Models for Dalitz plot analyses

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JPAC

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Recipes to build an amplitude

M. Mikhasenko, AP, J. Nys *et al.* (JPAC), EPJC78, 3, 229
AP, J. Nys, M. Mikhasenko *et al.* (JPAC), EPJC78, 9, 727

The literature abounds with discussions on the optimal approach to construct the amplitudes for the hadronic reactions

- Helicity formalism

  Jacob, Wick, Annals Phys. 7, 404 (1959)

- Covariant tensor formalisms

  Chung, PRD48, 1225 (1993)
  Chung, Friedrich, PRD78, 074027 (2008)
  Filippini, Fontana, Rotondi, PRD51, 2247 (1995)
  Anisovich, Sarantsev, EPJA30, 427 (2006)

The common lore is that the former one is nonrelativistic, especially when expressed in terms of LS couplings and the latter takes into account the proper relativistic corrections

How helicity formalism works

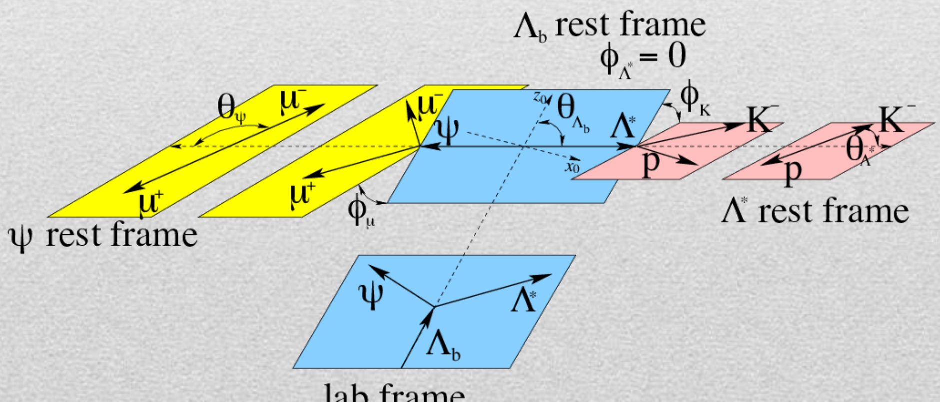
- Helicity formalism enforces the constraints about rotational invariance
- It allows us to fix the angular dependence of the amplitude
- What about energy dependence?

Example: $B \to \psi K^* \to \pi K$

$$\mathcal{M}^{K^*}_{\Delta\lambda_\mu} \equiv \sum_n \sum_{\lambda_{K^*}} \sum_{\lambda_\psi} \mathcal{H}^{B\to K_n^*\psi}_{\lambda_{K^*},\,\lambda_\psi}\, \delta_{\lambda_{K^*},\lambda_\psi}$$

$$\mathcal{H}^{K_n^*\to K\pi}\, D^{J_{K_n^*}}_{\lambda_{K^*},0}(\phi_K,\theta_{K^*},0)^*$$

$$R_{K^*_n}(m_{K\pi})\, D^{1}_{\lambda_\psi,\,\Delta\lambda_\mu}(\phi_\mu,\theta_\psi,0)^*,$$



LHCb, PRL115, 072001 (2015)

Each set of angles is defined in a different reference frame

How tensor formalism works

The method is based on the construction of explicitly covariant expressions.

- To describe the decay $a \to bc$, we first consider the polarization tensor of each particle, $\varepsilon^{i}_{\mu_1\dots\mu_{j_i}}(p_i)$
- We combine the polarizations of b and c into a "total spin" tensor $S_{\mu_1\dots\mu_S}(\varepsilon_b,\varepsilon_c)$
- Using the decay momentum, we build a tensor $L_{\mu_1\dots\mu_L}(p_{bc})$ to represent the orbital angular momentum of the bc system, orthogonal to the total momentum of p_a
- We contract S and L with the polarization of a

Tensor $\times R_X(m)$ which contain resonances and form factors

What do we know?

- Energy dependence is not constrained by symmetry
- Still, there are some known properties one can enforce

$$R_X(m) = B'_{L^X_{\Lambda_b^0}}(p, p_0, d)\left(\frac{p}{M_{\Lambda_b^0}}\right)^{L^X_{\Lambda_b^0}}$$

$$\mathrm{BW}(m|M_{0X}, \Gamma_{0X})\, B'_{L_X}(q, q_0, d)\left(\frac{q}{M_{0X}}\right)^{L_X}$$

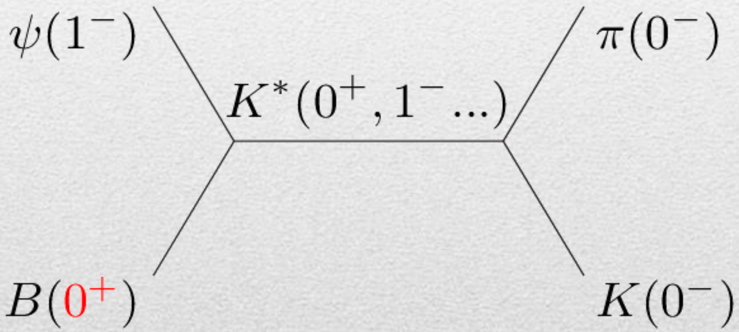
- Kinematical singularities: e.g. barrier factors (known)
- Left hand singularities (need model, e.g. Blatt-Weisskopf)
- Right hand singularities = resonant content (Breit Wigner, K-matrix...)

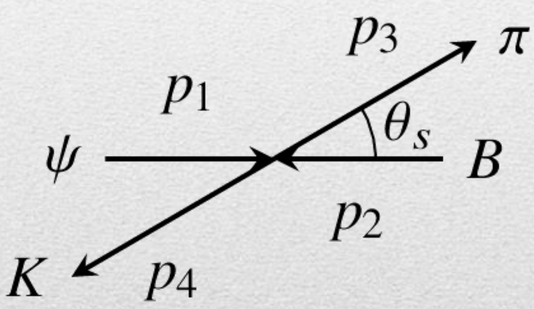
Kinematics

- Kinematical singularities appear because of the spin of the external particle involved
- We can write the most general covariant parametrization of the amplitude as
  tensor of external polarizations $\otimes$ scalar amplitudes
- Scalar amplitudes must be kinematical singularities free
- They can be matched to the helicity amplitudes
- We can get the minimal energy dependent factor
- Any other additional energy factor would be model-dependent

$B \to \psi\, \pi K$

To consider the effect of spin, let's consider $B \to \psi\pi K$
We focus on the parity violating amplitude for the K^* isobars, scattering kinematics





$$p = \text{incoming 3-momentum in the COM} = \frac{\lambda_{12}^{1/2}}{2\sqrt{s}}$$

$$= \frac{\sqrt{[s-(m_1+m_2)^2]\,[s-(m_1-m_2)^2]}}{2\sqrt{s}}$$

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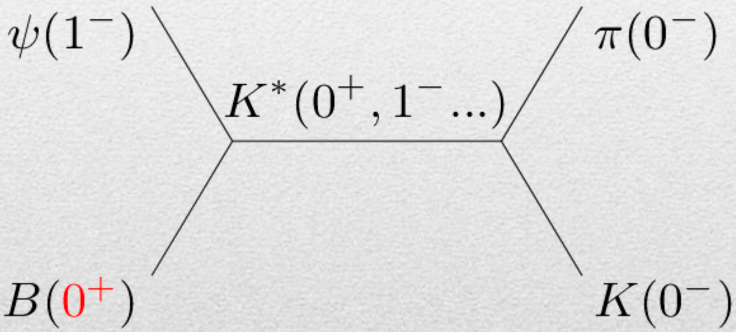

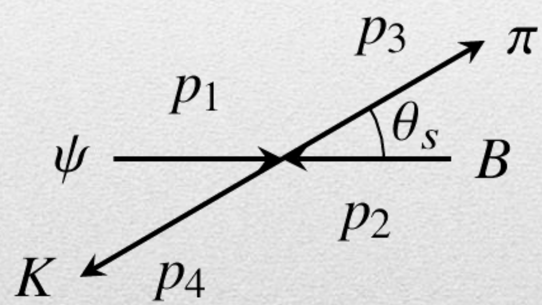

$$q = \text{outgoing 3-momentum in the COM} = \frac{\lambda_{34}^{1/2}}{2\sqrt{s}}$$

$$= \frac{\sqrt{[s-(m_3+m_4)^2]\,[s-(m_3-m_4)^2]}}{2\sqrt{s}}$$

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To consider the effect of spin, let's consider $B \to \psi\pi K$
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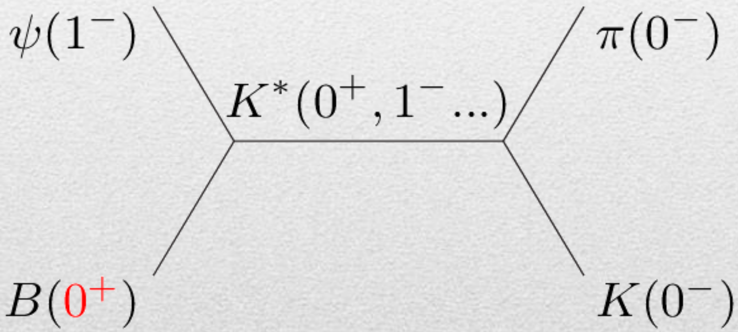
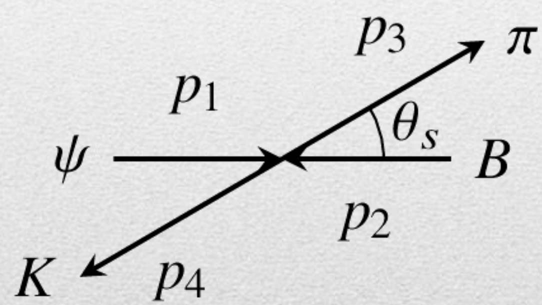




$$z_s = \text{cosine of the scatt. angle in the COM}$$
$$= \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\lambda_{12}^{1/2}\lambda_{34}^{1/2}} = \frac{\text{polynomial}}{\lambda_{12}^{1/2}\lambda_{34}^{1/2}}$$

Helicity amplitudes

$$A_\lambda = \frac{1}{4\pi}\sum_{j=|\lambda|}^{\infty}(2j+1)A^j_\lambda(s)\, d^j_{\lambda 0}(z_s)$$

$$d^j_{\lambda 0}(z_s) = \hat{d}^j_{\lambda 0}(z_s)\xi_{\lambda 0}(z_s), \qquad \xi_{\lambda 0}(z_s) = \left(\sqrt{1-z_s^2}\right)^\lambda$$

$\hat{d}^j_{\lambda 0}(z_s)$ is a polynomial of order $j-|\lambda|$ in z_s,
The kinematical singularities of $A^j_\lambda(s)$ can be isolated by writing

$$A^j_0 = \frac{m_1}{p\sqrt{s}}\,(pq)^j\ \hat{A}^j_0 \qquad \text{for } j \geq 1,$$

$$A^j_\pm = q\ (pq)^{j-1}\ \hat{A}^j_\pm \qquad \text{for } j \geq 1,$$

$$A^0_0 = \frac{p\sqrt{s}}{m_1}\,\hat{A}^0_0 \qquad \text{for } j = 0,$$

Identify covariants

Two helicity couplings $\to$ two independent covariant structures
Important: we are not imposing any intermediate isobar

$$A_\lambda(s,t) = \varepsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3 + p_4)^\mu\right] C(s,t) + \varepsilon_\mu(\lambda, p_1)(p_3 + p_4)^\mu B(s,t)$$

$$C(s,t) = \frac{1}{4\pi\sqrt{2}}\sum_{j>0}(2j+1)(pq)^{j-1}\hat{A}^j_+(s)\,\hat{d}^j_{10}(z_s)$$

$$B(s,t) = \frac{1}{4\pi}\hat{A}^0_0 + \frac{1}{4\pi}\frac{4m_1^2}{\lambda_{12}}\sum_{j>0}(2j+1)(pq)^j\left[\hat{A}^j_0(s)\hat{d}^j_{00}(z_s) + \frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2}\hat{A}^j_+(s)\,z_s\hat{d}^j_{10}(z_s)\right]$$

Everything looks fine but the λ_{12} in the denominator
The brackets must vanish at $\lambda_{12} = 0 \Rightarrow s = s_\pm = (m_1 \pm m_2)^2$,
$\hat{A}^j_+$ and $\hat{A}^j_0$ cannot be independent

General expression and comparison

$$\hat{A}^j_+ = \langle j-1,0;1,1|j,1\rangle g_j(s) + f_j(s)$$

$$\hat{A}^j_0 = \langle j-1,0;1,0|j,0\rangle \frac{s+m_1^2-m_2^2}{2m_1^2} g'_j(s) + f'_j(s)$$

$g_j(s_\pm) = g'_j(s_\pm)$, and $f_j(s), f'_j(s) \sim O(s-s_\pm)$
All these four functions are free of kinematic singularity.

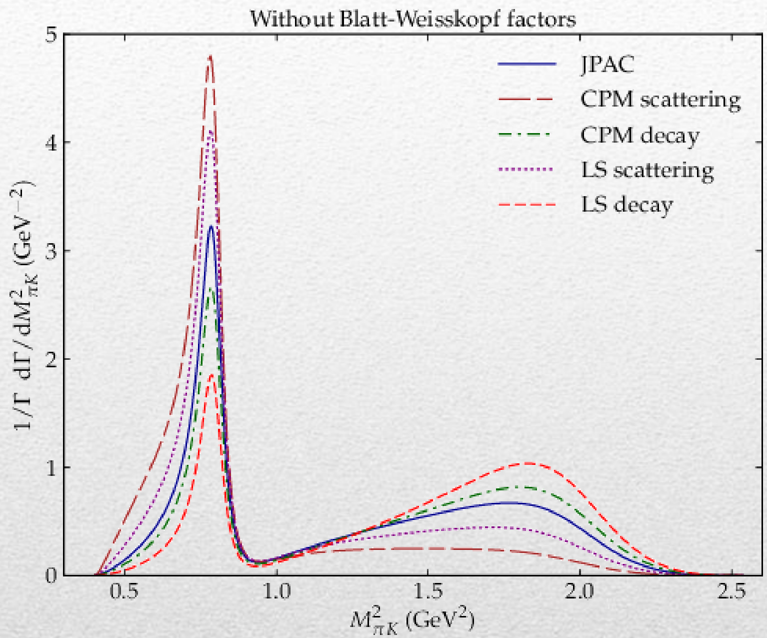
Comparison with tensor formalisms ($j=1$)

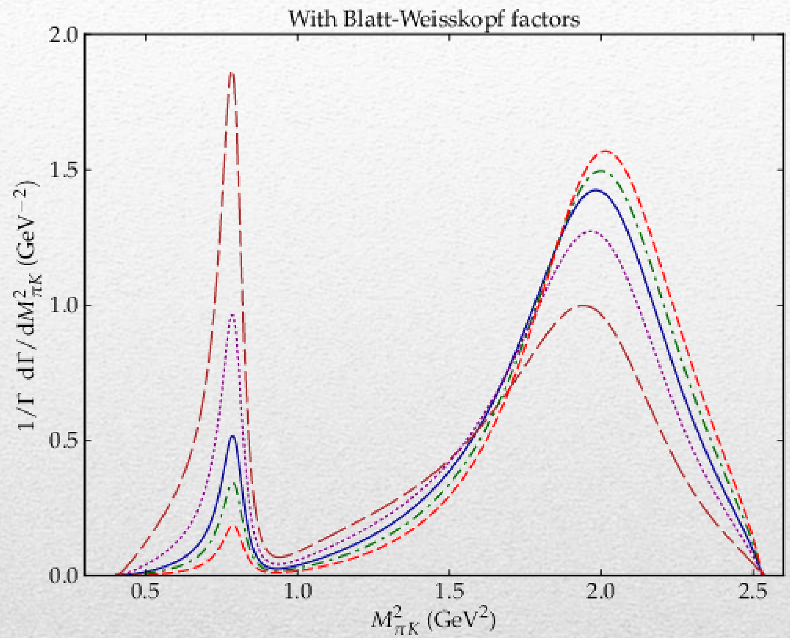
$$g_1 = g'_1 = \frac{4\pi}{3} g_S, \quad f_1 = \frac{2\pi\lambda_{12}}{3s} g_D, \quad f'_1 = -\frac{4\pi\lambda_{12}}{3s}\frac{s+m_1^2-m_2^2}{m_1^2} g_D.$$

If the g_S, g_D are the usual Breit-Wigner, the g', f' are fine

There is no unique recipe to build the right amplitude, but one can ensure the right singularities to be respected

General expression and comparison



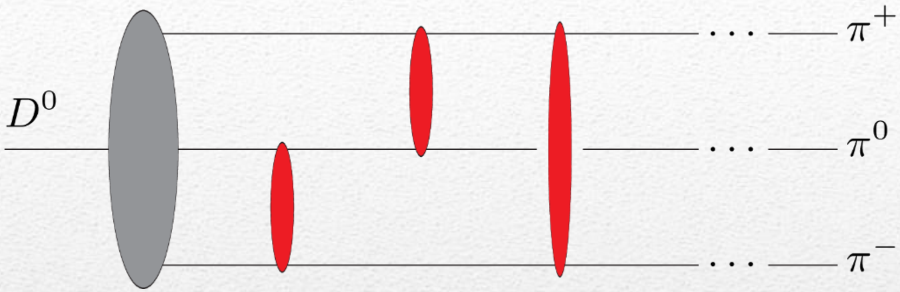


We consider the example of two intermediate $K^*(892)$ and $K^*(1410)$

We set $g_S(s) = 0$ and $g_D(s) =$ sum of Breit-Wigner

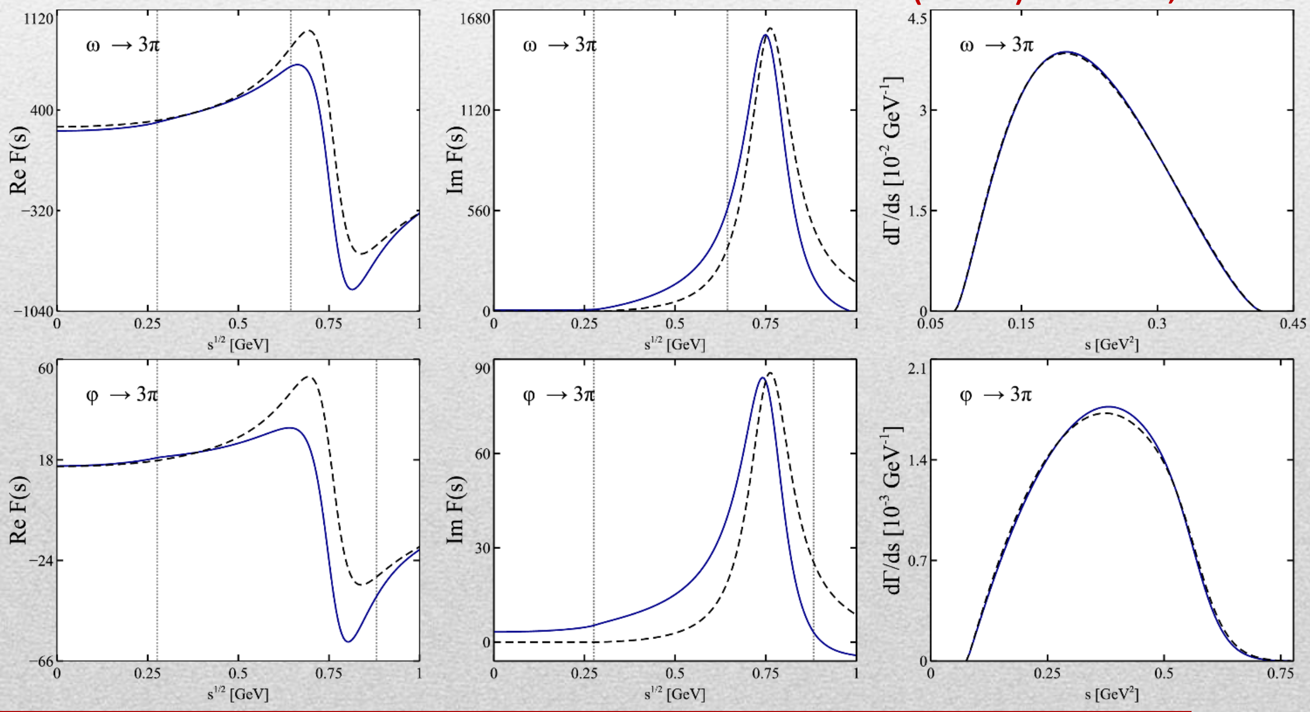
For the plot on the right we multiply the amplitudes by the Blatt-Weisskopf barrier factors

3-body interaction in Dalitz plots

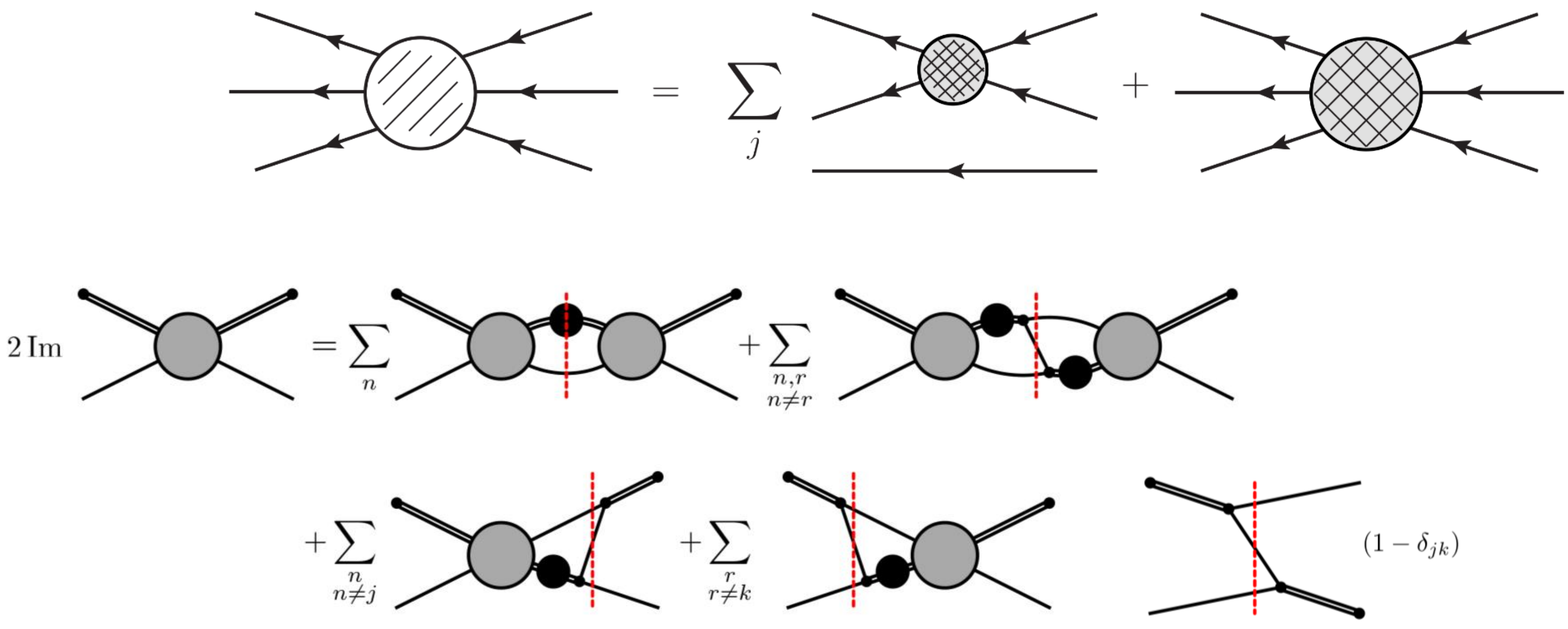


I. Danilkin *et al.* (JPAC) PRD91, 094029

The rescattering with the bachelor particle is known to modify the isobar lineshape
(Khuri-Treiman equations)

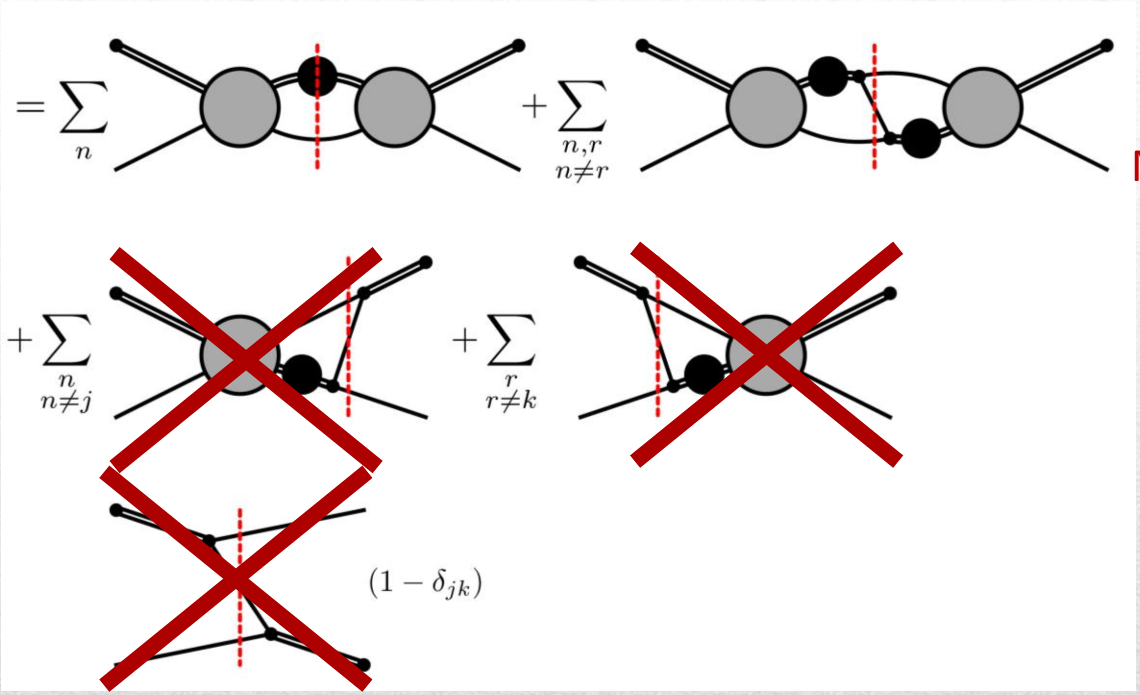


3-body unitarity



M. Mai, B. Hu, M. Doring, AP, A. Szczepaniak EPJA53, 9, 177
A. Jackura, ..., AP, *et al.* (JPAC), EPJC79, 1, 56

Factorizable model



M. Mikhasenko, AP, *et al.* (JPAC), PRD98 (2018) 9, 096021
M. Mikhasenko *et al.* (JPAC) to appear

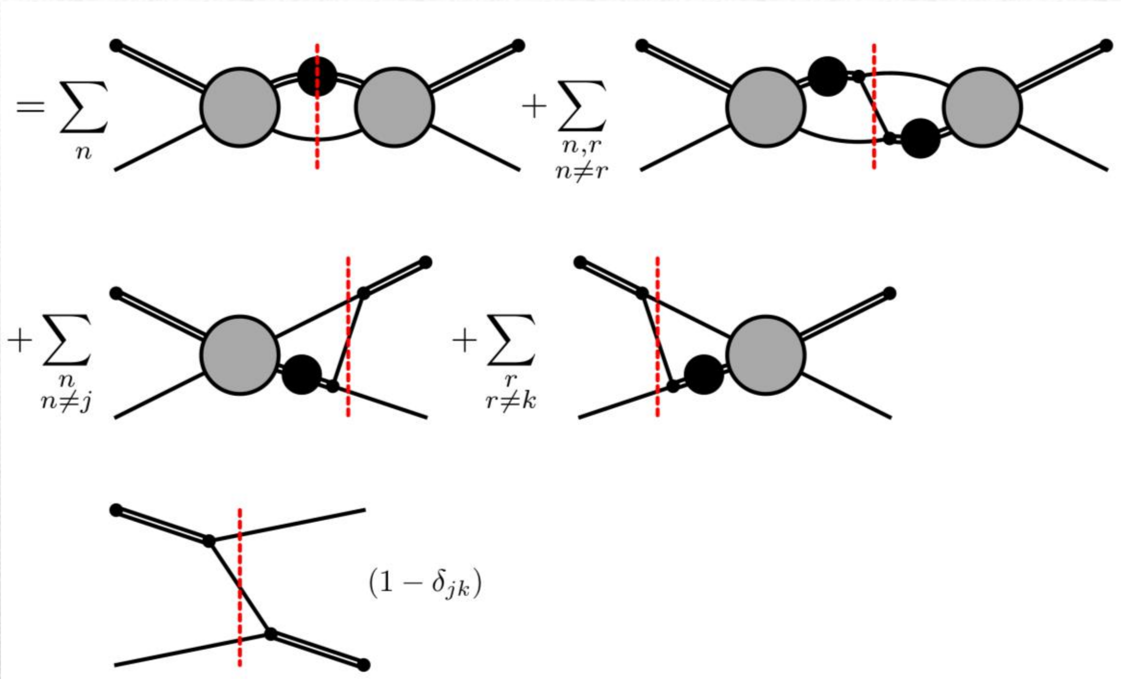
If one neglects the disconnected diagrams, one can suppress the dependence on the 2-body invariant masses

The unitarity equation is now algebraic and easier to handle

$$\mathrm{Im}\,\hat{A}(s) = \hat{A}(s)\,\hat{A}(s)^\dagger \int d\Phi_3 \left|\sum_j f(\sigma_j)\right|^2$$

Integral over the Dalitz plot (*aka* quasi 2-body)

Factorizable model



M. Mikhasenko, AP, *et al.* (JPAC), PRD98 (2018) 9, 096021
M. Mikhasenko *et al.* (JPAC) to appear

The approximation can be relaxed, but one can still obtain a (complicated) factorized form

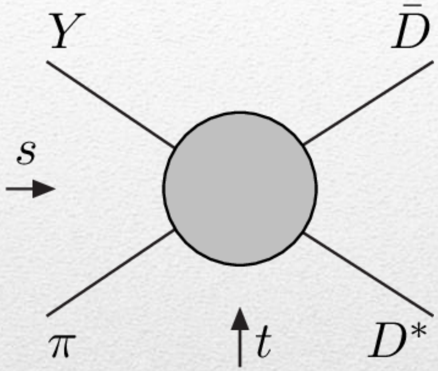
The quasi 2-body gets corrections from rescattering

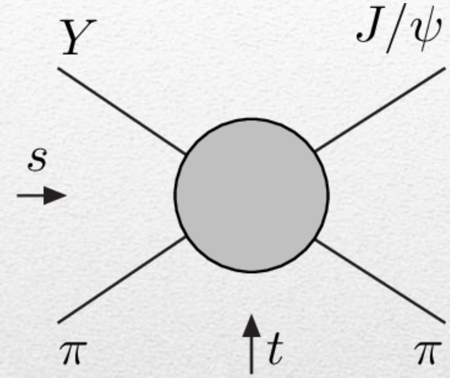
$$\mathrm{Im}\,\hat{A}(s) = \hat{A}(s)\,\hat{A}(s)^\dagger \int d\Phi_3 \left| \sum_j f(\sigma_j) \right|^2$$

Amplitude analysis for $Z_c(3900)$

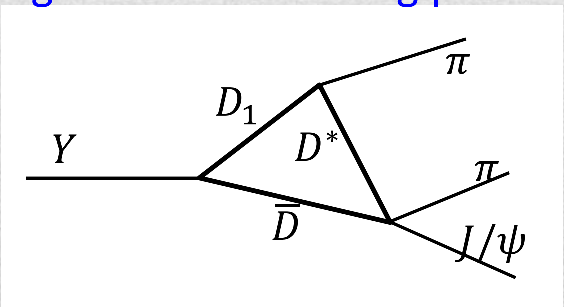
One can test different parametrizations of the amplitude, which correspond to
different singularities → different natures

AP *et al.* (JPAC), PLB772, 200





Triangle rescattering,
logarithmic branching point

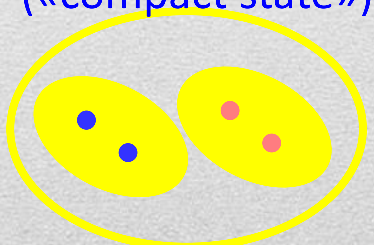


Szczepaniak, PLB747, 410-416
Szczepaniak, PLB757, 61-64
Guo *et al.* PRD92, 071502

(anti)bound state,
II/IV sheet pole
(«molecule»)

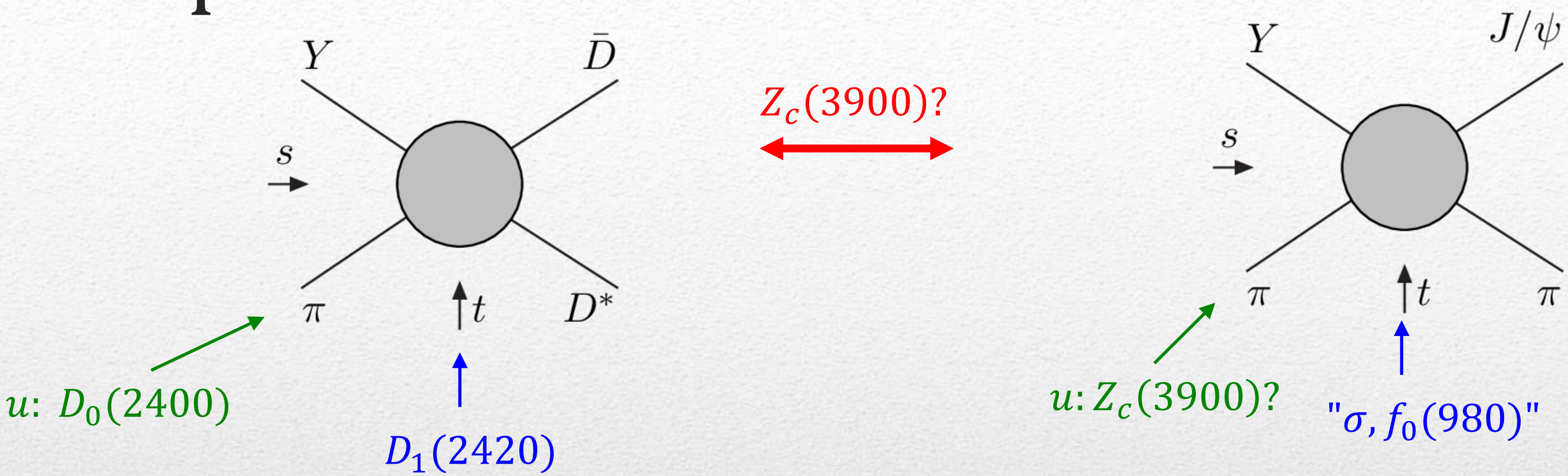
Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart *et al.* PRL111, 132003

Resonance,
III sheet pole
(«compact state»)



Maiani *et al.*, PRD71, 014028
Faccini *et al.*, PRD87, 111102
Esposito *et al.*, Phys.Rept. 668

Amplitude model



$$f_i(s,t,u) = 16\pi \sum_{l=0}^{L_{\max}} (2l+1)\left(a_{l,i}^{(s)}(s)P_l(z_s) + a_{l,i}^{(t)}(t)P_l(z_t) + a_{l,i}^{(u)}(u)P_l(z_u)\right)$$

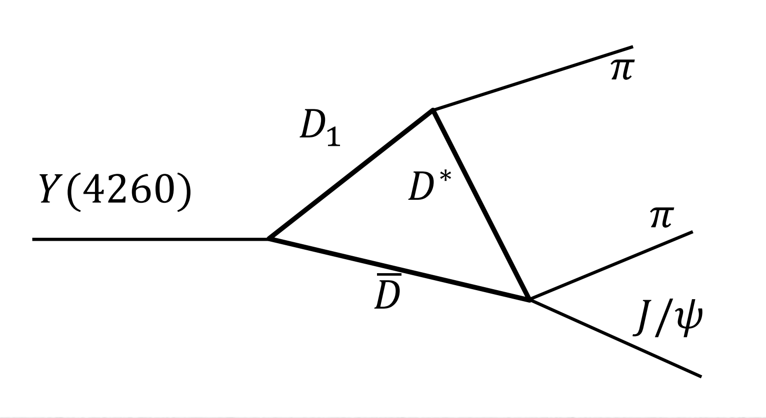
Khuri-Treiman

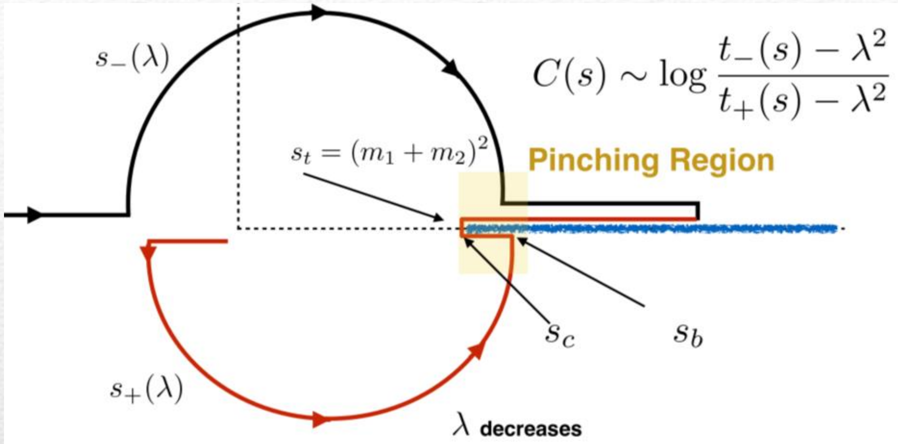
$$f_{0,i}(s) = \frac{1}{32\pi}\int_{-1}^{1} dz_s\, f_i\left(s, t(s,z_s), u(s,z_s)\right) = a_{0,i}^{(s)} + \frac{1}{32\pi}\int_{-1}^{1} dz_s \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u)\right) \equiv a_{0,i}^{(s)} + b_{0,i}(s)$$

$$f_{l,i}(s) = \frac{1}{32\pi}\int_{-1}^{1} dz_s\, P_l(z_s)\left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u)\right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s)\frac{1}{\pi}\int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'-s},$$

$$f_i(s,t,u) = 16\pi\left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s)\left(c_j + \frac{s}{\pi}\int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'(s'-s)}\right)\right],$$

Triangle singularity





Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in very special kinematical conditions (Coleman and Norton, Nuovo Cim. 38, 438), However, this effects cancels in Dalitz projections, no peaks (Schmid, Phys.Rev. 154, 1363)

$$f_{0,i}(s) = b_{0,i}(s) + \frac{t_{ij}}{\pi} \int_{s_i}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' - s}$$

...but the cancellation can be spread in different channels, you might still see peaks in other channels only!

Szczepaniak, PLB747, 410-416
Szczepaniak, PLB757, 61-64
Guo, Meissner, Wang, Yang PRD92, 071502

Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities.
  This also limits the number of free parameters

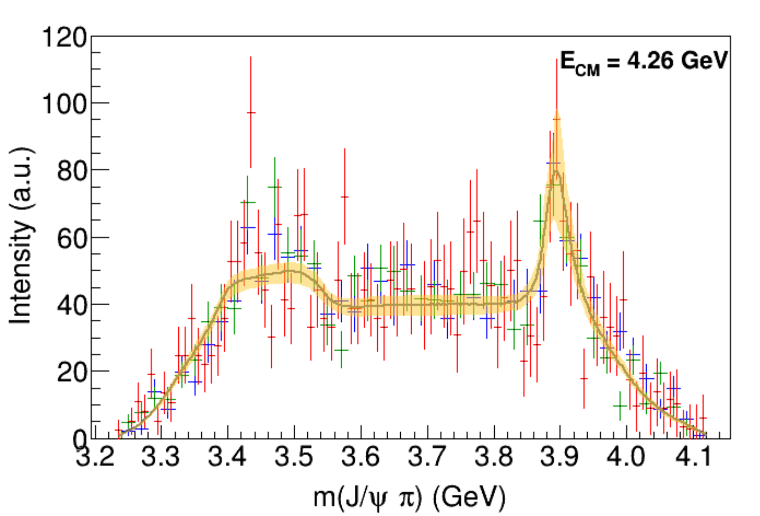
$$f_i(s,t,u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s'(s'-s)} \right) \right],$$

The scattering matrix is parametrized as $(t^{-1})_{ij} = K_{ij} - i\,\rho_i\,\delta_{ij}$

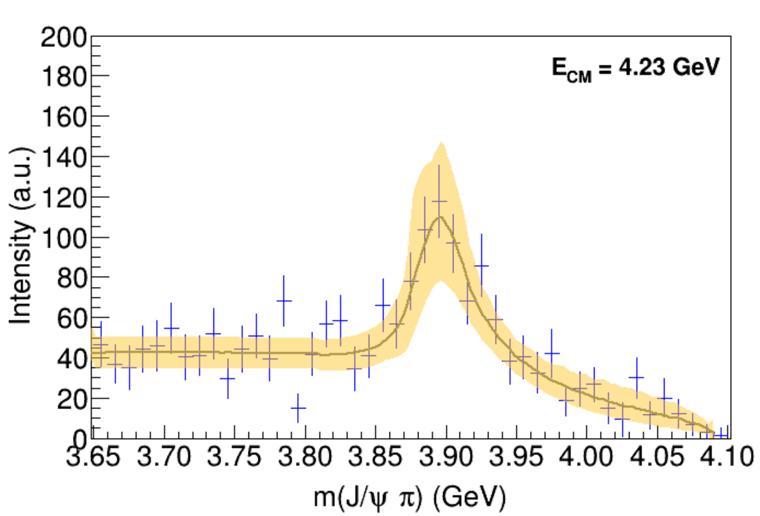
Four different scenarios considered:

- «III»: the K matrix is $\frac{g_i\, g_j}{M^2 - s}$, this generates a pole in the closest unphysical sheet the rescattering integral is set to zero
- «III+tr.»: same, but with the correct value of the rescattering integral
- «IV+tr.»: the K matrix is constant, this generates a pole in the IV sheet
- «tr.»: same, but the pole is pushed far away by adding a penalty in the χ^2

Fit: III



E_{CM} = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



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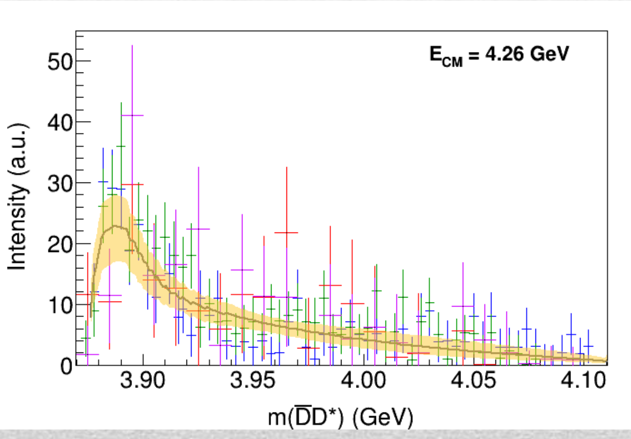
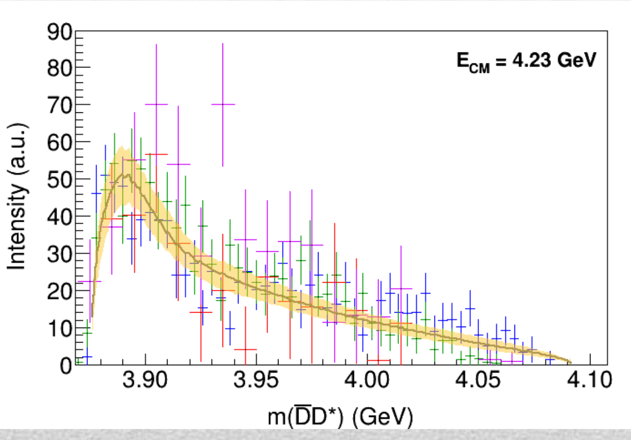


E_{CM} = 4.26 GeV
Intensity (a.u.)
m($\overline{D}$D*) (GeV)



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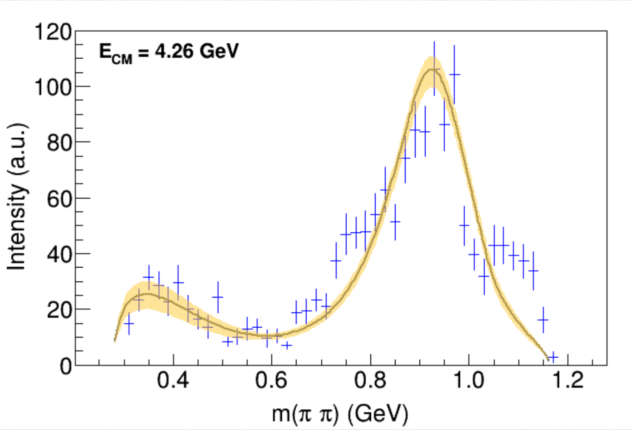
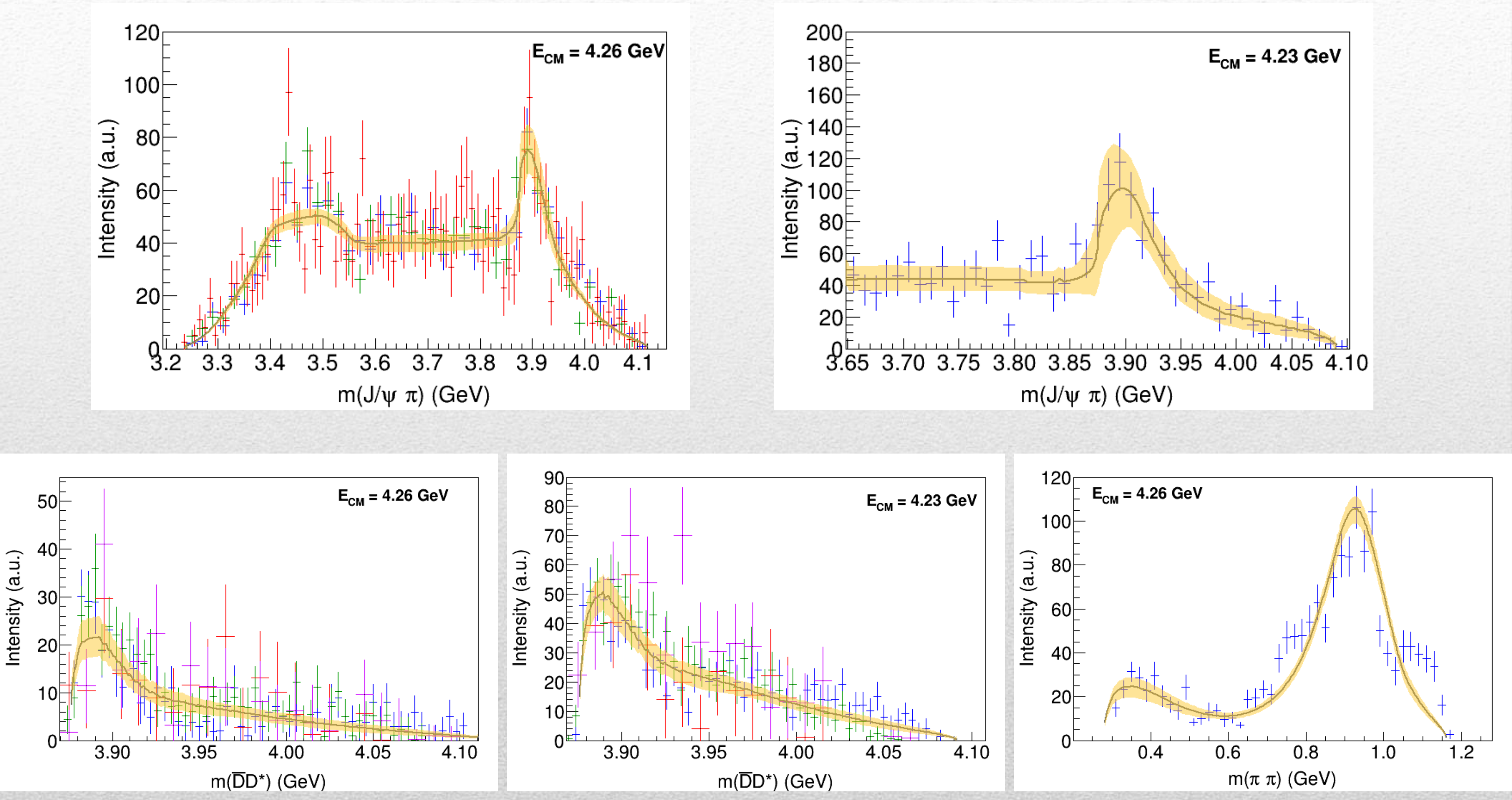


E_{CM} = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Fit: III+tr.



E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
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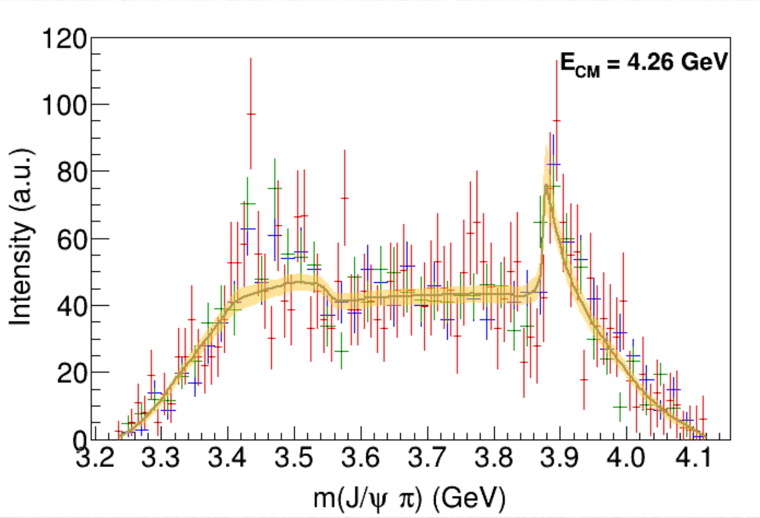
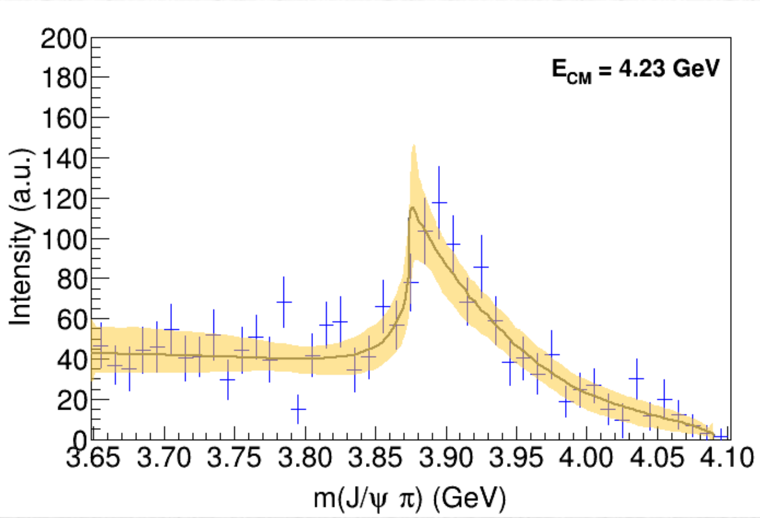
Intensity (a.u.)
m($\bar{D}$D*) (GeV)
E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Fit: IV+tr.



E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)



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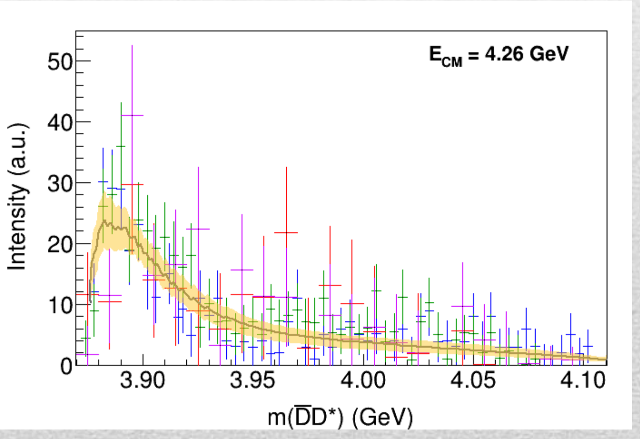

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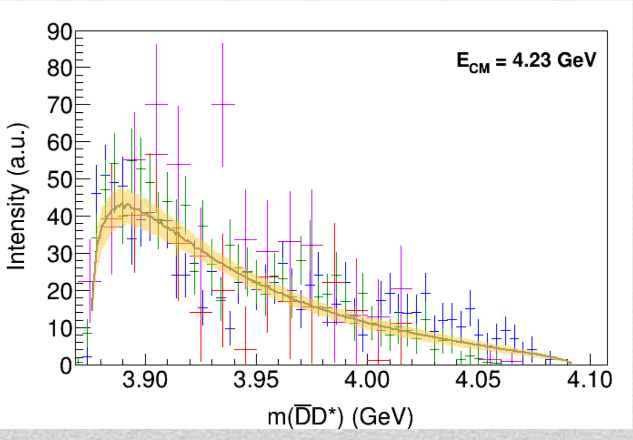
m(D̅D*) (GeV)



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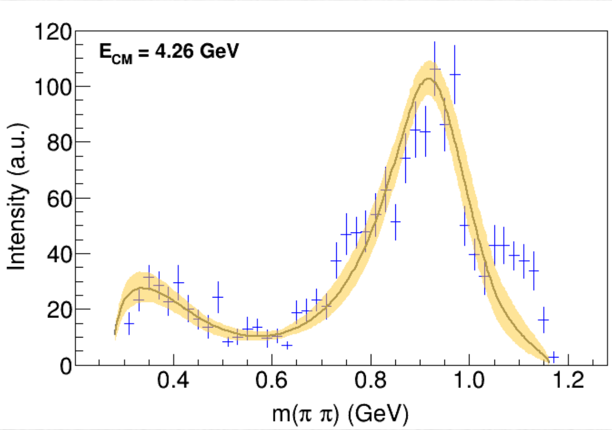
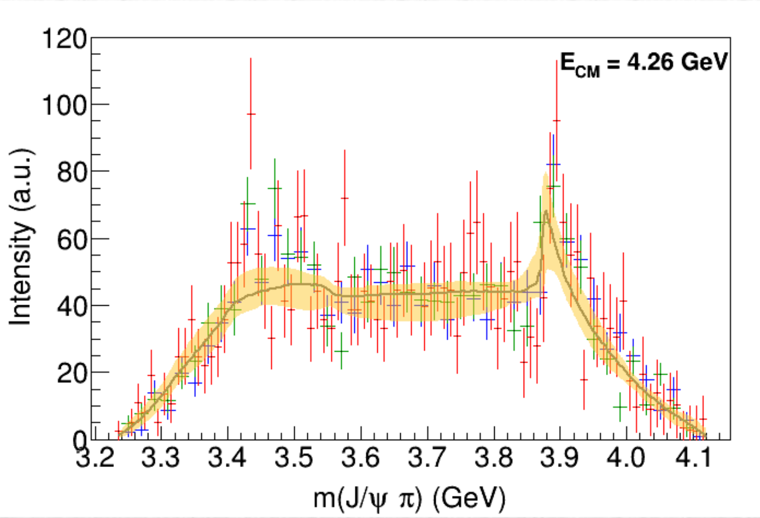
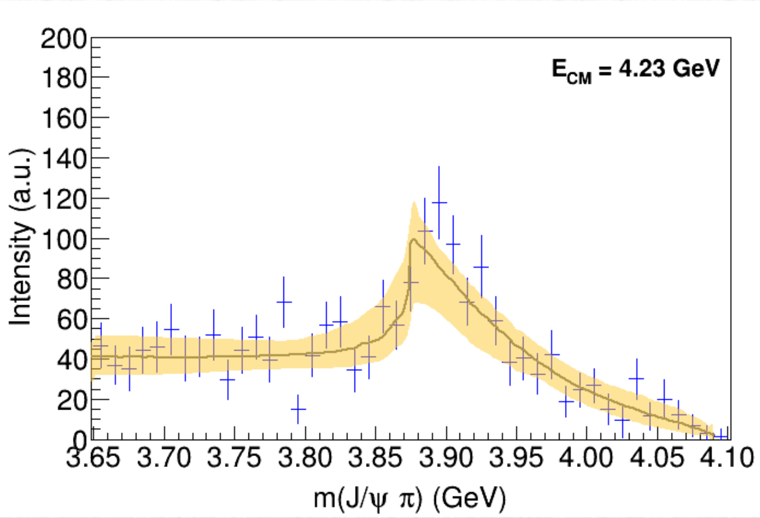
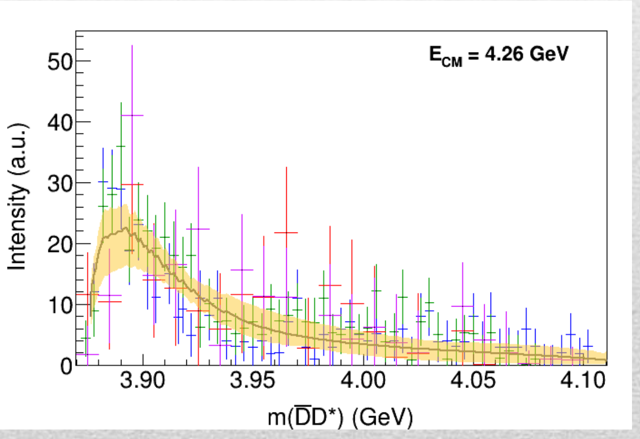


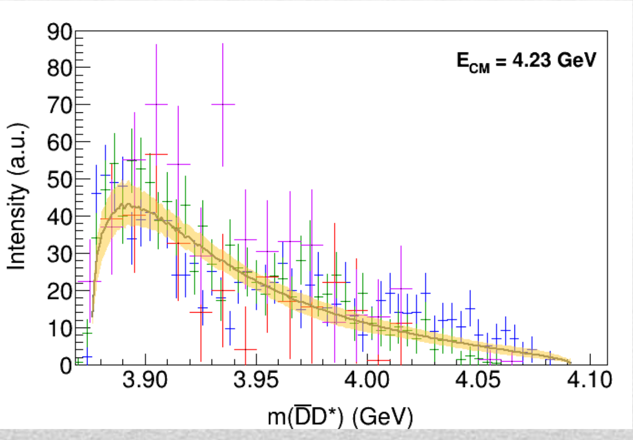
E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

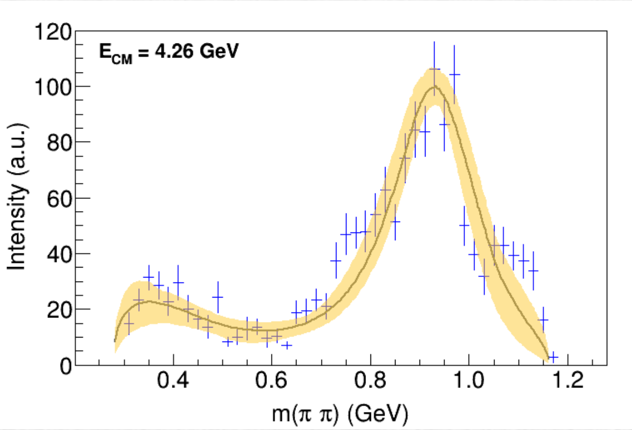
Fit: tr.


E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)

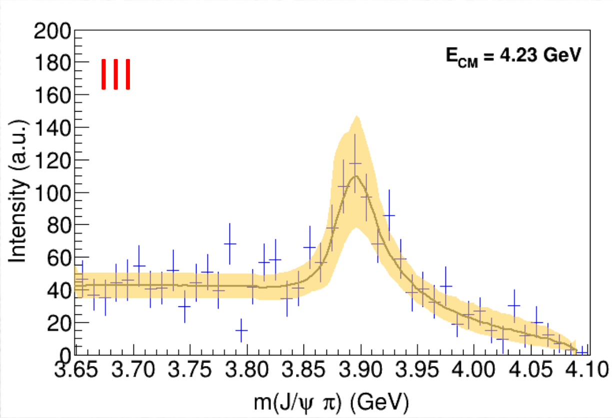

E_CM = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)

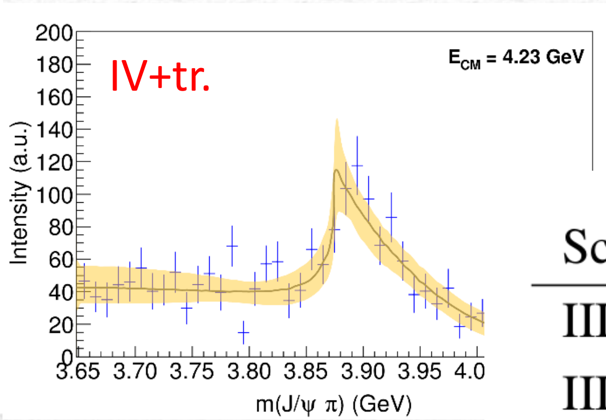

E_CM = 4.26 GeV
Intensity (a.u.)
m(D̄D*) (GeV)

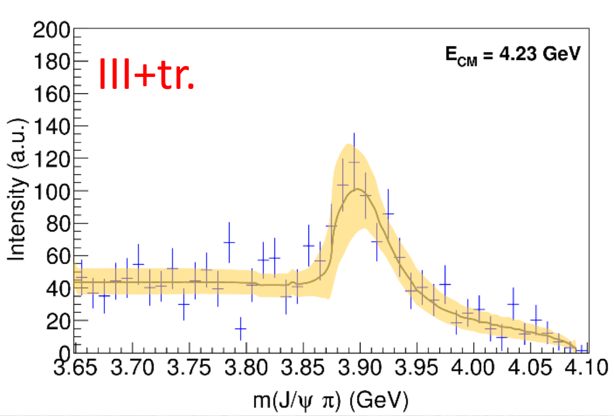

E_CM = 4.23 GeV
Intensity (a.u.)
m(D̄D*) (GeV)

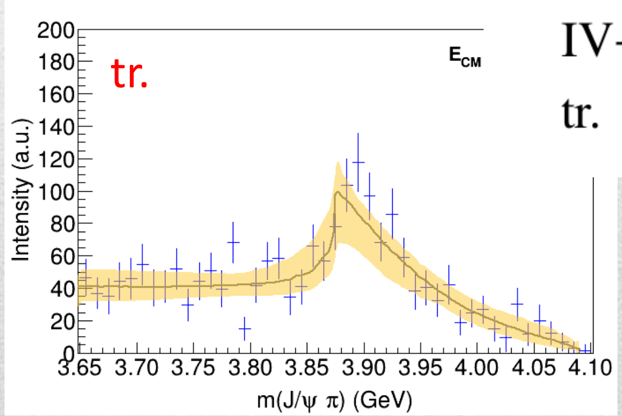

E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Fit summary





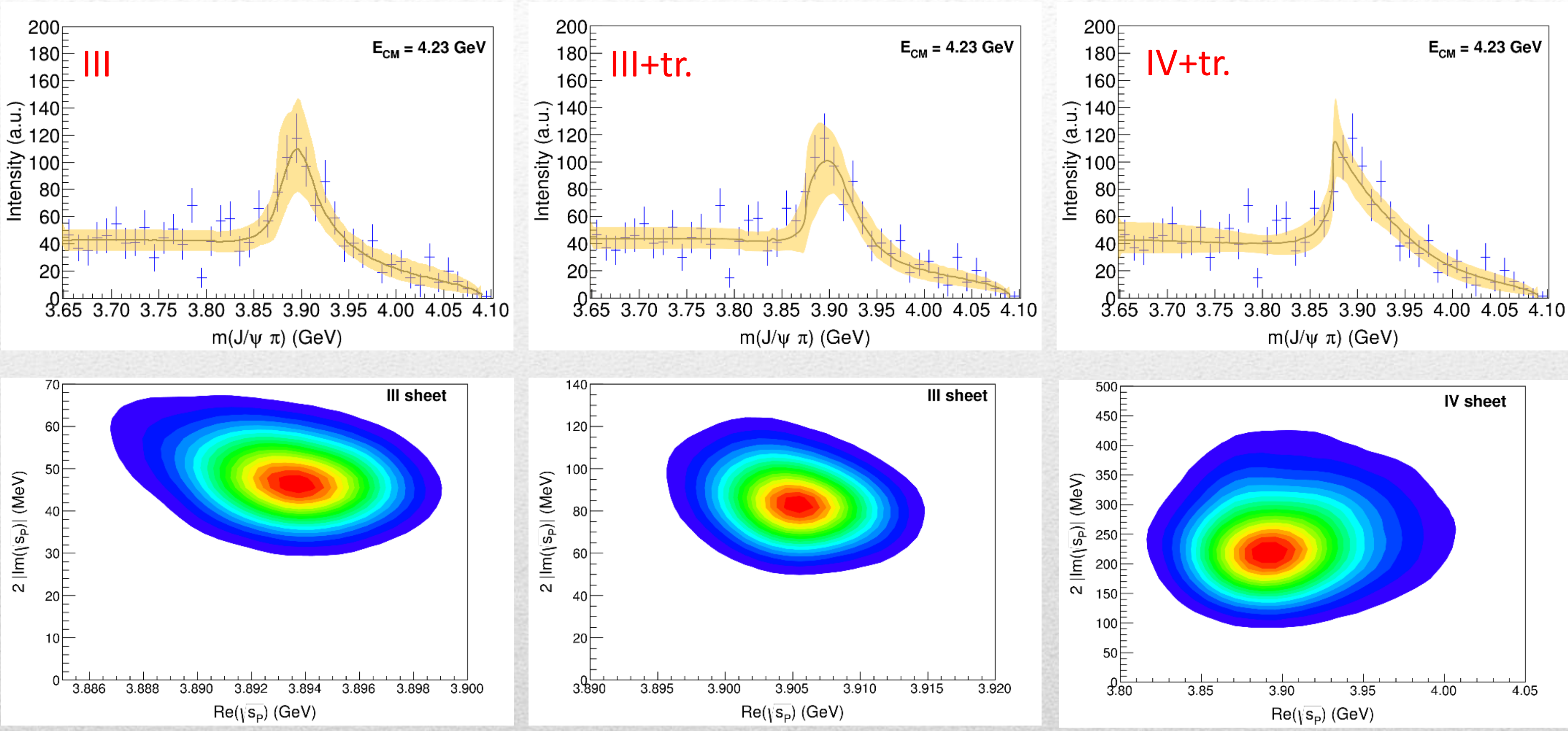




Scenario	χ^2	DOF	χ^2/DOF
III	644	532	1.21
III+tr.	642	532	1.21
IV+tr.	666	532	1.25
tr.	695	532	1.31

Naive loglikelihood ratio test give a $\sim 4\sigma$ significance of the scenario III+tr. over IV+tr., looking at plots it looks too much – better using some more solid test

Pole extraction



Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	"2.4σ" ("1.4σ")
III+tr.	–	1.5σ (3.1σ)	"2.6σ" ("1.3σ")
IV+tr.	–	–	"2.1σ" ("0.9σ")

	III	III+tr.	IV+tr.
M (MeV)	$3893.2^{+5.5}_{-7.7}$	3905^{+11}_{-9}	3900^{+140}_{-90}
Γ (MeV)	48^{+19}_{-14}	85^{+45}_{-26}	240^{+230}_{-130}

Not conclusive at this stage

How to improve the model

Molnar, Danilkin, Vanderhaeghen, 1903.08458

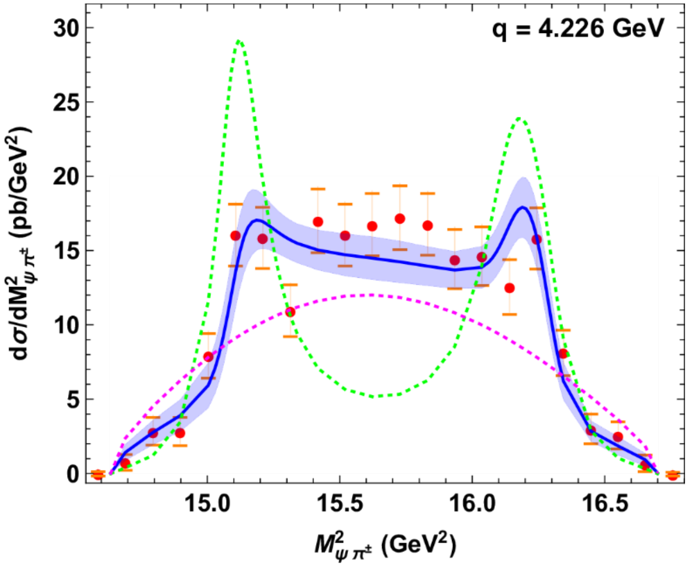
Ex. $e^+e^- \to \psi'\pi\pi$
Khuri-Treiman with spin, Omnès function for $\pi\pi$

$$\mathcal{H}_{\lambda_1\lambda_2}(s,t,u) \approx \sum_{J\ \text{even}}^{J_{max}} (2J+1) \times \left\{ h^{(J),s}_{\lambda_1\lambda_2}(s)\, d^{(J)}_{\Lambda,0}(\theta_s) + h^{(J),t}_{\lambda_1\lambda_2}(t)\, d^{(J)}_{\Lambda,0}(\theta_t) + h^{(J),u}_{\lambda_1\lambda_2}(u)\, d^{(J)}_{\Lambda,0}(\theta_u) \right\}$$

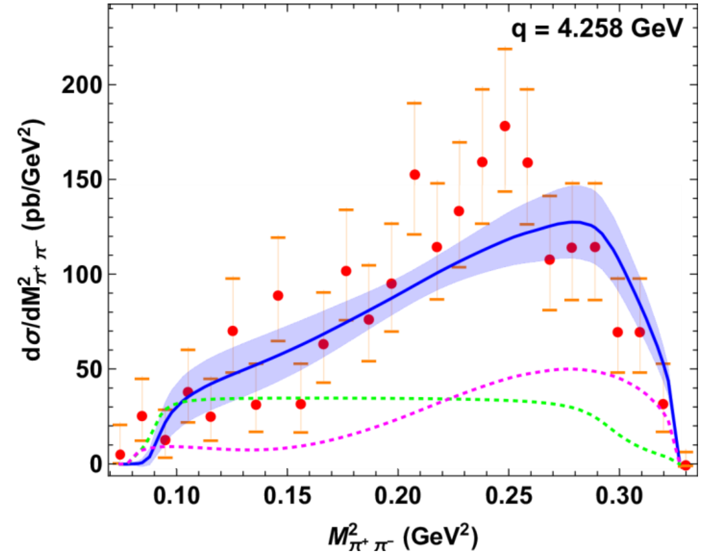
$$\mathcal{H}_{\lambda_1\lambda_2}(s,t,u) = h^{(0),t}_{\lambda_1\lambda_2}(t) + h^{(0),u}_{\lambda_1\lambda_2}(u) + \Omega^{(0)}(s) \left\{ a + b\, s - \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\text{Disc}\,(\Omega^{(0)})^{-1}(s')\, h^{(0),L}_{\lambda_1\lambda_2}(s')}{s' - s} \right\}$$

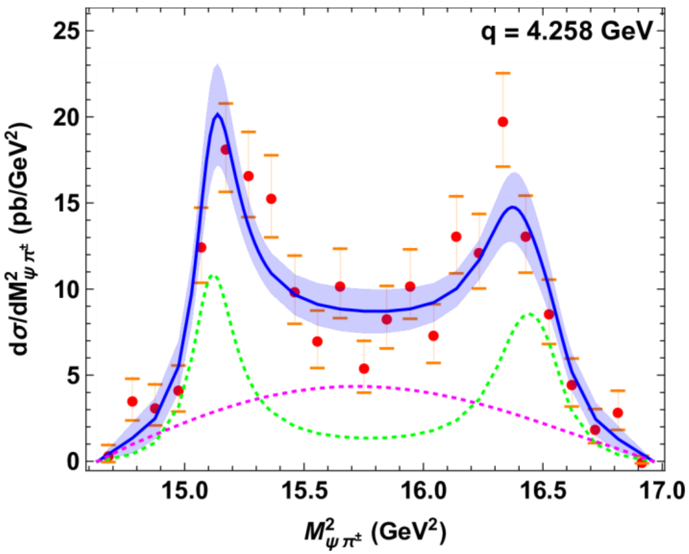
How to improve the model

Molnar, Danilkin, Vanderhaeghen, 1903.08458

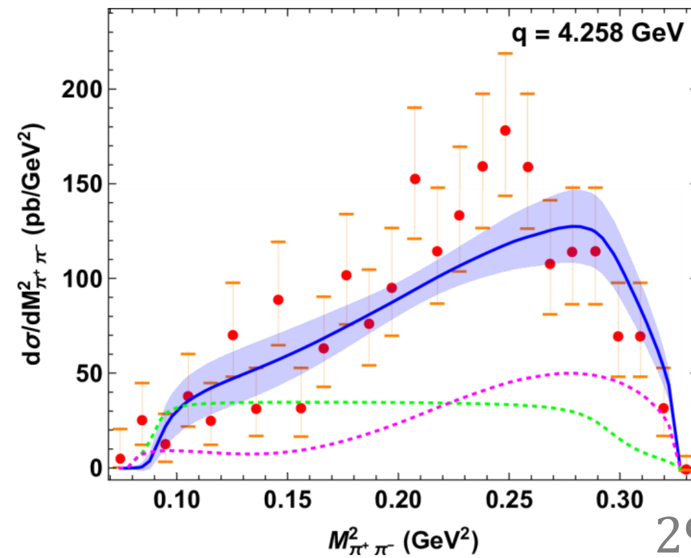


$Z_c(3900)$



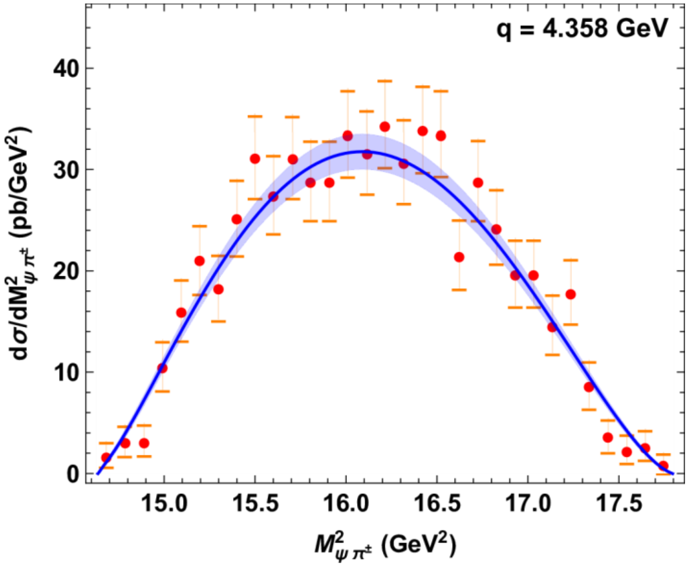


$Z_c(3900)$

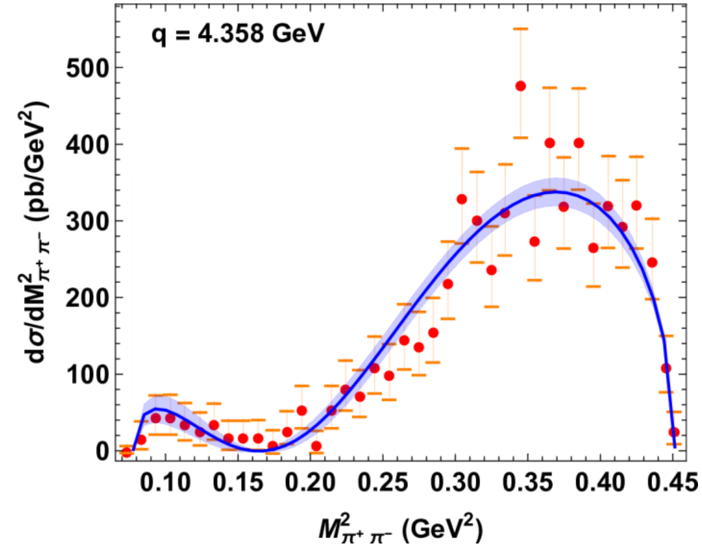


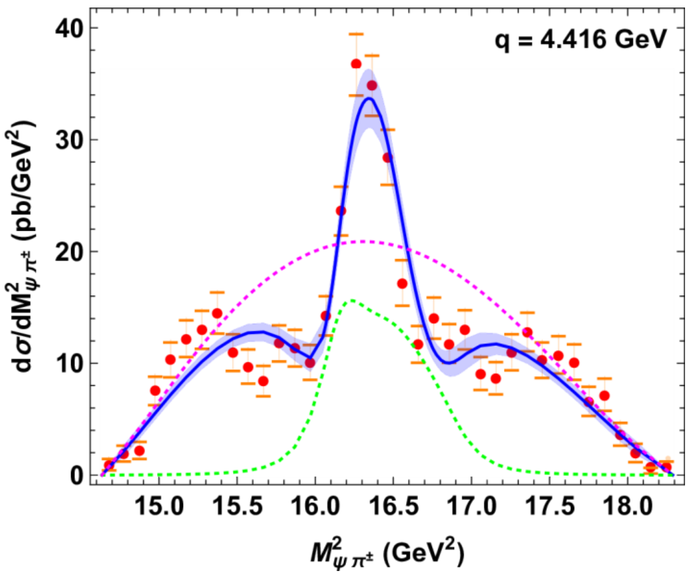
How to improve the model

Molnar, Danilkin, Vanderhaeghen, 1903.08458

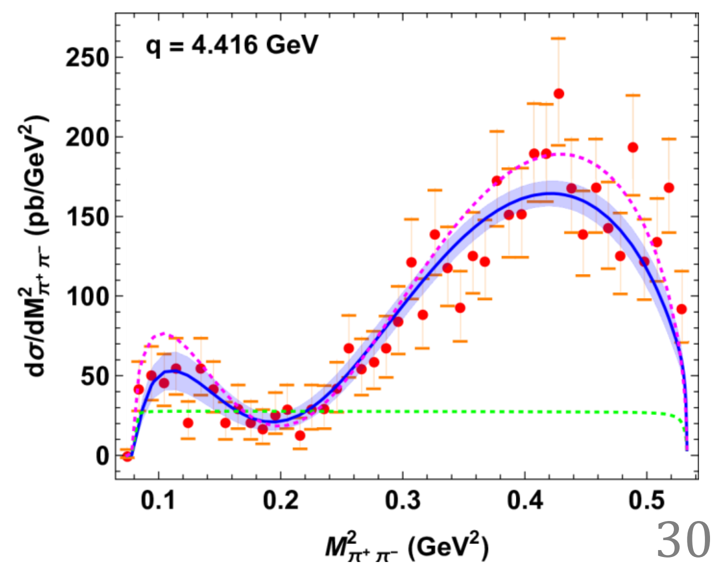


No Z_c





Broad Z_c'



Conclusions

- The study of kinematic singularities provides the «minimal» energy dependence of partial waves
- Isobar model remains the most effective method to describe Dalitz plots
- 3-body effects can be introduced in a flexible way, depending on the dominant effect expected

Thank you