宇宙线 3D 各项异性扩散初步结果

刘伟

合作者：林苏杰、郭义庆、胡红波

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Outline

Theoretical motivations

Implementation of Anisotropic diffusion

Preliminary Results

Conclusion

Theoretical motivations

Conventional transport model

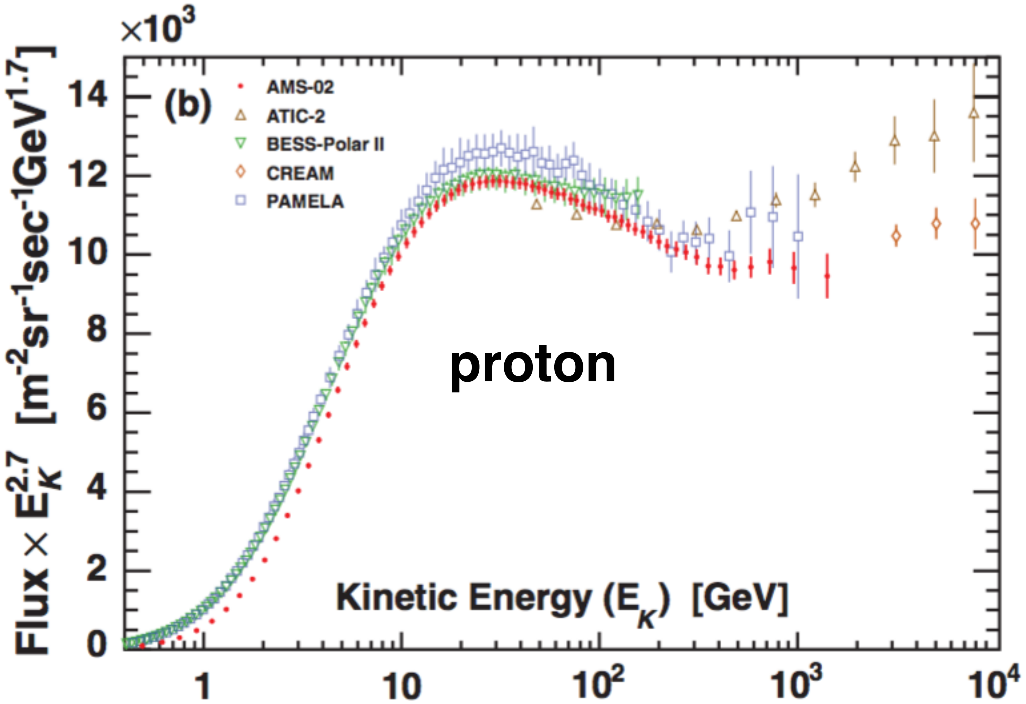
$$\frac{\partial n_i}{\partial t} - \vec{\nabla} \cdot \left(D_{xx} \cdot \vec{\nabla} n_i - \vec{u} n_i \right) - \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} n_i = Q_{\rm inj} + Q_{\rm losses} + Q_{\rm spall/dec}$$

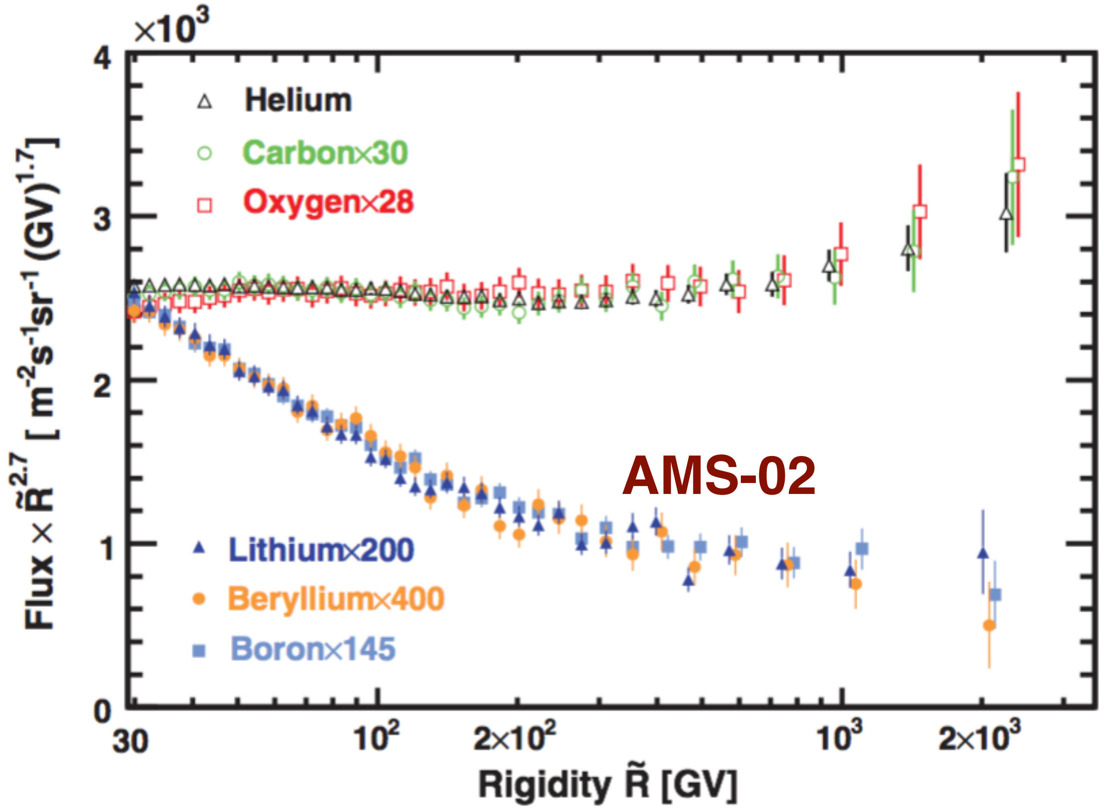
Diffusion coefficient is spatial-dependent, which is only a function of rigidity, i.e.

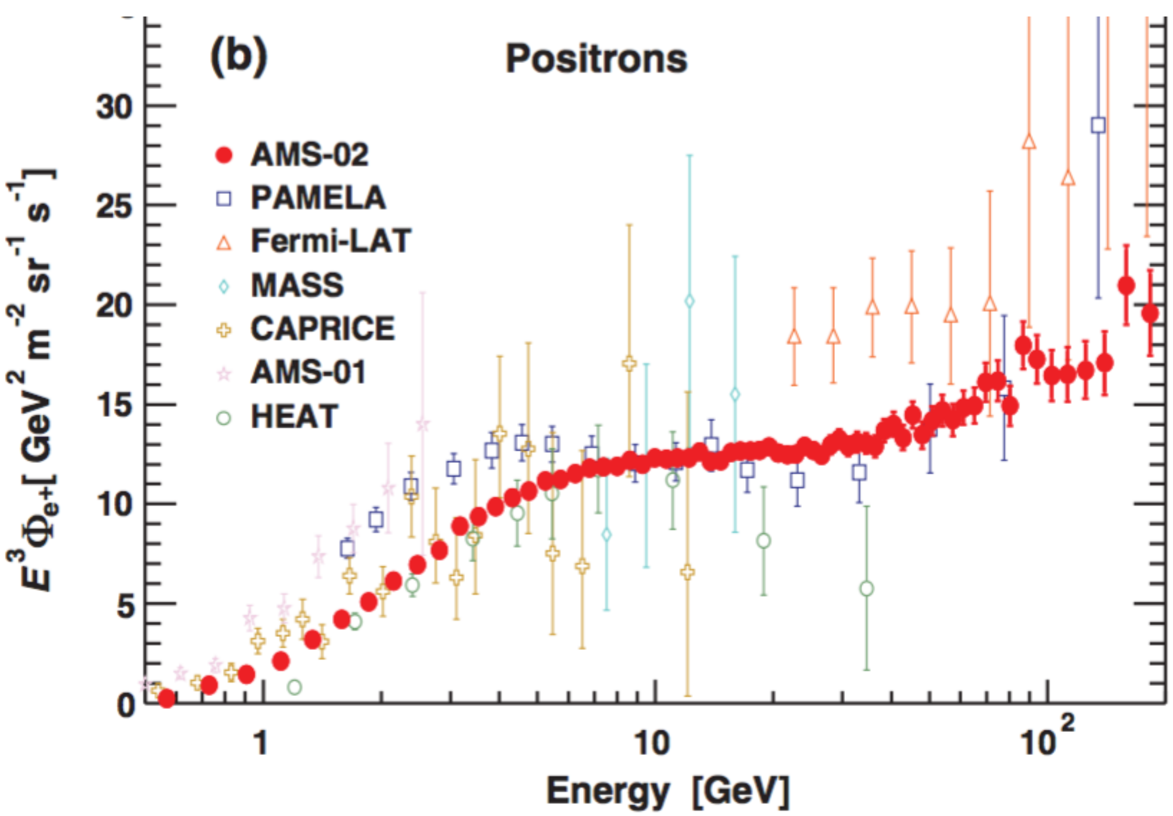
$$D_{xx} \propto (p/Z)^{\delta}$$

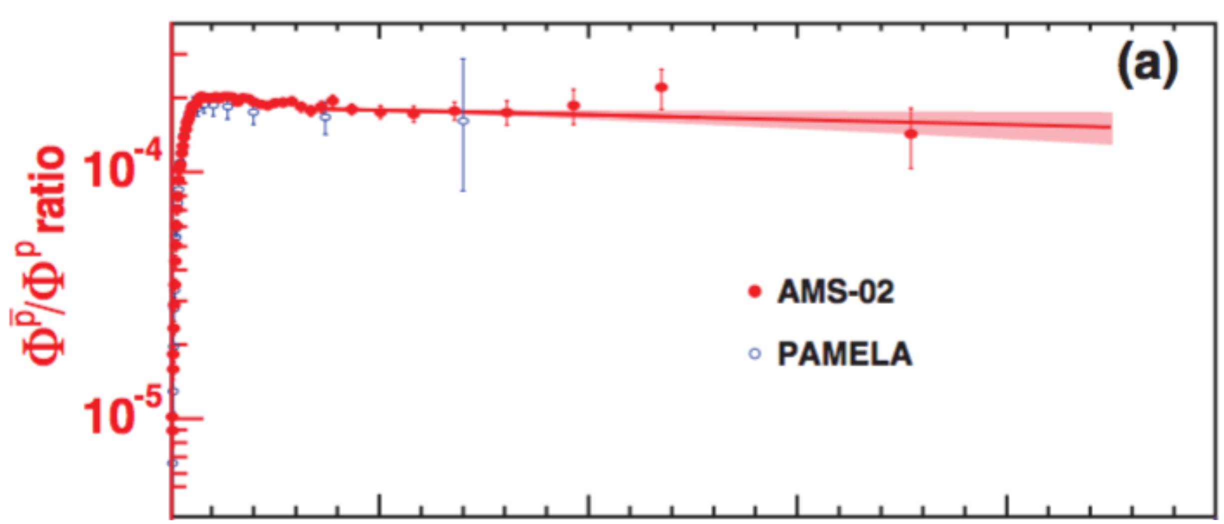
But high precision observations are challenging the convent-ional transport model

Challenge 1: Cosmic-ray energy spectrum



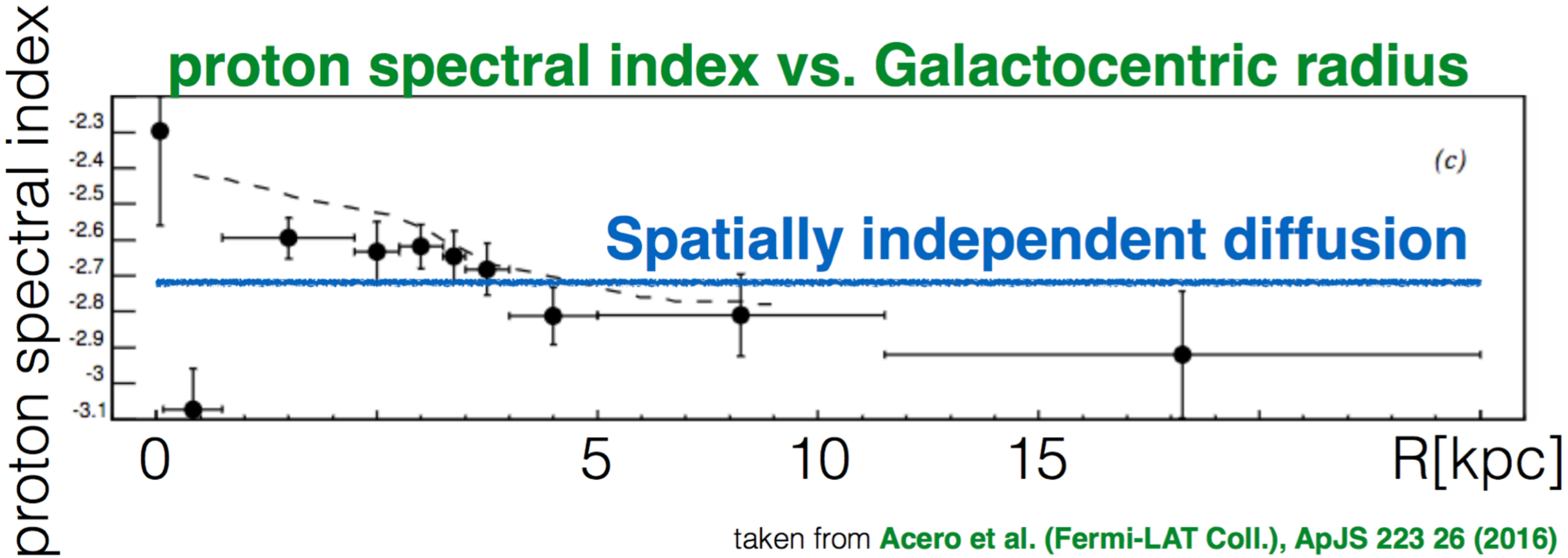






Challenge 2: Cosmic-ray spectral index

Non-local observables (e.g **gamma-rays**)



taken from **Acero et al. (Fermi-LAT Coll.), ApJS 223 26 (2016)**

Challenge 2: Cosmic-ray spectral index

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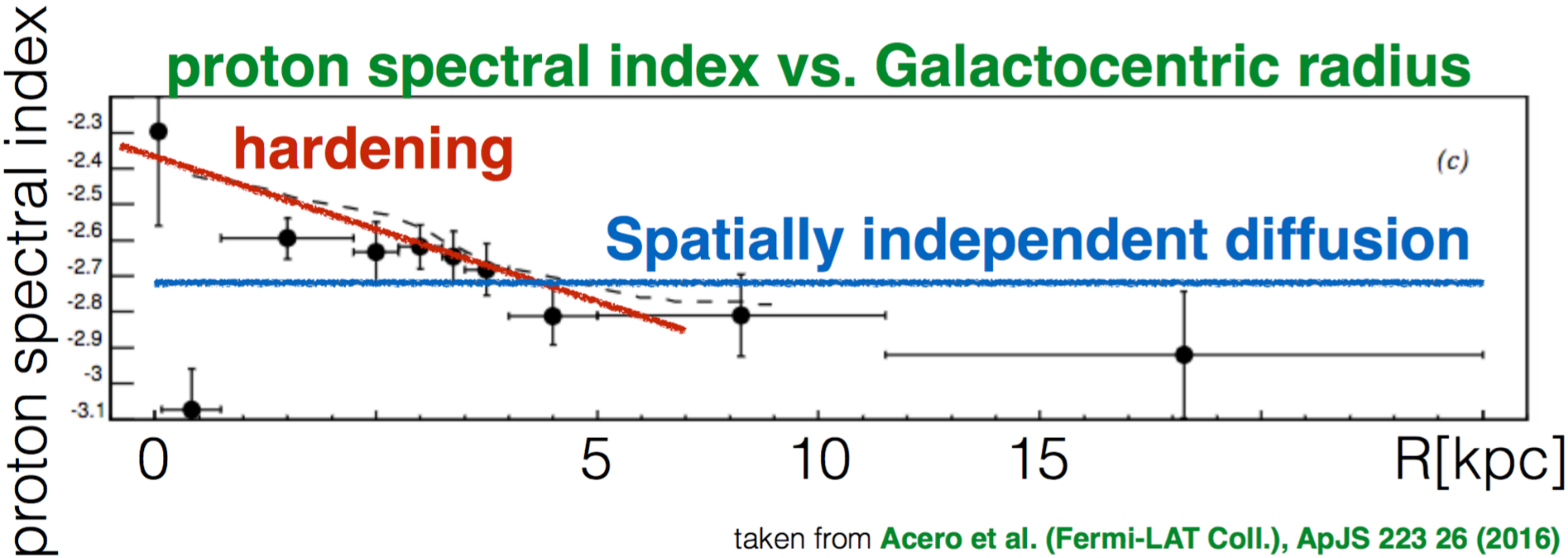


One of the reasonable scenarios is spatial-dependent propagation model

$$D_{xx} = D_0(r,z)\left(\frac{p}{Z}\right)^{\delta(r,z)}$$

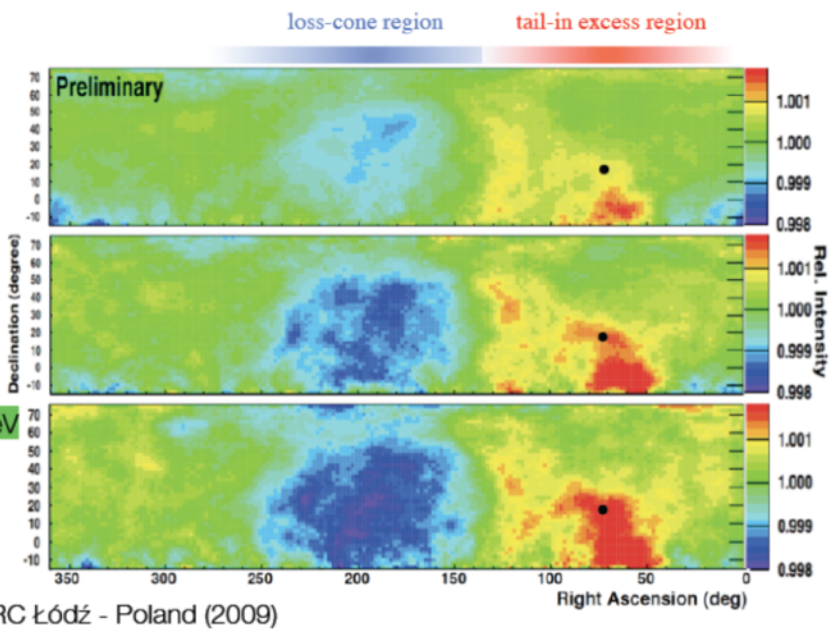
Guo, Yi-Qing; Yuan, Qiang, 2018PhRvD..97f3008G

Challenge3: Cosmic-ray anisotropy

Energy dependence of 2D anisotropy

ARGO-YBJ

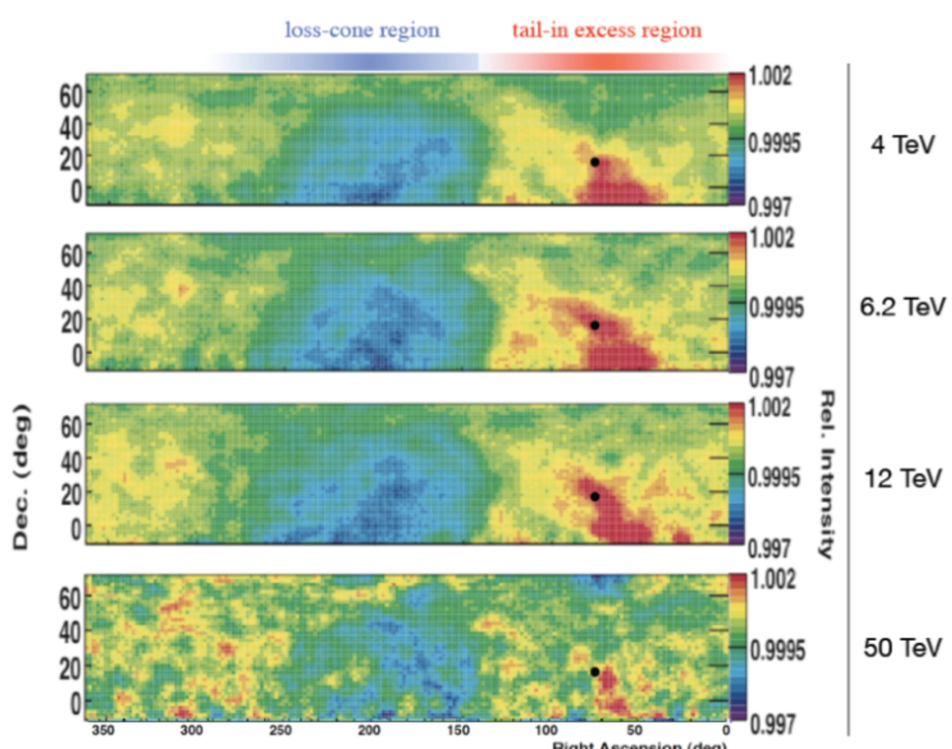
- data from 2008
- 365 days livetime
- $6.5 \cdot 10^{10}$ events
- median CR energy ~ 1.1 TeV



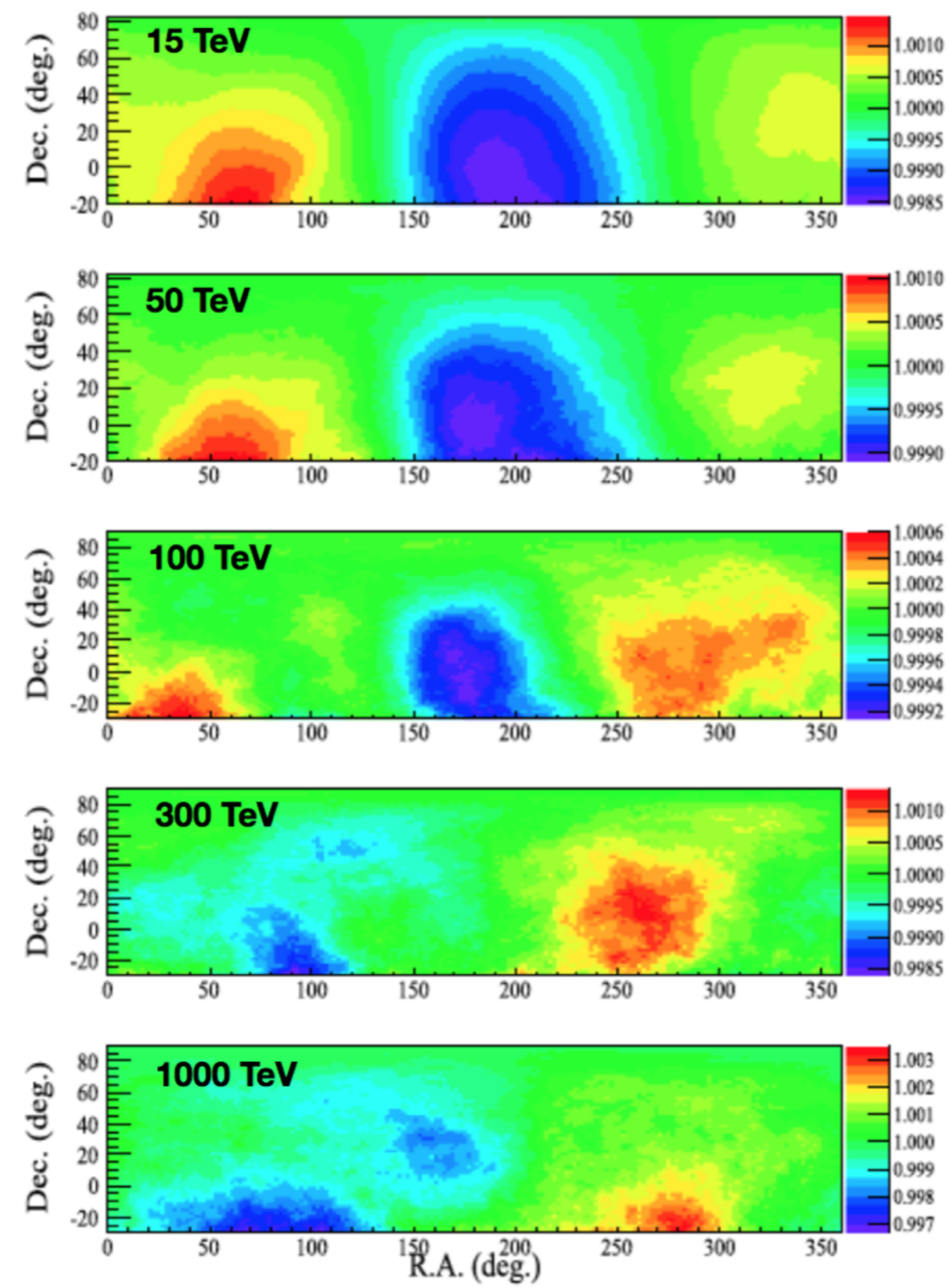
J.L. Zhang et al., 31st ICRC Łódź - Poland (2009)

Tibet-III

- data from 1997 to 2005
- 1874 days livetime
- $3.7 \cdot 10^{10}$ events
- angular resolution ~ 0.9°
- modal CR energy ~ 3 TeV



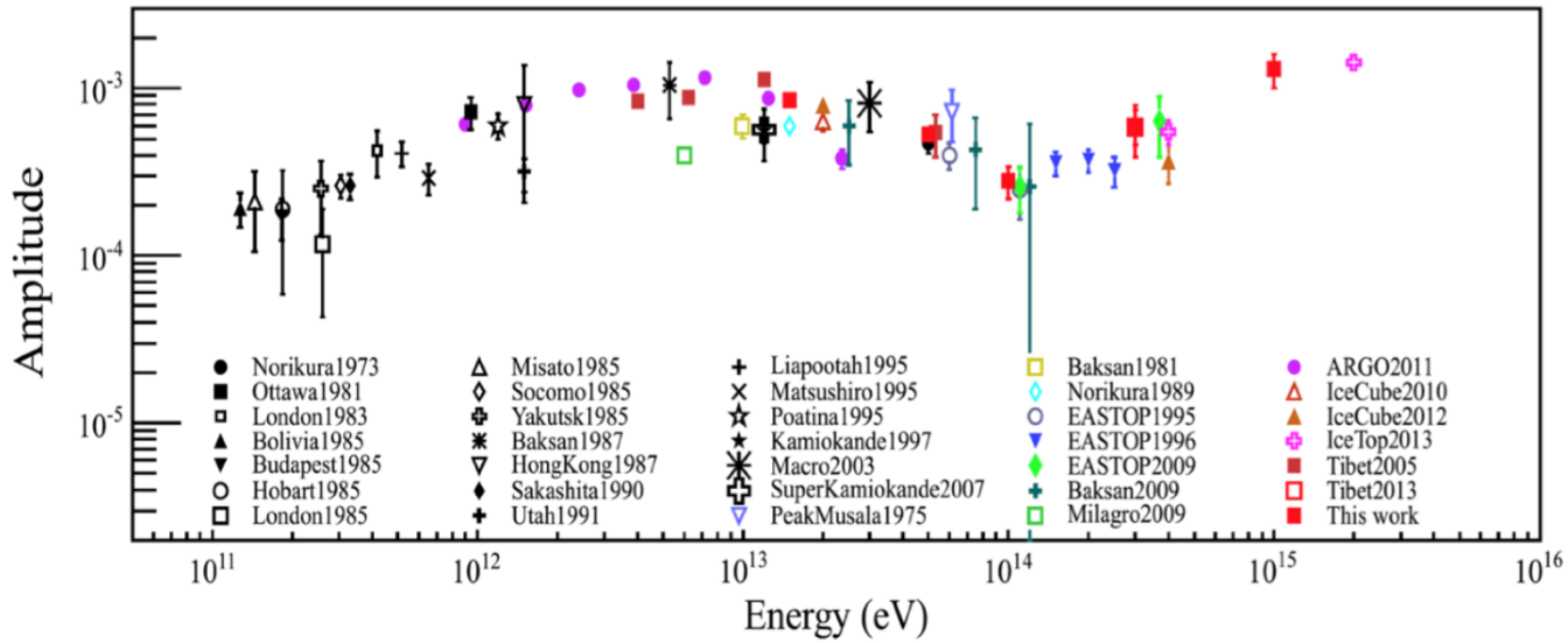
Amenomori et al., Science Vol. 314, pp. 439 (2006)



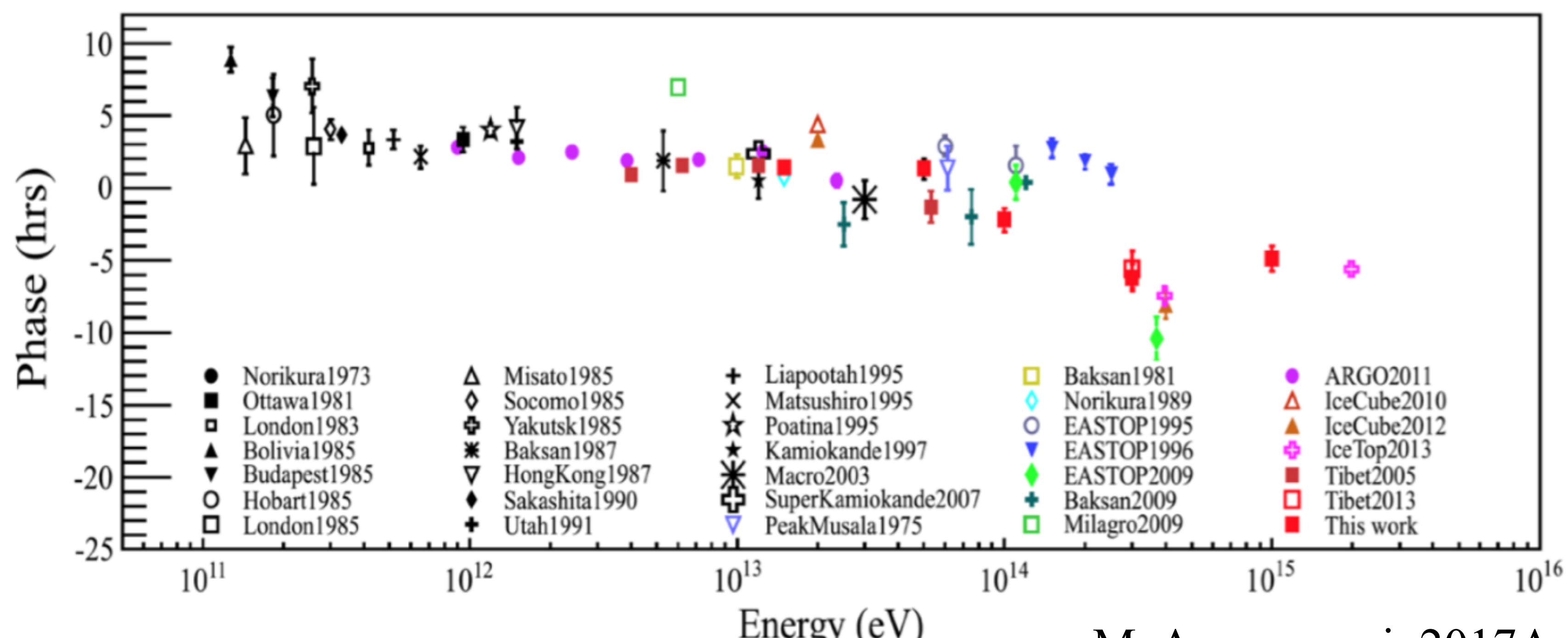
M. Amenomori, 2017ApJ…836..153A

Cosmic-ray anisotropy

A_1



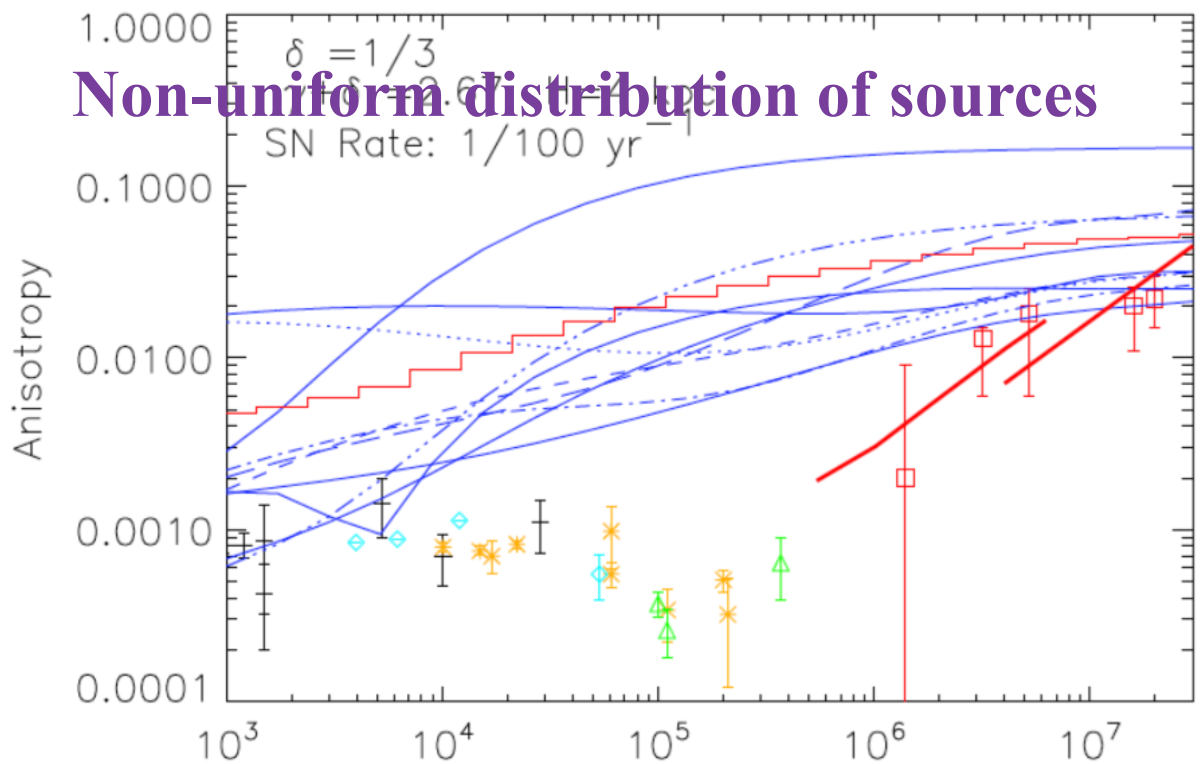
ϕ_1

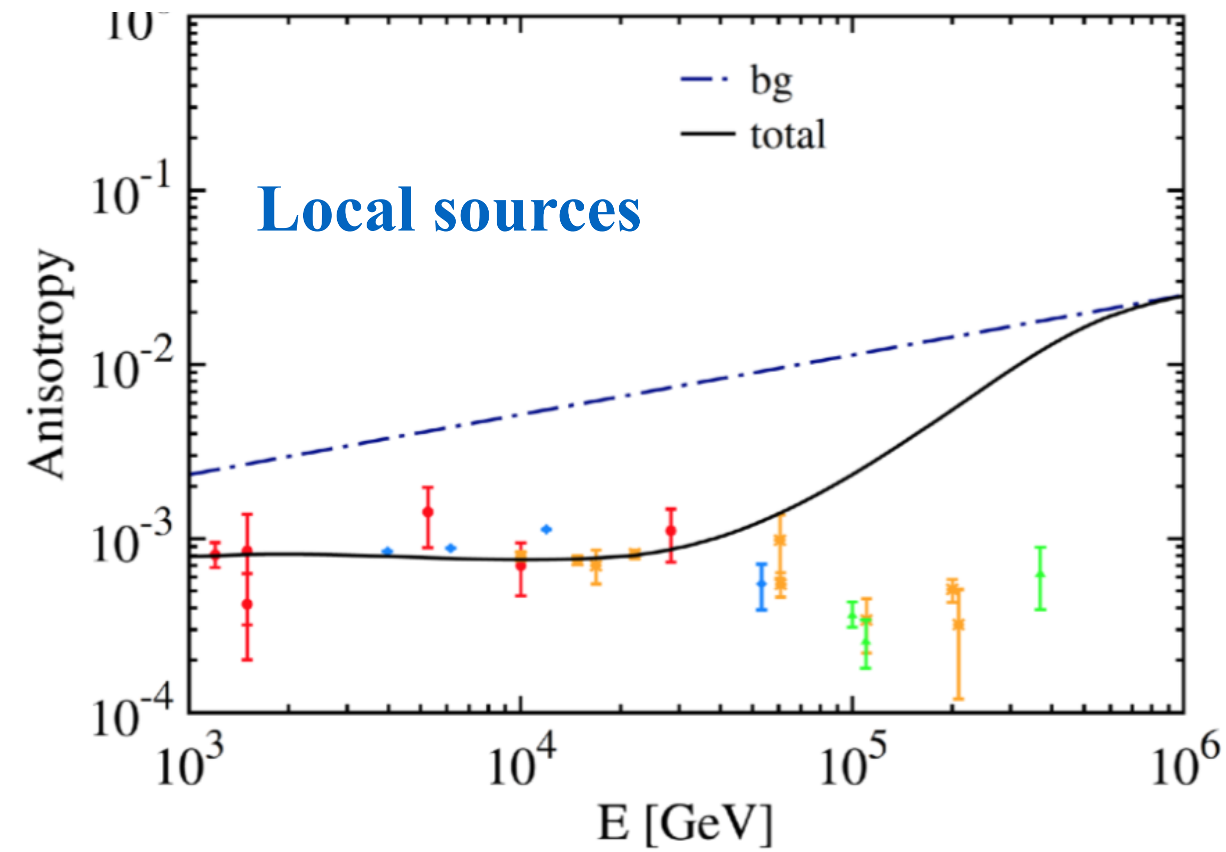


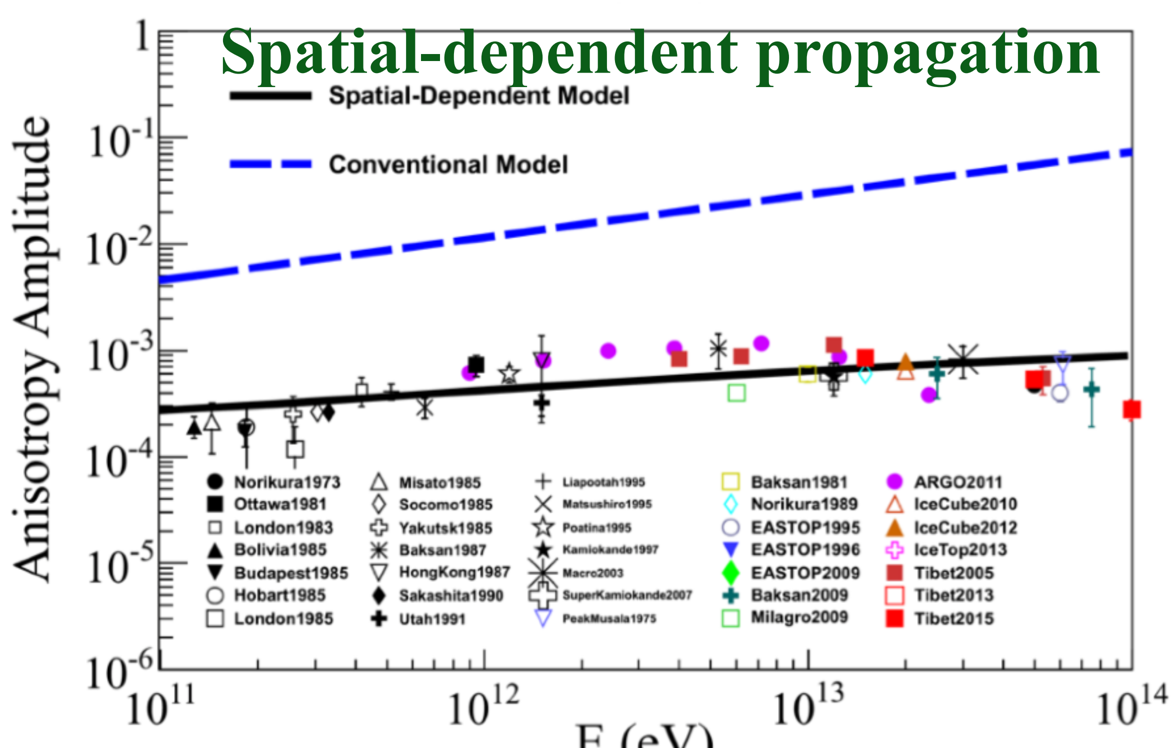
M. Amenomori, 2017ApJ...836..153A

Possible origin of dipole anisotropy

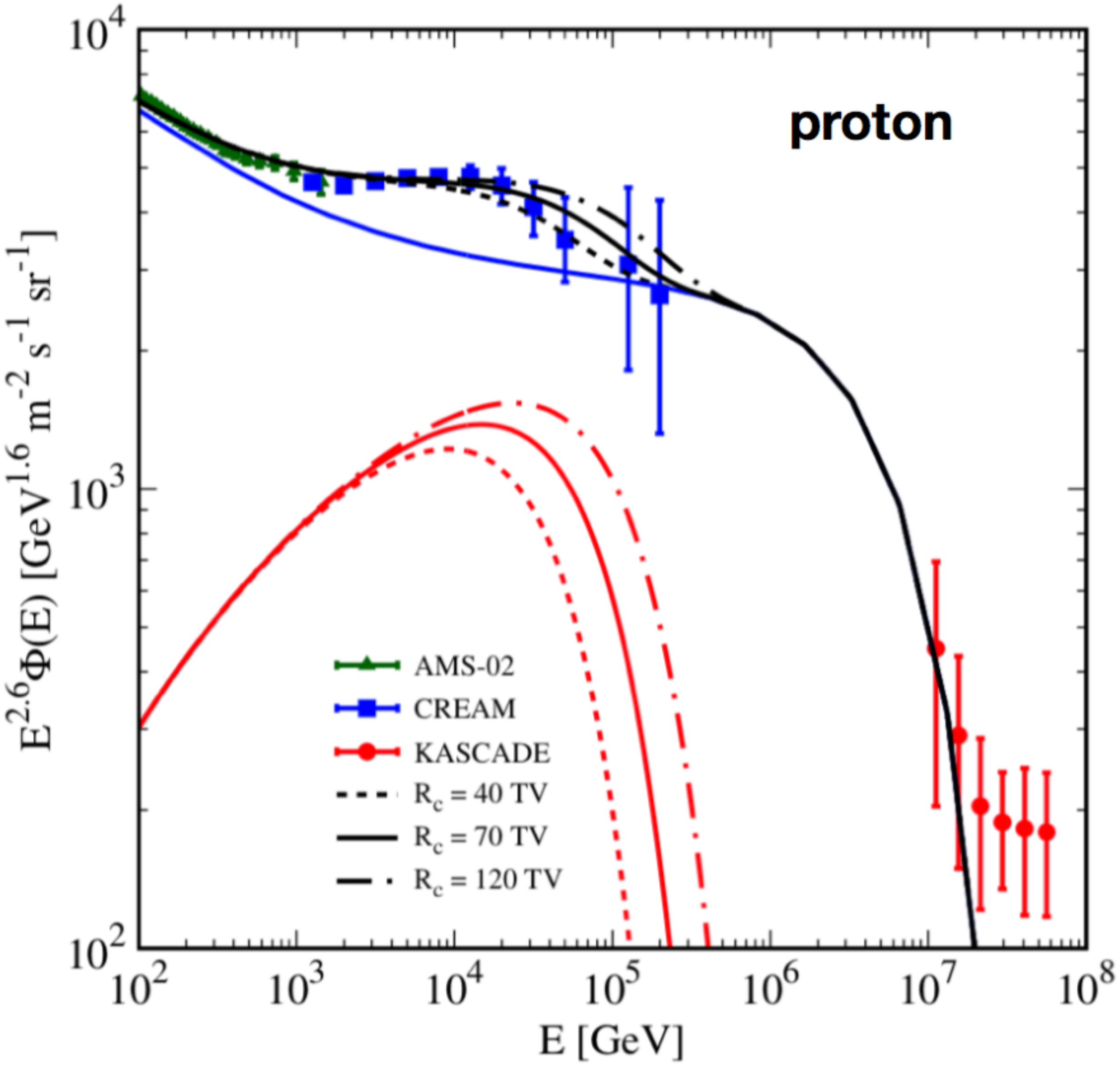
dipole anisotropy is defined as $\delta \equiv \frac{f_{\max} - f_{\min}}{f_{\max} + f_{\min}} = \frac{3\mathbf{K}}{v} \frac{\nabla n_{\mathrm{CR}}}{n_{\mathrm{CR}}}$

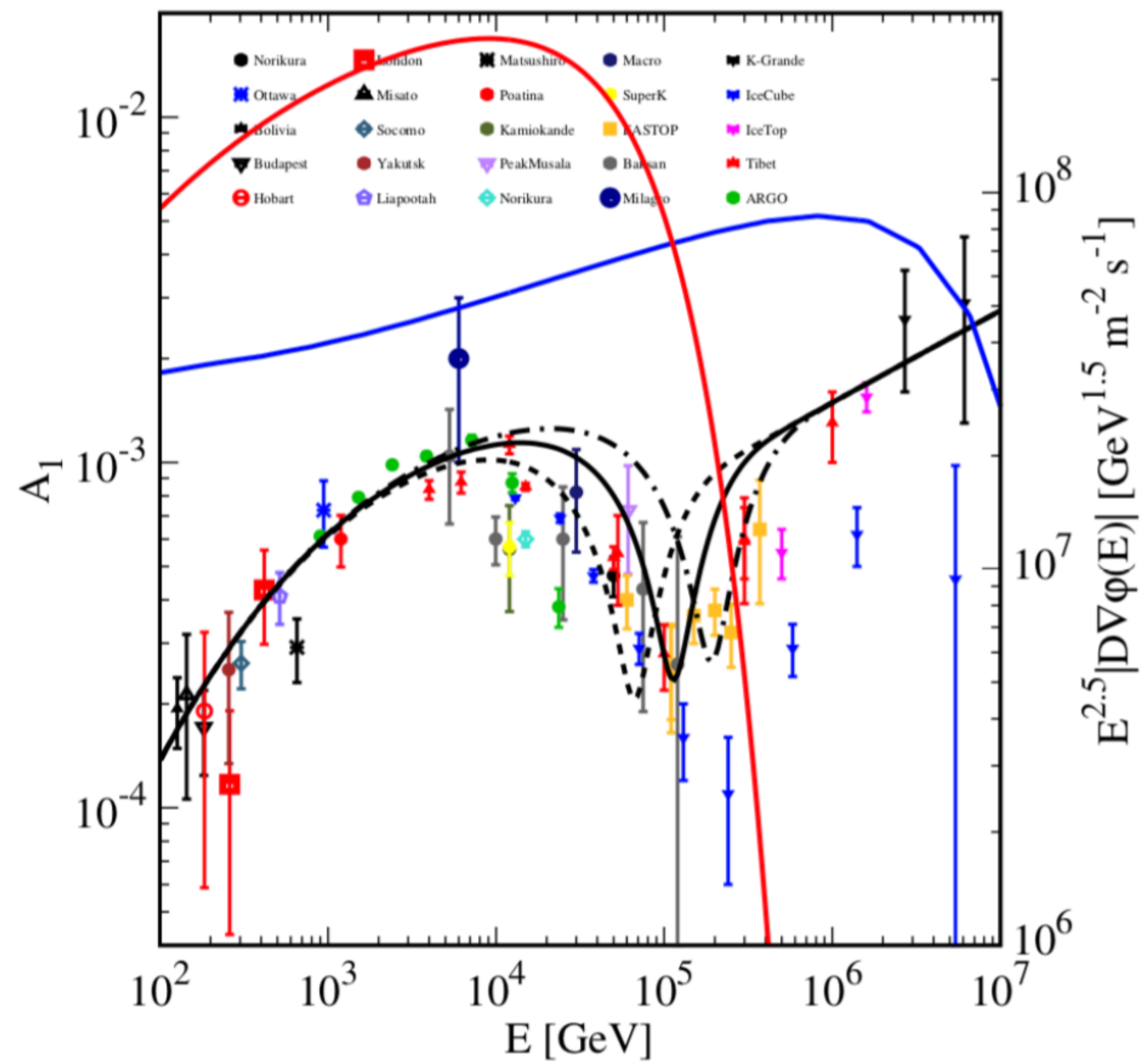






spatial-dependent propagation + local source





Wei Liu, Yi-Qing Guo and Qiang Yuan, arXiv:1812.09673

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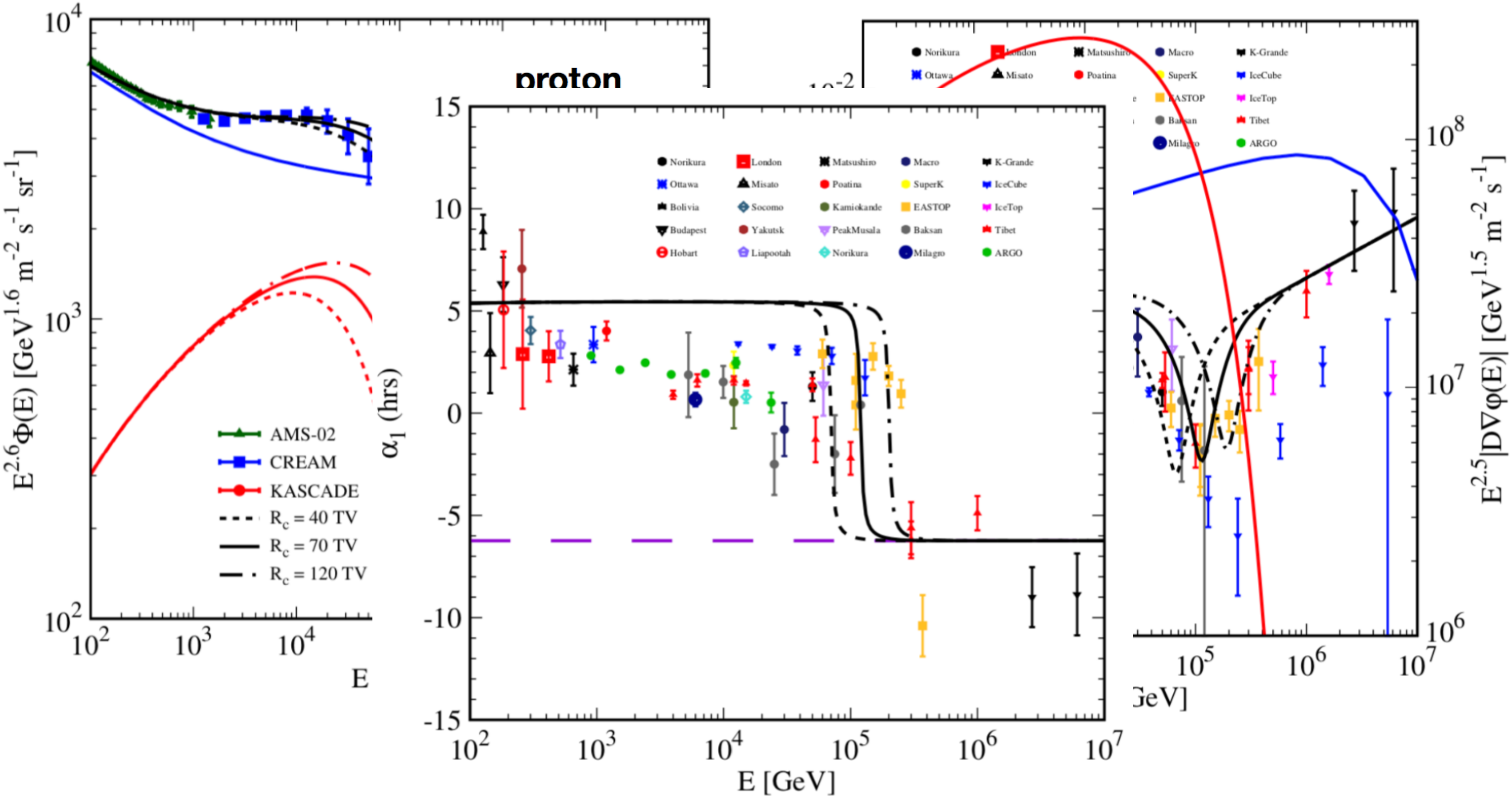


Local ordered magnetic field

- strong ordered magnetic fields in the local environment

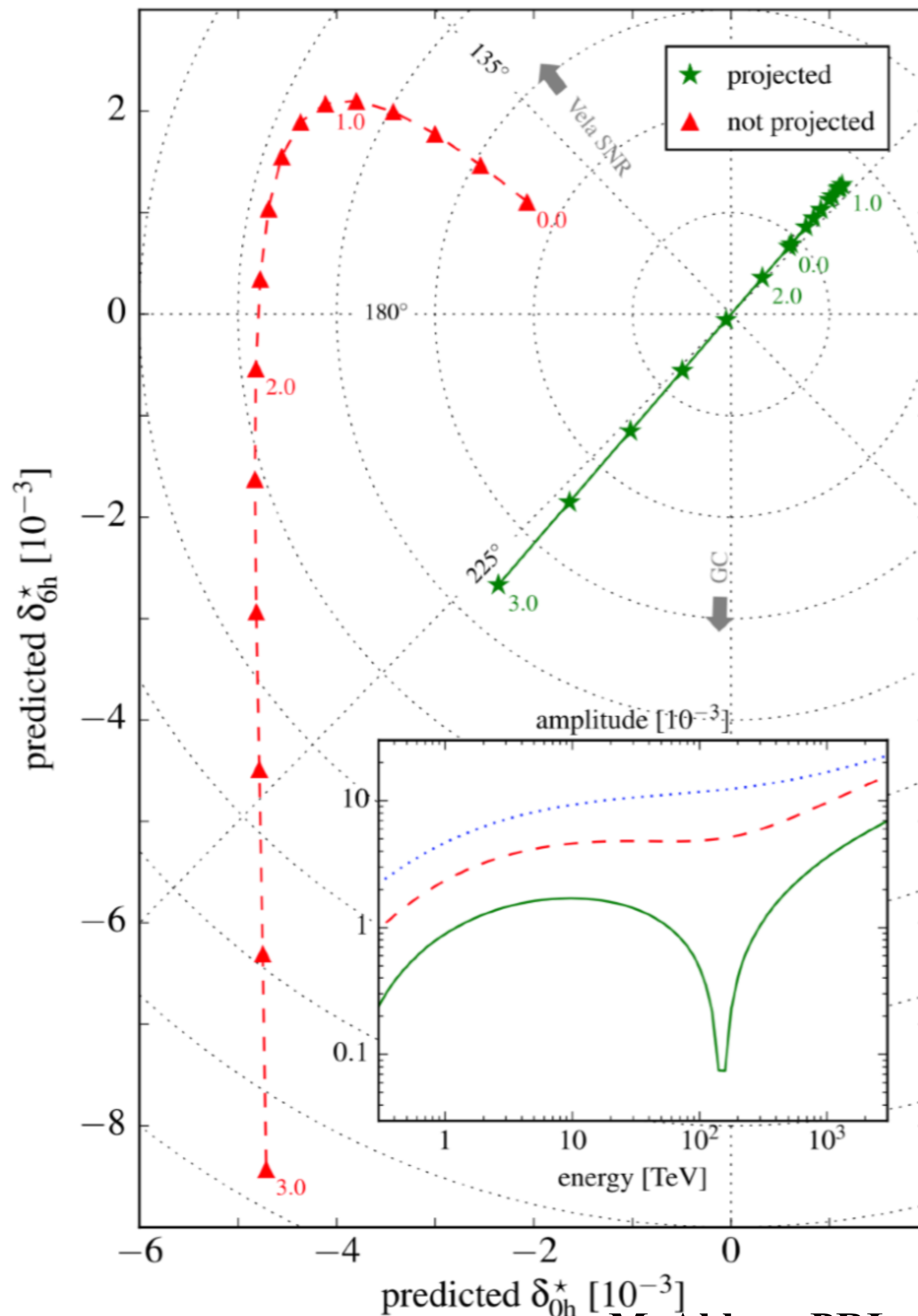
➜ diffusion tensor reduces to projector:

$$K_{ij} \to \frac{\hat{B}_i \hat{B}_j}{3\nu_{\parallel}}$$

- TeV–PeV dipole data consistent with magnetic field direction inferred by IBEX data [McComas *et al.*'09]
- 1–100 TeV phase indicates a local gradient within longitudes:

$$120^\circ \lesssim l \lesssim 300^\circ$$

- phase flip induced by Vela SNR? [MA'16]
- or a luminous 2Myr old SNR? [Savchenko, Kachelrieß & Semikoz'15]



M. Ahlers, PRL, 2016

Implementation of Anisotropic diffusion

Large-scale regular magnetic field

$$B = B_{halo} + B_{disk} + B_{poloidal}$$

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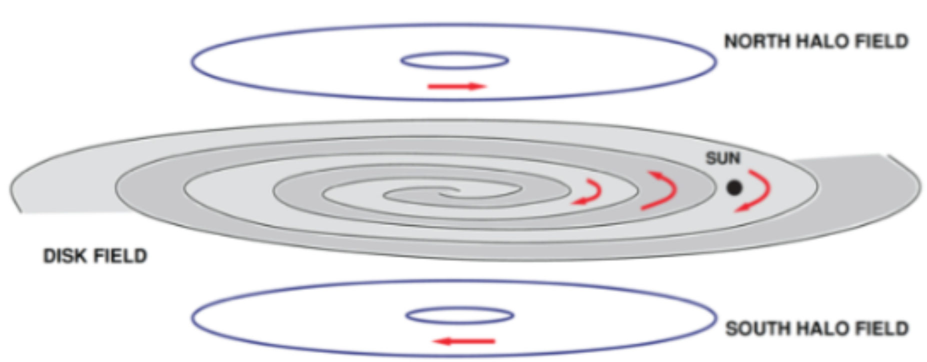
Toroidal fields in Galactic disk and halo

$$B_\phi^{disk}(R,z) = \begin{cases} B_{D0}\, e^{-|z|/z_0} & (R < R_{cD}) \\ B_{D0}\, e^{-|z|/z_0}\, e^{-(R-R_\odot)/R_0} & (R > R_{cD}) \end{cases},$$

$$B_\phi^{halo}(R,z) = B_{H0}\left[1 + \left(\frac{|z|-z_0^H}{z_1^H}\right)\right]^{-1} \frac{R}{R_O^H}\, e^{\left(1-\frac{R}{R_0^H}\right)}$$



X. H. Sun, W. Reich, A. Waelkens, T. A. Enßlin, 2008A&A...477..573S; M. S. Pshirkov, et. al. 2011ApJ...738..192P

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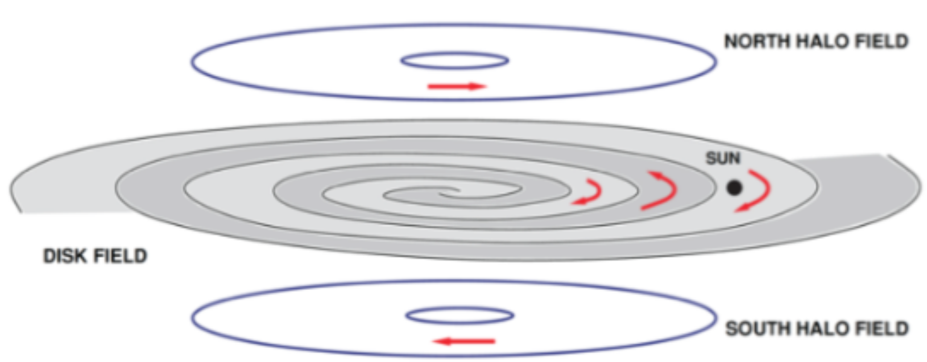

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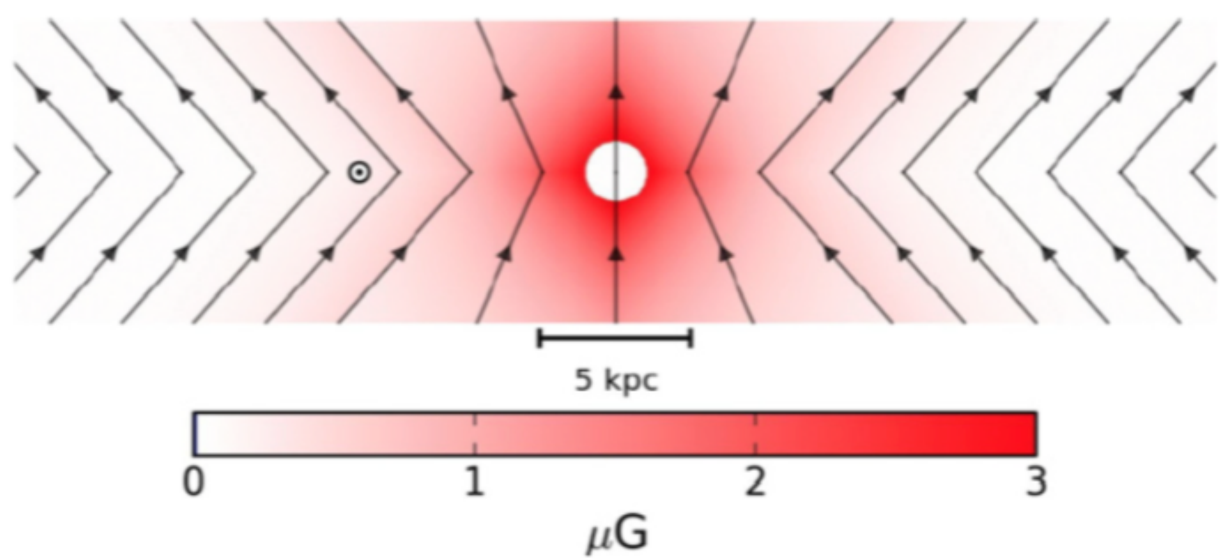
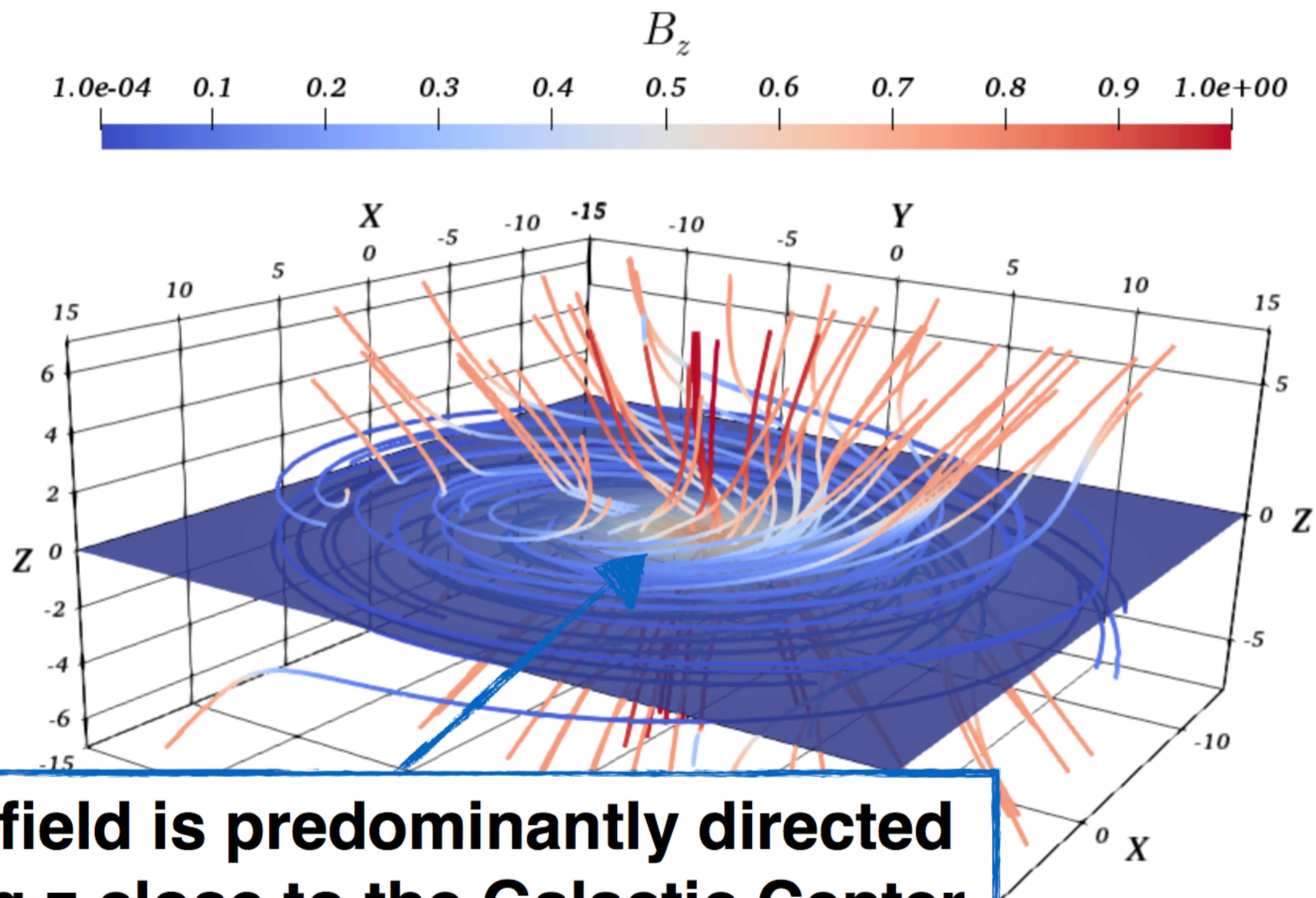
Poloidal fields

$$B_z^{pol}(R,z) = B_X(R,z)\cos\left[\Theta_X(R,z)\right],$$

$$B_R^{pol}(R,z) = B_X(R,z)\sin\left[\Theta_X(R,z)\right],$$

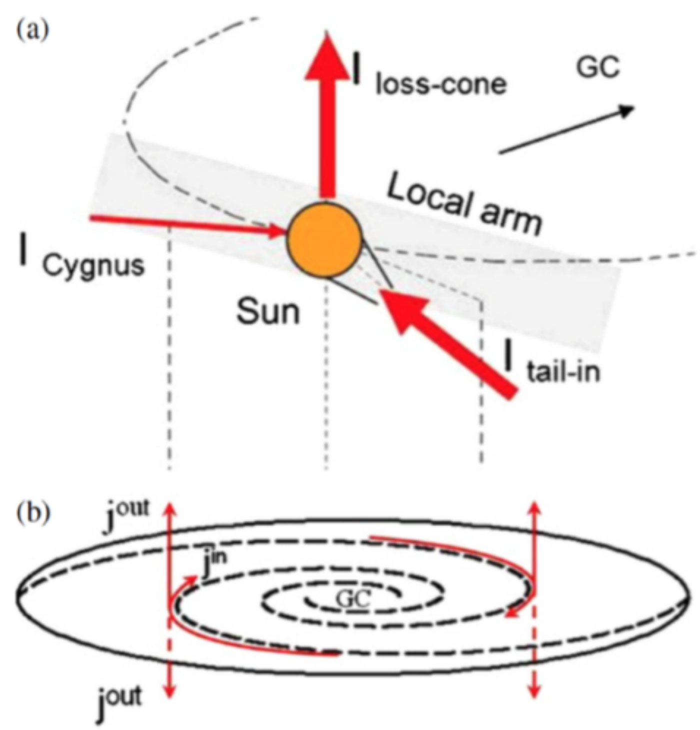


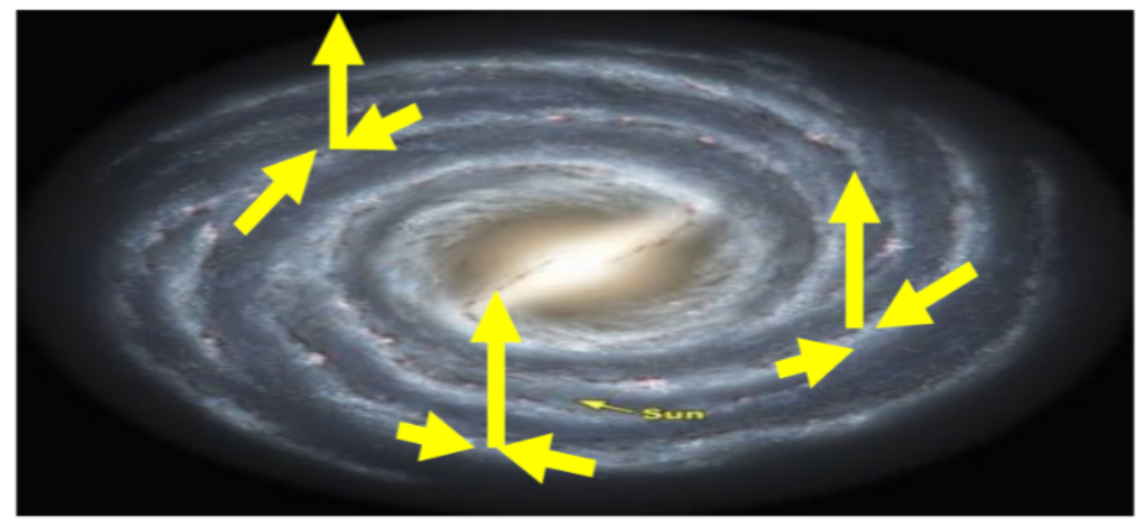


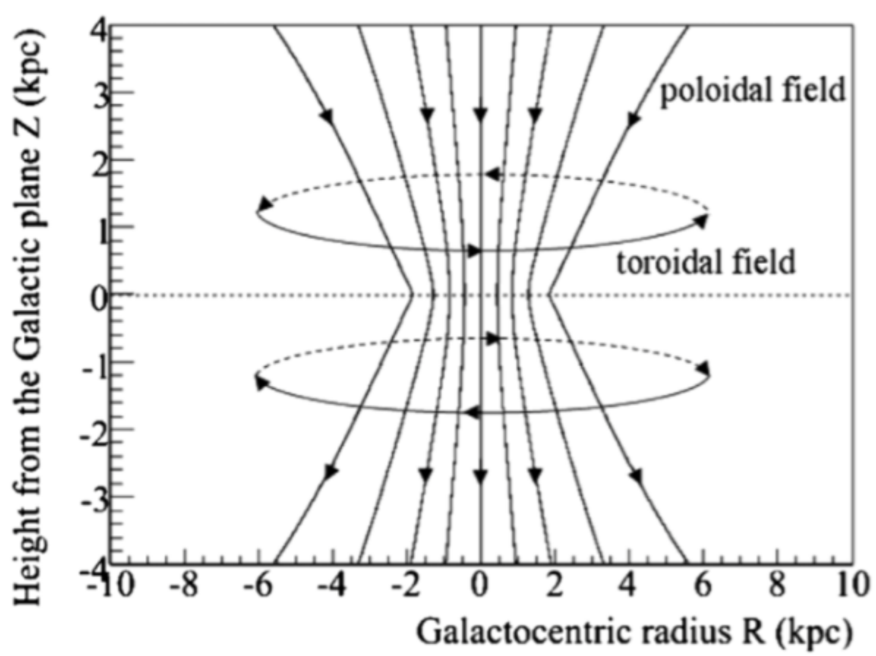
The field is predominantly directed along z close to the Galactic Center and predominantly azimuthal at large distances from it

Large-scale regular magnetic field induced by cosmic ray flows

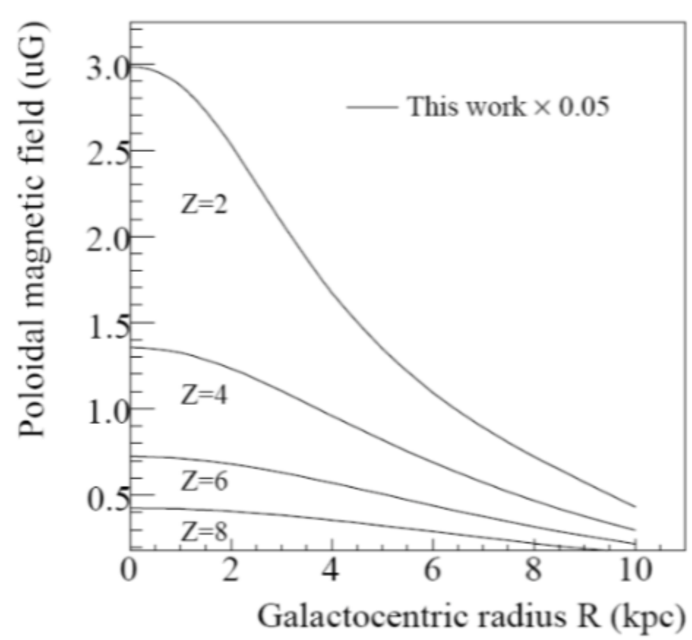
Extend the anisotropy image observed in the solar vicinity to the whole Galaxy.

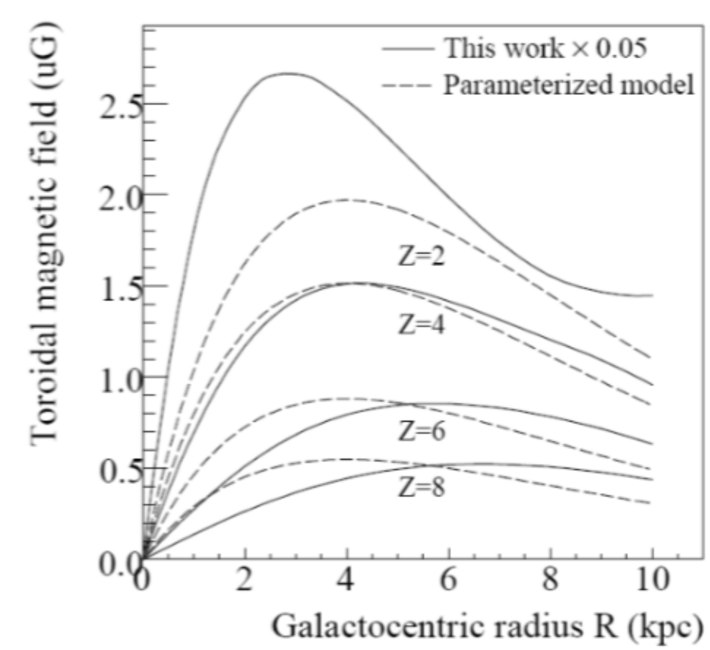






X. B. Qu, Y. Zhang, L. Xue, C. Liu, H. B. Hu, 2012ApJ…750L..17Q





Parameterized model: Sun, X. H. & Reich,W. 2010, Research in Astronomy and Astrophysics, 10, 1287

Cosmic ray anisotropic diffusion equation

$$\frac{\partial n_i}{\partial t} - \vec{\nabla} \cdot \left(D_{xx} \cdot \vec{\nabla} n_i - \vec{u} n_i \right) - \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} n_i = Q_{\mathrm{inj}} + Q_{\mathrm{losses}} + Q_{\mathrm{spall/dec}}$$

D_{xx} **is called diffusion tensor**

$$D = \begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix},$$

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D_{xx} **is called diffusion tensor**

$$D_{ij} \equiv D_{\perp} \delta_{ij} + \left(D_{\parallel} - D_{\perp} \right) b_i b_j \,, \qquad b_i \equiv \frac{B_i}{|\mathbf{B}|} \,,$$

B is the i-th component of the regular magnetic field

Cosmic ray anisotropic diffusion equation

$$\frac{\partial n_i}{\partial t} - \vec{\nabla} \cdot \left(D_{xx} \vec{\nabla} n_i - \vec{u} n_i \right) - \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} n_i = Q_{\rm inj} + Q_{\rm losses} + Q_{\rm spall/dec}$$

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with

$$D_\parallel = D_{0\parallel} \left(\frac{p}{Z} \right)^{\delta_\parallel} \quad \text{and} \quad D_\perp = D_{0\perp} \left(\frac{p}{Z} \right)^{\delta_\perp} \equiv \epsilon_D \, D_{0\parallel} \left(\frac{p}{Z} \right)^{\delta_\perp} \,,$$

diffusion parallel to B **diffusion perpendicular to B**

Iteration algorithm

$$\frac{\partial \psi}{\partial t} - \frac{\partial}{\partial x}\left(D_{xx}\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(D_{yy}\frac{\partial \psi}{\partial y}\right) - \frac{\partial}{\partial z}\left(D_{zz}\frac{\partial \psi}{\partial z}\right) = Q'(\vec{r}, p, t)$$

$$Q'(\vec{r}, p, t) = Q(\vec{r}, p, t) + Q_{\rm pseudo}(\vec{r}, p, t)$$

pseudo source term

$$\begin{aligned} Q_{\rm pseudo}(\vec{r}, p, t) = {} & 2D_{xy}\frac{\partial^2 \psi}{\partial x \partial y} + 2D_{xz}\frac{\partial^2 \psi}{\partial x \partial z} + 2D_{yz}\frac{\partial^2 \psi}{\partial y \partial z} \\ & + \frac{\partial D_{xy}}{\partial x}\frac{\partial \psi}{\partial y} + \frac{\partial D_{yz}}{\partial y}\frac{\partial \psi}{\partial z} + \frac{\partial D_{xz}}{\partial x}\frac{\partial \psi}{\partial z} \\ & + \frac{\partial D_{yx}}{\partial y}\frac{\partial \psi}{\partial x} + \frac{\partial D_{zy}}{\partial z}\frac{\partial \psi}{\partial y} + \frac{\partial D_{zx}}{\partial z}\frac{\partial \psi}{\partial x} \end{aligned}$$

迭代步骤如下:

第一步，忽略扩散张量中的非对角部分，计算传统的扩散方程得到分布$\psi^{(0)}$，

$$\frac{\partial \psi^{(0)}}{\partial t}-\frac{\partial}{\partial x}\left(D_{xx}\frac{\partial \psi^{(0)}}{\partial x}\right)-\frac{\partial}{\partial y}\left(D_{yy}\frac{\partial \psi^{(0)}}{\partial y}\right)-\frac{\partial}{\partial z}\left(D_{zz}\frac{\partial \psi^{(0)}}{\partial z}\right)=Q(x,y,p),$$

第二步，用分布$\psi^{(0)}$计算非对角项，然后合并到源项$Q(x,y,p)$，构成源项

$Q'^{(1)}(x,y,p)$，

$$Q'^{(1)}(x,y,p)=Q(x,y,p)+2D_{xy}\frac{\partial^2\psi^{(0)}}{\partial x\partial y}+2D_{yz}\frac{\partial^2\psi^{(0)}}{\partial y\partial z}+2D_{zx}\frac{\partial^2\psi^{(0)}}{\partial z\partial x}$$
$$+\frac{\partial D_{xy}}{\partial x}\frac{\partial \psi^{(0)}}{\partial y}+\frac{\partial D_{yx}}{\partial y}\frac{\partial \psi^{(0)}}{\partial x}+\frac{\partial D_{yz}}{\partial y}\frac{\partial \psi^{(0)}}{\partial z}+\frac{\partial D_{zy}}{\partial z}\frac{\partial \psi^{(0)}}{\partial y}$$
$$+\frac{\partial D_{zx}}{\partial z}\frac{\partial \psi^{(0)}}{\partial x}+\frac{\partial D_{xz}}{\partial x}\frac{\partial \psi^{(0)}}{\partial z},$$

第三步，将$Q'^{(1)}(x,y,p)$作为新的源项，计算传播方程得到分布$\psi^{(1)}$，

$$\frac{\partial \psi^{(1)}}{\partial t}-\frac{\partial}{\partial x}\left(D_{xx}\frac{\partial \psi^{(1)}}{\partial x}\right)-\frac{\partial}{\partial y}\left(D_{yy}\frac{\partial \psi^{(1)}}{\partial y}\right)-\frac{\partial}{\partial z}\left(D_{zz}\frac{\partial \psi^{(1)}}{\partial z}\right)=Q'^{(1)}(x,y,p),$$

第四步，用分布$\psi^{(1)}$计算非对角项，得到新的源项$Q'^{(2)}$，

$$
\begin{aligned}
Q'^{(2)}(x,y,p) = Q(x,y,p) &+ 2D_{xy}\frac{\partial^2\psi^{(1)}}{\partial x\partial y} + 2D_{yz}\frac{\partial^2\psi^{(1)}}{\partial y\partial z} + 2D_{zx}\frac{\partial^2\psi^{(1)}}{\partial z\partial x} \\
&+ \frac{\partial D_{xy}}{\partial x}\frac{\partial\psi^{(1)}}{\partial y} + \frac{\partial D_{yx}}{\partial y}\frac{\partial\psi^{(1)}}{\partial x} + \frac{\partial D_{yz}}{\partial y}\frac{\partial\psi^{(1)}}{\partial z} + \frac{\partial D_{zy}}{\partial z}\frac{\partial\psi^{(1)}}{\partial y} \\
&+ \frac{\partial D_{zx}}{\partial z}\frac{\partial\psi^{(1)}}{\partial x} + \frac{\partial D_{xz}}{\partial x}\frac{\partial\psi^{(1)}}{\partial z},
\end{aligned}
$$

第五步，用$Q'^{(2)}$作为新的源项计算传播方程得到分布$\psi^{(2)}$，

$$
\frac{\partial\psi^{(2)}}{\partial t} - \frac{\partial}{\partial x}\left(D_{xx}\frac{\partial\psi^{(2)}}{\partial x}\right) - \frac{\partial}{\partial y}\left(D_{yy}\frac{\partial\psi^{(2)}}{\partial y}\right) - \frac{\partial}{\partial z}\left(D_{zz}\frac{\partial\psi^{(2)}}{\partial z}\right) = Q'^{(2)}(x,y,p),
$$

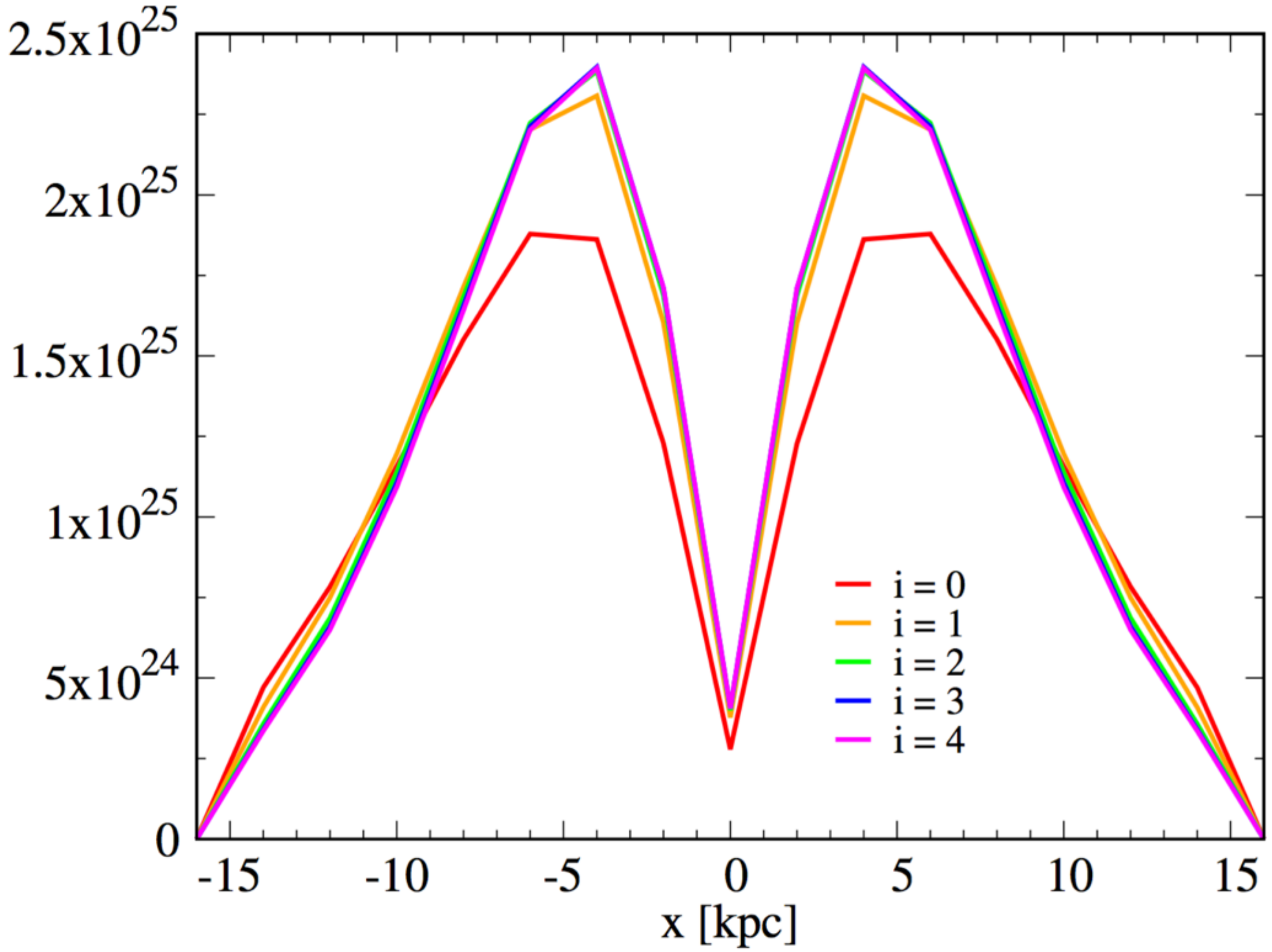
如此反复迭代，直到分布$\psi^{(n)}$不再发生变化为止。

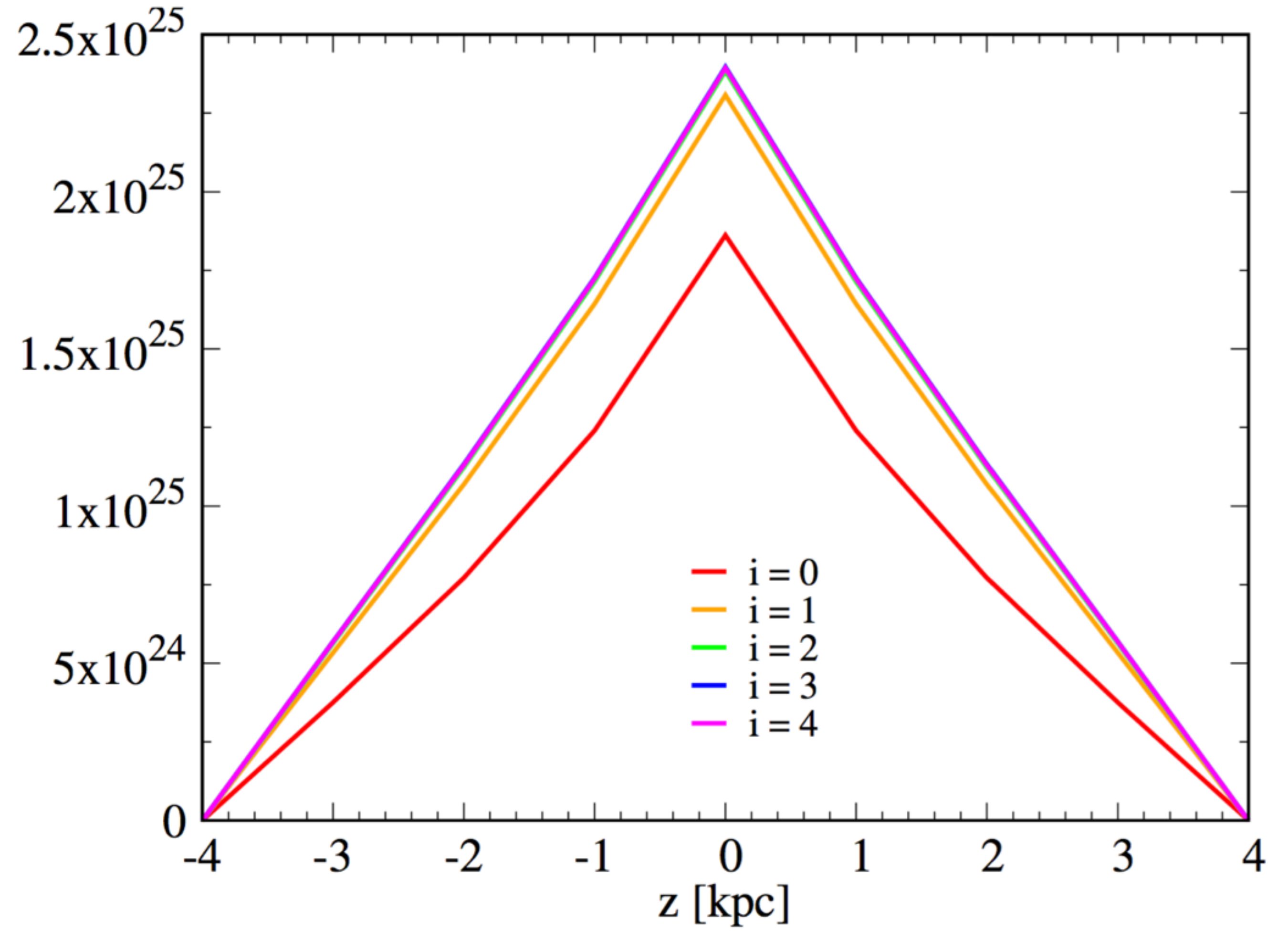
Results

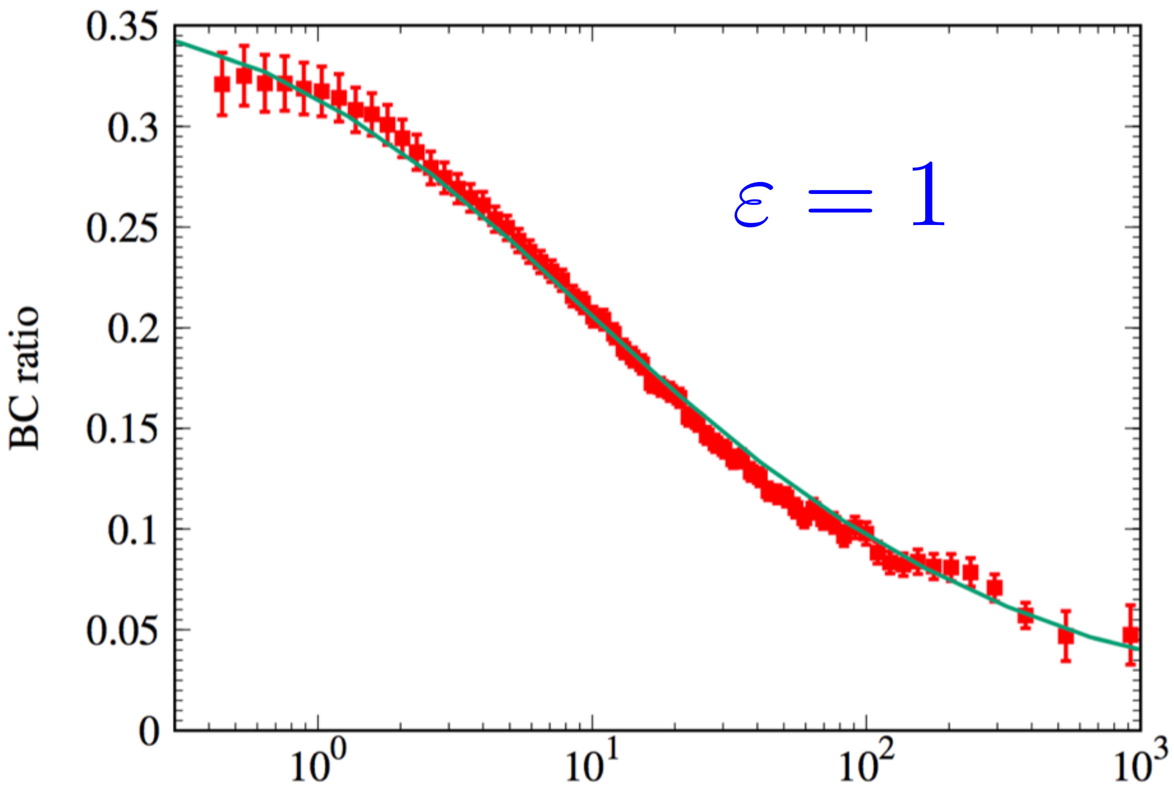
iteration check

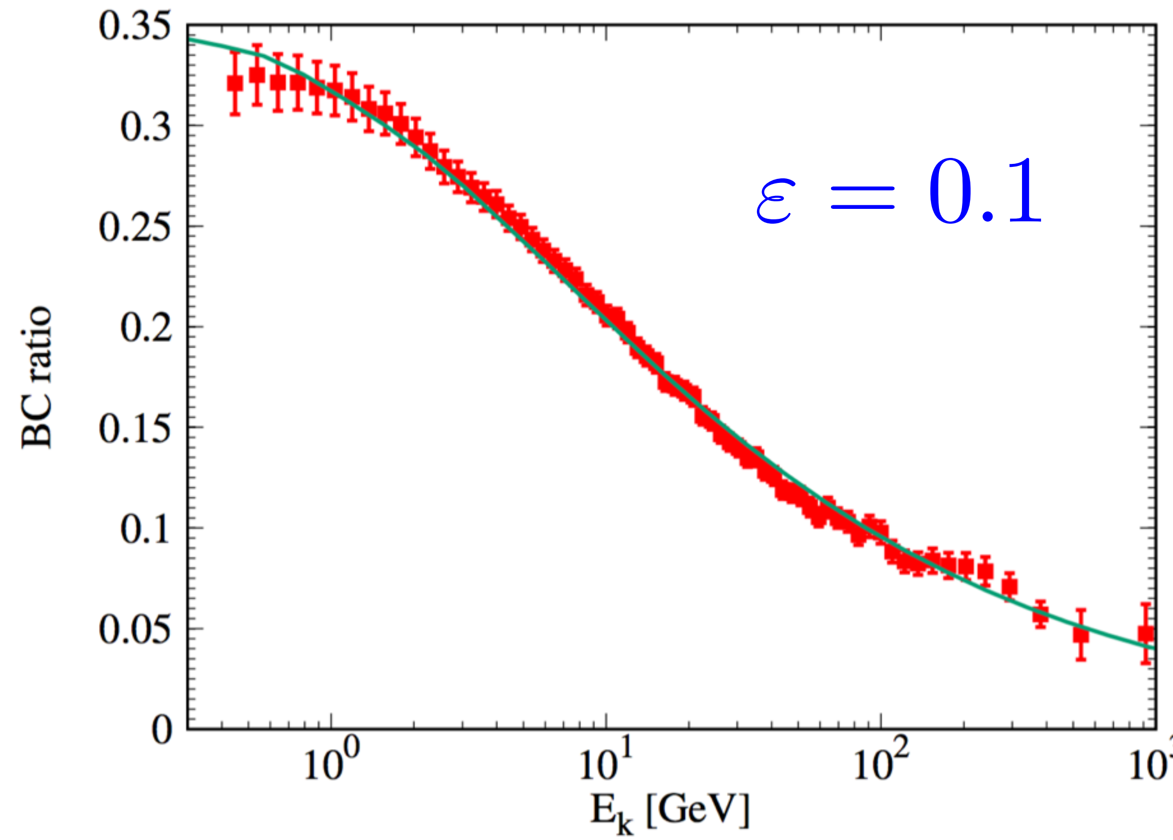
spatial distribution along x (y = 0, z = 0) and z (x=0,y=4) directions

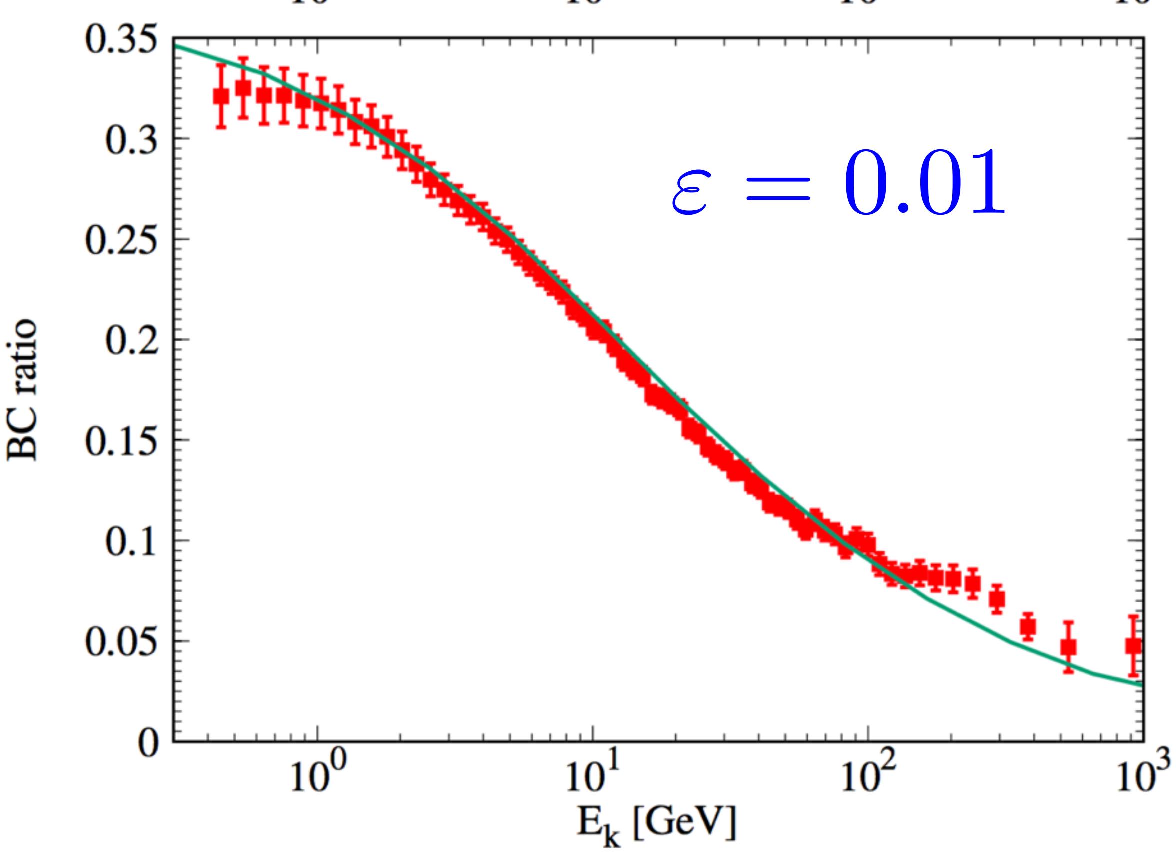
4 times are enough



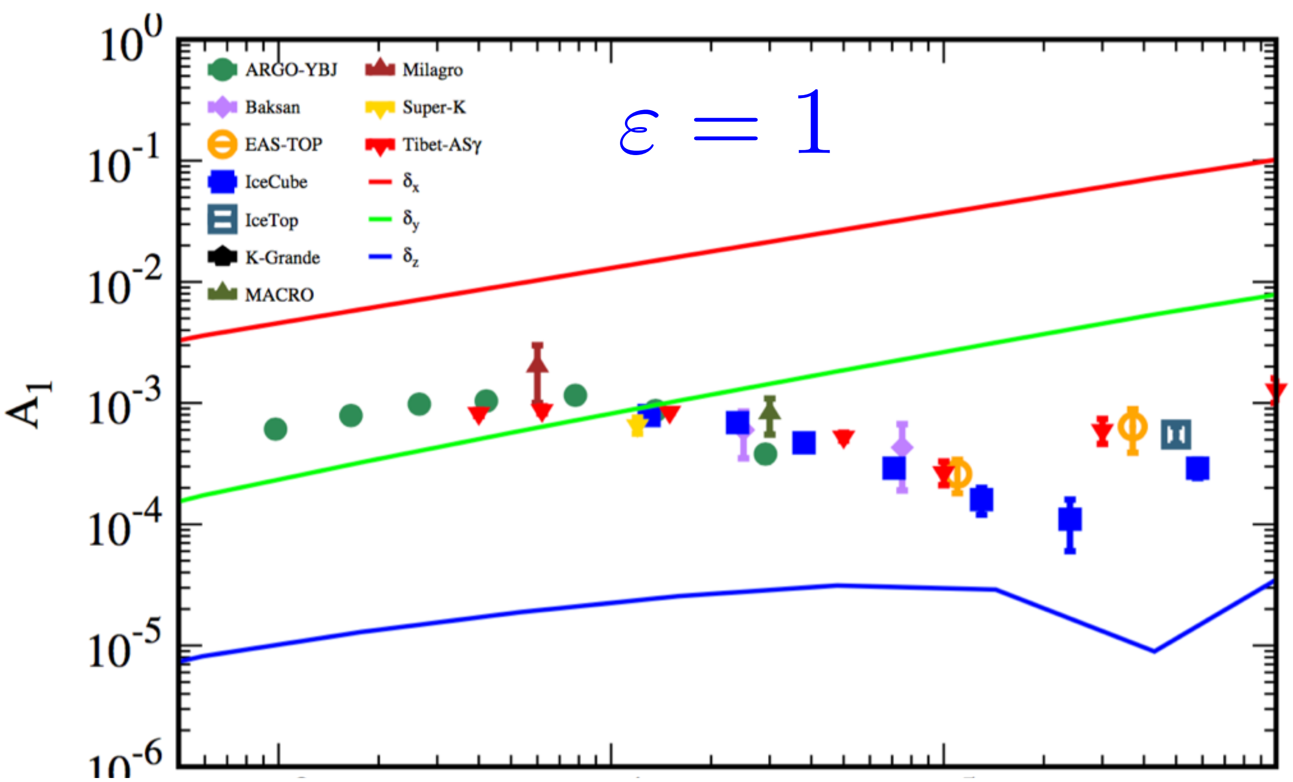


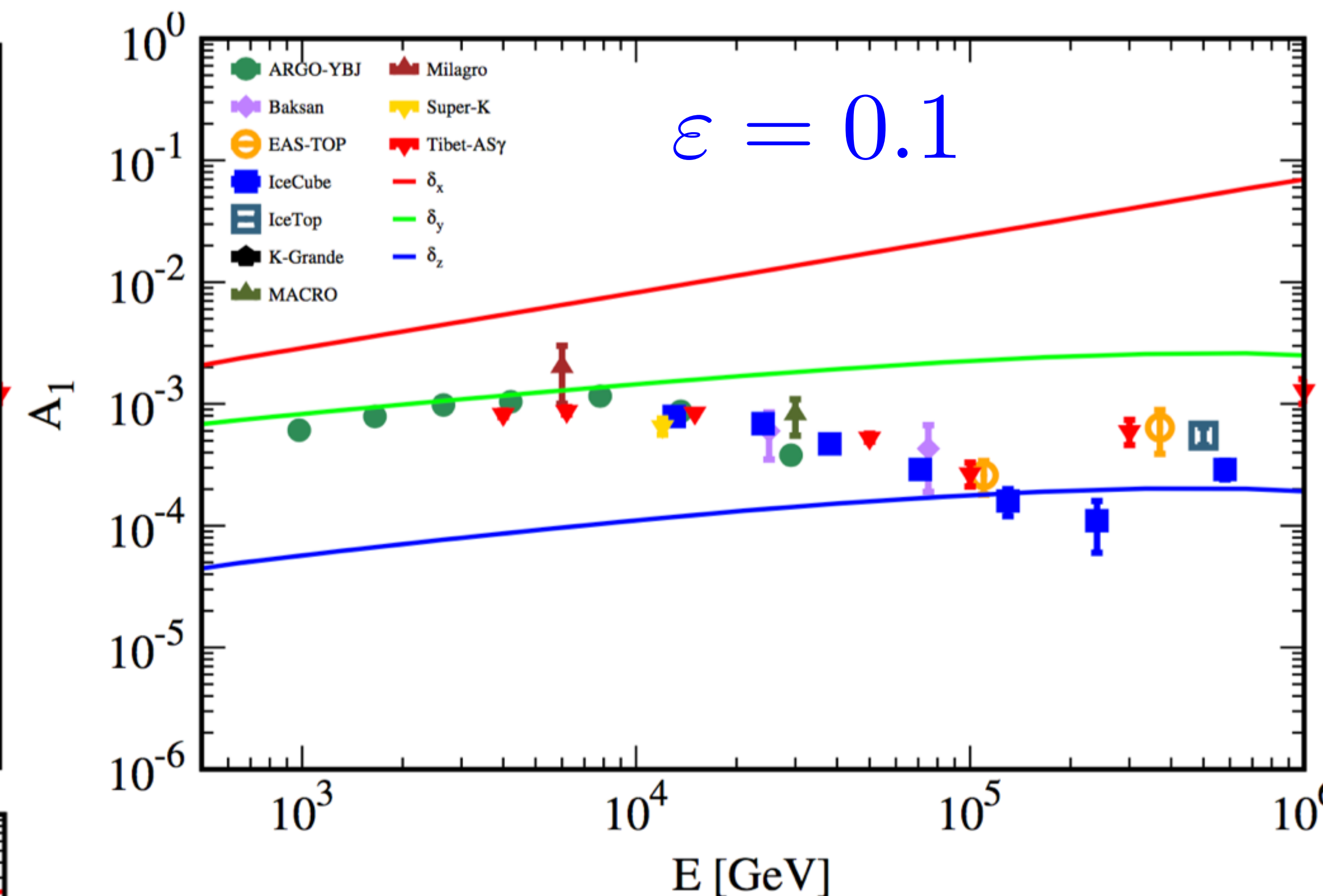


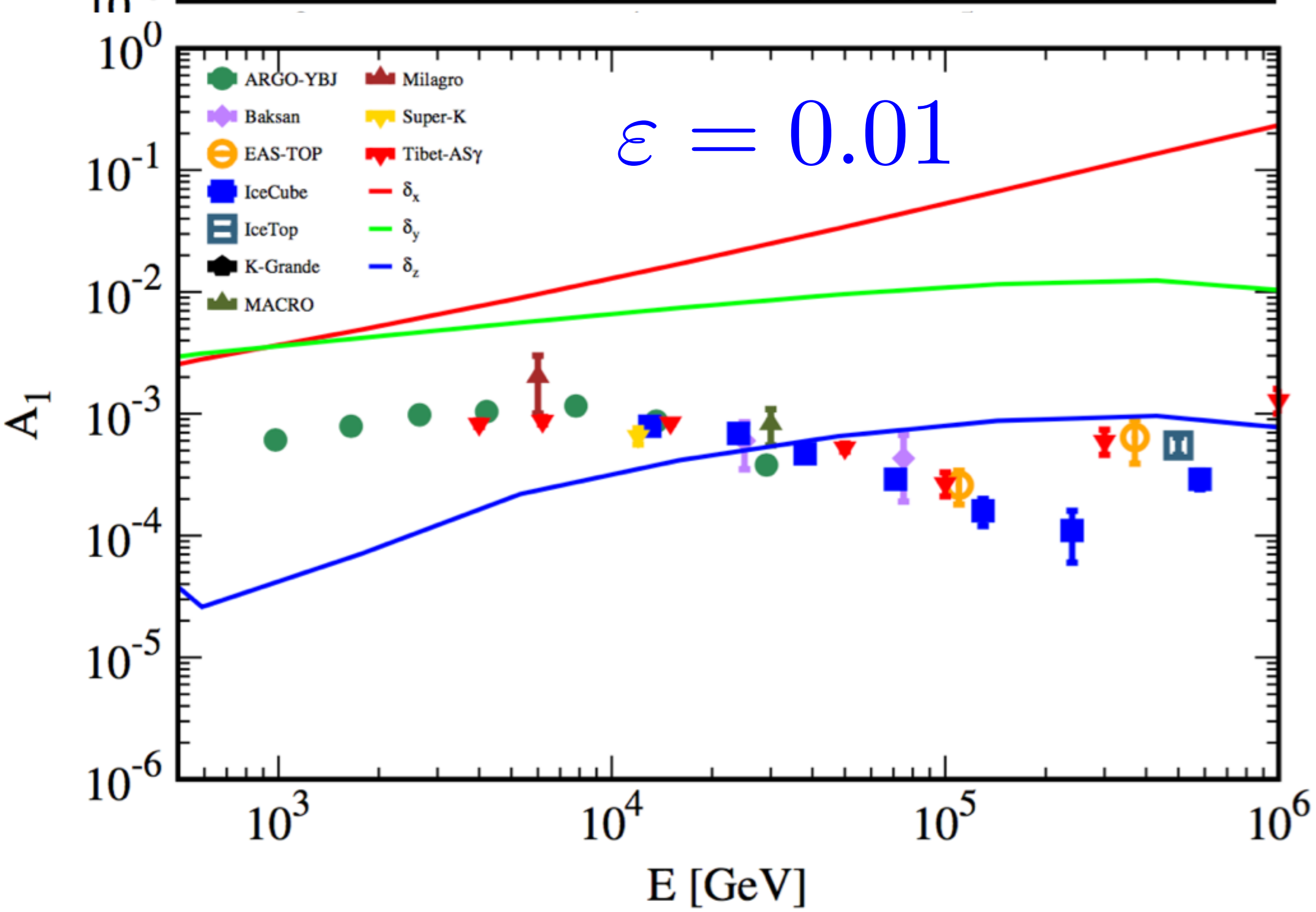




ε	$D_{0\parallel}$ [cm^2/s]	$\delta_{0\parallel}$	$\delta_{0\perp}$	η
1	5.6×10^{28}	0.3	0.44	0.5
0.1	3.6×10^{29}	0.3	0.46	0.5
0.01	1.3×10^{30}	0.3	0.64	0.5







red： x direction

green： y direction

blue： z direction

Conclusion

We have numerically implemented the 3D anisotropic diffusion based on an iteration algorithm.

The B/C ratio, cosmic ray energy spectrum and anisotropy under anisotropic diffusion have been computed preliminarily, which is going to be studied in-deep.

Thank you for your attention

$$\frac{\partial \psi^{(0)}}{\partial t} - \frac{\partial}{\partial x}\left(D_{xx}\frac{\partial \psi^{(0)}}{\partial x}\right) - \frac{\partial}{\partial y}\left(D_{yy}\frac{\partial \psi^{(0)}}{\partial y}\right) - \frac{\partial}{\partial z}\left(D_{zz}\frac{\partial \psi^{(0)}}{\partial z}\right) = Q(\vec{r}, p, t)$$

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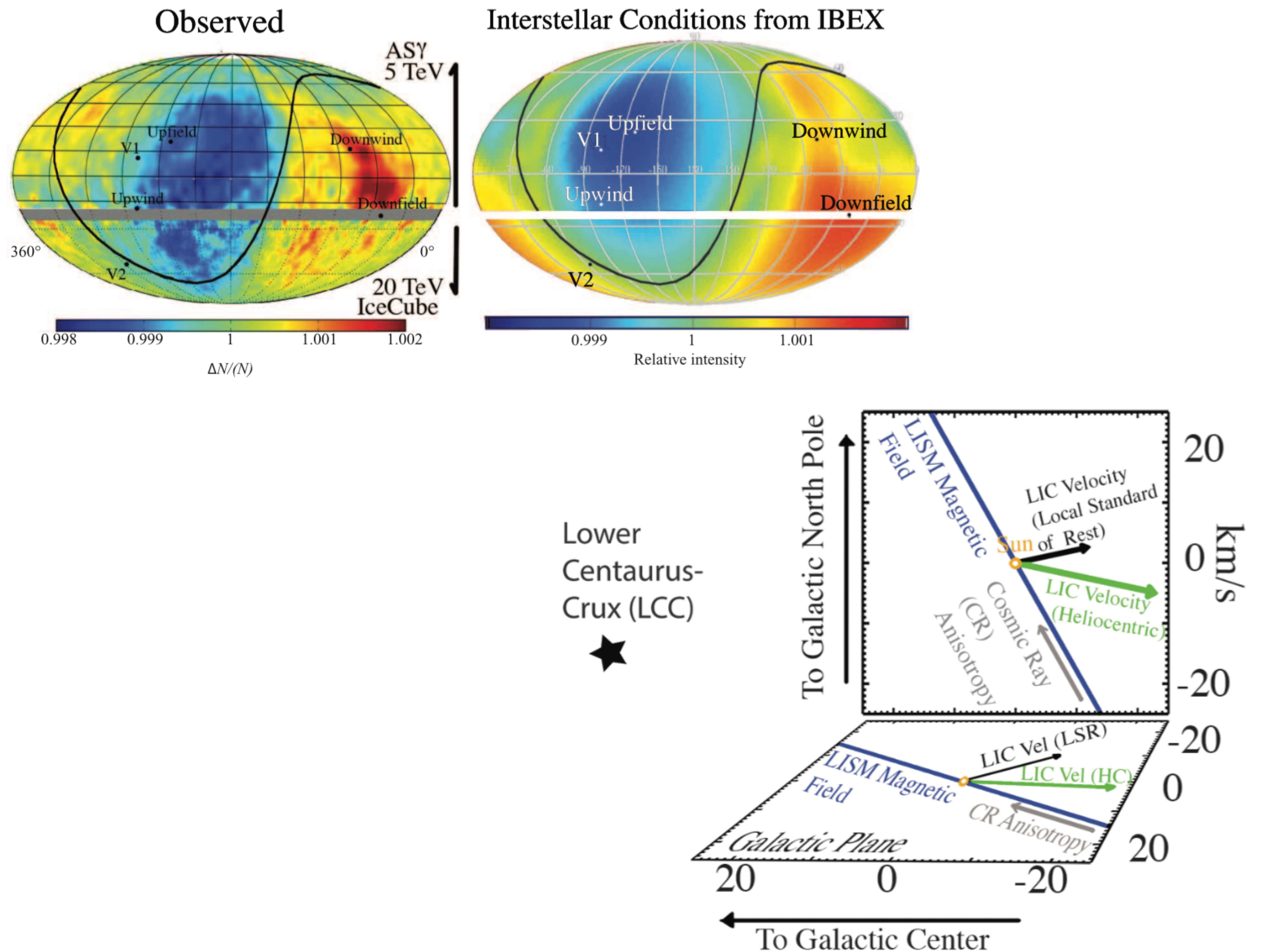
$$\vdots$$

$$\vdots$$

$$\frac{\partial \psi^{(n)}}{\partial t} - \frac{\partial}{\partial x}\left(D_{xx}\frac{\partial \psi^{(n)}}{\partial x}\right) - \frac{\partial}{\partial y}\left(D_{yy}\frac{\partial \psi^{(n)}}{\partial y}\right) - \frac{\partial}{\partial z}\left(D_{zz}\frac{\partial \psi^{(n)}}{\partial z}\right) = Q'^{(n)}(\vec{r}, p, t)$$



Observed
ASγ
5 TeV
Upfield
V1
Downwind
Upwind
Downfield
360°
0°
V2
20 TeV
IceCube
0.998
0.999
1
1.001
1.002
ΔN/(N)
Interstellar Conditions from IBEX
Upfield
V1
Downwind
Upwind
Downfield
V2
0.999
1
1.001
Relative intensity
Lower
Centaurus-
Crux (LCC)
To Galactic North Pole
LISM Magnetic
Field
LIC Velocity
(Local Standard
of Rest)
Sun
LIC Velocity
(Heliocentric)
Cosmic Ray
(CR)
Anisotropy
20
0
-20
km/s
LIC Vel (LSR)
LISM Magnetic
Field
LIC Vel (HC)
CR Anisotropy
Galactic Plane
-20
0
20
20
0
-20
To Galactic Center