

Neutron Stars in the Multi-Messenger Era

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Lecture 1: Basic notions of dense matter. Nuclear interactions and nuclear matter, effective field theory.

Lecture 2: Mass and radius. Linear response, proto-neutron star evolution, supernova neutrino emission and detection.

Lecture 3: Late neutron star cooling: Thermal and transport properties of degenerate matter, cooling of isolated neutron stars, heating and cooling in accreting neutron stars. Observational constraints.

Lecture 4: Neutron stars as laboratories for particle physics: Dark matter candidates (axions and other light weakly interacting particles, WIMPs, compact dark objects). Constraints from observations of neutron star masses, radii and cooling.

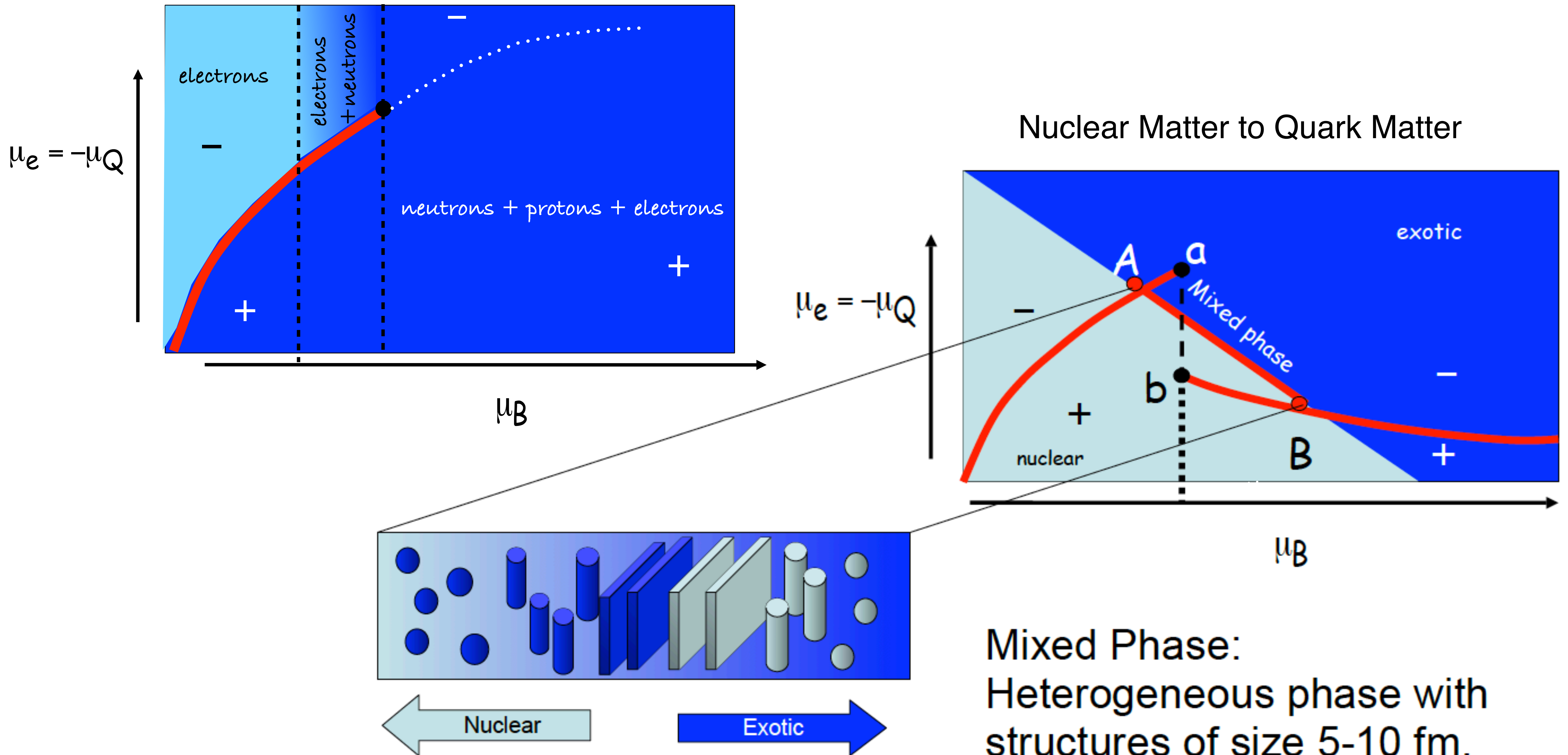


Neutrino Scattering in Novel High Density Phase in the Core

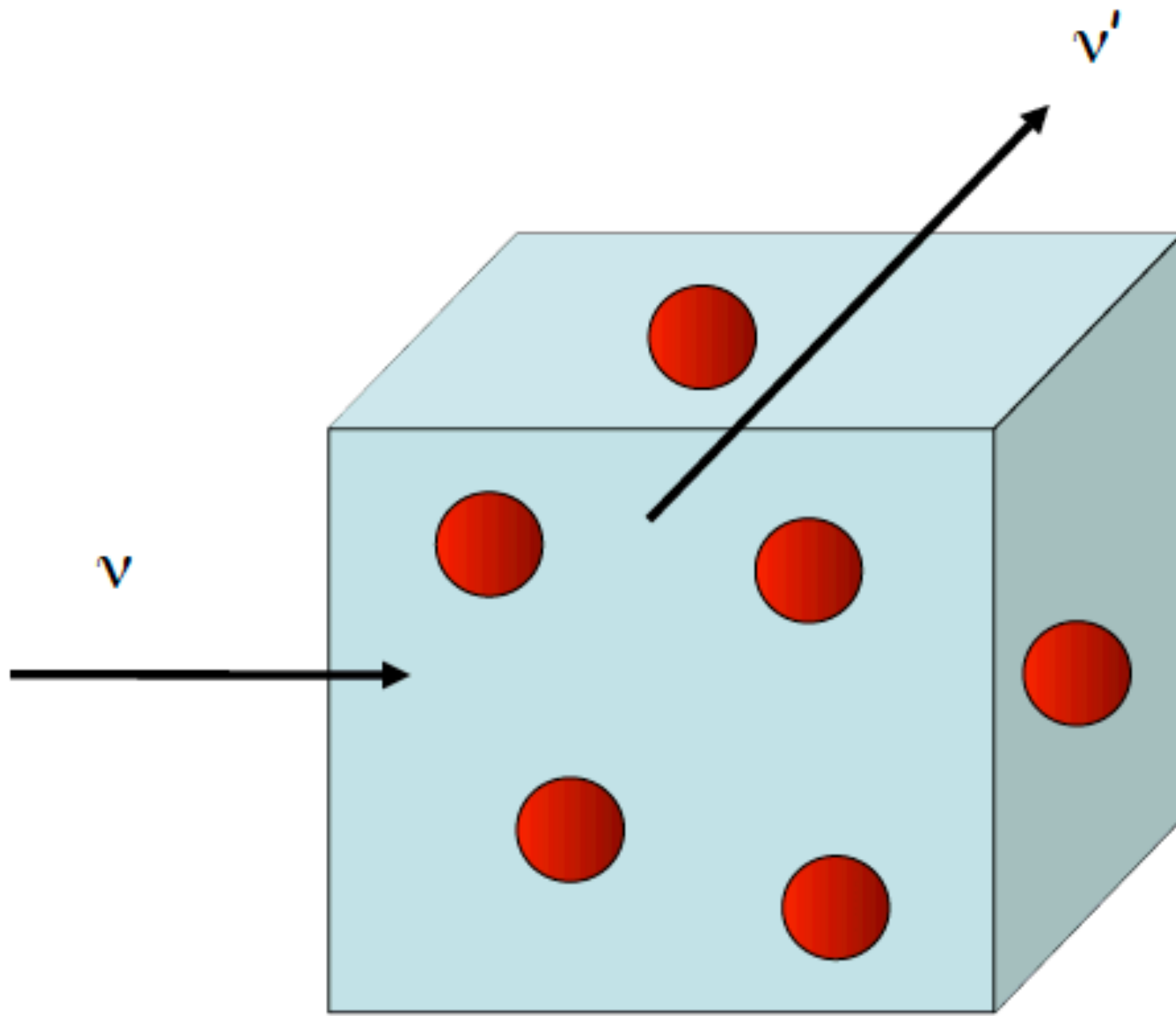
Two examples:

- Generic first-order transitions.
- Superconducting quark matter.

Mixed Phase are Generic to First-order Transitions



Neutrino Scattering in the Mixed Phase



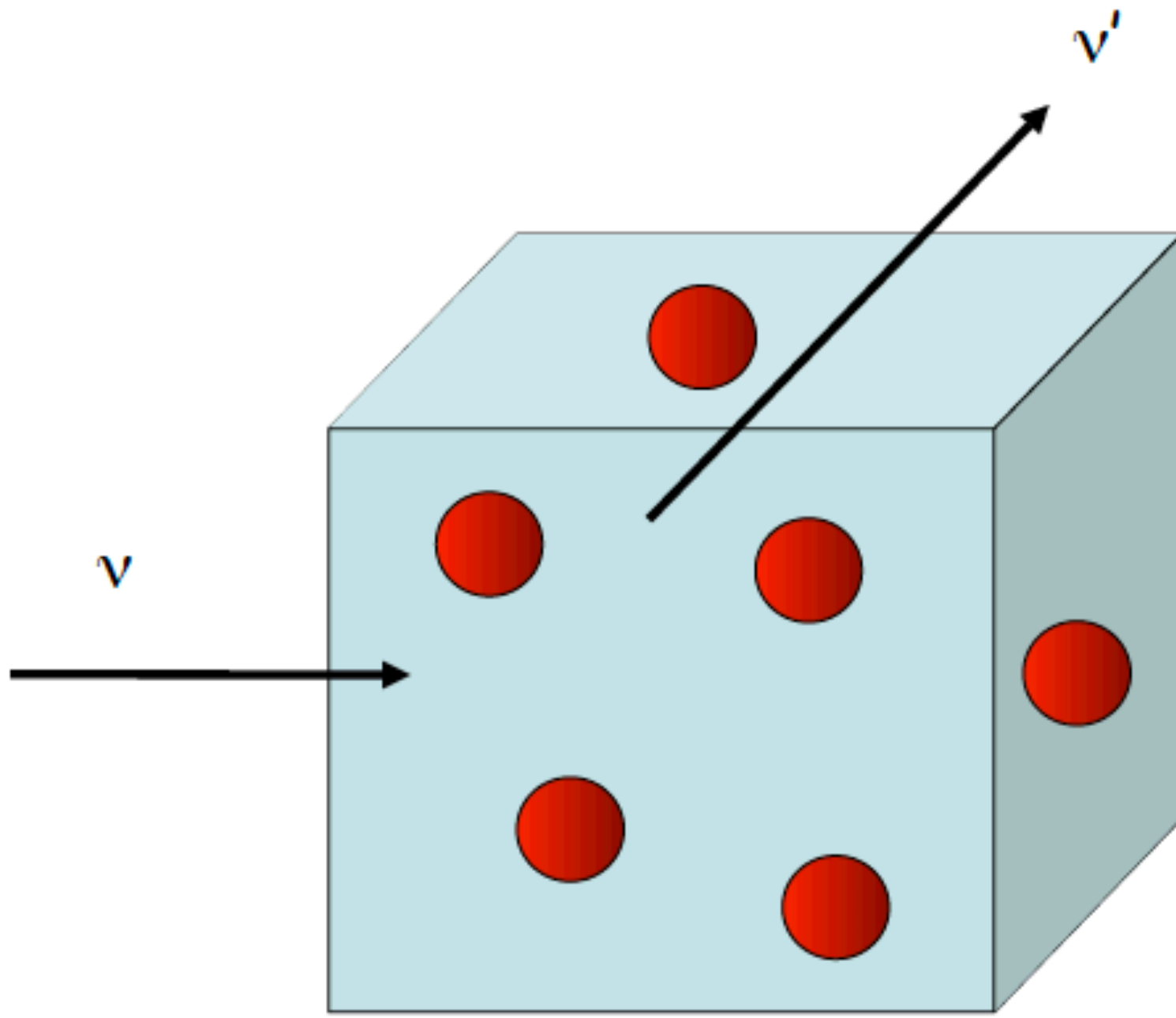
quark droplets in nuclear matter.

$$\frac{d\sigma}{d\cos(\theta)} = N_D \frac{G_F^2}{16\pi} S_q Q_W^2 E_\nu^2 (1 + \cos(\theta))$$

number density of droplets

weak charge of the droplet.

Neutrino Scattering in the Mixed Phase

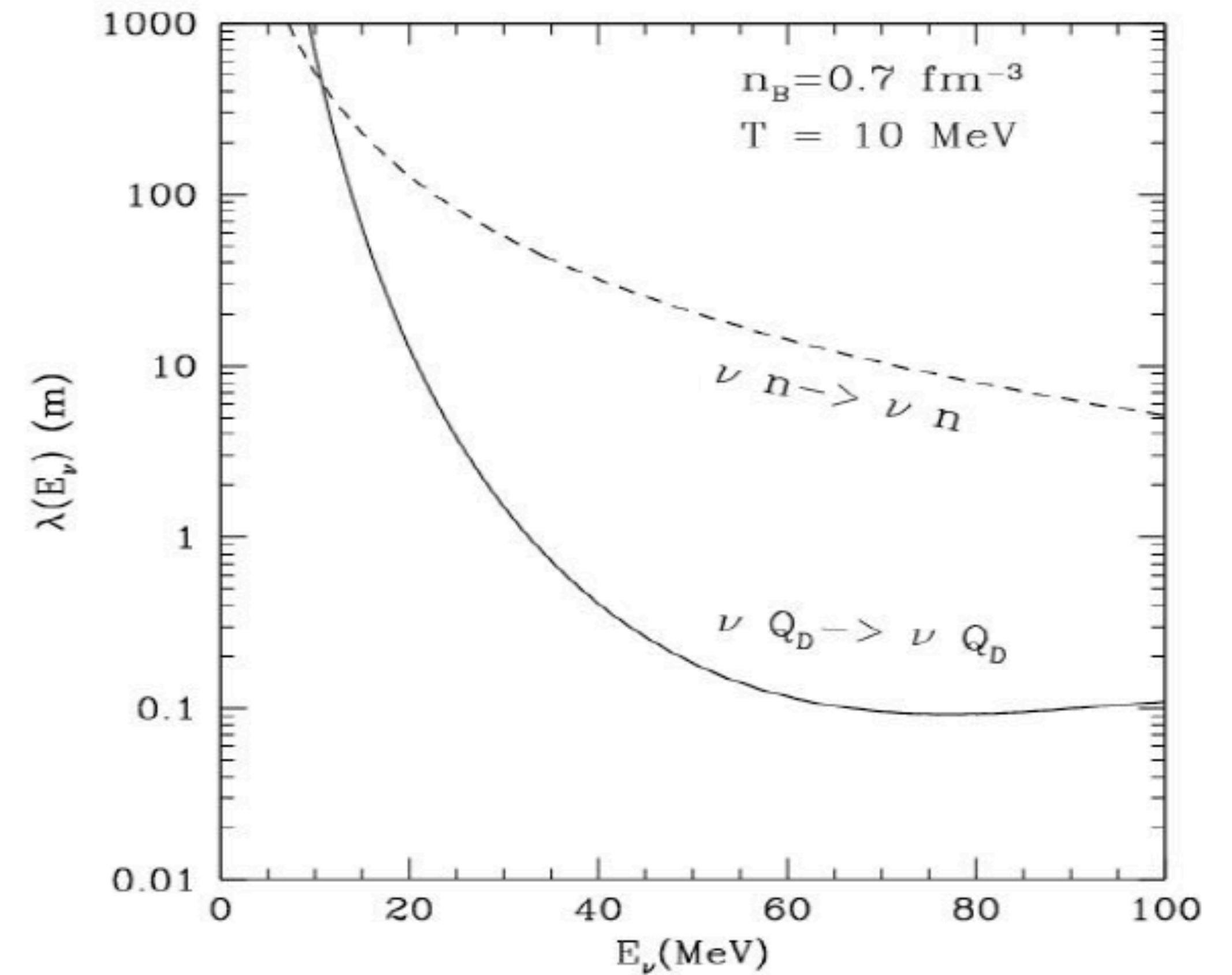


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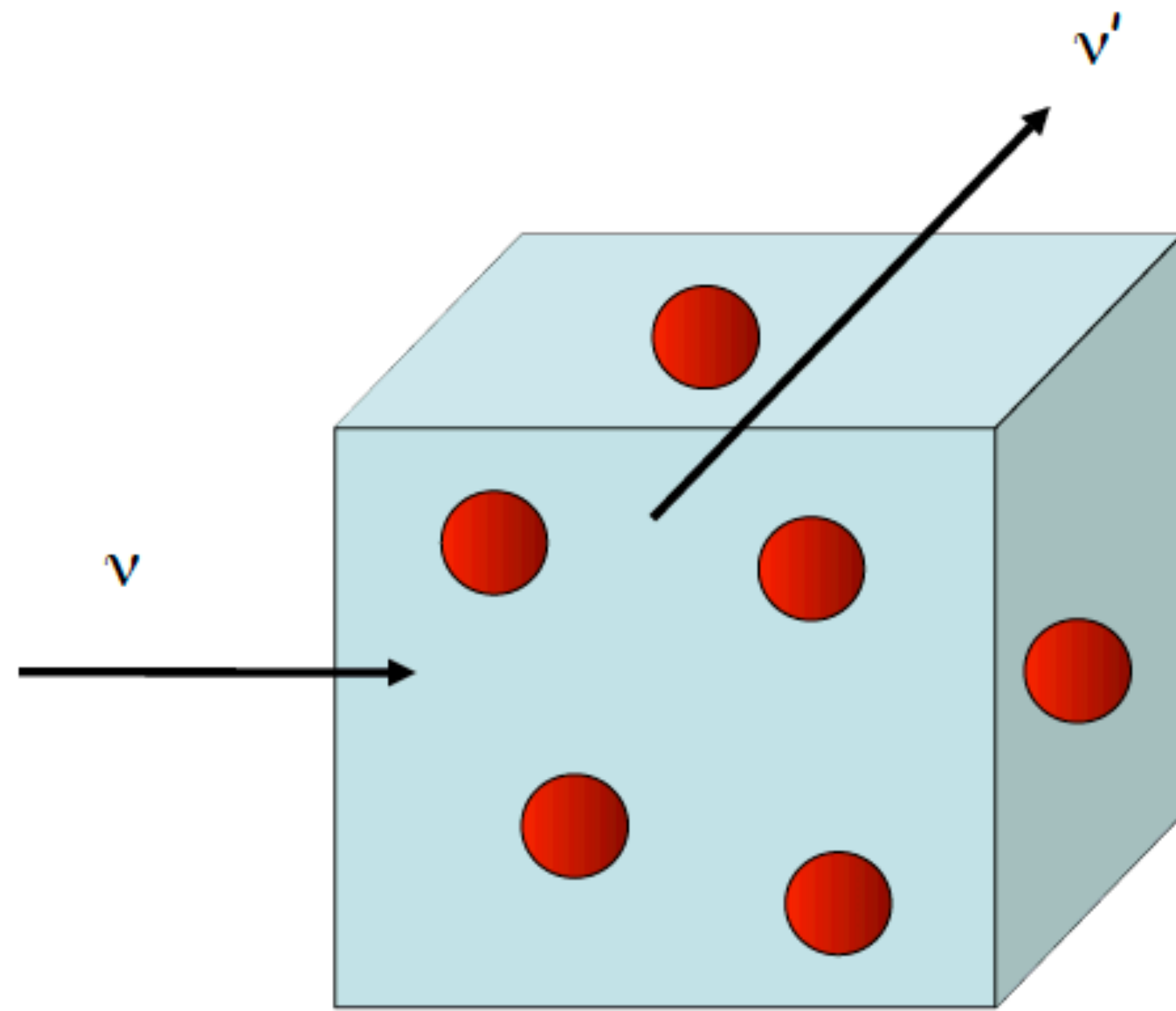
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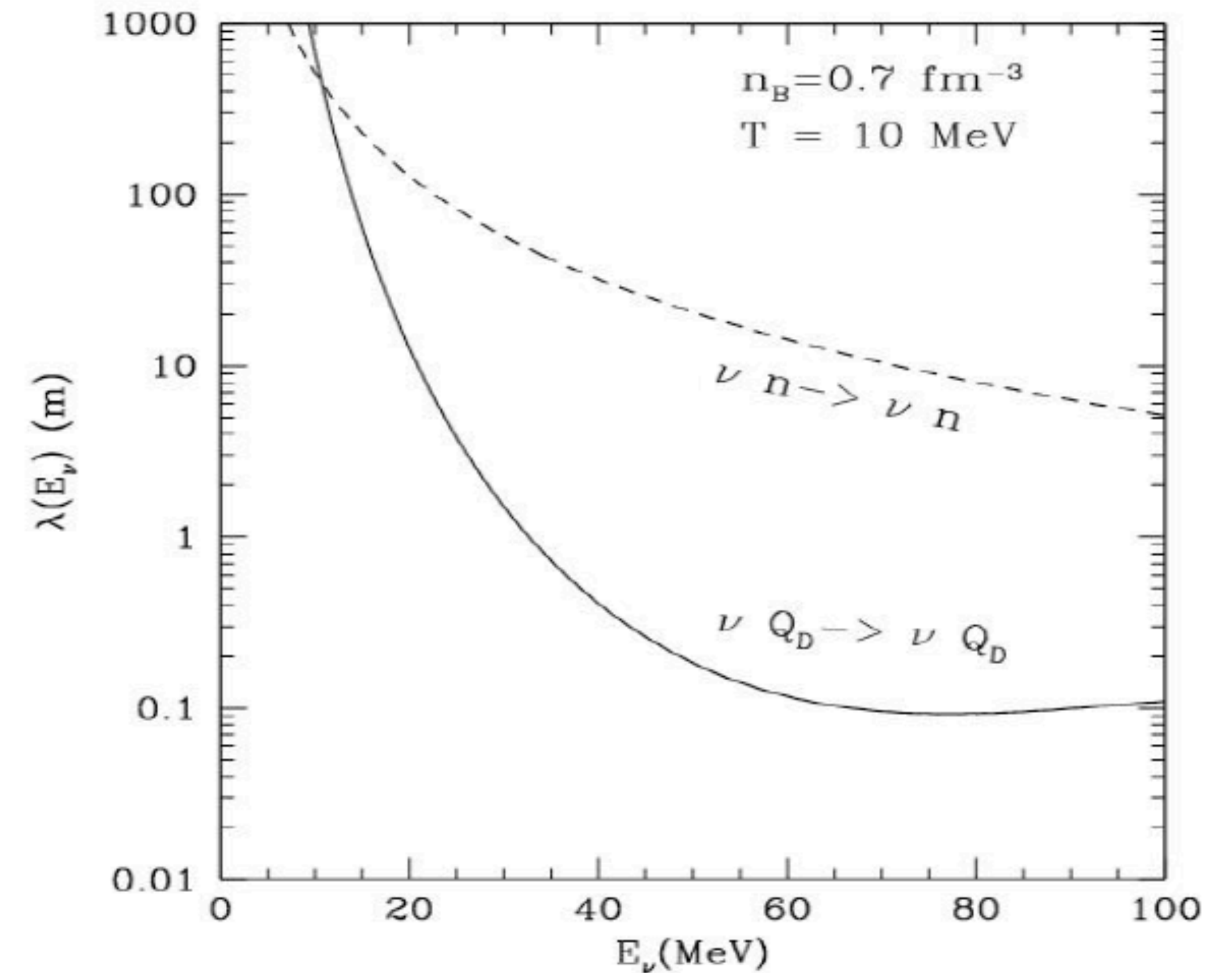


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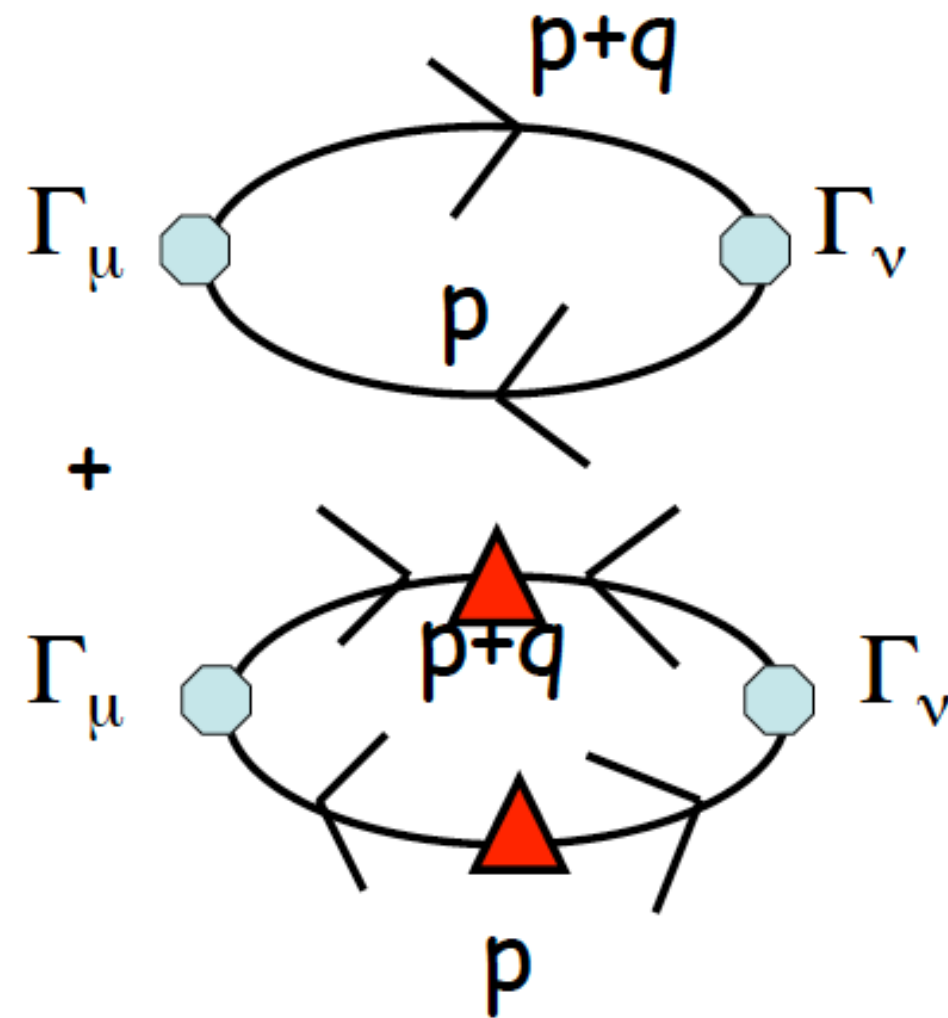
↑
weak charge of the droplet.



Coherent scattering from the droplets is large. Greatly reduces the neutrino mean free paths.

Neutrino Scattering in Superconducting Quark Matter.

$$\Pi_{\mu\nu}(q_0, q) = -i \int d^4p \text{Tr}[G(p+q)\Gamma_\mu G(p)\Gamma_\nu]$$

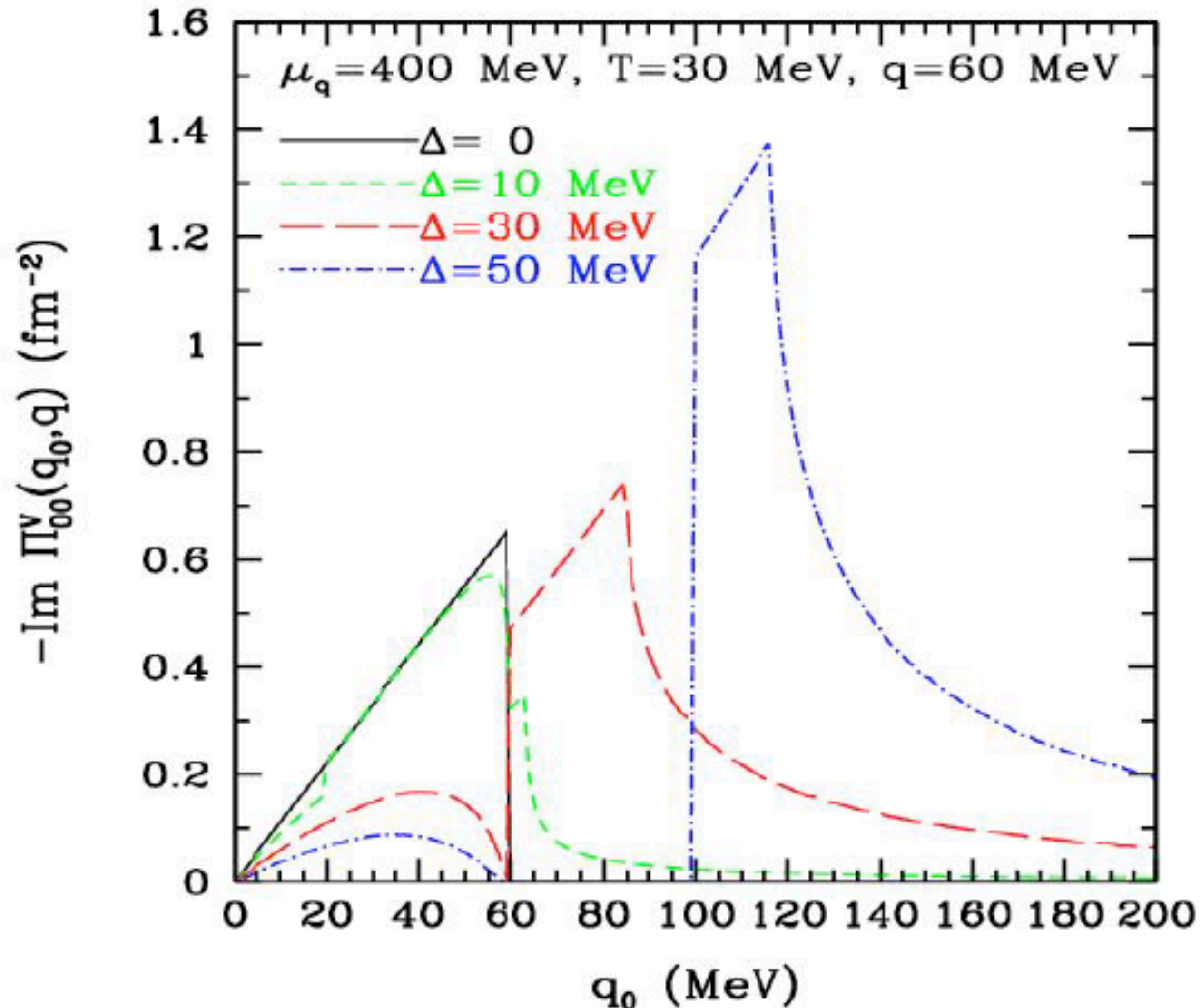


Pairing modifies particle propagation. Particles can be absorbed or emitted from the condensate.

Energy gap modifies the energy spectrum.

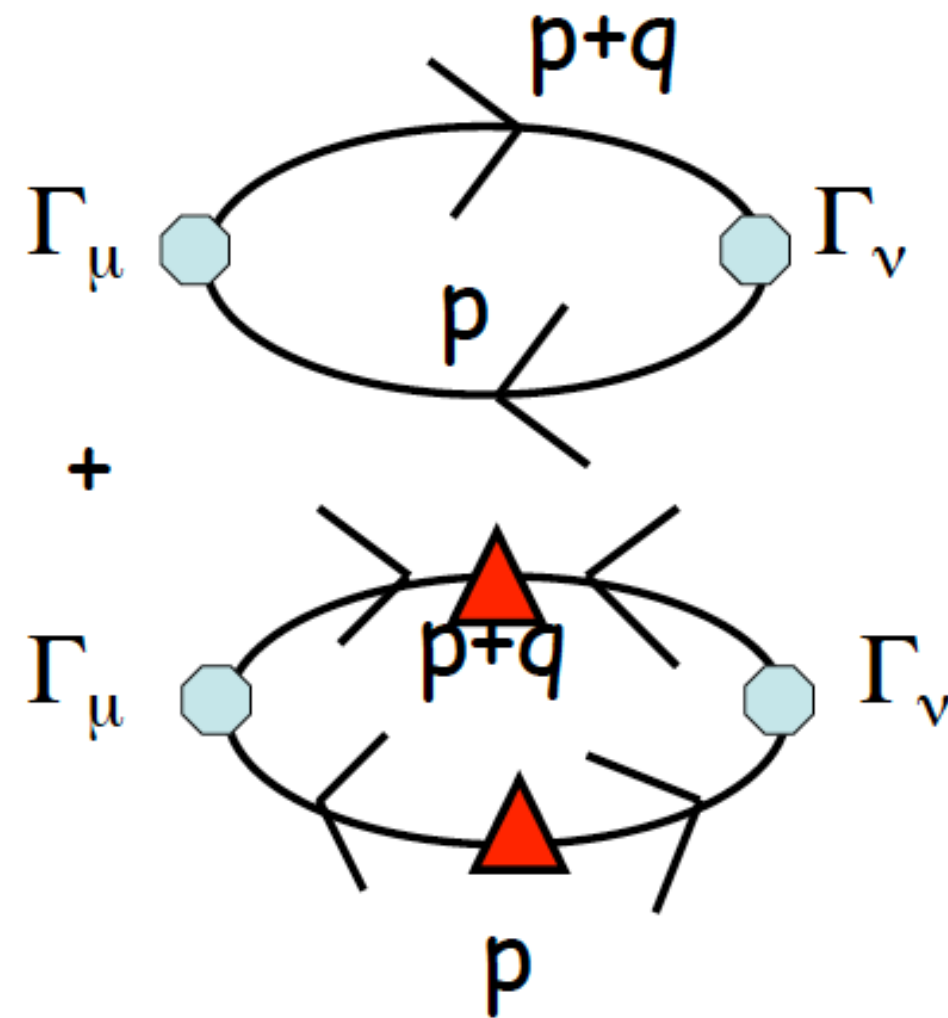
Response moves to high energy (time-like).

Neutrino scattering is exponentially suppressed.



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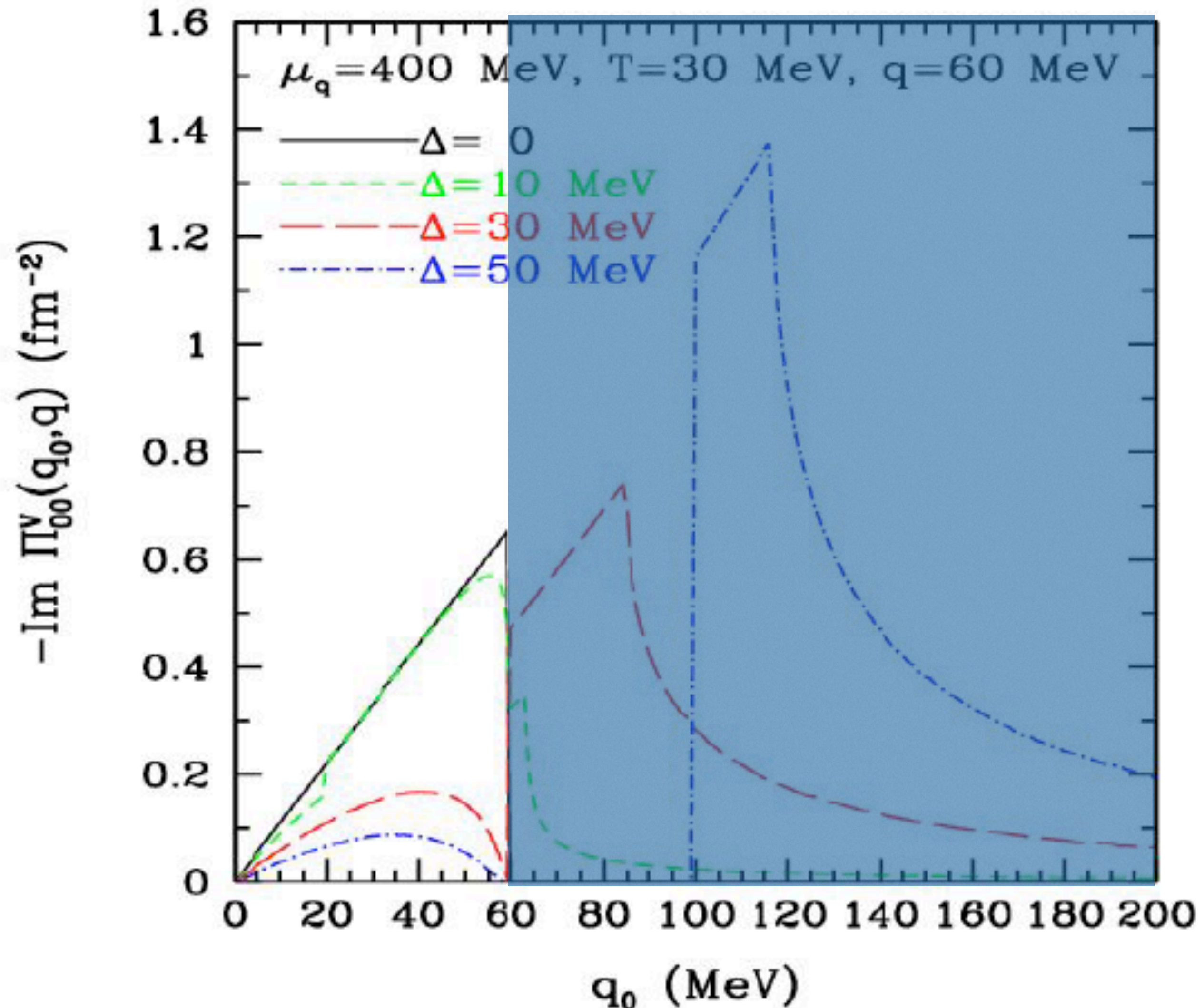


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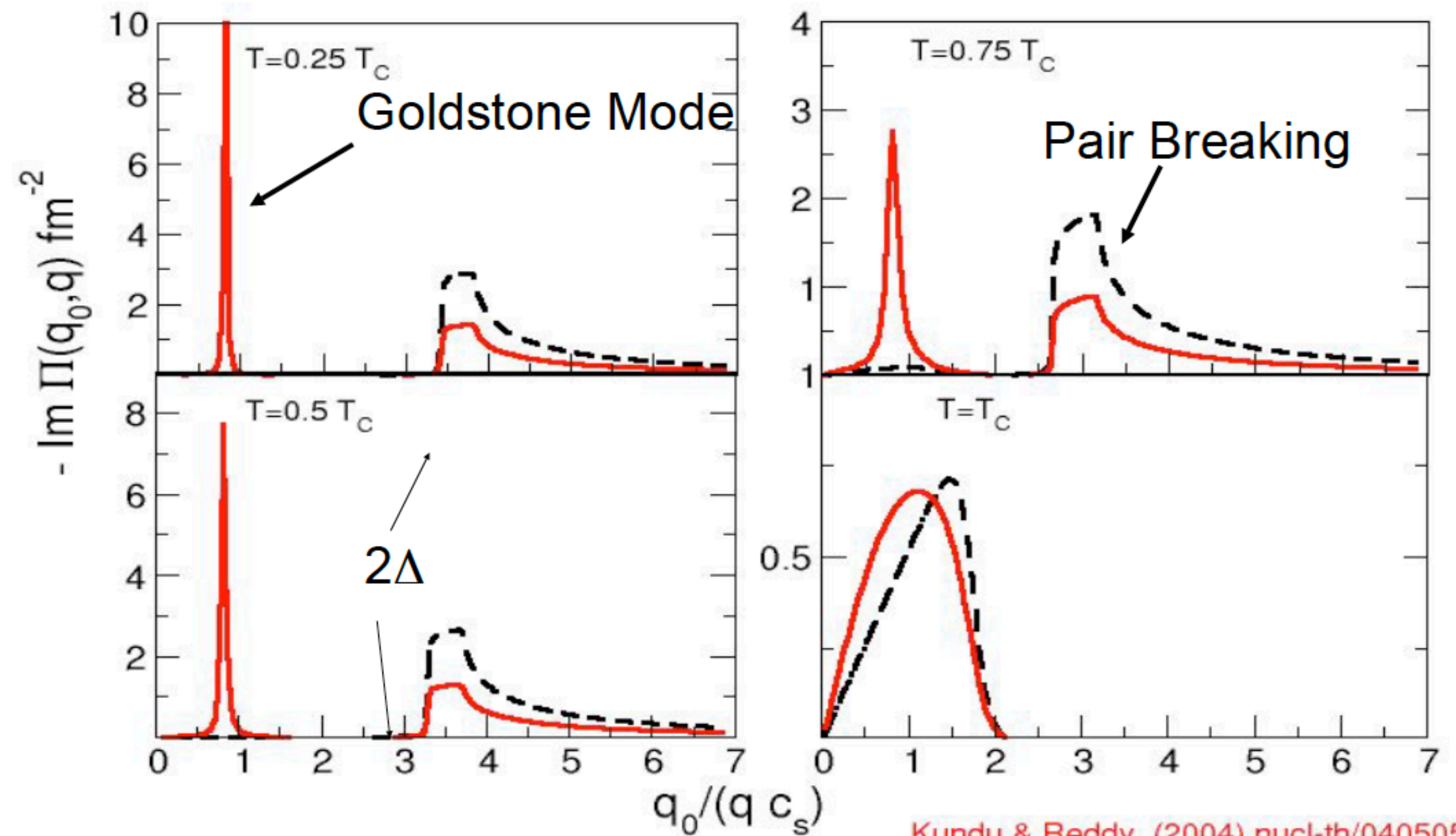
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Neutrino Scattering in Superfluid Quark Matter.

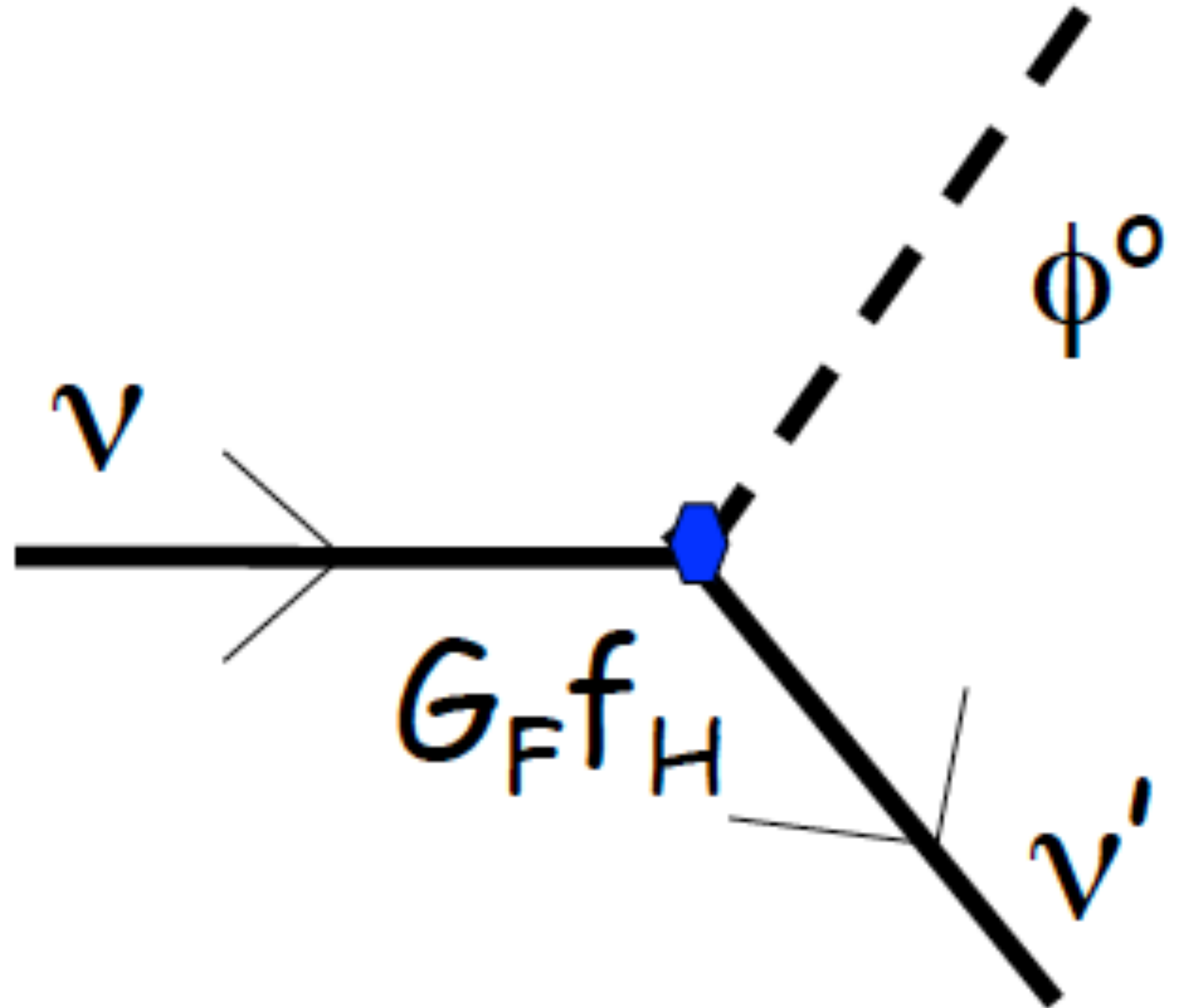
- Superfluid state has a Goldstone boson.
- Neutrinos couple to these modes.
- Arises naturally in RPA.
- At $T \ll T_c$ this is the only relevant mode for neutrino scattering.



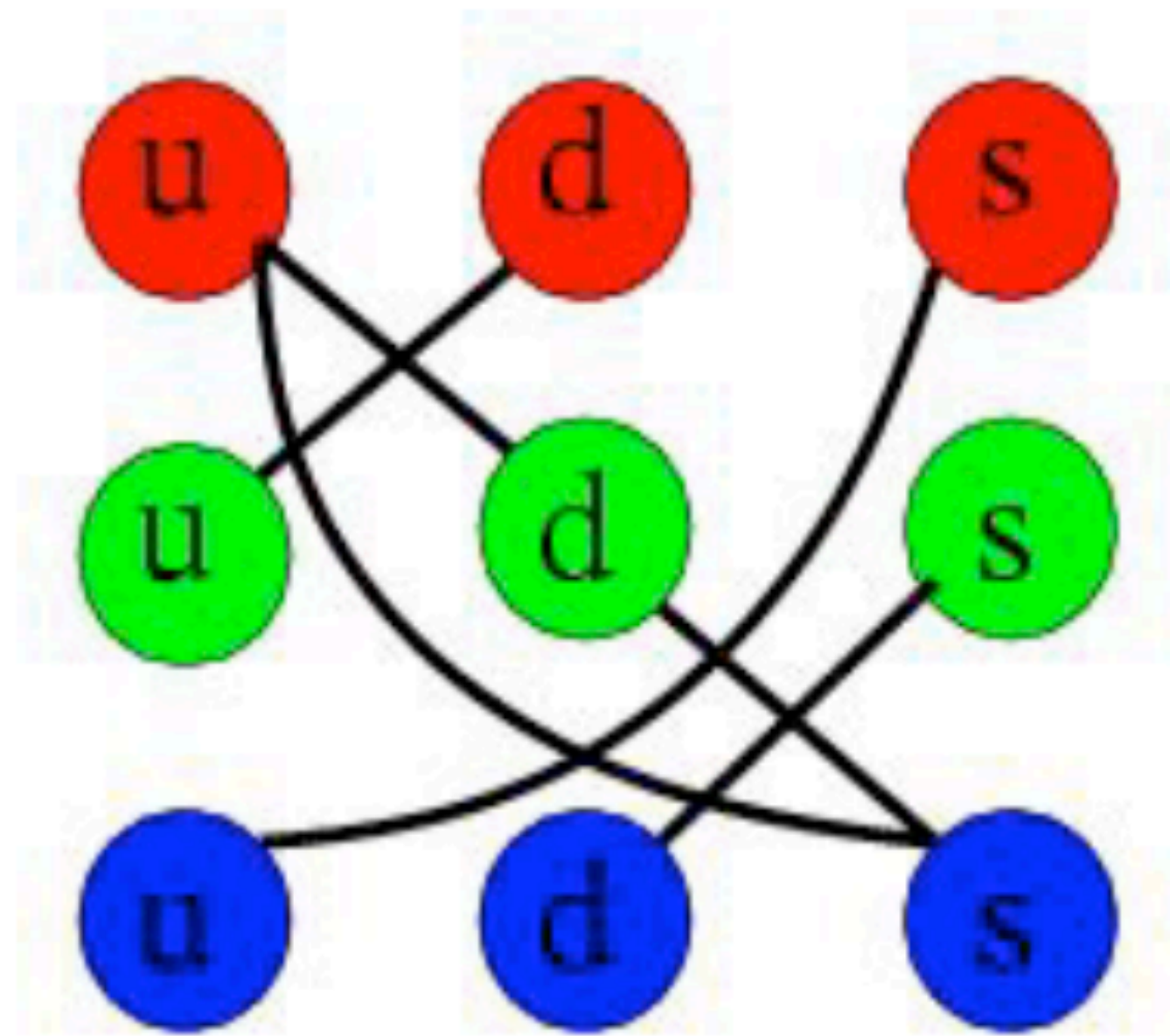
Kundu & Reddy, (2004) nucl-th/0405055

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Ultra Dense Matter is Opaque to Neutrinos but Transparent to Photons!

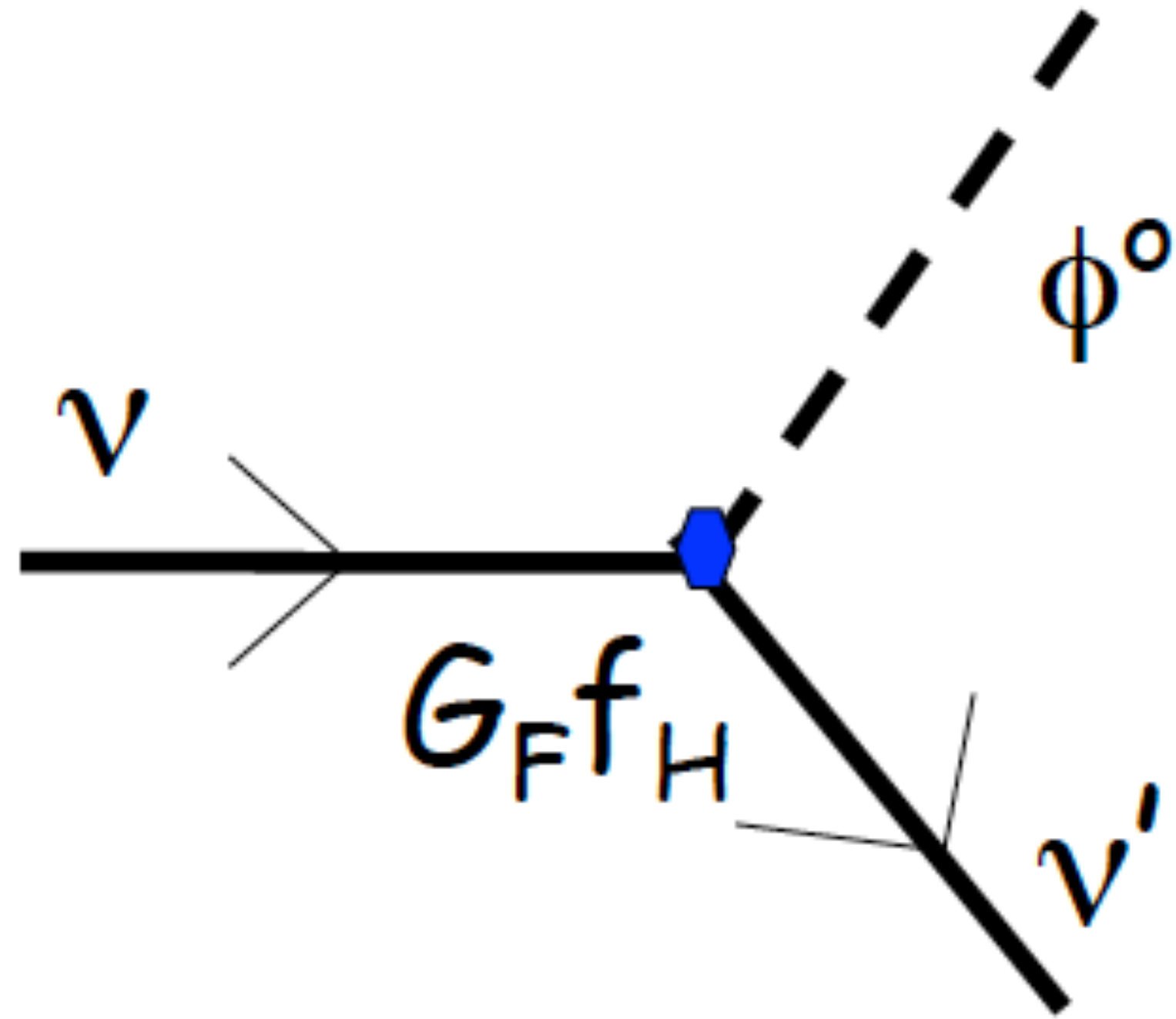


$$SU(3)_{\text{color}} \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_B$$



$$SU(3)_{\text{color}+L+R} \otimes Z_2$$

Ultra Dense Matter is Opaque to Neutrinos but Transparent to Photons!

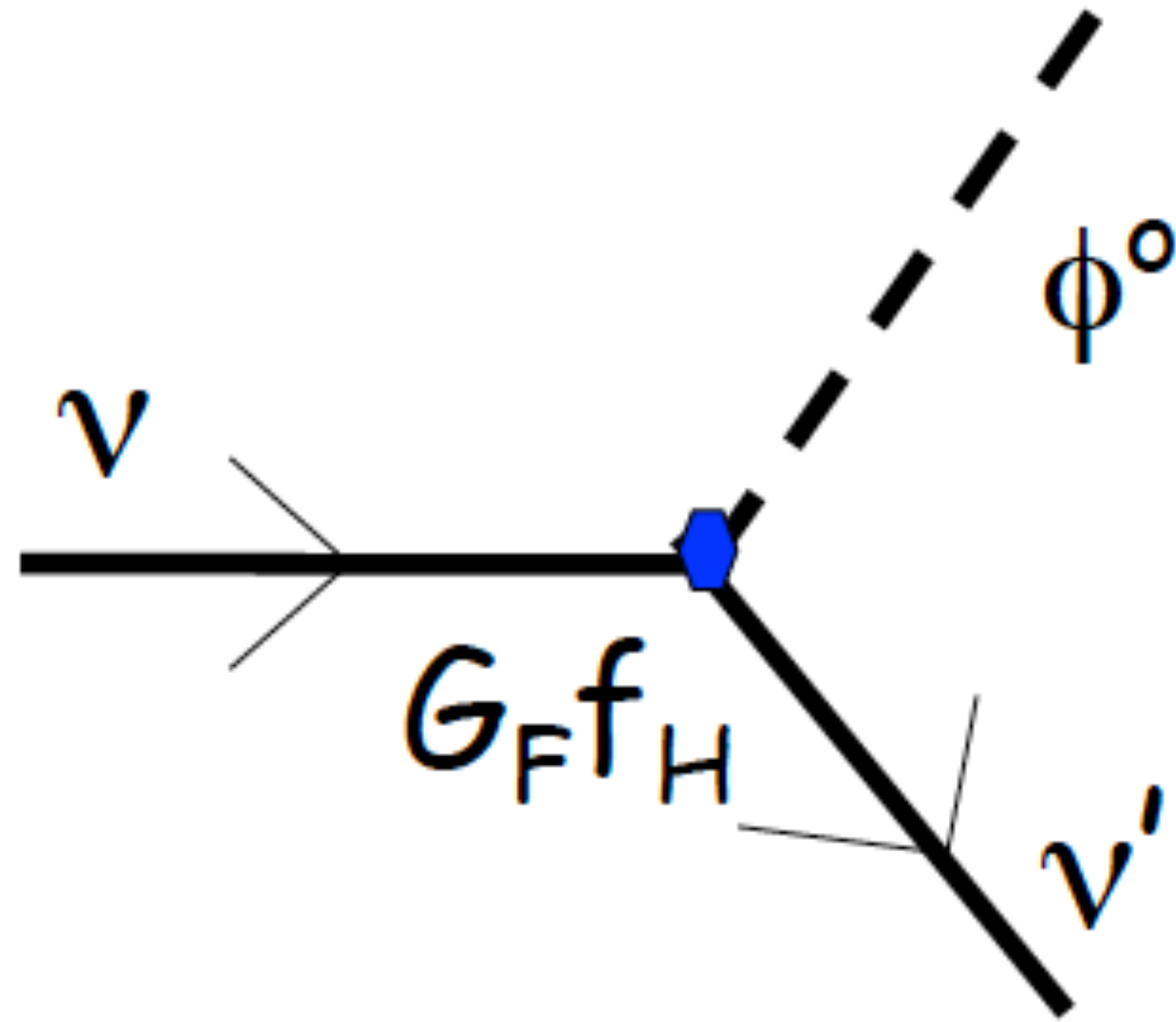


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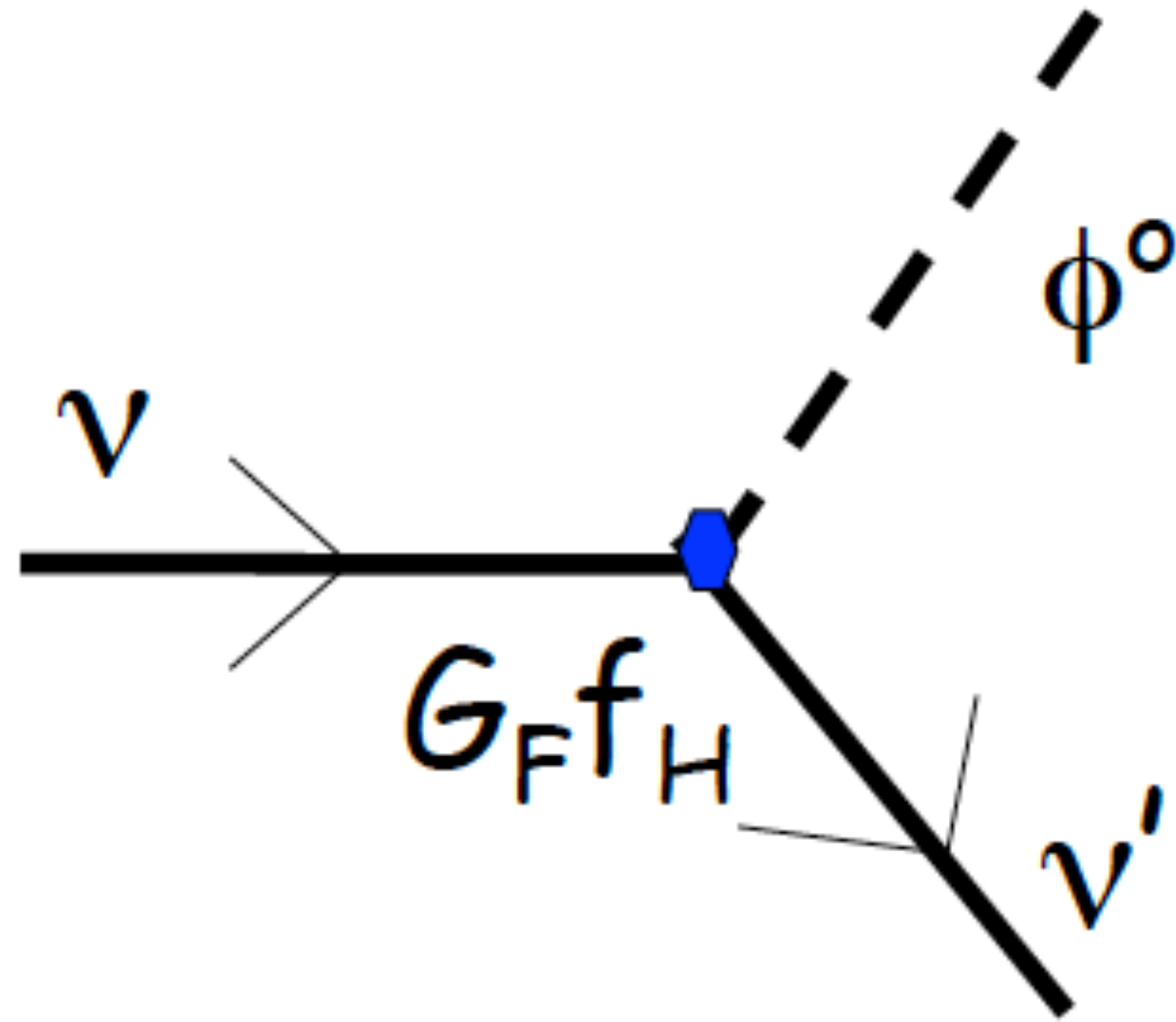
$$\frac{1}{\lambda_{\nu \rightarrow H\nu}(E_\nu)} = \frac{256}{45\pi} \left[\frac{v(1-v)^2(1+\frac{v}{4})}{(1+v)^2} \right] G_F^2 f_H^2 E_\nu^3$$

$$SU(3)_{\text{color}} \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_B$$

\Downarrow

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$$\begin{aligned} &SU(3)_{\text{color}} \otimes SU(3)_L \otimes SU(3)_R \otimes U(1)_B \\ &\quad \downarrow \\ &SU(3)_{\text{color}+L+R} \otimes Z_2 \end{aligned}$$

More Opaque than the Normal Phase !

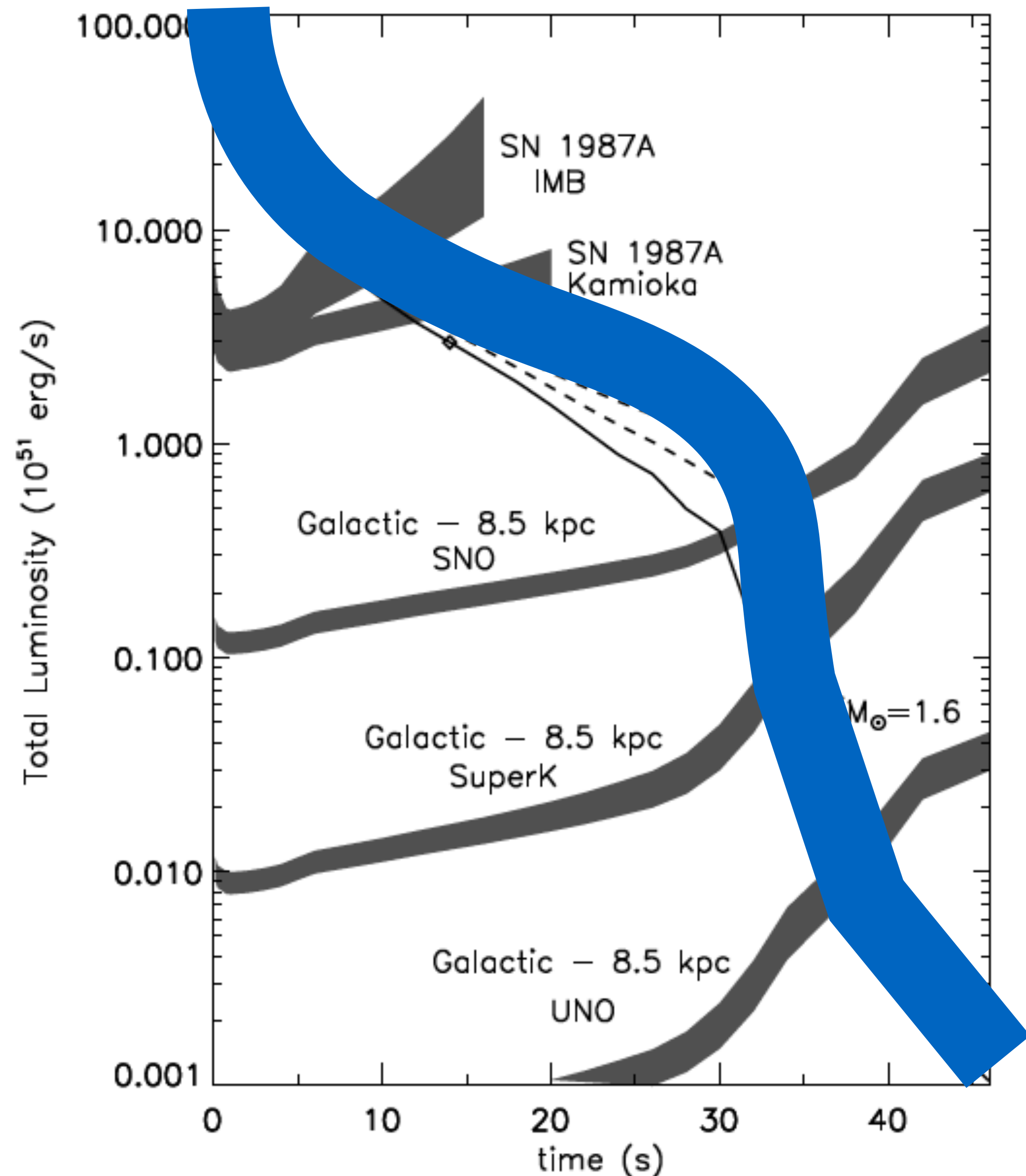
phase	process	$\lambda(T=5 \text{ MeV})$	$\lambda(T=30 \text{ MeV})$
Nuclear Matter	$\nu n \rightarrow \nu n$	200 m	1 cm
	$\nu_e n \rightarrow e^- p$	2 m	4 cm
Unpaired Quarks	$\nu q \rightarrow \nu q$	350 m	1.6 m
	$\nu d \rightarrow e^- u$	120 m	4 m
CFL	λ_{3B}	100 m	70 cm
	$\nu \phi \rightarrow \nu \phi$	>10 km	4 m

Neutron Star Tomography with Supernova Neutrinos

Temporal features in the late time neutrino signal contains valuable information about the core.

May be the only direct probe of the densest matter in the universe.

Its about time we had a galactic supernovae!

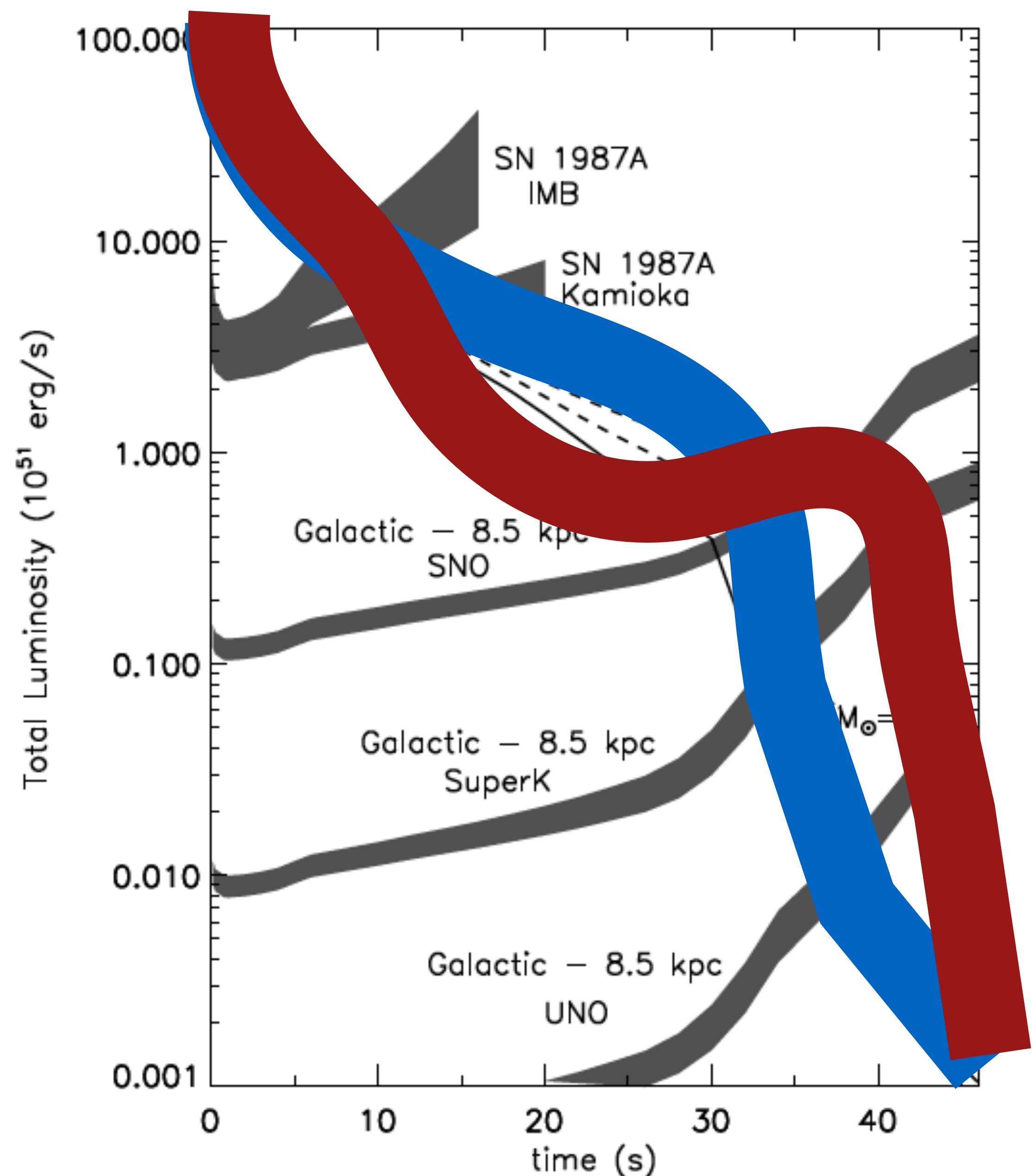


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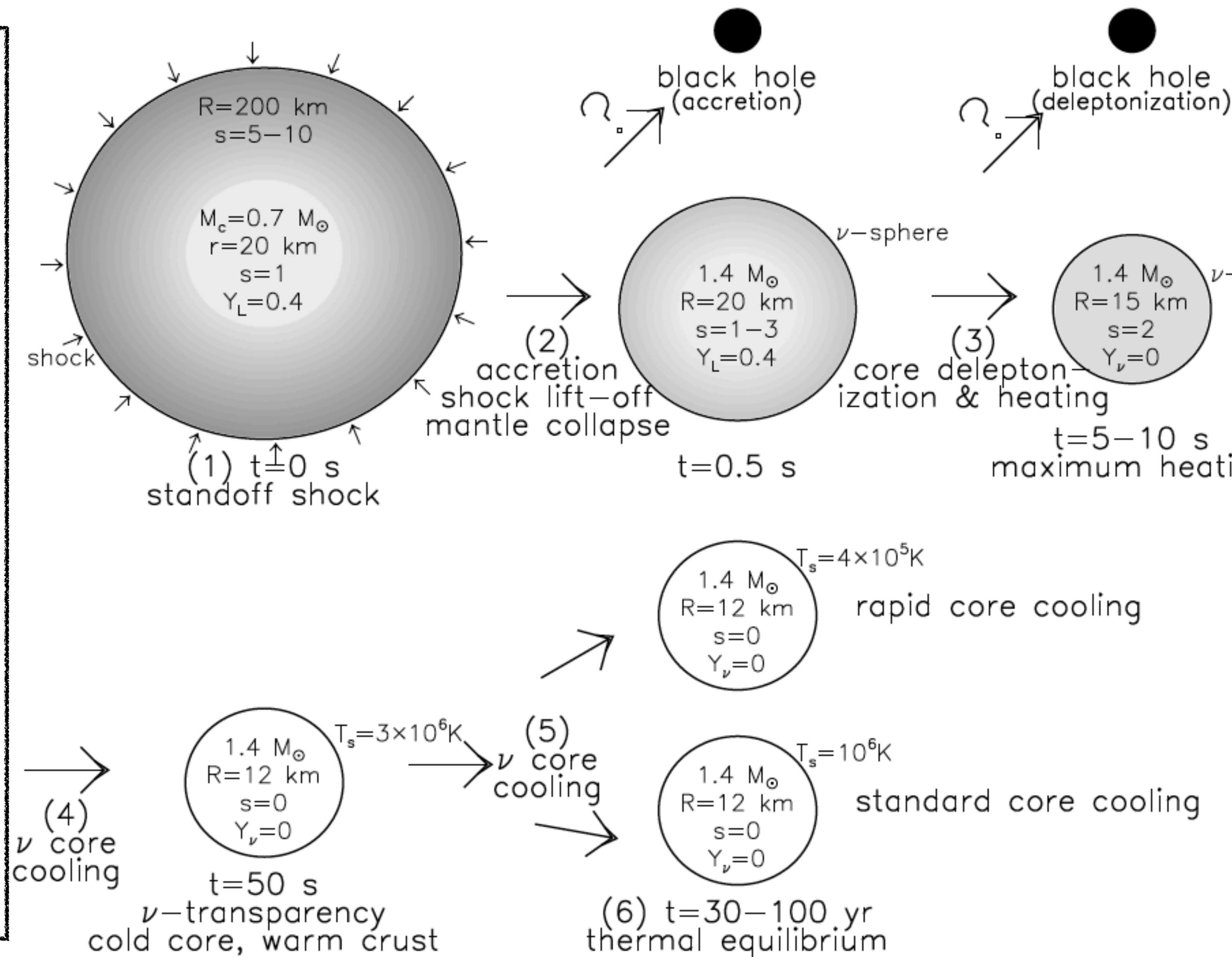
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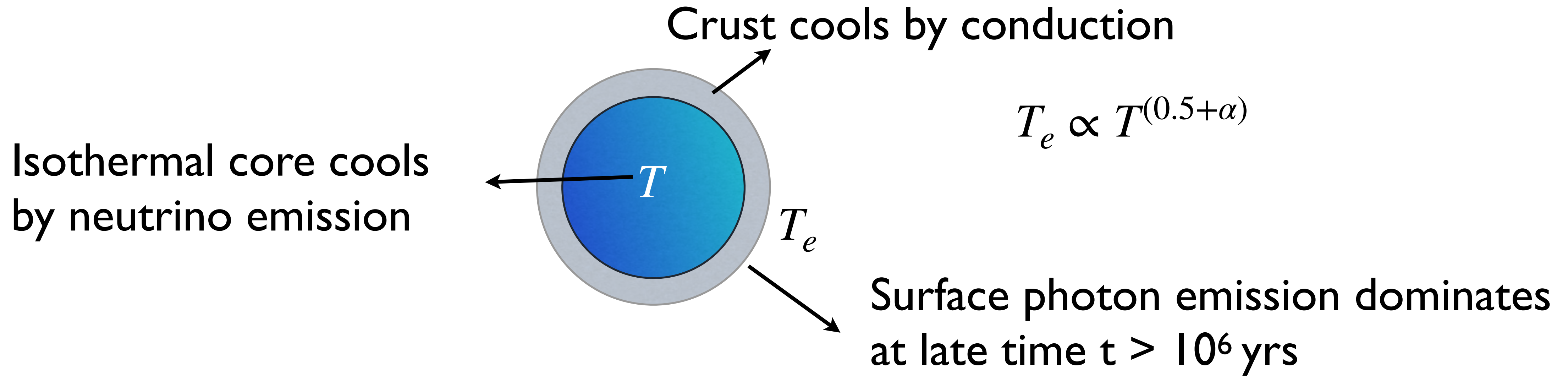
Thermal evolution of an isolated neutron star from birth to old age.

Direct detection of neutrinos is only possible during the first minute or so from a galactic supernova.

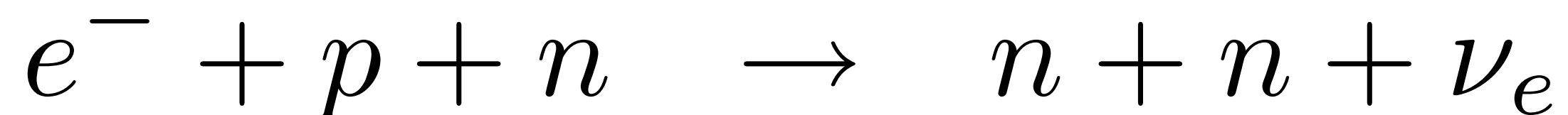
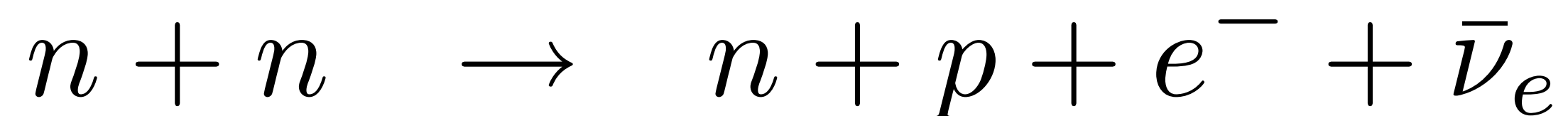
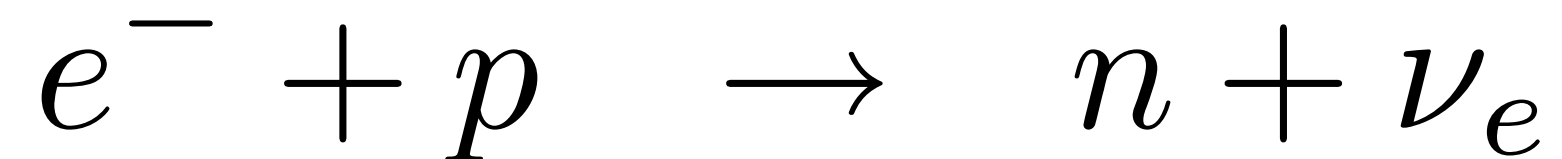
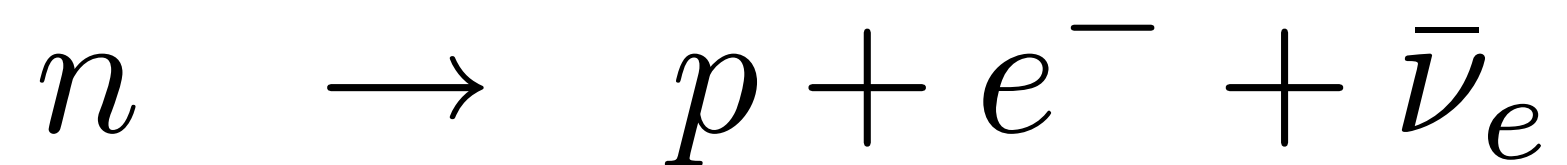
X-ray observations from the surface of a population of neutron stars informs us about late time thermal evolution.



Neutron Star Cooling



Basic neutrino reactions:



$$\dot{\epsilon}_\nu|_{\rho=\rho_o} \simeq 10^{25} T_9^6 \frac{\text{ergs}}{\text{cm}^3 \text{ s}}$$

Fast: Direct URCA

$$\dot{\epsilon}_\nu|_{\rho=\rho_o} \simeq 10^{22} T_9^8 \frac{\text{ergs}}{\text{cm}^3 \text{ s}}$$

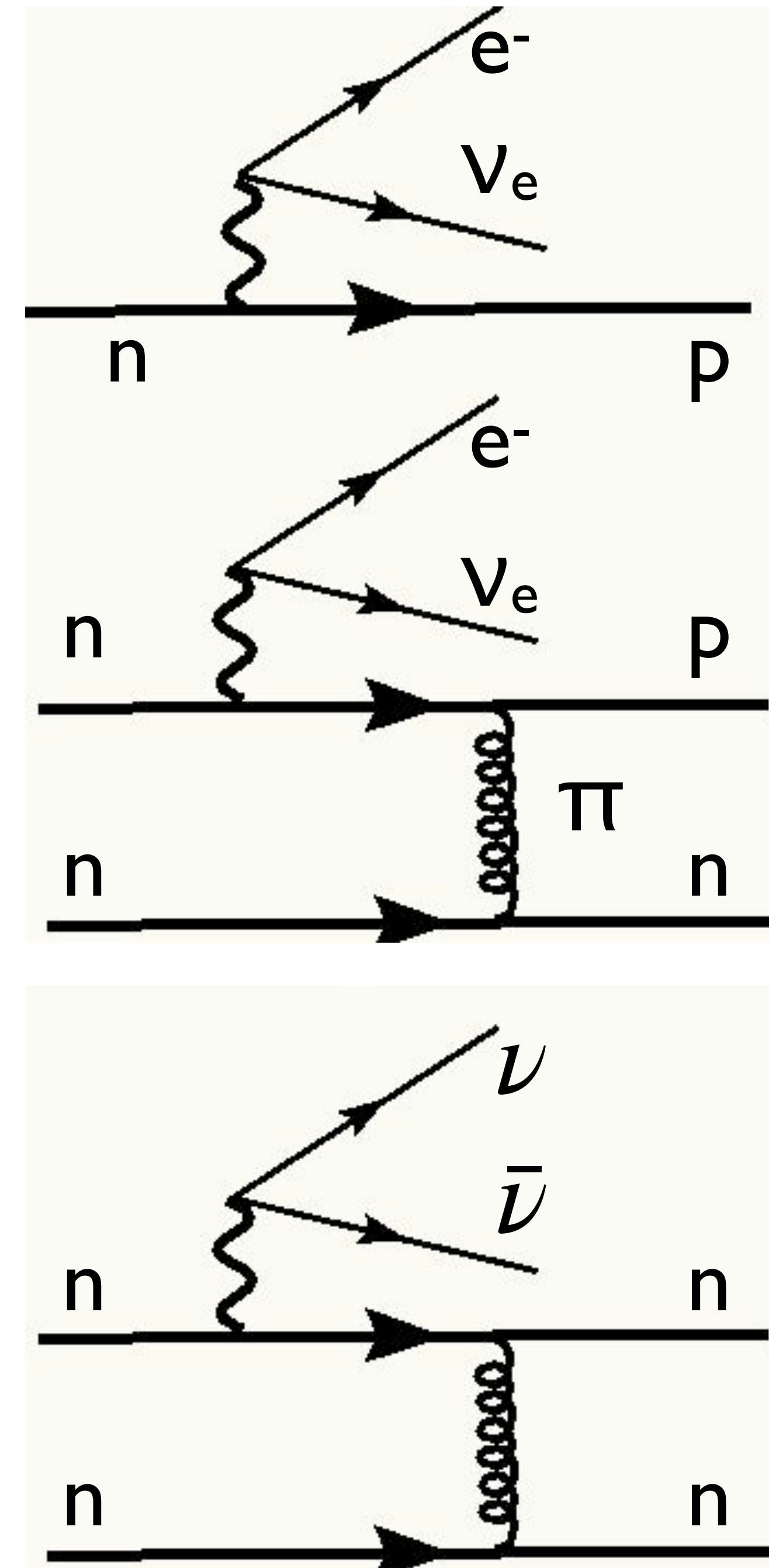
Slow: Modified URCA

Neutrino Emissivity in the Normal Phase

Single -particle reactions are fast. Need unstable particles- beta decay is the only reaction - “Direct Urca”

Multi-particle reactions are slow. “Modified Urca” can be thought of as beta-decay in the presence of a companion.

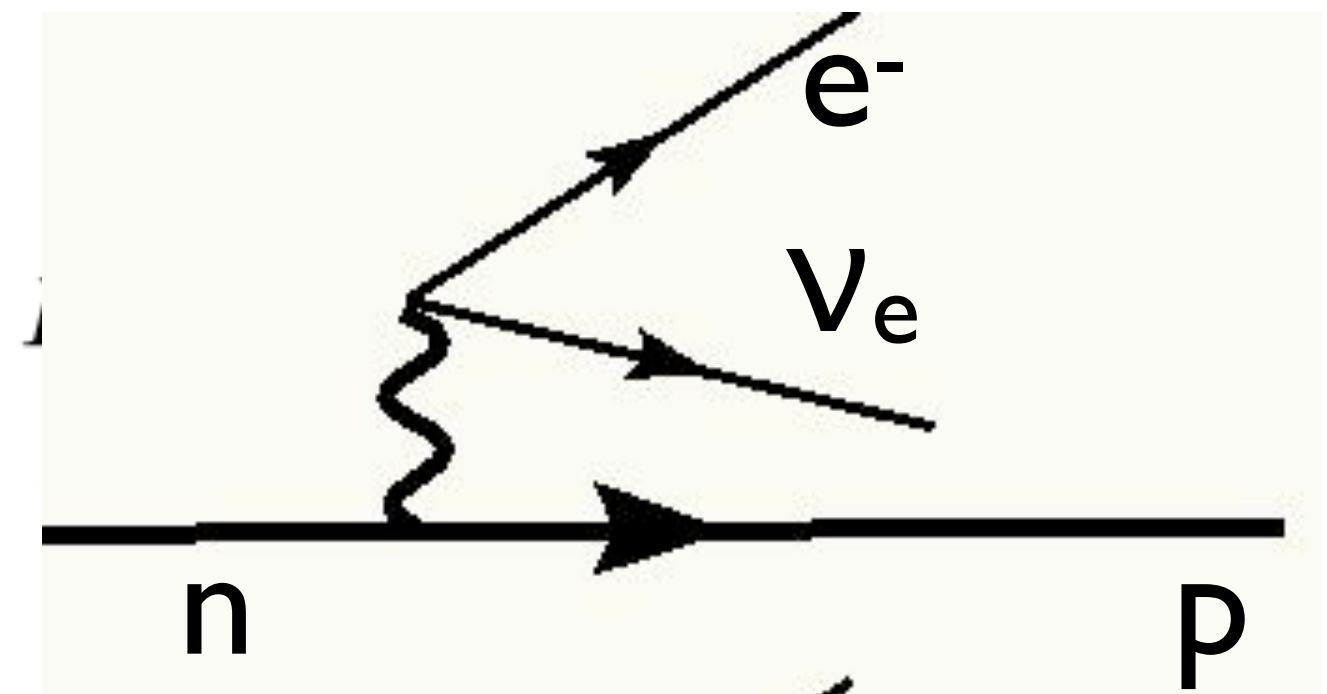
Bremsstrahlung reactions are even slower. Because the neutrino momenta are much smaller than that of electrons.



Direct URCA

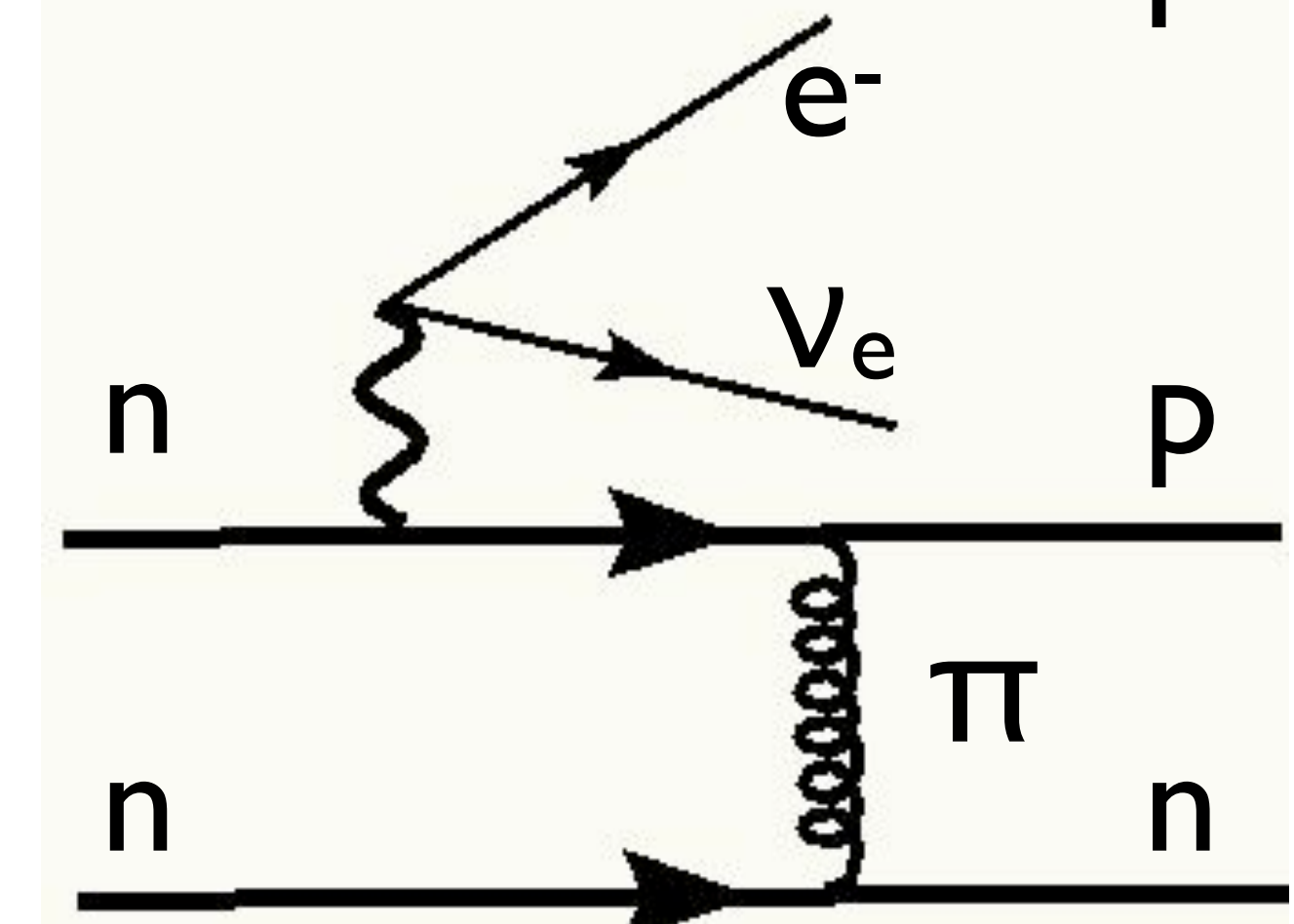
$$\epsilon^{\text{DUrca}} = \iiint \frac{d^3 p_\nu}{(2\pi)^3} \frac{d^3 p_e}{(2\pi)^3} \frac{d^3 p_p}{(2\pi)^3} \frac{d^3 p_n}{(2\pi)^3} (1 - f_e)(1 - f_p)f_n \cdot (2\pi)^4 \delta^4(P_f - P_i) |M_{fi}|^2 \cdot \dots$$

$$\epsilon^{\text{DUrca}} = \frac{457\pi}{10,080} G_F^2 \cos^2 \theta_C (1 + 3g_A^2) \frac{m_n^* m_p^* m_e^*}{\hbar^{10} c^3} (k_B T)^6$$



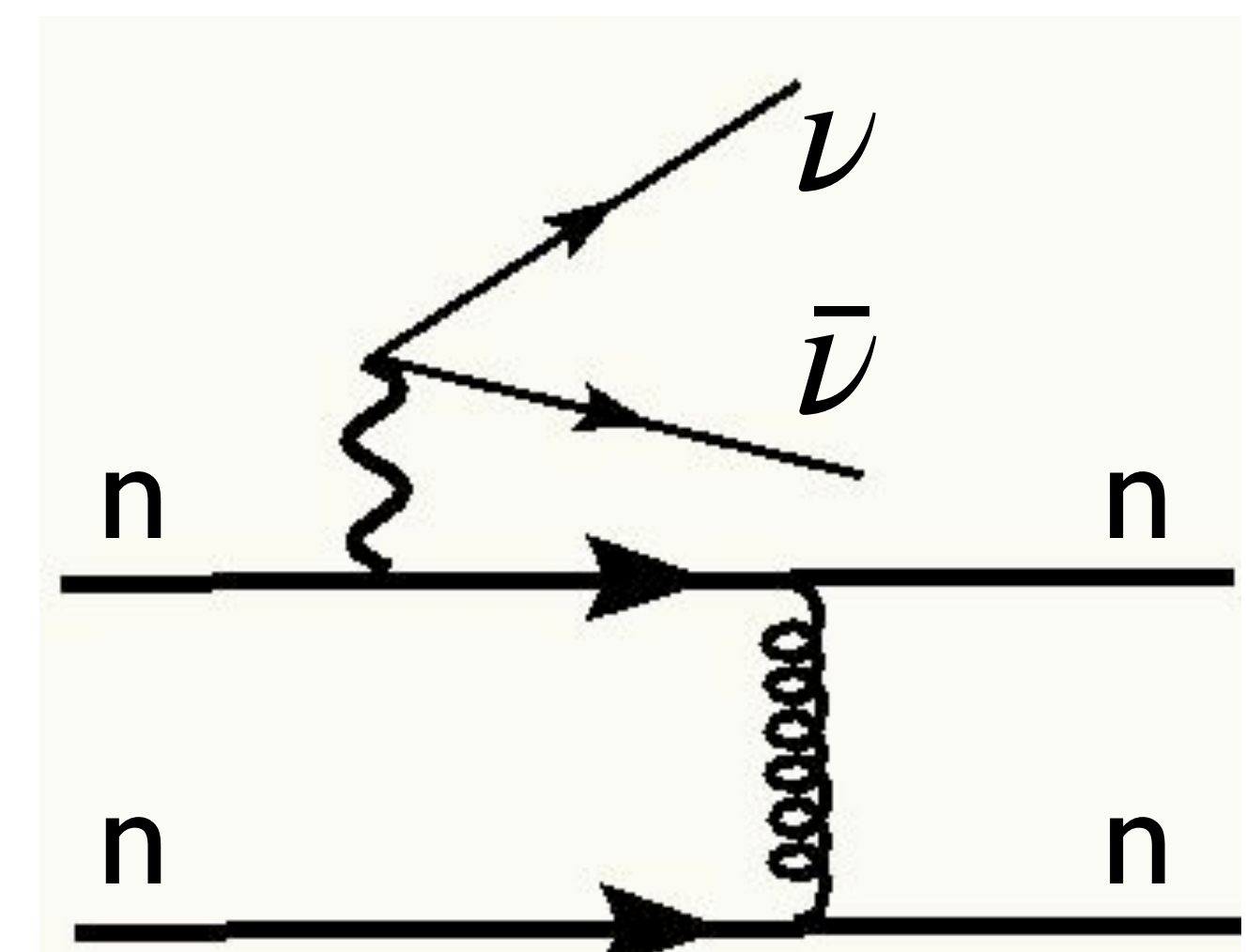
Modified URCA

$$\epsilon^{\text{MUrca,n}} = \frac{11513}{30240} \frac{G_F^2 \cos^2 \theta_C g_A^2}{2\pi} \frac{m_n^{*3} m_p^* p_{Fe}}{\hbar^{10} c^8} \left(\frac{f_\pi}{m_\pi} \right)^4 \alpha_n \beta_n (k_B T)^8$$



Bremsstrahlung

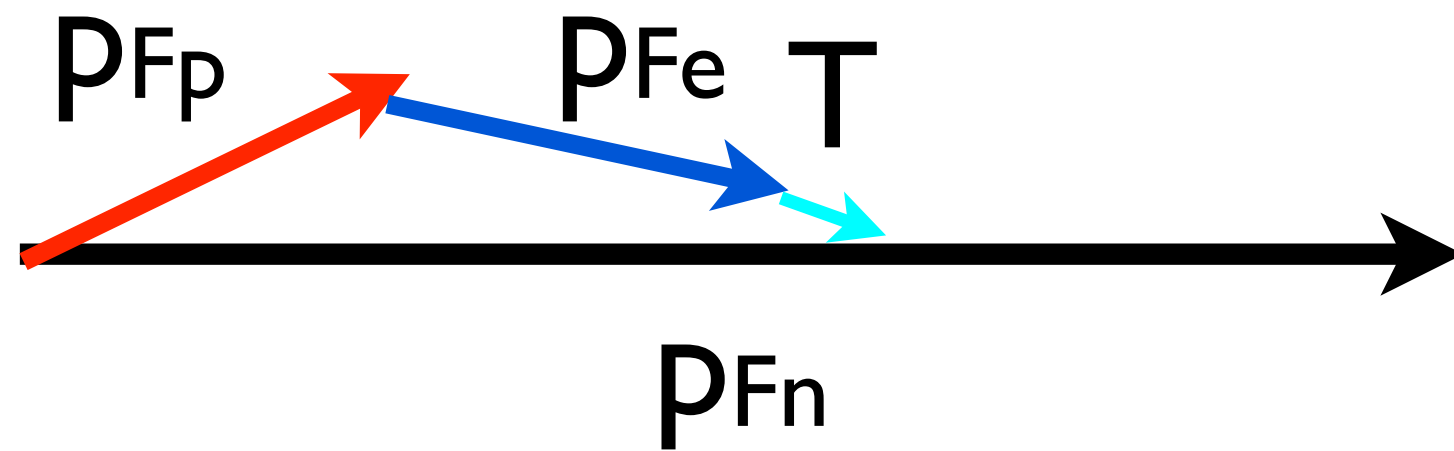
$$\epsilon^{\text{Brem,nn}} = \frac{41}{14175} \frac{G_F^2 g_A^2}{2\pi} \frac{m_n^{*4} p_{Fn}}{\hbar^{10} c^8} \left(\frac{f_\pi}{m_\pi} \right)^4 \alpha_{nn} \beta_{nn} \mathcal{N}_\nu (k_B T)^8$$



Neutrino Emission Rates in Normal Nuclear Matter

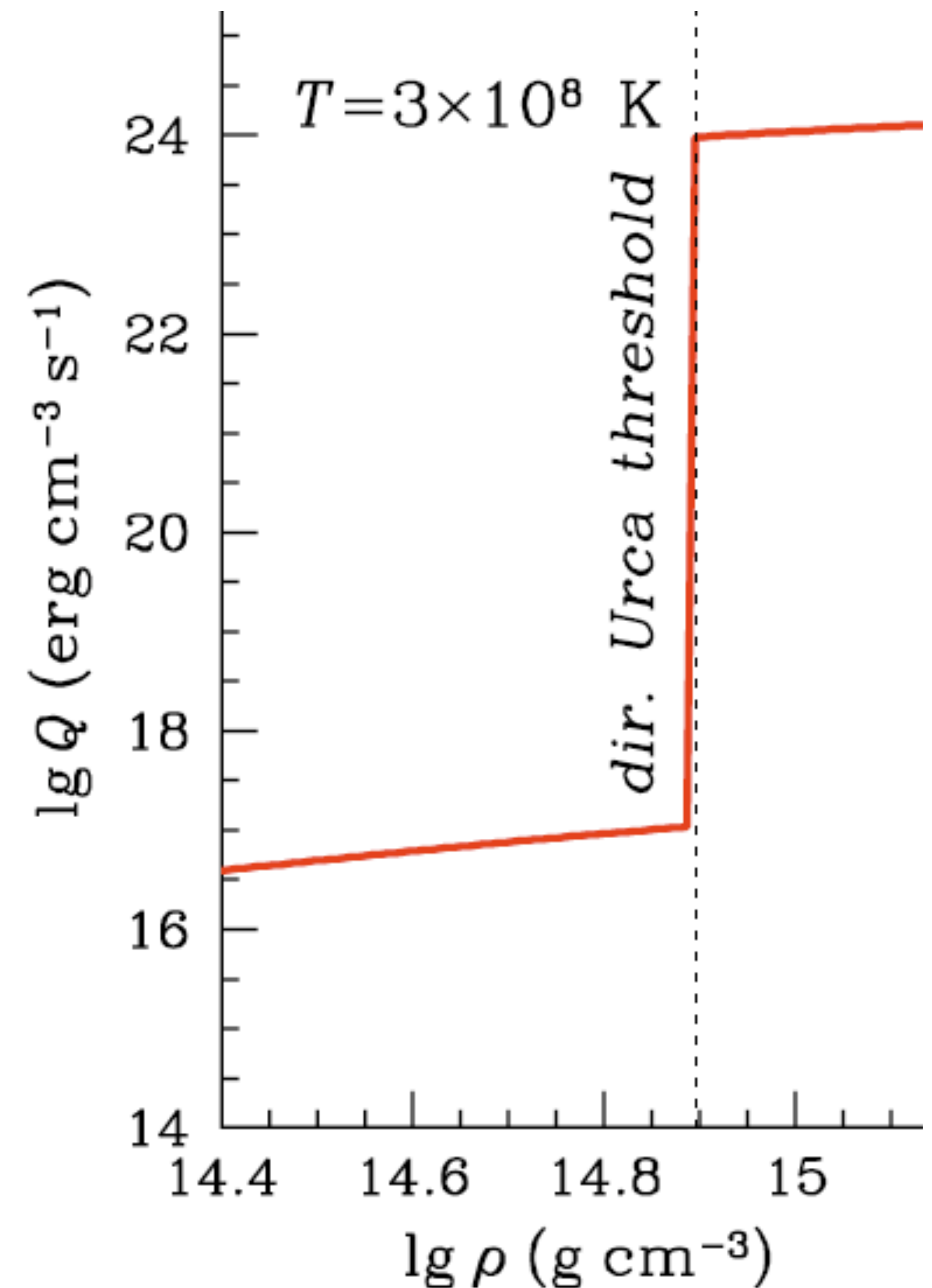
Name	Process	Emissivity (erg cm ⁻³ s ⁻¹)	
Modified Urca cycle (neutron branch)	$n + n \rightarrow n + p + e^- + \bar{\nu}_e$ $n + p + e^- \rightarrow n + n + \nu_e$	$\sim 2 \times 10^{21} R T_9^8$	Slow
Modified Urca cycle (proton branch)	$p + n \rightarrow p + p + e^- + \bar{\nu}_e$ $p + p + e^- \rightarrow p + n + \nu_e$	$\sim 10^{21} R T_9^8$	Slow
Bremsstrahlung	$n + n \rightarrow n + n + \nu + \bar{\nu}$ $n + p \rightarrow n + p + \nu + \bar{\nu}$ $p + p \rightarrow p + p + \nu + \bar{\nu}$	$\sim 10^{19} R T_9^8$	Slow
Direct Urca cycle	$n \rightarrow p + e^- + \bar{\nu}_e$ $p + e^- \rightarrow n + \nu_e$	$\sim 10^{27} R T_9^6$	Fast

Proton Fraction & Direct URCA



Neutron decay at the Fermi surface cannot conserve momentum if $x_p \sim (p_{Fp} / p_{Fn})^3 < 0.1$

More massive stars have larger central densities and hence a larger proton fraction.



Neutron Star Cooling - Normal Nucleons

Energy Balance Equation:

$$\frac{dE_{th}}{dt} = C_V \frac{dT}{dt} = -L_\gamma - L_\nu + H$$

Specific Heat:

$$C_V = \sum_i C_{Vi} \quad c_{Vi} = N(0) \frac{\pi^2}{3} k_B^2 T \quad \text{with} \quad N(0) = \frac{m^* p_F}{\pi^2 \hbar^3}$$
$$C_V = \iiint c_V dV \simeq 10^{38} - 10^{39} \times T_9 \text{ erg K}^{-1} \equiv CT$$

Neutrino Luminosity
(Modified URCA)

$$L_\nu = \iiint \epsilon_\nu dV \simeq 10^{38} - 10^{40} \times T_9^8 \text{ erg K}^{-1} \equiv NT^8$$

Photon Luminosity
(surface)

$$L_\gamma = 4\pi R^2 \sigma T_e^4 = ST^{2+4\alpha}$$

Analytic Model for Neutron Star Cooling

$$\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu$$

$$C_v = CT \quad L_\nu = NT^8 \quad L_\gamma = ST^{2+4\alpha}$$

- **Neutrino Cooling Era:** $L_\nu \gg L_\gamma$

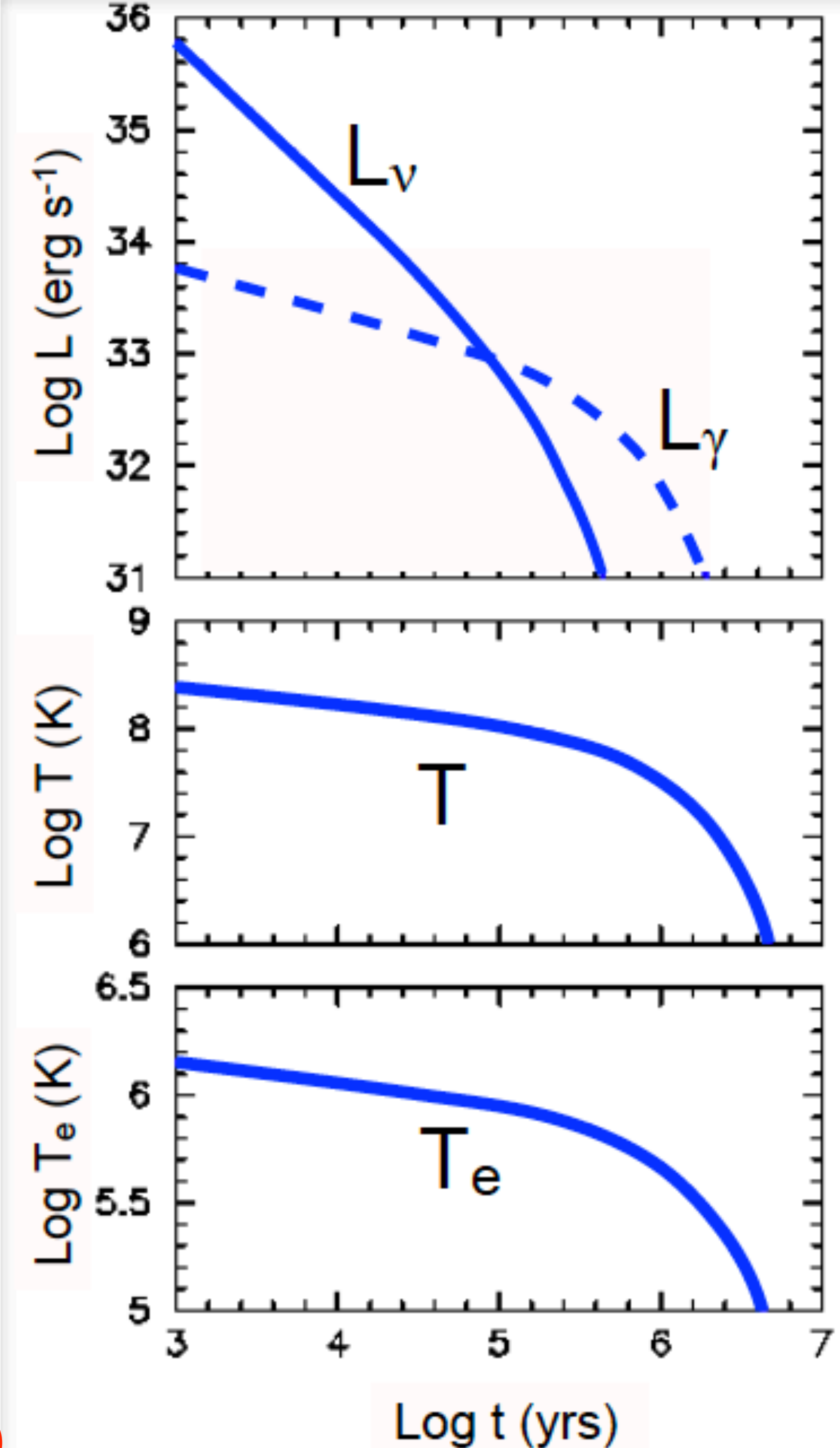
$$\frac{dT}{dt} = -\frac{N}{C}T^7 \Rightarrow t - t_0 = A \left[\frac{1}{T^6} - \frac{1}{T_0^6} \right]$$

$$T \propto t^{-1/6} \quad \text{and} \quad T_e \propto t^{-1/12}$$

- **Photon Cooling Era:** $L_\gamma \gg L_\nu$

$$\frac{dT}{dt} = -\frac{N}{S}T^{1+\alpha} \Rightarrow t - t_0 = A \left[\frac{1}{T^\alpha} - \frac{1}{T_0^\alpha} \right]$$

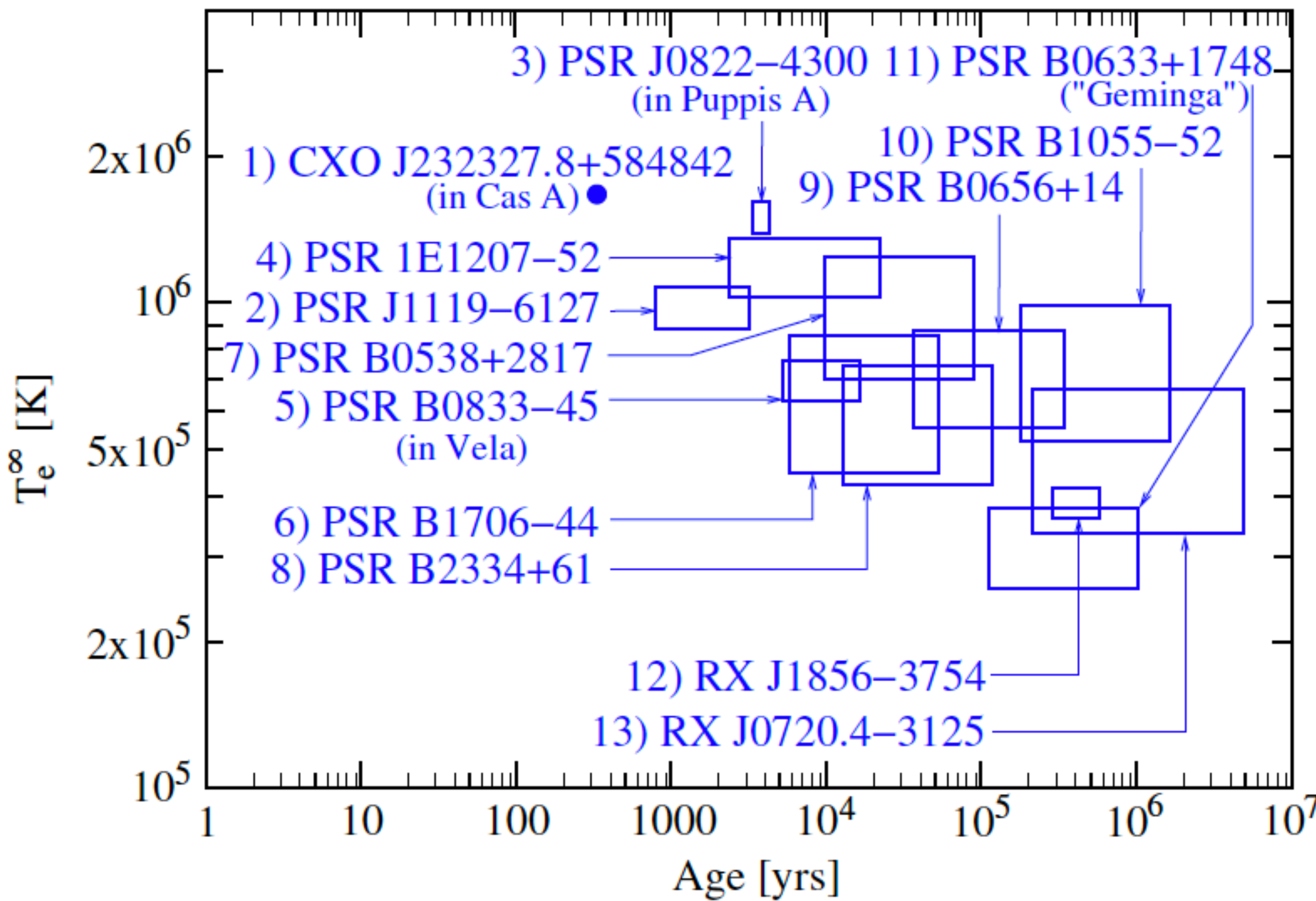
$$T \propto t^{-1/\alpha} \quad \text{and} \quad T_e \propto t^{-1/2\alpha}$$



Neutron Star Cooling Data: Isolated Neutron Stars with Thermal Emission.

Ages are estimated either from pulsar spin-down or by association with a supernova. Age uncertainties are estimated.

Uncertainty in the temperature due to atmosphere and magnetic fields are estimated.

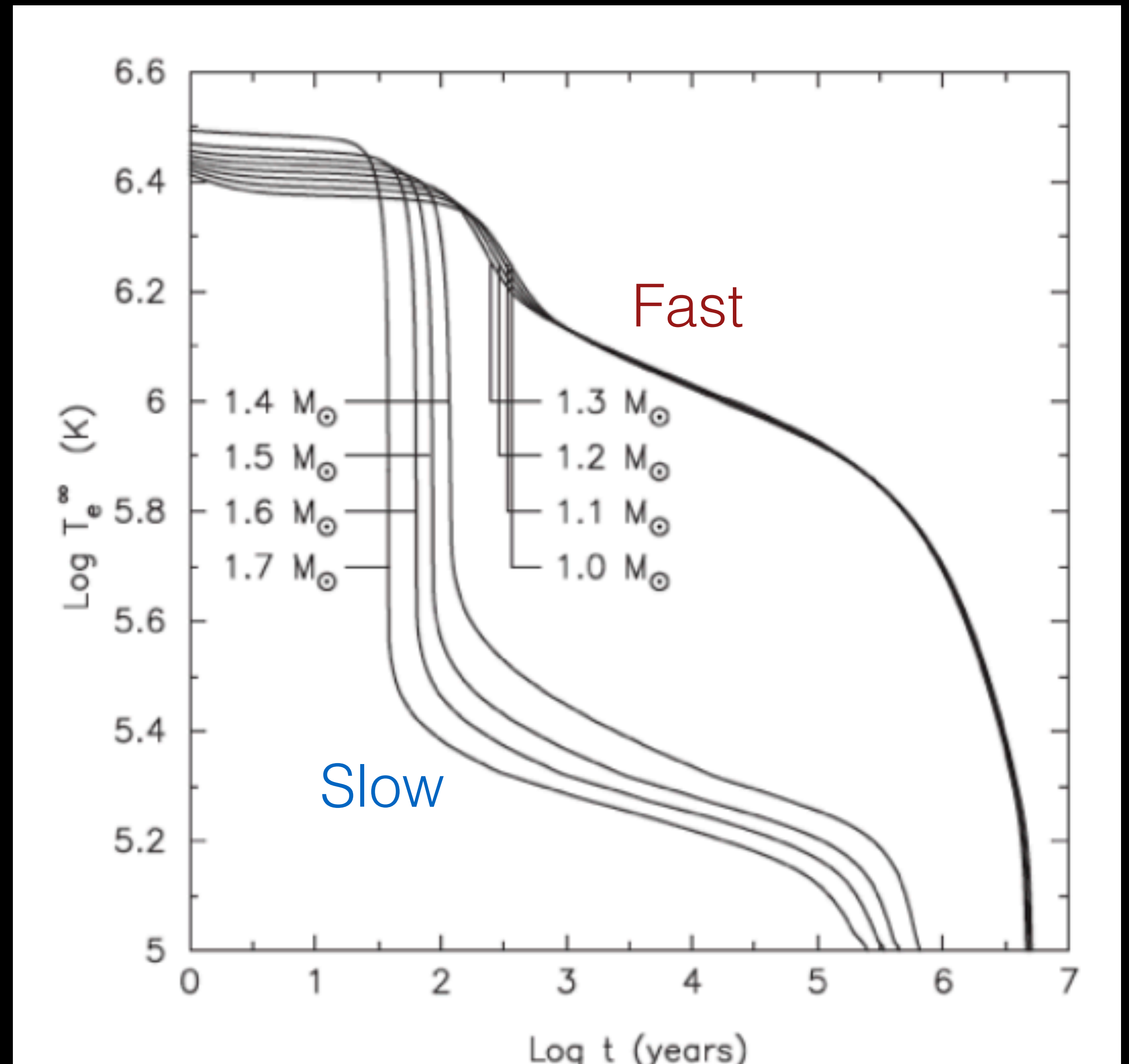


Cooling Models with only Normal Neutrons is Inadequate

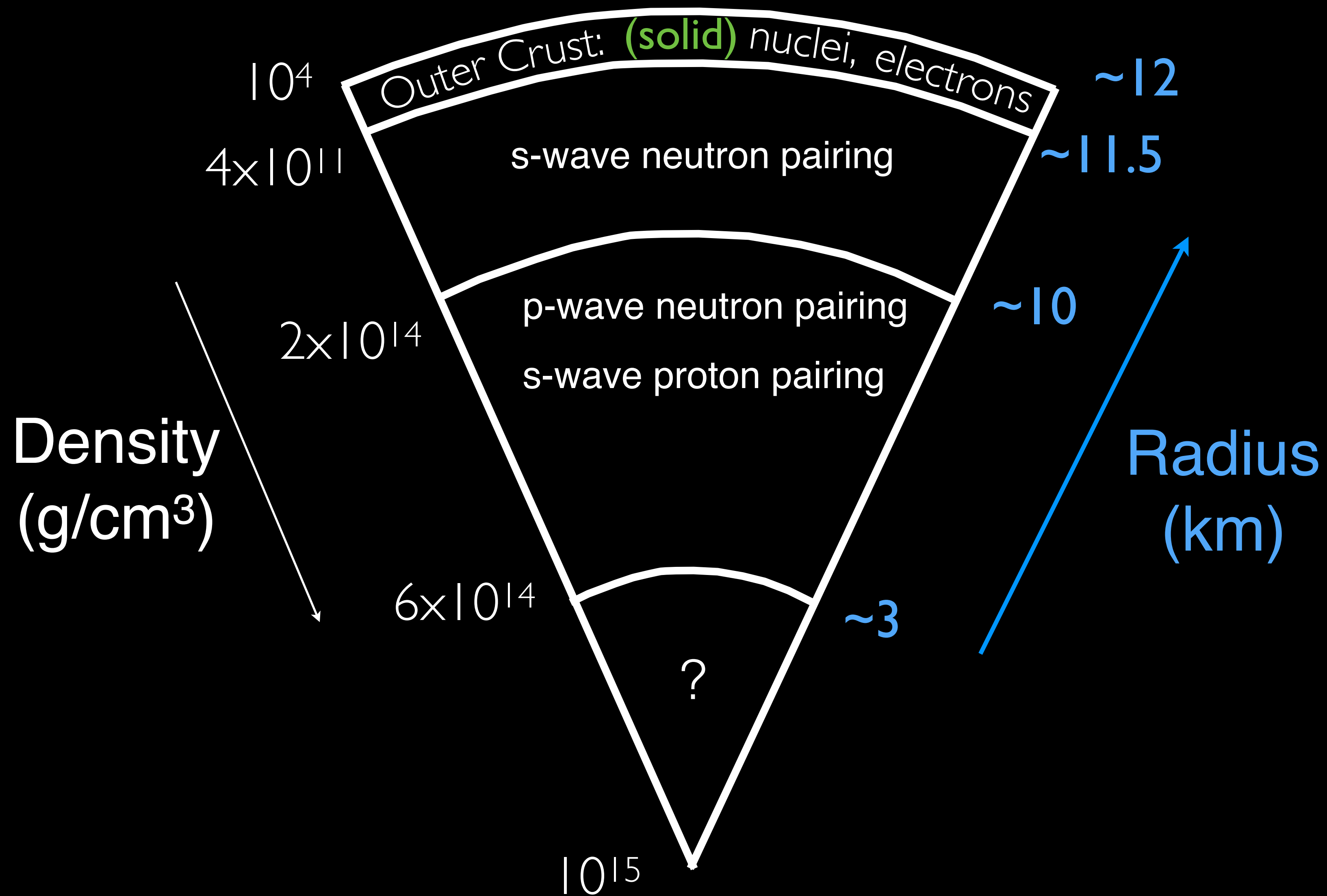
Even a small fraction of the normal core with DURCA can lead to very rapid cooling.

This is incompatible with the observed trends in the neutron stars population.

There is a need for an intermediate process between DURCA and MURCA.



Phases of Cold Dense Matter in Neutron Stars



Pairing

1. Too hot for electron pairing:

$$T_c \approx \omega_p^{\text{ion}} \exp \left(-\frac{v_{Fe}}{\alpha_{\text{em}}} \right) \quad \text{Ginzburg (1969)}$$

Relativistic electrons move too quickly to feel the phonon induced attraction.

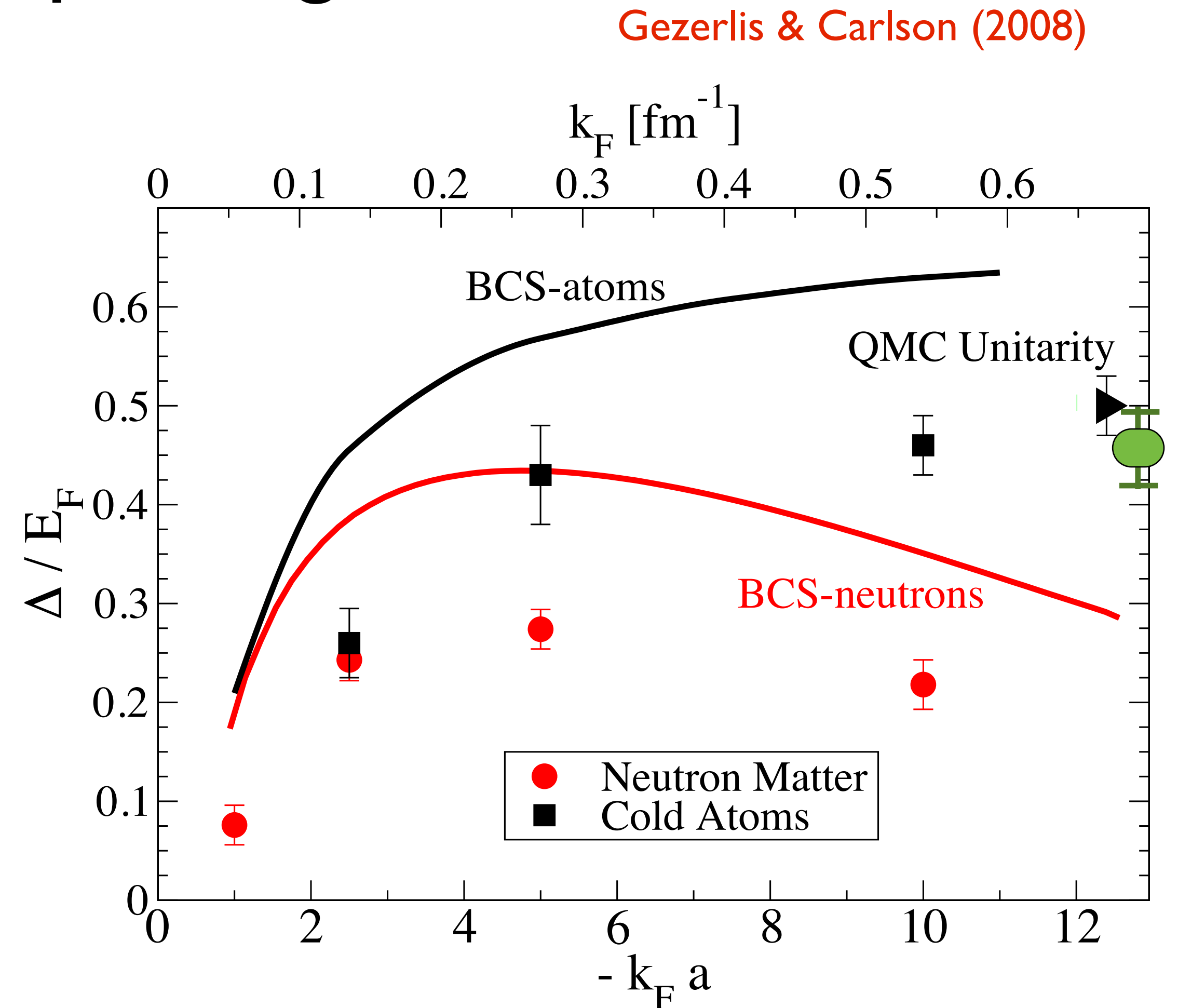
2. Pairing between nucleons is inevitable.

$$T_c \approx E_{Fn} \exp \left(-\frac{\pi}{2k_{Fn} a_{nn}} \right) \quad \begin{array}{l} \text{Bohr, Mottelson, Pines (1958)} \\ \text{Migdal (1959)} \end{array}$$

Typical energy scale is MeV ($\sim 10^{10}$ K)

S-wave pairing

- The nucleon-nucleon interaction is known up to relative momenta ~ 350 MeV.
- Perturbation theory fails, but Quantum Monte Carlo and lattice methods may be reliable.
- Best estimates for the gap indicate that it reaches a maximum value ~ 1 MeV in the crust.

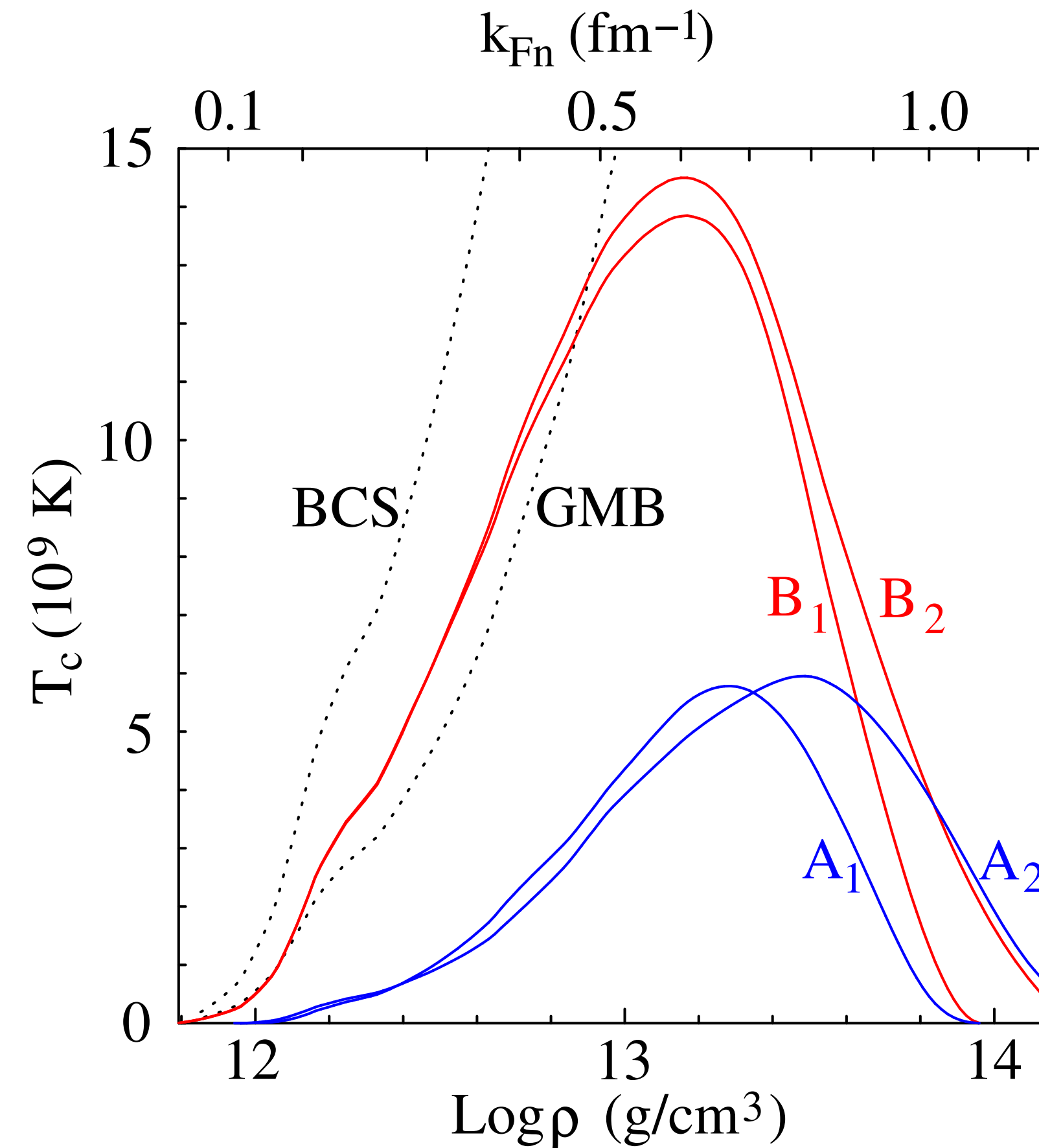


Cold atom experiments help validate many-body theory of strong short-range interactions.

Bulgac, Carlson, Drut, Gandolfi, Forbes, Reddy ..

S-wave pairing

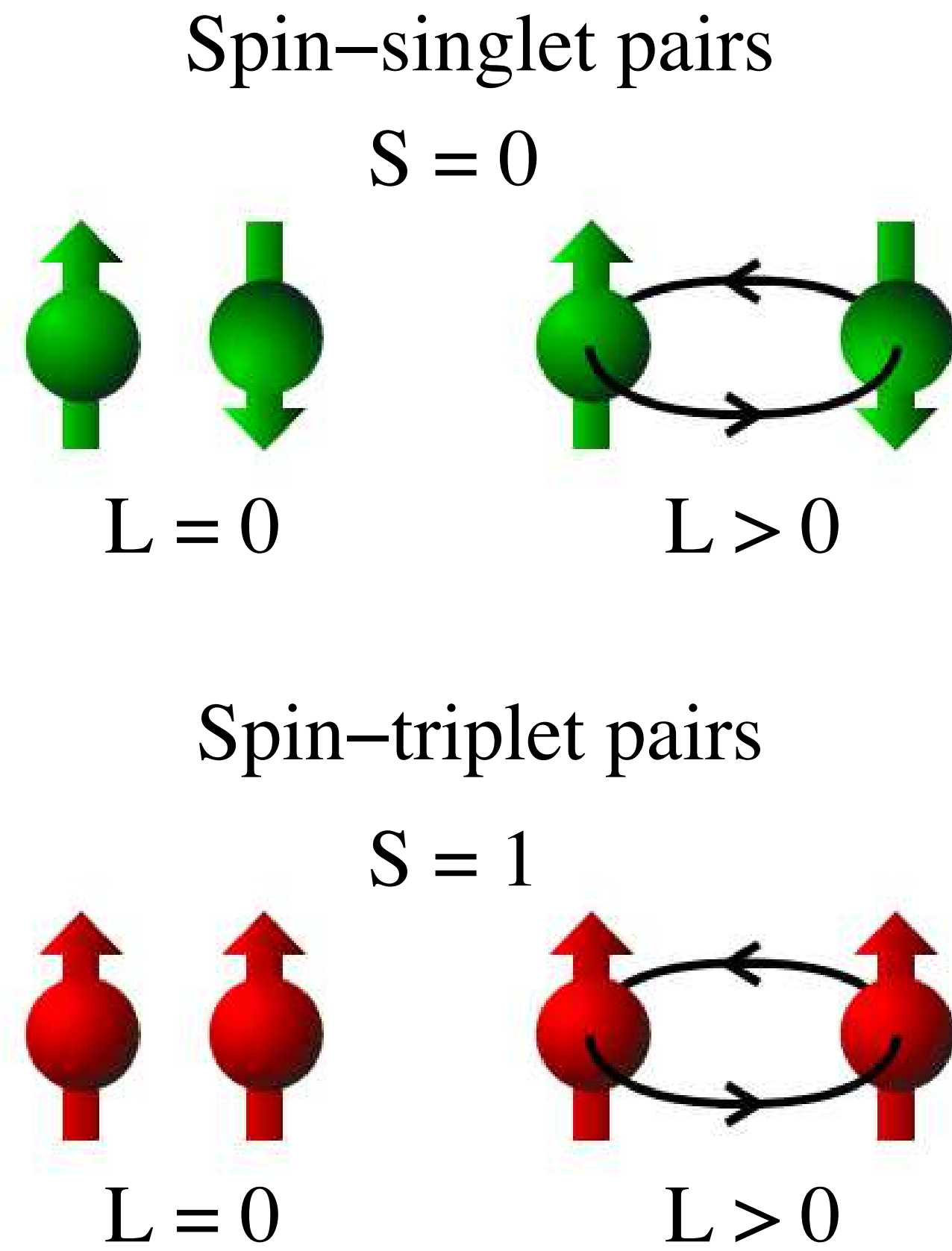
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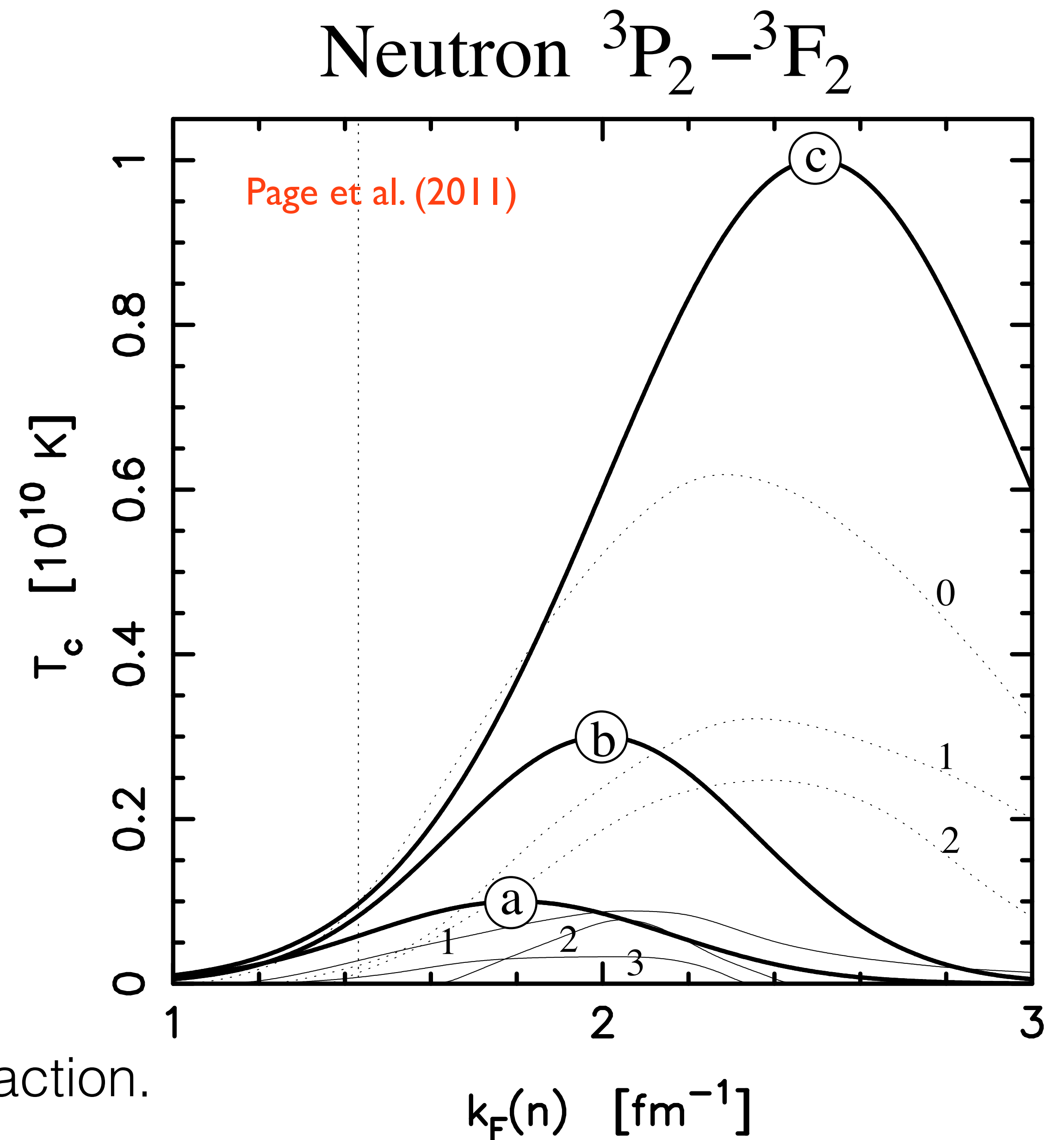
Bulgac, Carlson, Drut, Gandolfi, Forbes, Reddy ..

P-wave Triplet Pairing



S-wave interaction is repulsive at high density.

Attraction is in spin-1 channel due to P-wave interaction.



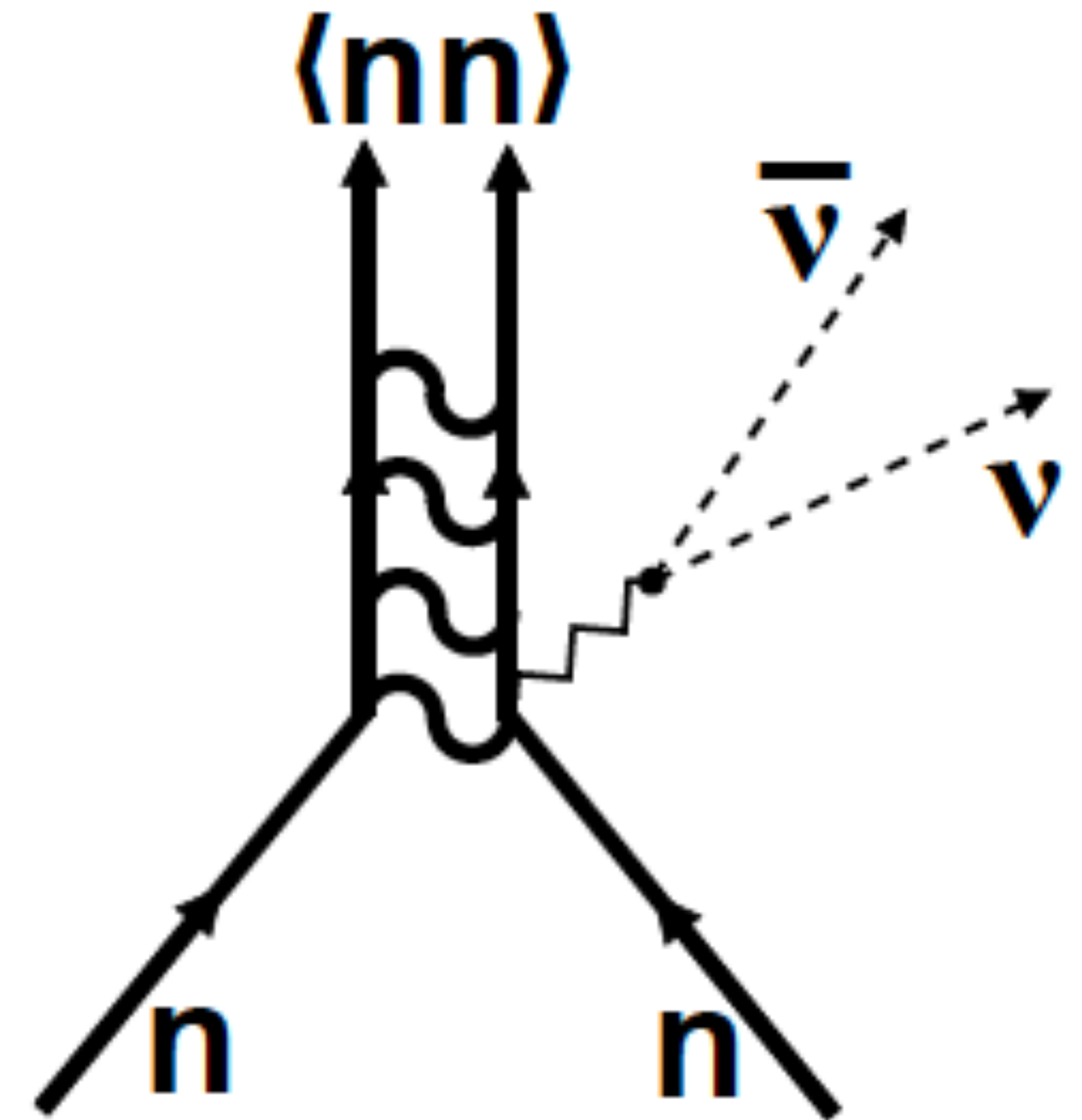
Neutrino Emission in Superfluid Neutron Matter

$$\epsilon_{\nu}^{nn} = \frac{G_F^2}{60 \pi^4} \int_0^\infty d\omega \omega^6 S_{\sigma}(\omega)$$

Near the critical temperature Cooper pairs form and dissociate due to thermal fluctuations.
(Recall pair-breaking discussed earlier.)

When two neutron quasi-particles combine to form a pair, the binding energy is radiated as neutrino- anti-neutrino pairs.

This process is called the PBF process - Pair Breaking and Formation.



Cooper pair
formation

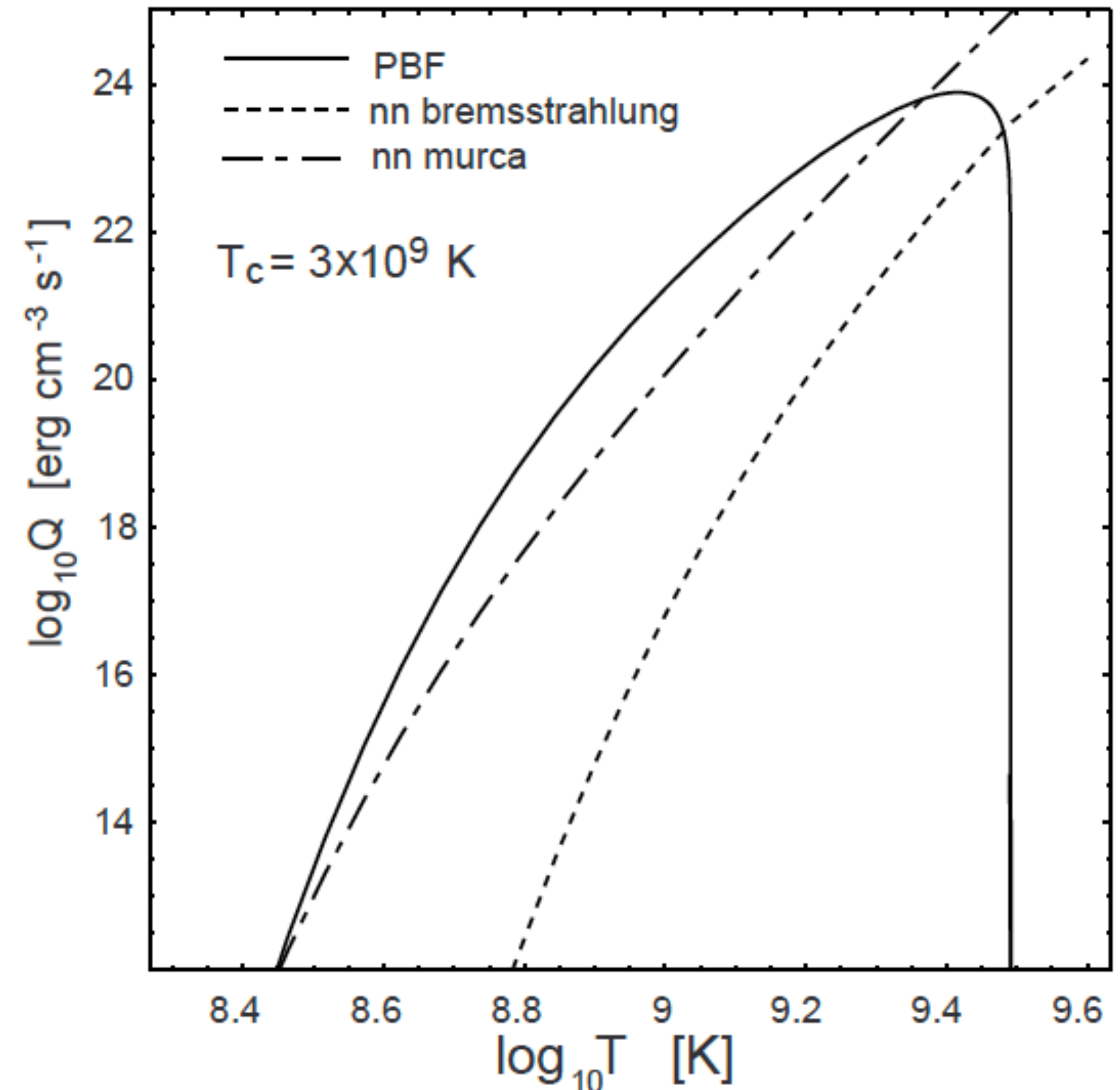
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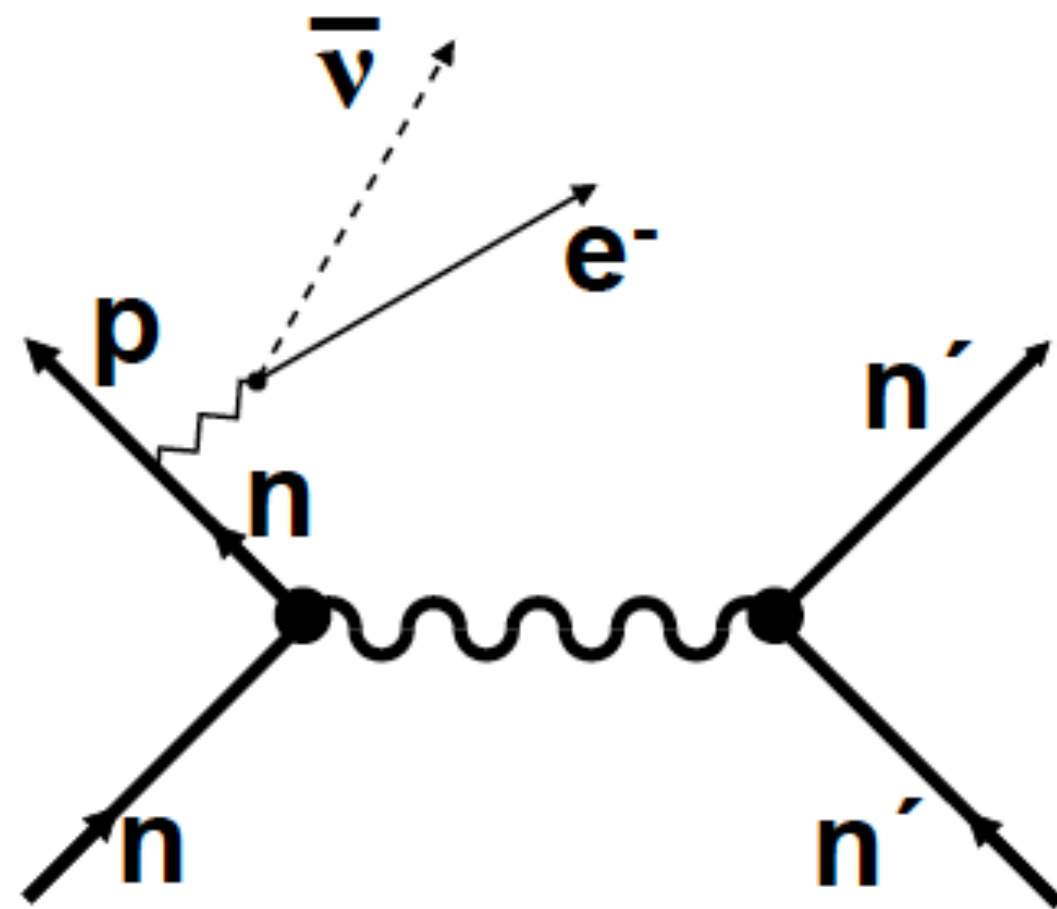
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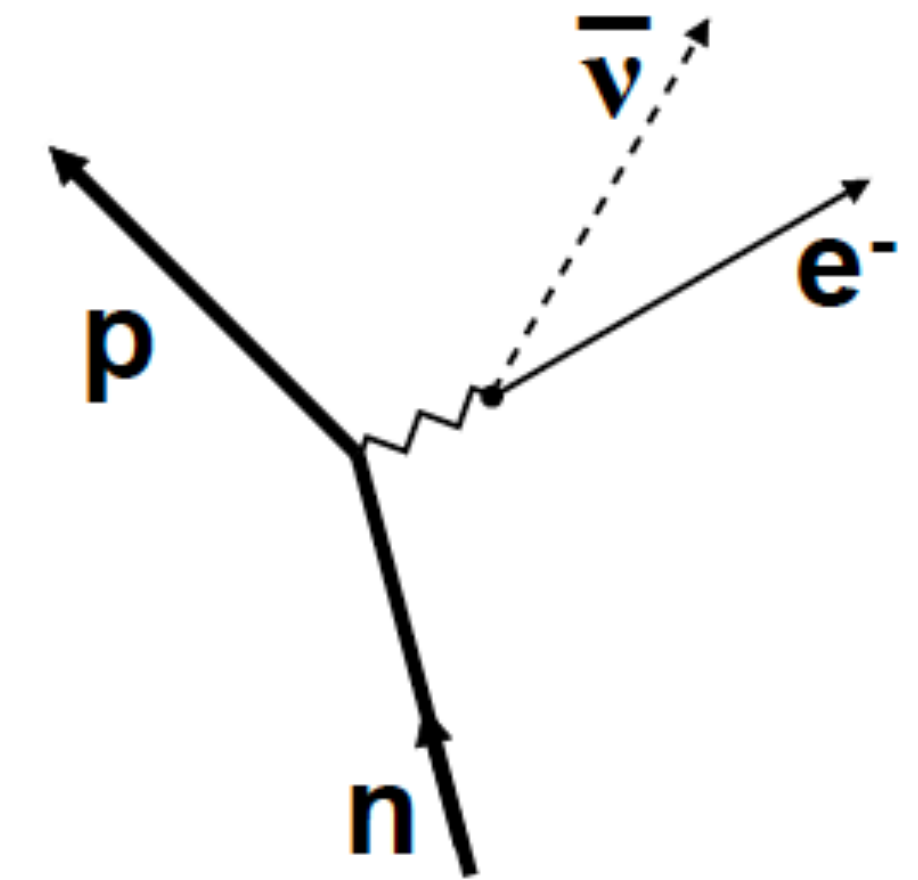
SLOW



Modified Urca

?

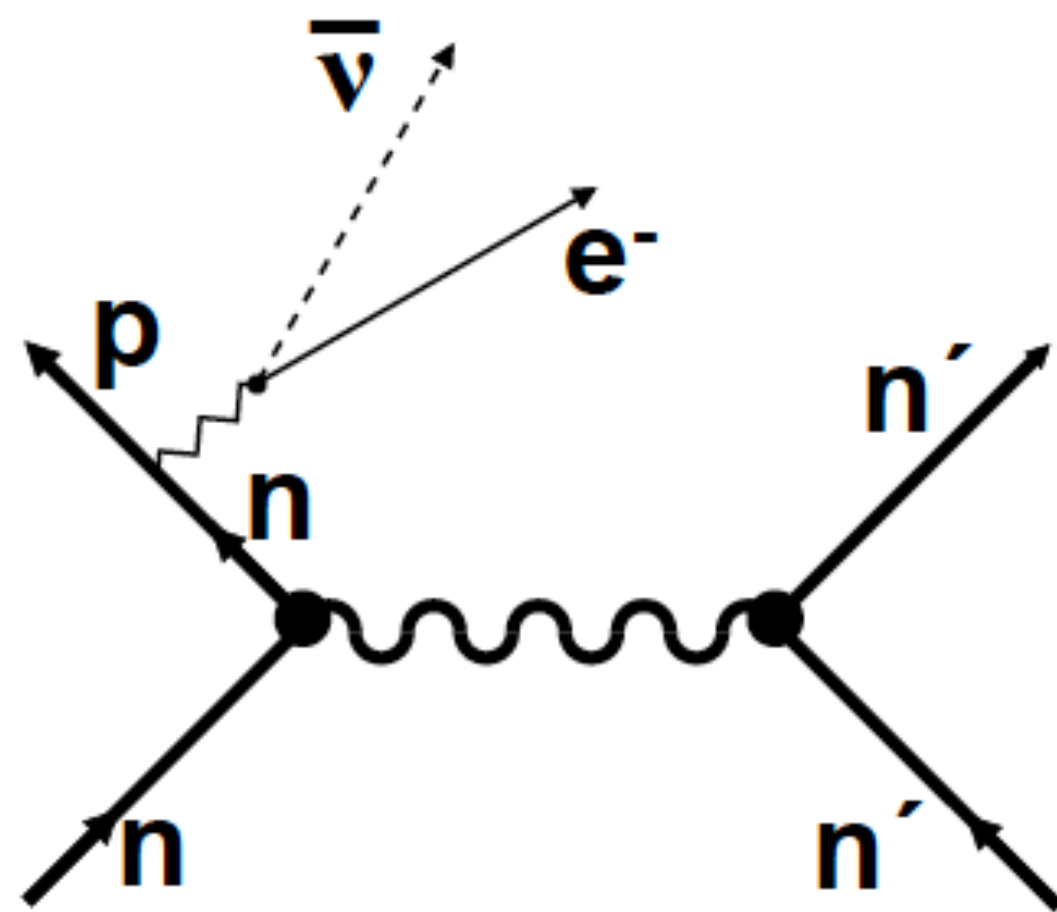
FAST



Direct Urca

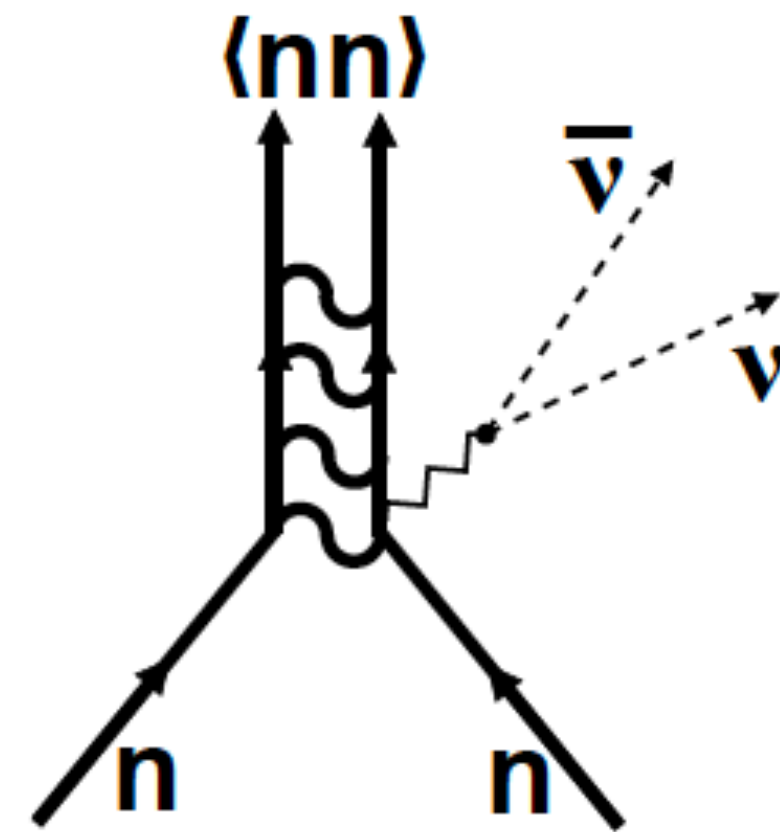
Neutrino Emission in Superfluid Neutron Matter

SLOW



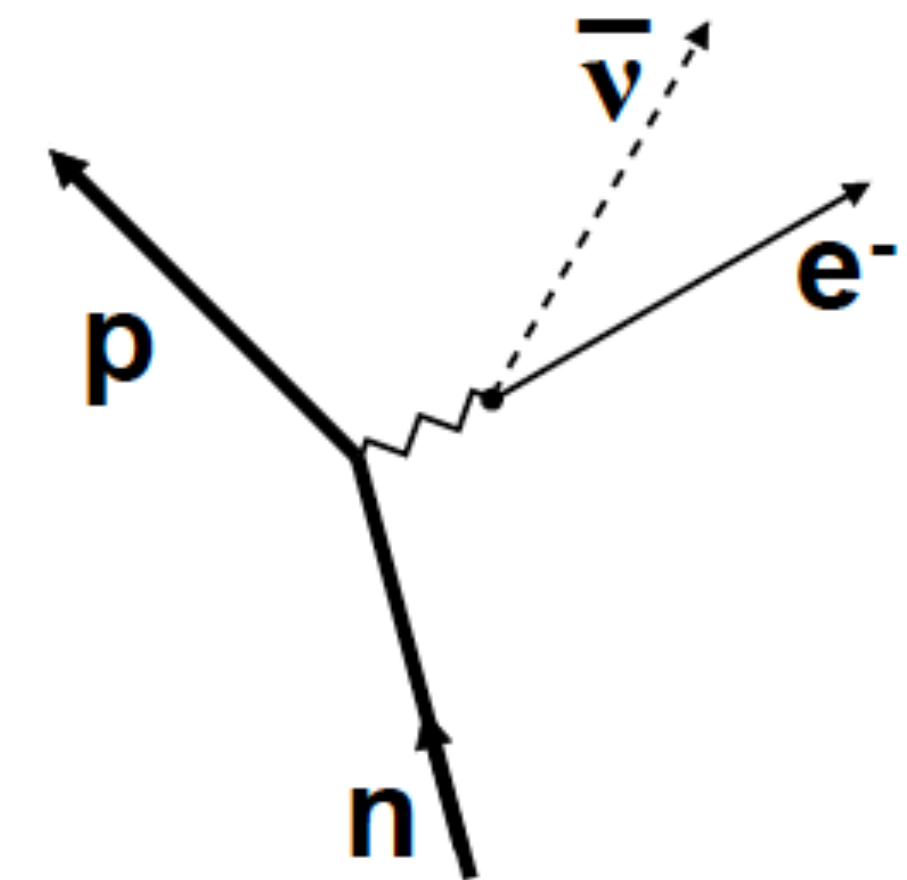
Modified Urca

MEDIUM



Cooper pair
formation
("PBF")

FAST



Direct Urca

Neutrino Emission Rates in Normal Nuclear Matter

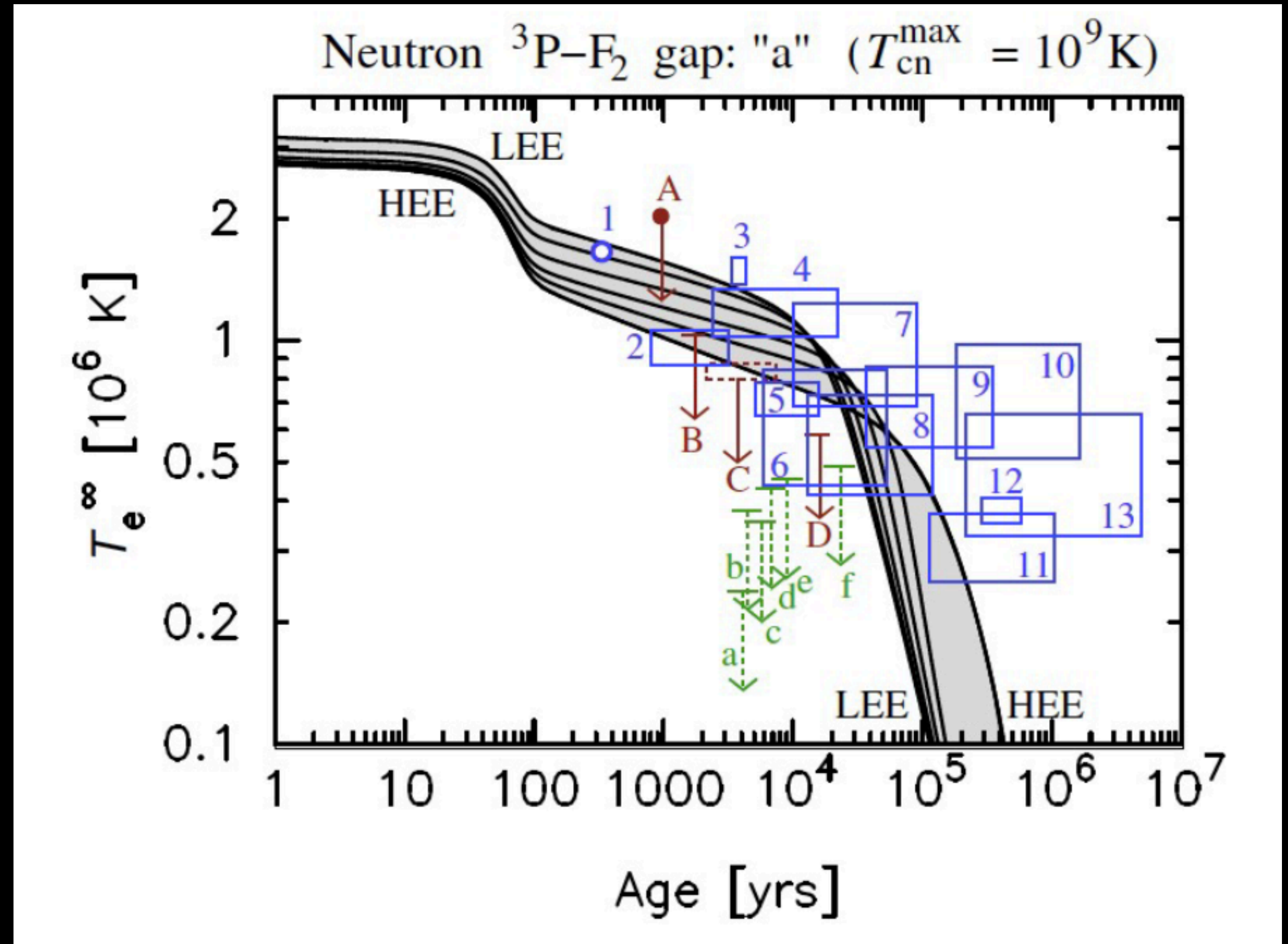
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Modified Urca cycle (proton branch)	$p + n \rightarrow p + p + e^- + \bar{\nu}_e$ $p + p + e^- \rightarrow p + n + \nu_e$	$\sim 10^{21} R T_9^8$	Slow
Bremsstrahlung	$n + n \rightarrow n + n + \nu + \bar{\nu}$ $n + p \rightarrow n + p + \nu + \bar{\nu}$ $p + p \rightarrow p + p + \nu + \bar{\nu}$	$\sim 10^{19} R T_9^8$	Slow
Direct Urca cycle	$n \rightarrow p + e^- + \bar{\nu}_e$ $p + e^- \rightarrow n + \nu_e$	$\sim 10^{27} R T_9^6$	Fast
Cooper pair formations	$n + n \rightarrow [nn] + \nu + \bar{\nu}$ $p + p \rightarrow [pp] + \nu + \bar{\nu}$	$\sim 5 \times 10^{21} R T_9^7$ $\sim 5 \times 10^{19} R T_9^7$	Medium

A Finely Tuned Model with Neutron Pairing

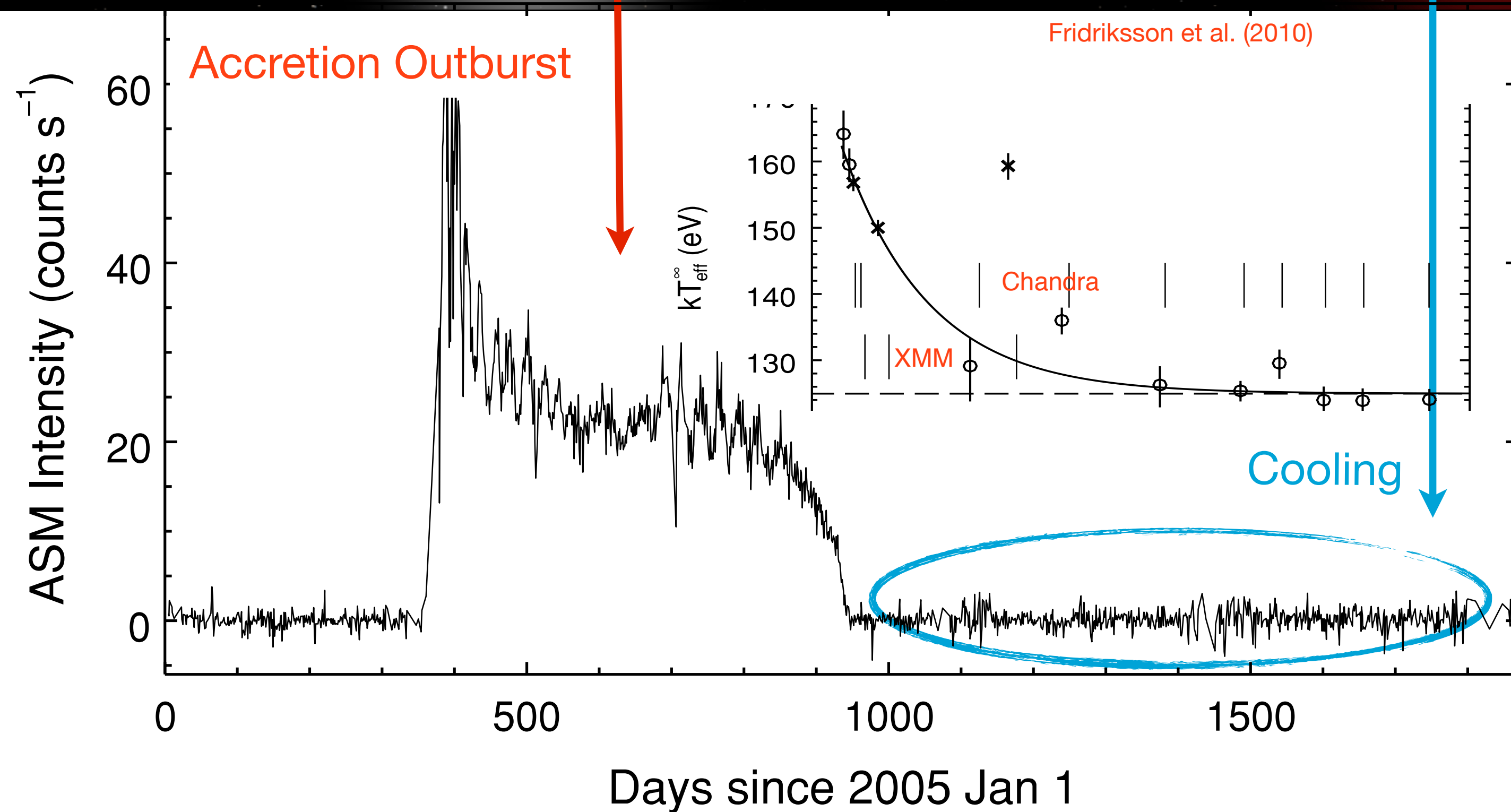
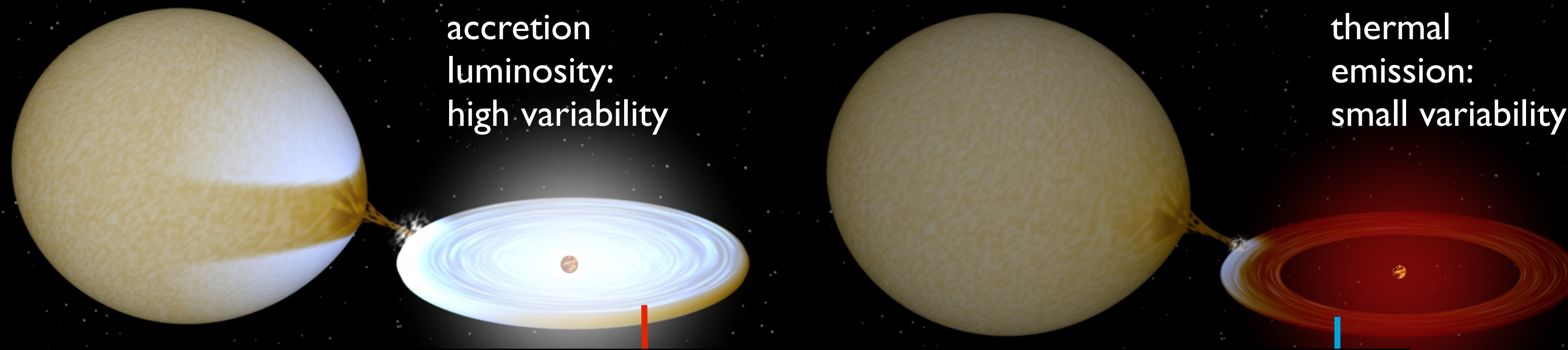
Minimal Cooling Paradigm D. Page et al. (2011)

Neutron stars in their mid-life are roughly compatible with a finely tuned model of neutrinos cooling with 3P_2 neutron superfluidity.

The situation (in my opinion) suggests that there is something missing.

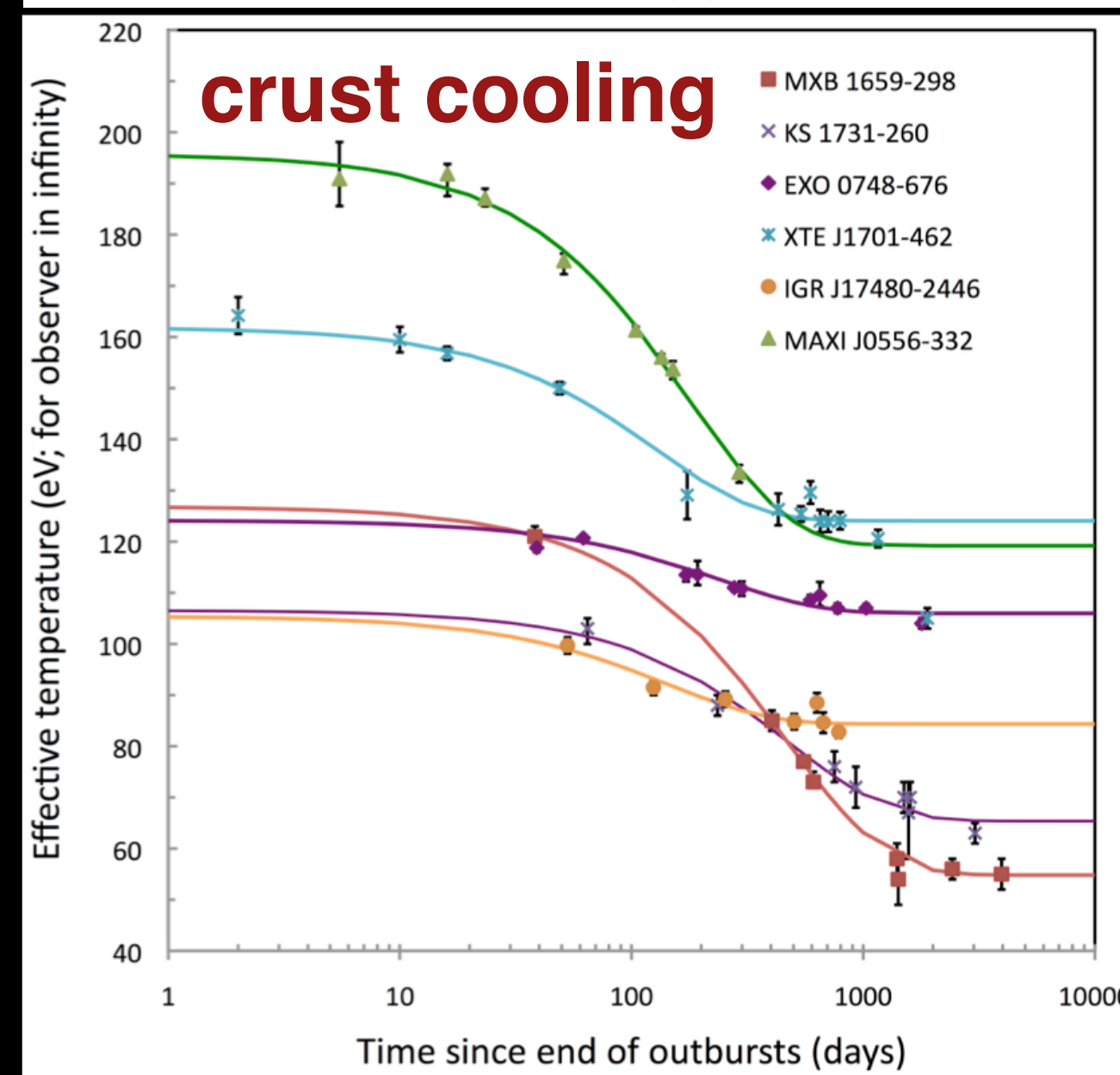
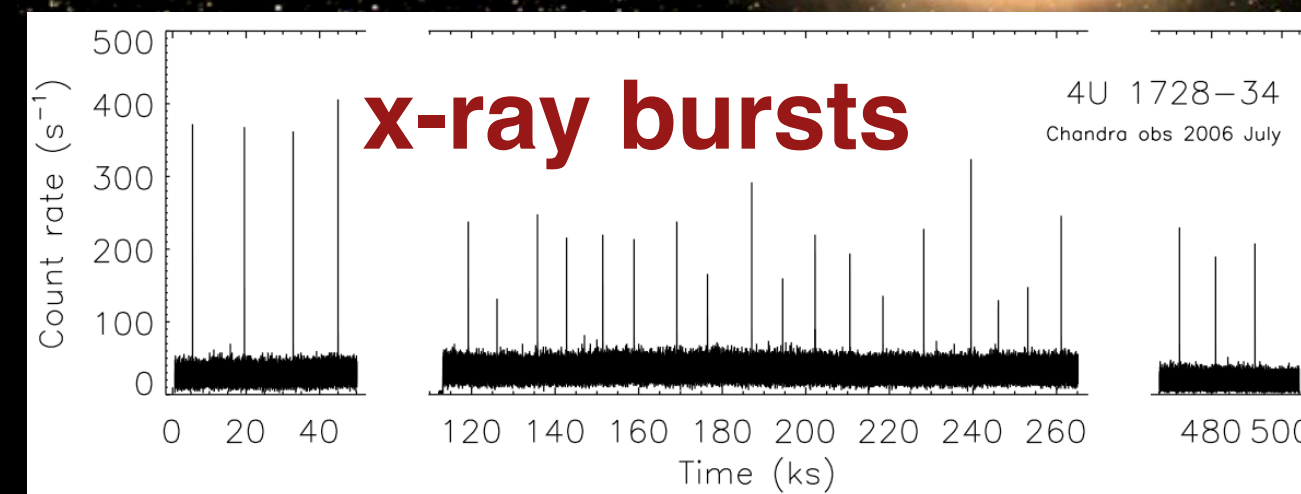
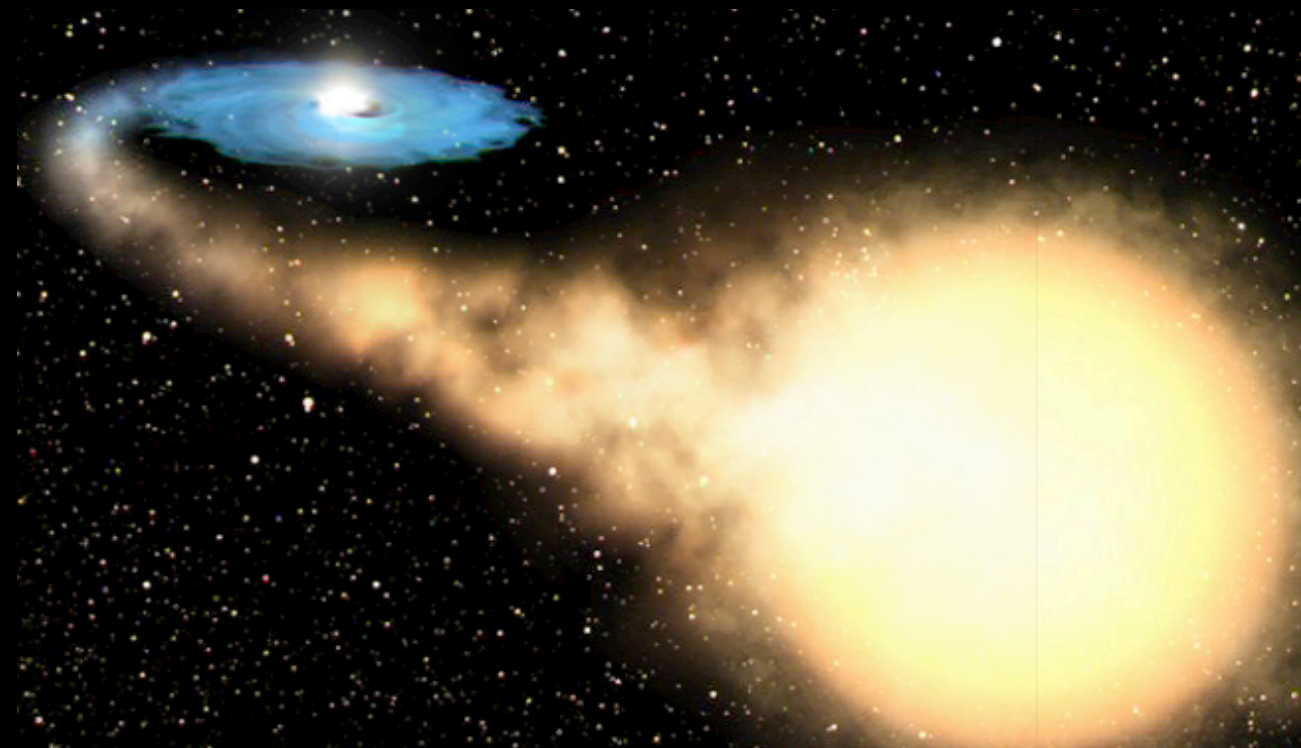


Transiently Accreting Neutron Stars



Physical Processes in Accreting Neutron Stars

- Accreting neutron stars host phenomena that uniquely probe the physics of its ultra dense interior.
- It is a data driven field.
- Interpreting this data requires a coordinated effort that combines theory, experiment and observations. JINA-CEE has played a key role.



density (g/cm^3)

depth (m)

0.1

1

10^5

10^8

10^{11}

10^{13}

10^{14}

envelope

H/He
burning
r-p process

^{12}C burning

e^- capture
 β^- decay

n emission
& capture
fusion

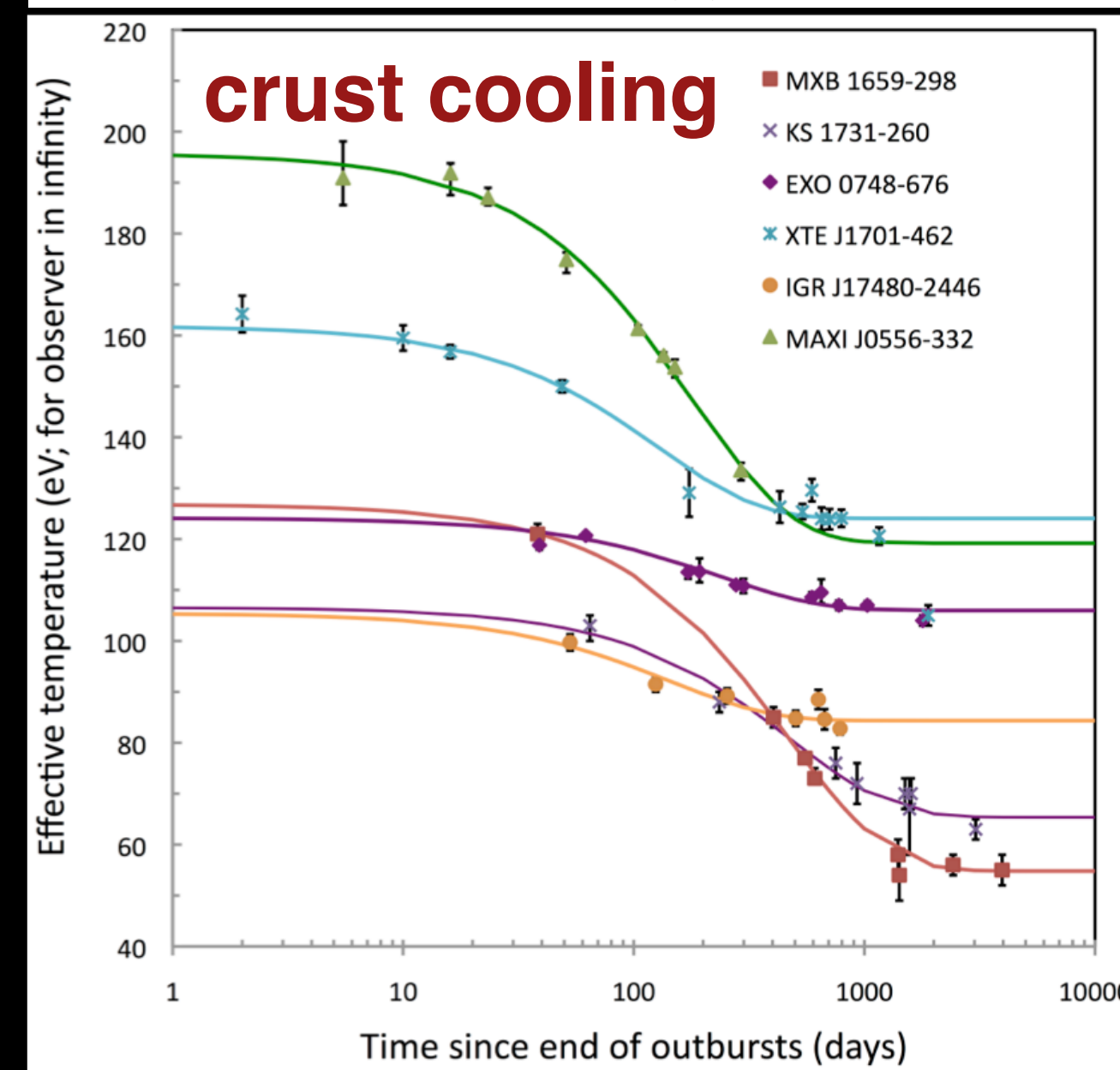
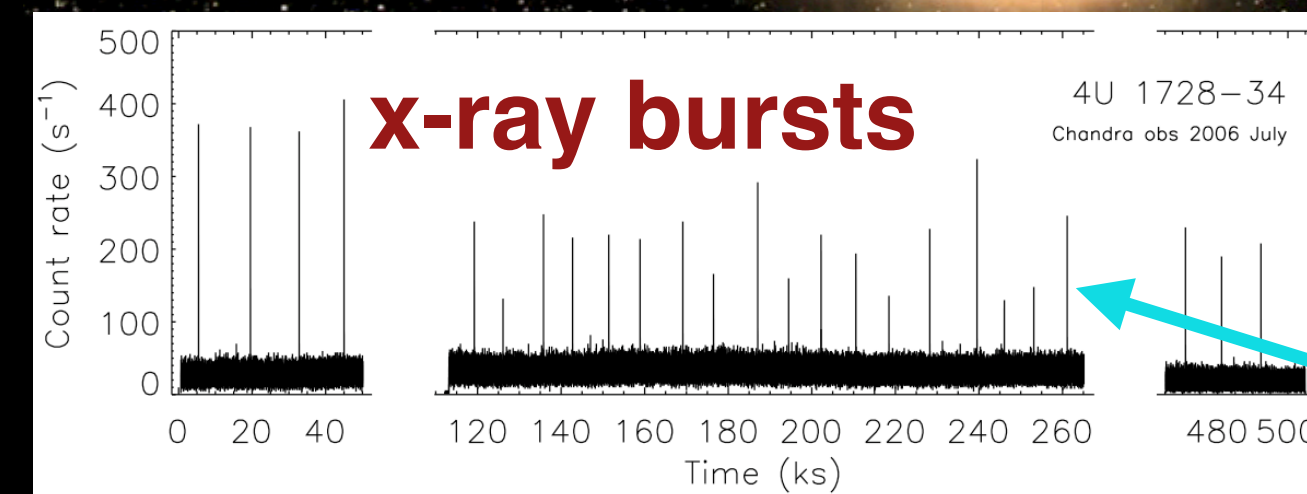
neutron drip
superfluid
neutrons

nuclear pasta

core

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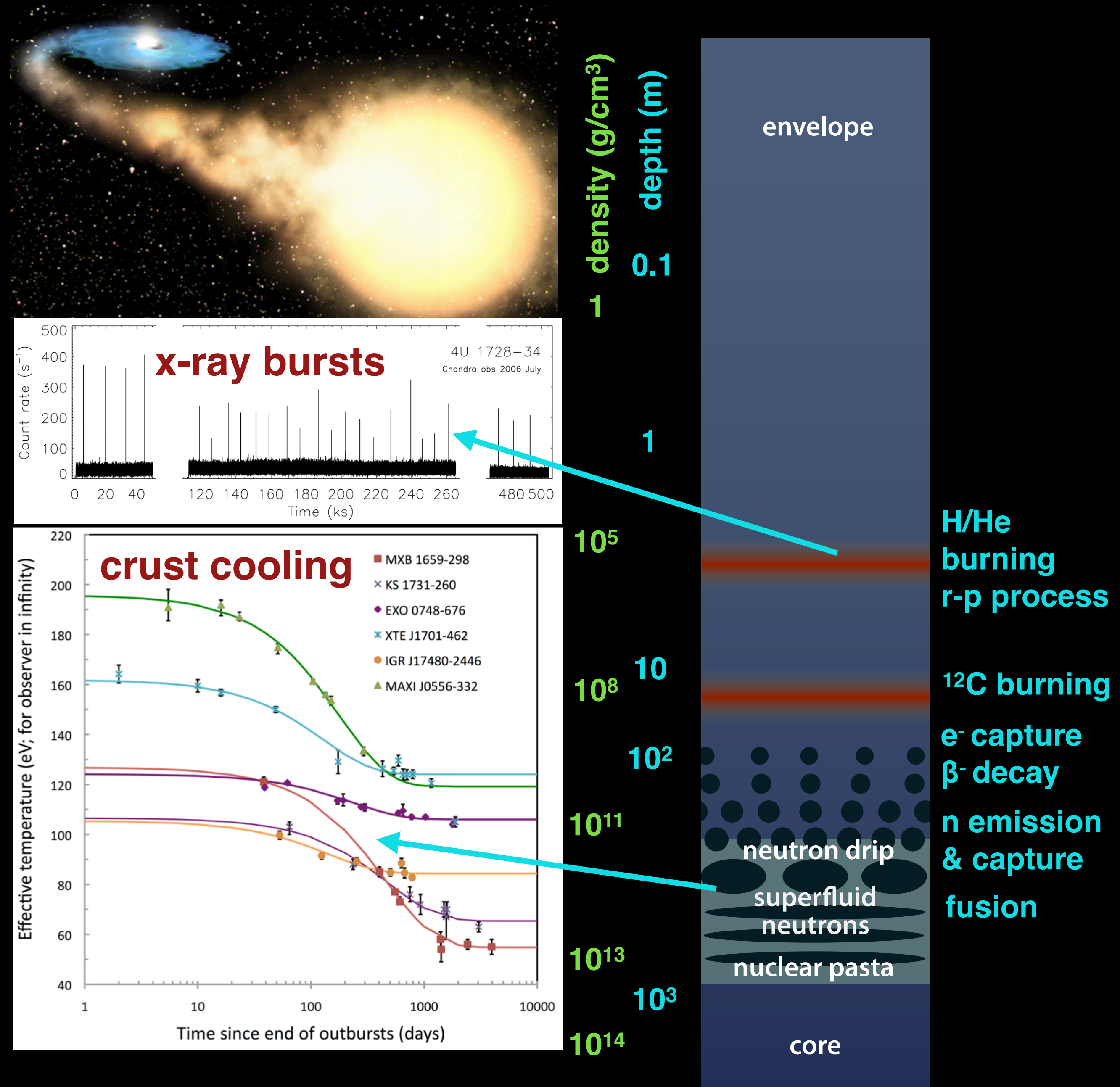
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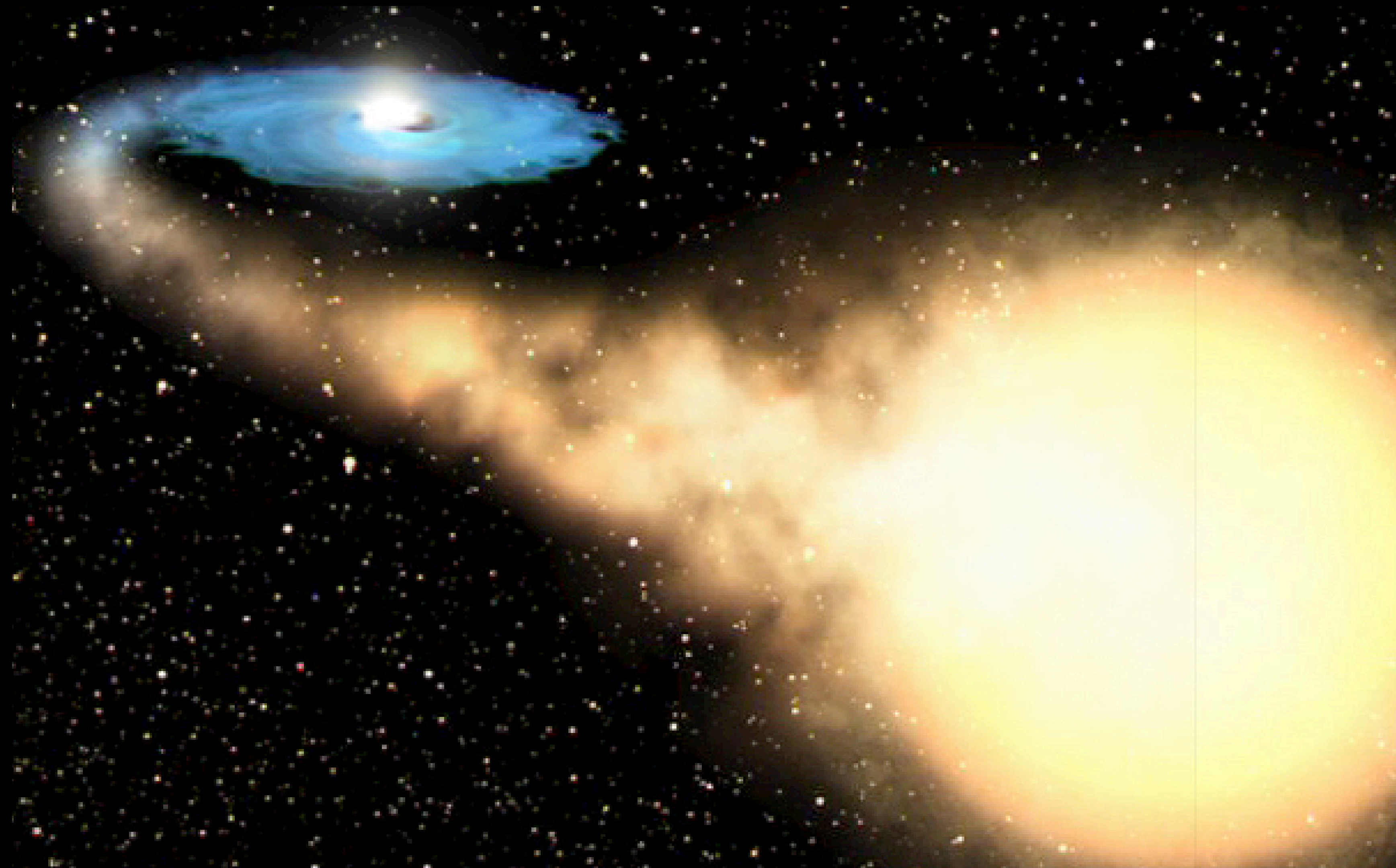
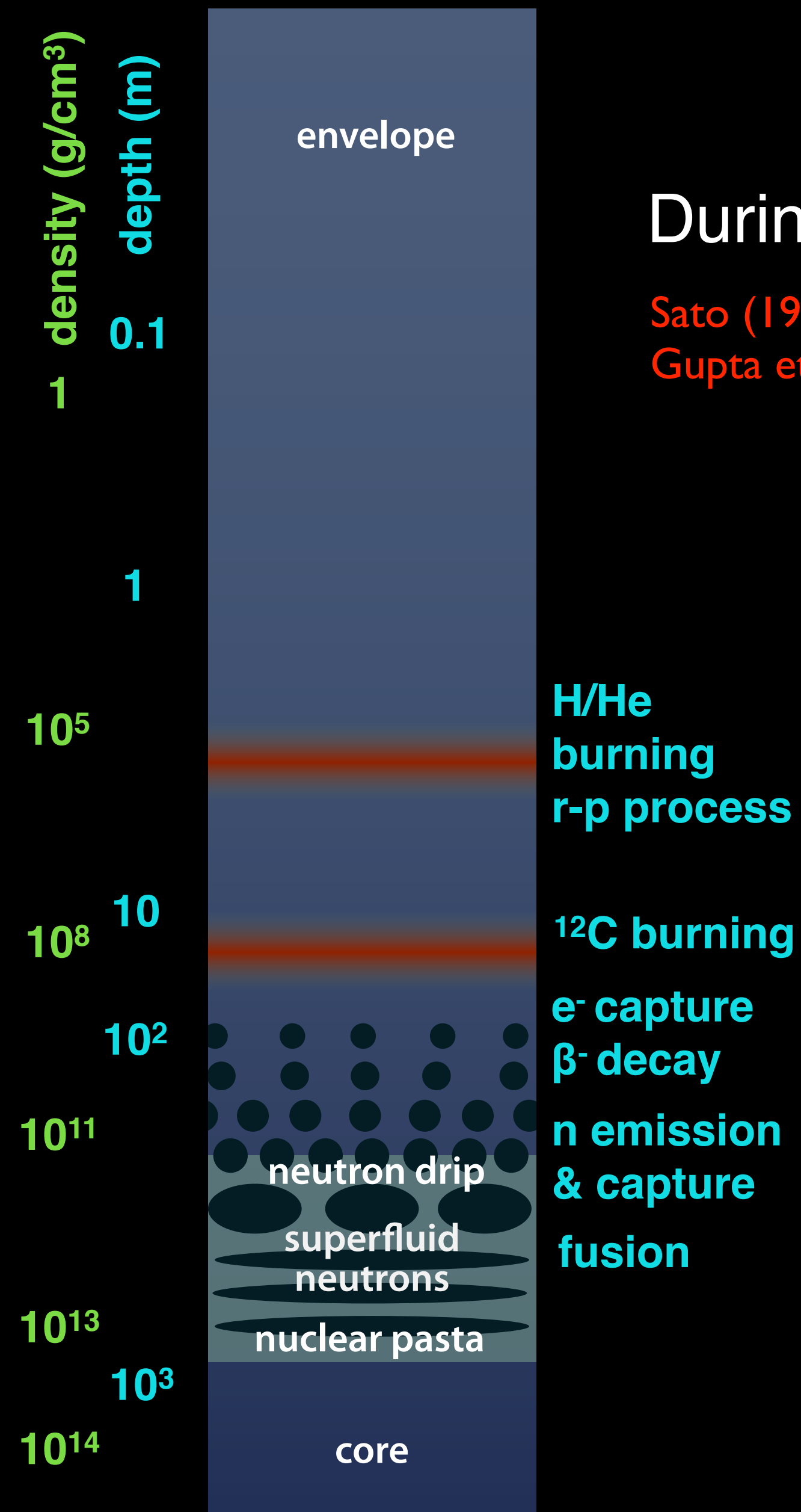
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Deep Crustal Heating

During accretion nuclear reactions release: $\sim 2\text{-}4 \text{ MeV / nucleon}$

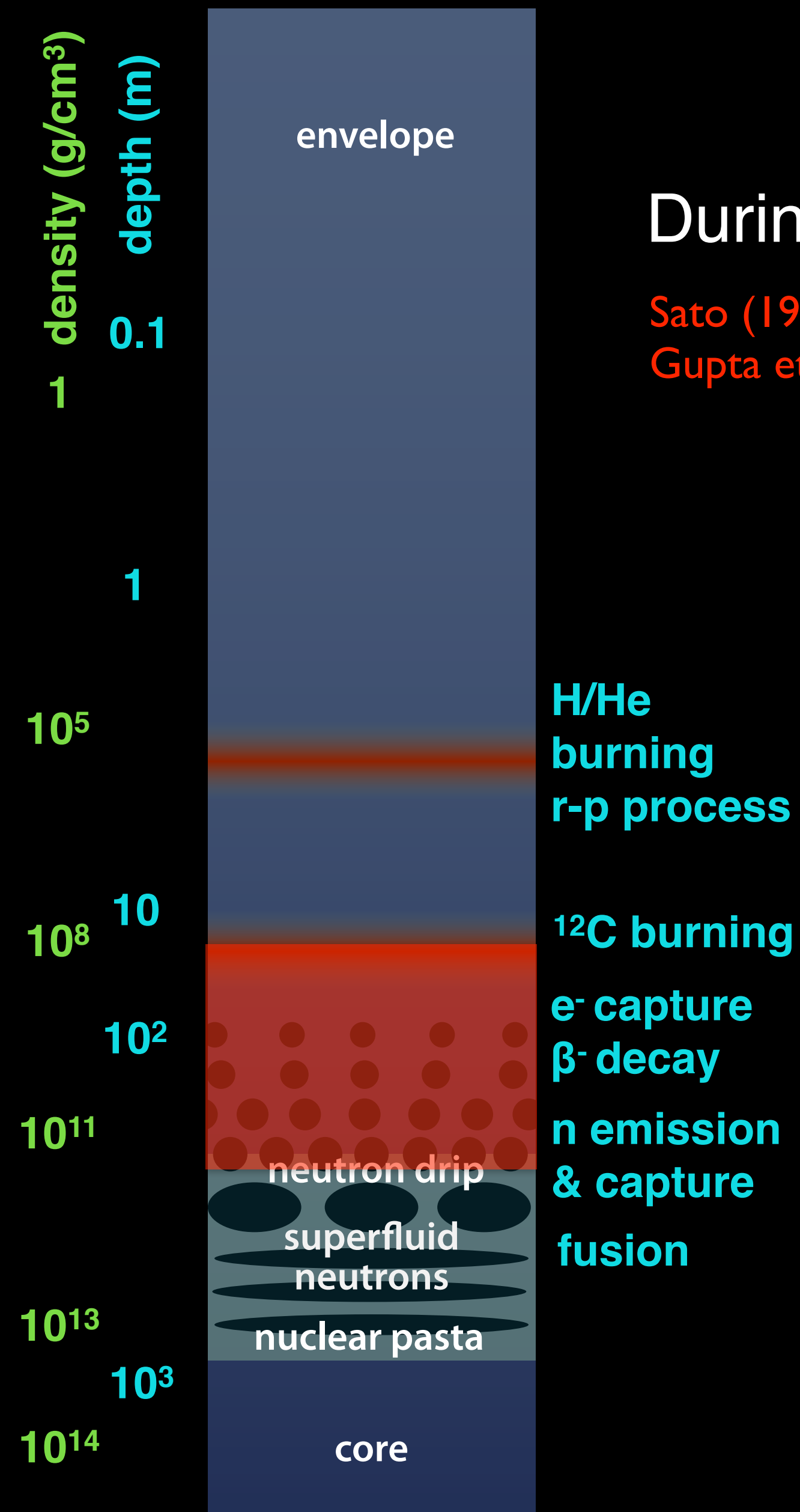
Sato (1974), Haensel & Zdunik (1990), Brown, Bildsten Rutledge (1998)
Gupta et al (2007,2011).



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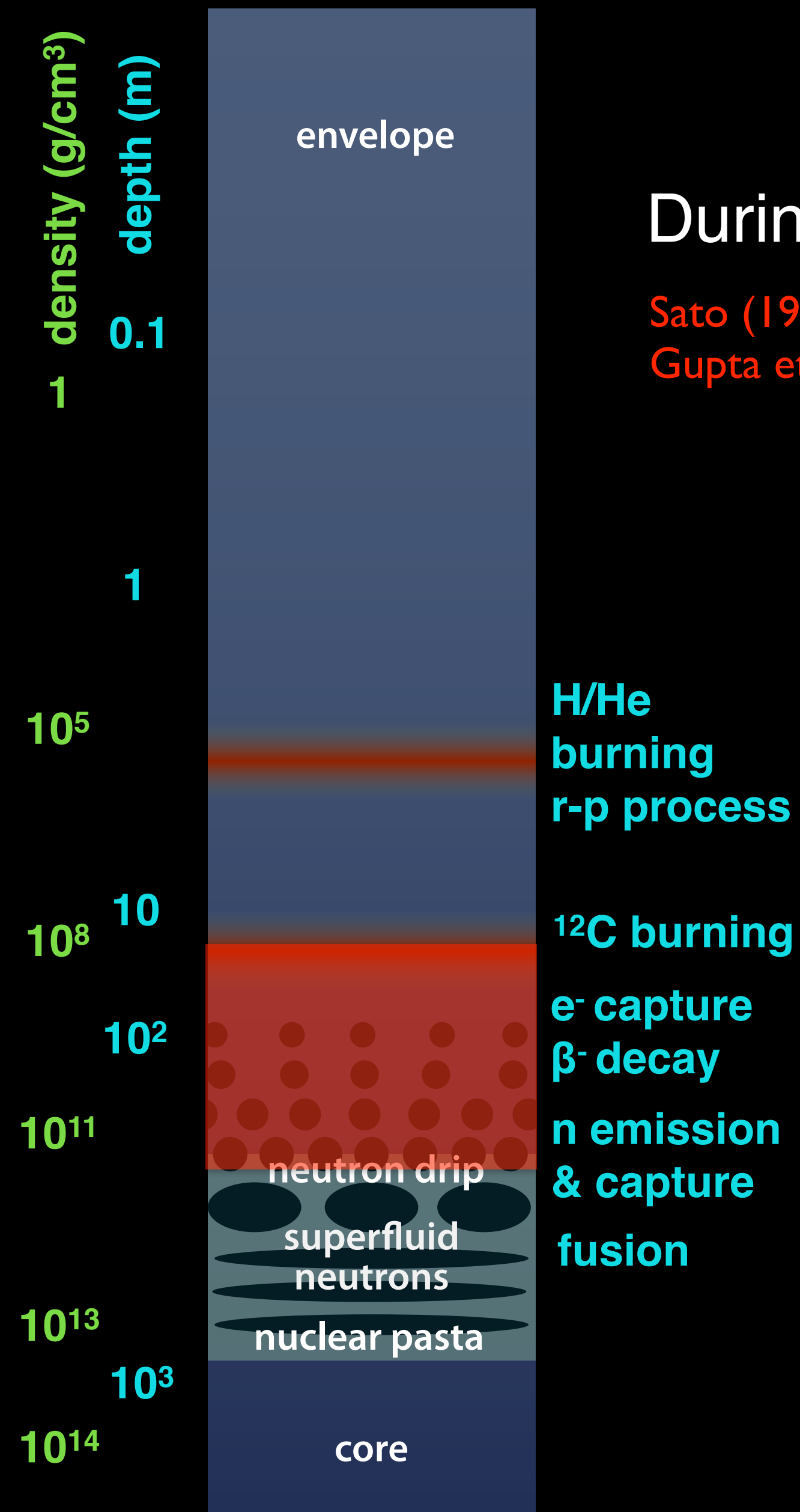
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Warms up old neutron stars

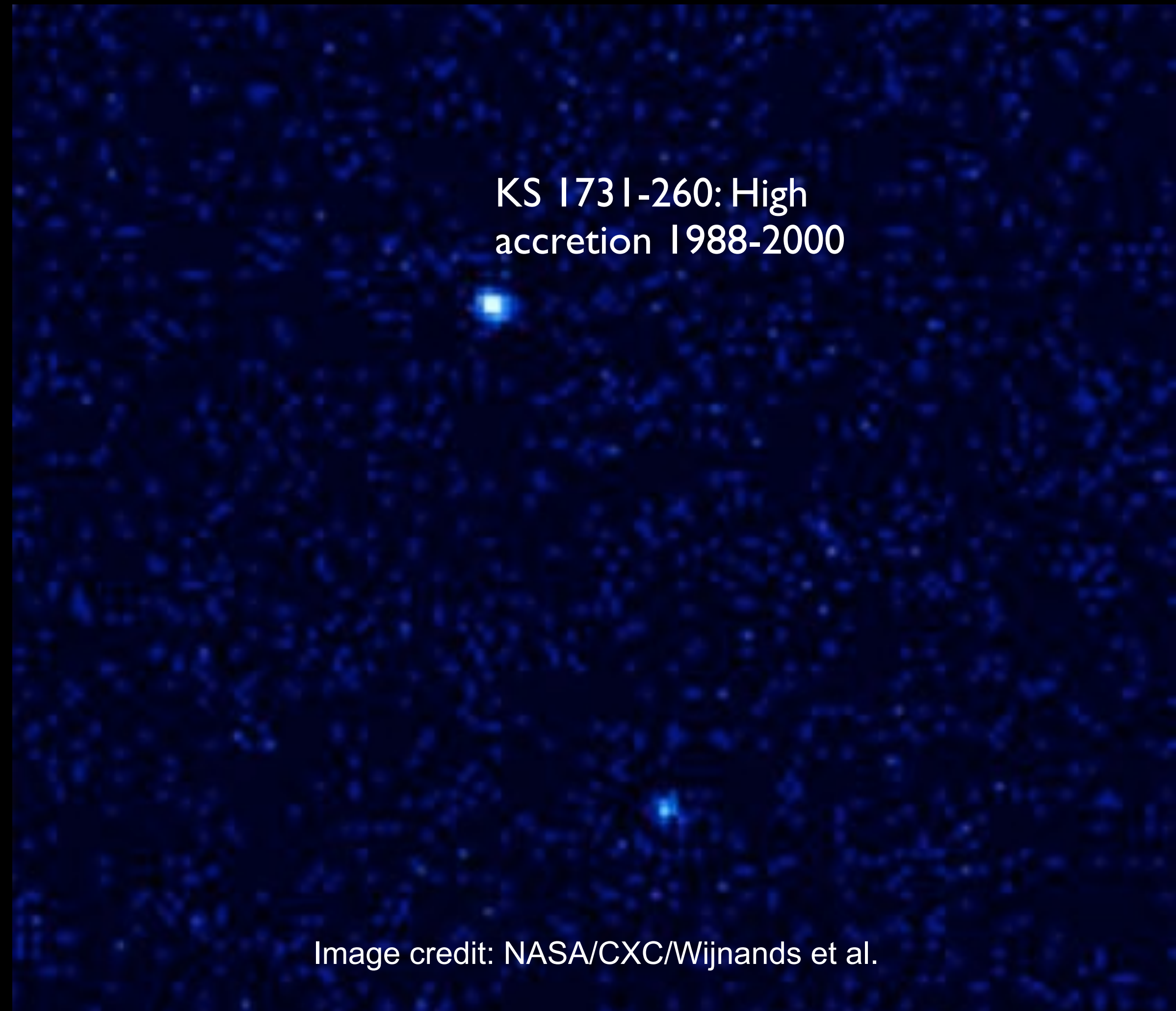


Image credit: NASA/CXC/Wijnands et al.

Cooling Post Accretion

All known Quasi-persistent sources show cooling after accretion

- After a period of intense accretion the neutron star surface cools on a time scale of ~ 1000 days.
- This relaxation was first discovered in 2001 and 6 sources have been studied to date.
- Expected rate of detecting new sources $\sim 1/\text{year}$.

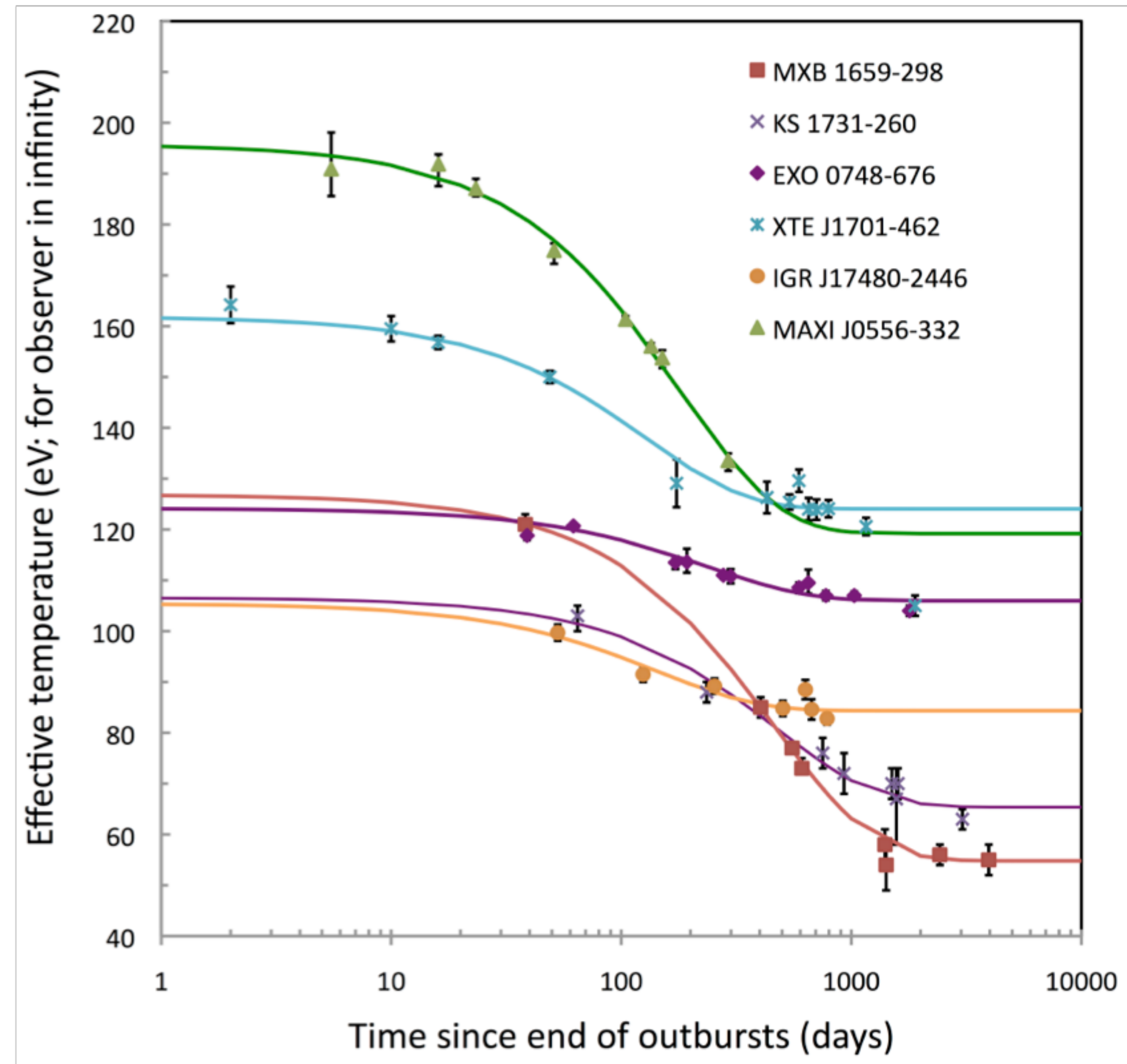


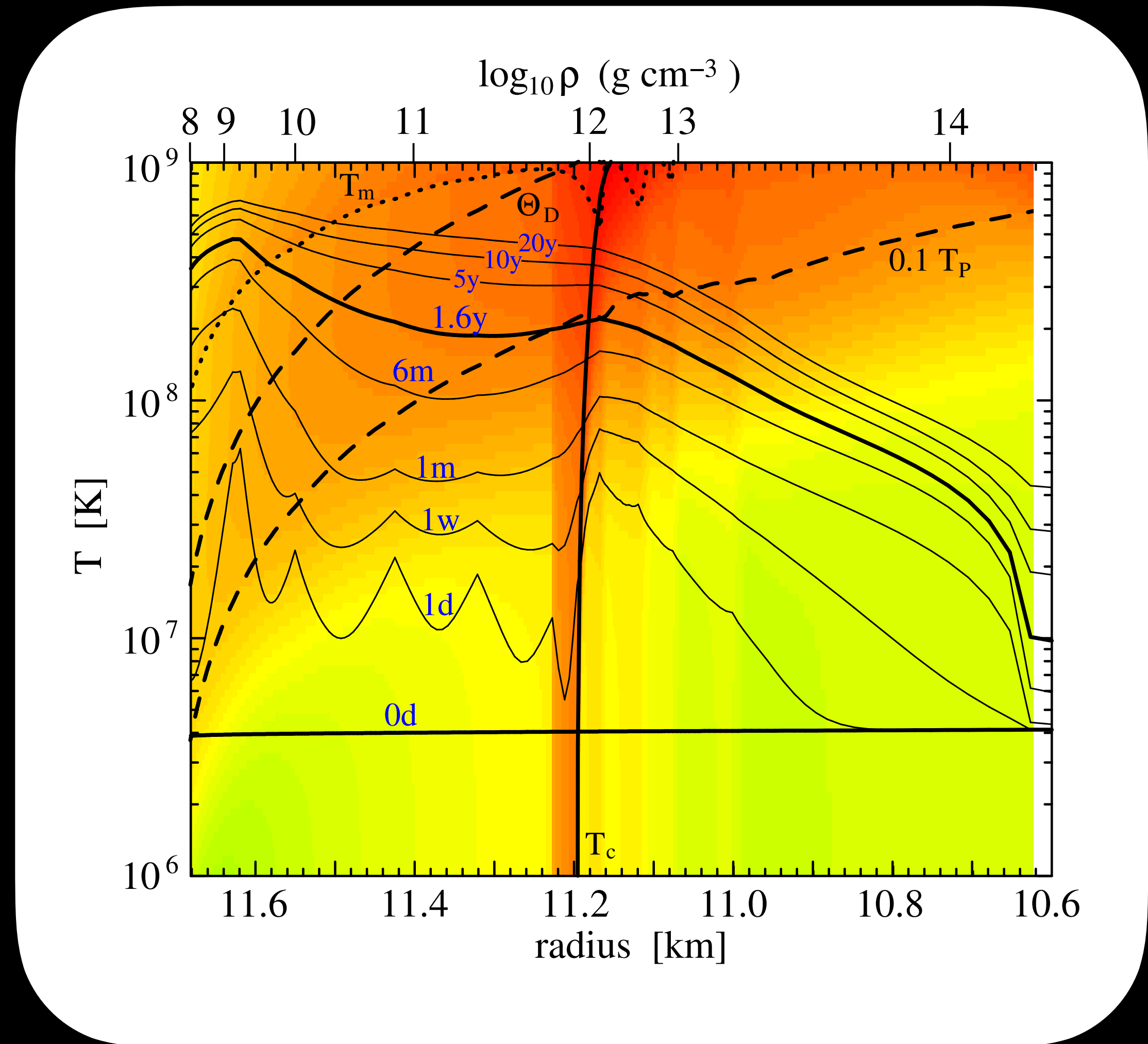
Figure from Rudy Wijnands (2013)

Thermal Evolution of the Crust

Temperature profile in the crust depends on the duration of the accretion phase.

When accretion ends heat flows into the core and is radiated away as neutrinos.

Timescale for cooling is set by the heat diffusion time.

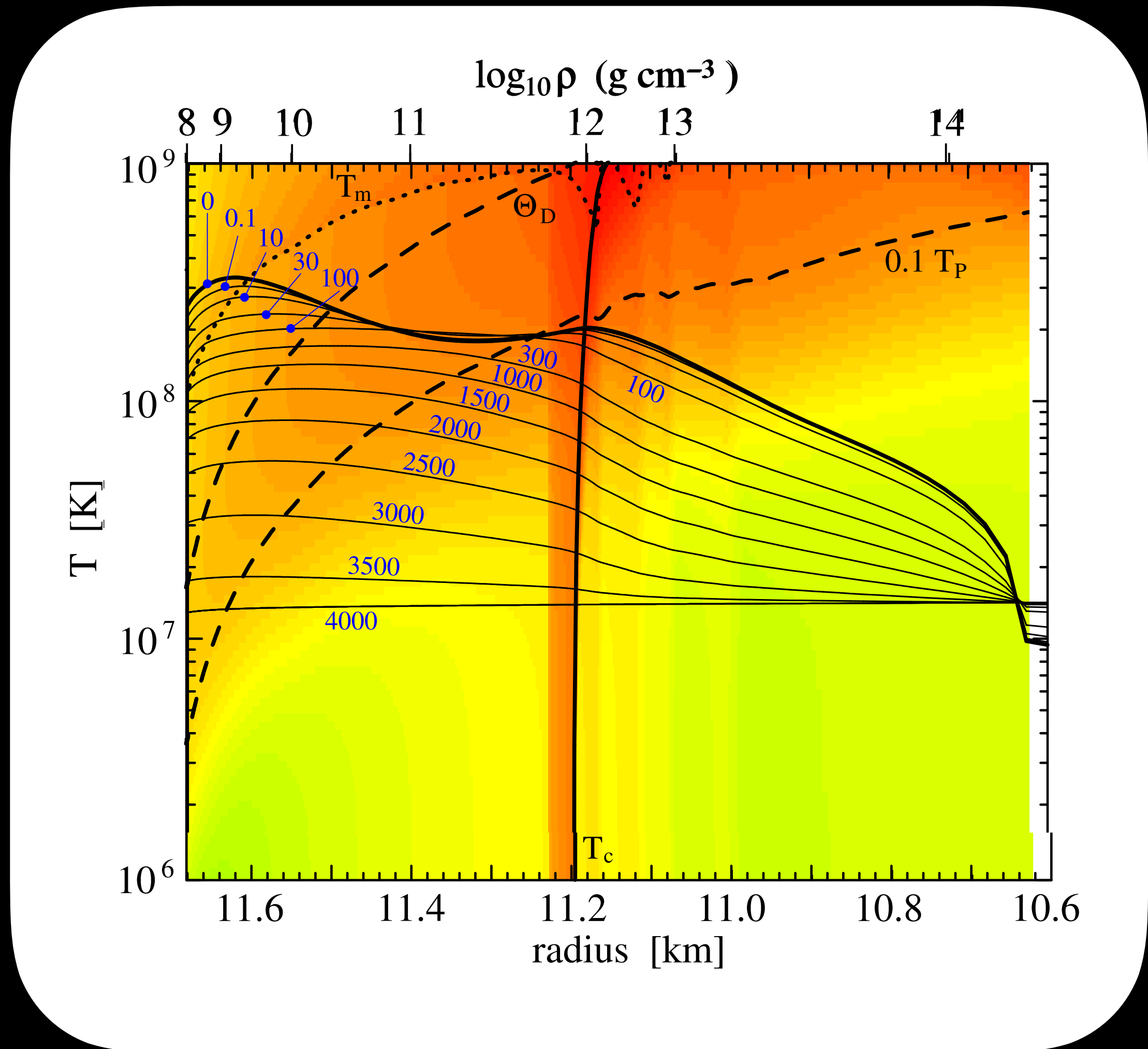


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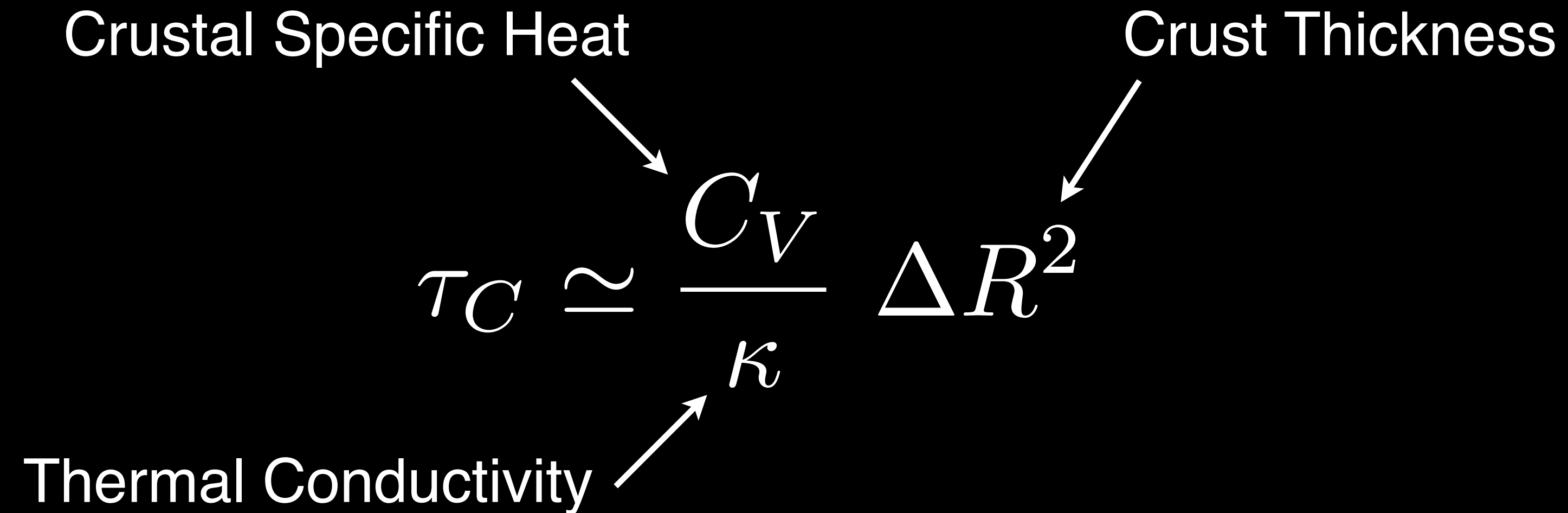
Connecting to Crust Microphysics

Crustal Specific Heat

Crust Thickness

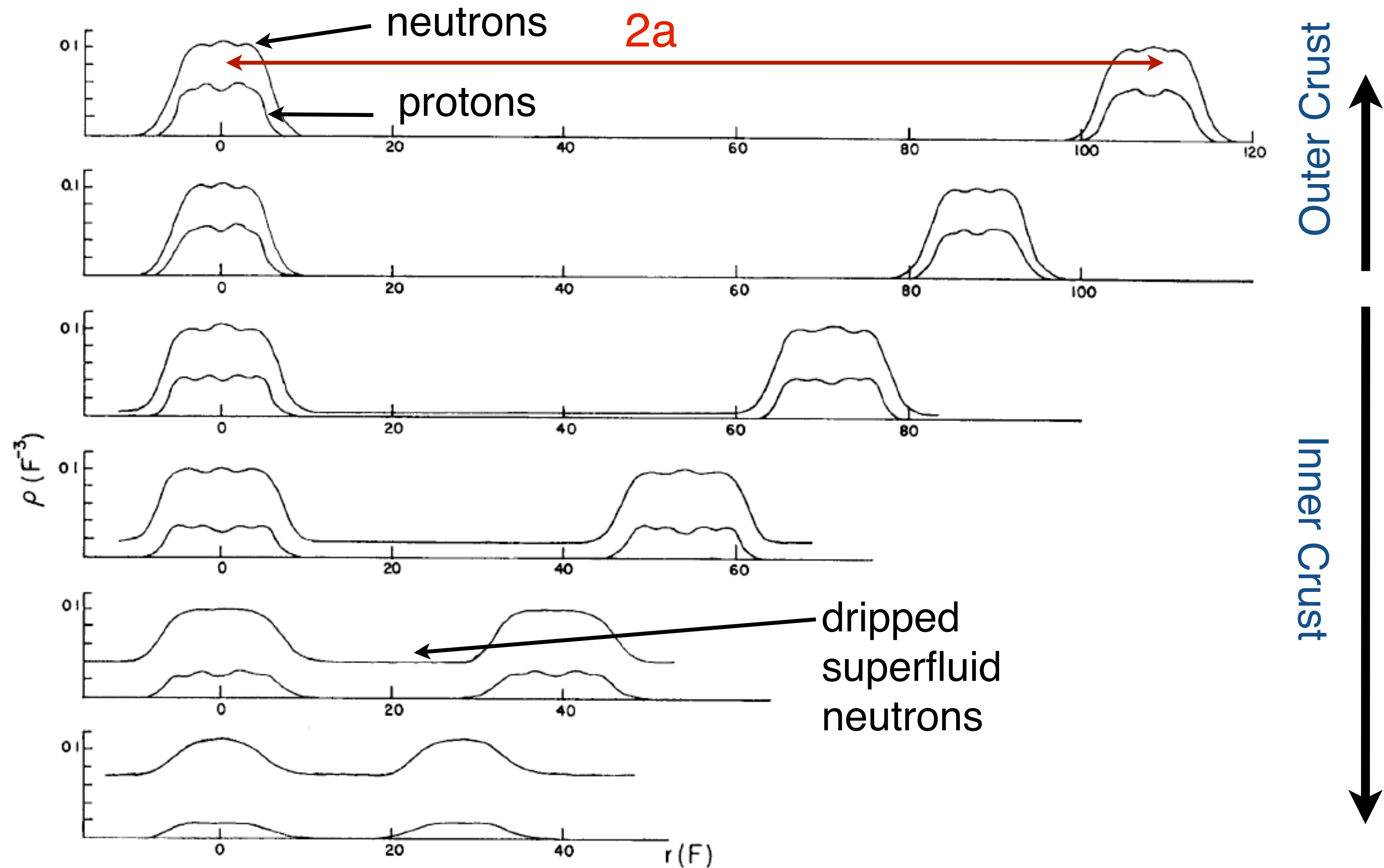
$$\tau_C \simeq \frac{C_V}{\kappa} \Delta R^2$$

Thermal Conductivity



- Observed timescales are short.
- Requires small specific heat and large thermal conductivity.
- Favors a solid (with small impurity fraction) and superfluid inner crust.

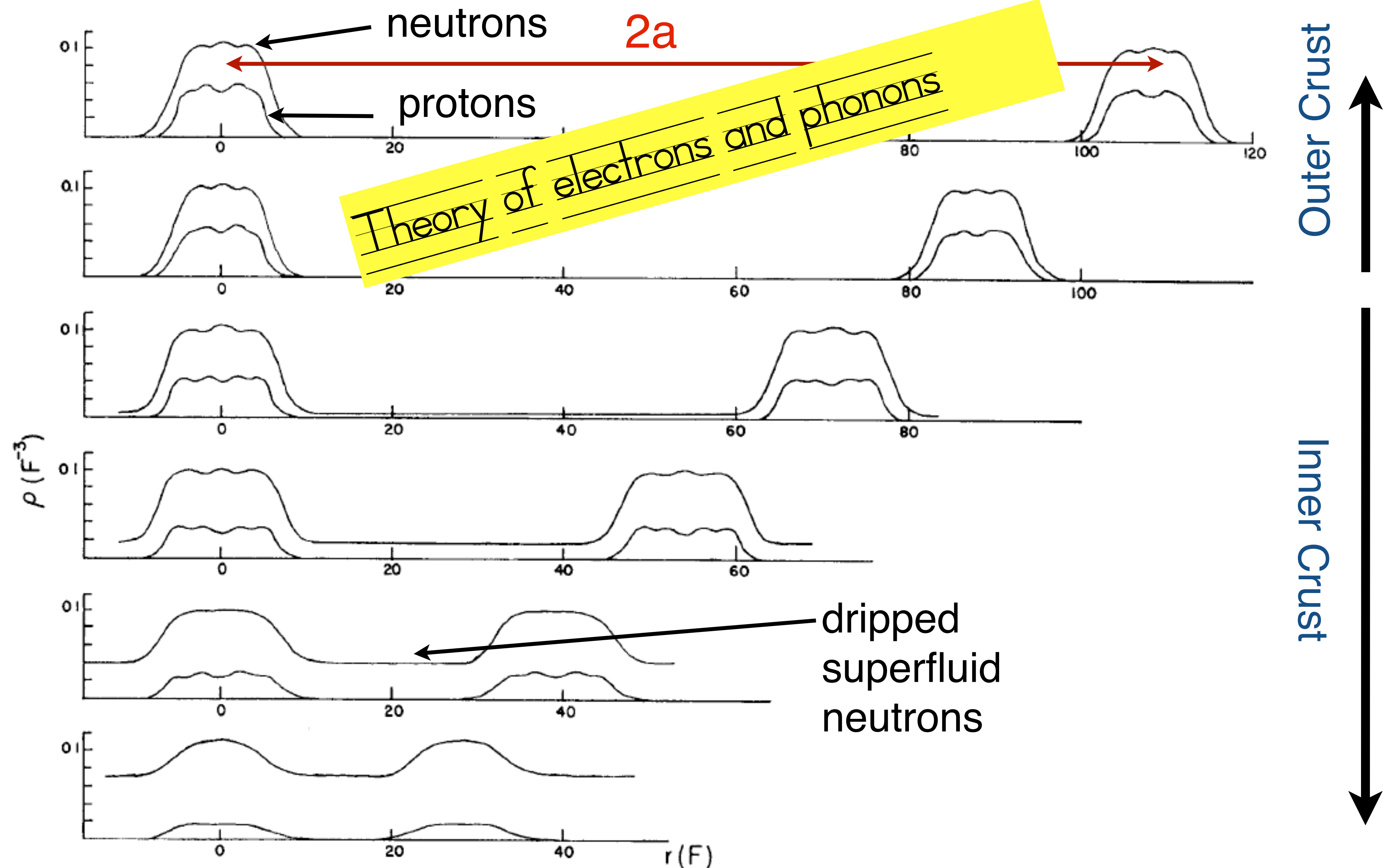
Low Energy Excitations in the Crust



Baym Pethick & Sutherland (1971)

Negele & Vautherin (1973)

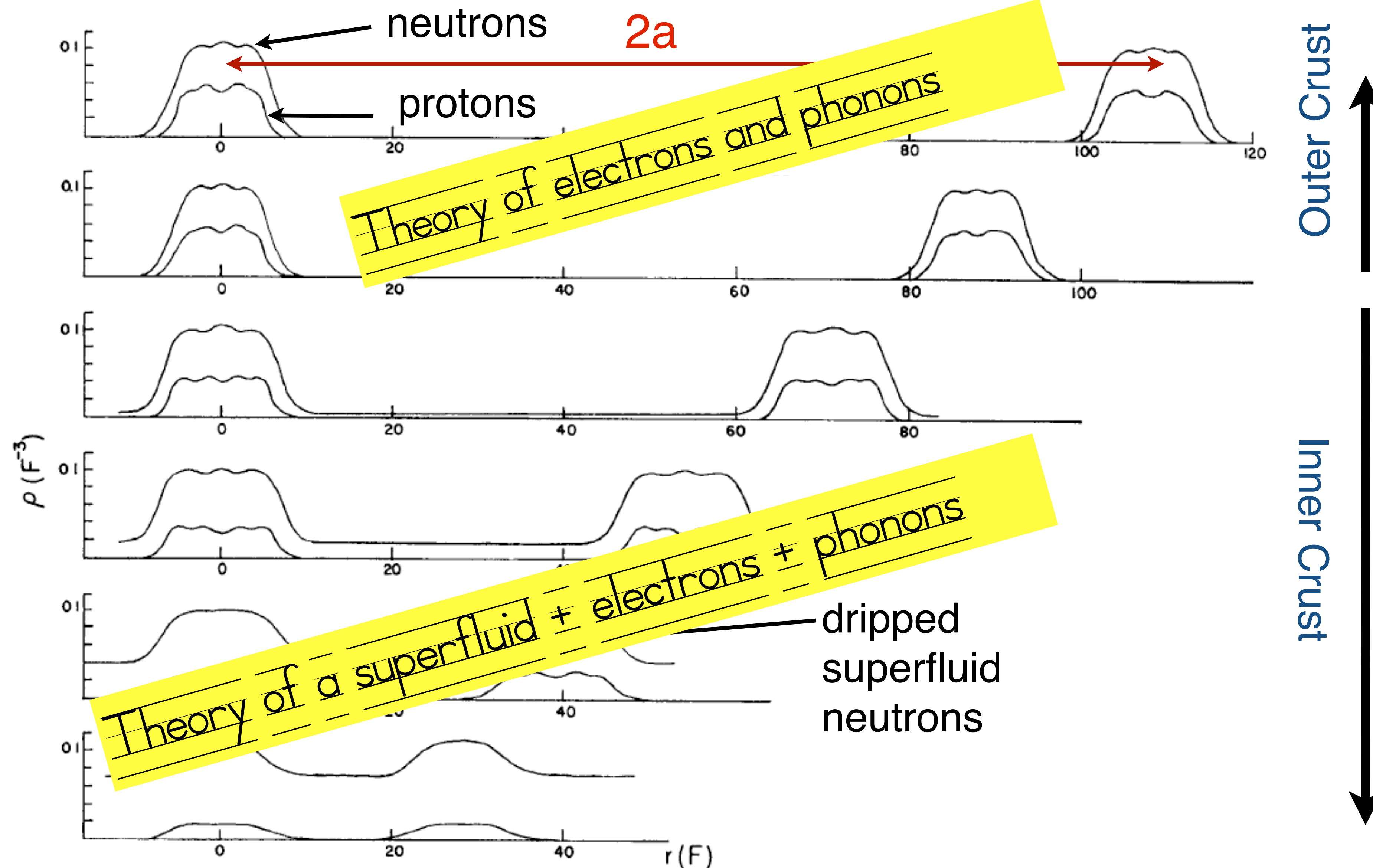
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Low Energy Excitations in the Crust



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Phonons in the Inner Crust



Proton (clusters) move collectively on lattice sites.
Displacement is a good collective coordinate.

Neutron superfluid: Goldstone excitation is the
fluctuation of the phase of the condensate.

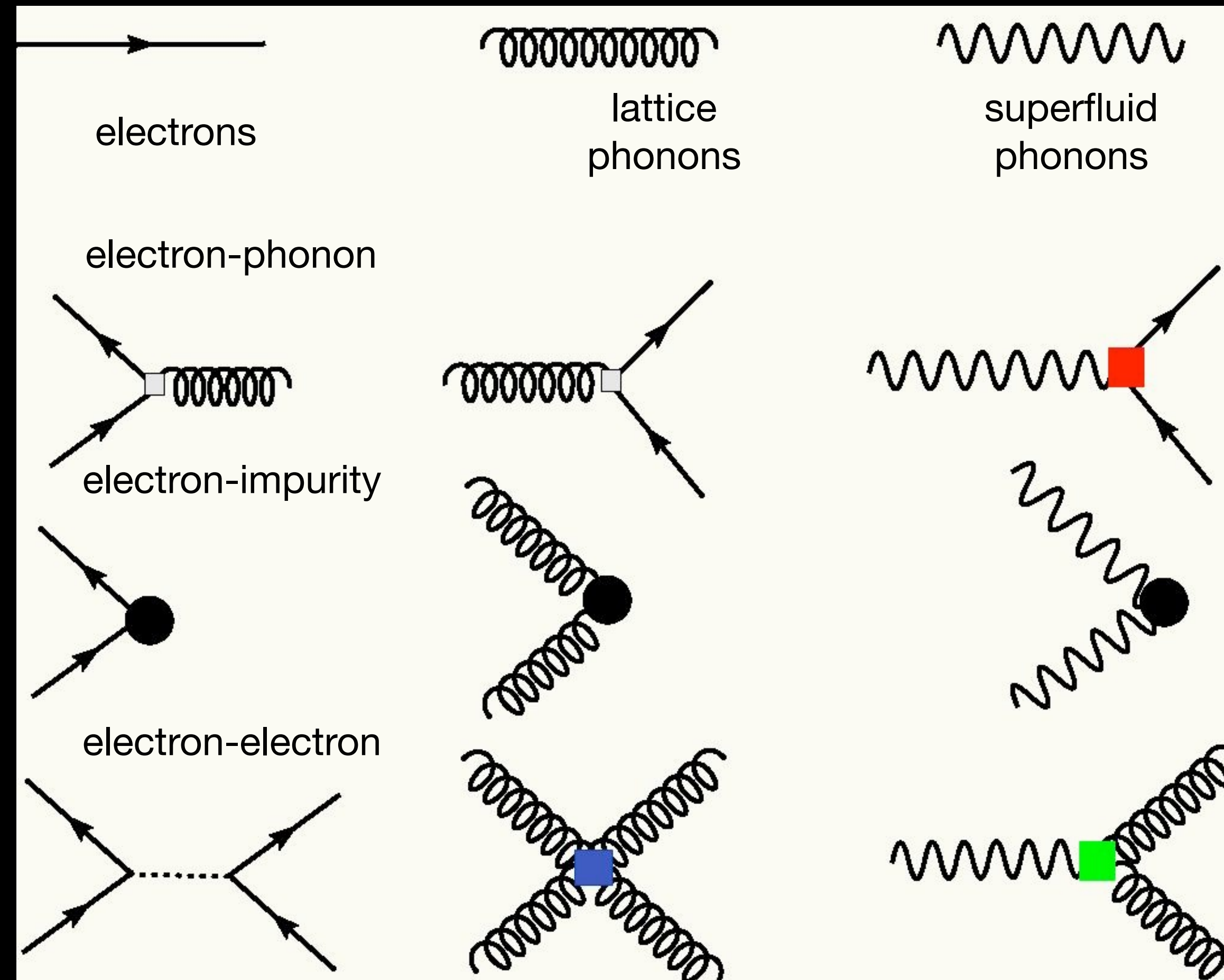
Vector Field: $\xi_i(r, t)$
Scalar Field: $\phi(r, t)$

Excitations and Interactions in the Inner Crust

Electrons and 2 longitudinal and 2 transverse phonons are the relevant excitations.

Thermal and transport properties of the solid and superfluid crust can be calculated using an effective field theory.

Mixing between phonons leads to strong Landau damping. Phonon conduction is highly suppressed.

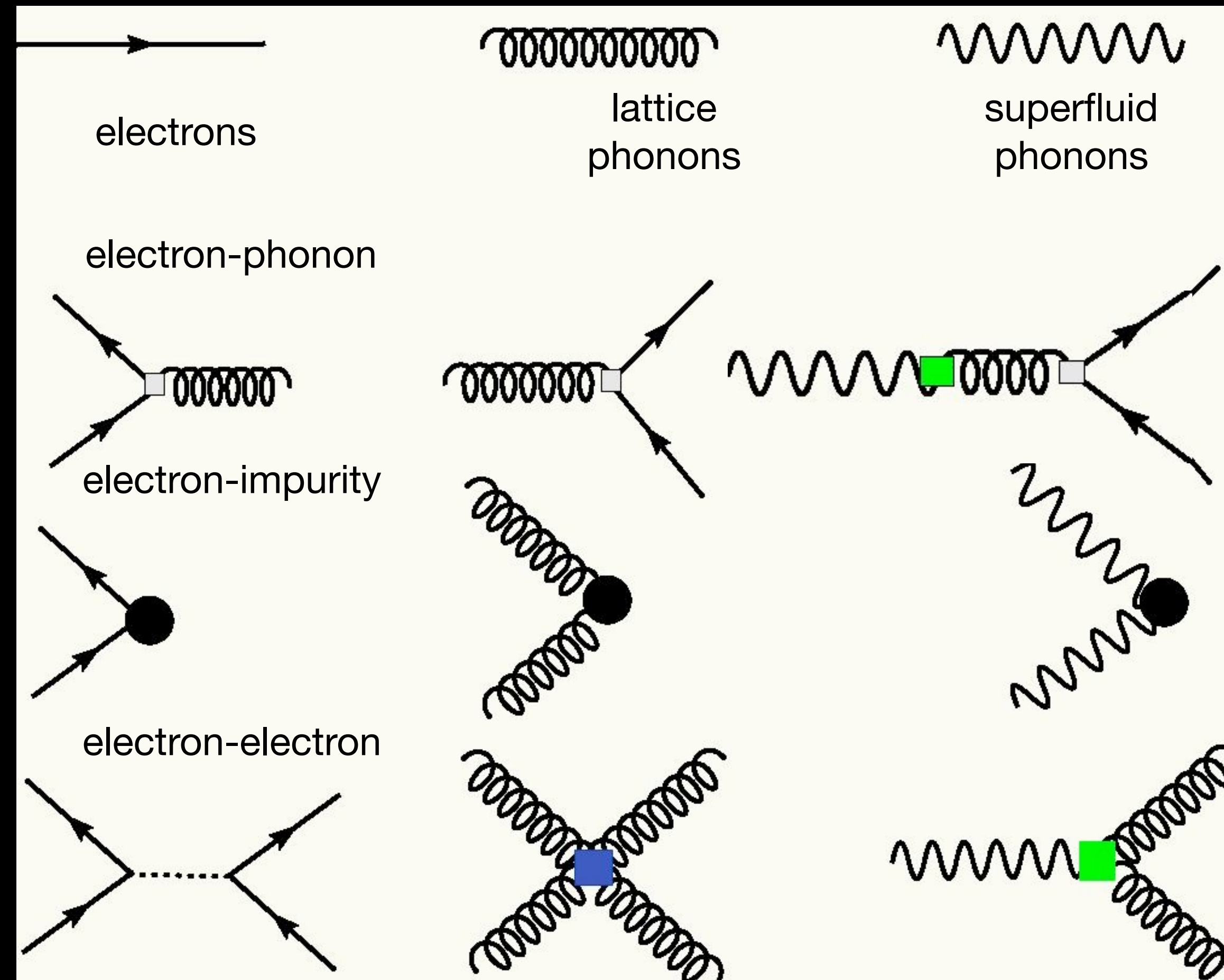


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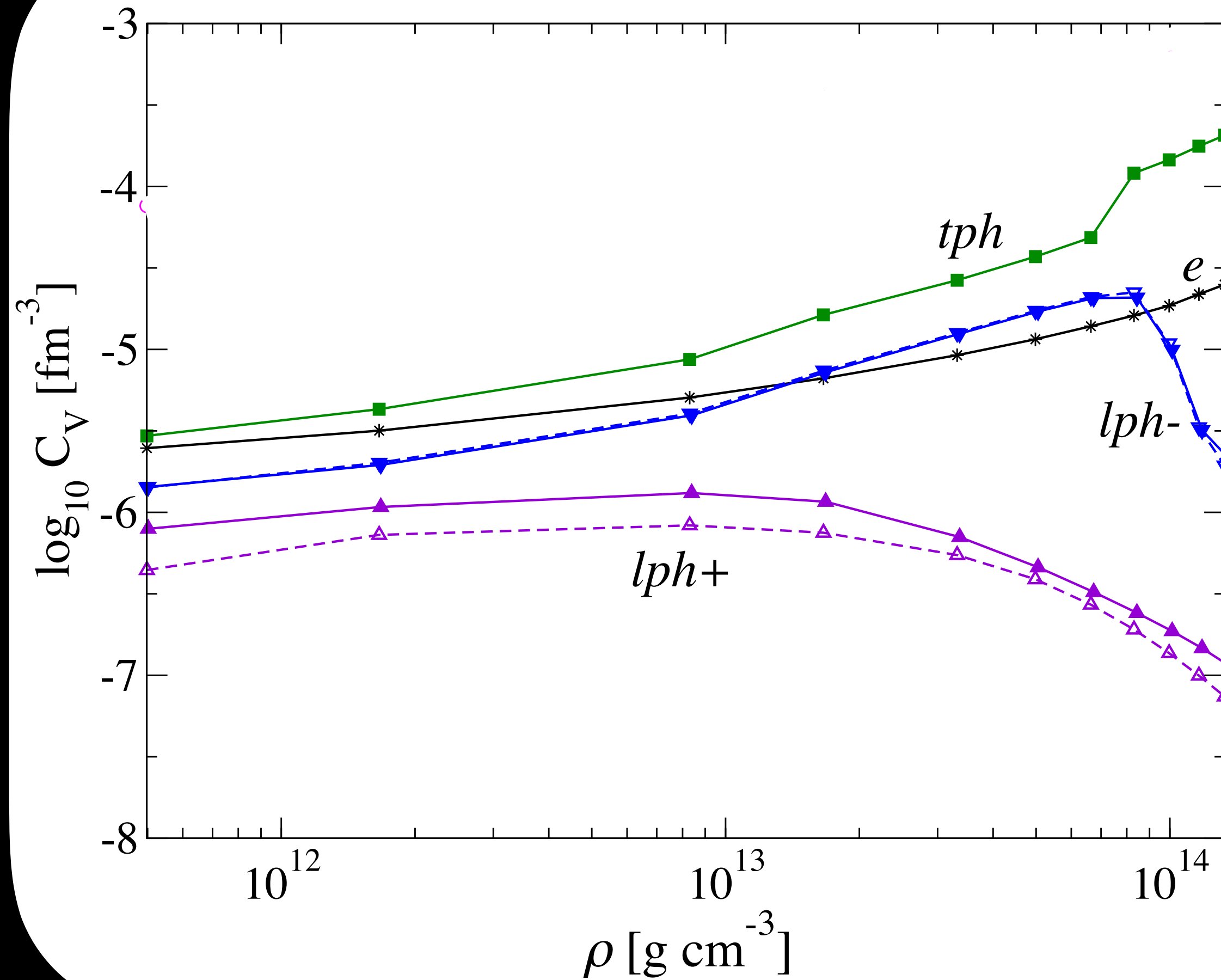
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Crustal Specific Heat



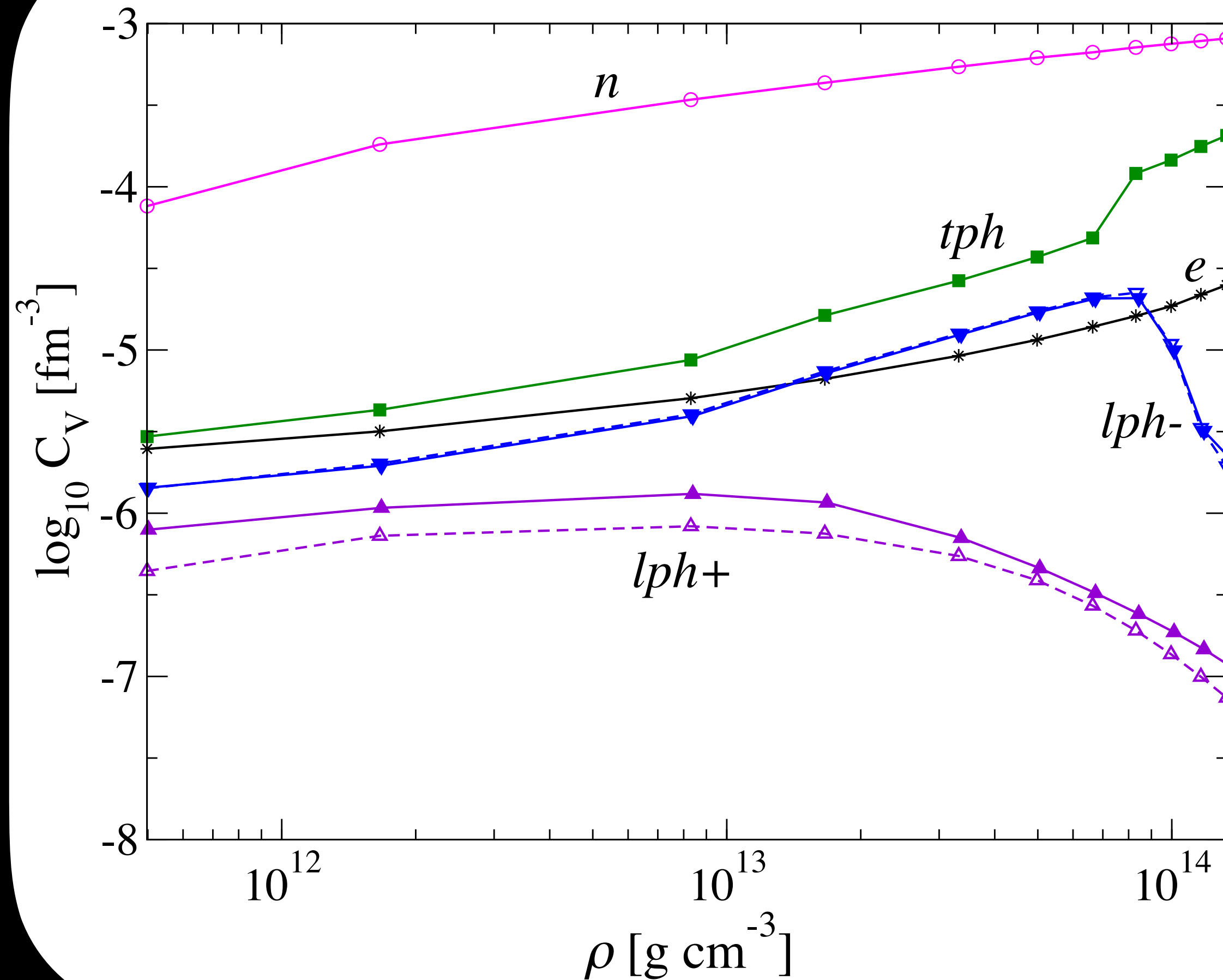
Electrons:

$$C_V^e \simeq \mu_e^2 T$$

Phonons:

$$C_V^i \simeq \frac{T^3}{v_i^3}$$

Crustal Specific Heat



Electrons:

$$C_V^e \simeq \mu_e^2 T$$

Phonons:

$$C_V^i \simeq \frac{T^3}{v_i^3}$$

If neutrons were normal

$$C_V^n \simeq M k_{Fn} T$$

their contribution would overwhelm.

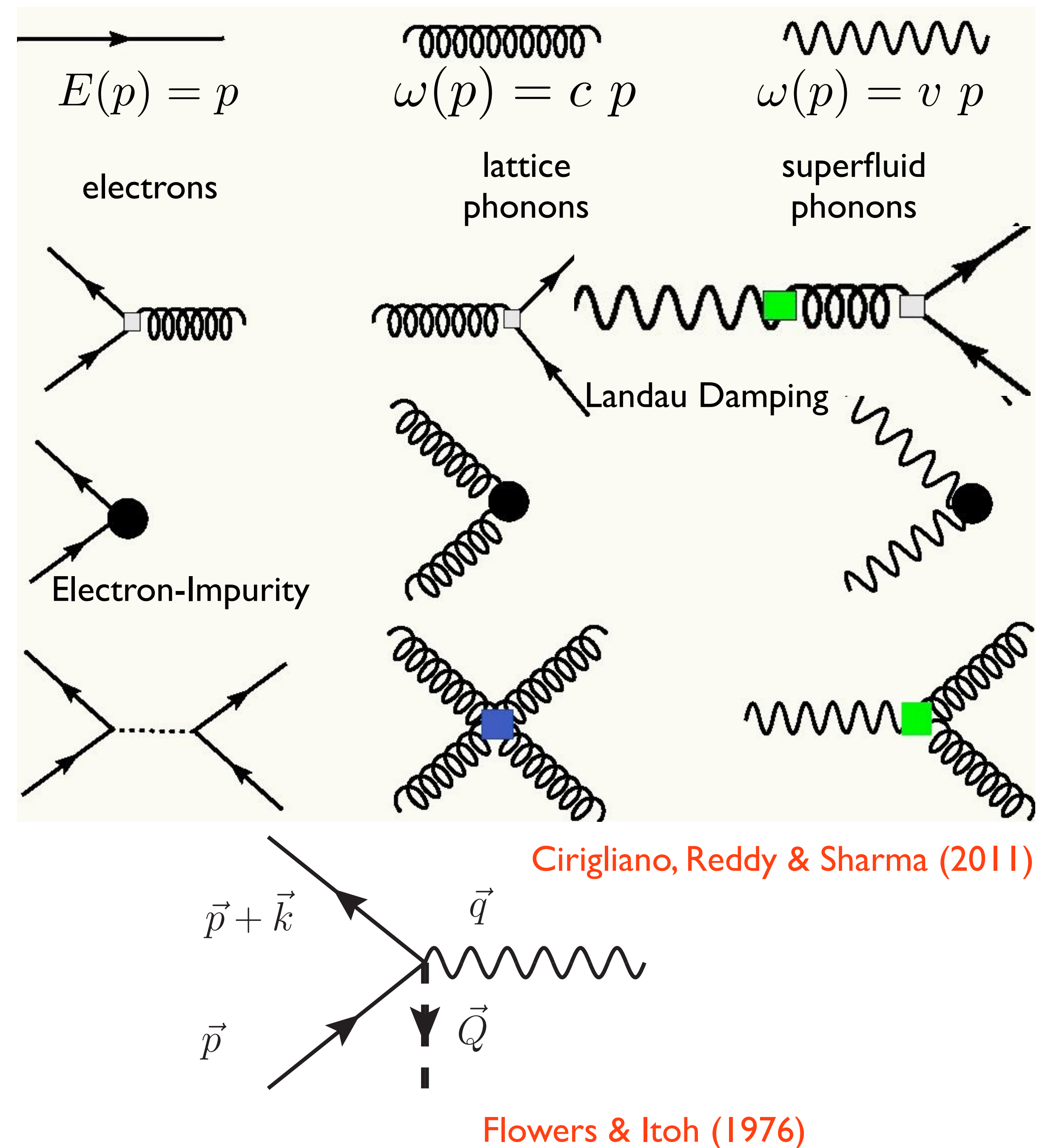
Thermal Conduction

$$\kappa = \frac{1}{3} C_v \times v \times \lambda$$

- Dissipative processes:
- Umklapp is important:

$$\frac{k_{\text{Fe}}}{q_{\text{D}}} = \left(\frac{Z}{2} \right)^{1/3} > 1$$

Electron Bragg scatters and emits a transverse phonon.

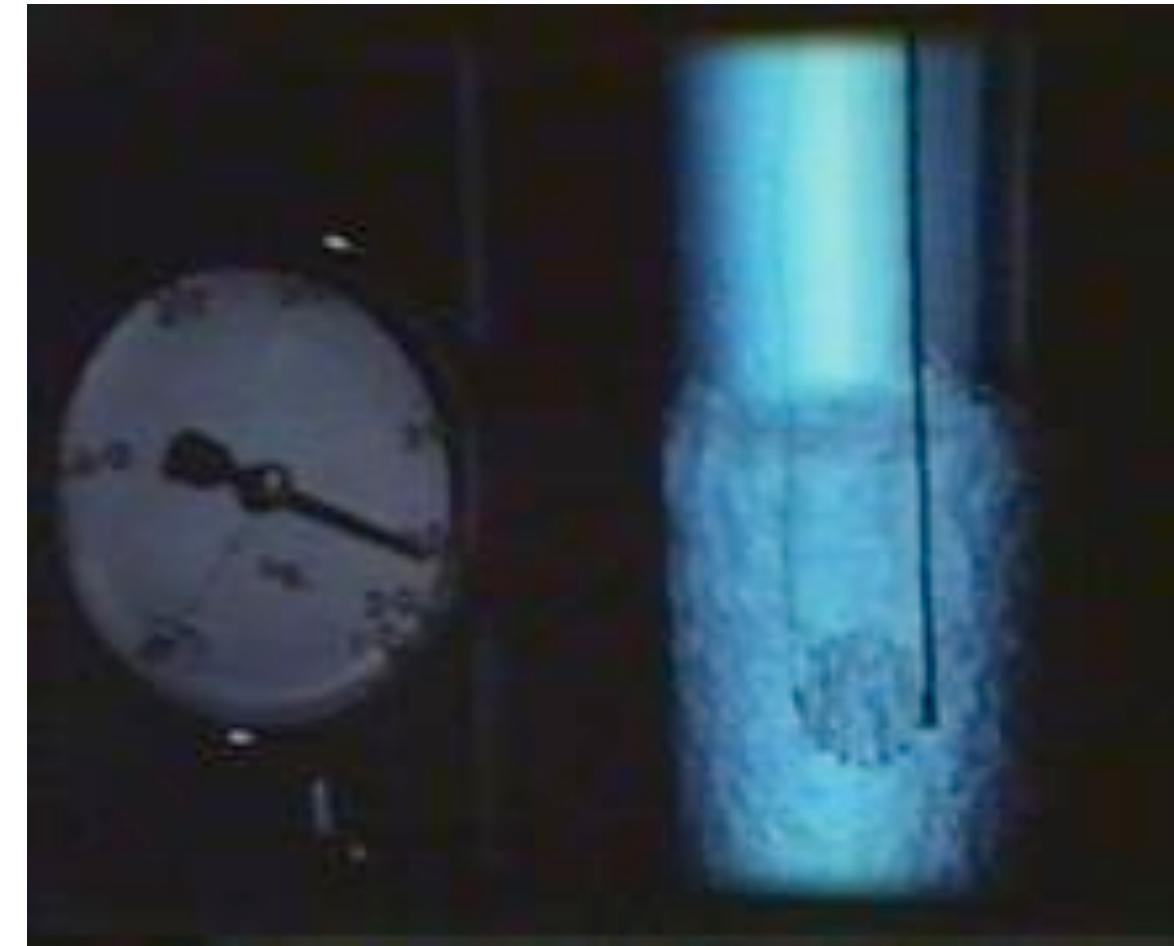


Superfluid Heat Conduction

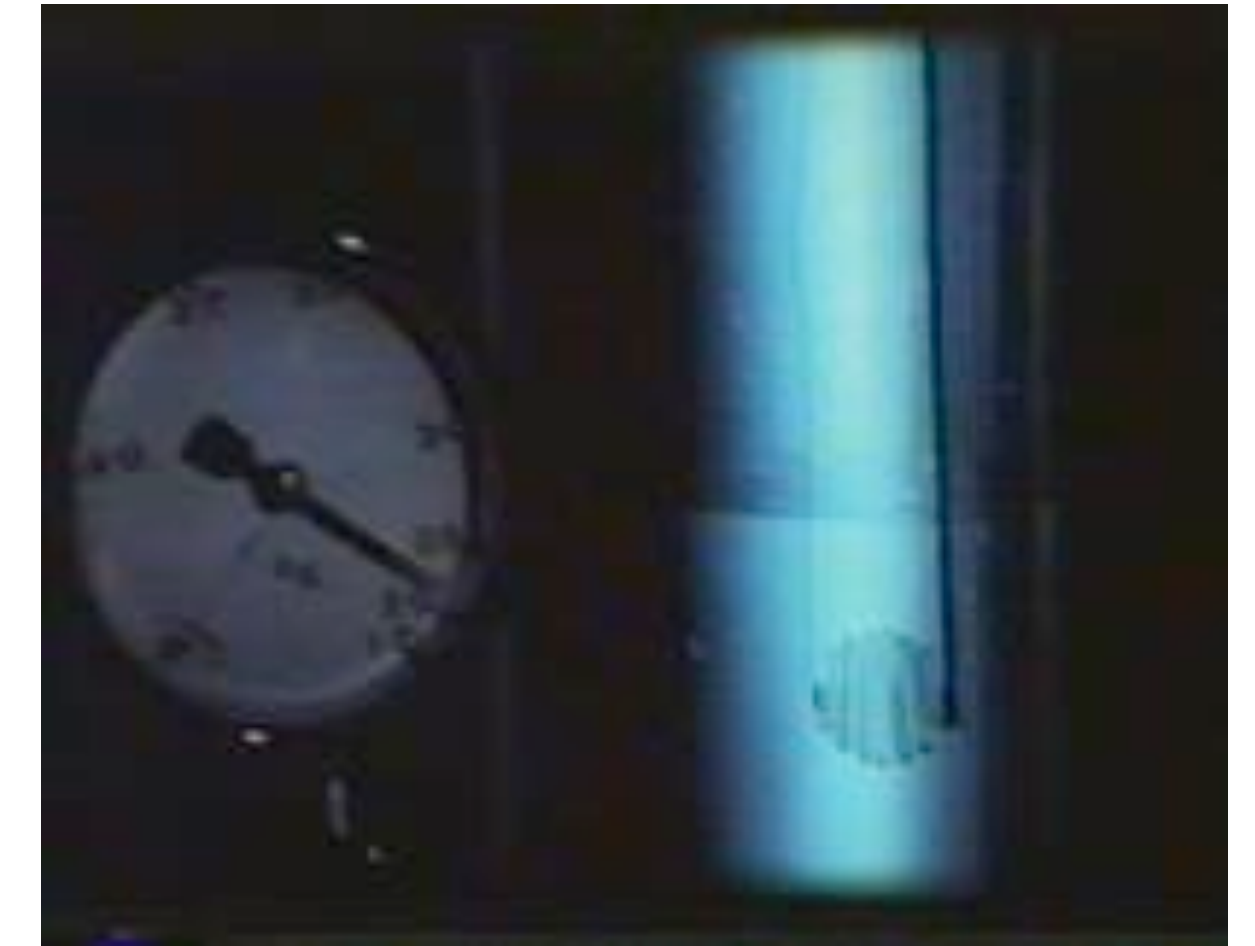
Photographs: JF Allen and JMG Armitage (St Andrews University 1982).

Its impossible to sustain a temperature gradient in bulk superfluid helium !

$$\vec{Q} = S^{(\text{sPh})} T \vec{v}_n$$
$$S^{(\text{sPh})} = \frac{1}{3} C_v^{(\text{sPh})} = \frac{2\pi^2}{15 c_s^3} T^3$$



$T > T_c$



$T < T_c$

Two fluid model: Counter-flow transports heat.
(Its the superfluid phonon fluid)

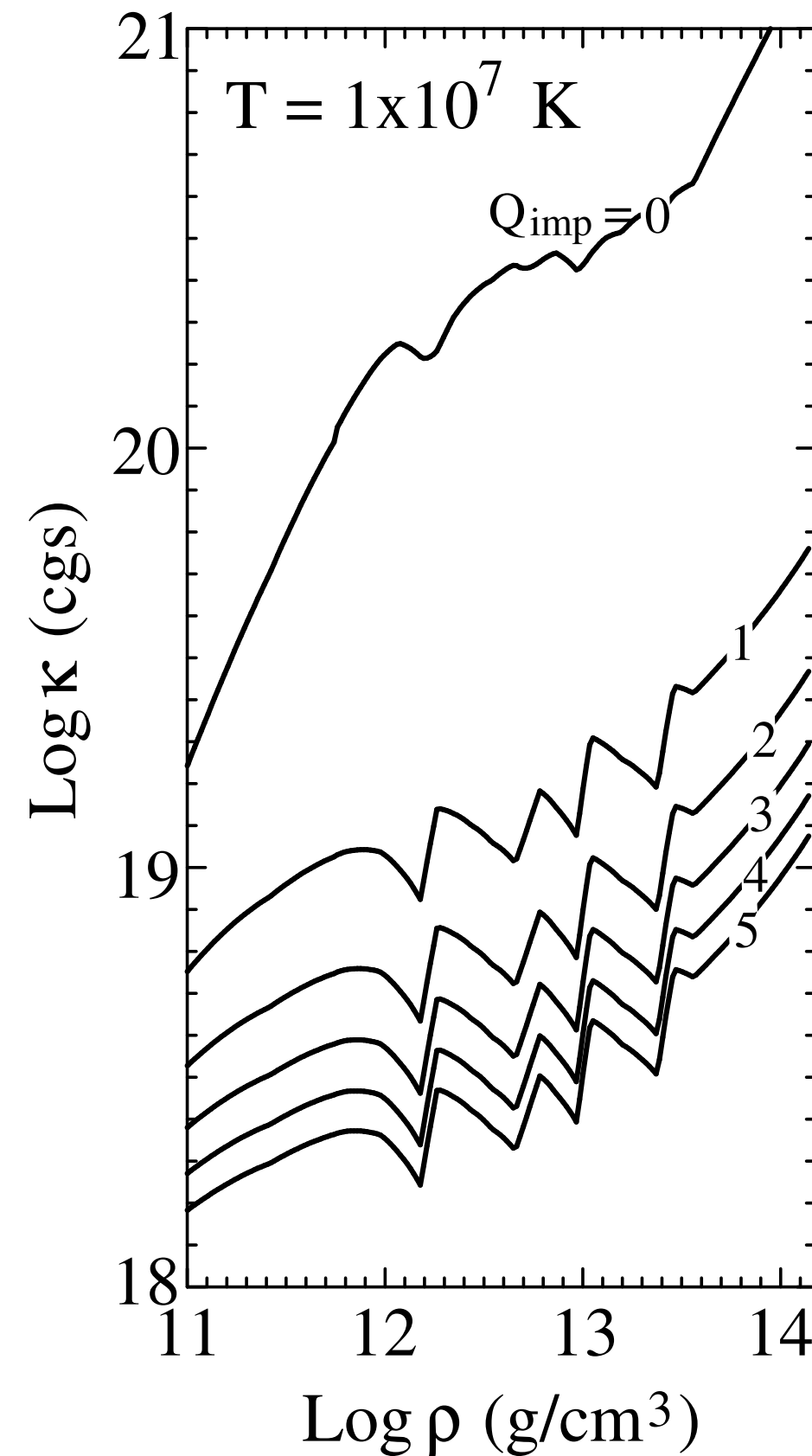
The velocity is limited only by fluid dynamics: (i) boundary shear viscosity or
(ii) superfluid turbulence.

Why does this not occur in neutron stars ?

Answer: Fluid motion is damped by electrons.

Electron Conduction

$$\kappa_e = \frac{1}{9} \mu_e^2 T \lambda_e$$



Electron-phonon:

$$\begin{cases} \lambda_e^{\text{ph}} \propto v_t^3 / T^2 & T \geq T_{\text{um}} \\ \lambda_e^{\text{ph}} \propto v_l^4 / T^3 & T \ll T_{\text{um}} \end{cases}$$

$$T_{\text{um}} = (4e^3 / 9\pi) v_t k_{\text{Fe}}$$

Electron-impurity:

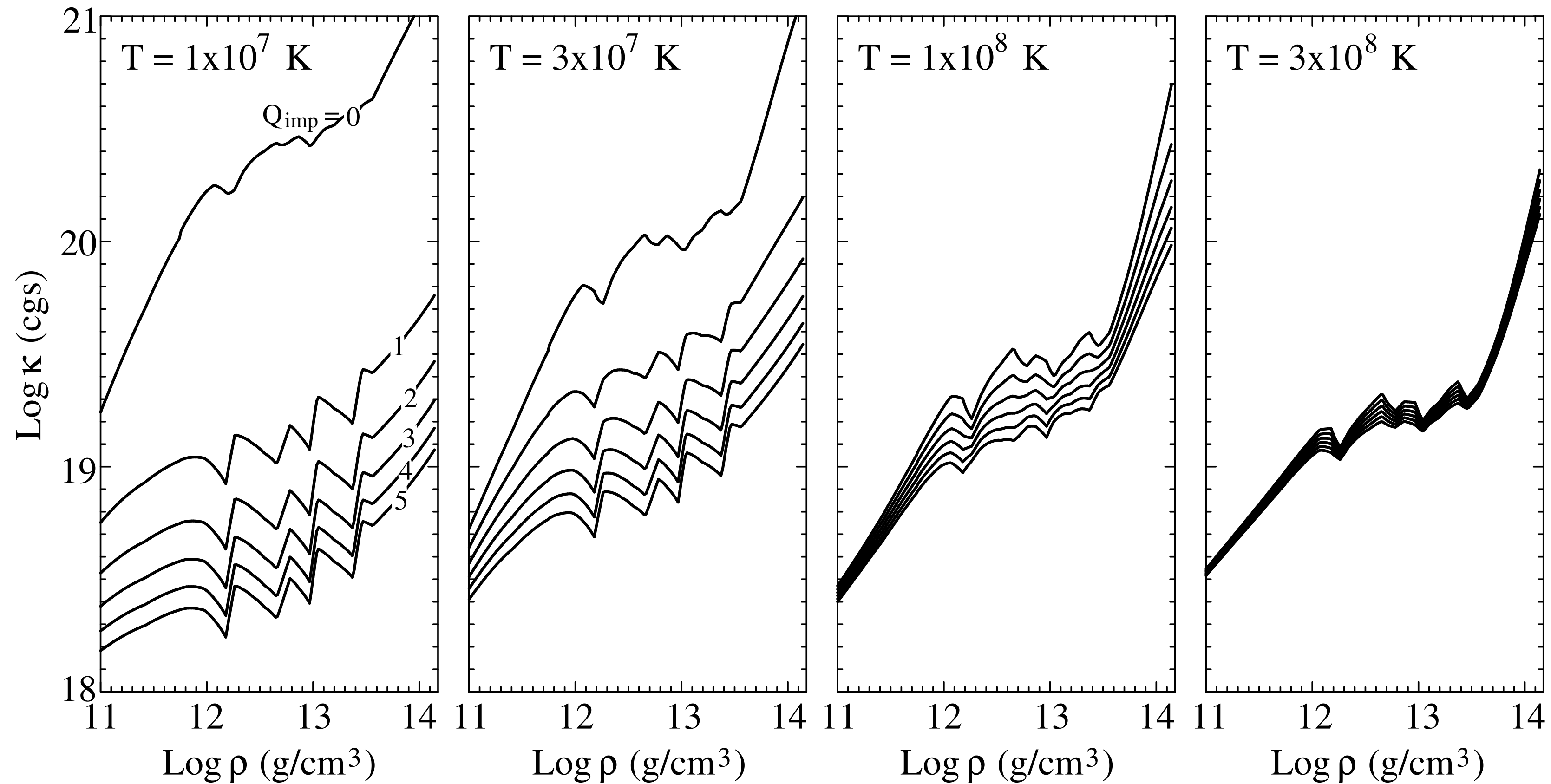
$$\lambda_e^{\text{imp}} = \frac{3\pi \langle Z \rangle}{4e^4 Q_{\text{imp}} k_{\text{Fe}}} \Lambda^{-1}$$

$$Q_{\text{imp}} = \frac{1}{n_{\text{ion}}} \sum_i n_i (Z_i - \langle Z \rangle)^2$$

Impurity scattering is important at low temperature.

Electron Conduction

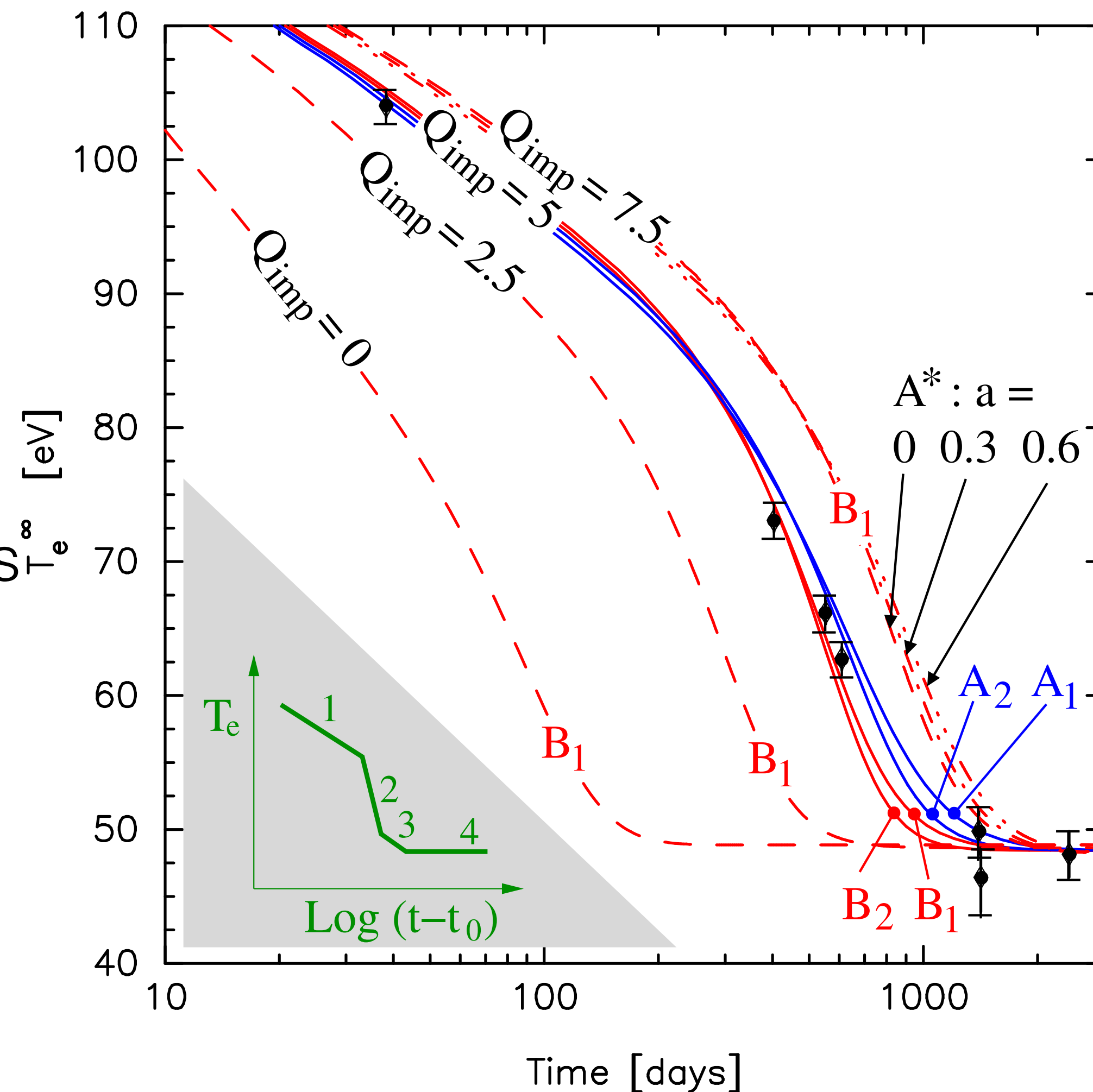
$$\kappa_e = \frac{1}{9} \mu_e^2 T \lambda_e$$



Impurity scattering is important at low temperature.

Unraveling Thermal Relaxation

- Late time signal is sensitive to inner crust thermal and transport properties.
- Impurity parameter can be fixed at earlier times.
- Variations in the pairing gap (changes the fraction of normal neutrons) are discernible !
- If neutrons were unpaired the cooling time scale would be too large.



Measuring the Heat Capacity of the Core

Heat the star, allow it to relax, and observe the change in temperature:

$$C_{NS} dT = dQ$$



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Heat the star, allow it to relax, and observe the change in temperature:

$$C_{NS} dT = dQ$$

When $C_{NS} = \alpha T$: $\frac{\alpha}{2} (T_f^2 - T_i^2) = \Delta Q$

Lower limit: $C_{NS}(T_f) > 2 \frac{\Delta Q}{T_f}$

$$\Delta Q = \dot{H} \times t_H - L_\nu \times (t_H + t_{obs})$$

heating
rate

duration
of heating

neutrino
cooling rate

time of observation
(after heating ceases)



Observations of KS 1731-260

Quiescent Surface Temperature (post relaxation): $T_s = 63.1$ eV

Accretion Phase: 12 yrs at $dM/dt \approx 10^{17}$ g/s

Thermal Relaxation: $t \approx 8$ yrs

Wijnands et al. (2002) Cackett et al. (2010)

Inferred Core Temperature:

Insulating envelope supports a temperature gradient near the surface.

Heavy element envelope: $T_c^\infty = 7.0 \times 10^7$ K $\left(\frac{T_s^\infty}{63.1 \text{ eV}} \right)^{1.82}$

Light element envelope: $T_c^\infty = 3.1 \times 10^7$ K $\left(\frac{T_s^\infty}{63.1 \text{ eV}} \right)^{1.65}$

Inferred Energy Deposition:

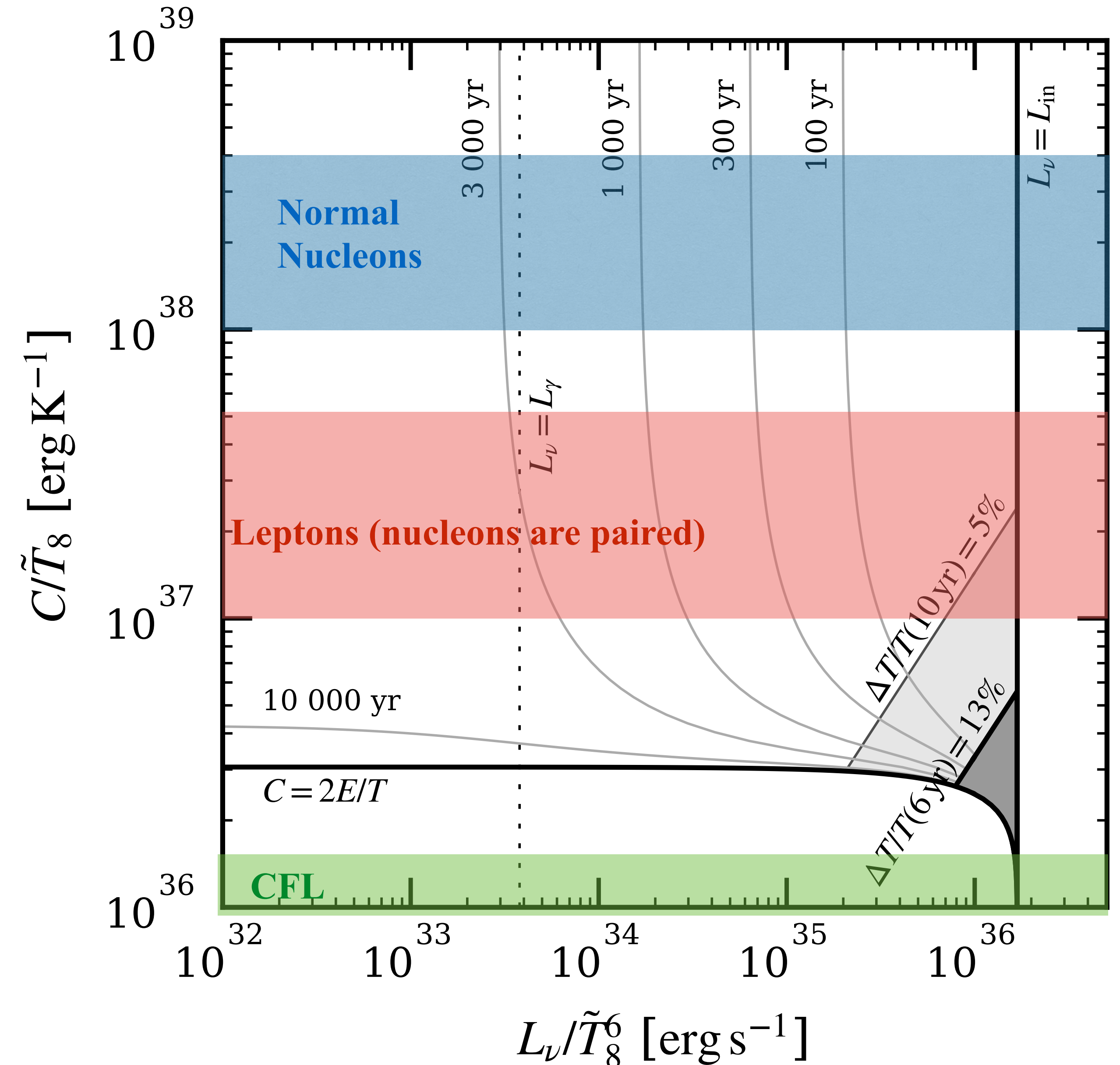
$$\Delta Q = \dot{H} \times t_H = 6 \times 10^{43} \text{ ergs} \left(\frac{Q_{nuc}}{2 \text{ MeV}} \right) \left(\frac{\dot{M}}{10^{17} \text{ g/s}} \right) \left(\frac{t_H}{10 \text{ yrs}} \right)$$

Lower Limit on the Core Specific Heat: Current & Future

The limit is compatible with most models of dense matter.

One exception is a neutron star core made entirely of CFL quark matter.

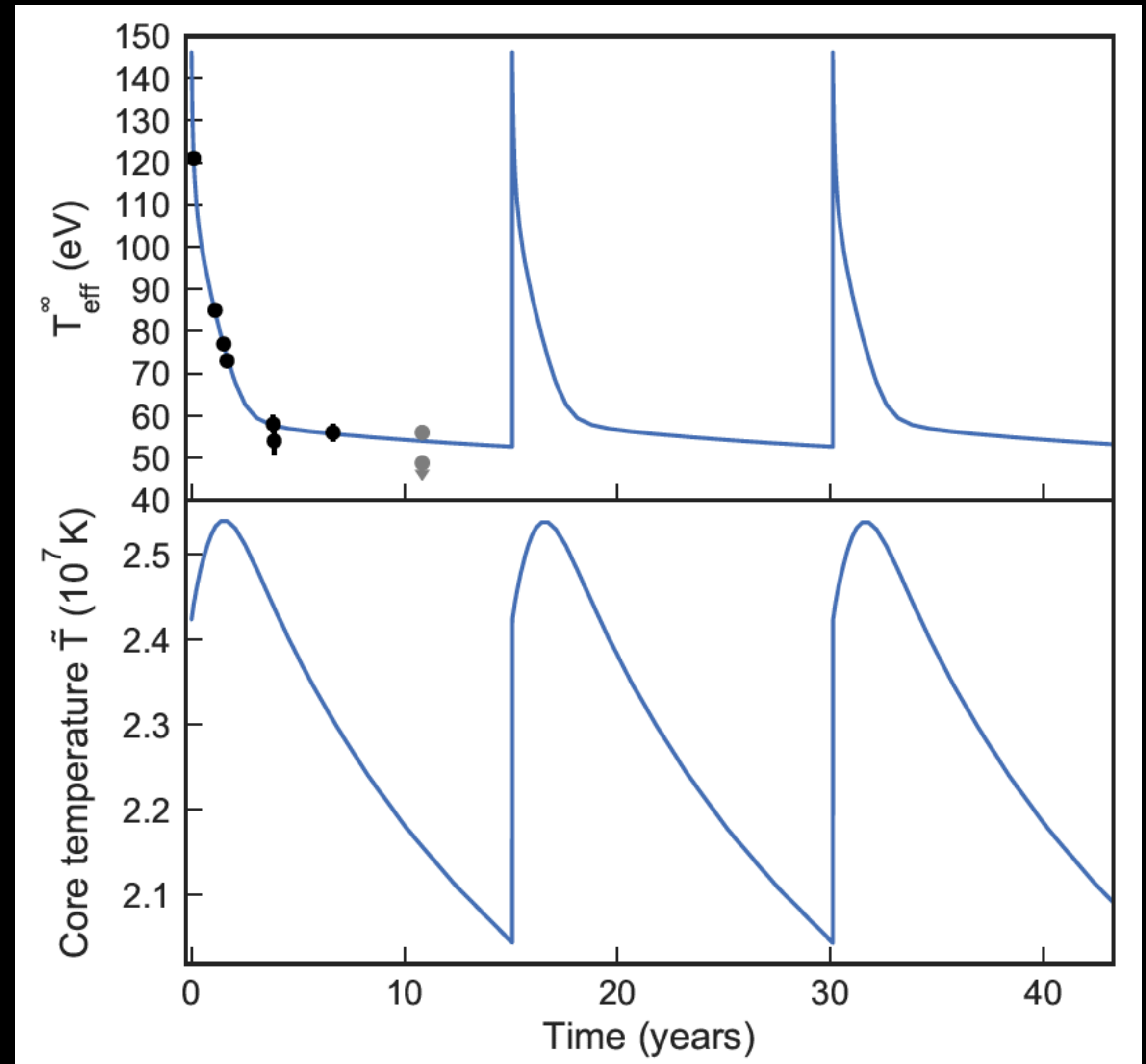
If temperature variation is observed on a 10 year time scale, it would imply some form of exotic matter in which most baryons are frozen!



Long Term Evolution of Accreting Neutron Stars

Balance between neutrino luminosity and crustal heating sets the average core temperature.

If we know the heating and accretion rate on average then a measurement of the neutron star surface temperature provides a constraint on the core neutrino luminosity!



Rapid neutrino cooling in the neutron star MXB 1659-29

Edward F. Brown,^{1,*} Andrew Cumming,^{2,†} Farrukh J. Fattoyev,^{3,‡}

C. J. Horowitz,^{3,§} Dany Page,^{4,¶} and Sanjay Reddy^{5,**}

Phys.Rev.Lett. 120 (2018) no.18, 182701

Evidence for diversity. Not all neutron star cores are the same!

If we observe cooling on a 10 year timescale we can obtain an upper bound on the core specific heat as well!

