Novel method for precisely measuring the $X(3872)$ mass

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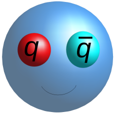
Based on: FKG, Phys. Rev. Lett. 122, 202002 (2019) [arXiv:1902.11221]

- XYZ states, $X(3872)$
- Kinematic singularities: threshold cusp, triangle singularity
- New method for measuring the $X(3872)$ mass

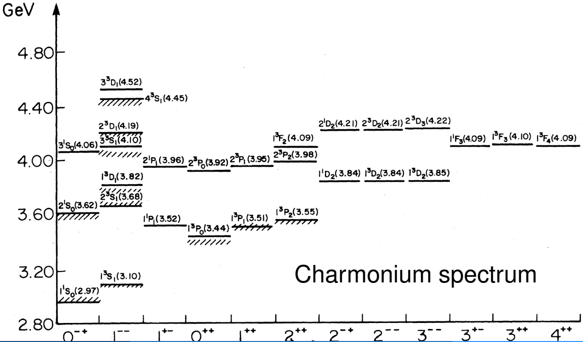
Godfrey–Isgur quark model

Mesons in a Relativized Quark Model with Chromodynamics
S. Godfrey, Nathan Isgur (Toronto U.). 1985. 43 pp.
Published in **Phys.Rev. D32 (1985) 189-231**
DOI: 10.1103/PhysRevD.32.189



$$\left(\sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} + V\right)|\Psi\rangle = E|\Psi\rangle$$

Potential V: One-gluon exchange + linear confinement + relativistic effects



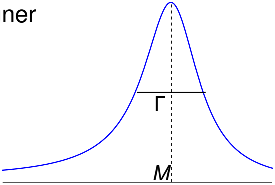
New discoveries since 2003

Many new hadron resonances observed in experiments since 2003

- Inactive: BaBar, Belle, CDF, CLEO-c, D0, ...
- Running: Belle-II, BESIII, COMPASS, LHCb, ...
- Under construction/discussion: PANDA, EIC, EicC, ...

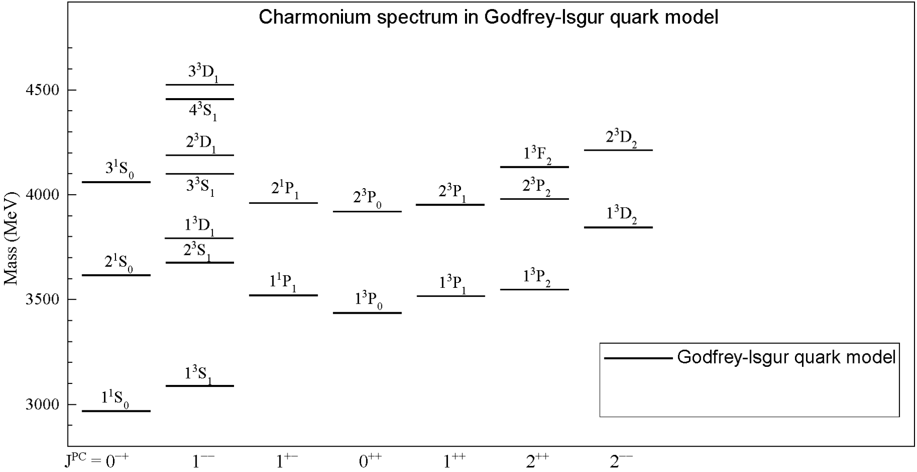
Common strategy: search for peaks, fit with Breit–Wigner

$$\propto \frac{1}{(s-M^2)^2 + s\,\Gamma^2(s)}$$



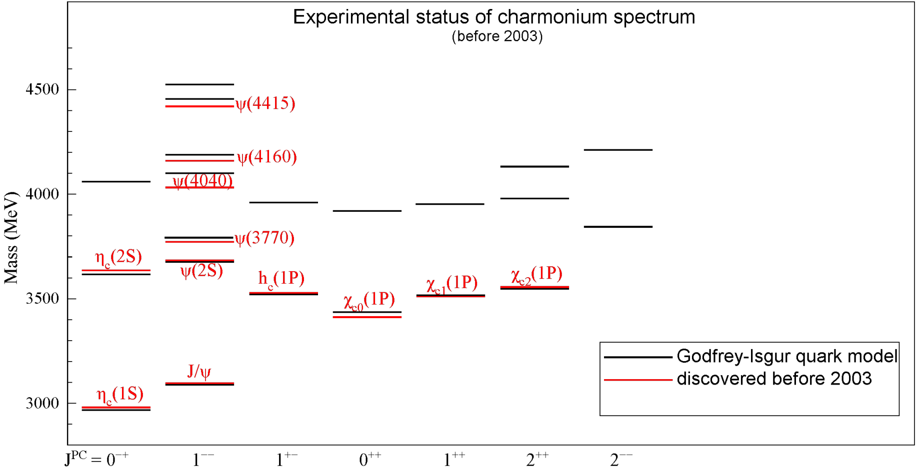
Lots of mysteries right now ...

XYZ states



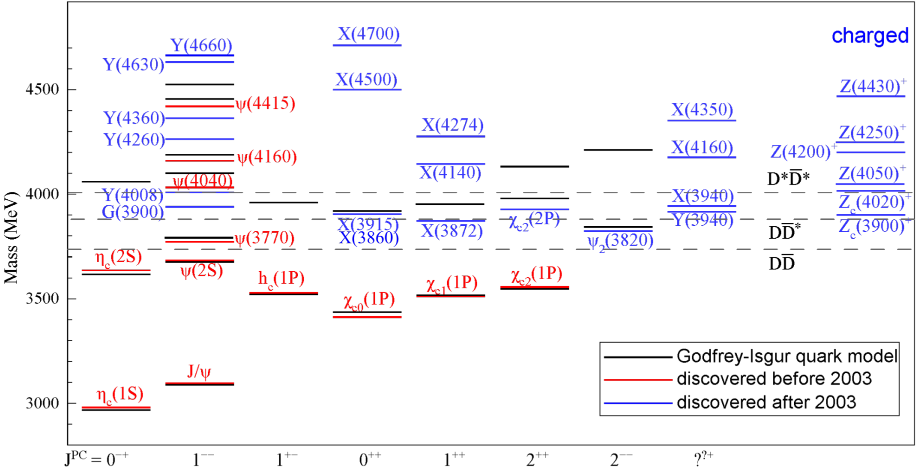
Charmonium spectrum in Godfrey-Isgur quark model
Mass (MeV)
4500
4000
3500
3000
3^3D_1
4^3S_1
2^3D_1
3^3S_1
1^3D_1
2^3S_1
1^3S_1
3^1S_0
2^1S_0
1^1S_0
2^1P_1
1^1P_1
2^3P_0
1^3P_0
2^3P_1
1^3P_1
1^3F_2
2^3P_2
1^3P_2
2^3D_2
1^3D_2
Godfrey-Isgur quark model
$J^{PC} = 0^{-+}$
1^{--}
1^{+-}
0^{++}
1^{++}
2^{++}
2^{--}

XYZ states



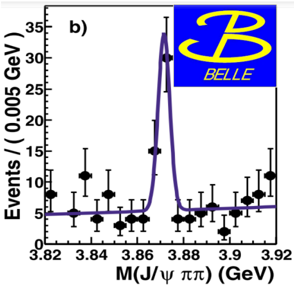
Experimental status of charmonium spectrum
(before 2003)
Mass (MeV)
4500
4000
3500
3000
ψ(4415)
ψ(4160)
ψ(4040)
ψ(3770)
ψ(2S)
J/ψ
η_c(2S)
η_c(1S)
h_c(1P)
χ_c0(1P)
χ_c1(1P)
χ_c2(1P)
Godfrey-Isgur quark model
discovered before 2003
J^PC = 0^-+
1^--
1^+-
0^++
1^++
2^++
2^--

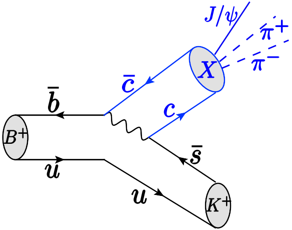
XYZ states



Note: J^{PC} of $X(3915)$ might also be 2^{++}

Belle, PRL91(2003)262001





- The beginning of the XYZ story, discovered in $B^{\pm} \to K^{\pm} J/\psi\pi\pi$

  $M_X = (3871.69 \pm 0.17)$ MeV
- $\Gamma < 1.2$ MeV   Belle, PRD84(2011)052004
- Confirmed in many experiments: Belle, BaBar, BESIII, CDF, CMS, D0, LHCb, ...
- 10 years later, $J^{PC} = 1^{++}$

  LHCb, PRL110(2013)222001

  $\Rightarrow$ S-wave coupling to $D\bar{D}^*$

Mysterious properties:

- Mass coincides with the $D^0\bar{D}^{*0}$ threshold:

  $M_{D^0} + M_{D^{*0}} - M_X = (0.00 \pm 0.18)$ MeV

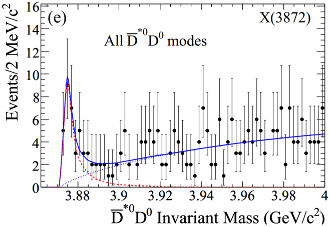
$X(3872)$: properties (2)

Mysterious properties (cont.):

- Large coupling to $D^0\bar{D}^{*0}$:

$\mathcal{B}(X \to D^0\bar{D}^{*0}) > 30\%$ Belle, PRD81(2010)031103

$\mathcal{B}(X \to D^0\bar{D}^0\pi^0) > 40\%$ Belle, PRL97(2006)162002



- No isospin partner observed $\Rightarrow I = 0$

but, large isospin breaking:

$$\frac{\mathcal{B}(X \to \omega J/\psi)}{\mathcal{B}(X \to \pi^+\pi^- J/\psi)} = 0.8 \pm 0.3$$

BaBar, PRD77(2008)011102

$$C(X) = +, C(J/\psi) = - \quad \Rightarrow C(\pi^+\pi^-) = - \quad \Rightarrow I(\pi^+\pi^-) = 1$$

- Radiative decays:

$$\frac{\mathcal{B}(X \to \gamma\psi')}{\mathcal{B}(X \to \gamma J/\psi)} = 2.6 \pm 0.6$$

PDG18 average of BaBar(2009) and LHCb(2014)

Upper limit at Belle: < 2.1 at 90% C.L. Belle, PRL107(2011)091803

Notice: New BESIII result: < 0.59 at 90% C.L. see C.-Z. Yuan, talk at Lattice2019

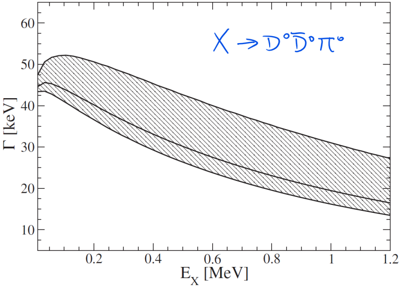
$X(3872)$: important observables

$$\delta \equiv M_{D^0} + M_{D^{*0}} - M_X = (0.00 \pm 0.18)\ \text{MeV}$$

- Long-distance wave function (below th.) given by $D^0\bar{D}^{*0}$: Braaten, Voloshin, ...

$$\psi_X(k^2) = \frac{\sqrt{8\pi\gamma_X}}{k^2+\gamma_X^2} \qquad \text{binding momentum}: \ \gamma_X \equiv \sqrt{2\delta\frac{M_{D^{*0}}M_{D^0}}{M_{D^{*0}}+M_{D^0}}}$$

- Line shapes, mass and width are important to understand the $X(3872)$
  Hanhart et al., PRD76(2007)034007; Artoisenet, Braaten, Kang, PRD82(2010)014013; ...
- Width: sensitivity of $\lesssim 100$ keV at PANDA
  PANDA, EPJA55(2019)42
- Mass $\Rightarrow$ Coupling of X to $D^0\bar{D}^{*0}$ for the molecular component
  Is the X above or below the $D^0\bar{D}^{*0}$ threshold? High precision!



Prediction in XEFT

Fleming et al., PRD76(2007)034006

$X(3872)$ mass

PDG2018 average from the $J/\psi\pi\pi$ and $J/\psi\pi\pi\pi$ modes

$\chi_{c1}(3872)$ MASS FROM $J/\psi X$ MODE

INSPIRE search

VALUE (MeV)	*EVTS*		*DOCUMENT ID*		*TECN*	*COMMENT*
$\mathbf{3871.69 \pm 0.17}$	**OUR AVERAGE**					
$3871.9 \pm 0.7 \pm 0.2$	20 ± 5		ABLIKIM	2014	BES3	$e^+ e^- \to J/\psi\pi^+\pi^-\gamma$
$3871.95 \pm 0.48 \pm 0.12$	0.6k		AAIJ	2012H	LHCB	$p\,p \to J/\psi\pi^+\pi^- X$
$3871.85 \pm 0.27 \pm 0.19$	$\sim$ 170	1	CHOI	2011	BELL	$B \to K\pi^+\pi^- J/\psi$
$3873\ ^{+1.8}_{-1.6} \pm 1.3$	27 ± 8	2	DEL-AMO-SANCH..	2010B	BABR	$B \to \omega J/\psi K$
$3871.61 \pm 0.16 \pm 0.19$	6k	3, 2	AALTONEN	2009AU	CDF2	$p\,\overline{p} \to J/\psi\pi^+\pi^- X$
$3871.4 \pm 0.6 \pm 0.1$	93.4		AUBERT	2008Y	BABR	$B^+ \to K^+ J/\psi\pi^+\pi^-$
$3868.7 \pm 1.5 \pm 0.4$	9.4		AUBERT	2008Y	BABR	$B^0 \to K^0_S\ J/\psi\pi^+\pi^-$
$3871.8 \pm 3.1 \pm 3.0$	522	4, 2	ABAZOV	2004F	D0	$p\,\overline{p} \to J/\psi\pi^+\pi^- X$

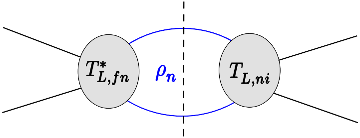
D^0, D^{*0} masses (PDG AVERAGE):

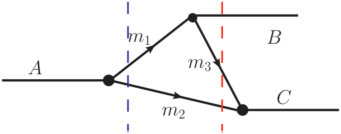
$$M_{D^0} = (1864.84 \pm 0.05)\ \text{MeV}, \quad M_{D^{*0}} = (2006.85 \pm 0.05)\ \text{MeV}$$

Two- and three-body Landau singularities

- Landau singularities (kinematic, fixed by the involved kinematic variables: masses, energies)
  - normal two-body threshold cusp
  - triangle singularity
  - ...

  traps/tools in hadron spectroscopy





Unitarity and threshold cusp

- Unitarity of the S-matrix: $S\,S^\dagger = S^\dagger S = \mathbb{1}$, $S_{fi} = \delta_{fi} - i(2\pi)^4\delta^4(p_f - p_i)T_{fi}$

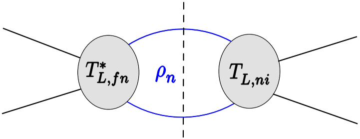
  T-matrix: $T_{fi} - T^\dagger_{fi} = -i(2\pi)^4 \underbrace{\sum_n}_{\text{all physically accesible states}} \delta(p_n - p_i) T^\dagger_{fn} T_{ni}$

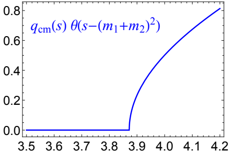
  assuming all intermediate states are two-body, partial-wave unitarity relation:

  $$\boxed{\text{Im}\, T_{L,fi}(s) = -\sum_n T^*_{L,fn}\,\rho_n(s)\,T_{L,ni}}$$

  2-body phase space factor: $\rho_n(s) = q_{\text{cm},n}(s)/(2\sqrt{s})\theta(\sqrt{s} - m_{n1} - m_{n2})$,

  $q_{\text{cm},n}(s) = \sqrt{[s-(m_{n1}+m_{n2})^2][s-(m_{n1}-m_{n2})^2]}/(2\sqrt{s})$



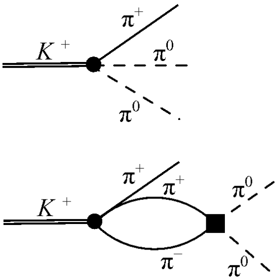


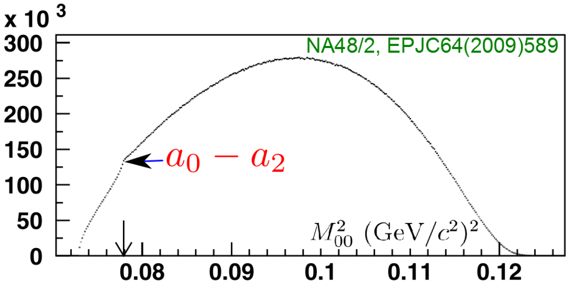
- There is always a cusp at an S-wave threshold

Threshold cusp: a well-known example

- Cusp effect as a useful tool for precise measurement:
  - ☞ example of the cusp in $K^\pm \to \pi^\pm\pi^0\pi^0$
  - ☞ strength of the cusp measures the interaction strength!

Meißner, Müller, Steininger (1997); Cabibbo (2004); Colangelo, Gasser, Kubis, Rusetsky (2006); . . .



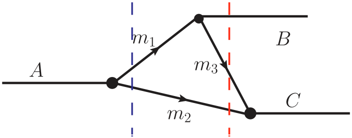


$\sim$ threshold, only sensitive to scattering length, $(a_0 - a_2)M_{\pi^+} = 0.2571 \pm 0.0056$

- Very prominent cusp $\Rightarrow$ large scattering length $\Rightarrow$ likely a nearby pole

effective range expansion: $f(k) = \dfrac{1}{1/a + rk^2/2 - ik}$

Triangle singularity



$$\frac{1}{2m_A}\sqrt{\lambda(m_A^2, m_1^2, m_2^2)} \equiv \boxed{p_{2,\text{left}} = p_{2,\text{right}}} \equiv \gamma\,(\beta\, E_2^* - p_2^*)$$

on-shell momentum of m_2 at the left and right cuts in the A rest frame

$\beta = |\vec{p}_{23}|/E_{23}, \gamma = 1/\sqrt{1-\beta^2}$ Bayar et al., PRD94(2016)074039

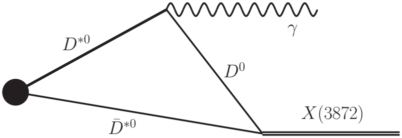
- $p_2 > 0$, $p_3 = \gamma\,(\beta\, E_3^* + p_2^*) > 0 \Rightarrow m_2$ and m_3 move in the same direction
- velocities in the A rest frame: $v_3 > \beta > v_2$

$$v_2 = \beta\,\frac{E_2^* - p_2^*/\beta}{E_2^* - \beta\, p_2^*} < \beta\,, \qquad v_3 = \beta\,\frac{E_3^* + p_2^*/\beta}{E_3^* + \beta\, p_2^*} > \beta$$

- Conditions (Coleman–Norton theorem): Coleman, Norton (1965); Bronzan (1964)
  - ☞ all three intermediate particles can go on shell simultaneously
  - ☞ $\vec{p}_2 \parallel \vec{p}_3$, particle-3 can catch up with particle-2 (as a classical process)
- needs very special kinematics ⇒ process dependent! (contrary to pole position)

New method for measuring the $X(3872)$ mass (1)

FKG, PRL122(2019)202002



Use of triangle singularity:

- Massless photon
- TS for the $X\gamma$ invariant mass ($\delta = M_{D^{*0}} + M_{D^0} - M_X$):

$$E_{X\gamma}^{\text{TS}} \simeq 2M_{D^{*0}} + \frac{1}{2M_{D^0}}\left(M_{D^{*0}} - M_{D^0} - 2\sqrt{-M_{D^0}\delta} + \delta\right)^2$$

$\Rightarrow$ TS in $E_{X\gamma}$ around the $D^{*0}\bar{D}^{*0}$ thr. (S-wave $D^{*0}\bar{D}^{*0}$ with $J^PC = 1^{+-}$)

- D^{0*} width is tiny:

$\Gamma(D^{*\pm}) = (83.4 \pm 1.8)$ keV, $\mathcal{B}(D^{*\pm} \to \pi^0 D^\pm) = (67.7 \pm 0.5)\%$,

$\mathcal{B}(D^{*0} \to \pi^0 D^0) = (64.7 \pm 0.9)\%$

$\Rightarrow \Gamma(D^{*0}) = (55.3 \pm 1.4)$ keV

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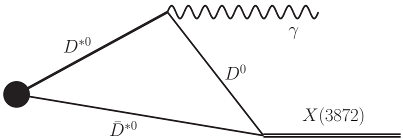
FKG, PRL122(2019)202002



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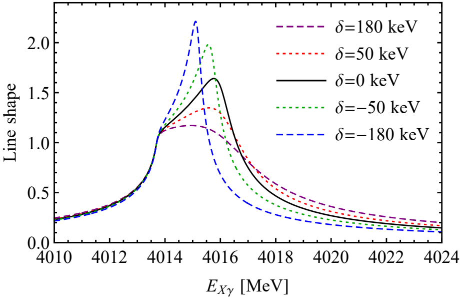
New method for measuring the $X(3872)$ mass (2)

$$E_{X\gamma}^{\text{TS}} \simeq 2M_{D^{*0}} + \frac{1}{2M_{D^0}}\left(M_{D^{*0}} - M_{D^0} - 2\sqrt{-M_{D^0}\delta} + \delta\right)^2$$



- For X blow $D^0\bar{D}^{*0}$ threshold, $\delta > 0$, TS is complex, smooth shape
- For X above $D^0\bar{D}^{*0}$ threshold, $\delta < 0$, TS is real $\Rightarrow$ logarithmic divergent peak if neglecting D^{*0} width
- Sharper peak when $\delta < 0$

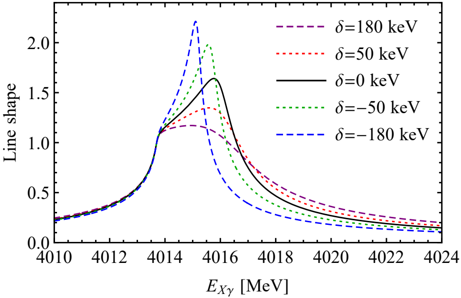
Line shape normalized at the $D^{*0}\bar{D}^{*0}$ threshold:

$$F(E_{X\gamma}) = \frac{|I(E_{X\gamma})|^2}{|I(2m_*)|^2}\frac{E_\gamma^3}{\left[(4m_*^2 - m_X^2)/(4m_*)\right]^3}$$

here $I(E_{X\gamma})$: the triangle loop integral

New method for measuring the $X(3872)$ mass (2)

$$E^{\rm TS}_{X\gamma} \simeq 2M_{D^{*0}} + \frac{1}{2M_{D^0}}\left(M_{D^{*0}} - M_{D^0} - 2\sqrt{-M_{D^0}\delta} + \delta\right)^2$$



- Cusp fixed at the $D^{*0}\bar{D}^{*0}$ threshold
- Peak fixed at the TS energy:

δ (keV)	$E^{\rm TS}_{X\gamma}$ (MeV)
-180	$4015.2 - i0.1$
-50	$4015.7 - i0.2$
0	$4016.0 - i0.4$

Line shape normalized at the $D^{*0}\bar{D}^{*0}$ threshold:

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here $I(E_{X\gamma})$: the triangle loop integral

Sensitivity study (1)

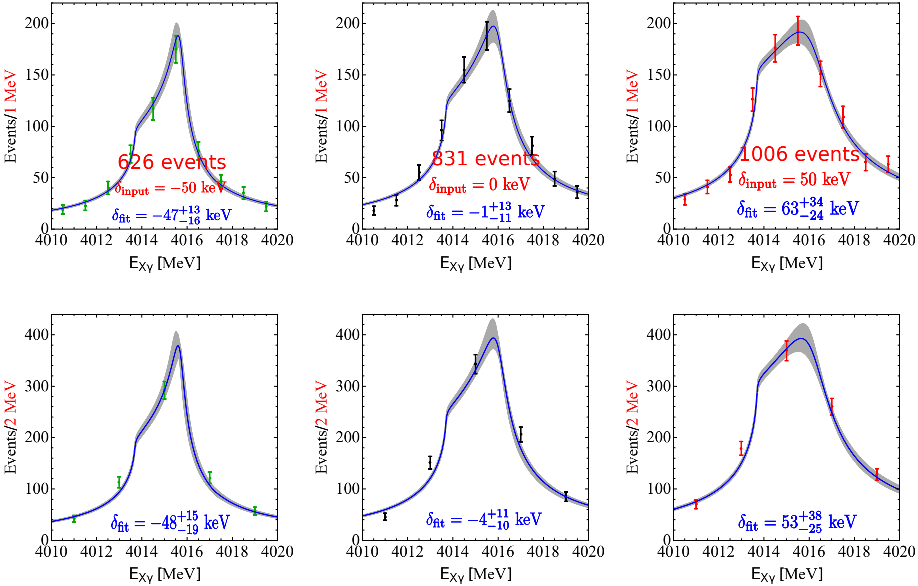
Sensitivity study from a simple Monte Carlo simulation:

(1) Generate synthetic events following the distribution

$$F(E_{X\gamma}) = \frac{|I(E_{X\gamma})|^2}{|I(2m_*)|^2} \frac{E_\gamma^3}{\left[(4m_*^2 - m_X^2)/(4m_*)\right]^3}$$

(2) Fit to the synthetic data treating δ as a free parameter

Sensitivity study (2)



Sensitivity study (3)

A	$\delta_{\text{in}} = -50$ keV (127 events)	$\delta_{\text{in}} = 0$ (164 events)	$\delta_{\text{in}} = 50$ keV (192 events)
10 bins	-24^{+24}_{-28}	11^{+31}_{-20}	22^{+41}_{-23}
5 bins	-17^{+24}_{-27}	30^{+64}_{-29}	40^{+67}_{-31}
B	$\delta_{\text{in}} = -50$ keV (626 events)	$\delta_{\text{in}} = 0$ (831 events)	$\delta_{\text{in}} = 50$ keV (1006 events)
10 bins	-47^{+13}_{-16}	-1^{+13}_{-11}	63^{+34}_{-24}
5 bins	-48^{+15}_{-19}	-4^{+11}_{-10}	53^{+38}_{-25}
C	$\delta_{\text{in}} = -50$ keV (3133 events)	$\delta_{\text{in}} = 0$ (4027 events)	$\delta_{\text{in}} = 50$ keV (5015 events)
10 bins	-53^{+7}_{-8}	-2 ± 5	55^{+13}_{-11}
5 bins	-52^{+7}_{-8}	-2^{+7}_{-6}	61^{+17}_{-14}

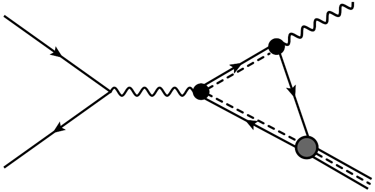
10 bins: 1 MeV/bin

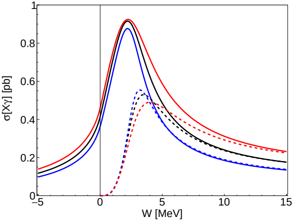
5 bins: 2 MeV/bin

Triangle singularity for $e^+e^- \to \gamma X(3872)$

- There can also be triangle singularity for $e^+e^- \to D^{*0}\bar{D}^{*0} \to \gamma X(3872)$
- $D^{*0}\bar{D}^{*0}$ in P-wave, thus suppressed at threshold, no threshold cusp
- But, still enhanced cross section and a narrow peak at about 2.2 MeV above the $D^{*0}\bar{D}^{*0}$ threshold

E. Braaten, L.-P. He, K. Ingles, arXiv:1904.12915





- For BESIII: to measure cross section for $e^+e^- \to \gamma X(3872)$ between 4009 MeV and 4020 MeV

Summary

- Unprecedented data allow us to study analytic structures of QFT other than the resonance poles
- Triangle singularity as a tool:
  - ☞ TS $\Rightarrow$ enhanced production of near-threshold states
  - ☞ New method for precisely measuring the $X(3872)$ mass:

    to measure the $X\gamma$ line shape between 4.01 and 4.02 GeV
- A few possibilities:
  - ☞ BESIII + STCF + Belle-II (ISR):

$$e^+e^- \to \pi^0 D^{*0}\bar{D}^{*0} \to \pi^0\gamma X(3872) \ @ \ \sqrt{s}_{e^+e^-} \sim 4.4 \text{ GeV}$$

  - ☞ B factories: $B \to K D^{*0}\bar{D}^{*0} \to K\gamma X(3872)$
  - ☞ PANDA: $p\bar{p} \to D^{*0}\bar{D}^{*0} \to \gamma X(3872)$

THANK YOU FOR YOUR ATTENTION!

Backup slides