Coupled-Channel Dalitz Plot Analysis of $D^+ \to K^- \pi^+ \pi^+$ Decay

Phys. Rev. D 93, 014005 (2016)

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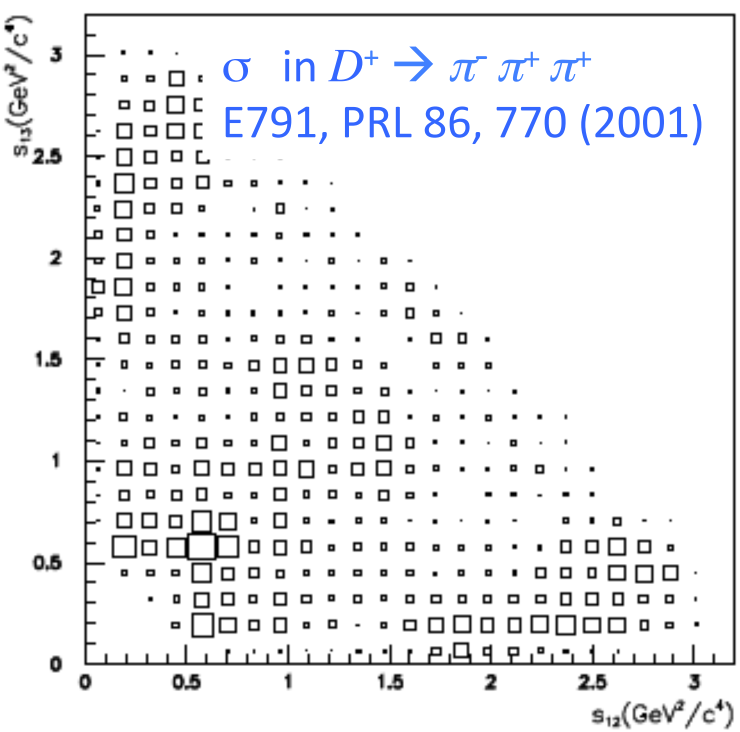
Introduction

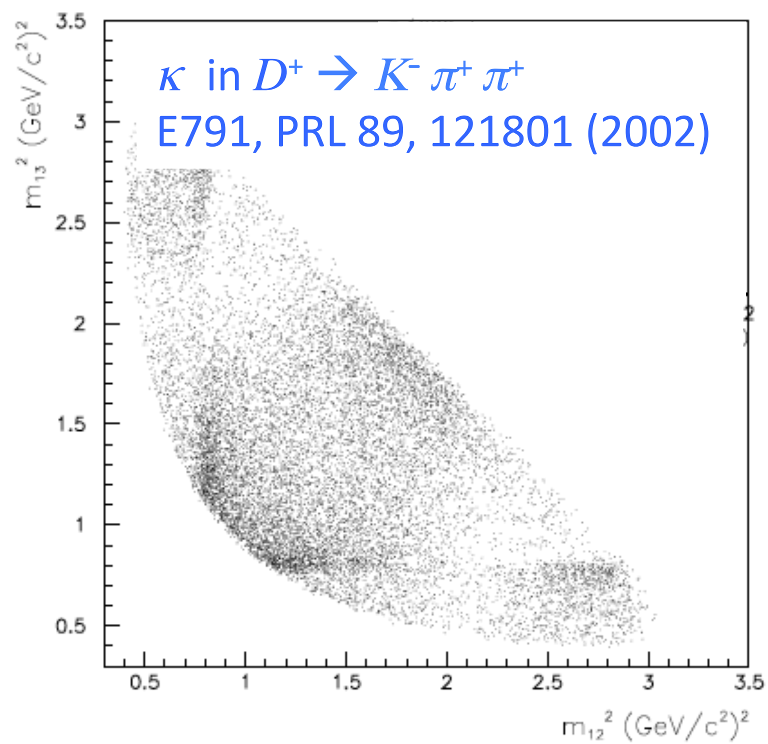
Lots of charm meson decay data from **Charm & B factories** (BES, Belle, Babar, LHCb, etc.)

→ Partial wave decay amplitudes can be extracted

→ Information of hadron interactions and resonances thereby

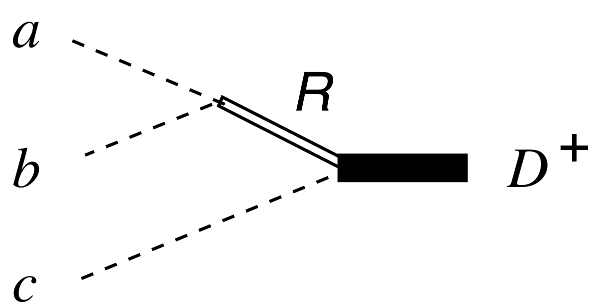
→ CPV analysis (B meson decays), CKM matrix, new physics search

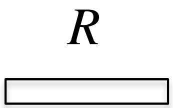




Conventional analysis method --- isobar model

Isobar amplitude





$$\sim \frac{1}{E - M + i\frac{\Gamma}{2}}$$

Decay amplitude = coherent sum of isobar amplitudes & flat background

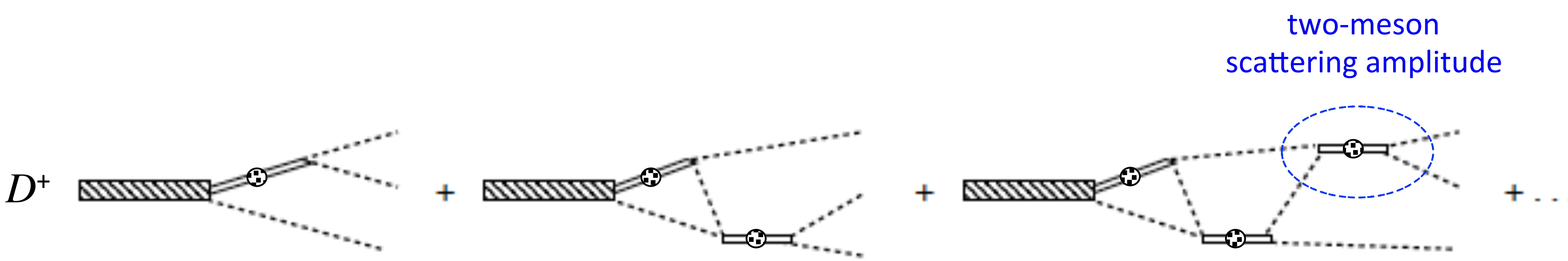
Strength and phase of each $D^+ \rightarrow Rc$ vertex are fitted to Dalitz plot

Questions on isobar models

- No explicit treatment of three-body scattering effects (how bad is it ?)
- Violation of Watson theorem in elastic region (need to be fixed)

In this work, these questions are addressed in $D^+ \rightarrow K^- \pi^+ \pi^+$ Dalitz plot analysis

This work



- Realistic $D^+ \rightarrow K^- \pi^+ \pi^+$ Dalitz plot pseudo-data analyzed

  pseudo-data generated from E791's isobar model PRD 73, 032004 (2006)

- FSI is taken into account, using unitary coupled-channel model
- Demonstrate coupled-channel analysis is feasible for high-quality Dalitz plot data

  first coupled-channel analysis of $D \rightarrow$ three-light-mesons Dalitz plot

- Hadronic dynamics in FSI of $D^+ \rightarrow K^- \pi^+ \pi^+$ examined
- Examine the extent to which isobar model is valid in analyzing Dalitz plot data

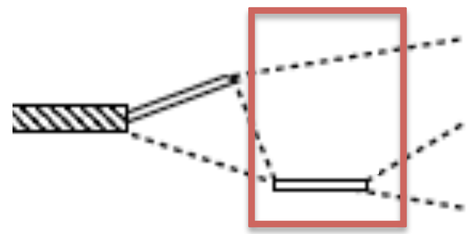
  (How reliably amplitudes are extracted from Dalitz plot)

Unitary coupled-channel model for $D^+ \to K^- \pi^+ \pi^+$

Model

Kamano, SXN, Lee, Sato, PRD 84, 114019 (2011)
SXN, PRD 93, 014005 (2016)

$D^+ \rightarrow K^- \pi^+ \pi^+$



Channels (partial wave) $\left(\bar{K}\pi\right)_S^{I=1/2}\pi\,,\ \left(\bar{K}\pi\right)_P^{I=1/2}\pi\,,\ \left(\bar{K}\pi\right)_D^{I=1/2}\pi\,,\ \left(\pi\pi\right)_P^{I=1}\bar{K}\,,\ \left(\bar{K}\pi\right)_S^{I=3/2}\pi\,,\ \left(\pi\pi\right)_S^{I=2}\bar{K}$

resonances (poles of amplitudes) κ, $K_0^*(1430)$ $K^*(892)$ $K_2^*(1430)$ $\rho(770)$

No flat background

First considered in Dalitz analysis of $D^+ \rightarrow K^- \pi^+ \pi^+$

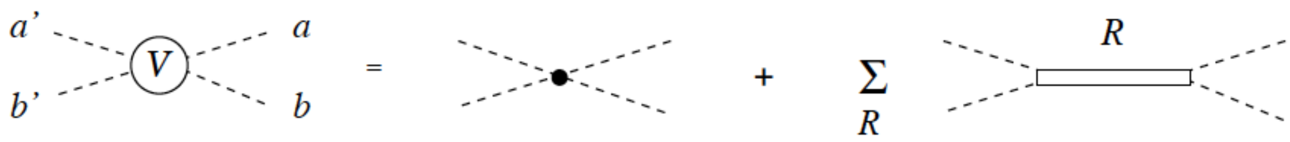
(i) Develop $\bar{K}\pi$ and $\pi\pi$ scattering model

(ii) Develop $\bar{K}\pi\pi$ scattering model based on (i)

(iii) Analyze $D^+ \rightarrow K^- \pi^+ \pi^+$ Dalitz plot data; no adjustment of resonance properties

cf. $K_0^*(1430)$ in previous analyses

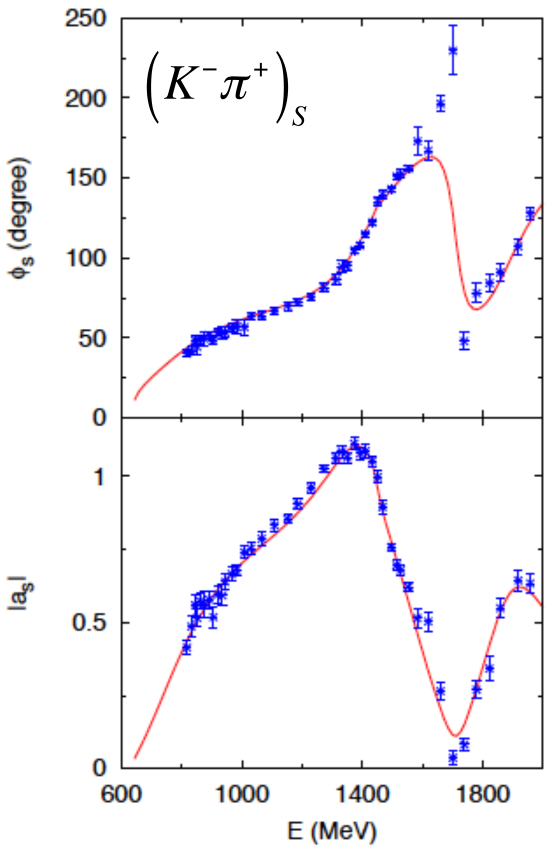
Unitary coupled-channel $\bar{K}\pi$ and $\pi\pi$ scattering model



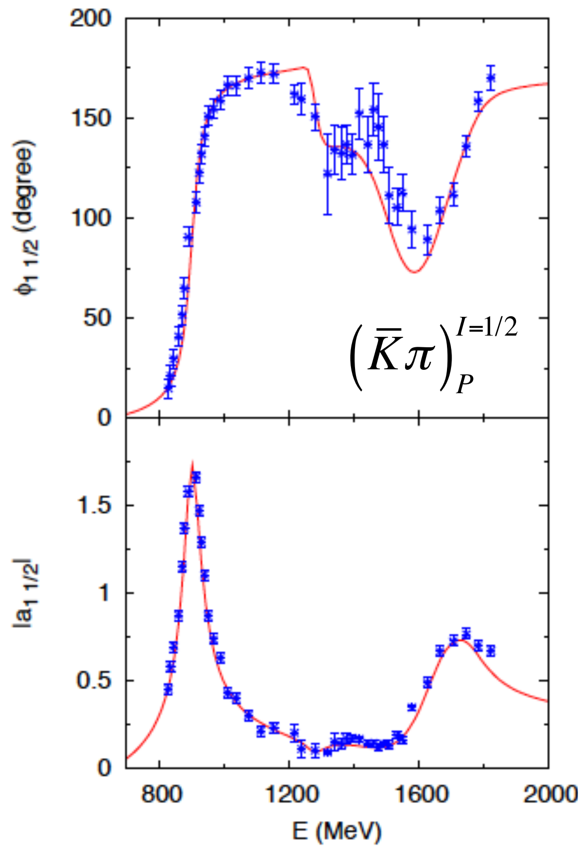
→ Lippmann-Schwinger equation

Coupled-channels

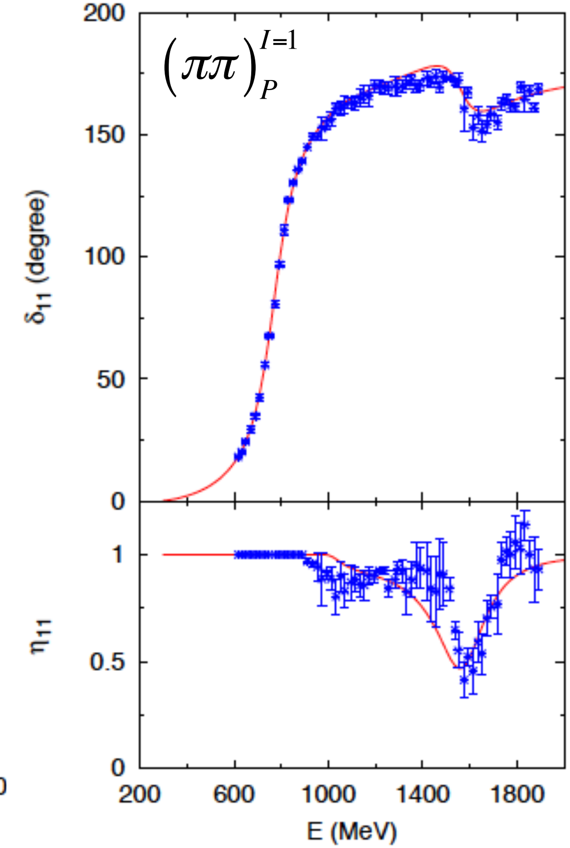
$K^-\pi^+ - \bar{K}^0\eta'$



$\bar{K}\pi - \bar{K}_x\pi_x$

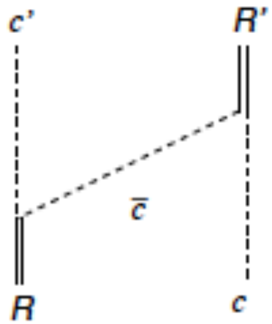


$\pi\pi - K\bar{K}$

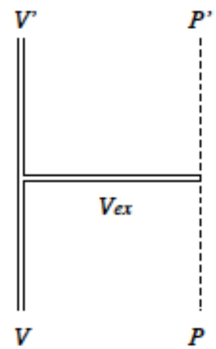


Unitary three-meson scattering model

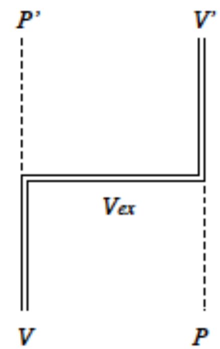
$V =$



$+$



$+$



$\rightarrow$ $T' = V + VGT'$

Z-diagram

no free parameter

'three-meson-force' based on hidden local symmetry model

Cutoff is fitted to Dalitz plot

Bando et al., Phys. Rept. (1988)

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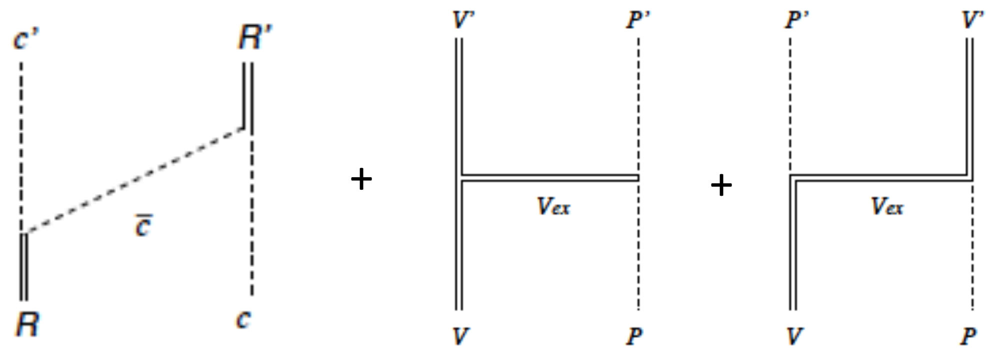
$V =$ 

→ $T' = V + VGT'$

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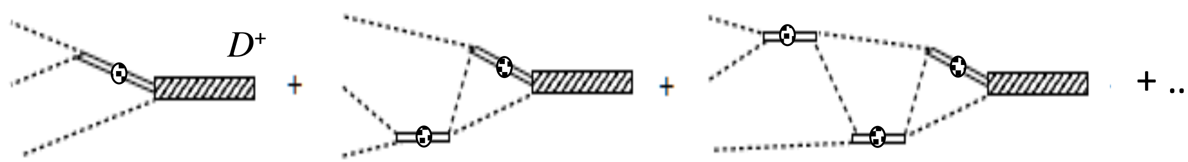
D^+ -decay amplitude



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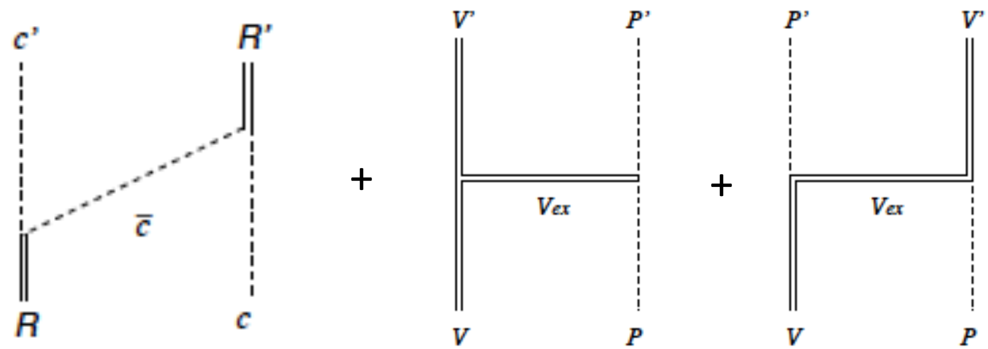

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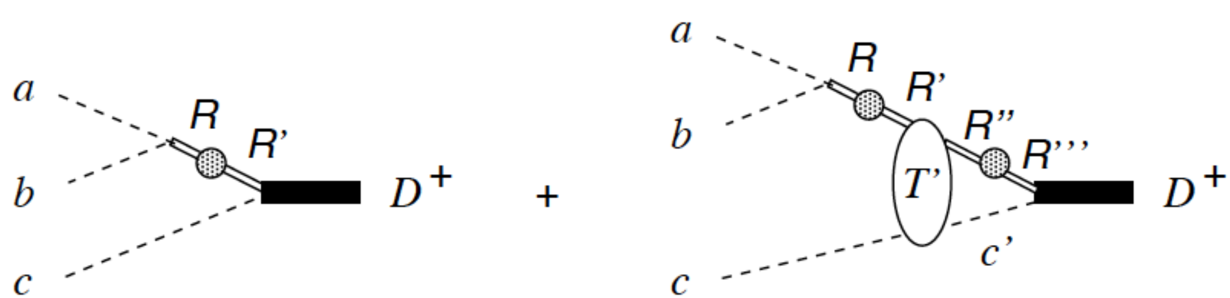


Strength and phase of $D^+ \rightarrow Rc$ vertices fitted to Dalitz plot (Watson theorem maintained)

Three models : Full, Z (no three-meson-force), Isobar (no rescattering)

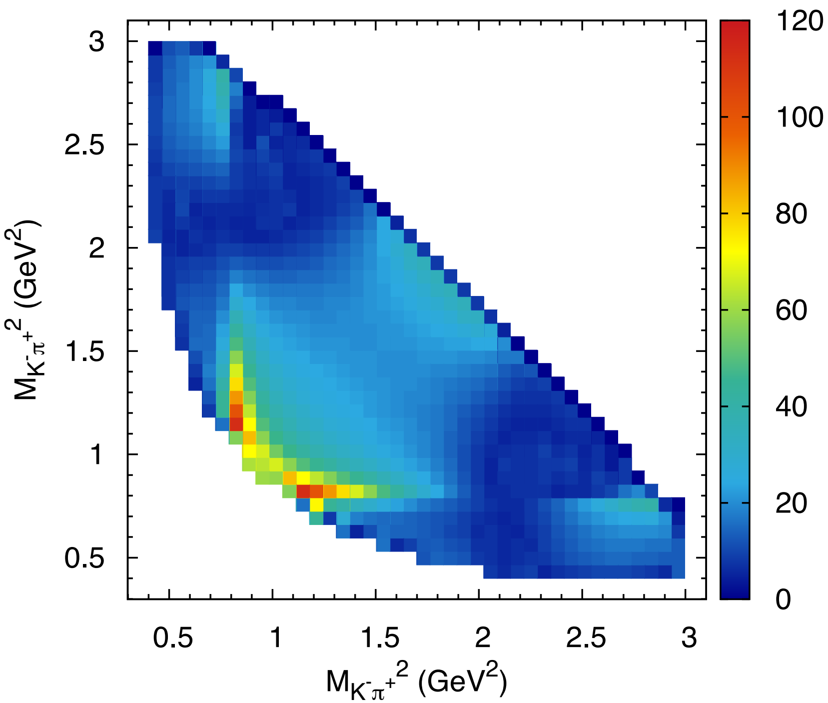
Numerical results for

$D^+ \to K^- \pi^+ \pi^+$ Dalitz plot analysis

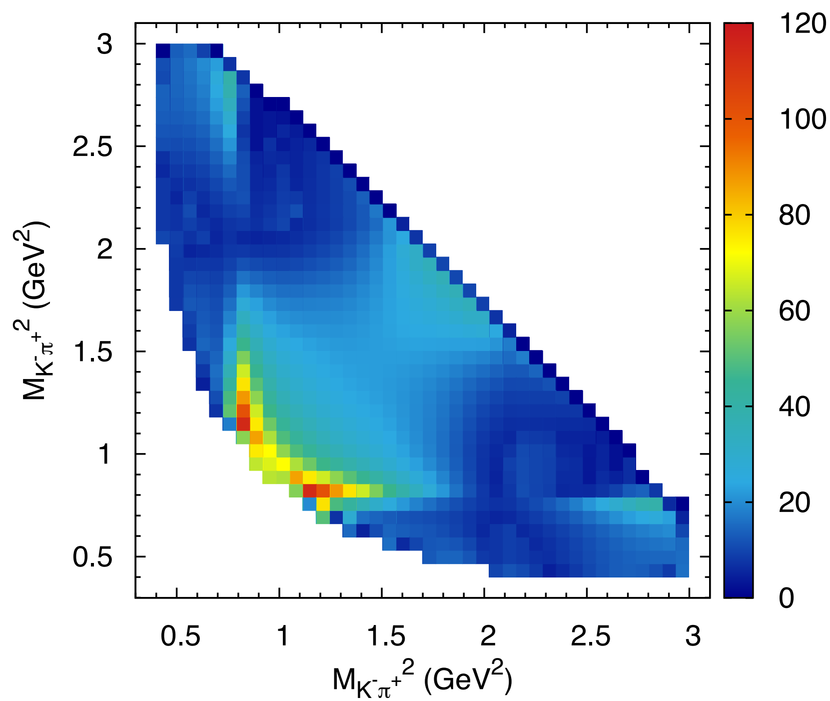
Results

SXN, Phys. Rev. D 93, 014005 (2016)

Pseudo-data (total # of bins: 725)



Fit (Full model)



$$\chi^2 = \sum_j \chi_j^2 = \sum_j \left(\frac{N_j^{th} - N_j^{exp}}{\Delta N_j^{exp}} \right)^2$$

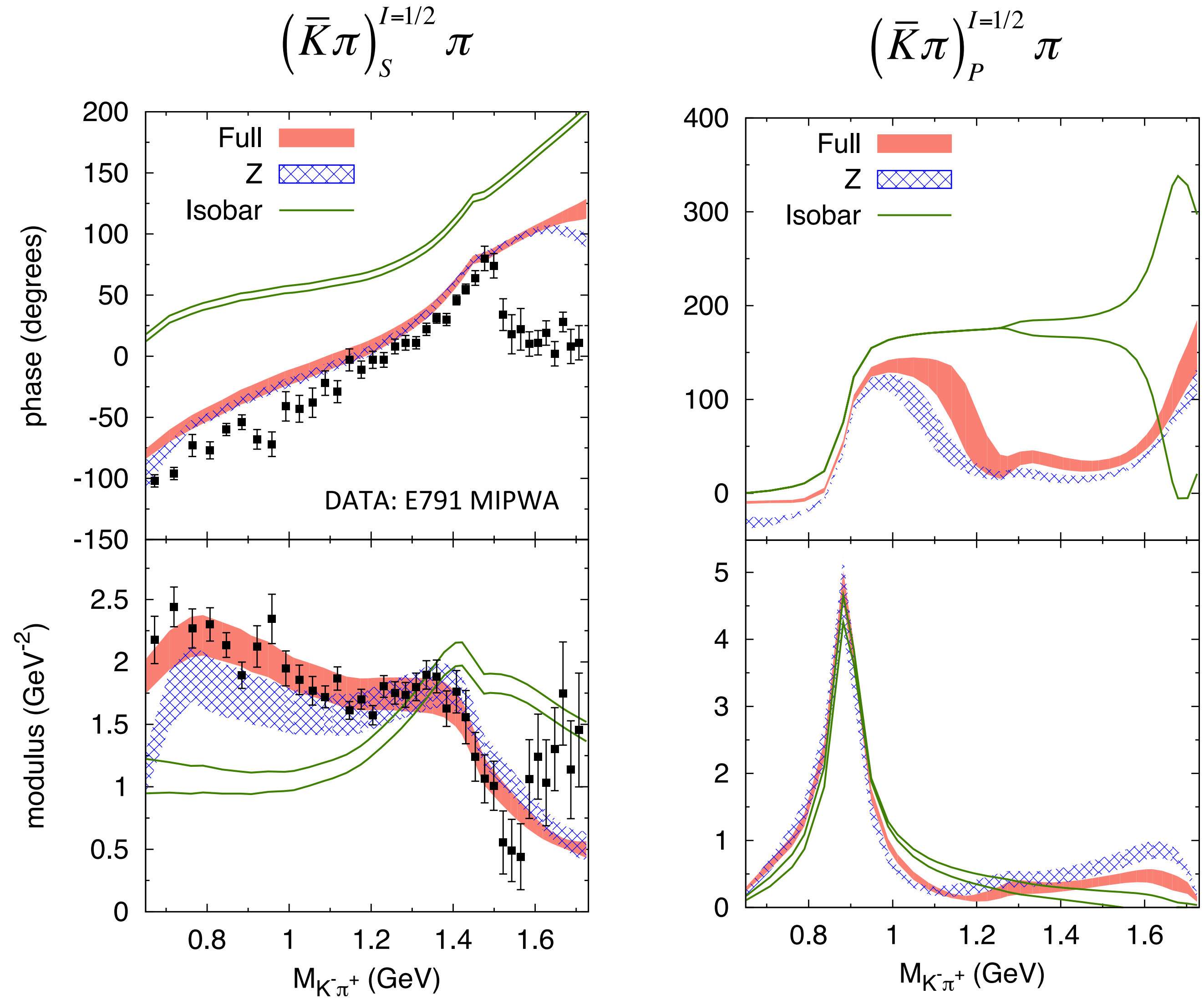
$$\Delta N_j^{exp} = \sqrt{N_j^{exp}}$$

Total # of events : 14,234

χ^2/d.o.f.

	Full	Z	Isobar
	0.22	0.16	0.42
# of fit parameters →	16	15	12

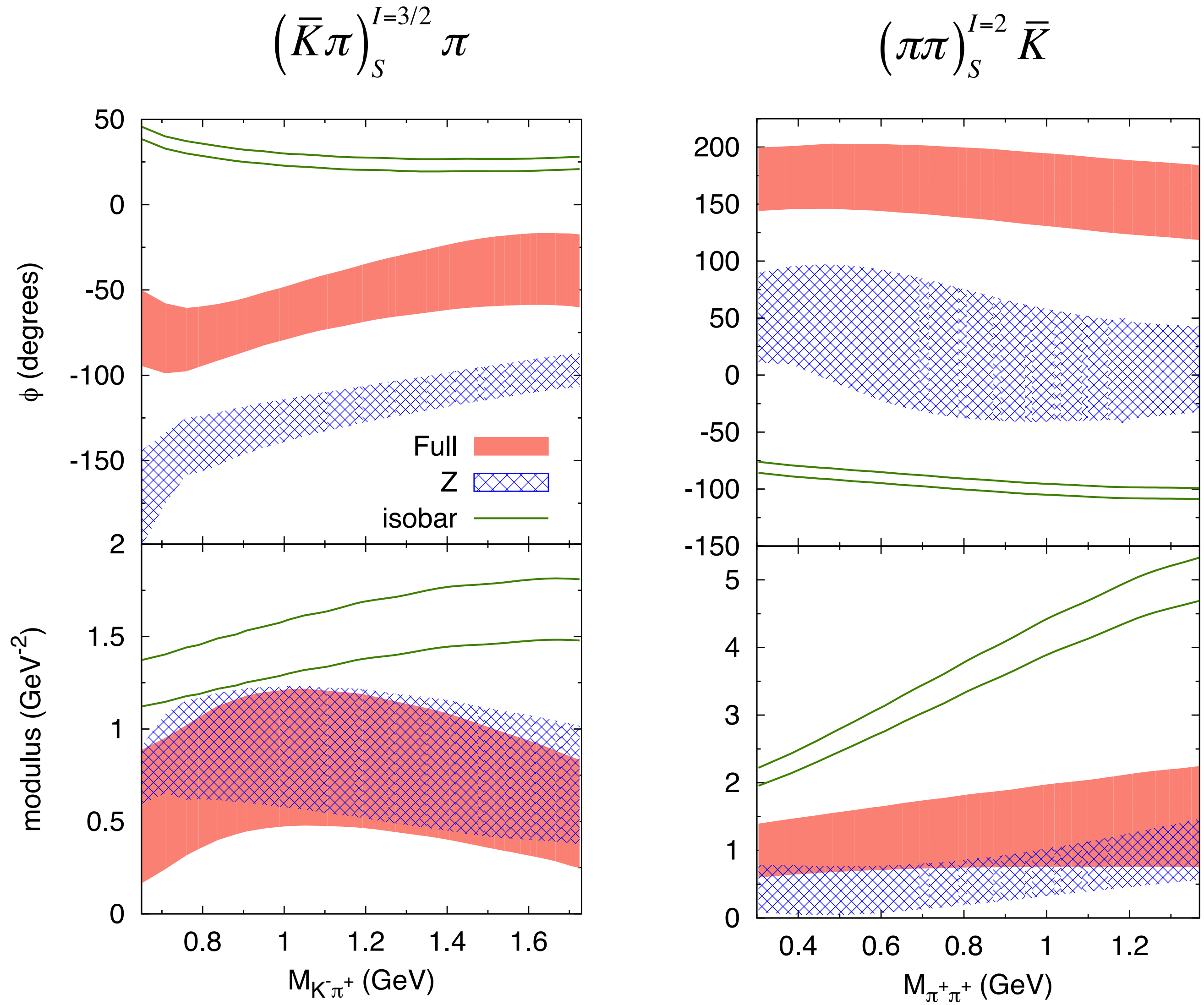
Partial wave amplitudes



- Full and Z (and MIPWA) reasonably agree, while Isobar is significantly different
- Phase in elastic region deviates from Watson theorem (LASS amplitude) ← rescattering effect

cf. interference between I=1/2 and 3/2, Edera and Pennington, PLB623 (2005); FOCUS, PLB 653 (2007)

Partial wave amplitudes



- Non-resonant amplitudes, $\left(\bar{K}\pi\right)_S^{I=3/2}\pi$ and $\left(\pi\pi\right)_S^{I=2}\bar{K}$ are not well determined by the data used
- Sum of s-waves, $\left(\bar{K}\pi\right)_S^{I=1/2}\pi$, $\left(\bar{K}\pi\right)_S^{I=3/2}\pi$ and $\left(\pi\pi\right)_S^{I=2}\bar{K}$ is well determined

Fit fractions

$$\frac{\Gamma_{D^+\to K^-\pi^+\pi^+}(\text{partial wave})}{\Gamma_{D^+\to K^-\pi^+\pi^+}(\text{total})} \times 100\ (\%)$$

E791, PRD 73 (2006)
FOCUS, PLB 681 (2009)

	Full	Z	Isobar	E791 [5] isobar	FOCUS [8] K-matrix
$(K^-\pi^+)_S^{I=1/2}\pi^+$	96.7 ± 6.1	78.9 ± 3.9	69.3 ± 5.1	$33.8 \pm \langle 10.8 \rangle$	207.25 ± 25.45
$(K^-\pi^+)_P^{I=1/2}\pi^+$	15.4 ± 1.0	16.3 ± 1.7	13.8 ± 1.1	$16.2 \pm \langle 1.6 \rangle$	$\langle 15.99 \pm 1.18 \rangle$
$(K^-\pi^+)_D^{I=1/2}\pi^+$	0.5 ± 0.1	0.3 ± 0.1	0.3 ± 0.1	0.6 ± 0.1	0.39 ± 0.09
$(K^-\pi^+)_S^{I=3/2}\pi^+$	27.7 ± 15.2	33.1 ± 19.1	111.9 ± 32.9	—	40.50 ± 9.63
$(\pi^+\pi^+)_S^{I=2}K^-$	22.2 ± 13.3	4.9 ± 5.4	175.7 ± 26.0	—	–
Background	—	—	—	17.2 ± 5.3	–
S-waves	(81.7 ± 0.9)	(82.3 ± 0.7)	(81.2 ± 0.7)	$(79.8 \pm \langle 12.0 \rangle)$	(83.23 ± 1.50)
Sum	162.5	133.5	371.0	66.3	264.13

- Full and Z models are fairly consistent while Isobar model is significantly different

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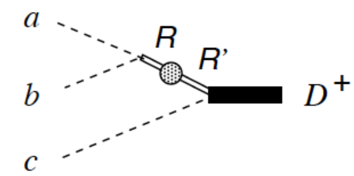
- FF of $(\bar{K}\pi)_S^{I=3/2}\pi$, $(\pi\pi)_S^{I=2}\bar{K}$ are not well determined with the data used
- All analyses agree on FF from coherent sum of s-waves with small error
- Large destructive interference between $I=1/2$ and $3/2$ in FOCUS analysis

Bare fit fractions



Def. $$FF = \frac{\Gamma_{D^+\to K^-\pi^+\pi^+ + \bar{K}^0\pi^+\pi^0}\text{(partial wave; no rescattering)}}{\text{(sum of bare partial wave decay widths)}} \times 100\ (\%)$$

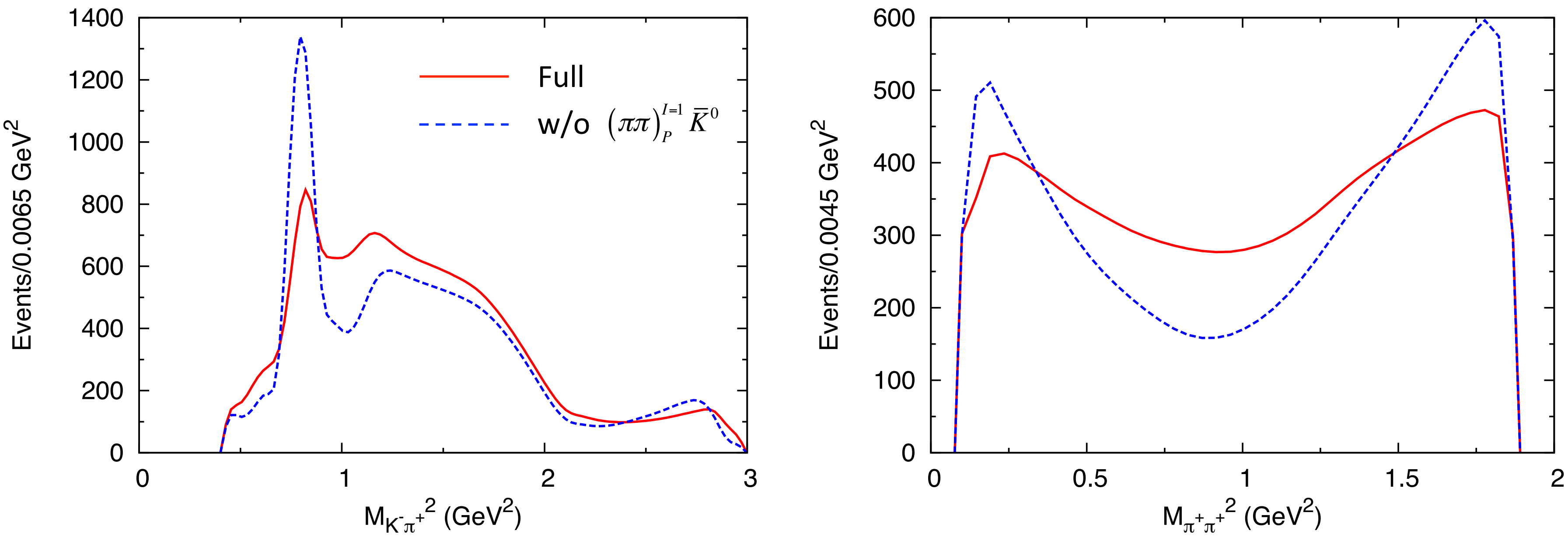
	Full	Z	Isobar
$(\bar{K}\pi)_S^{I=1/2}\pi^+$	45.3 ± 8.7	40.5 ± 3.8	—
$(\bar{K}\pi)_P^{I=1/2}\pi^+$	12.6 ± 1.1	5.7 ± 0.8	—
$(\bar{K}\pi)_D^{I=1/2}\pi^+$	0.3 ± 0.1	0.1 ± 0.0	—
$(\pi^+\pi^0)_P^{I=1}\bar{K}^0$	16.0 ± 0.9	42.6 ± 0.5	—
$(\bar{K}\pi)_S^{I=3/2}\pi$	8.4 ± 6.8	7.3 ± 5.9	—
$(\pi\pi)_S^{I=2}\bar{K}$	17.3 ± 13.3	3.9 ± 4.0	—

Large contribution of $(\pi\pi)_P^{I=1}\bar{K}^0$ where $\rho(770)$ plays a major role

- $(\pi\pi)_P^{I=1}\bar{K}^0$ contributes to $D^+ \to K^- \pi^+ \pi^+$ only through channel-coupling (cf. isobar model)
- Consistent with large $(\pi\pi)_P^{I=1}\bar{K}^0$ fit fraction (85%) in $D^+ \to \bar{K}^0 \pi^+ \pi^0$ analysis (BES III, 2014)
- Full and Z models have rather different $(\pi\pi)_P^{I=1}\bar{K}^0$ contribution

→ combined analysis of $D^+ \to K^- \pi^+ \pi^+$ and $D^+ \to \bar{K}^0 \pi^+ \pi^0$ would be needed

Effects of hadronic rescattering on $D^+ \to K^- \pi^+ \pi^+$ spectra



$(\pi\pi)_P^{I=1} \bar{K}^0$ contribution significantly change the shape of the spectra

$\rho(770)$ plays a major role

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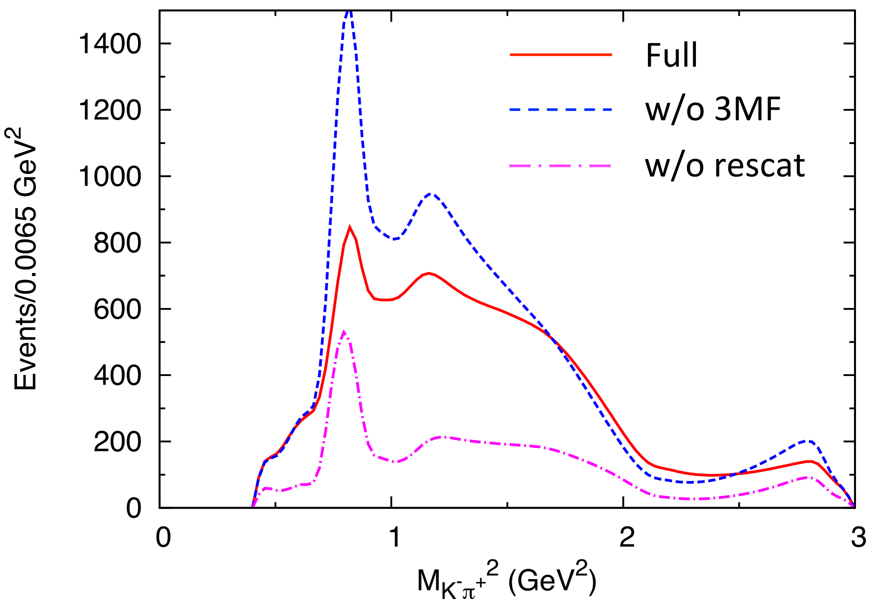
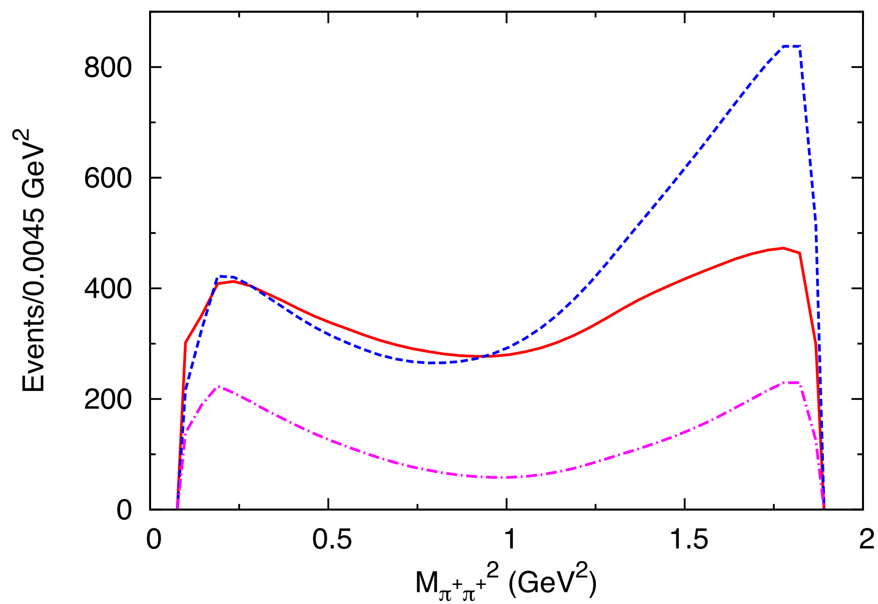
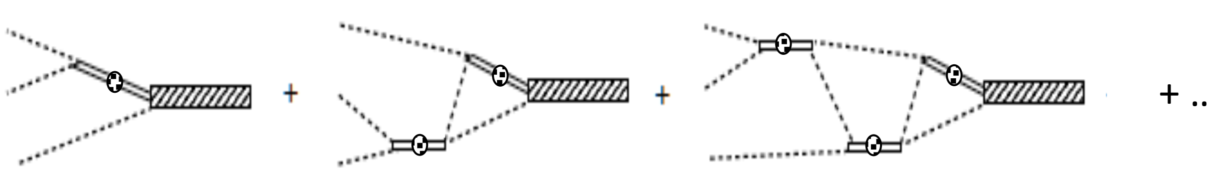
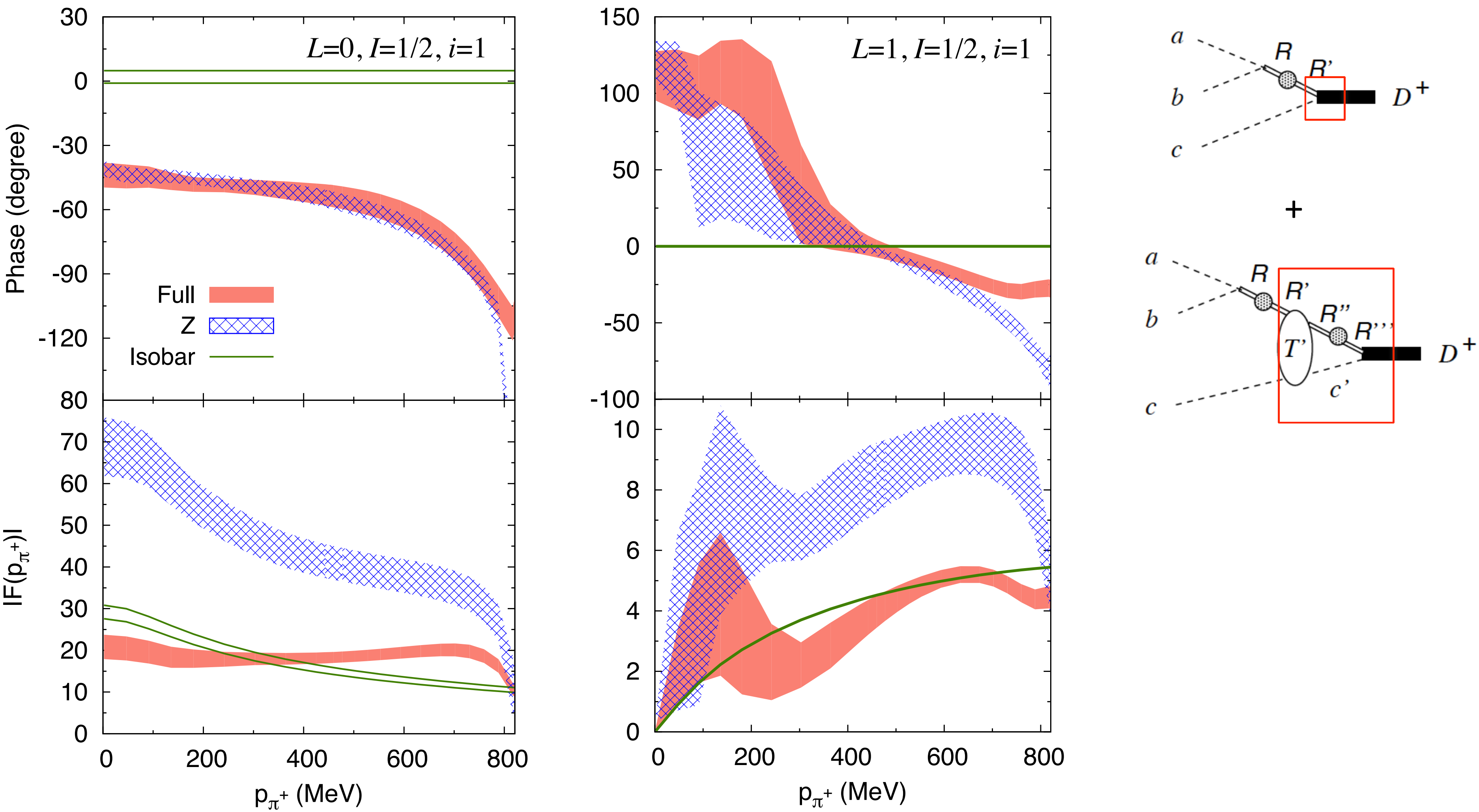




- Three-meson-force (3MF) significantly changes the shape of spectra
- Hadronic rescattering almost triplicate the decay width
- Very different picture of hadronic dynamics from isobar model



Dressed $D^+ \rightarrow R_i^{LI}\, c$ vertices



- Constant phase is assumed in isobar model
- Hadronic rescattering generates phase with non-trivial momentum dependence

Summary

Summary

What we did

First coupled-channel analysis of $D^+ \to K^- \pi^+ \pi^+$ Dalitz plot

Analyzed pseudo-data from E791's isobar model

Conclusion

- Good fits to data are obtained for Full, Z and Isobar models
- Full and Z are similar in partial wave amplitudes (fit fractions),

  while Isobar model is significantly different

  $\to$ importance of channel-couplings, rescattering effects

  $\to$ isobar model analysis should be viewed with caution
- Large hadronic rescattering effects

  Decay width triplicated, shape of spectra significantly changed

  $(\pi\pi)_P^{I=1} \bar{K}^0$ [$\rho(770)$] partial wave plays important role

Ongoing project

Collaborator: J.-J. Wu

Application of unitary coupled-channel model to

Radiative J/ψ decays to $\pi\pi\pi$, $\pi\pi\eta$, $\pi K\bar{K}$

relevant to $\eta(1405/1475)$

Puzzles about $\eta(1405/1475)$

$\eta(1405)$ observed in $\pi\pi\eta$ and πKK ($a_0(980)\pi$ dominant) final states

$\eta(1475)$ observed in πKK ($KK^*(892)$ dominant) final states

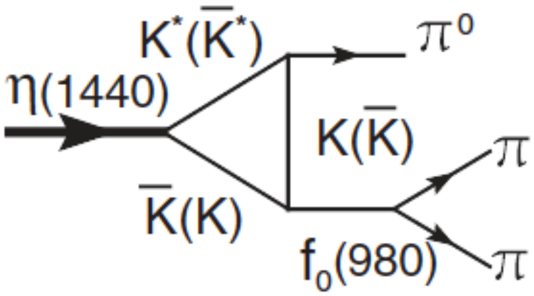
Q. Are $\eta(1405)$ and $\eta(1475)$ different states ?

$\eta(1405) \rightarrow f_0(980)\pi \rightarrow \pi\pi\pi$ observed BESIII, PRL 108 182001 (2012)

Q. What is causing the large isospin violation ?

Puzzles about $\eta(1405/1475)$

Promising answer Wu et al. PRL 108 081803 (2012)



- Triangle singularity enhances isospin-violating mechanism
- Triangle singularity shifts $\eta(1440)$ peak differently for different decay modes → single state solution

More elaborate analysis can be made with unitary coupled-channel model

Improvements

Tree + one-loop	→	full-order of hadronic rescattering (unitarity)
Breit-Wigner mass and width of $\eta(1440)$	→	pole of $\eta(1440)$ propagator that includes decay dynamics of the unitary coupled-channel model

Back up

Error estimation

Inverse of error matrix (Hessian)

$$H_{ij} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial \theta_i \partial \theta_j} \bigg|_{\{\theta\}=\{\bar{\theta}\}}$$

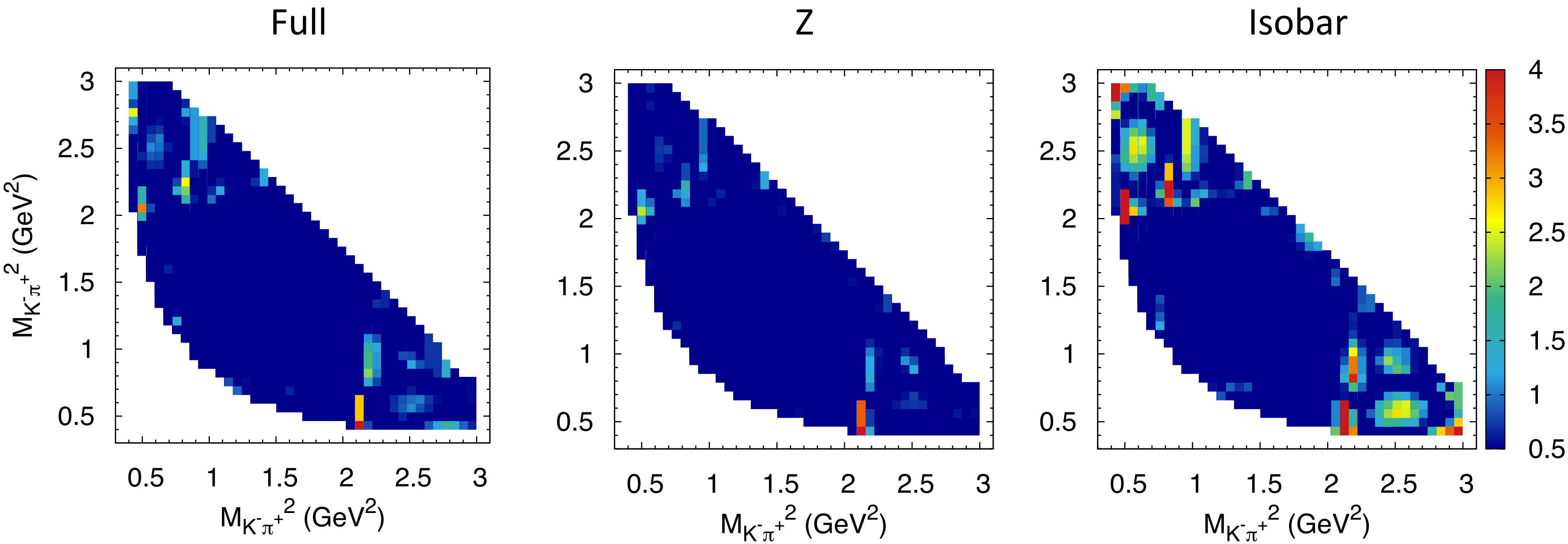
Error matrix

$$E_{ij} = (H^{-1})_{ij} \qquad \Delta\theta_i = \sqrt{E_{ii}}$$

Error propagation

$$[\delta X]^2 = \sum_{i,j} \frac{\partial X}{\partial \theta_i} \bigg|_{\{\theta\}=\{\bar{\theta}\}} E_{ij} \frac{\partial X}{\partial \theta_j} \bigg|_{\{\theta\}=\{\bar{\theta}\}}$$

χ^2 distribution



total χ^2/d.o.f.

Full	Z	Isobar
0.22	0.16	0.42

Comparison of similar analyses of $D^+ \to K^- \pi^+ \pi^+$

All analyses showed importance of FSI in describing $D^+ \to K^- \pi^+ \pi^+$

- Magalhães et al. (PRD 84, 094001 (2011), PRD 92, 094005 (2015))

  First analysis of rescattering effect on $\left(\bar{K}\pi\right)_S^{I=1/2}\pi$ amplitude

  Some relevant resonances and partial waves are missing

  Dalitz plot was not analyzed

- Niecknig and Kubis (JHEP 1510, 142 (2015))

  Dalitz plot analysis based on dispersion theory

  Include all relevant resonances and partial waves in elastic region

  Analysis was limited to elastic region (about half of phase space)

- This work (PRD 93, 014005 (2016))

  Dalitz plot analysis based on unitary coupled-channel model

  Include all relevant resonances and partial waves in elastic and inelastic region

  Analysis covers all phase space

Possible future directions with coupled-channel model

- Combined analysis of $D^+ \rightarrow K^- \pi^+ \pi^+$ and $D^+ \rightarrow \bar{K}^0 \pi^0 \pi^+$

→ Better determine partial wave amplitudes that have large errors in this work

combined analysis is common in analysis for baryon spectroscopy

(Suggestion to Alberto's talk on Monday to determine $K^+ K^-$ s-wave

→ combined analysis of $D^+ \rightarrow K^+ K^- K^+$ and $K^+ \pi^- \pi^+$)

- $D^0 \rightarrow K_S \pi^+ \pi^-$ dalitz plot analysis

→ determination of γ / ϕ_3 , exploration of new physics

More interesting and precise data to analyze with coupled-channel model

are expected in near future from B and charm factories