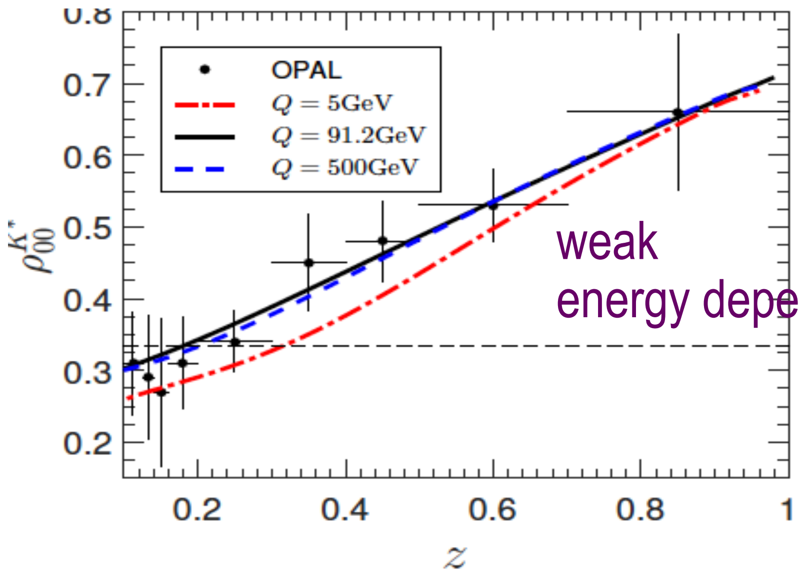
See K.B. Chen, W.H. Yang, Y.J. Zhou, & ZTL, Phys. Rev. D95, 034009 (2017).

Numerical estimations for inclusive processes

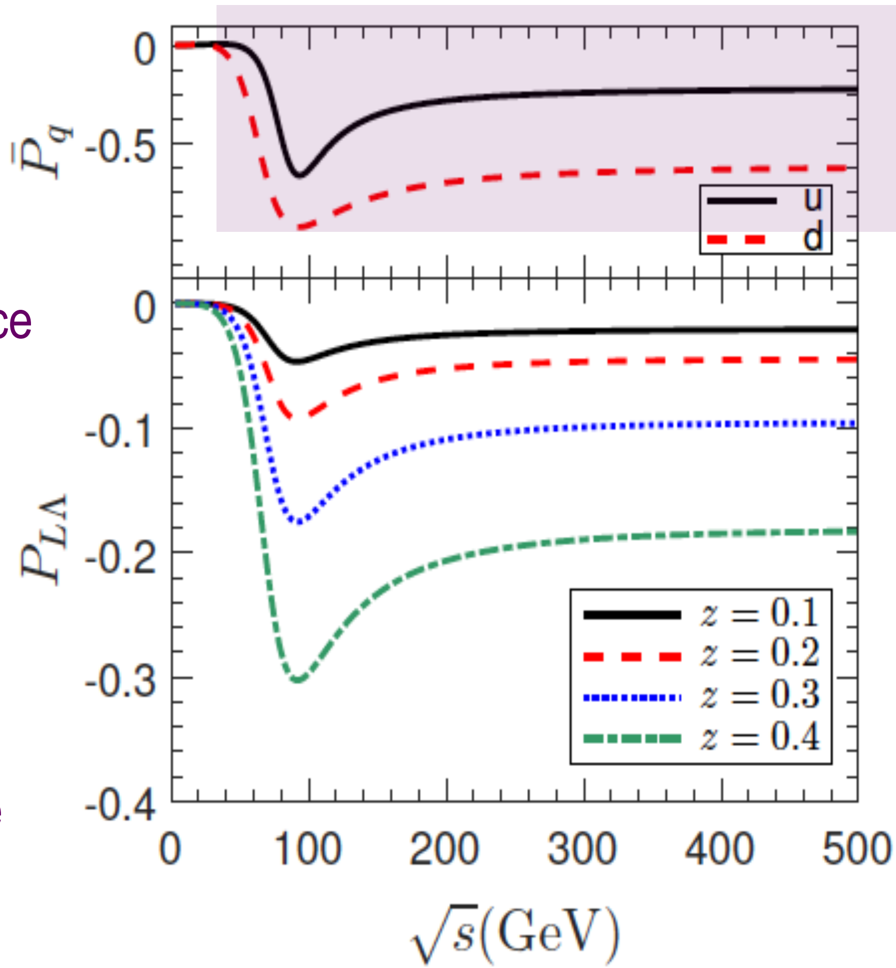
Leading twist and leading order pQCD evolution



strong
energy dependence



weak
energy dependence



Summary and Outlook

A systematic study of $e^+e^- \to V\pi X$, the best place to study spin dependent FFs

★A general and complete kinematic analysis for $e^+e^- \to V\pi X$

- There are in total 81 structure functions or basic Lorentz tensors:

$$\begin{pmatrix}\text{polarization dependent set}\end{pmatrix} = \begin{pmatrix}\text{polarization dependent} \\ \text{Lorentz (pseudo)scalar}\end{pmatrix} \times \begin{pmatrix}\text{the unpolarized set}\end{pmatrix}$$

★Complete parton model results up to twist-4 at LO pQCD for $e^+e^- \to V\bar{q}X$

- 18 non-vanishing structure functions at twist-2, 36 at twist-3, 27 at twist-4:

  (1) twist 2 & 4 $\Longleftrightarrow$ even number of φ & φ_S; twist-3 $\Longleftrightarrow$ odd number of φ & φ_S

  (2) wherever there is a twist-2 contribution, there is a twist-4 addendum to it.

- Hadron polarizations:

  (1) P_q-dependent, parity violating, strong energy dependence, e.g. $P_{L\Lambda}$;

  (2) P_q-independent, parity conserving, weak energy dependence, e.g. ρ_{00}^V

Thank you for your attention!