$Y(4260)$: a light quark perspective from a dispersive analysis

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Outline

- Motivation
- Theoretical framework
  - Heavy quark effective theory
  - Dispersion theory
- Phenomenology discussion
  - Characteristics of singlet and octet contributions
  - Numerical fits
- Conclusions

The nature of $Y(4260)$ has remained controversial since its discovery in 2005.

- hybrid state: $c\bar{c}g$

  [S.-L. Zhu, PLB'2005; F. E. Close, P. R. Page, PLB'2005]

- excited charmonium

  [F. J. Llanes-Estrada, PRD'2005; B.-Q. Li, K.-T. Chao, PRD'2009]

- hadrocharmonium

  [S. Dubynskiy, M. B. Voloshin, PLB'2008; X. Li, M. B. Voloshin, MPLA'2014]

- tetraquark state

  [L. Maiani *et al.*, PRD'2005, EPJC'2018; Z.-G. Wang EPJC'2018]

- hadronic molecule of $\bar{D}D_1(2420)$ or $\omega\chi_{c0}$

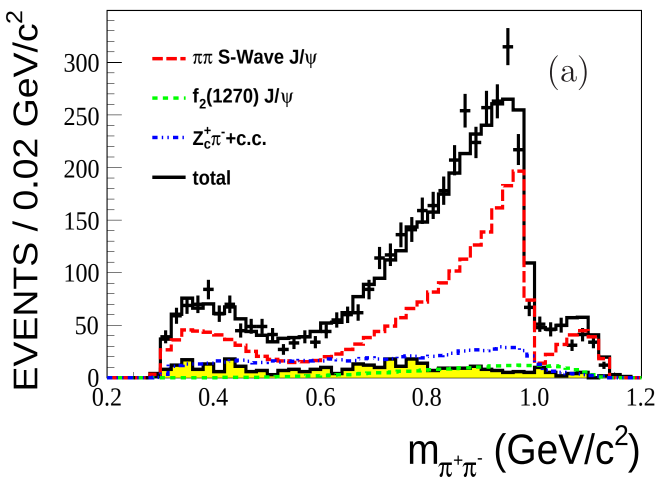
  [G.-J. Ding, PRD'2009; Q. Wang, C. Hanhart, Q. Zhao, PRL'2013; M. Cleven *et al.*, PRD'2014; L.-Y. Dai *et al.*, PRD'2014]

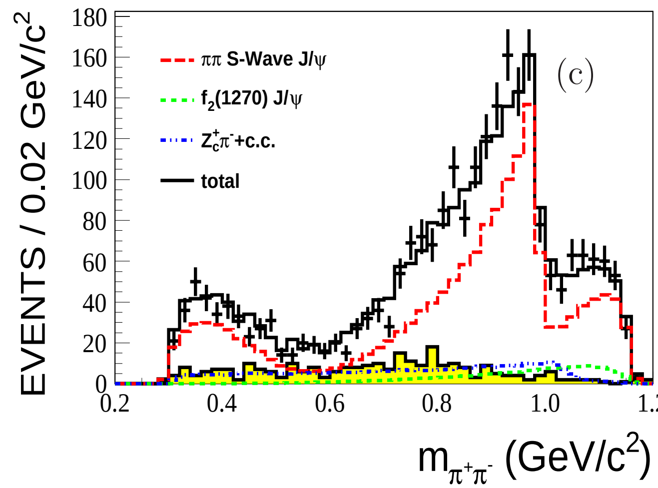
- interference effect

  [D.-Y. Chen, J, He, X. Liu, PRD'2011; D.-Y. Chen, X. Liu, T. Matsuki, EPJC'2018]

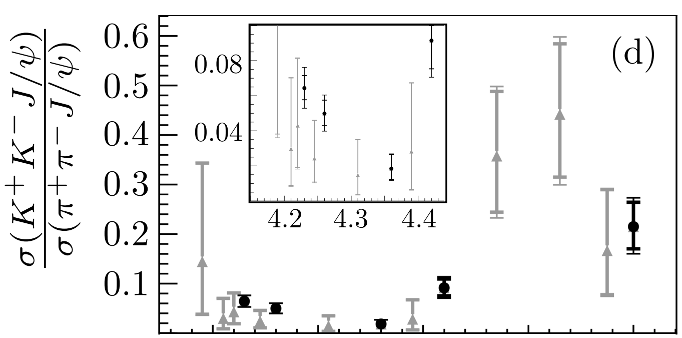
If the $Y(4260)$ contains no light quarks (as hybrid state or charmonium), the light-quark source provided by the $Y(4260)$ has to be an SU(3) singlet state.

- The $\pi\pi$ invariant mass distribution in the $e^+e^- \to Y(4260) \to J/\psi\pi^+\pi^-$ process, at $\sqrt{s} = 4.23$ GeV (left) and $\sqrt{s} = 4.26$ GeV (right). BESIII [PRL'2017]



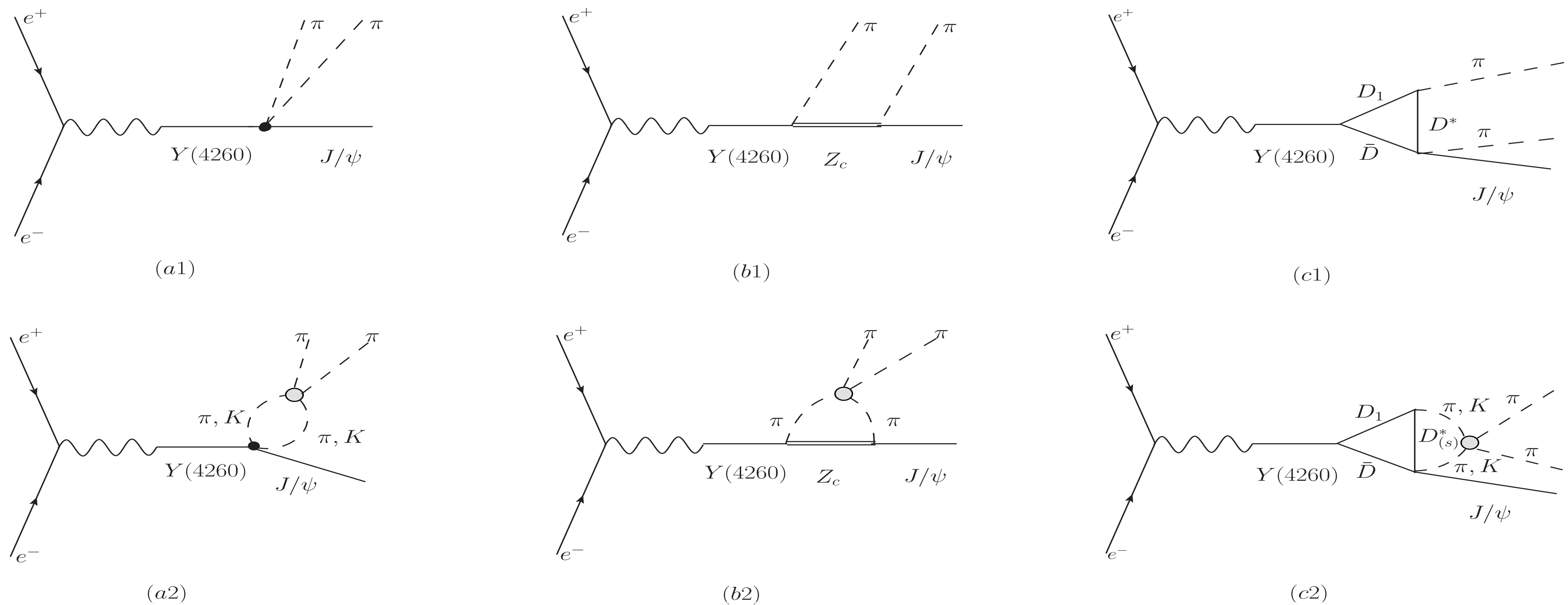


- The ratio of $\sigma(e^+e^- \to J/\psi K^+K^-)/\sigma(e^+e^- \to J/\psi\pi^+\pi^-)$. BESIII [PRD'2018]



Our strategy:

- dispersion theory, based on unitarity, analyticity and crossing symmetry $\Rightarrow$ account for the $\pi\pi$ rescattering and the $K\bar{K}$ coupled channel in the S-wave in a model-independent way
- consider the effects of the $Z_c(3900)$ and the triangle diagrams, which provide the left-hand-cut contribution
- the subtraction constants are obtained by matching the dispersive amplitudes to the heavy quark chiral effective theory



$$|Y(4260)\rangle = a|V_1\rangle + b|V_8\rangle$$

SU(3) singlet: $|V_1\rangle \equiv V_1^{light} \otimes V^{heavy} = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \otimes V^{heavy}$

SU(3) octet: $|V_8\rangle \equiv V_8^{light} \otimes V^{heavy} = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \otimes V^{heavy}$

$$\begin{aligned}\mathscr{L}_{Y\psi\Phi\Phi} \;=\;& g_1\langle V_1^{\alpha} J_{\alpha}^{\dagger}\rangle\langle u_\mu u^\mu\rangle + h_1\langle V_1^{\alpha} J_{\alpha}^{\dagger}\rangle\langle u_\mu u_\nu\rangle v^\mu v^\nu + g_8\langle J_{\alpha}^{\dagger}\rangle\langle V_8^{\alpha} u_\mu u^\mu\rangle \\ & + h_8\langle J_{\alpha}^{\dagger}\rangle\langle V_8^{\alpha} u_\mu u_\nu\rangle v^\mu v^\nu + \text{h.c.} \qquad (1)\end{aligned}$$

No strange partner of the Z_c states, thus the SU(3) singlet and octet components of the $Y(4260)$ are not distinguishable in the $Z_cY(4260)\pi$ interaction.

$$\mathscr{L}_{Z_cY\pi} = C_{Z_cY\pi}Y^i\langle Z_c^{i\dagger}u_\mu\rangle v^\mu + \text{h.c.}\,, \tag{2}$$

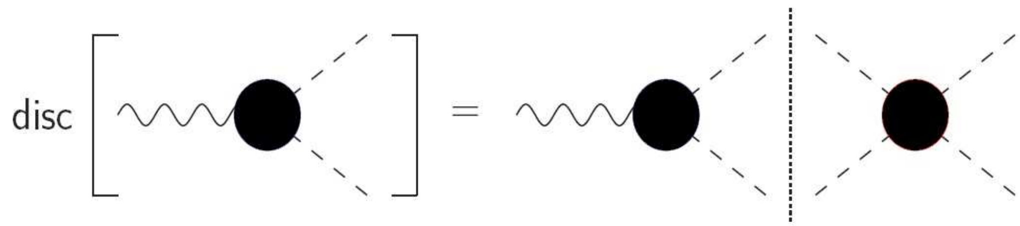
$$\mathscr{L}_{Z_c\psi\pi} = C_{Z_c\psi\pi}\psi^i\langle Z_c^{i\dagger}u_\mu\rangle v^\mu + \text{h.c.}\,, \tag{3}$$

$$\mathscr{L}_{YD_1D} = \frac{y}{\sqrt{2}}Y^i\left(\bar{D}_a^\dagger D_{1a}^{i\dagger} - \bar{D}_{1a}^{i\dagger}D_a^\dagger\right) + \text{h.c.}\,, \tag{4}$$

$$\mathscr{L}_{D_1D^*P} = i\frac{h'}{F}\left[3D_{1a}^i(\partial^i\partial^j\Phi_{ab})D_b^{*j\dagger} - D_{1a}^i(\partial^j\partial^j\Phi_{ab})D_b^{*i\dagger} + ...\right] + \text{h.c.}\,, \tag{5}$$

$$\mathscr{L}_{\psi D^*DP} = \frac{g_{\psi P}}{2}\langle\psi\bar{H}_a^\dagger H_b^\dagger\rangle u_{ab}^0\,, \tag{6}$$

- elastic unitarity (single channel, no left-hand cut)



$$\frac{1}{2i}\mathrm{disc}\, F_l(s) = \mathrm{Im}\, F_l(s) = F_l(s)\sin\delta_l^I(s) e^{-i\delta_l^I(s)}\,. \tag{7}$$

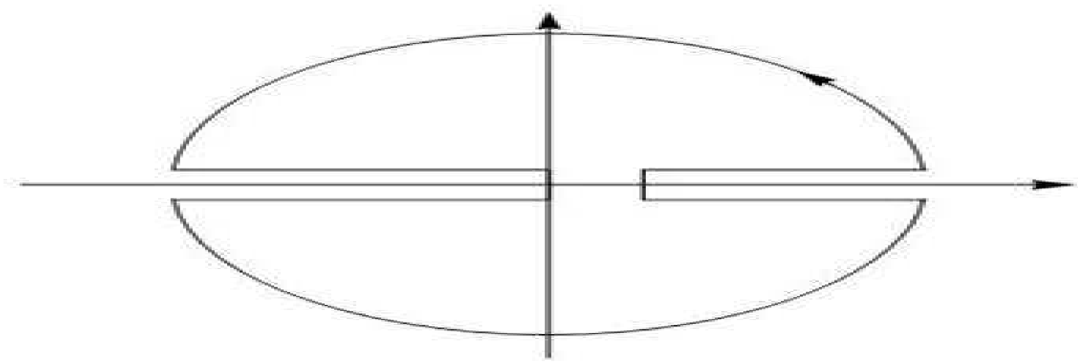
Watson's theorem: phase of $F_l(s)$ is just $\delta_l^I(s)$, the elastic $\pi\pi$ phase shift

- traditional solution to this homogeneous integral equation [Omnès, Nuovo Cim'1958]

$$F_l(s) = P_n(s)\Omega_l^I(s)\,, \qquad \Omega_l^I(s) = \exp\left\{\frac{s}{\pi}\int_{4m_\pi^2}^\infty \frac{dx}{x}\frac{\delta_l^I(x)}{x-s}\right\}. \tag{8}$$

$P_n(s)$: polynomial; $\Omega_l^I(s)$: Omnès function

- modified Omnès solution (right-hand and left-hand cuts are separated)
  [A. V. Anisovich and H. Leutwyler,, PLB'1996]



$$\mathrm{Im}\, M_l(s) = \left[M_l(s) + \hat{M}_l(s) \right] \sin \delta_l^I(s) e^{-i\delta_l^I(s)} \,. \tag{9}$$

  - $M_l(s)$: right-hand-cut contributions
  - $\hat{M}_l(s)$: left-hand-cut contributions, approximated by Z_c-exchange + triangle diagrams

$$M_l(s) = \Omega_l^I(s) \left\{ P_l^{n-1}(s) + \frac{s^n}{\pi} \int_{4m_\pi^2}^{\infty} \frac{dx}{x^n} \frac{\hat{M}_l(x) \sin \delta_l^I(x)}{|\Omega_l^I(x)|(x-s)} \right\} , \tag{10}$$

  - $P_l^{n-1}(s)$: subtraction polynomial determined by matching to heavy quark chiral effective theory

- two-channel unitarity conditions

$$\mathrm{Im}\,\mathbf{M}_0(s) = 2iT_0^{0*}(s)\Sigma(s)\left[\mathbf{M}_0(s) + \hat{\mathbf{M}}_0(s)\right] , \tag{11}$$

$$\mathbf{M}_0(s) = \begin{pmatrix} M_0^\pi(s) \\ \frac{2}{\sqrt{3}} M_0^K(s) \end{pmatrix}, \quad \hat{\mathbf{M}}_0(s) = \begin{pmatrix} \hat{M}_0^\pi(s) \\ \frac{2}{\sqrt{3}} \hat{M}_0^K(s) \end{pmatrix} . \tag{12}$$

$$T_0^0(s) = \begin{pmatrix} \frac{\eta_0^0(s)e^{2i\delta_0^0(s)}-1}{2i\sigma_\pi(s)} & |g_0^0(s)|e^{i\psi_0^0(s)} \\ |g_0^0(s)|e^{i\psi_0^0(s)} & \frac{\eta_0^0(s)e^{2i\left(\psi_0^0(s)-\delta_0^0(s)\right)}-1}{2i\sigma_K(s)} \end{pmatrix} \tag{13}$$

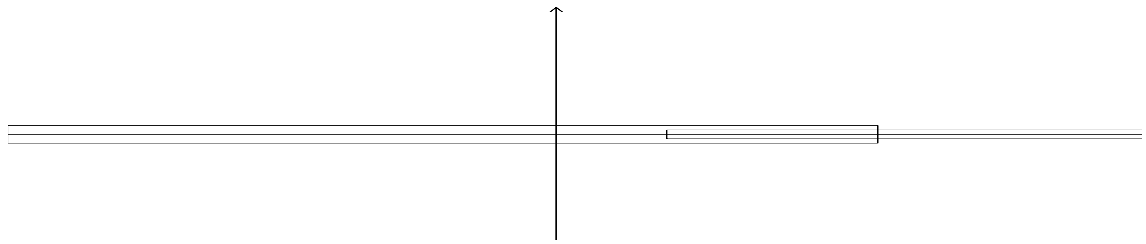
$$\Sigma(s) = diag\big(\sigma_\pi(s)\theta(s-4m_\pi^2), \sigma_K(s)\theta(s-4m_K^2)\big) . \tag{14}$$

  - $\delta_0^0(s)$: $\pi\pi$ S-wave isoscalar phase shift
  - $|g_0^0(s)|, \psi_0^0(s)$: modulus and phase of $\pi\pi \to K\bar{K}$ S-wave amplitude
  - $\eta_0^0(s)$: inelasticity, $= \sqrt{1 - 4\sigma_\pi(s)\sigma_K(s)|g_0^0(s)|^2\theta(s-4m_K^2)}$

Two different $T_0^0(s)$ matrices will be used: the Dai–Pennington (DP) and the Bern/Orsay (BO) parametrizations.

$$\mathbf{M}_0(s) = \Omega(s)\left\{\mathbf{M}_0^\chi(s) + \frac{s^3}{\pi}\int_{4m_\pi^2}^\infty \frac{dx}{x^3}\frac{\Omega^{-1}(x)T(x)\Sigma(x)\hat{\mathbf{M}}_0(x)}{x-s}\right\} , \tag{15}$$

- $Y(4260) \to J/\psi\pi^+\pi^-$: the crossed-channel exchanged Z_c and DD^* can be on-shell, the left-hand cut intersects and overlaps with the right-hand cut.



For example, the S-wave component of the Z_c-exchange amplitude reads

$$\hat{M}_0^{Z_c,\pi}(s) = -\frac{2\sqrt{M_Y M_\psi}M_{Z_c}}{F^2\kappa_\pi(s)} C_{Y\psi}\Big\{\Big(s+|\mathbf{q}|^2\Big)Q_0(y(s)) - |\mathbf{q}|^2\sigma_\pi^2\Big[y^2(s)Q_0(y(s)) - y(s)\Big]\Big\}, \quad (16)$$

where $y(s) \equiv (3s_0 - s - 2M_{Z_c}^2)/\kappa_\pi(s)$, and $Q_0(y) = \frac{1}{2}\log(y+1)/(y-1)$.

Two branch points in $Q_0(y(s))$,

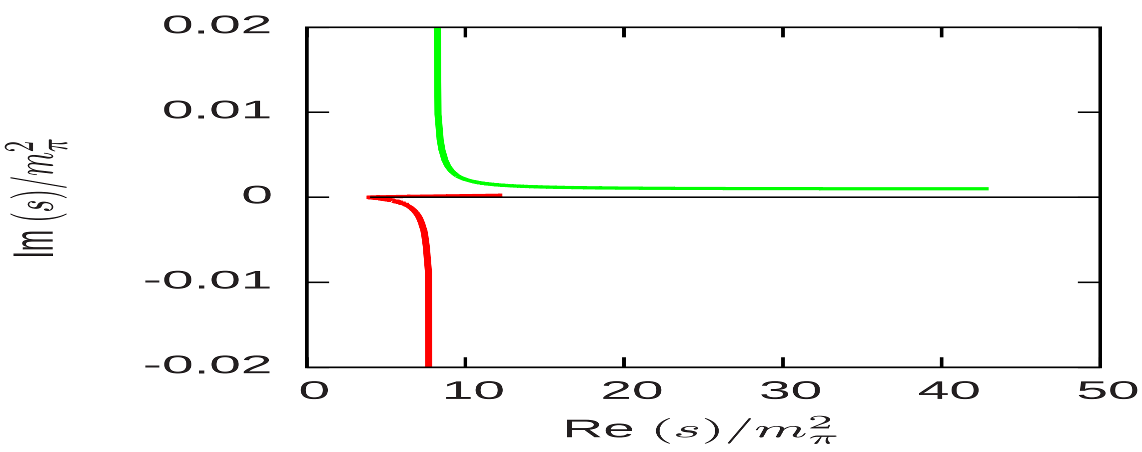
$$\begin{aligned} s_\pm &= \frac{1}{2M_{Z_c}^2}\Big\{(M_Y^2+M_\psi^2)(m_\pi^2+M_{Z_c}^2) - M_Y^2 M_\psi^2 - (M_{Z_c}^2 - m_\pi^2)^2 \\ &\quad \pm\lambda^{1/2}(M_Y^2, M_{Z_c}^2, m_\pi^2)\lambda^{1/2}(M_\psi^2, M_{Z_c}^2, m_\pi^2)\Big\} \\ &> 4m_\pi^2 \,. \end{aligned} \quad (17)$$

- Solution: using the spectral representation of the resonance propagator, and application of $q^2 \to q^2 + i\epsilon$. [B. Moussallam, EPJC'2013]

$$\widetilde{BW}_R(x) = \frac{1}{\pi}\int_{x_R^{thr}}^{\infty} dx' \frac{Im[BW_R(x')]}{x'-x}\,, \tag{18}$$

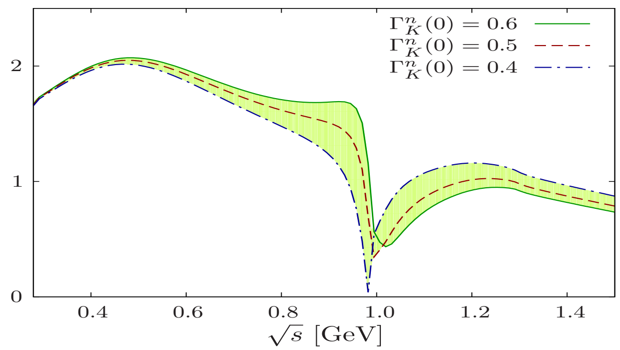
where $BW_R(x') = (M_R^2 - x' - iM_R\Gamma_R(x'))^{-1}$, and $x_{Z_c}^{thr} = (M_\psi + m_\pi)^2$.

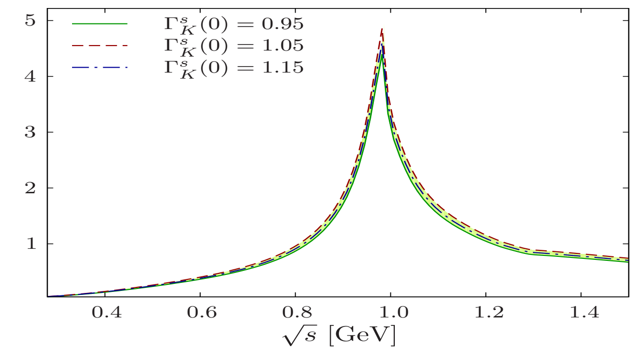
$$\hat{M}_0^{Z_c,\pi}(s) = -\frac{2\sqrt{M_Y M_\psi}M_{Z_c}}{\pi F^2 \kappa_\pi(s)} C_{Y\psi}\int_{x_{Z_c}^{thr}}^{\infty} dx' \frac{M_{Z_c}\Gamma_{Z_c}(x')}{(x'-M_{Z_c}^2)^2 + M_{Z_c}^2\Gamma_{Z_c}^2(x')}\Big\{\Big(s+|\mathbf{q}|^2\Big) Q_0(y(s,x')) - |\mathbf{q}|^2\sigma_\pi^2\Big[y^2(s,x')Q_0(y(s,x')) - y(s,x')\Big]\Big\}\,. \tag{19}$$



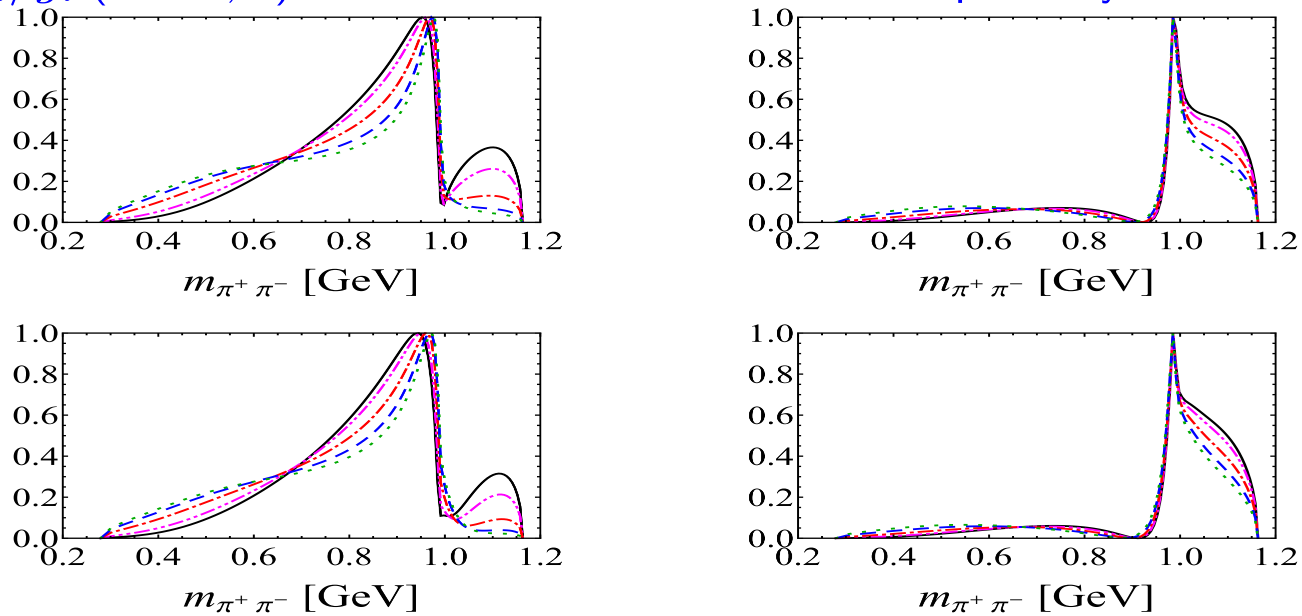
Locus of the branch points $s_\pm(s,x')$: upper branch lies above the unitarity cut; lower branch crosses the real axis close to but strictly below $4m_\pi^2$.

- Modulus of scalar pion nonstrange form factors $\langle 0|(\bar{u}u+\bar{d}d)|\pi^+\pi^-\rangle$ (left) and strange form factors $\langle 0|s\bar{s}|\pi^+\pi^-\rangle$ (right), depicted for three different normalizations.





- Shapes of the $\pi\pi$ mass spectra contributed from SU(3) singlet (left) and octet (right) contact terms using DP (top) or BO (bottom) parametrizations in $e^+e^- \to Y(4260) \to J/\psi\pi^+\pi^-$. h_i/g_i $(i = 1, 8)$ fixed at 0.1, 0.3, 1, 3, and 10, respectively.

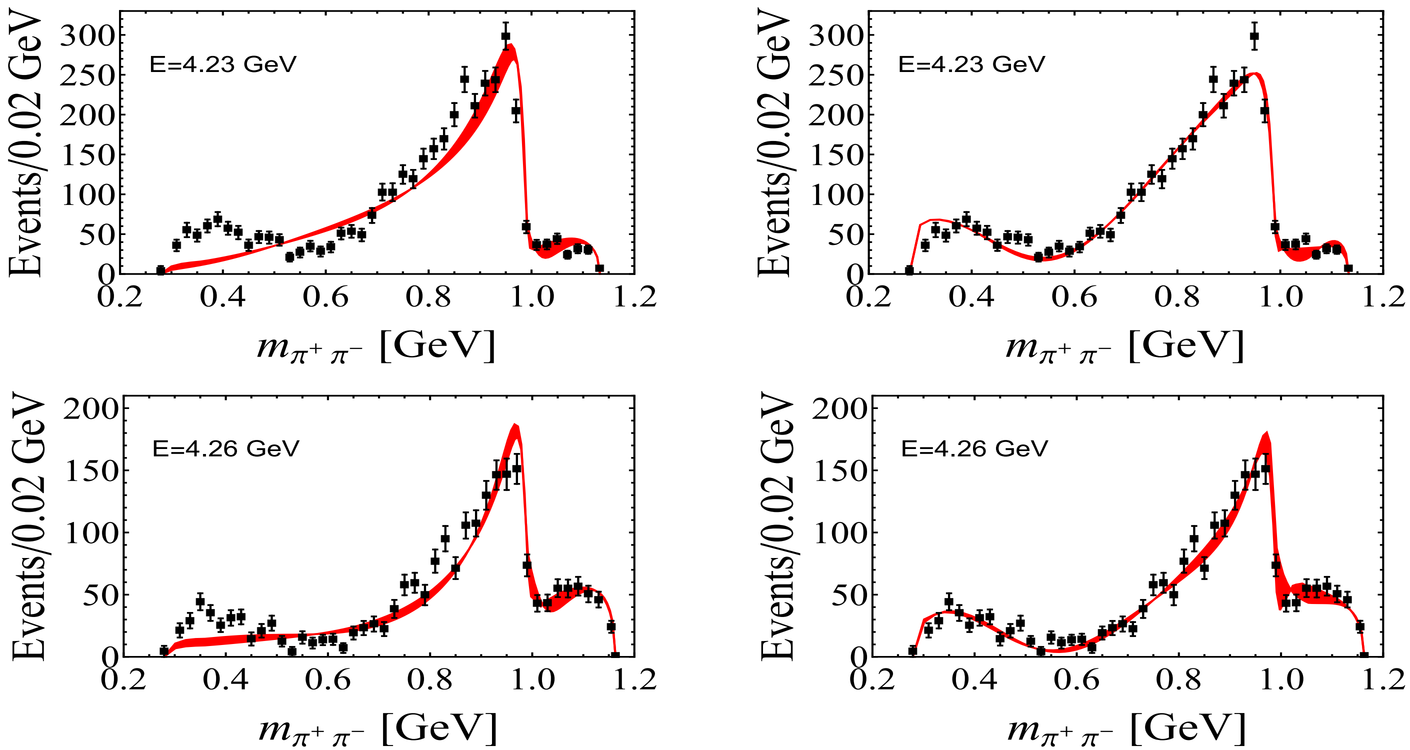


singlet: a bump below 1 GeV; around 1 GeV a dip for $h_1/g_1 \lesssim 1$

octet: little contribution below 0.9 GeV; a sharp peak around 1 GeV.

- ## Fitting to the BESIII data

Fit results of the $\pi\pi$ mass spectra in $e^+e^- \to J/\psi\pi^+\pi^-$.



$E = 4.23$ GeV:
Fit Ia (top left):
Only SU(3) singlet
Fit Ib (top right):
SU(3) singlet + octet

$E = 4.26$ GeV:
Fit IIa (bottom left):
Only SU(3) singlet
Fit IIb (bottom right):
SU(3) singlet + octet

	Experiment	Fit Ia, DP	Fit Ib, DP
$\frac{\sigma(J/\psi K^+K^-)}{\sigma(J/\psi\pi^+\pi^-)} \times 10^2$, $E = 4.23$ GeV	6.44 ± 1.15	7.82 ± 0.83	7.75 ± 1.10
	Experiment	Fit IIa, DP	Fit IIb, DP
$\frac{\sigma(J/\psi K^+K^-)}{\sigma(J/\psi\pi^+\pi^-)} \times 10^2$, $E = 4.26$ GeV	4.99 ± 1.10	4.46 ± 0.82	4.67 ± 0.98

Table 1: Fit parameters using the DP T-matrix parametrization.

	Fit Ia, DP	Fit Ib, DP	Fit IIa, DP	Fit IIb, DP
g_1 $[GeV^{-1}]$	-0.29 ± 0.04	1.87 ± 0.13	0.21 ± 0.04	-0.99 ± 0.11
h_1 $[GeV^{-1}]$	-0.29 ± 0.02	-0.31 ± 0.06	-0.32 ± 0.02	0.03 ± 0.04
g_8 $[GeV^{-1}]$	0 (fixed)	1.25 ± 0.11	0 (fixed)	-1.18 ± 0.03
h_8 $[GeV^{-1}]$	0 (fixed)	-1.96 ± 0.10	0 (fixed)	1.70 ± 0.18
$C_{Y\Psi}^{Zc} \times 10^2$	0.7 ± 0.6	2.0 ± 0.8	4.6 ± 0.3	6.9 ± 0.3
$C_{Y\Psi}^{loop}$ $[GeV^{-3}]$	4.5 ± 1.0	38.8 ± 2.5	12.5 ± 0.8	-19.4 ± 2.1
$\chi^2/\mathrm{d.o.f.}$	$\frac{405.1}{(44-4)} = 10.13$	$\frac{102.1}{(44-6)} = 2.69$	$\frac{182.7}{(46-4)} = 4.35$	$\frac{63.9}{(46-6)} = 1.60$

- Ratio of the SU(3) octet component relative to the SU(3) singlet component:
  - In the $\bar{D}D_1$ hadronic molecule scenario of $Y(4260)$: $1/\sqrt{2}$ since $|Y(4260)\rangle = \frac{1}{2}\left[|D_1^0\bar{D}^0\rangle + |D_1^+D^-\rangle\right] + \mathrm{c.c.}$, from which the light-quark component $|u\bar{u} + d\bar{d}\rangle/\sqrt{2} = (\sqrt{2}V_1^{light} + V_8^{light})/\sqrt{3}$.
  - Our results in Fit IIb, DP: $g_8/g_1 = 1.2 \pm 0.2$ and $h_8/h_1 = 57 \pm 76$

Assuming the strengths of the light-quark components from the $\bar{D}D_1$ hadronic molecule and the other SU(3) singlet source, e.g., from $|c\bar{c}\rangle$ or a hybrid, are α and β, respectively,

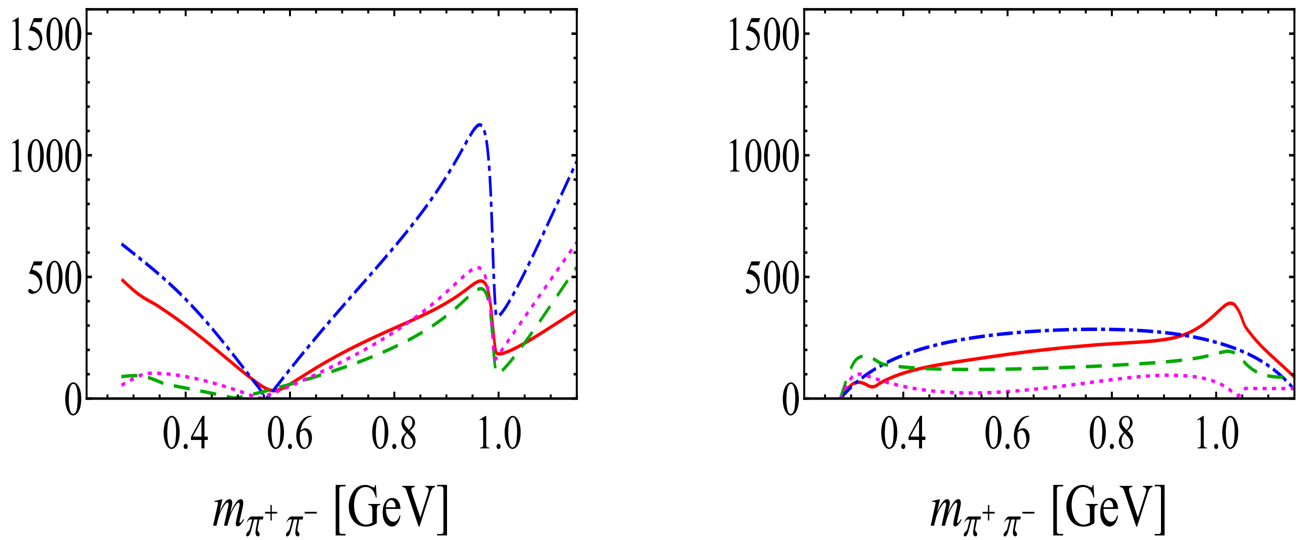
$$\frac{\alpha}{\sqrt{3}}\left(\sqrt{2}V_1^{light} + V_8^{light}\right) + \beta\, V_1^{light}\,, \qquad (20)$$

we can estimate the ratio of $\beta/\alpha = -0.30 \pm 0.05$ based on our results of g_8/g_1.

The $\bar{D}D_1$ component of the $Y(4260)$ may not be completely dominant.

$M_{Y(4260)} - M_D - M_{D_1} \simeq (4220 - 1870 - 2420)$ MeV $\simeq -70$ MeV

- Moduli of the S- (left) and D-wave (right) amplitudes for $e^+e^- \to J/\psi\pi^+\pi^-$.



red: best fit result
blue: contact terms
green: Z_c-exchange
magenta: triangle diag.

D-wave contribution is comparable to the S-wave contribution

- $Y(4260)$ cannot be a conventional charmonium state, for which the $\pi\pi$ S-wave should be dominant
- In the $\bar{D}D_1$ hadronic molecule interpretation, the $\pi\pi$ D-wave emerges naturally since the D_1 decays dominantly into D-wave $D^*\pi$

Conclusions

- Dispersion theory can consider pion-pion final-state interaction in a model-independent way
- $Y(4260)$ contains a large light-quark component, thus it is in all likelihood neither a hybrid nor a conventional charmonium state
- Our findings are consistent with the $Y(4260)$ having a sizeable $\bar{D}D_1$ component which, however, is not completely dominant

Thanks for your patience