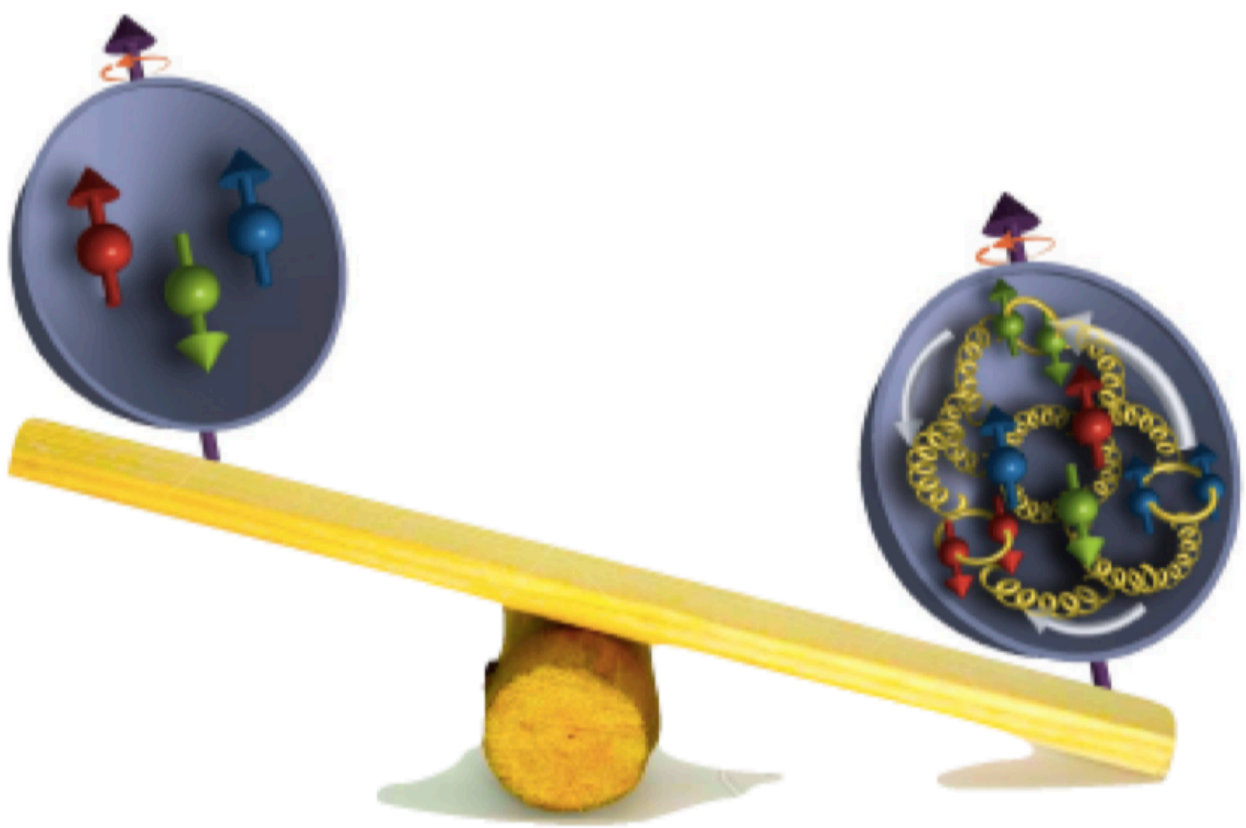
Nucleon from Lattice QCD

Yi-Bo Yang

Jun. 23rd, 2019, Changsha

How does the mass and spin of nucleon arise?



- Only 30% of the proton **spin** comes from the quark helicity, based on the experiments
- The gluon helicity and orbital angular momentum can have significant contribution to the. Proton spin

- The Higgs boson only provide ~2-5 MeV for the u/d quark mass at 2GeV MS-bar,
- But the **mass** of the proton is 938.272046(21) MeV.

http://www.latticeaverages.org

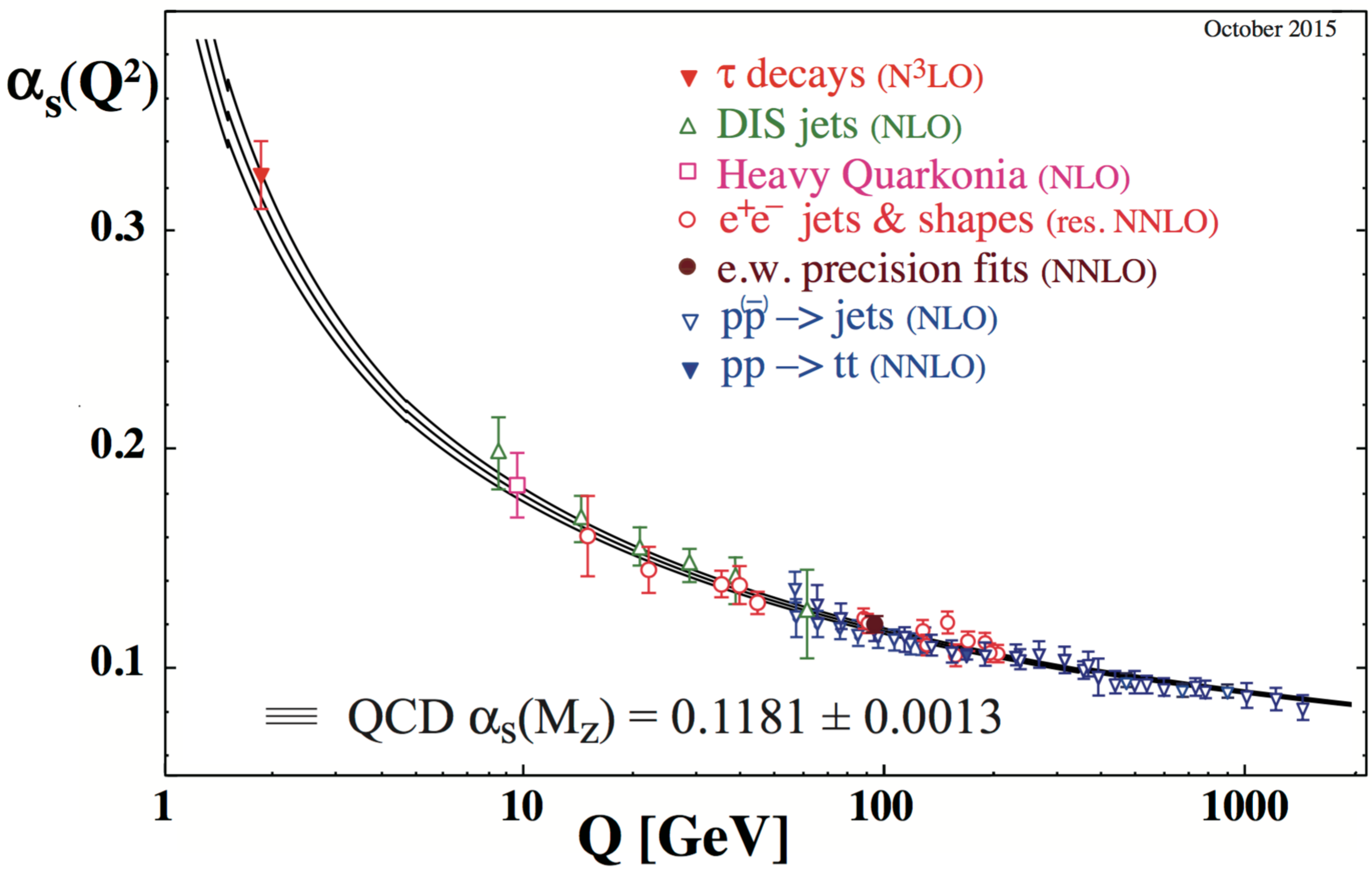
In order to take advantage of the full potential of the EIC, theory program to match its scope is essential, comparing both continuum and lattice QCD

— **The US National Academy of Science assessment of U.S.-based Electron-Ion Collider Science**

https://www.nap.edu/read/25171/chapter/9#92

Quantum Chromodynamics

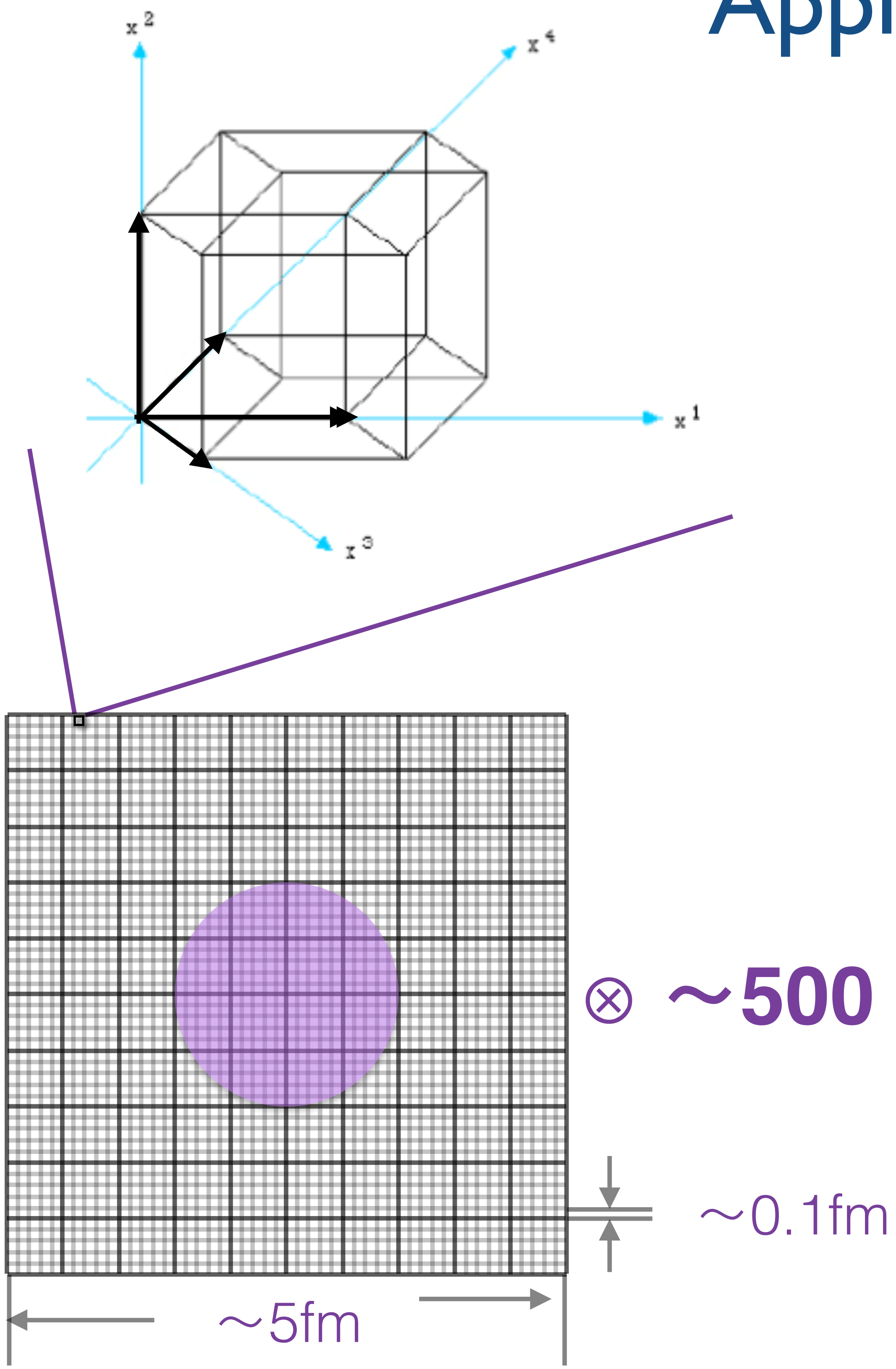
The perturbative calculation in the continuum



- **Very precise** at Q~100 GeV, the experiment confirmed the predictions;
- Limited application for nucleon properties due to **bad** convergence at Q≤2 GeV.
- The other possibilities?

Lattice QCD

Application of the statistics in QCD



- Discretization on the 4-D integral of the space-time
- Monte Carlo sampling in the integral of the gauge phase space

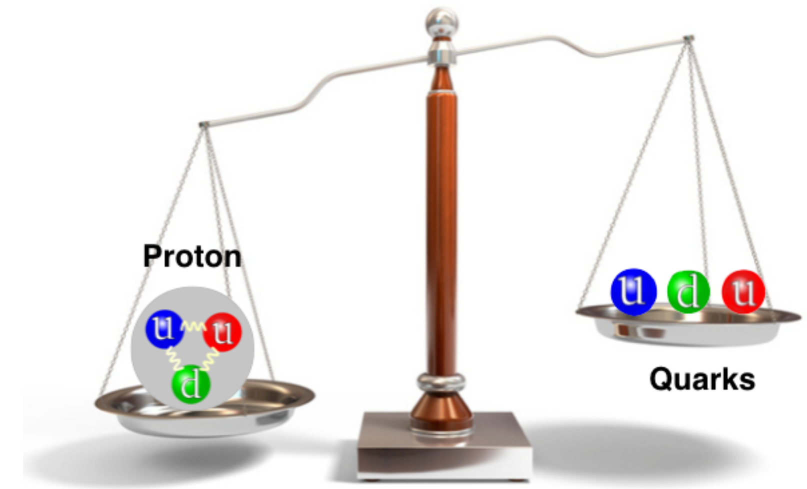
- A state-of-the-art calculation requires：

- 4-D lattice with ~5x proton size;
- With a lattice spacing ~1/10 proton size;
- ~500 samples with 1000 measurement each.
- Use 4k-8k cores per sample and handle different samples in parallel.

What can Lattice QCD help on the nucleon physics?

- **How does the mass of the nucleon arise?**
- How does the spin of the nucleon arise?
- How does the gluon in the nucleon look like?

Two sum rules of the nucleon mass



$$M = -\langle T_{44}\rangle = \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \frac{1}{4}\langle H_a\rangle,$$

$$\frac{1}{4}M = -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \frac{1}{4}\langle H_a\rangle$$

Xiangdong Ji, PRL 74 (1995) 1071-1074

With

$$H_m = \sum_{u,d,s\cdots}\int d^3x\, m\overline{\psi}\psi,$$

The quark mass

Gauge Invariant and scale independent combinations.

The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$

The glue anomaly

$$H_g^a = \int d^3x\, \frac{-\beta(g)}{g}(E^2+B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots}\int d^3x\, \gamma_m m\overline{\psi}\psi.$$

The quark mass anomaly

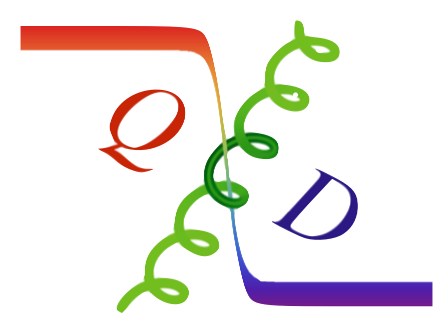
The total energy

$$H_E = \sum_{u,d,s\ldots}\int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2-E^2),$$

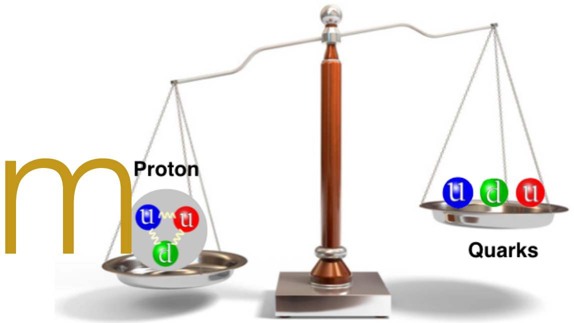
The glue field energy



YBY, et. al., χQCD collaboration, PRD 91 (2015) 074516

Proton mass decomposition

The quark mass term



Then we have

$$\begin{aligned} M &= -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle \\ &= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle, \\ \frac{1}{4}M &= -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \frac{1}{4}\langle H_a\rangle, \end{aligned}$$

in the rest frame.

$$H_m = \sum_{u,d,s\cdots} \int d^3x\, m\bar{\psi}\psi,$$

The quark mass

- Renormalization scheme/scale independent in continuum; also in discrete case when the chiral fermion is used.

$\sigma_{\pi N} = \langle H_m(u)+H_m(d)\rangle = 45.9(7.4)(2.8)$ MeV

$f_s^N M_N = \langle H_m(s)\rangle = 40.2(11.7)(3.5)$ MeV

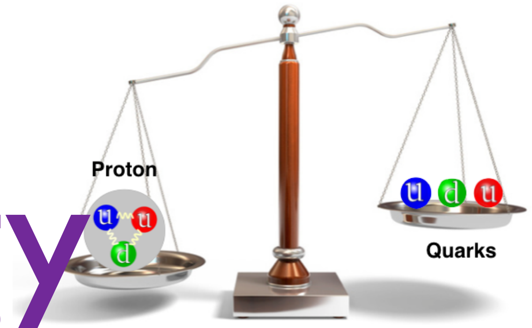
$\langle \mathbf{H_m(u,d,s)}\rangle \,/\, \mathbf{M_N} = \mathbf{9(2)\%}$

The best lattice result free of the systematic uncertainty from the explicit chiral symmetry breaking

YBY, et. al., χQCD collaboration, PRD 91 (2015) 074516

Proton mass decomposition

The quark/gluon energy



Then we have

$$
\begin{aligned}
M &= -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle \\
&= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle, \\
\frac{1}{4}M &= -\langle \hat{T}_{44}\rangle = \frac{1}{4}\langle H_m\rangle + \frac{1}{4}\langle H_a\rangle
\end{aligned}
$$

- The quark/glue energy can be deduced from the momentum fraction,

$$
\begin{aligned}
\langle H_E\rangle &= \frac{3}{4}\langle x\rangle_q M - \frac{3}{4}\langle H_m\rangle \\
\langle H_q\rangle &= \frac{3}{4}\langle x\rangle_q M + \frac{1}{4}\langle H_m\rangle
\end{aligned}
\qquad
\langle H_g\rangle = \frac{3}{4}\langle x\rangle_g M.
$$

- The renormalization of the quark momentum fraction is much more trivial, which is just mixed with the glue one.

- **It is more straightforward to obtain the quark/glue momentum fraction first, and convert it to the quark/glue energy.**

The total energy

$$
H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,
$$

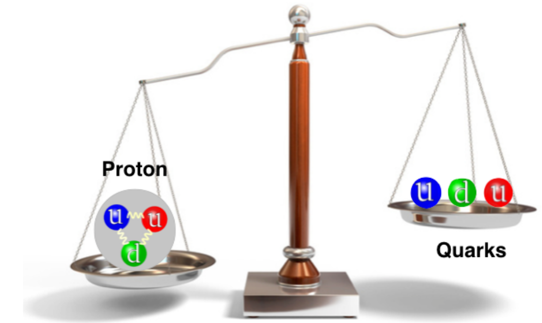
The quark energy

$$
H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),
$$

The glue field energy

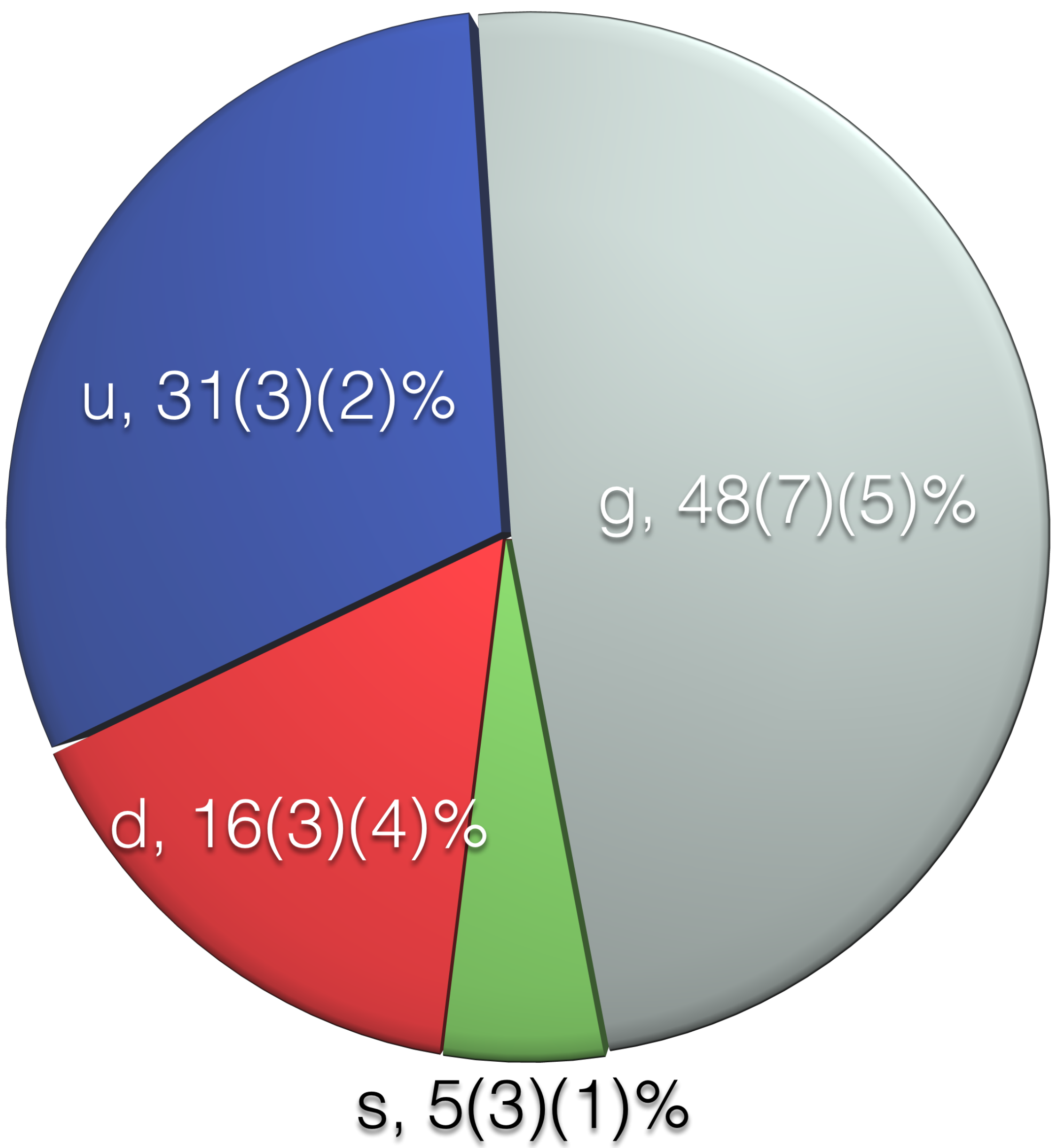
Proton mass decomposition

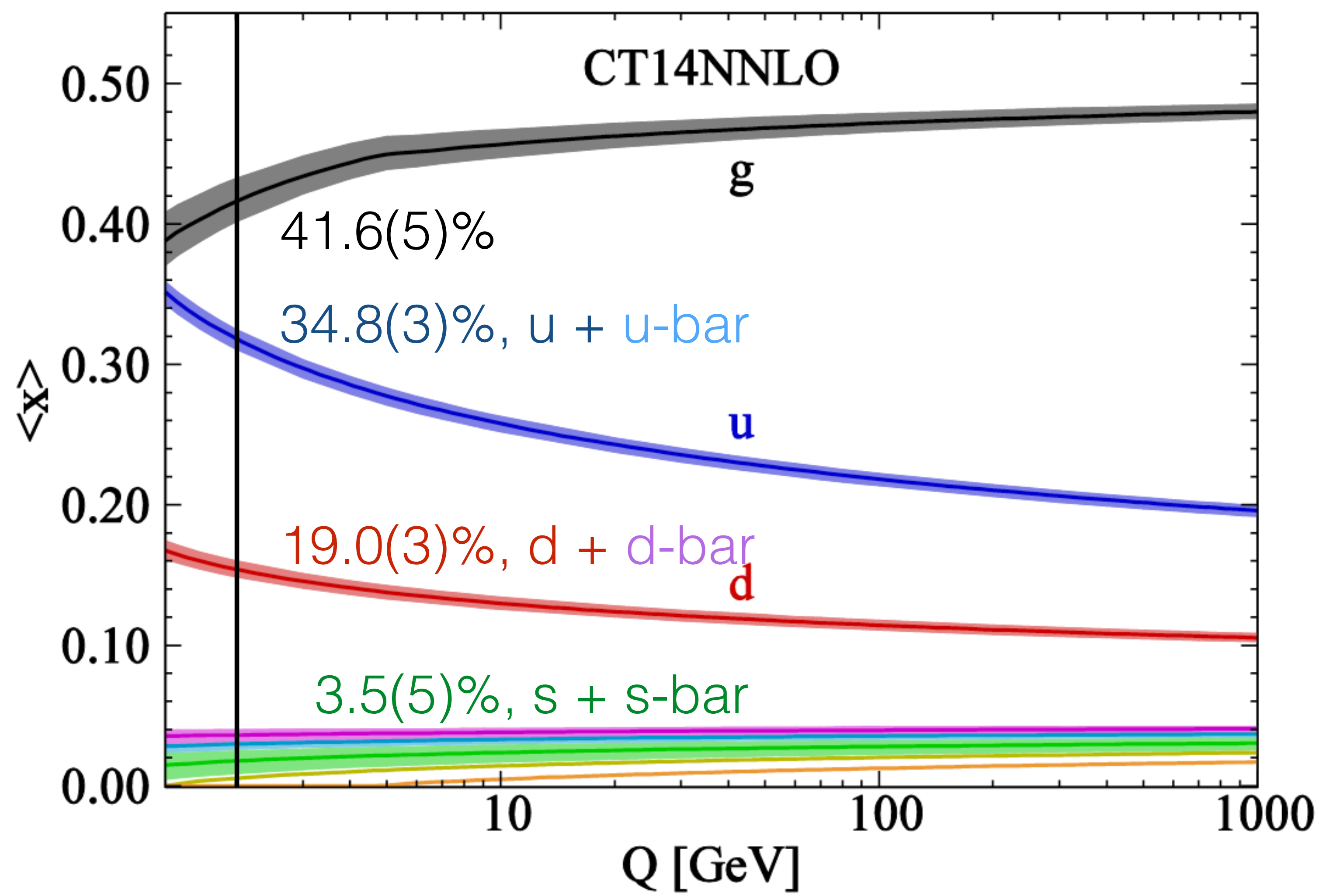
Comparing the momentum fractions



YBY, J. Liang, et. al., χQCD collaboration, PRL121(2018) 212001, 1808.08677
ViewPoint and Editor’s suggestion

from the experiment

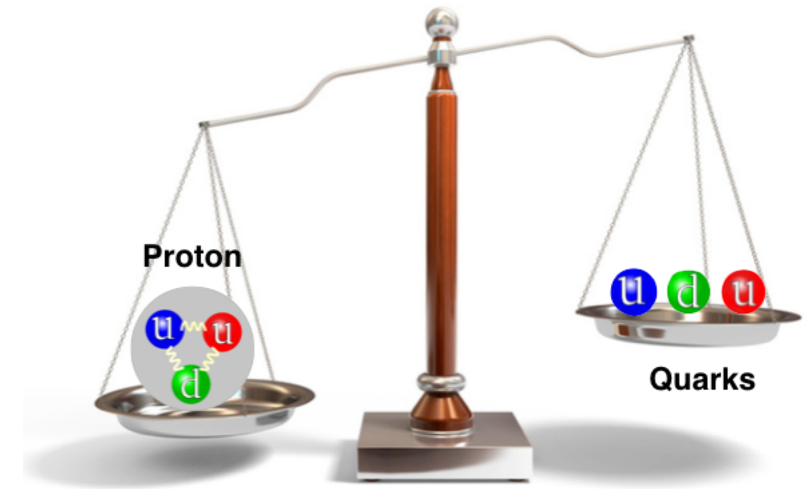




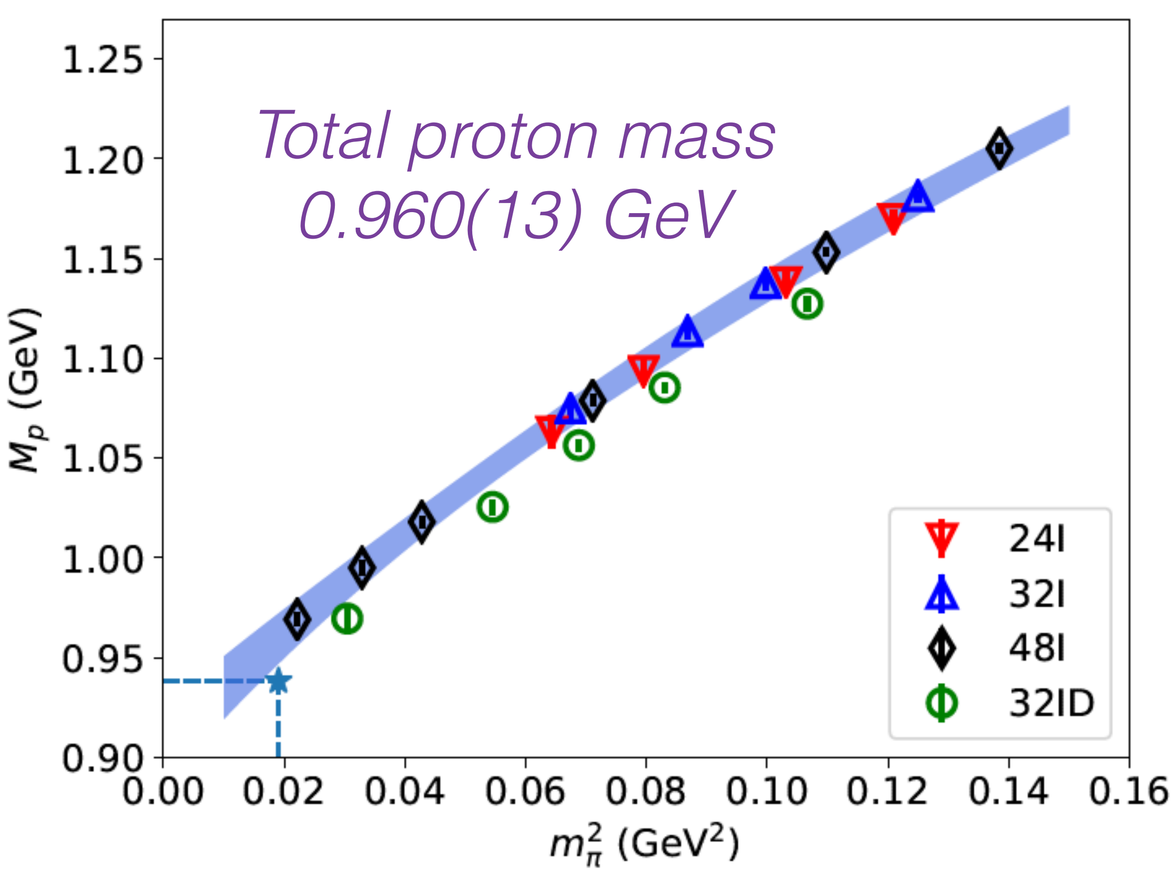
S. Dulat et al, Phys. Rev. D 93 (2016), 033006

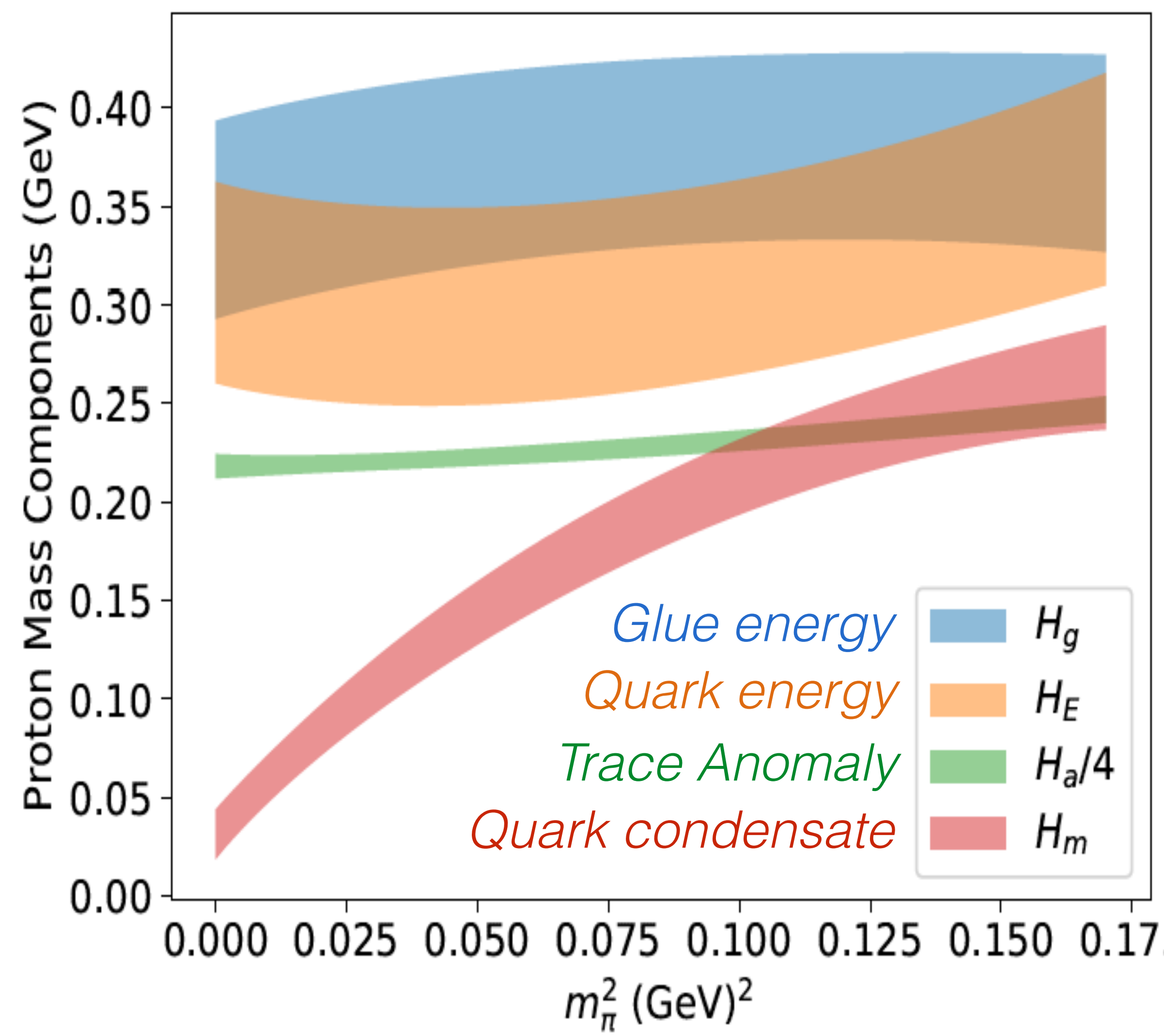
- Direct calculation of the quark/glue momentum fraction with non-perturbative renormalization and normalization.

- Trace anomaly contribution deduced by the direct calculation of the quark scalar condensate in nucleon, based on the sum rule



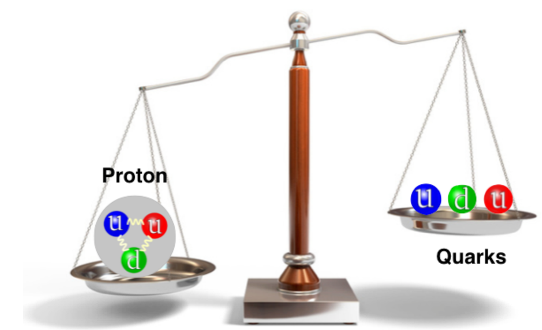
$$\frac{1}{4}M = -\langle \hat{T}_{44} \rangle = \frac{1}{4}\langle H_m \rangle + \frac{1}{4}\langle H_a \rangle$$





YBY, J. Liang, et. al., χQCD collaboration, PRL121(2018) 212001, 1808.08677
ViewPoint and Editor's suggestion

Proton mass decomposition



Trace anomaly?

Under the dimensional regularization, the QCD EMT can be decomposed into the trace part and the traceless part:

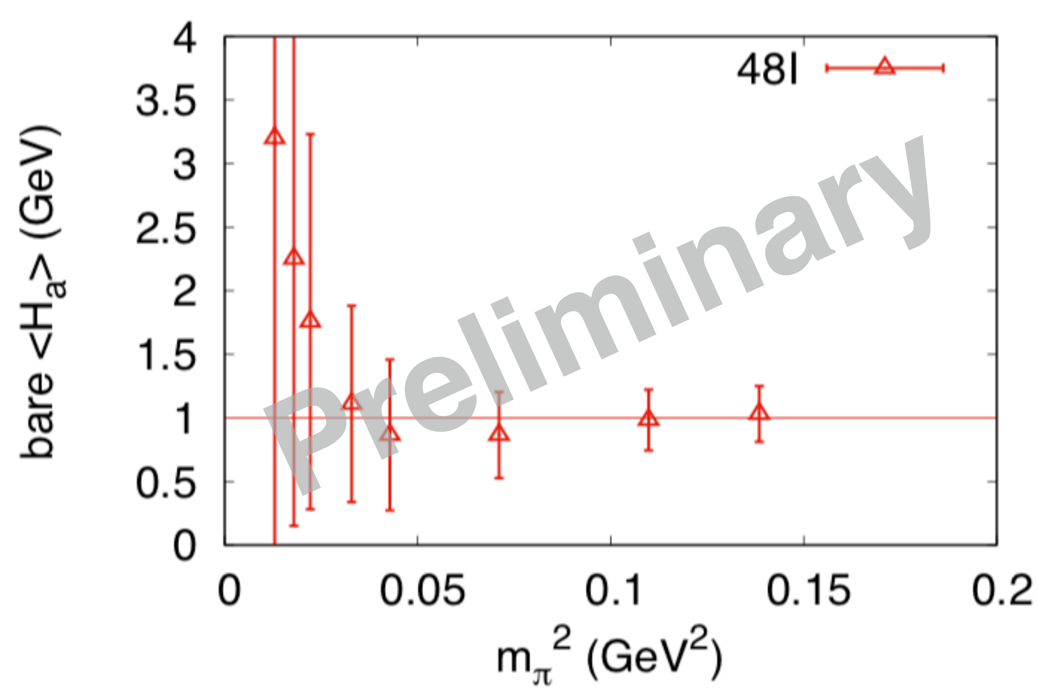
$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}F^2$$

$$= \left(T_{\mu\nu} - \frac{g_{\mu\nu}}{d}T^{\alpha}_{\alpha}\right) + \frac{g_{\mu\nu}}{d}T^{\alpha}_{\alpha} \equiv \bar{T}_{\mu\nu} + \hat{T}_{\mu\nu},$$

$$T^{\alpha}_{\alpha} = m\bar{\psi}\psi - 2\epsilon\frac{F^2}{4} + \mathcal{O}(\epsilon^2) = (1+\gamma_m^R)\left(m\bar{\psi}\psi\right)_R + \frac{\beta_R}{2g_R}F_R^2 .$$

Under the other regularizations:

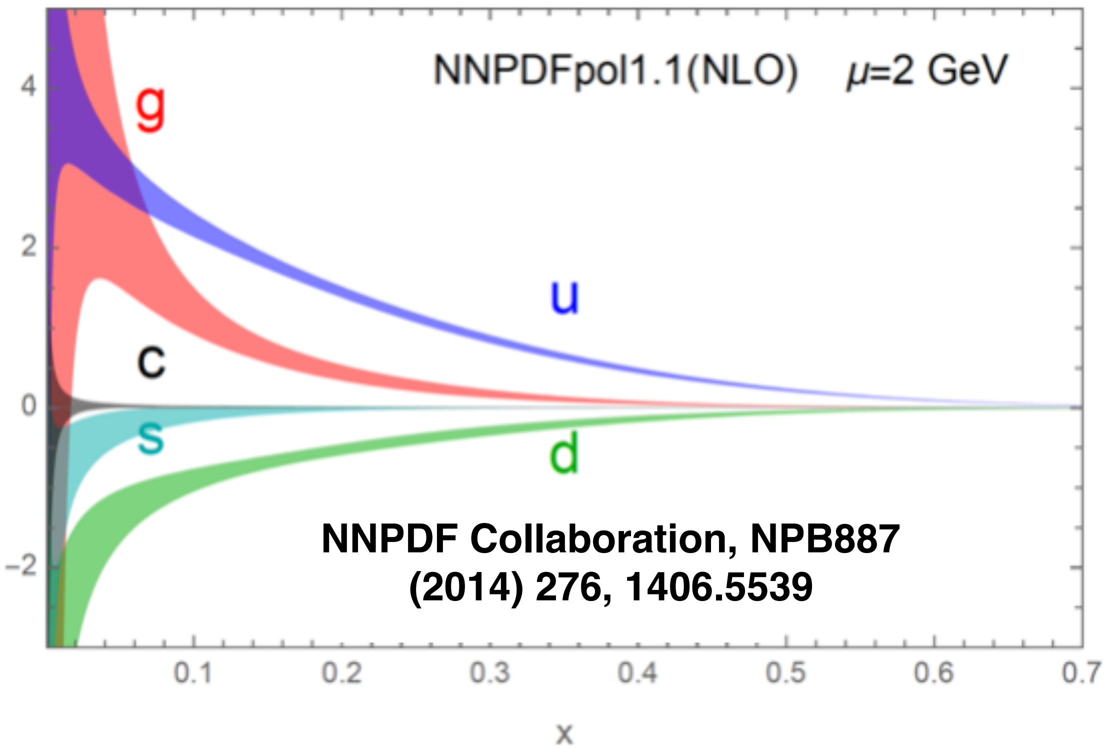
- The regularization introduces some tiny trace on the traceless QCD EMT;
- Such a trace vanishes when the UV cutoff vanishes;
- But it become finite when the quantum corrections are included.

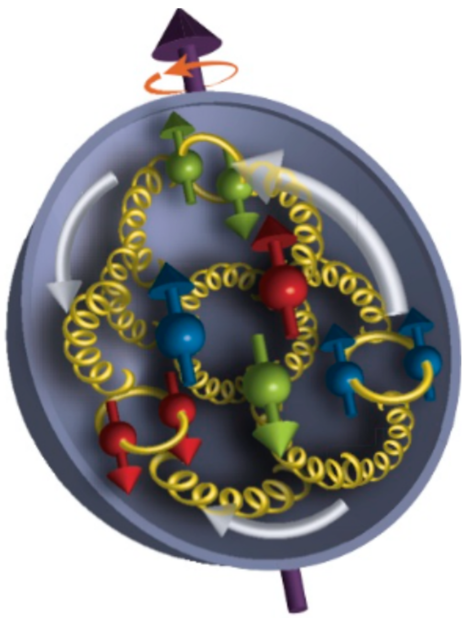


What can Lattice QCD help on the nucleon physics?

- How does the mass of the nucleon arise?
- **How does the spin of the nucleon arise?**
- How does the gluon in the nucleon look like?

How does the spin of nucleon arise?





*Spin/helicity (**u**,**d**,**s**…,g): the **integration** of the polarized parton distribution function (PDF)*

$$\Delta q = \int_0^1 dx \Delta q(x)$$

$$\Delta G = \int_0^1 dx \Delta g(x)$$

- The quark model (agrees with the lattice simulation at heavy quark limit): $\Delta u \rightarrow 4/3$, $\Delta d \rightarrow -1/3$, $\Delta s \rightarrow 0$, $\Delta g \rightarrow 0$;

As shown in the introduction of X. Zhang and B. Ma, PRD85(2012)114048

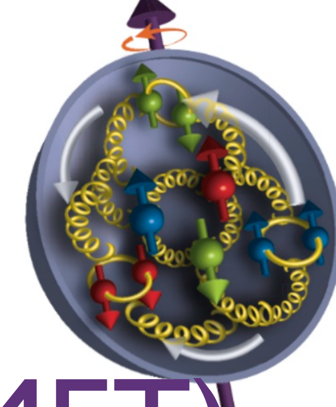
- **The phenomenology fit of quark distribution based on Exp.:** $\Delta u \sim 0.8$, $\Delta d \sim -0.4$, $\Delta s \sim -0.1$, $\Delta g \sim 0.4$;

  E. R. Nocera, et.al. (NNPDF), NPB887(2014)276, 1406.5539

- **The experiments** are quite **different** from the **naive theoretical understanding,** just becomes the **quark masses** in the real world are actually **light!**

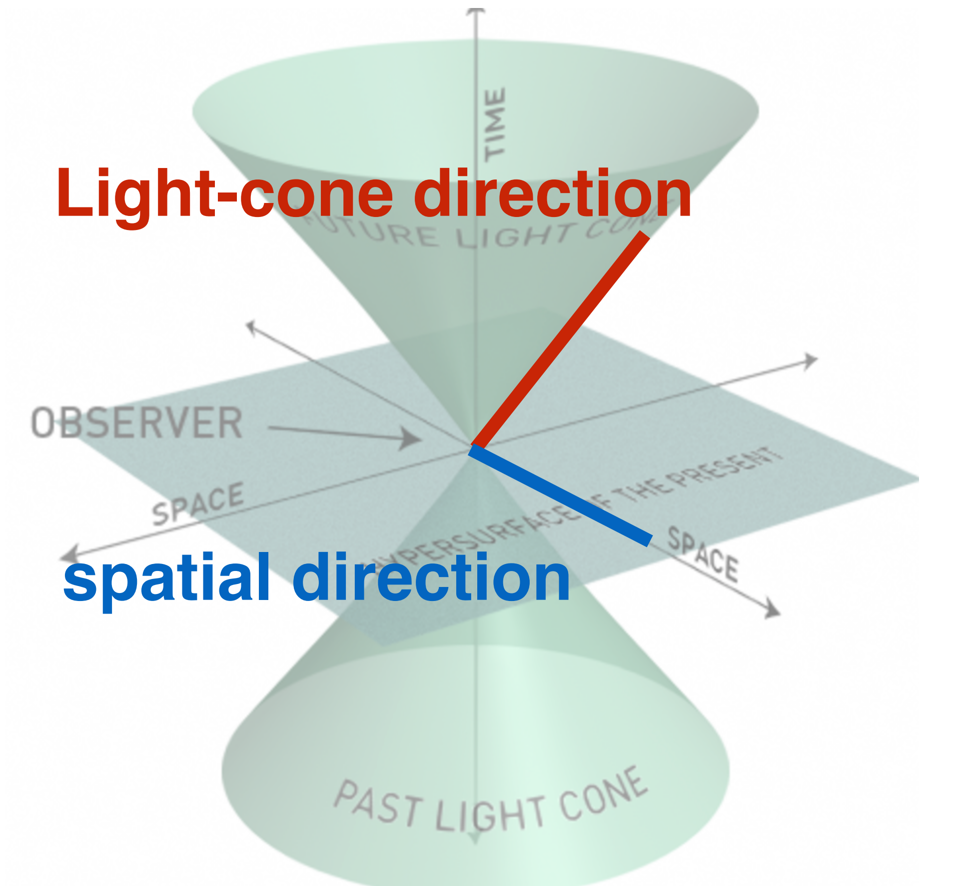
YBY, χQCD collaboration, 1904.04138

Glue spin



Large momentum effective theory (LaMET)

$$O_{\Delta_G} = \left[\vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla} A^{+,b})\mathcal{L}^{ba}(\xi^-, 0)) \right]^z = \vec{E}_{LC} \times \vec{A}_{LC},\ A^+_{LC} = 0$$



When nucleon is boosted:

- The Coulomb and Temporal gauge conditions become the light-cone one.
- **Glue spin** below becomes **glue helicity,** the integration of the glue polarized PDF, **at tree level**.

$$\boxed{O_{S^c_G} = \vec{E}^c \times \vec{A}^c,\ \partial_i A^c_i = 0}$$ ***Coulomb gauge***

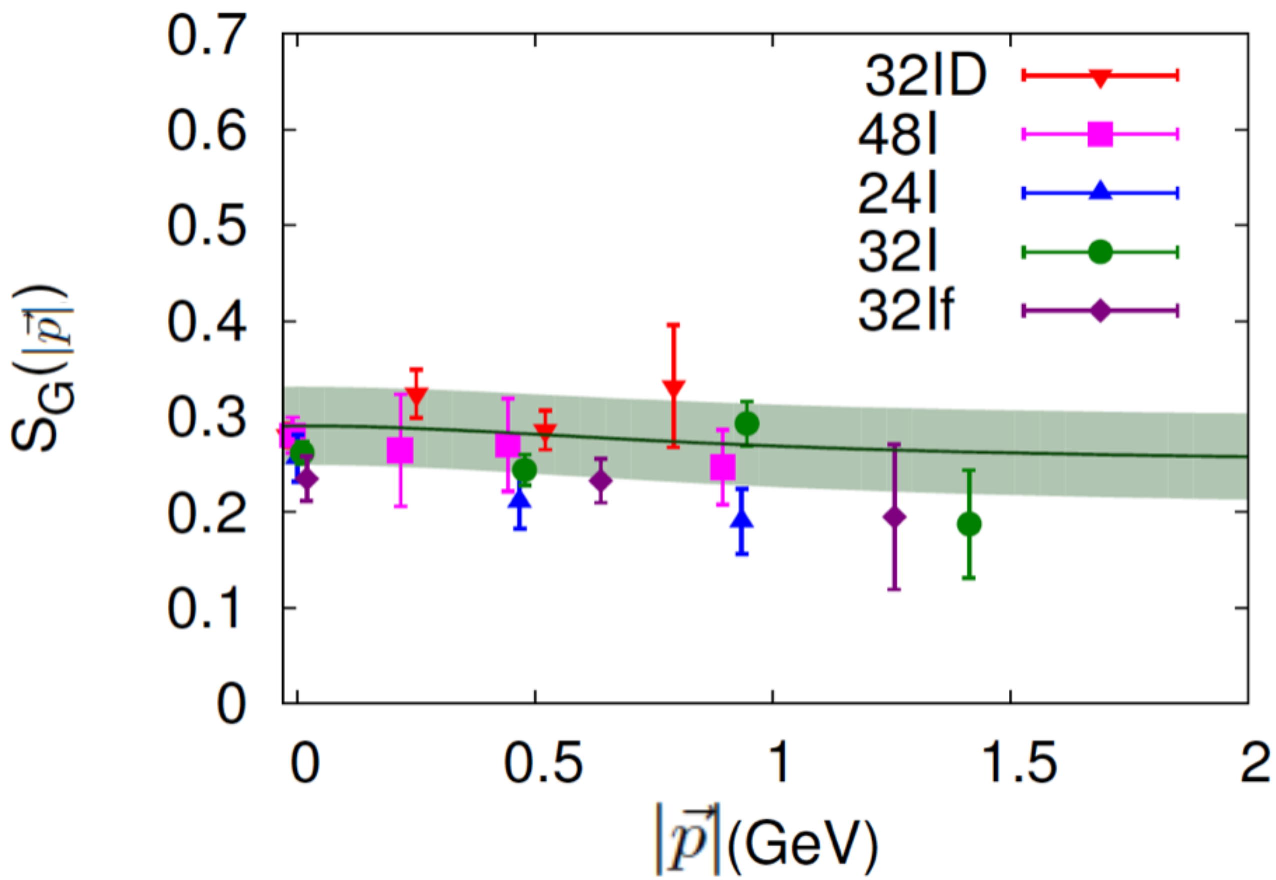
or $$\boxed{O_{S^t_G} = \vec{E}^t \times \vec{A}^t,\ A^t_0 = 0}$$ ***Temporal gauge***

X. Ji, J. Zhang, and Y. Zhao, PRL111 (2013)112002, 1304.6708

Glue spin

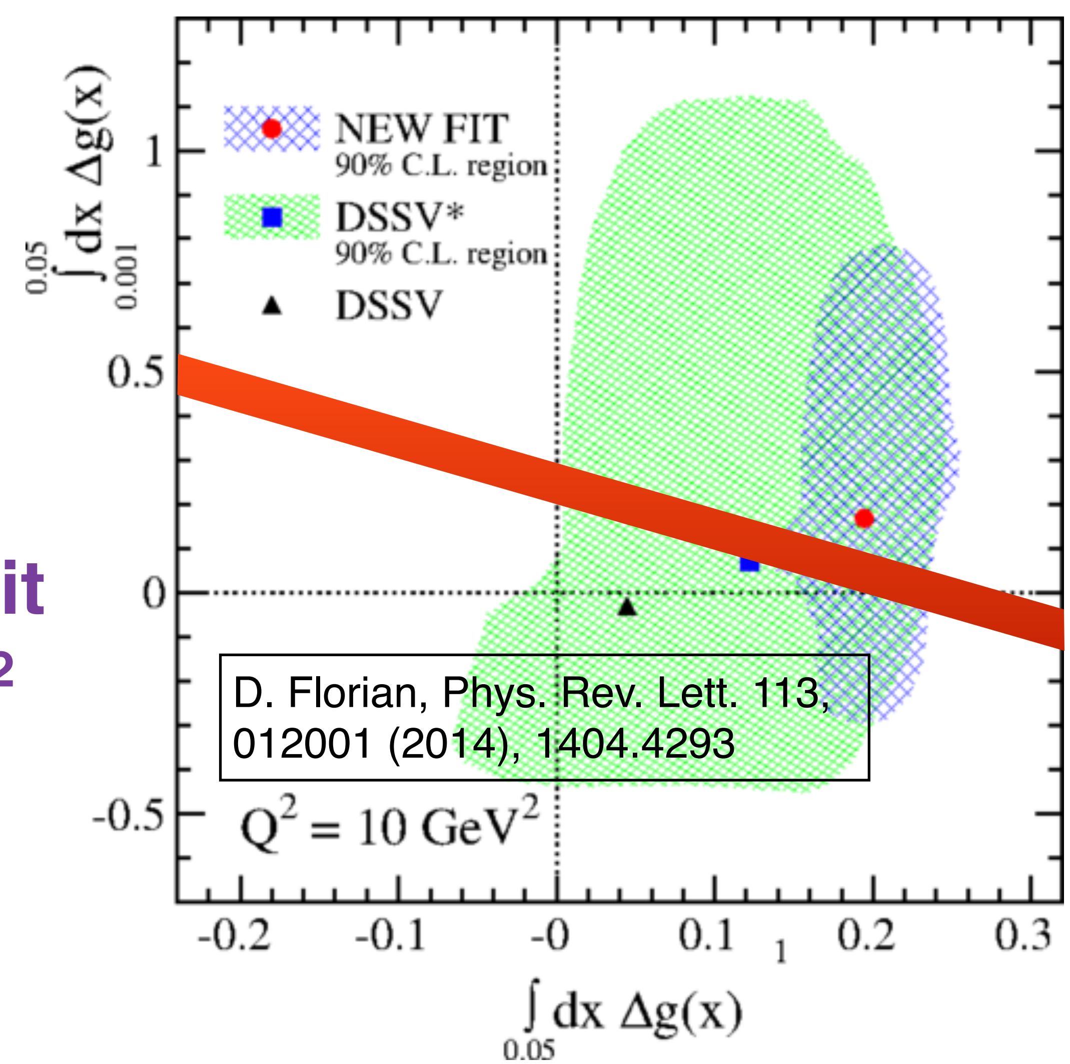
YBY, R. Sufian, et. al., χQCD collaboration, PRL118(2017)042001, 1609.05937
ViewPoint and Editor's suggestion

Results



Neglect the matching and apply an empirical form to fit the data,

$$\int_{0.001}^{0.05} \mathrm{d}x \Delta g(x) + \int_{0.05}^{1} \mathrm{d}x \Delta g(x) \simeq S_g$$

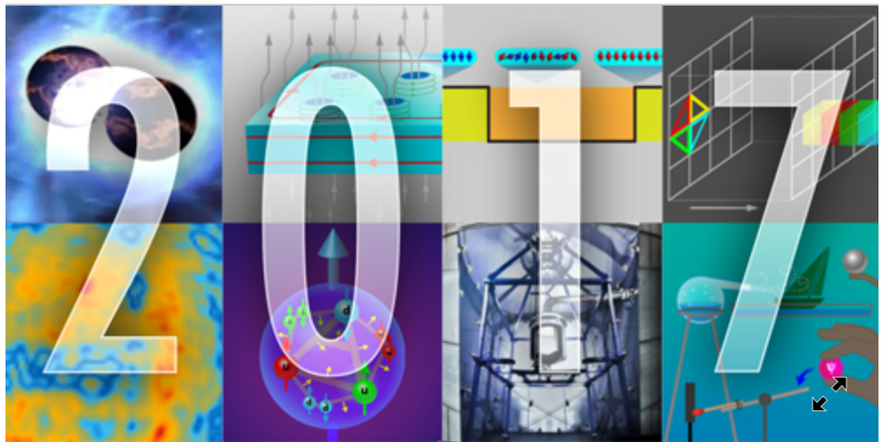


the glue spin at the large momentum limit for the renormalized value at μ^2=10GeV^2 will be

S_G=0.251(47)(16).

One of **eight**

YBY, R. Sufian, et. al., χQCD collaboration, PRL118(2017)042001, 1609.05937
ViewPoint and Editor's suggestion



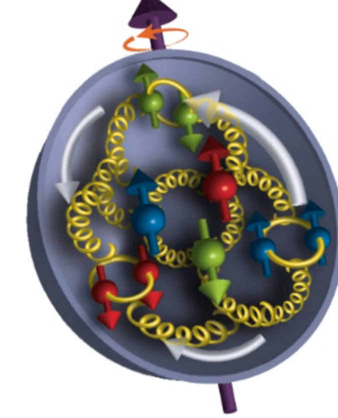
APS Highlights of 2017

https://physics.aps.org/articles/v10/137

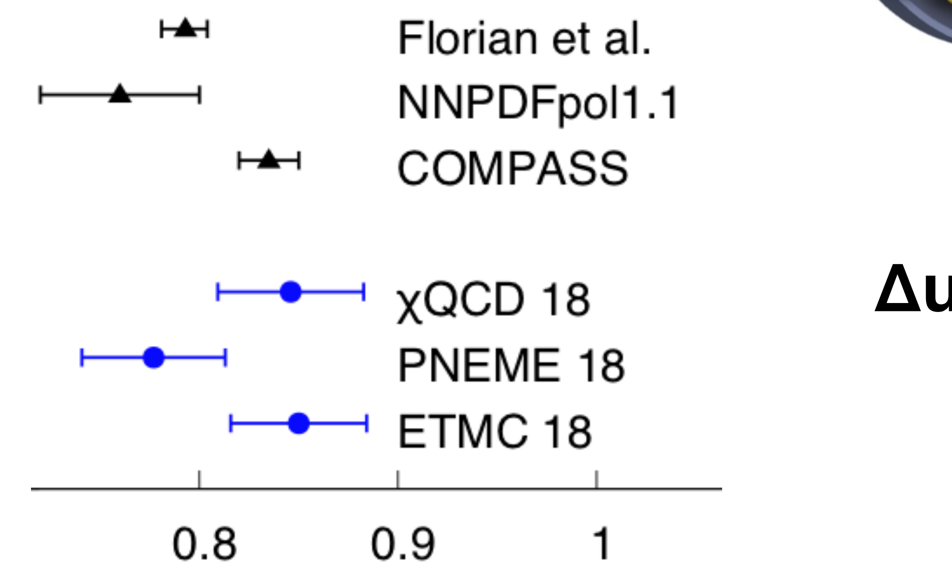
Gluons Provide Half of the Proton's Spin

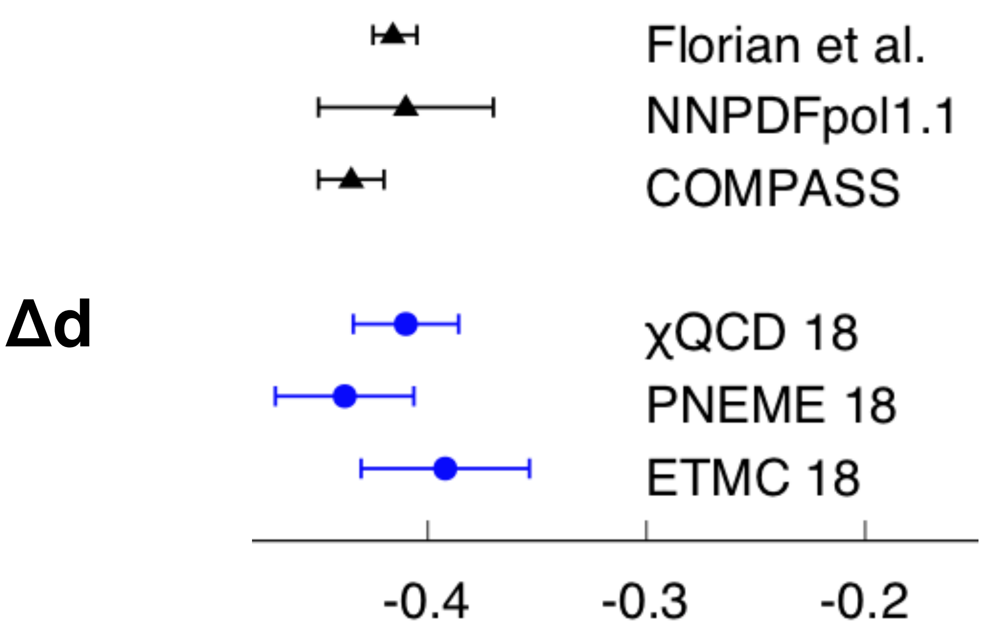
The gluons that bind quarks together in nucleons provide a considerable chunk of the proton's total spin. That was the conclusion reached by Yi-Bo Yang from the University of Kentucky, Lexington, and colleagues (see Viewpoint: **Spinning Gluons in the Proton**). By running state-of-the-art computer simulations of quark-gluon dynamics on a so-called spacetime lattice, the researchers found that 50% of the proton's spin comes from its gluons. The result is in agreement with recent experiments and shows how such lattice simulations can now accurately predict an increasing number of particle properties. The simulations also indicate that, despite being substantial, the gluon spin contribution is too small to play a major part in "screening" the quark spin contribution—which according to experiments is only 30%—through a quantum effect called the axial anomaly. The remaining 20% of the proton spin is thought to come from the orbital angular momentum of quarks and gluons.

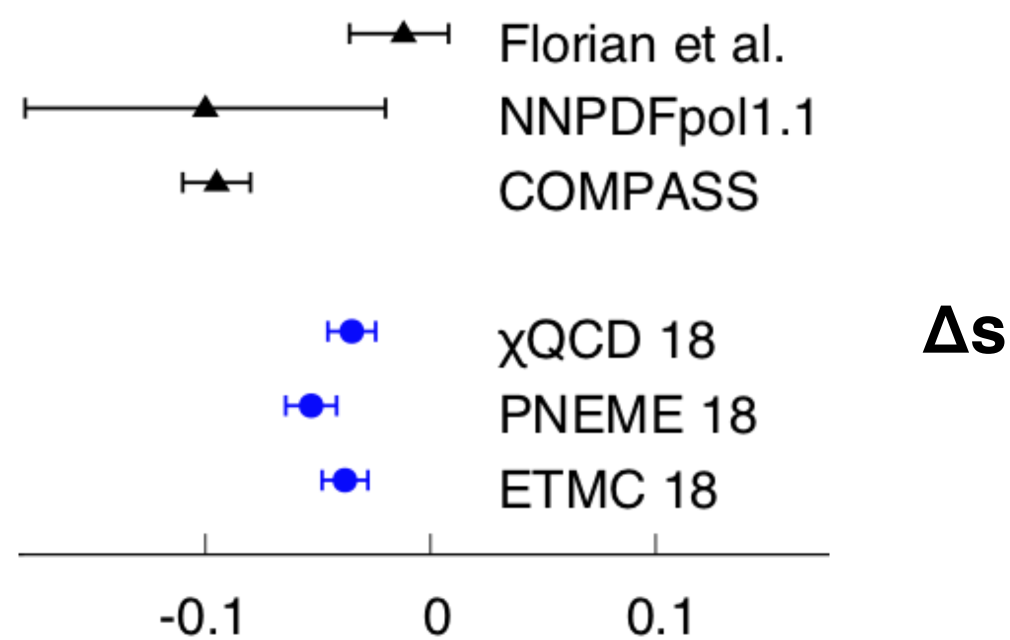
Quark spin: Present summary



	N_f	Disc	m_π	FV	Ren	ESC
χQCD 18	2+1	o	o	★	★	★
PNDME 18	2+1+1	o	★	★	■	★
ETMC 18	2+1+1	■	★	★	★	★







D. Florian, et.al, PRL113 (2014) 012001, 1404.4293

E. Nocera, et.al, NNPDF Collaboration, NPB887 (2014) 276, 1406.5539

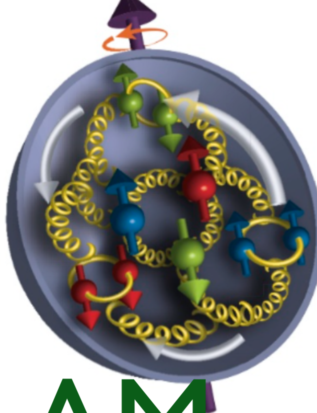
C. Adoph, et.al, COMPASS Collaboration, PLB753 (2016) 18, 1503.08935

J Liang, **YBY**, et.al, χQCD Collaboration, PRD98 (2018) 074505, 1806.08366

H. Lin, et.al, PNEME Collaboration, PRD98 (2018) 094512, 1806.10604

C. Alexandrou, et.al, ETMC collaboration, PRL119 (2017) 142002, 1706.02973, with 2018 updates

Proton spin

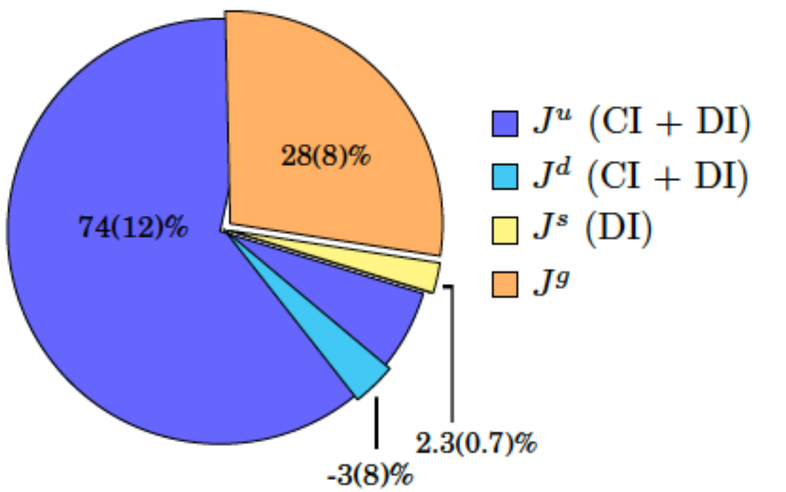


Lattice result of Ji AM

Quark **AM** — Glue **AM**

$$\vec{J} = \underbrace{\int d^3x \, \tfrac{1}{2} \overline{\psi} \vec{\gamma} \gamma^5 \psi + \int d^3x \psi^\dagger \{\vec{x} \times (i\vec{D})\} \psi}_{\text{Quark AM}} + \underbrace{\int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}}_{\text{Glue AM}}$$

Quenched result

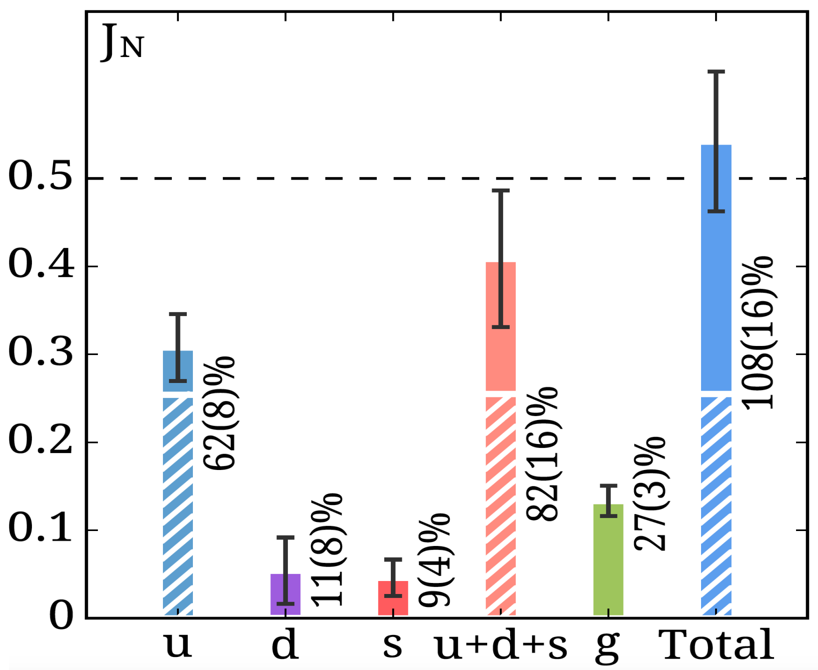


M. Deka, T. Doi, **YBY**, et. al., χQCD collaboration, PRD91(2015)014505, 1312.4816

2-flavor result

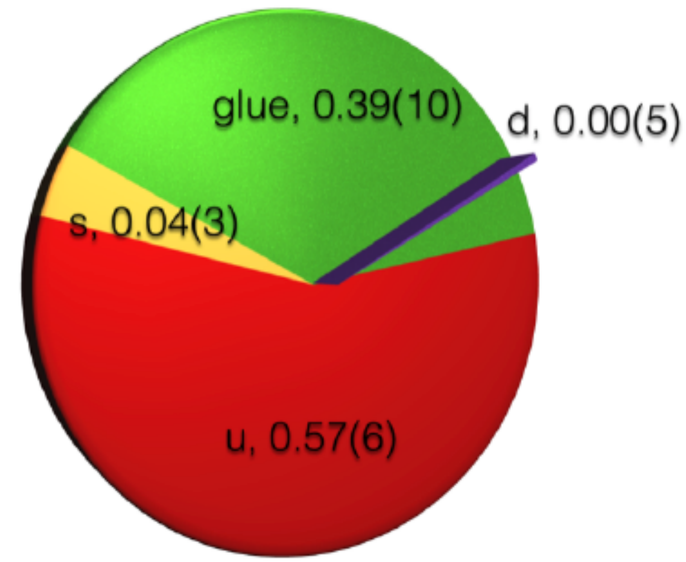
(neglecting the difference between the glue momentum and AM fractions)

EMTC 17: PRL119(2017), 1706.02973



Perturbatively *renormalized*

Preliminary *2+1 flavors result*



YBY, χQCD collaboration, 1904.04138

Non-Perturbatively *renormalized*

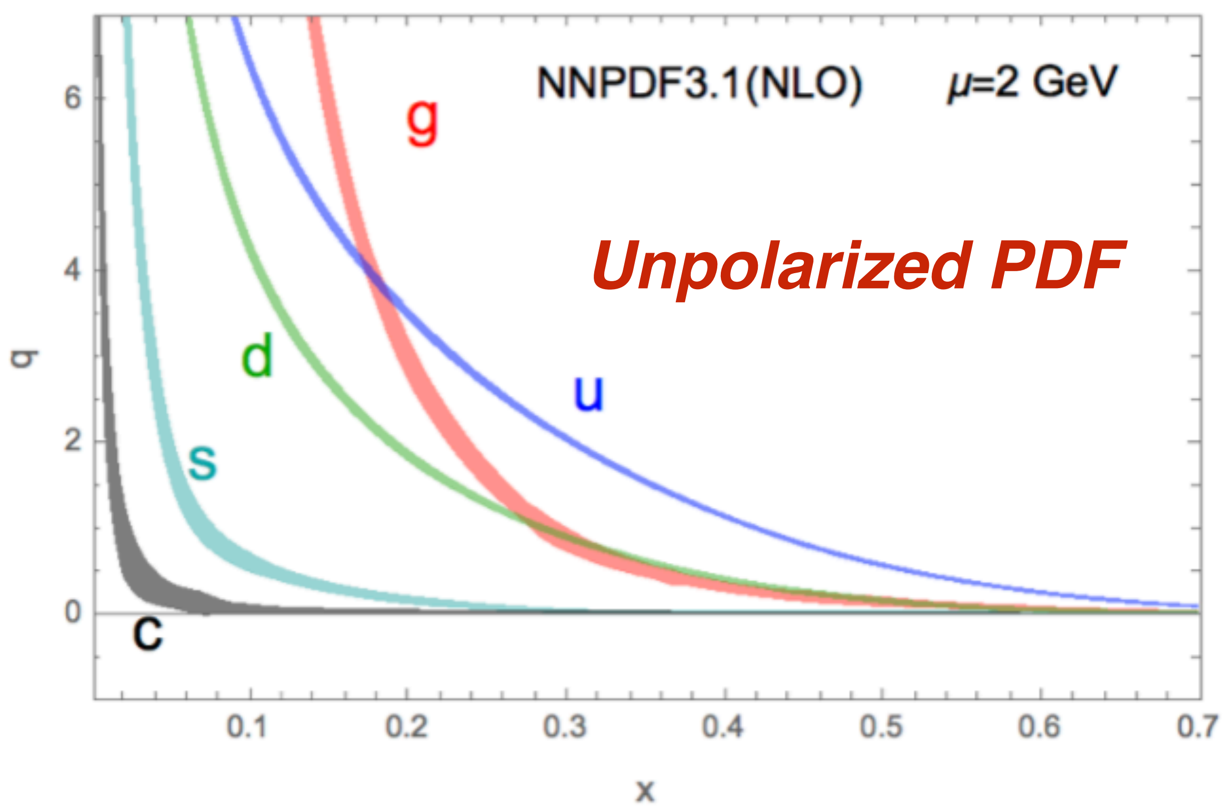
What can Lattice QCD help on the nucleon physics?

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- **How does the gluon in the nucleon look like?**

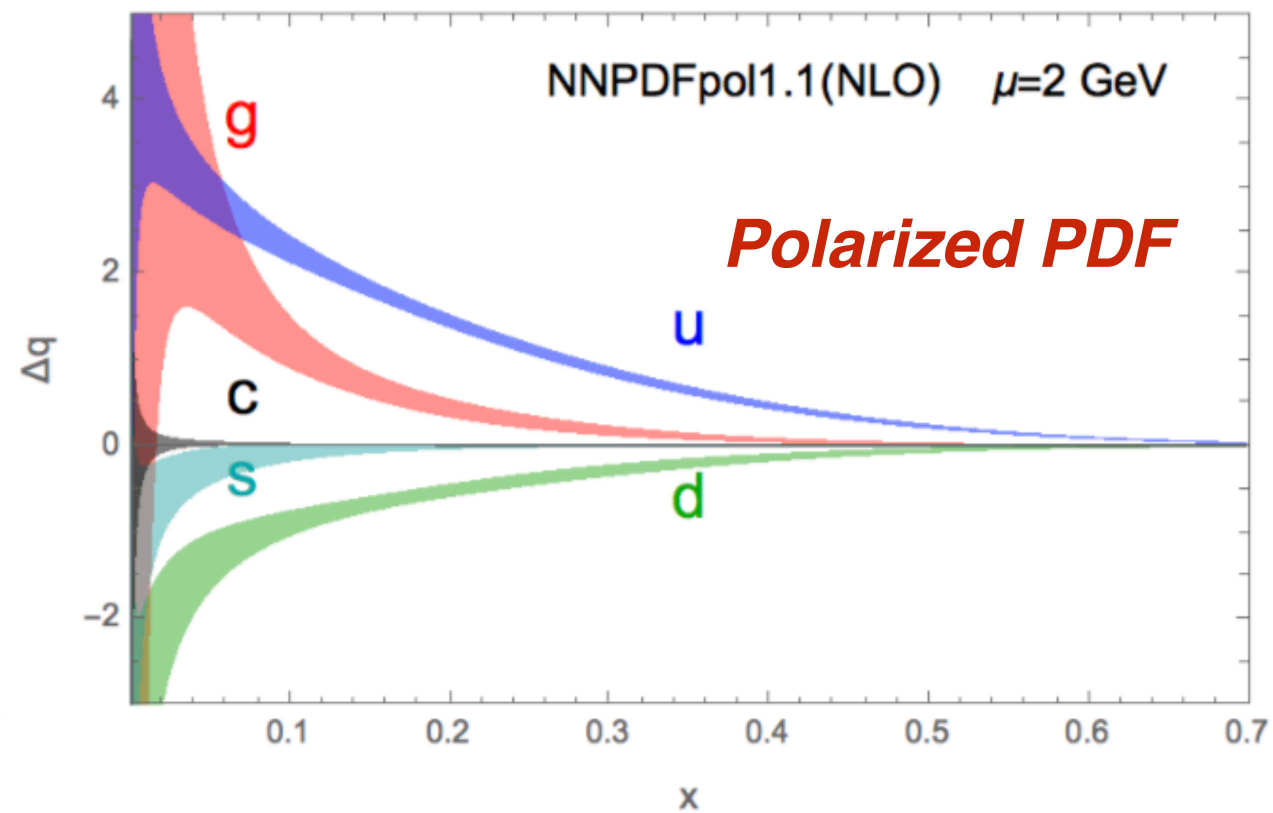
Less known part

of the parton distribution function

R. D. Ball, et al. (NNPDF), EPJC77, 663 (2017), 1706.00428



E. R. Nocera, et.al. (NNPDF), NPB887, 276 (2014), 1406.5539

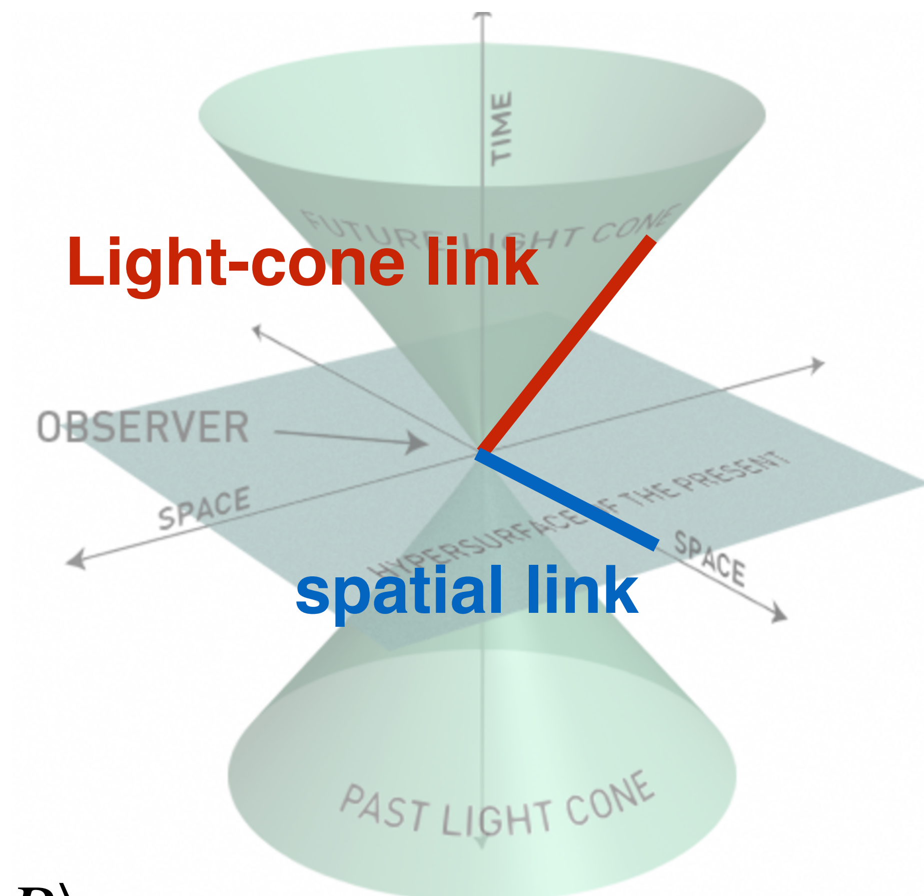


- If we consider the first-moment of the unpolarized PDF and the zeroth-moment of the polarized one, the values of the gluon case are comparable with the quark case.
- But their x-dependence are very different.
- **The gluon PDF is much less unknown from the experiment.**

From quasi-PDF to PDF

The original polarized quark PDF defined in the light front frame is,

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P | \bar{\psi}(\xi^-)\gamma^+ \times \exp\Big(-ig \int_0^{\xi^-}) d\eta A^+(\eta^-) \Big) \psi(0) | P \rangle$$



And the quasi-PDF is defined by

$$\tilde{q}(x, P_z, \tilde{\mu}) = \int \frac{dz}{2\pi} e^{-ixP_z z} \langle P | O(z) | P \rangle$$

With $O(z) = \langle P | \bar{\psi}(z) \exp\Big(-ig \int_0^z) dz' A^z(z') \Big) \psi(0) | P \rangle$

Their IR behaviors (or **non-perturbative properties**) are **the same** and the difference in UV can be matched with **Large Momentum Effective Theory**

X.D. Ji, PRL110 (2013) 262002, 1305.1539
X. Xiong, et.al, PRD90 (2014) 014051, 1310.7471

Gluon PDF and its moments

The gluon PDF is defined by:

$$g(x,\mu) = \int \frac{\mathrm{d}\xi^-}{\pi x} e^{-ix\xi^- P^+} \langle P|F_\mu^{+}(\xi^-)U(\xi^-,0)F^{\mu+}(0)|P\rangle\,,$$

And its odd moments can be defined through local operators, likes the first moment:

$$\begin{aligned}\langle x\rangle_g \equiv \int_0^1 x\ g(x)\mathrm{d}x &= \frac{1}{P^+}\langle P|F_\mu^{+}(0)F^{\mu+}(0)|P\rangle \\ &= \frac{1}{P_z}\langle P|\overline{T}^{tz}(0)|P\rangle \\ &= \frac{P_0\langle P|\overline{T}^{zz}(0)|P\rangle}{\frac{1}{4}P_0^2+\frac{3}{4}P_z^2} = \frac{P_0\langle P|\overline{T}^{tt}(0)|P\rangle}{\frac{3}{4}P_0^2+\frac{1}{4}P_z^2},\end{aligned}$$

All the operators belong to the traceless part of the gauge EMT.

Gluon quasi-PDF

So the gluon quasi-PDF can be defined through the quasi-PDF matrix elements $\tilde{H}_0$ **:**

$$\tilde{g}(x, P_z^2, \mu) = \int \frac{\mathrm{d}z}{\pi x} e^{-ixzP_z} \tilde{H}_0^R(z, P_z, \mu),$$

where $\tilde{H}_0(z, P_z) = \langle P|\mathcal{O}_0(z)|P\rangle$ *and* $\mathcal{O}_0$ *is defined by*

$$\mathcal{O}_0 \equiv \frac{P_0\left(\mathcal{O}(F^t{}_\mu, F^{\mu t}; z) - \frac{1}{4}g^{tt}\mathcal{O}(F^\mu{}_\nu, F^\nu{}_\mu; z)\right)}{\frac{3}{4}P_0^2 + \frac{1}{4}P_z^2},$$

or the other alternative choice:

$$\mathcal{O}_1(z) \equiv \frac{1}{P_z}\mathcal{O}(F_{t\mu}, F_{z\mu}; z),$$

$$\mathcal{O}_2(z) \equiv \frac{P_0\left(\mathcal{O}(F_{z\mu}, F_{\mu z}; z) - \frac{1}{4}g^{zz}\mathcal{O}(F_{\mu\nu}, F_{\nu\mu}; z)\right)}{\frac{1}{4}P_0^2 + \frac{3}{4}P_z^2},$$

Z. Fan, **YBY**, et.al, PRL121, 242001 (2018), 1808.02077

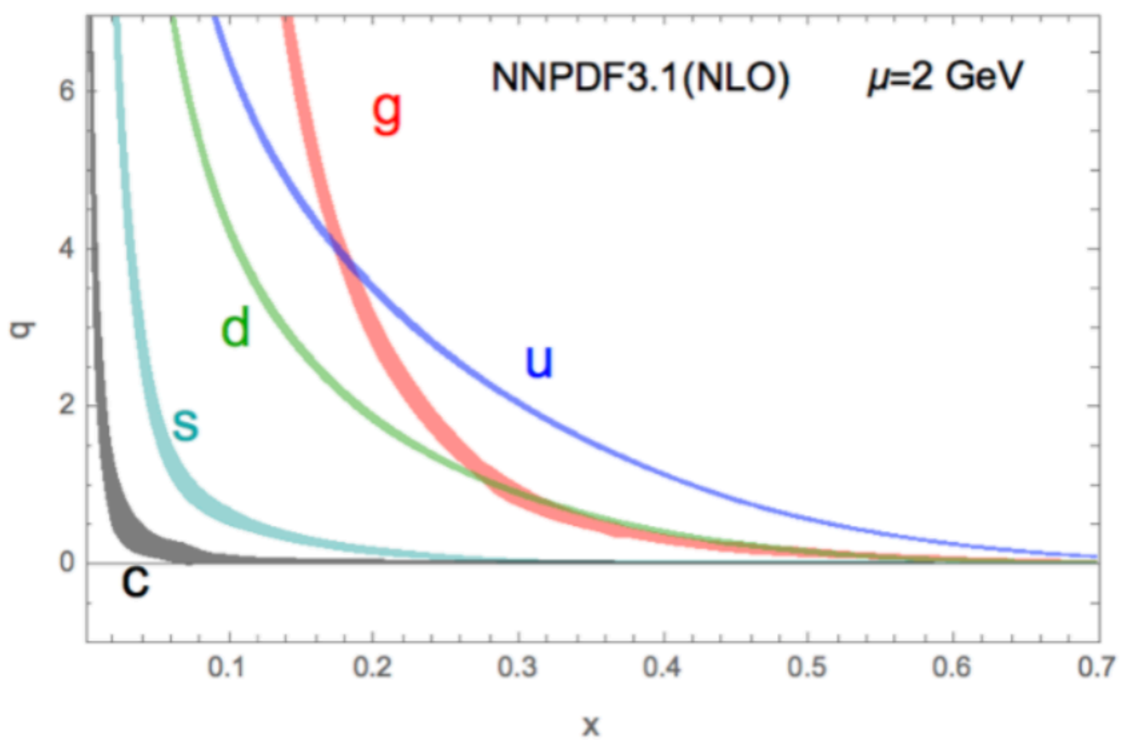
all of them provide ***the same first moment*** *of the gluon PDF, while only* $\mathcal{O}_1$ *can be multiplicative renormalizable.*

Jian-Hui Zhang, et.al, PRL122, 142001 (2019), 1808.10824

Zheng-Yang Li, et.al, PRL122, 062002 (2019), 1809.01836

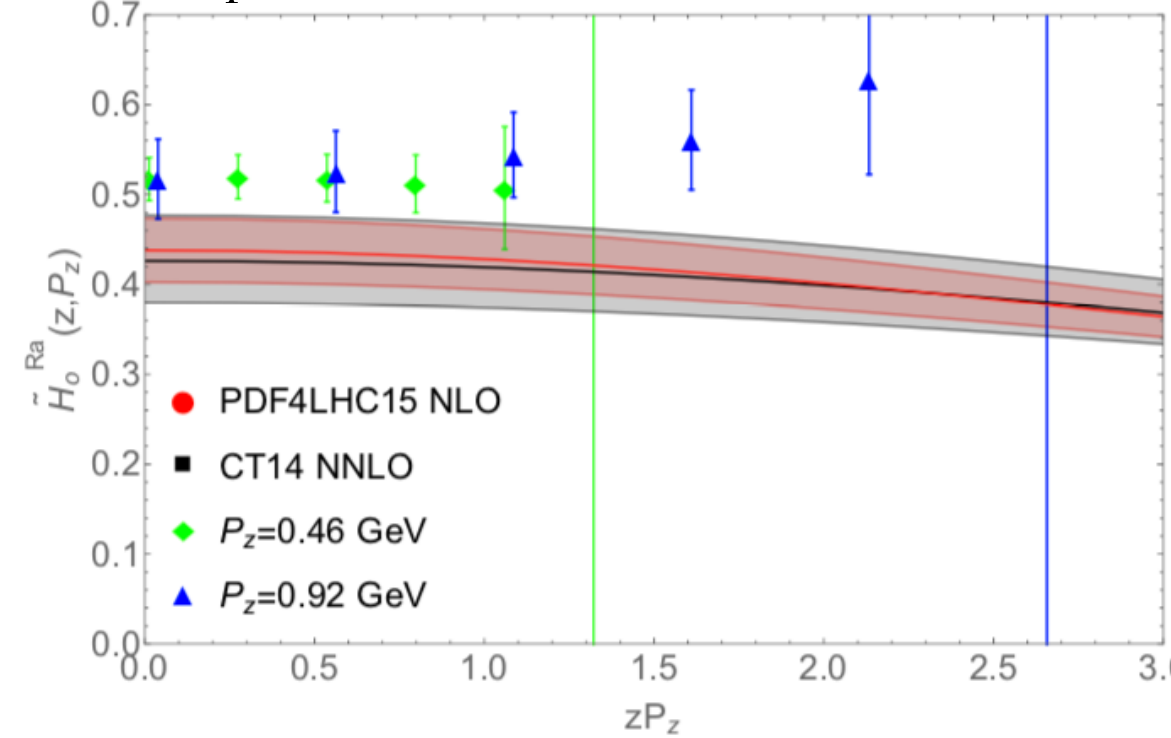
First Lattice QCD attempt on the gluon quasi-PDF

$$u(x), d(x), \ldots g(x)$$



R. D. Ball, et al. (NNPDF), EPJC77, 663 (2017), 1706.00428, replotted by Yu-Sheng Liu

$$\int_{-1}^{1} dx e^{ixzP_z} xg(x,\mu) \quad \text{v.s.} \quad H_0^{Ra}(z, P_z, \mu)$$

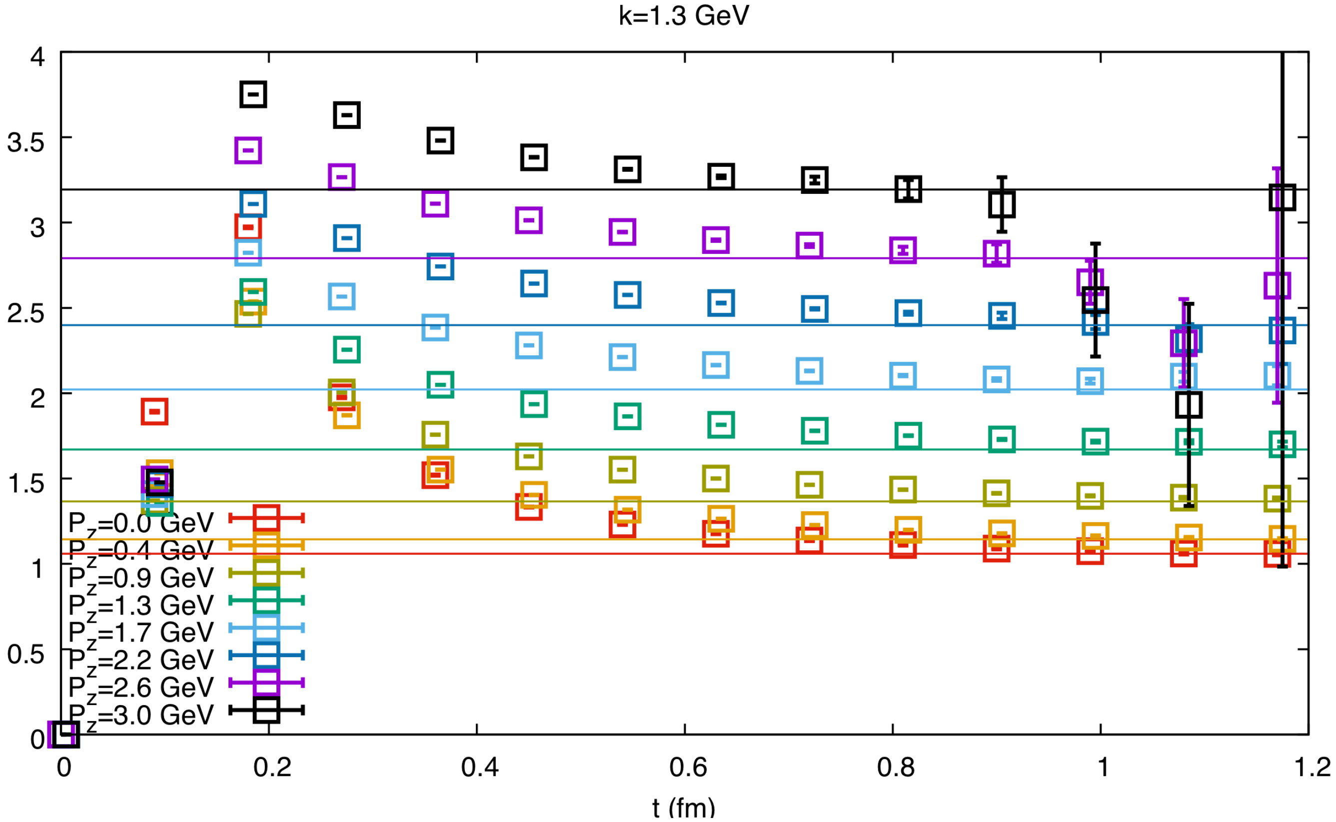


Z. Fan, **YBY**, et al, PRL 121 (2018) 212001, 1808.08677

- The gluon quasi-PDF still suffer from the large statistical uncertainty.
- The coverage of the zP_z region should be enlarged significantly to constraint gluon PDF.
- The major challenge is reaching a large P_z with good signal.

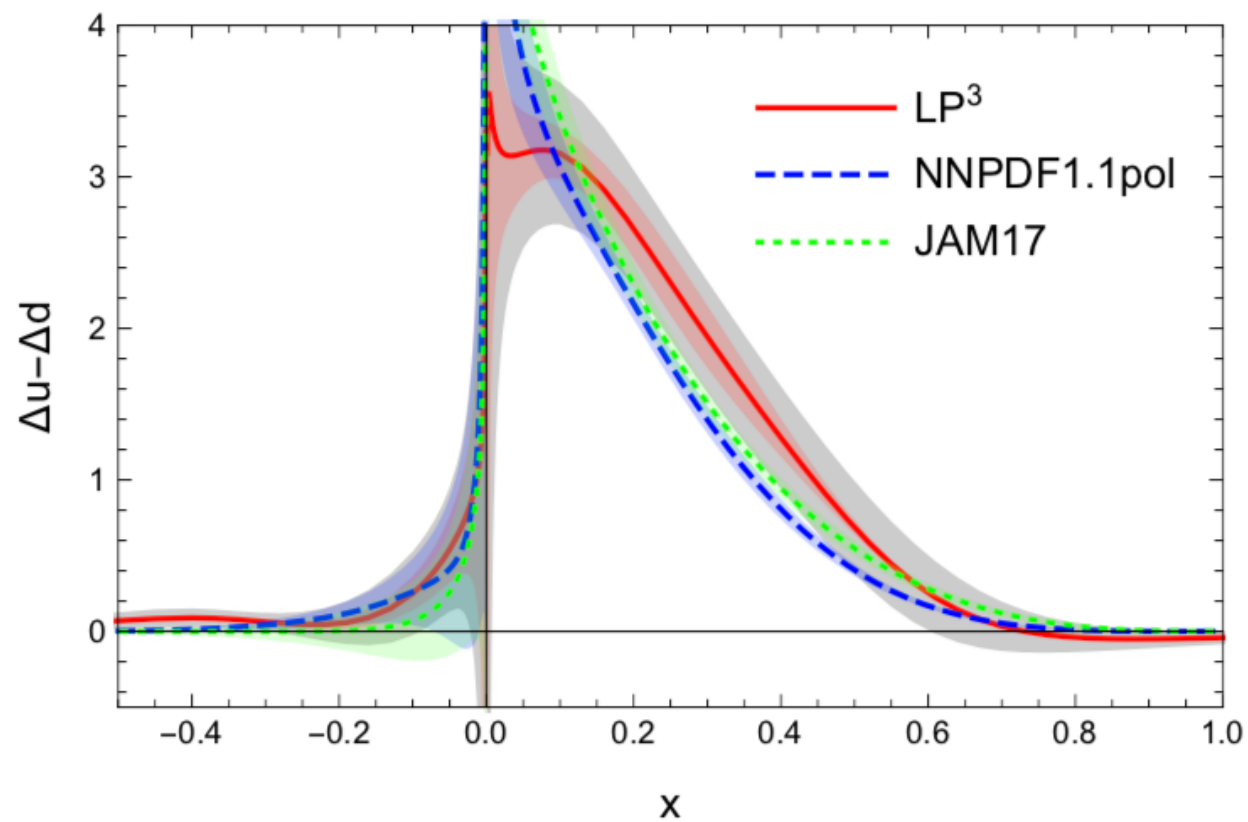
Attempts to reach large P_z

$$E^{eff}(P_z, t) \xrightarrow[t \to \infty]{} E_N = \sqrt{m_N^2 + P_z^2}$$



Based on the momentum smearing technique introduced by
G. S. Bali, et.al, PRD93, 094515 (2016), 1602.05525

$\Delta u(x) - \Delta d(x)$, based on the quasi PDF calculation at $P_z = 3.0$ GeV



H. Lin, et al, LP[3] Collaboration, PRL 121(2018) 242003, 1807.07431

- The momentum smearing technique provides the possibility to reach a large P_z with good signal.

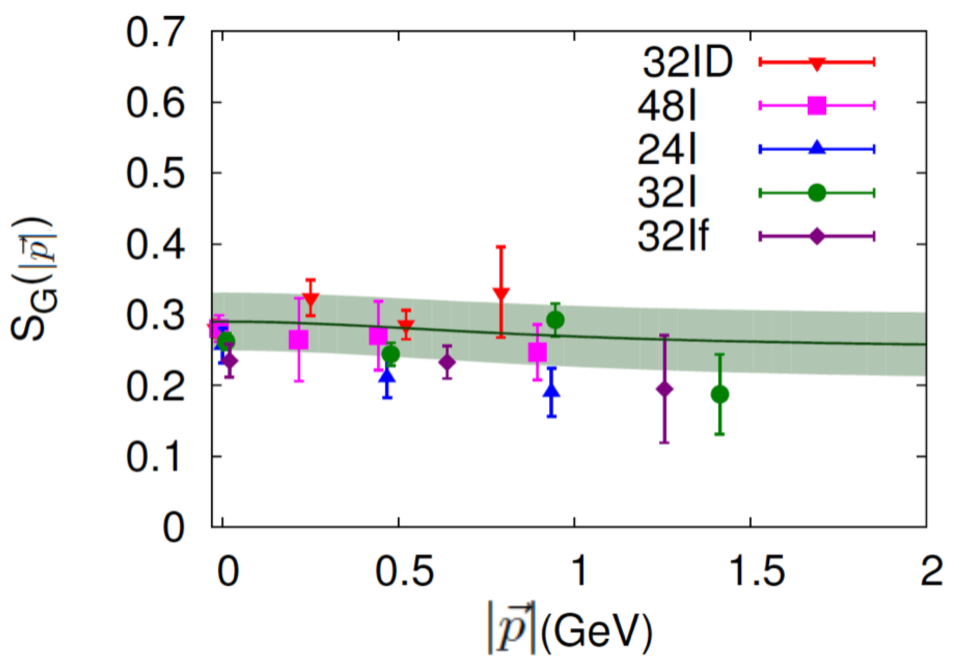
G. S. Bali, et.al, PRD93, 094515 (2016), 1602.05525

- The same technique can be used for the gluon case.

Gluon polarized PDF from Lattice QCD

Revisit the zeroth moment

YBY, R. Sufian, et al., (χQCD), PRL118, 042001(2017), 1609.05937

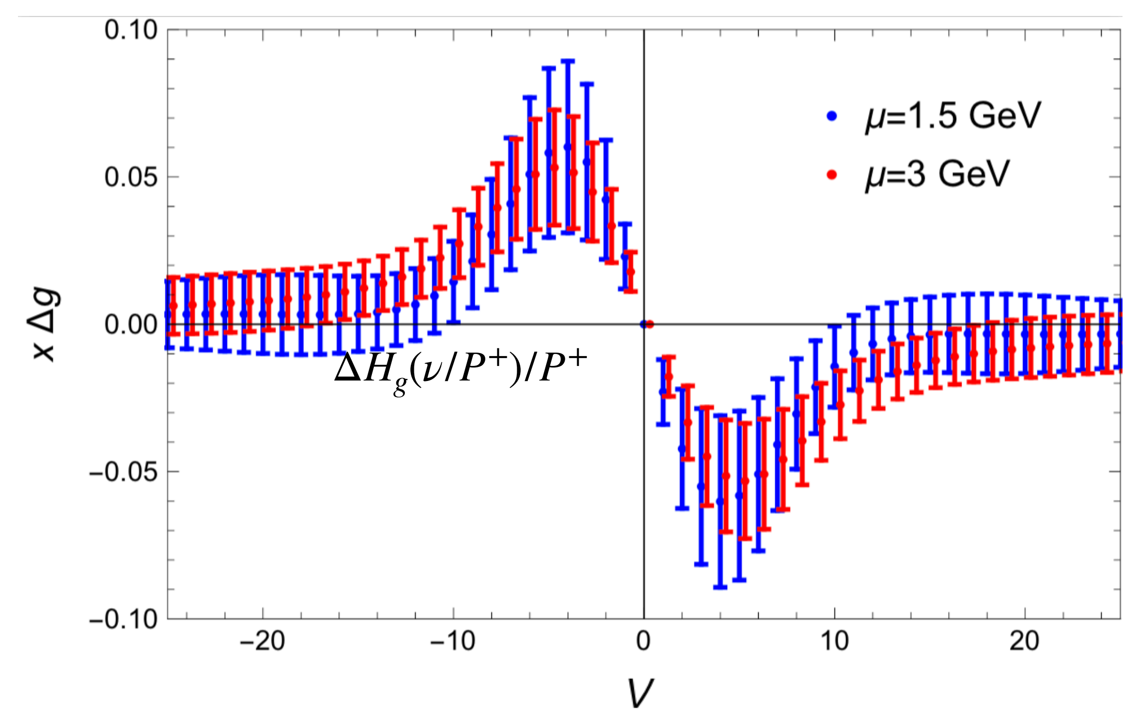


Gluon spin under the Coulomb gauge

- *Sizable contribution to the proton spin;*
- *Convergence of the LaMET matching is poor at 1-loop level;*
- *Gauge dependence should be checked with the calculation under the other gauge conditions.*

Figure from Yu-Sheng Liu. Based on

E. R. Nocera, et.al. (NNPDF), NPB887, 276 (2014), 1406.5539



Gluon spin can also be obtained through the following gauge invariant definition:

$$\Delta\tilde{g} = \int_0^\infty dz \Delta\tilde{H}_g(z)|_{P_z\to\infty} = \int_0^\infty dz \Delta H_g(z) + \mathcal{O}(\alpha_s) = \Delta g + \mathcal{O}(\alpha_s)\,.$$

$$\Delta\tilde{H}_g(z) = \sum_{i=x,y} \langle PS|F_{iz,a}(z)(e^{\int_0^z igA_z(z')dz'})_{ab}\tilde{F}_{iz,b}(0)|PS\rangle$$

$$\Delta H_g(z) = \sum_{i=x,y} \langle PS|F_{+\mu,a}(\xi^-)(e^{\int_0^{\xi^-} igA^+(\eta^-)d\eta^-})_{ab}\tilde{F}^{+,b}_{\mu}(0)|PS\rangle = \int dx P^+ x e^{ix\xi^- P^+} g(x)$$

Summary

- Lattice QCD can provide a systematic way to predict the non-perturbative characters of nucleon and many results have come out recently;
- Gluon provide significant contribution to nucleon mass and spin;
- We are on the way to determine the distribution of gluon in the nucleon, from Lattice QCD.