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Chiral Vortices and Pseudoscalar Condensation in Rotating Strongly Interacting Matter

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Outline

- Introduction： Rotating strongly interacting matter
- Self-consistent treatment: Bogoliubov-de Gennes formalism applied to chiral condensate
- Results: Finite-size effect, temperature and rotation effects, chiral vortex state and pseudoscalar condensation

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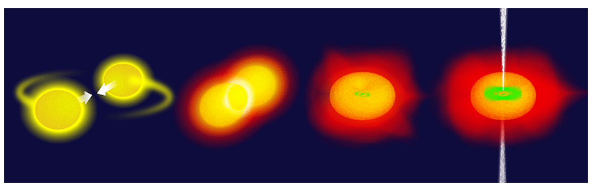
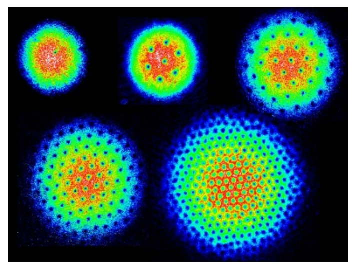
Rotation

There are many systems of interest that are rotating: hot matter in heavy ion collisions, neutron stars, cold atoms, …



from NASA/AEI/ZIB/M.Koppitz and L.Rezzolla

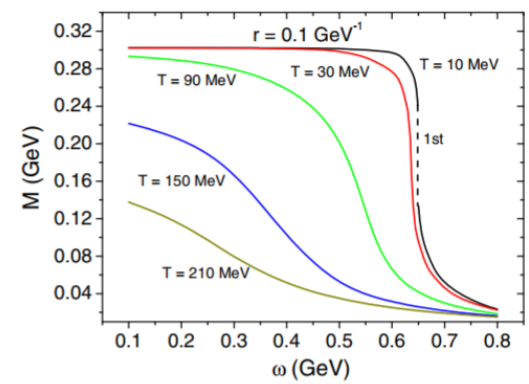


from JILA

Rotation leads to many interesting phenomena in hot and dense QCD matter. What is the effect on the chiral condensate?

STAR Collaboration, Nature 548, 62 (2017)

$\omega \sim 10\times 10^{21}$ Hz



Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016)

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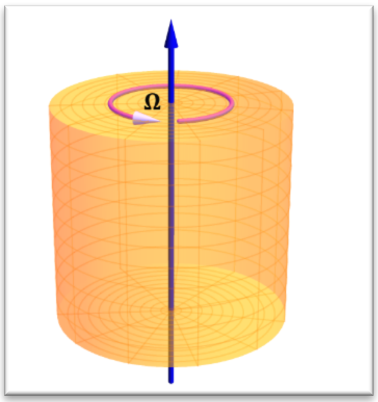
[1] H.-L. Chen, K. Fukushima, X.-G. Huang, and K. Mameda, PRD 93, 104052 (2016).
[2] S. Ebihara, K. Fukushima, and K. Mameda, PLB 764, 94 (2017).
[3] M. N. Chernodub and S. Gongyo, JHEP 201701, 136 (2017).
[4] X. Wang, M. Wei, Z. Li, and M. Huang, PRD 99, 016018 (2019).

QCD Phase Transition due to Rotation



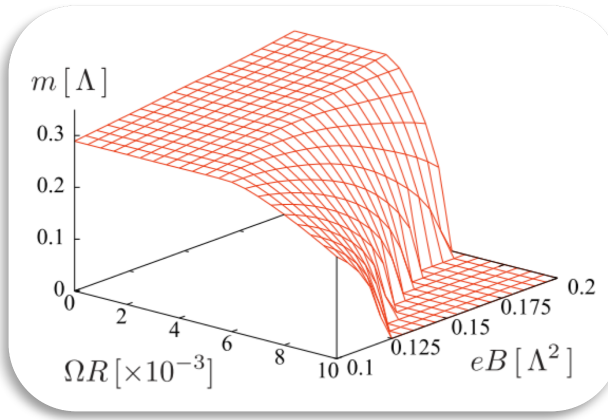
Rotating Frame

$$\widetilde{E}_j = E_j - \Omega\left(m + \frac{1}{2}\right) \equiv E_j - \Omega\mu_m$$

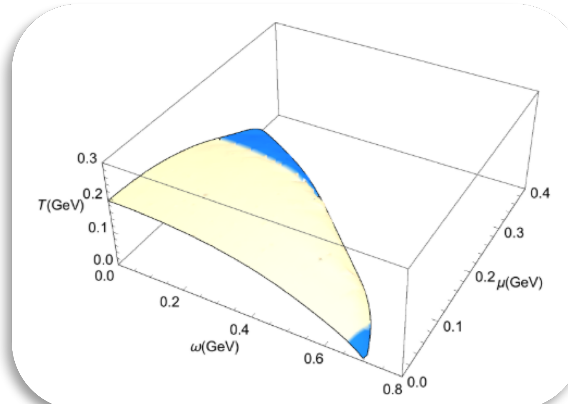
$$E_j \equiv E_{ml}(k_z, M) = \pm\sqrt{k_z^2 + \frac{q_{ml}^2}{R^2} + M^2}$$

Finite Size + Rotation:

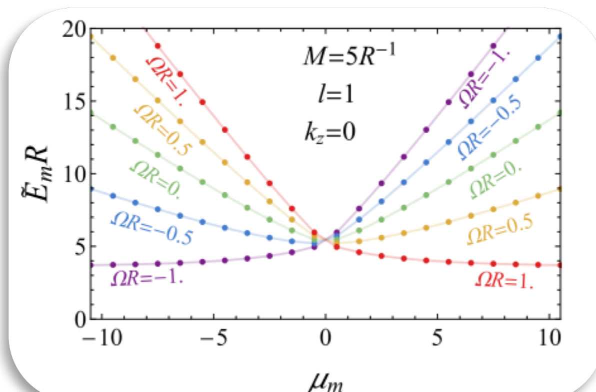
Inhomogeneous !



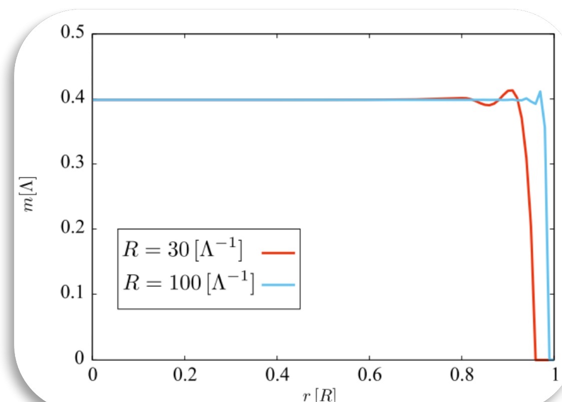
PRD 93, 104052 (2016)



PRD 99, 016018



JHEP 201701, 136 (2017)



PLB 764, 94 (2017)

NJL model with U(1) chiral symmetry in rotating frame

$$\mathcal{S}[\psi^\dagger,\psi] = \int dx \sqrt{-\det g_{\mu\nu}}\,\left[\psi^\dagger \partial_\tau \psi + \mathcal{H}(\psi^\dagger,\psi)\right]$$

$$\mathcal{H} = \psi^\dagger \hat{K}_0 \psi - \frac{G}{2N}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\right] \qquad \hat{K}_0 = -i\gamma^0\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} - \boldsymbol{\omega}\cdot\hat{\mathbf{J}}$$

Mean-Field Approximation

$$\sigma(\mathbf{r}) = -G\langle\bar{\psi}\psi\rangle, \qquad \pi(\mathbf{r}) = -G\langle\bar{\psi}i\gamma_5\psi\rangle$$

$$\mathcal{H}_{\mathrm{MF}} = \psi^\dagger \hat{K}[\sigma,\pi]\psi + \frac{N}{2G}\left(\sigma^2 + \pi^2\right) \qquad \hat{K} = \hat{K}_0 + \gamma^0(\sigma + i\gamma_5\pi)$$

Eigenvalue Equation

$$\hat{K}\Psi_n(\mathbf{r}) = \varepsilon_n \Psi_n(\mathbf{r})$$

Complex Chiral Condensate with U(1) Phase

$$\Delta(\mathbf{r}) = \sigma(\mathbf{r}) + i\pi(\mathbf{r}) = M(\mathbf{r})e^{i\phi(\mathbf{r})}$$

Bogoliubov-de Gennes Theory: Brute-force diagnolization

Effective potential

$$\frac{U_{\text{eff}}}{N} = \frac{1}{2G}\int d\mathbf{r}|\Delta(\mathbf{r})|^2 - \sum_n \left[\frac{\varepsilon_n}{2} + \frac{1}{\beta}\ln\left(1+e^{-\beta\varepsilon_n}\right)\right]$$

Self-consistent Bogoliubov-de Gennes theory:

$$\hat{K}\Psi_n(\mathbf{r}) = \varepsilon_n\Psi_n(\mathbf{r})$$

$$\frac{\delta U_{\text{eff}}[\Delta(\mathbf{r})]}{\delta\Delta(\mathbf{r})} = 0$$

Very hard to solve if we do not use any approximation…

People normally use the Local Density Approximation (LDA)

Beyond LDA: Chiral Vortices

Complex Chiral Condensate with U(1) Phase:

$$\Delta(\mathbf{r}) = \sigma(\mathbf{r}) + i\pi(\mathbf{r}) = M(\mathbf{r})e^{i\phi(\mathbf{r})}$$

Chiral Vortex Solution:

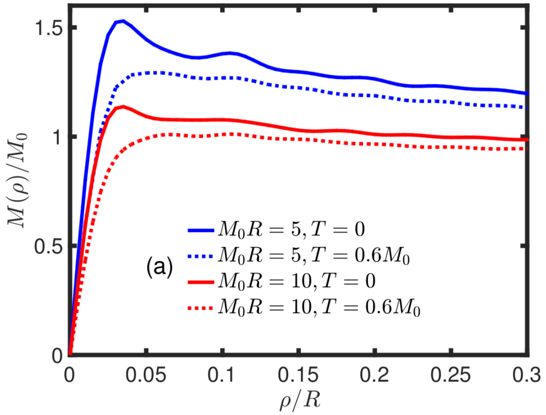
$$\Delta(\mathbf{r}) = M(\rho)e^{i\kappa\theta}, \qquad \kappa \in \mathbb{Z}.$$

Induced stripe-like pseudoscalar condensate:

$$\sigma(\mathbf{r}) = M(\rho)\cos(\kappa\theta)$$

$$\pi(\mathbf{r}) = M(\rho)\sin(\kappa\theta)$$

Chiral vortex state: $\kappa \neq 0$



NJL Model in 2+1 Dimensions: Renormalizable

At the leading order in 1/N, only the bare coupling should be fine tuned

$$\frac{1}{G(\Lambda)} - \frac{1}{G_c} = -\frac{M_0}{\pi}\mathrm{sgn}(G - G_c) \qquad G_c = \pi/\Lambda$$

The emergent quantity M_0 serves as the only mass scale of the theory.

$G > G_c$: **spontaneous chiral symmetry occurs in vacuum effective fermion mass given by** $M_* = M_0$

BdG Eigenvalue equation in (2+1) D

$$\begin{pmatrix} \hat{K}_{11} & \hat{K}_{12} \\ \hat{K}_{21} & \hat{K}_{22} \end{pmatrix} \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix} = \varepsilon_n \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix}$$

In Chiral Basis:

$$\hat{K}_{11} = \hat{K}_{22} = -i(\sigma_2\partial_x + \sigma_1\partial_y) - \omega(\hat{l}_z + \sigma_3/2)$$

$$\hat{K}_{12} = \sigma_3\Delta(\mathbf{r}) \qquad \hat{K}_{21} = \sigma_3\Delta^*(\mathbf{r})$$

Rotational symmetry: Different l sectors decouple

$$\Delta(\mathbf{r}) = M(\rho)e^{i\kappa\theta}, \qquad \kappa \in \mathbb{Z}.$$

Because of the rotational symmetry, we write

$$u_n(\mathbf{r}) = \sum_l \frac{e^{il\theta}}{\sqrt{2\pi}} \begin{pmatrix} u^{\uparrow}_{nl}(\rho)e^{i\theta} \\ u^{\downarrow}_{nl}(\rho) \end{pmatrix},$$

$$v_n(\mathbf{r}) = \sum_l \frac{e^{i(l-\kappa)\theta}}{\sqrt{2\pi}} \begin{pmatrix} v^{\uparrow}_{nl}(\rho)e^{i\theta} \\ v^{\downarrow}_{nl}(\rho) \end{pmatrix},$$

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Boundary condition (infinitely deep potential well)

We choose a close boundary condition, such that there is no incoming and outgoing flux at the spatial boundary

$$\int_0^{2\pi} d\theta \bar{\Psi}_n \gamma^\rho \Psi_n|_{\rho=R} = 0$$

$$\gamma^\rho \equiv \gamma^1 \cos\theta + \gamma^2 \sin\theta$$

$$\phi_{j,l}(\rho) = \frac{\sqrt{2} J_l(\alpha_{j,l}\rho/R)}{R J_{l+1}(\alpha_{j,l})]}$$

$\Rightarrow$

$$u^\uparrow_{nl}(\rho) = \sum_j c^\uparrow_{nj} \phi_{j,l+1}(\rho)$$

$$u^\downarrow_{nl}(\rho) = \sum_j c^\downarrow_{nj} \phi_{j,l}(\rho)$$

$$v^\uparrow_{nl}(\rho) = \sum_j d^\uparrow_{nj} \phi_{j,l-\kappa+1}(\rho)$$

$$v^\downarrow_{nl}(\rho) = \sum_j d^\downarrow_{nj} \phi_{j,l-\kappa}(\rho)$$

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The BdG eigenvalue equation decouples into different l sectors

For a given angular quantum number l, we have a matrix problem

$$\sum_{j'} \begin{pmatrix} -K_{l+1}^{jj'} & S_l^{jj'} & \Delta_{l+1}^{jj'} & 0 \\ S_l^{j'j} & K_{-l}^{jj'} & 0 & -\Delta_l^{jj'} \\ \Delta_{l+1}^{j'j} & 0 & -K_{l-\kappa+1}^{jj'} & S_{l-\kappa}^{jj'} \\ 0 & -\Delta_l^{j'j} & S_{l-\kappa}^{j'j} & K_{-(l-\kappa)}^{jj'} \end{pmatrix} \begin{pmatrix} c_{nj'}^{\uparrow} \\ c_{nj'}^{\downarrow} \\ d_{nj'}^{\uparrow} \\ d_{nj'}^{\downarrow} \end{pmatrix} = \varepsilon_{nl} \begin{pmatrix} c_{nj}^{\uparrow} \\ c_{nj}^{\downarrow} \\ d_{nj}^{\uparrow} \\ d_{nj}^{\downarrow} \end{pmatrix}$$

$$K_l^{jj'} = \omega(l - 1/2)\delta_{jj'}$$

$$S_l^{jj'} = \int d\rho\, \phi_{j,l+1}(\rho) \left(l - \rho \frac{\partial}{\partial \rho} \right) \phi_{j',l}(\rho)$$

$$\Delta_l^{jj'} = \int \rho d\rho M(\rho) \phi_{j,l}(\rho) \phi_{j',l-\kappa}(\rho)$$

- The eigenvalue equation should be solved with a gap equation that minimizes the effective potential

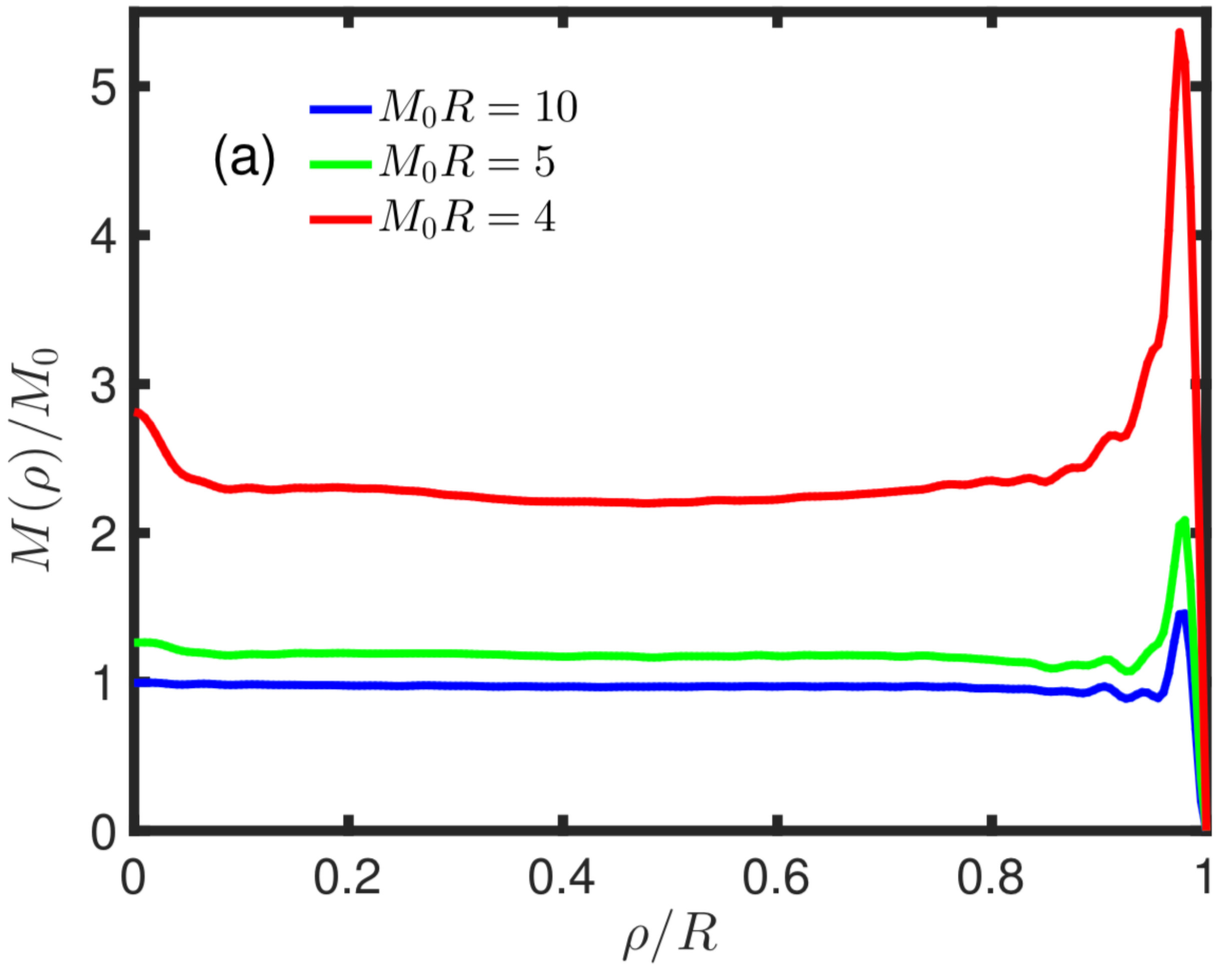
$$\frac{M(\rho)}{G(\Lambda)} = \sum_{n,l} \left[u^{\uparrow}_{nl} v^{\uparrow}_{nl}(\rho) - u^{\downarrow}_{nl} v^{\downarrow}_{nl}(\rho) \right] (1 - 2n_{\mathrm{F}}(\varepsilon_{nl}))$$

The gap equation couples different l sectors.

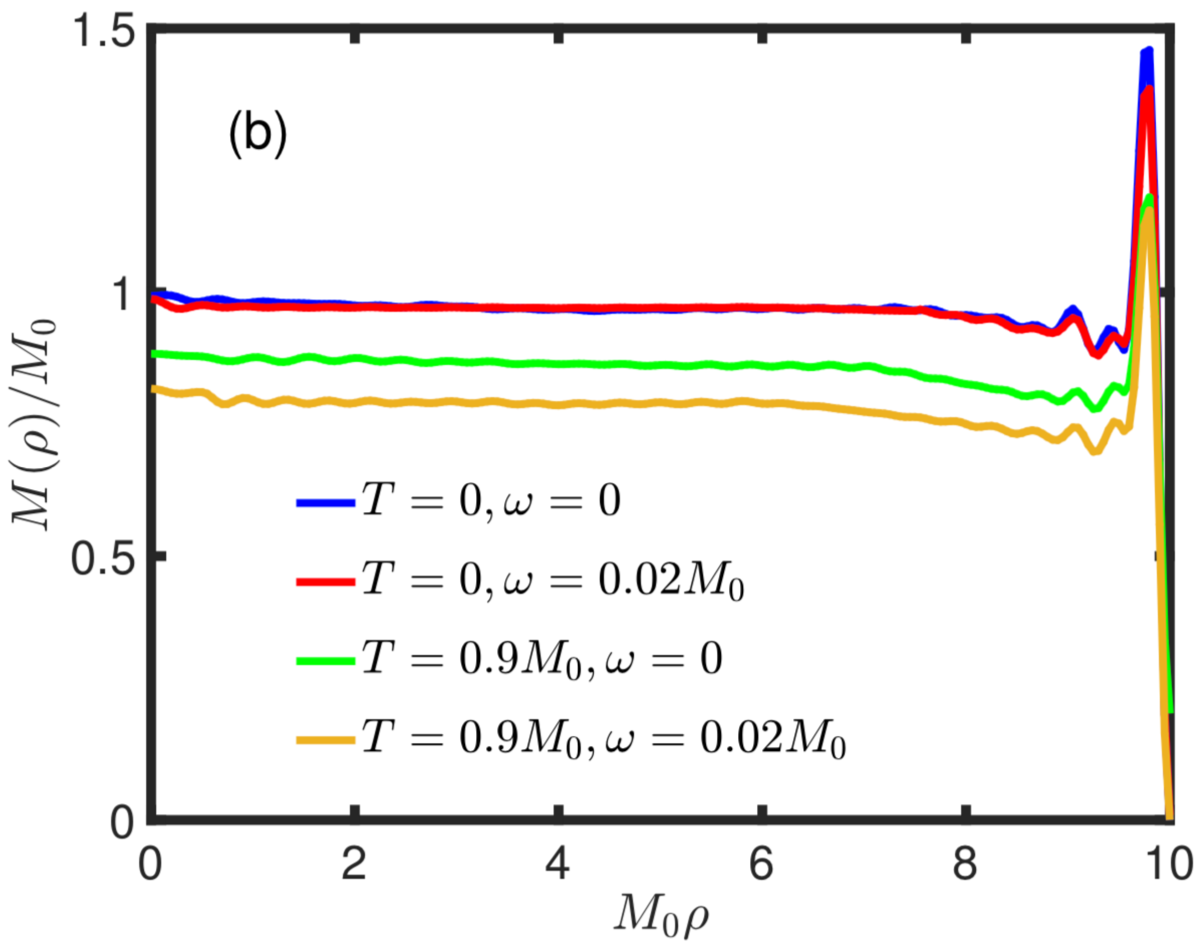
Since the theory is renormalizable, the cutoff dependence finally disappears when we set $\Lambda \to \infty$

There is no model parameter dependence!

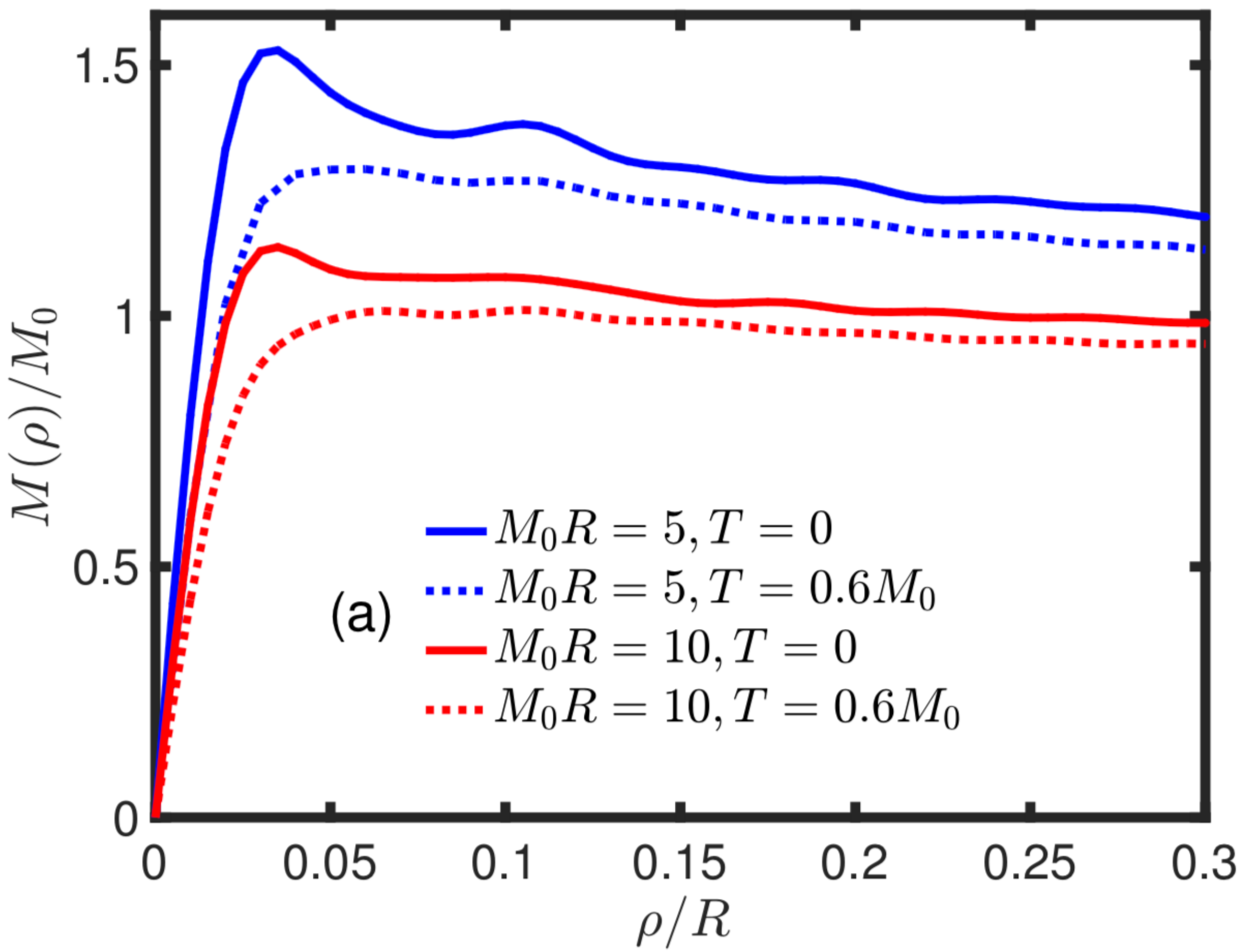
Results: Finite-size effect



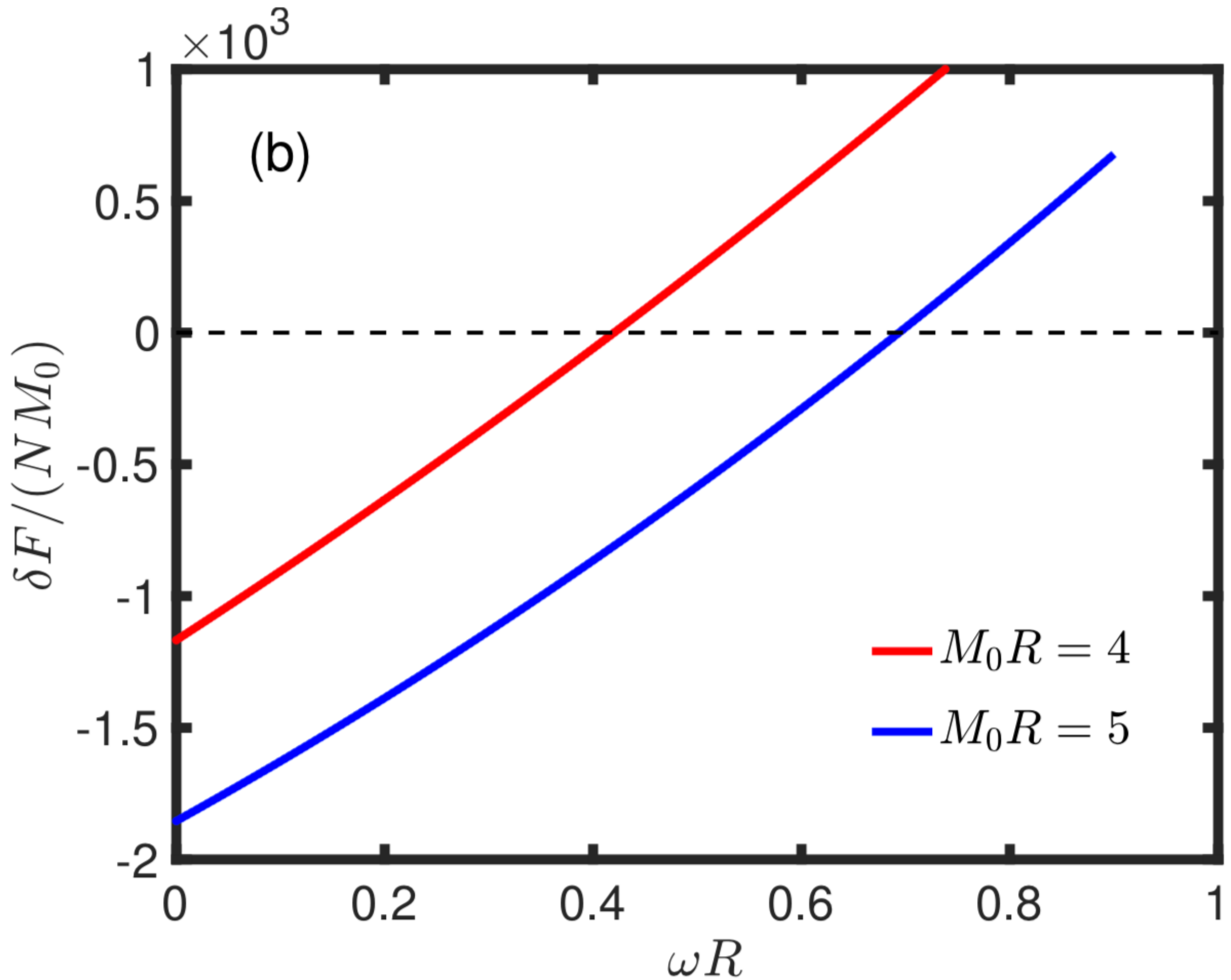
Results: Temperature and rortation effects



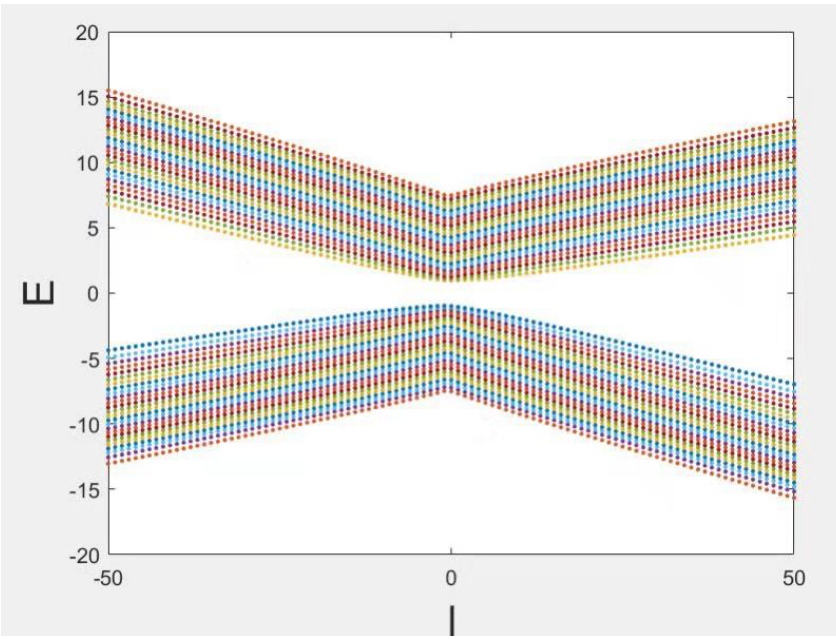
Results: Chiral vortex state

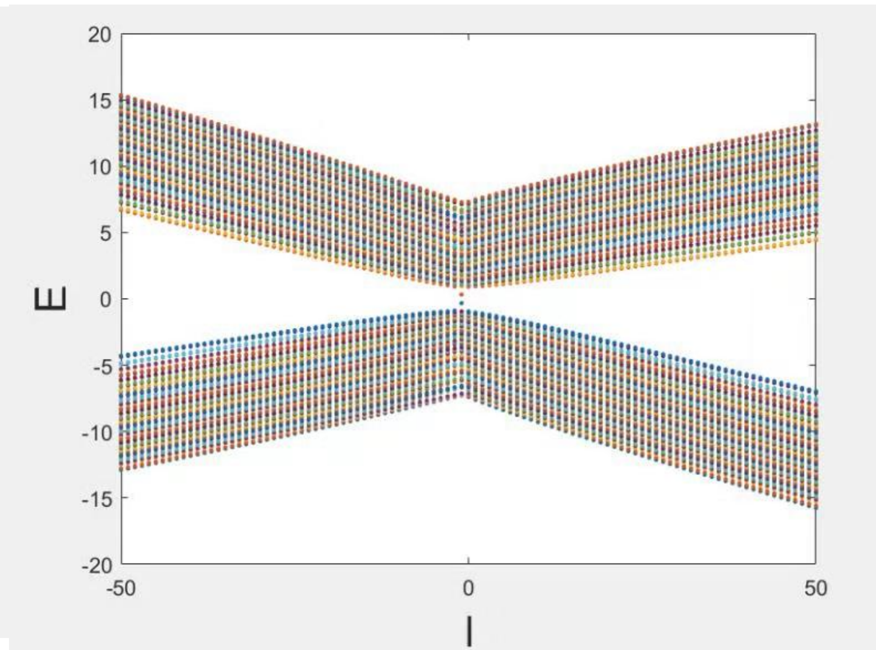


Results: Energy Comparison



Results: Excitation Spectrum





non-vortex state

vortex state

Summary and outlook

- Self-consistent treatment of inhomogeneous chiral condensate in rotating four-fermion models
- For U(1) chiral symmetry, the chiral vortex state can be favored ground state at large vorticity
- Inclusion of magnetic field, (3+1)-dimensions, charged pion condensation, …

Thank You!