第十八届全国中高能核物理大会

Heavy Flavor Hadrons from Multi-Body Dirac Equations

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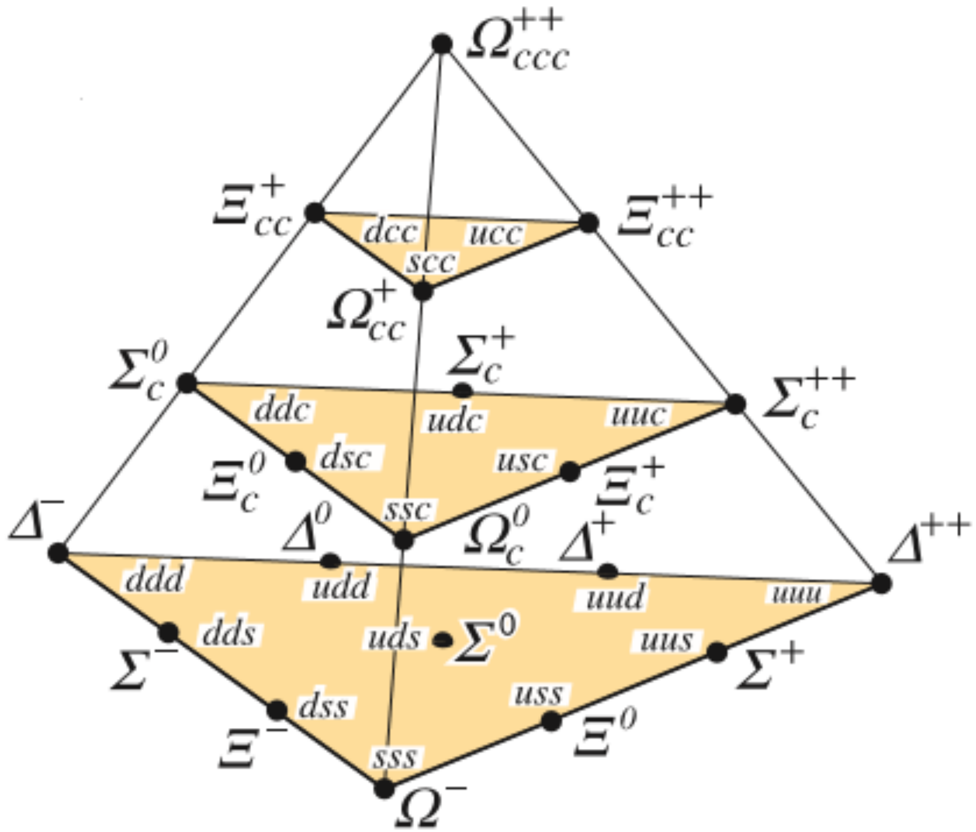
Tsinghua University.

In collab. with: Shuzhe Shi, Pengfei Zhuang

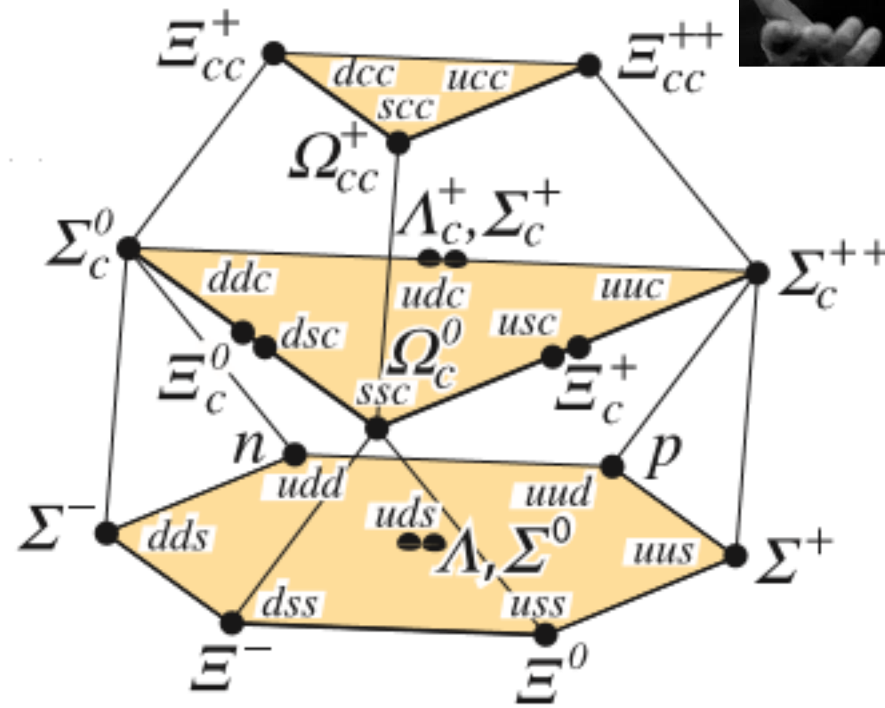
arXiv: 1905.10627

21-25 June, 2019.

SU(4) Quark Model



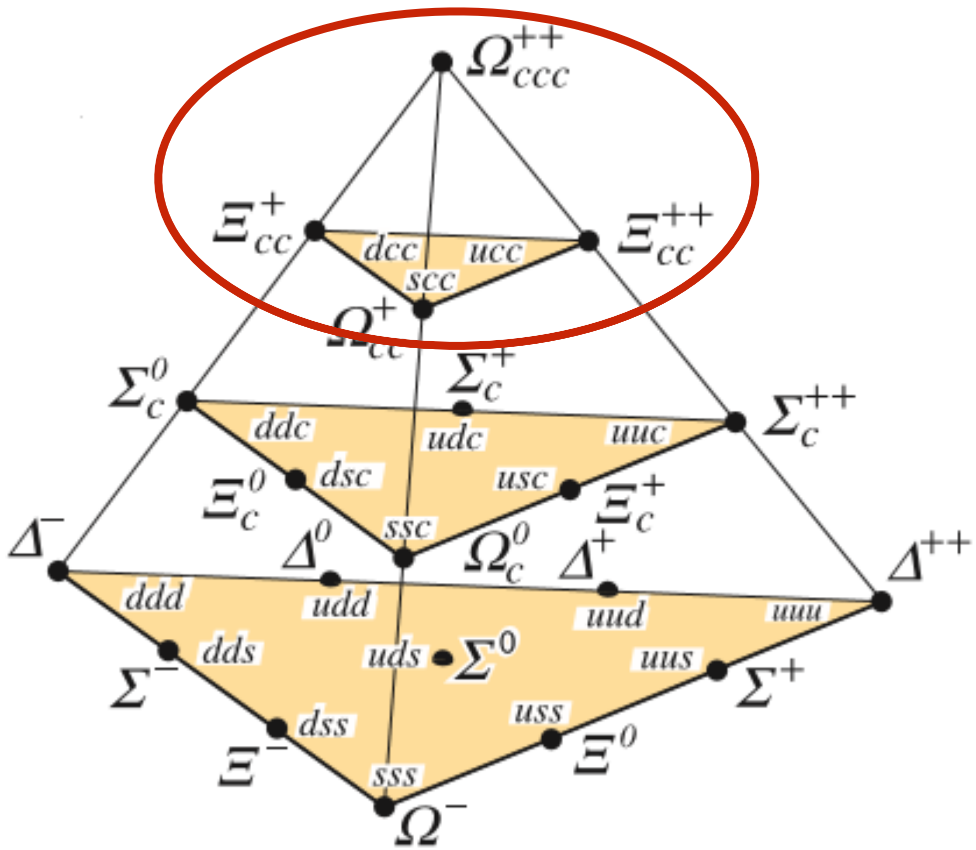
20-plet with SU(4) decuplet.



20-plet with SU(4) octet.

The flavor SU(4) quark model predict 22 charmed baryons,
but some of them are not yet discovered.

SU(4) Quark Model



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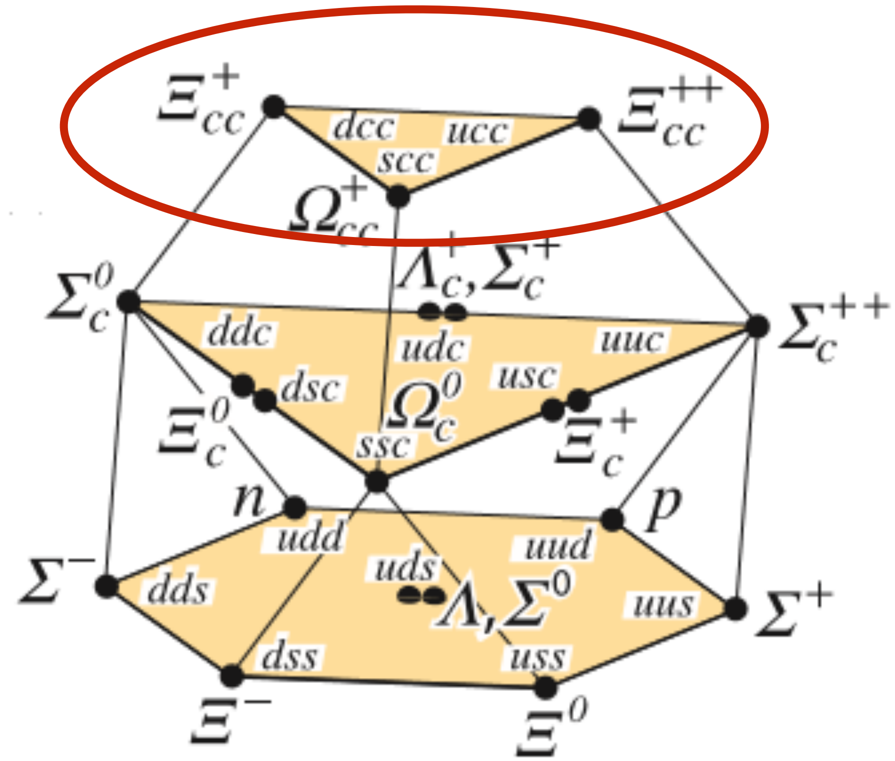


20-plet with SU(4) octet.

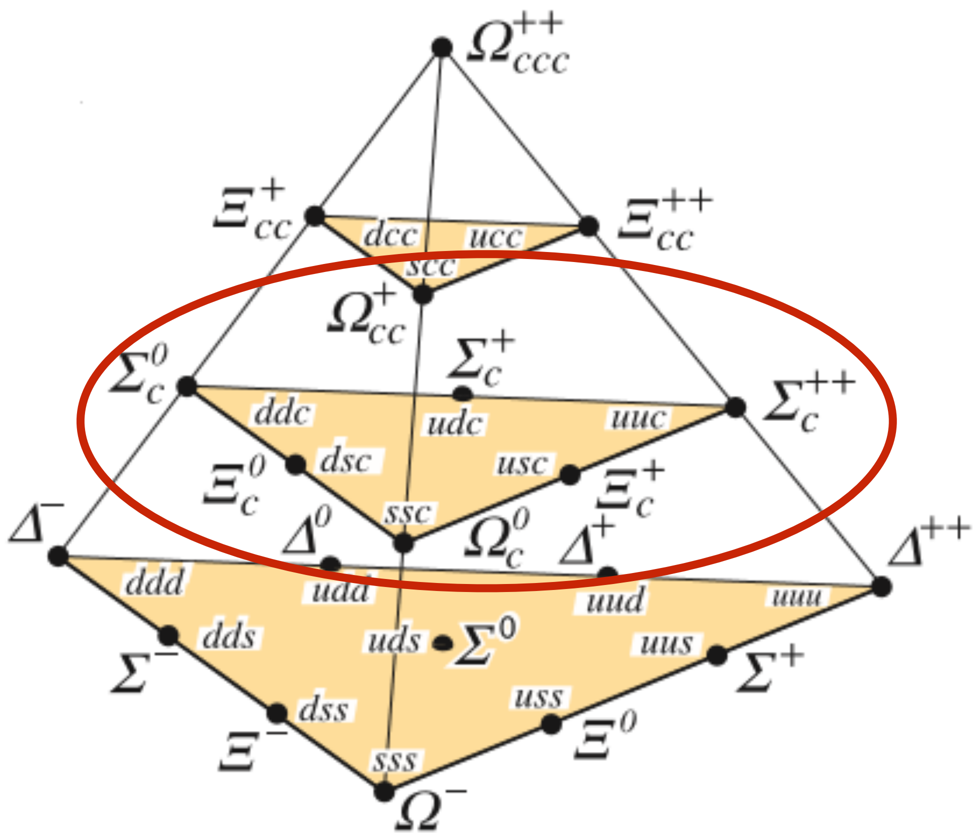
The flavor SU(4) quark model predict 22 charmed baryons, but some of them are not yet discovered.

1. For baryons contain 3/2 heavy quarks.

We have studied them via Non-relativistic potential model and predicted their yield would be dramatically enhanced in Heavy Ion Collisions!

JZhao, Hang He and Pengfei Zhuang, Phys. Lett. B771,349(2017).
Hang He, Yunpeng Liu and Pengfei Zhuang, Phys. Lett. B746,59(2017).

SU(4) Quark Model



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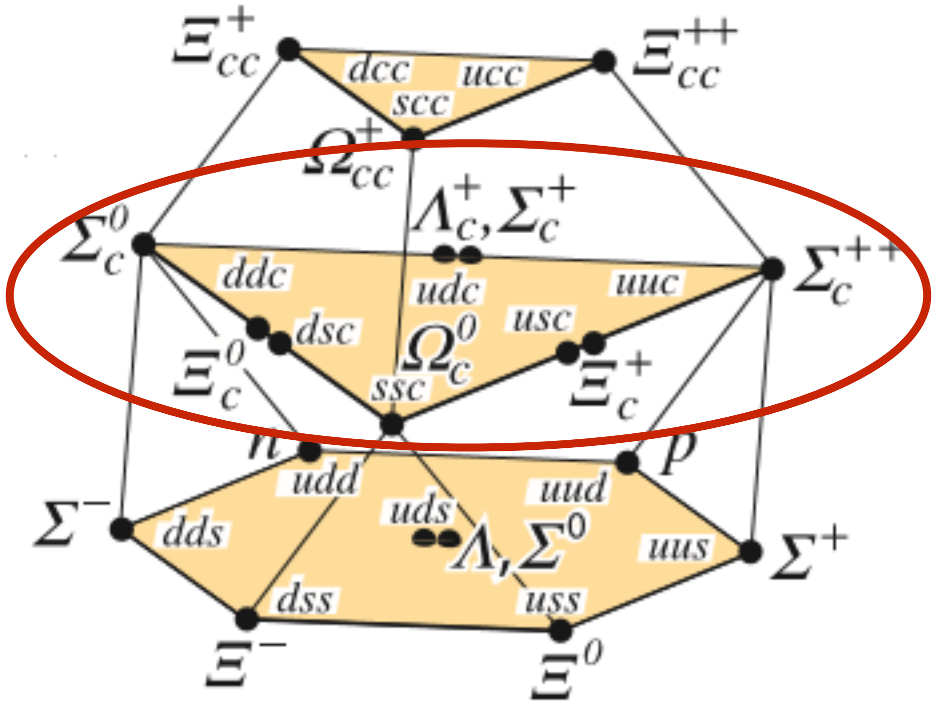

20-plet with SU(4) octet.

The flavor SU(4) quark model predict 22 charmed baryons, but some of them are not yet discovered.

2. For baryons contain 1 light quarks, especially two light quarks.

Need to include whole relativistic correction:
kinematics(which we have considered before)
spin(more important in multi-quark state and external field).

Beyond Non-relativistic Quark Model:

1. Schroedinger-like quasipotential equation

$$(\frac{b^2(M)}{2\mu} - \frac{p^2}{2\mu})\Psi(p) = \int \frac{d^3q}{(2\pi)^3} V(p,q;M)\Psi(q)$$

$$b^2(M) = \frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2}$$

A. A. Logunov, Nuovo Cimento(1963)

D. Ebert, Phys. Lett. B635, 93(2006)

D. Ebert, Phys. Rev D66, 014008(2002)

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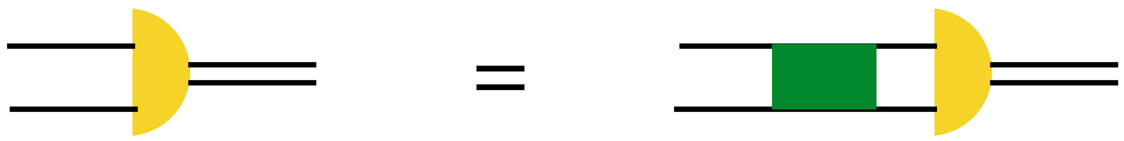
2. Bethe-Salpeter equation

$$G = S_a S_b + S_a S_b K_{ab} G$$

E. E. Salpeter and H. A. Bethe, Phys. Rev 84, 1232(1951)



Bound state appear as poles in the Green function

The 3-D truncated BS Equation have been proposed for the relativistic 2-body problem

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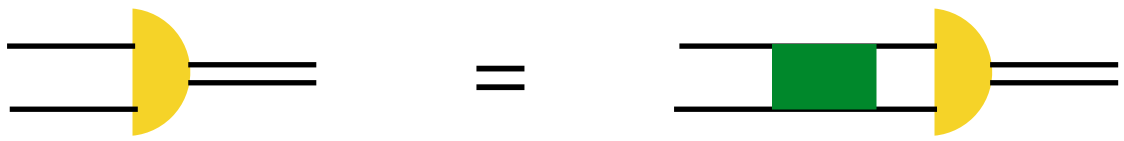


Bound state appear as poles in the Green function

The 3-D truncated BS Equation have been proposed for the relativistic 2-body problem

3. Two Body Dirac Equation (TBDE)

Provide a covariant 3-D truncation !

P. V. Alstine and H. W. Crater. J. Math. Phys. 23(1982)

TBDE have dual origins:

1. one of quasipotential reductions of the BS Equation
2. covariant Hamiltonian formalism with constraints

P. A. M. Dirac, Yeshiva University, New York, 1964

Framework:

N-Body Bound State Relativistic Wave Equations

H. Sazdjian

Division de Physique Théorique, Institut de Physique Nucléaire, Université Paris XI, F-91406, Orsay Cedex, France*

Received July 17, 1988; revised December 21, 1988

covariant formalism with constraints is used to describe the dynamic of N interacting spin 0, 1/2 particles!

two-fermion case:

$$\Psi = \Psi_{\alpha_1,\alpha_2}(x_1 \cdot x_2) \qquad (\alpha_1, \alpha_2 = 1, ..., 4),$$

$$H_1 \Psi \equiv [\gamma_1 \cdot p_1 - m_1 - V_{12}]\, \Psi = 0,$$

$$H_2 \Psi \equiv [\gamma_2 \cdot p_2 - m_2 - V_{21}]\, \Psi = 0. \qquad \{H_1, H_2\}\Psi \approx 0$$

generalize to N-fermion case:

$$\Psi = \Psi_{\alpha_1 \cdots \alpha_N}(x_1, ..., x_N) \qquad (\alpha_1, ..., \alpha_N = 1, ..., 4),$$

and satisfies N independent wave equations:

$$H_a \Psi \equiv (\gamma_a \cdot p_a - m_a - V_a)\, \Psi = 0, \qquad (a = 1, ..., N).$$

$$\left\{(p^2)^{1/2} - \sum_{a=1}^{N} \left[\gamma_{aL} m_a - \gamma_{aL} \gamma_a^T \cdot p_a^T + \sum_{\substack{b=1 \\ b \neq a}}^{N} \gamma_{aL} V_{ab}\right]\right\} \Psi = 0, \qquad N =/> 2$$

Framework:

ANNALS OF PHYSICS 148, 57–94 (1983)

2BDE & 3BDE by Crater et al.

Two-Body Dirac Equations

HORACE W. CRATER

PHYSICAL REVIEW D 89, 014023 (2014)

Baryon spectrum analysis using Dirac's covariant constraint dynamics

Joshua F. Whitney and Horace W. Crater

(Received 10 October 2013; revised manuscript received 16 December 2013; published 30 January 2014)

We present a relativistic quark model for the baryons that combines three related relativistic formalisms. The three-bo … or the three pai … d state energie … ac equations of … ar interactions … al quasipotenti … ar dynamics us … he Richardson … d and several different algorithms, including a gradient approach, and a Monte Carlo method.

Applications of Two Body Dirac Equations to Hadron and Positronium Spectroscopy

arXiv:1403.6466

H. W. Crater*, J. Schiermeyer, J. Whitney
The University of Tennessee Space Institute

C. Y. Wong
Oak Ridge National Laboratory

March 27, 2014

Framework:

$$
\begin{aligned}
\mathcal{H}_1\psi &= \left[p_1^2+m_1^2+\Phi_{12}\right]\psi=0,\\
\mathcal{H}_2\psi &= \left[p_2^2+m_2^2+\Phi_{12}\right]\psi=0,\\
\varepsilon_1 &= [w+\left(m_1^2-m_2^2\right)/(\varepsilon_1+\varepsilon_2)]/2,\\
\varepsilon_2 &= [w+\left(m_2^2-m_1^2\right)/(\varepsilon_1+\varepsilon_2)]/2,\\
\varepsilon_1+\varepsilon_2 &= w,\\
[\mathcal{H}_1,\mathcal{H}_2]\psi &= 0\to\Phi_{12}=\Phi_{12}(x_{12\perp}),\\
(p_\perp^2+\Phi_{12})\psi &= (\varepsilon_1^2-m_1^2)\psi=(\varepsilon_2^2-m_2^2)\psi=b^2(w)\psi.
\end{aligned}
$$

$$
\begin{aligned}
&\Phi_{ab}(\mathbf{r}_{ab},m_a,m_b,w_{ab},\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b)\\
&\quad=2m_{w_{ab}}S+S^2+2\varepsilon_{w_{ab}}A-A^2+2\varepsilon_{w_{ab}}V-V^2+\Phi_D\\
&\qquad+\mathbf{L}_{ab}\cdot(\boldsymbol{\sigma}_a+\boldsymbol{\sigma}_b)\Phi_{SO}+\boldsymbol{\sigma}_a\cdot\hat{\mathbf{r}}_{ab}\boldsymbol{\sigma}_b\cdot\hat{\mathbf{r}}_{ab}\mathbf{L}_{ab}\cdot(\boldsymbol{\sigma}_a+\boldsymbol{\sigma}_b)\Phi_{SOT}\\
&\qquad+\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b\Phi_{SS}+(3\boldsymbol{\sigma}_a\cdot\hat{\mathbf{r}}_{ab}\boldsymbol{\sigma}_b\cdot\hat{\mathbf{r}}_{ab}-\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b)\Phi_T\\
&\qquad+\mathbf{L}_{ab}\cdot(\boldsymbol{\sigma}_a-\boldsymbol{\sigma}_b)\Phi_{SOD}+i\mathbf{L}_{ab}\cdot\boldsymbol{\sigma}_a\times\boldsymbol{\sigma}_b\Phi_{SOX},\\
&w_{ab}=\varepsilon_a+\varepsilon_b.
\end{aligned}
$$

P. V. Alstine and H. W. Crater. J. Math. Phys. 23(1982)

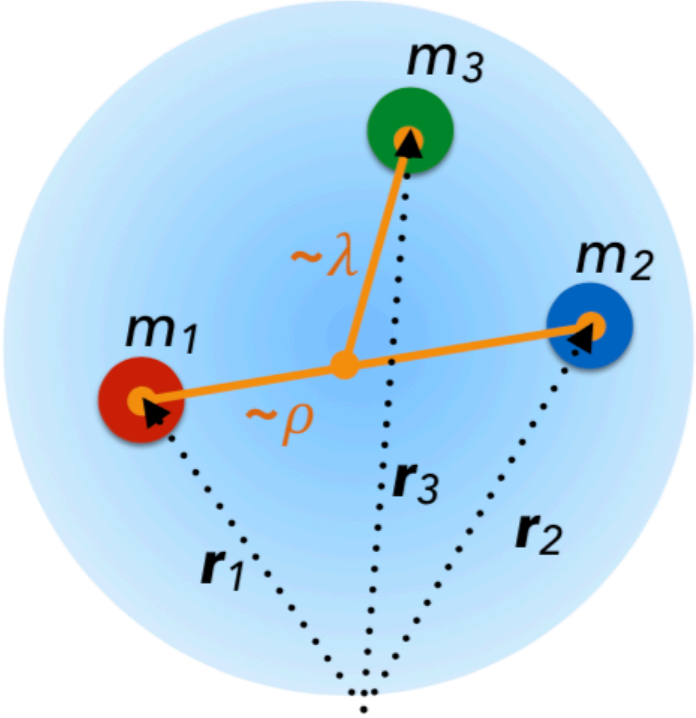
Framework:

$$\begin{aligned}
\mathcal{H}_1\psi &= \left[p_1^2+m_1^2+\Phi_{12}+\Phi_{31}\right]\psi=0,\\
\mathcal{H}_2\psi &= \left[p_2^2+m_2^2+\Phi_{23}+\Phi_{12}\right]\psi=0,\\
\mathcal{H}_3\psi &= \left[p_3^2+m_3^2+\Phi_{31}+\Phi_{23}\right]\psi=0,\\
\varepsilon_1 &= \left[w+\left(m_1^2-m_2^2\right)/(\varepsilon_1+\varepsilon_2)+\left(m_1^2-m_3^2\right)/(\varepsilon_1+\varepsilon_3)\right]/3,\\
\varepsilon_2 &= \left[w+\left(m_2^2-m_3^2\right)/(\varepsilon_2+\varepsilon_3)+\left(m_2^2-m_1^2\right)/(\varepsilon_2+\varepsilon_1)\right]/3,\\
\varepsilon_3 &= \left[w+\left(m_3^2-m_1^2\right)/(\varepsilon_3+\varepsilon_1)+\left(m_3^2-m_1^2\right)/(\varepsilon_3+\varepsilon_1)\right]/3,\\
\varepsilon_1+\varepsilon_2+\varepsilon_3 &= w\\
\Phi_{12} &= \Phi_{12}(x_{12\perp}),\Phi_{23}=\Phi_{23}(x_{23\perp}),\Phi_{31}=\Phi_{31}(x_{31\perp}),\\
x_{ij\perp}^{\mu} &= (x_i^{\mu}-x_j^{\mu})+\hat{P}^{\mu}\hat{P}\cdot(x_i-x_j),
\end{aligned}$$

$$\begin{aligned}
\mathcal{H}\psi &\equiv \frac{1}{F}\left(\frac{p_{1\perp}^2+\Phi_{12}+\Phi_{13}}{2\varepsilon_1(w,m_1,m_2,m_3)}+\frac{p_{2\perp}^2+\Phi_{23}+\Phi_{12}}{2\varepsilon_2(w,m_1,m_2,m_3)}+\frac{p_{3\perp}^2+\Phi_{31}+\Phi_{23}}{2\varepsilon_3(w,m_1,m_2,m_3)}\right)\psi\\
&= (w-m_1-m_2)\psi,
\end{aligned}$$

$$\begin{aligned}
&\Phi_{ab}(\mathbf{r}_{ab},m_a,m_b,w_{ab},\boldsymbol{\sigma}_a,\boldsymbol{\sigma}_b)\\
&\quad=2m_{w_{ab}}S+S^2+2\varepsilon_{w_{ab}}A-A^2+2\varepsilon_{w_{ab}}V-V^2+\Phi_D\\
&\qquad+\mathbf{L}_{ab}\cdot(\boldsymbol{\sigma}_a+\boldsymbol{\sigma}_b)\Phi_{SO}+\boldsymbol{\sigma}_a\cdot\hat{\mathbf{r}}_{ab}\boldsymbol{\sigma}_b\cdot\hat{\mathbf{r}}_{ab}\mathbf{L}_{ab}\cdot(\boldsymbol{\sigma}_a+\boldsymbol{\sigma}_b)\Phi_{SOT}\\
&\qquad+\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b\Phi_{SS}+(3\boldsymbol{\sigma}_a\cdot\hat{\mathbf{r}}_{ab}\boldsymbol{\sigma}_b\cdot\hat{\mathbf{r}}_{ab}-\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b)\Phi_T\\
&\qquad+\mathbf{L}_{ab}\cdot(\boldsymbol{\sigma}_a-\boldsymbol{\sigma}_b)\Phi_{SOD}+i\mathbf{L}_{ab}\cdot\boldsymbol{\sigma}_a\times\boldsymbol{\sigma}_b\Phi_{SOX},\\
&w_{ab}=\varepsilon_a+\varepsilon_b.
\end{aligned}$$

Coordinate Transformation:



$$
\begin{aligned}
\mathbf{R} &= \frac{\epsilon_1 \mathbf{r}_1 + \epsilon_2 \mathbf{r}_2 + \epsilon_3 \mathbf{r}_3}{\epsilon_1 + \epsilon_2 + \epsilon_3}, \\
\boldsymbol{\rho} &= \sqrt{\frac{\epsilon_1 \epsilon_2}{(\epsilon_1 + \epsilon_2)\overline{m}}}(\mathbf{r}_1 - \mathbf{r}_2), \\
\boldsymbol{\lambda} &= \sqrt{\frac{\epsilon_3}{\overline{m}(\epsilon_1 + \epsilon_2)(\epsilon_1 + \epsilon_2 + \epsilon_3)}}\left[\epsilon_1(\mathbf{r}_3 - \mathbf{r}_1) + \epsilon_2(\mathbf{r}_3 - \mathbf{r}_2)\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{P} &= \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3, \\
\mathbf{p} &= \sqrt{\frac{\overline{m}\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}\left(\frac{\mathbf{p}_1}{\epsilon_1} - \frac{\mathbf{p}_2}{\epsilon_2}\right), \\
\mathbf{q} &= \sqrt{\frac{\overline{m}\epsilon_3(\epsilon_1 + \epsilon_2)}{\epsilon_1 + \epsilon_2 + \epsilon_3}}\left(-\frac{\mathbf{p}_1 + \mathbf{p}_2}{\epsilon_1 + \epsilon_2} + \frac{\mathbf{p}_3}{\epsilon_3}\right)
\end{aligned}
$$

Numerical Method (SHO Basis Expansion):

We use SHO basis, It not only increase the precision but also can be used to study excited states.

With

$$|n_\rho, l_\rho, m_\rho; n_\lambda, l_\lambda, m_\lambda\rangle \equiv \Psi_{n_\rho, l_\rho, m_\rho}(\boldsymbol{\rho})\Psi_{n_\lambda, l_\lambda, m_\lambda}(\boldsymbol{\lambda})$$

$$\Psi_{n,l,m}(\boldsymbol{r}) \equiv \sqrt{\frac{2\,\Gamma(n+1)}{\Gamma(n+l+3/2)}}\alpha^{l+3/2} r^l e^{-\frac{\alpha^2 r^2}{2}} L_n^{l+1/2}(\alpha^2 r^2) Y_l^m(\theta, \varphi)$$

one obtains the Hamiltonian matrix:

$$\left\langle n'_\rho, l'_\rho, m'_\rho; n'_\lambda, l'_\lambda, m'_\lambda \middle| \widehat{H} \middle| n_\rho, l_\rho, m_\rho; n_\lambda, l_\lambda, m_\lambda \right\rangle$$

then the lowest eigenvalue (ground state) as *E*

minimize energy ***E*** by varying width parameter α

Optimal Parameters:

Cornell Potential:

Meson: $$V(r_{ij}) = -\frac{\alpha_{q\bar{q}}}{r_{ij}} + \sigma_{q\bar{q}} r_{ij}$$

Baryon: $$V(r_{ij}) = -\frac{\alpha_{qq}}{r_{ij}} + \sigma_{qq} r_{ij}$$

$$m_u = m_d = 0.150 \text{ GeV}$$
$$m_s = 0.301 \text{ GeV}$$
$$m_c = 1.458 \text{ GeV}$$
$$m_b = 4.824 \text{ GeV}$$
$$\alpha_{q\bar{q}} = 2\alpha_{qq} = 0.50$$
$$\sigma_{q\bar{q}} = 2\sigma_{qq} = 0.17 \text{ GeV}^2$$

Take a universal set of quark mass and coupling parameters for all hadrons!

Results 1

Heavy Flavor Mesons

Meson	J^P	M_E (GeV)	M_T (GeV)	D_R	r_{rms} (fm)
D^0	0^-	1.865	1.908	2.3%	0.41
D^{*0}	1^-	2.007	2.057	2.5%	0.48
D^+	0^-	1.870	1.908	2.0%	0.41
D^{*+}	1^-	2.010	2.057	2.3%	0.48
D_s	0^-	1.968	2.006	1.9%	0.39
D_s^*	1^-	2.112	2.165	2.5%	0.46
η_c	0^-	2.984	2.966	0.6%	0.29
$\eta_c(2S)$	0^-	3.637	3.580	1.6%	0.63
h_{c1}	1^+	3.525	3.525	0.0%	0.55
J/ψ	1^-	3.097	3.149	1.7%	0.37
ψ	1^-	3.686	3.697	0.3%	0.69
χ_{c0}	0^+	3.415	3.440	0.7%	0.47
χ_{c1}	1^+	3.511	3.520	0.3%	0.54
χ_{c2}	2^+	3.556	3.542	0.4%	0.57
B^-	0^-	5.279	5.310	0.2%	0.44
B^{-*}	1^-	5.325	5.365	0.8%	0.47
B^0	0^-	5.280	5.310	0.6%	0.44
B^{0*}	1^-	5.325	5.365	0.8%	0.47
B_s	0^-	5.367	5.402	0.7%	0.41
B_s^*	1^-	5.415	5.467	1.0%	0.44
η_b	0^-	9.399	9.284	1.2%	0.16
$\eta_b(2S)$	0^-	9.999	9.948	0.5%	0.42
h_{b1}	1^+	9.899	9.939	0.4%	0.37
$\Upsilon(1S)$	1^-	9.460	9.500	0.4%	0.21
$\Upsilon(2S)$	1^-	10.023	10.031	0.1%	0.47
χ_{b0}	0^+	9.859	9.887	0.3%	0.33
χ_{b1}	1^+	9.893	9.932	0.4%	0.37
χ_{b2}	2^+	9.912	9.953	0.4%	0.38

Results 1

Meson	J^P	M_E (GeV)	M_T (GeV)	D_R	r_{rms} (fm)
D^0	0				
D^{*0}	1				
D^+	0				
D^{*+}	1				
D_s	0				
D_s^*	1				
η_c	0				
$\eta_c(2S)$	0				
h_{c1}	1				
J/ψ	1				
ψ	1				
χ_{c0}	0				
χ_{c1}	1				
χ_{c2}	2				
B^-	0				
B^{-*}	1				
B^0	0				
B^{0*}	1				
B_s	0				
B_s^*	1				
η_b	0				
$\eta_b(2S)$	0				
h_{b1}	1				
$\Upsilon(1S)$	1				
$\Upsilon(2S)$	1				
χ_{b0}	0				
χ_{b1}	1				
χ_{b2}	2				

Heavy Flavor Baryons

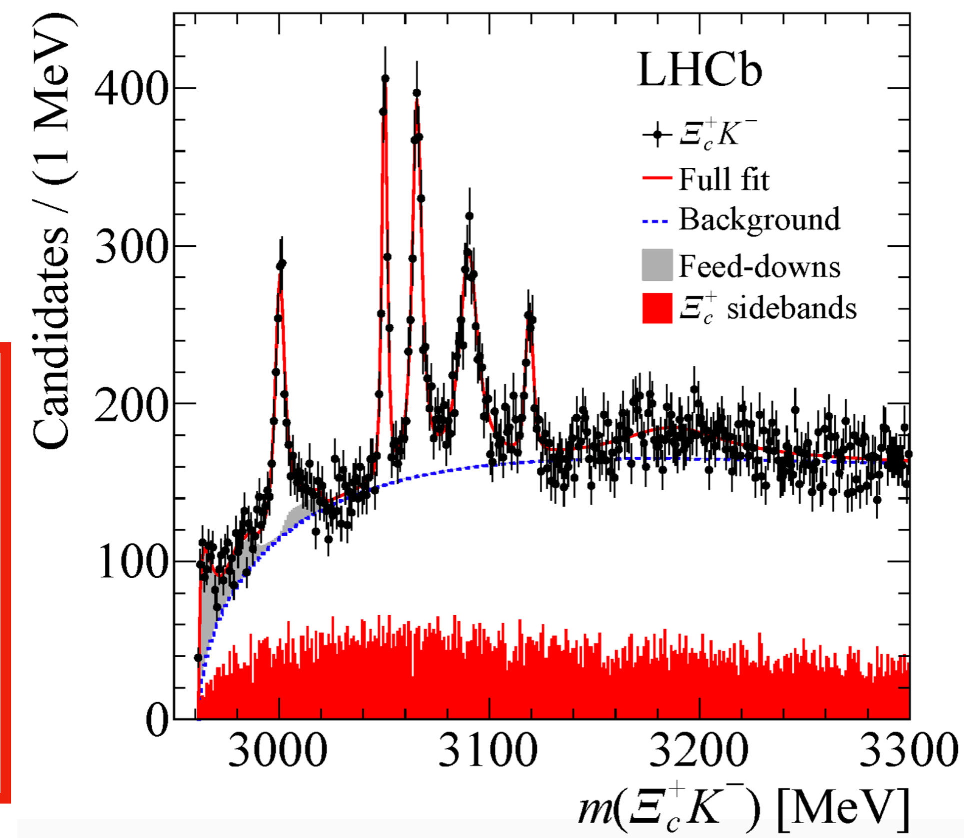
Baryon	J^P	M_E (GeV)	M_T (GeV)	D_R
Λ_c^+	$(1/2)^+$	2.286	2.383	4.2%
Σ_c^{++}	$(1/2)^+$	2.454	2.356	4.0%
Σ_c^+	$(1/2)^+$	2.453	2.356	4.0%
Σ_c^0	$(1/2)^+$	2.454	2.356	4.0%
Ξ_c^+	$(1/2)^+$	2.468	2.517	1.9%
Ξ_c^0	$(1/2)^+$	2.471	2.517	1.8%
$\Xi_c^{\prime +}$	$(1/2)^+$	2.577	2.532	1.8%
$\Xi_c^{\prime 0}$	$(1/2)^+$	2.579	2.532	1.9%
Ω_c^0	$(1/2)^+$	2.695	2.660	1.3%
Ξ_{cc}^{++}	$(1/2)^+$	3.621	3.616	0.1%
Ξ_{cc}^+	$(1/2)^+$	3.619	3.616	0.1%
Ω_{cc}^+	$(1/2)^+$		3.746	
Σ_c^{*++}	$(3/2)^+$	2.518	2.369	6.3%
Σ_c^{*+}	$(3/2)^+$	2.518	2.369	6.3%
Σ_c^{*0}	$(3/2)^+$	2.518	2.369	6.3%
Ξ_c^{*+}	$(3/2)^+$	2.646	2.527	4.5%
Ξ_c^{*0}	$(3/2)^+$	2.646	2.527	4.5%
Ω_c^{*0}	$(3/2)^+$	2.766	2.669	3.5%
Ξ_{cc}^{*++}	$(3/2)^+$		3.627	
Ξ_{cc}^{*+}	$(3/2)^+$		3.627	
Ω_{cc}^{*+}	$(3/2)^+$		3.754	
Ω_{ccc}^{++}	$(3/2)^+$		4.790	

Λ_b^0	$(1/2)^+$	5.620	5.744	2.3%
Σ_b^+	$(1/2)^+$	5.811	5.720	1.5%
Σ_b^0	$(1/2)^+$		5.720	
Σ_b^-	$(1/2)^+$	5.816	5.720	1.6%
Ξ_b^0	$(1/2)^+$	5.792	5.871	1.4%
Ξ_b^-	$(1/2)^+$	5.795	5.871	1.3%
Ω_b^-	$(1/2)^+$	6.046	6.007	0.6%
Ξ_{bb}^+	$(1/2)^+$		10.195	
Ξ_{bb}^0	$(1/2)^+$		10.195	
Ω_{bb}^-	$(1/2)^+$		10.318	
Σ_b^{*+}	$(3/2)^+$	5.832	5.736	1.6%
Σ_b^{*0}	$(3/2)^+$		5.736	
Σ_b^{*-}	$(3/2)^+$	5.835	5.736	1.6%
Ξ_b^{*0}	$(3/2)^+$		5.886	
Ξ_b^{*-}	$(3/2)^+$		5.886	
Ω_b^{*-}	$(3/2)^+$		6.021	
Ξ_{bb}^{*+}	$(3/2)^+$		10.210	
Ξ_{bb}^{*0}	$(3/2)^+$		10.210	
Ω_{bb}^{*-}	$(3/2)^+$		10.332	
Ω_{bbb}^-	$(3/2)^+$		14.440	

Results 2

Prediction on [css] Excitations

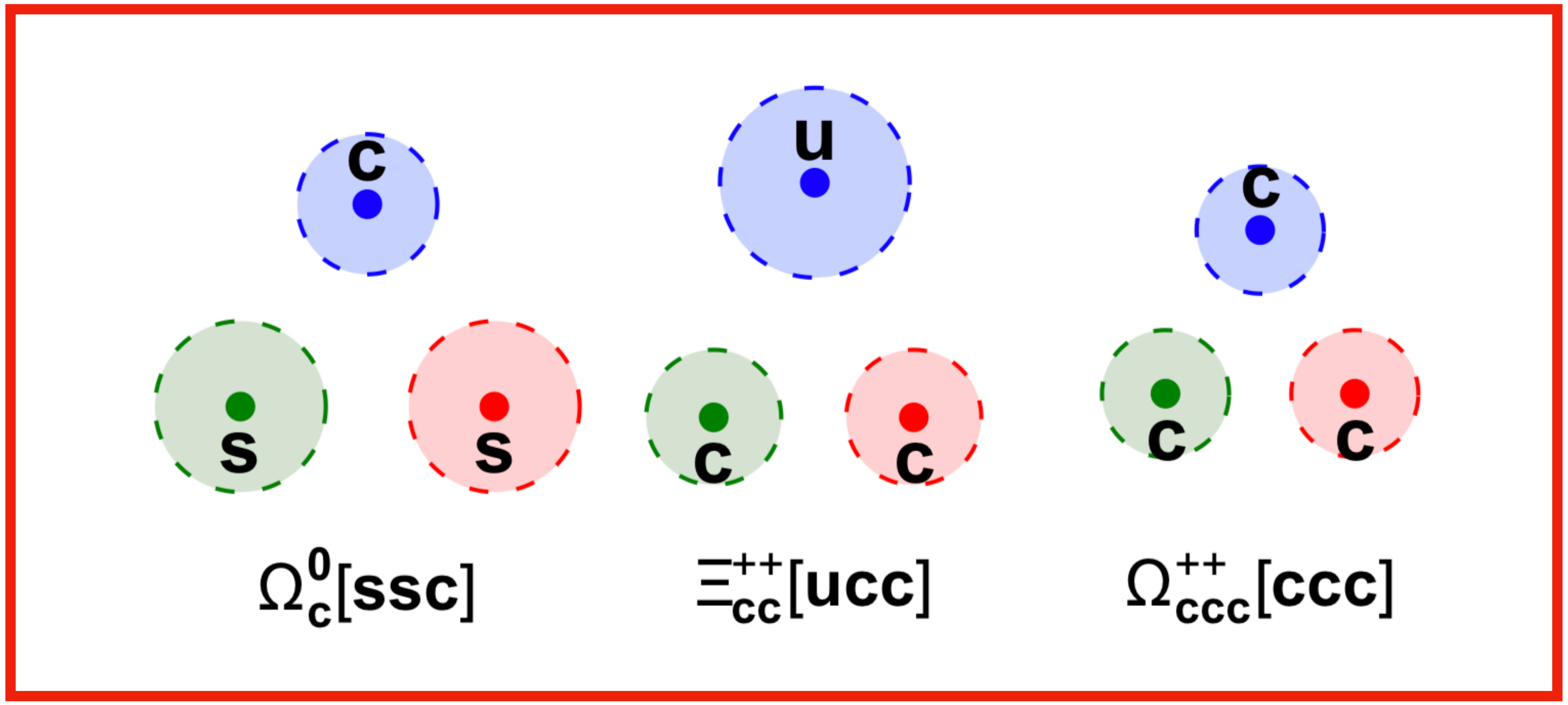
Baryon	Experiment J^P	Experiment M_E (GeV)	Model J^P	Model M_T (GeV)	D_R
Ω_c^0	$(1/2)^+$	2.695	$(1/2)^+(1S)$	2.660	1.3%
$\Omega_c^*(2770)^0$	$(3/2)^+$	2.766	$(3/2)^+(1S)$	2.669	3.5%
$\Omega_c(3000)^0$		3.000	$(1/2)^-(1P)$	2.965	1.2%
$\Omega_c(3050)^0$		3.050	$(3/2)^-(1P)$	3.042	0.3%
$\Omega_c(3065)^0$	?	3.065	$(1/2)^-(1P)$	3.053	0.3%
$\Omega_c(3090)^0$		3.090	$(3/2)^-(1P)$	3.220	4.2%
$\Omega_c(3120)^0$		3.119	$(5/2)^-(1P)$	3.292	5.5%



Results 3

Root-Mean-Squared radius and spatial profile

Baryon	r_{rms}	$\langle r_{12}^2 \rangle^{1/2}$	$\langle r_{13}^2 \rangle^{1/2}$	$\langle r_{23}^2 \rangle^{1/2}$
$\Lambda_c^+, \Sigma_c, \Sigma_c^*$	0.29	0.58	0.56	0.56
Ξ_c, Ξ_c^*	0.29	0.58	0.55	0.54
$\Omega_c^0, \Omega_c^{*0}$	0.29	0.57	0.53	0.53
Ξ_{cc}, Ξ_{cc}^*	0.28	0.56	0.56	0.45
$\Omega_{cc}^{++}, \Omega_{cc}^{*++}$	0.27	0.53	0.53	0.44
Ω_{ccc}^{++}	0.24	0.42	0.42	0.42



Summary and Outlook

Study the heavy flavor hadrons via improved multi-body Dirac equation with a universal set of quark mass and coupling parameters.

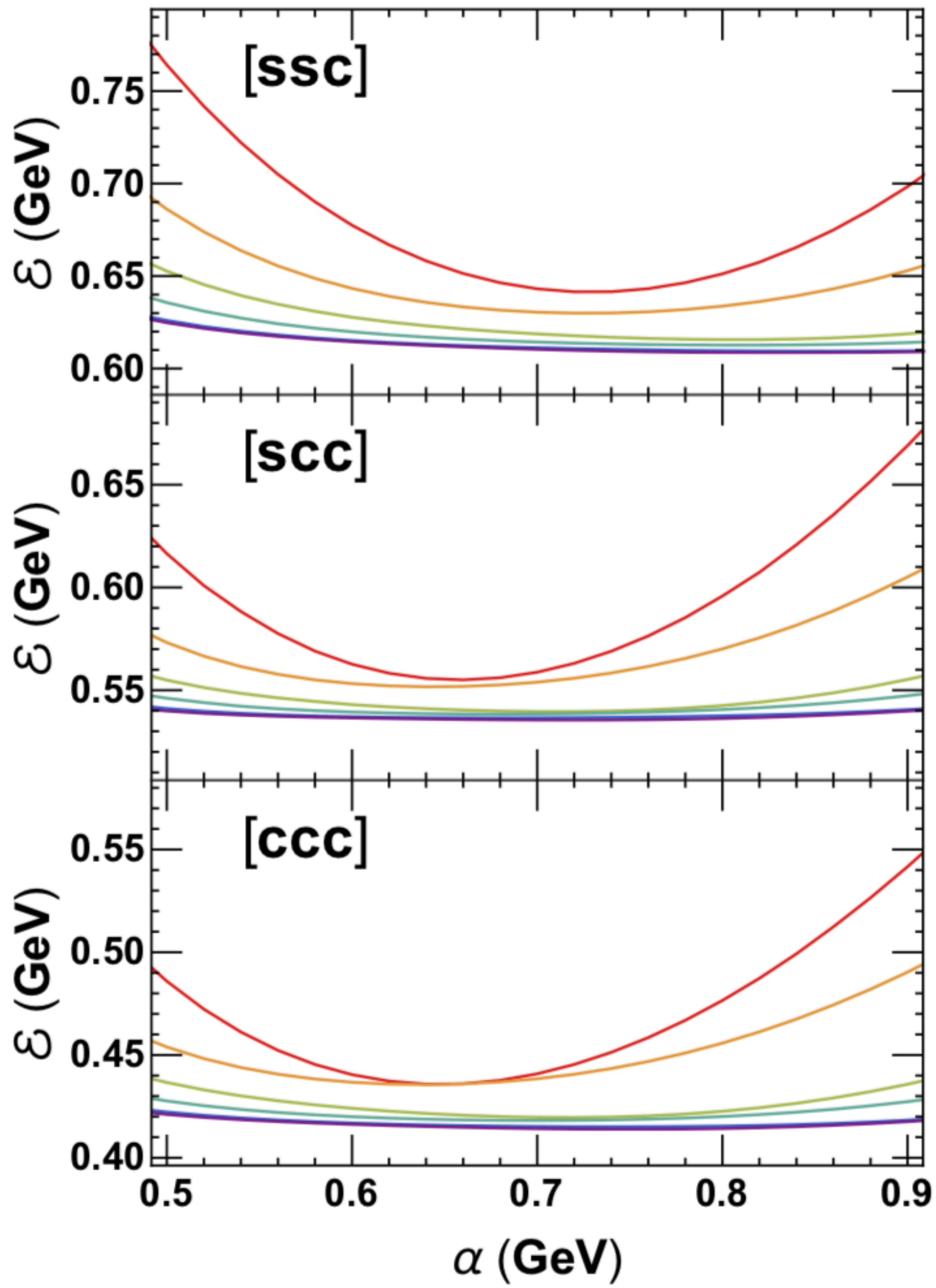
1. Agree well with the experimental data, with relative difference $< 2.5\%$ for mesons and $< 6.3\%$ for baryons.
2. Not only for ground state but also excited states.
3. Construct Wigner function from wavefunctions.
4. More powerful and predictable for multi-quark states.

Summary and Outlook

1. Extend to Tetraquark or Pentaquark states.
2. Extend to finite temperature and finite baryon density.

Thank you !

Backup



Here we test the convergency of basis-expansion for [J=3/2] states with including all basis with $N \leq$ **0**, **2**, **4**, **6**, **8** [$N = 2n_\rho + 2n_\lambda + l_\rho + l_\lambda$]
Correspondingly, # of included basis N_{basis} = **1**, **8**, **34**, **108**, **259.**