Neutron stars and the properties of matter under extreme conditions:

quarks in the interior

Gordon Baym
University of Illinois, Urbana

8th Huada School on QCD
Central China Normal University
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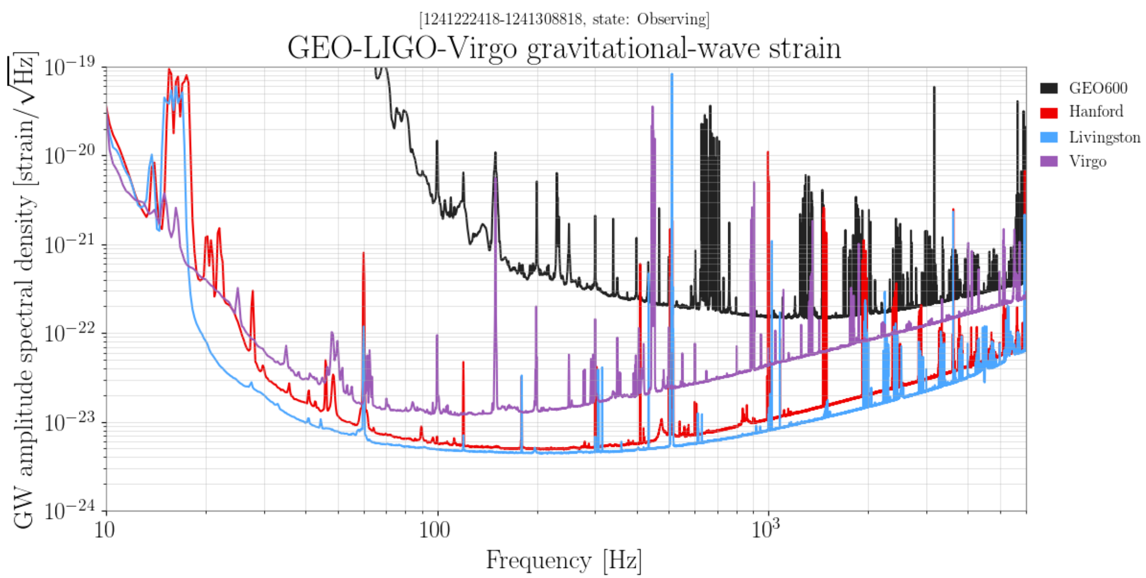
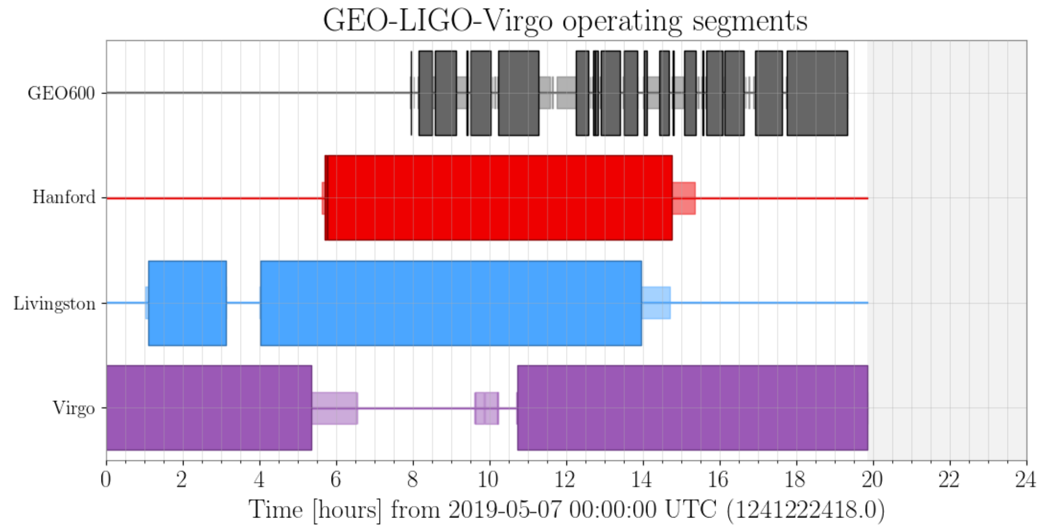
CENTRAL CHINA NORMAL UNIVERSITY

The plots shown below characterize the sensitivity and status of each of the LIGO interferometers as well as the Virgo (http://www.virgo-gw.eu/) detector in Cascina, Italy and the GEO600 (http://www.geo600.org) detector in Hanover, Germany.

For more information about the plots listed below, click on an image to read the caption. Use the tabs in the navigation bar at the top of the screen for more detailed information about the LIGO, Virgo, and GEO interferometers.

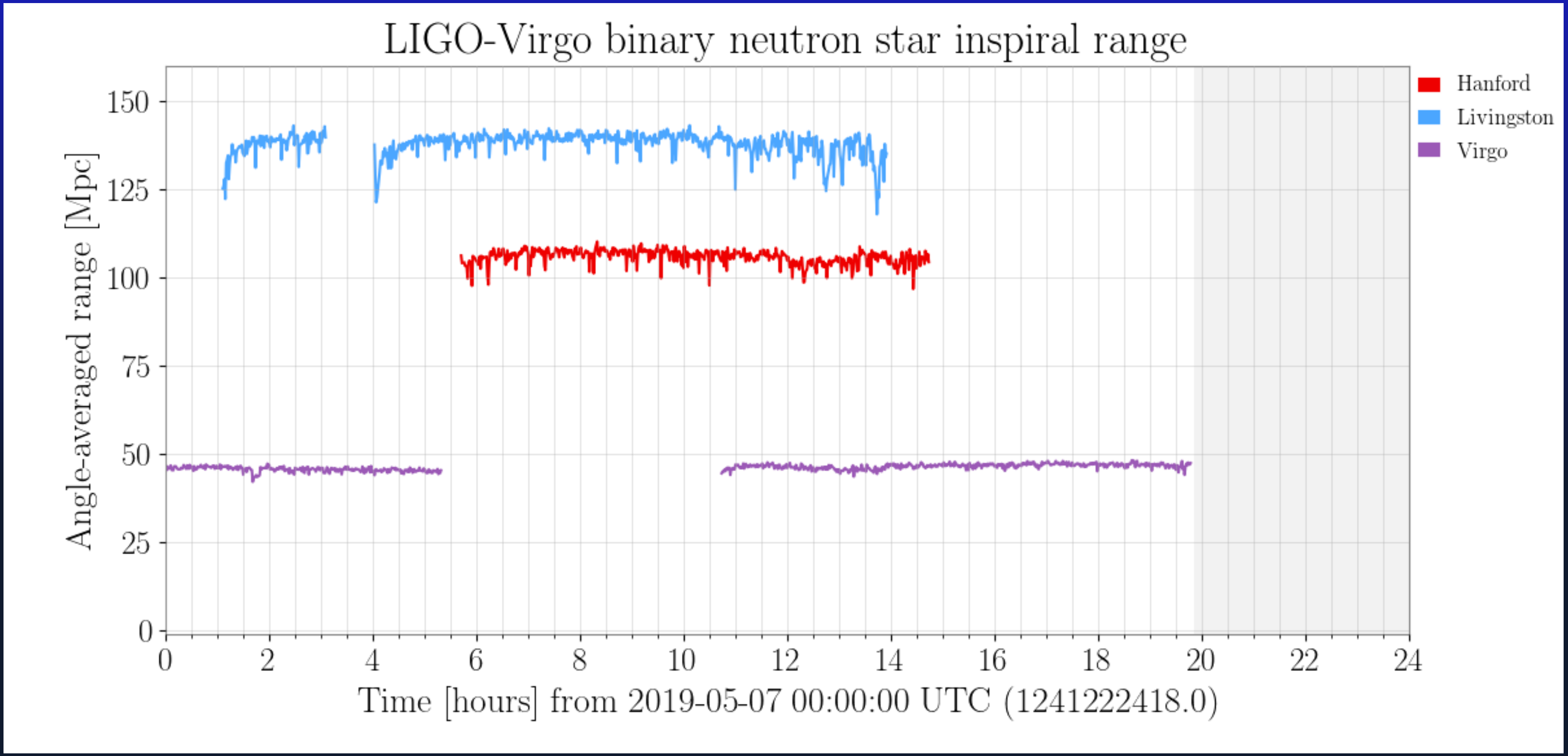


(/s/summary_pages/detector_status/cache/day/20190507/G1H1L1V1-OBSERVING_HOFT_SPECTRUM-1241222418-86400.png)



https://gracedb.ligo.org/ Gravitational Wave Candidate Event Data Base

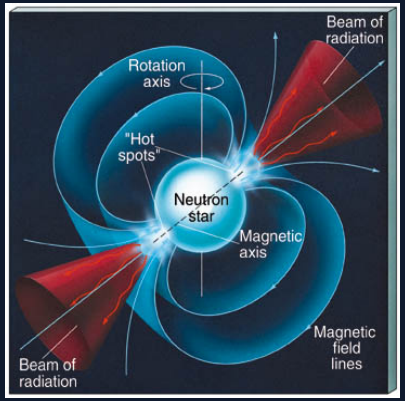
https://www.gw-openscience.org/detector_status/day/20190507/

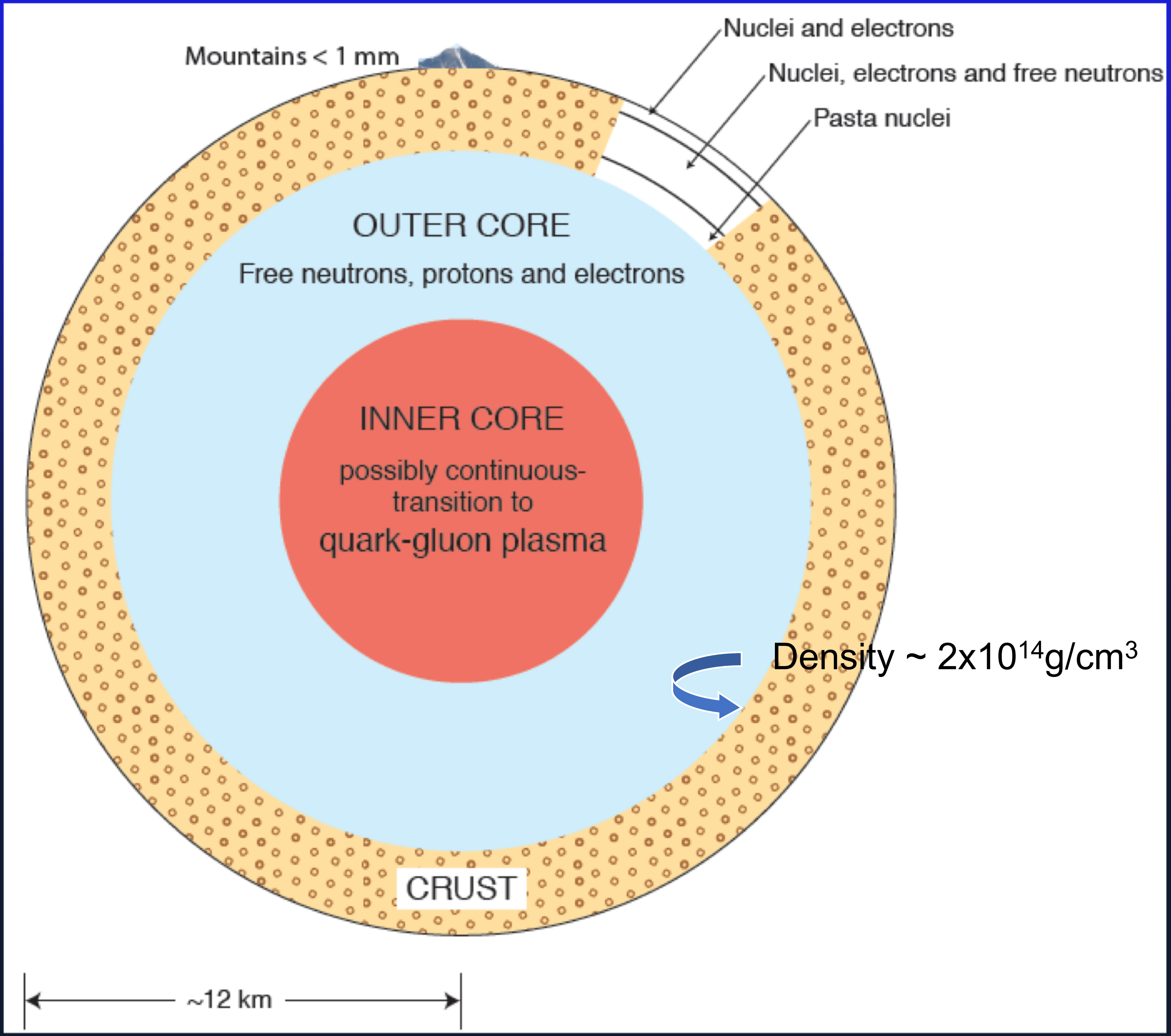


Neutron star interior

Mass ~ 1.4-2+ M_{sun}
Radius ~ 10-12 km
Temperature
~ 10^6-10^9 K

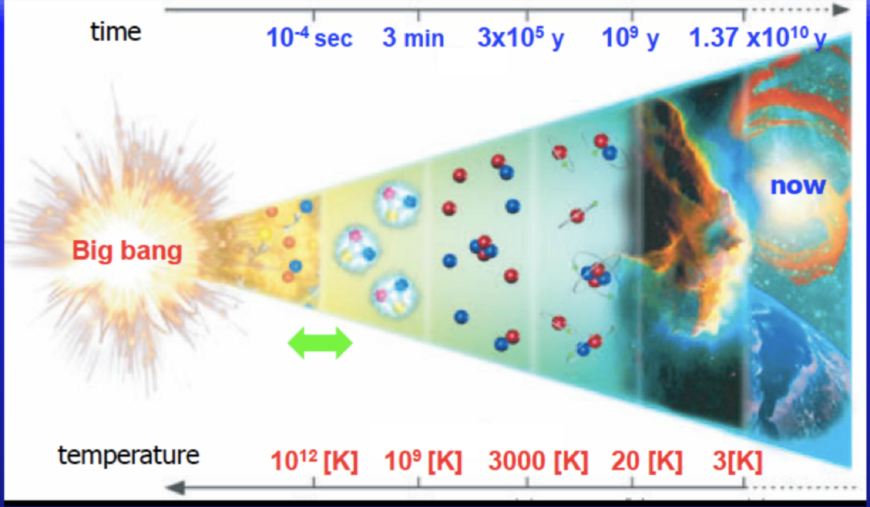
Surface gravity
~10^{11} that of Earth
Surface binding
~ 1/10 mc^2

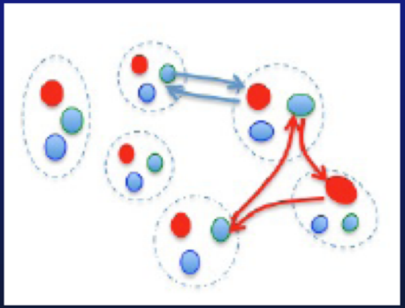




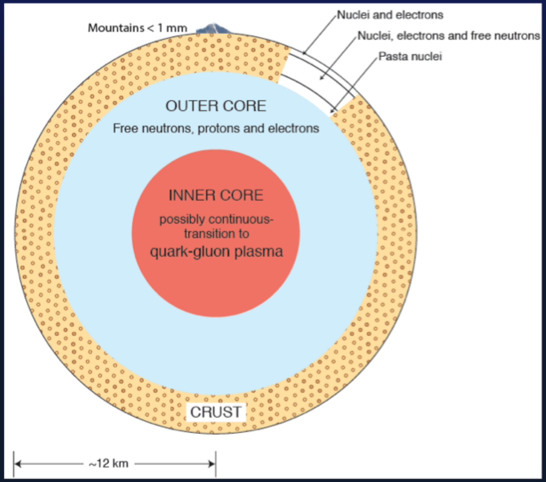
Quarks in dense matter

The early universe before one microsecond after the big bang -- hot quark gluon plasma

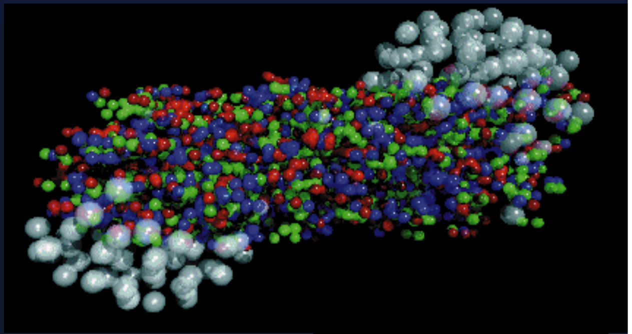


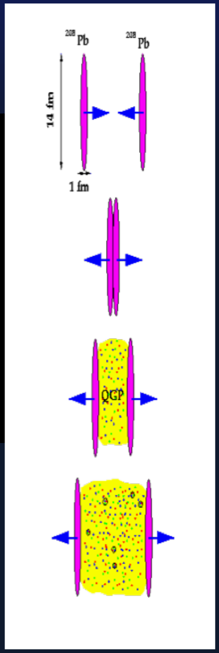


Quarks (and gluons) in nuclei, mapped by future Electron-Ion Collider



The cores of quiescent neutron stars – cold quark matter





Ultrarelativistic heavy ion collisions –
hot quark gluon plasma

Higher densities: cold quark matter

Fundamental limitations of eq. of state based on NN interactions alone

Accurate for $n \sim n_0$.   But for $n >> n_0$:

-can forces be described with static few-body potentials?

-Force range $\sim 1/2m_\pi$ => relative importance of 3 (and higher) body forces $\sim n/(2m_\pi)^3 \sim 0.4 n_{fm^{-3}}$.

-No well defined expansion in terms of 2,3,4,...body forces.

-Can one even describe system in terms of well-defined ``asymptotic'' laboratory particles?   Early percolation of nucleonic volumes!   Wrong degrees of freedom!!

Strongly interacting system, but cannot do numerical simulations at zero temperature, finite density via lattice QCD calculations, owing to the fermion sign problem.

Equation of state based on NN interactions alone
yields nice looking models, but with faulty input physics
Wrong degrees of freedom at high density

That it works does not make it right!

Cancelling the six's

$$\frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$$

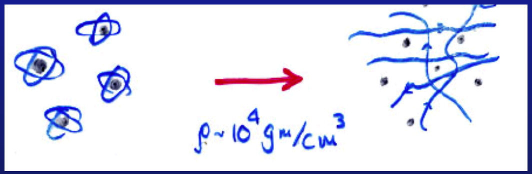
$$\frac{19}{95} = \frac{1\cancel{9}}{\cancel{9}5} = \frac{1}{5}$$

$$\frac{18}{85} = \frac{1\cancel{8}}{\cancel{8}5} \simeq \frac{1}{5}$$

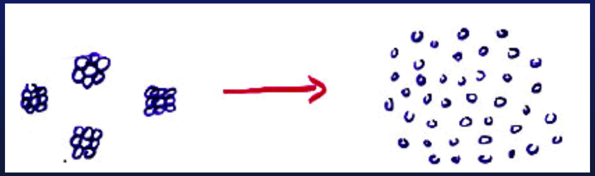
New degrees of freedom at higher densities

Squeeze:  => 

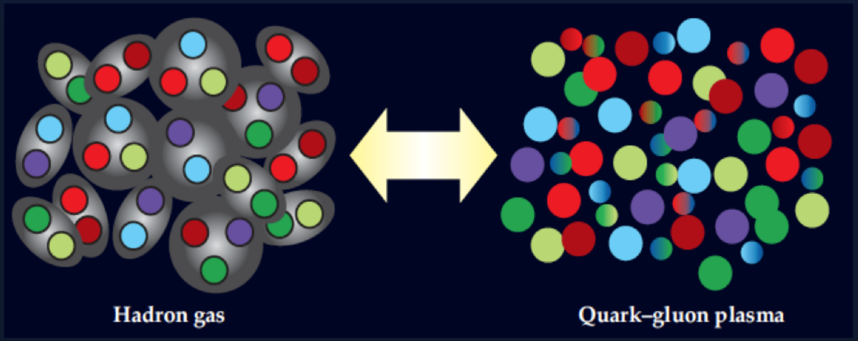
Atoms -> plasma



Nuclei -> nuclear matter



Hadrons (n,p,...) -> quark matter



Given all information on Nb+Nb atomic scattering could one predict that metallic Nb is a superconductor?

Quarks, rather than hadrons, become the correct degrees of freedom at higher densities

Squeeze:  => 

Existence of quarks was (my mistake) approached by looking for a (first order) phase transition between hadronic matter and quark matter. This process, by its very construction eliminates the possibility a stiff quark matter equation of state that could permit two-solar-mass neutron stars to exist.

Three neutron stars in binary orbits with white dwarfs:

PSR J1614-2230 : $M_{\text{neutron star}} = 1.93 \pm 0.02 M_\odot$

PSR J0348+0432: $M_{\text{neutron star}} = 2.01 \pm 0.04 M_\odot$

PSR J0740+6620 : $M_{\text{neutron star}} = 2.17 \pm 0.1 M_\odot$

These require a very stiff equation of state!

How can quark matter give stiff eq. of state, to explain large masses?

Earliest phase diagram

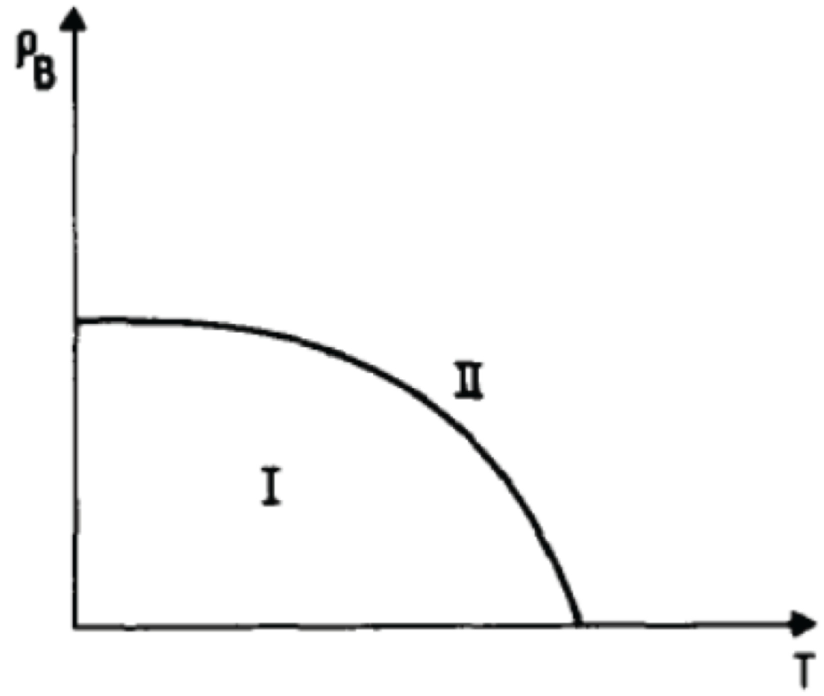
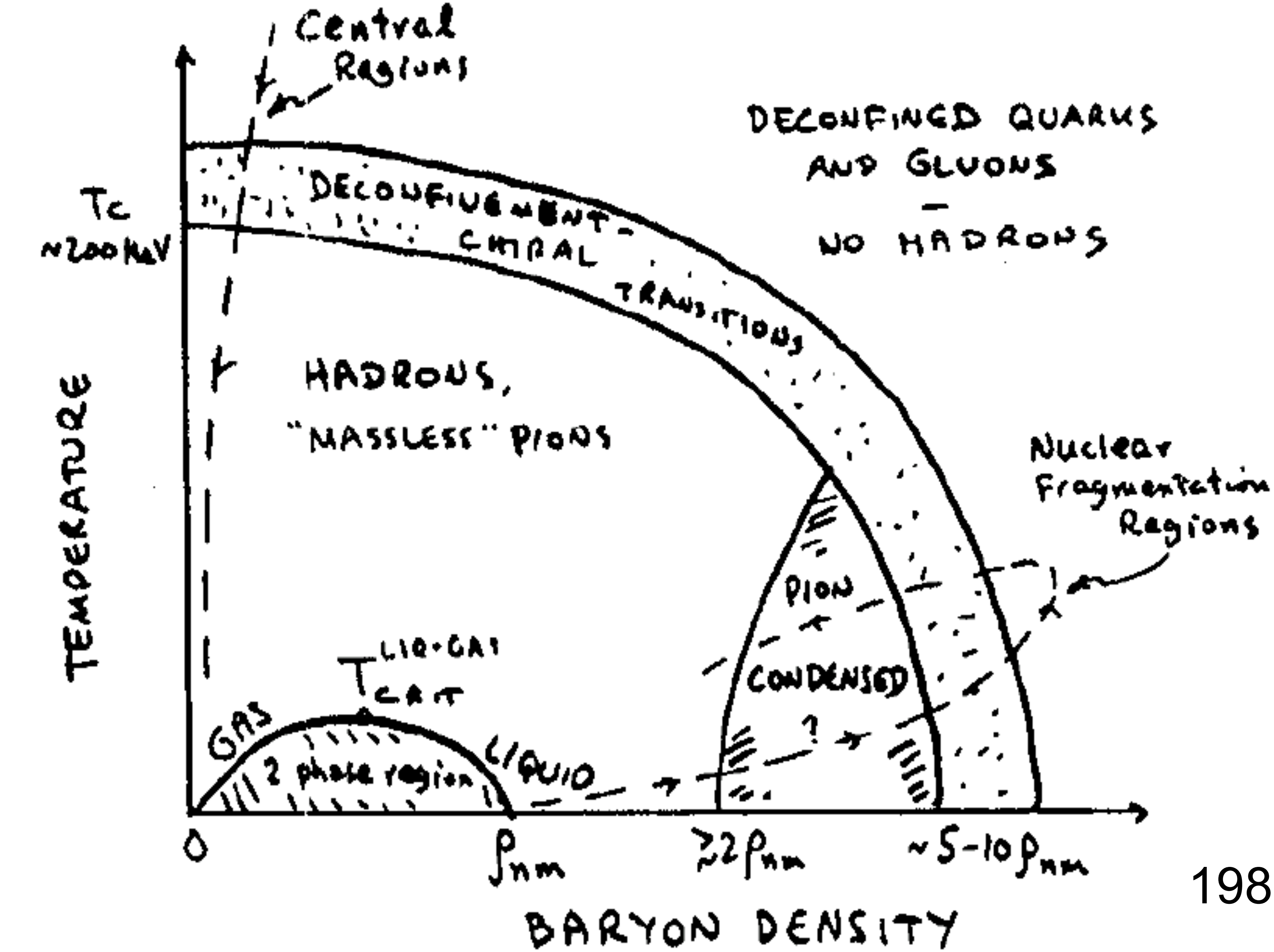


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

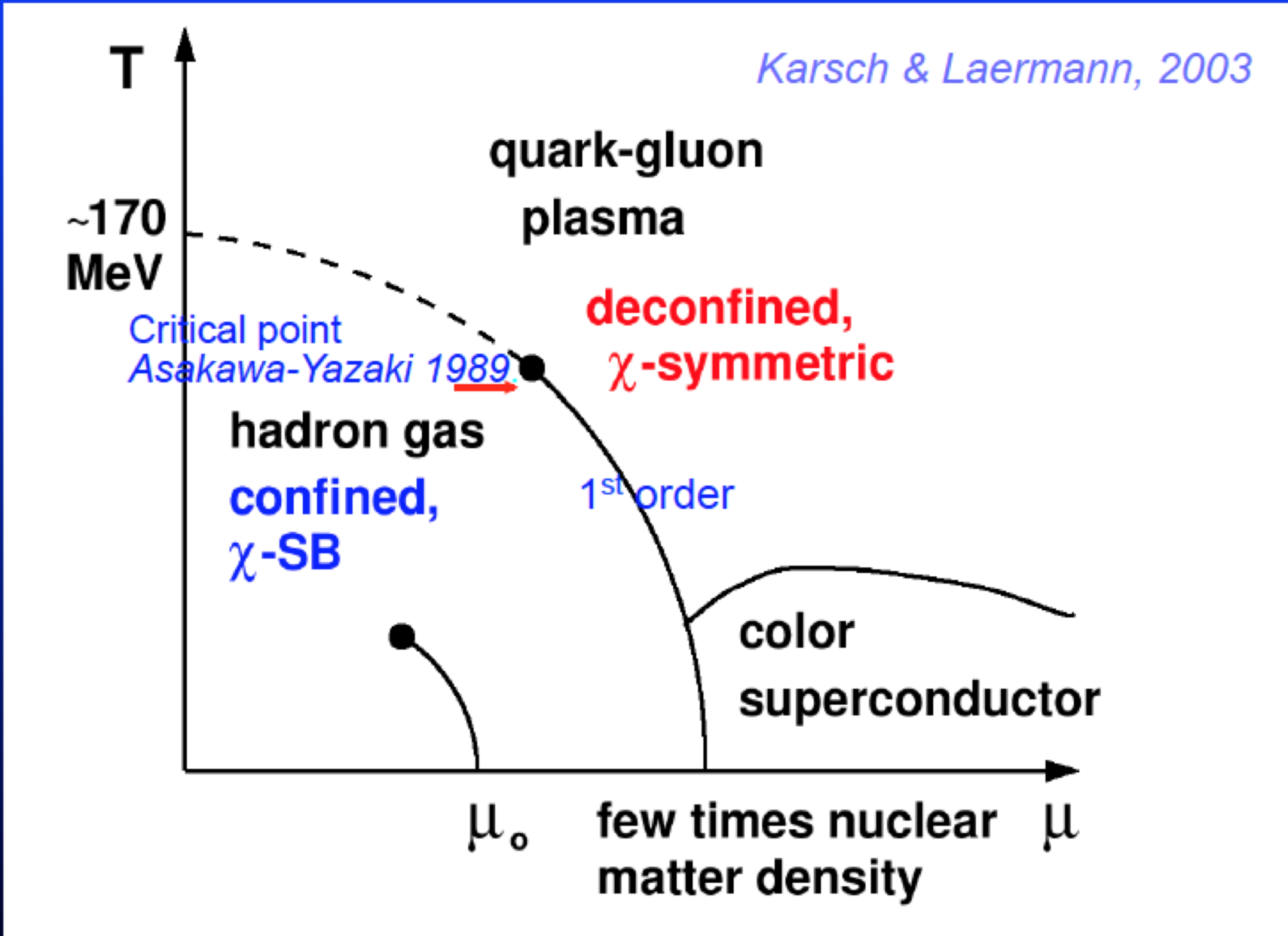
"Quark liberation" from Phase I to Phase II

PHASE DIAGRAM OF NUCLEAR MATTER



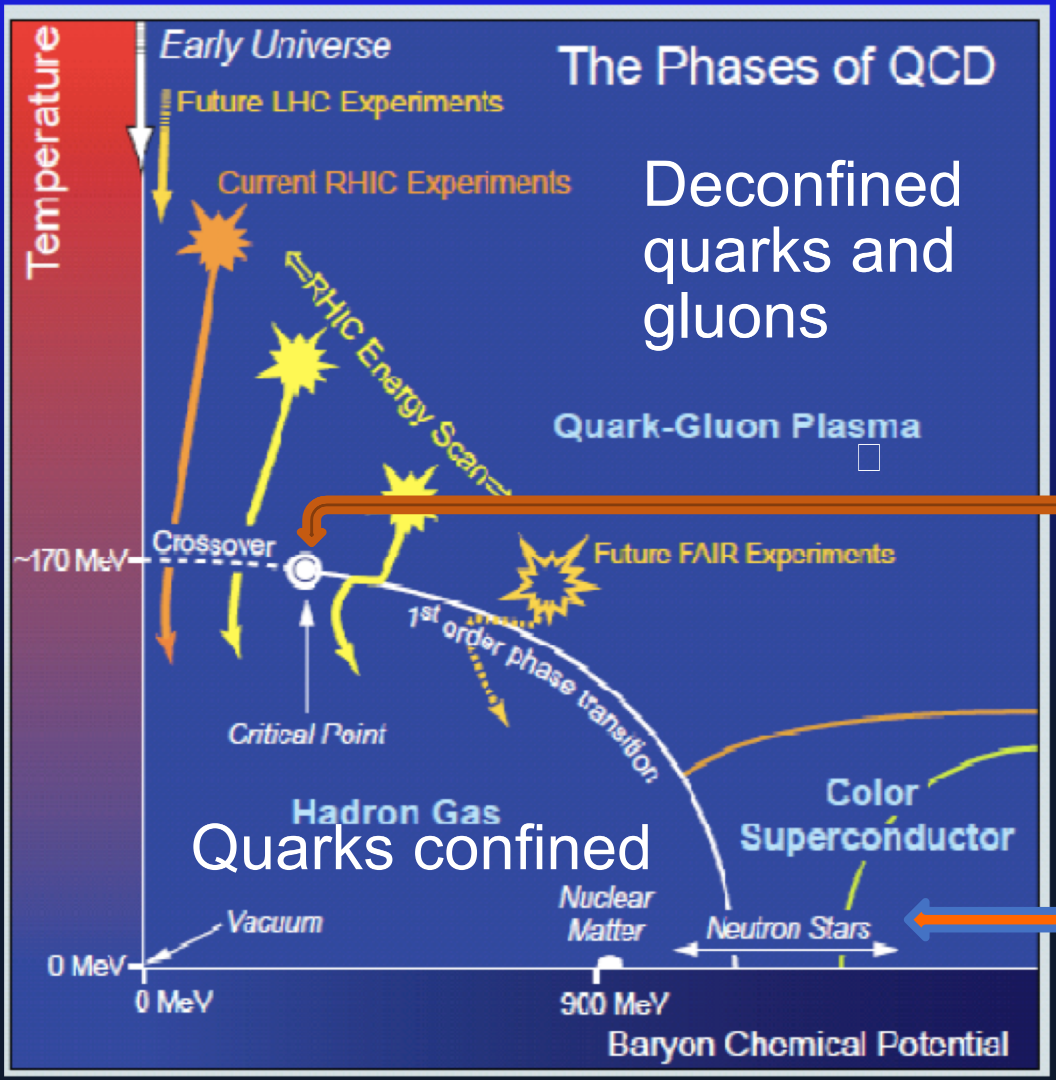
1983

Arrival of the Asakawa-Yazaki critical point



Note that the baryon chemical potential repaces the baryon density

Modern phase diagram



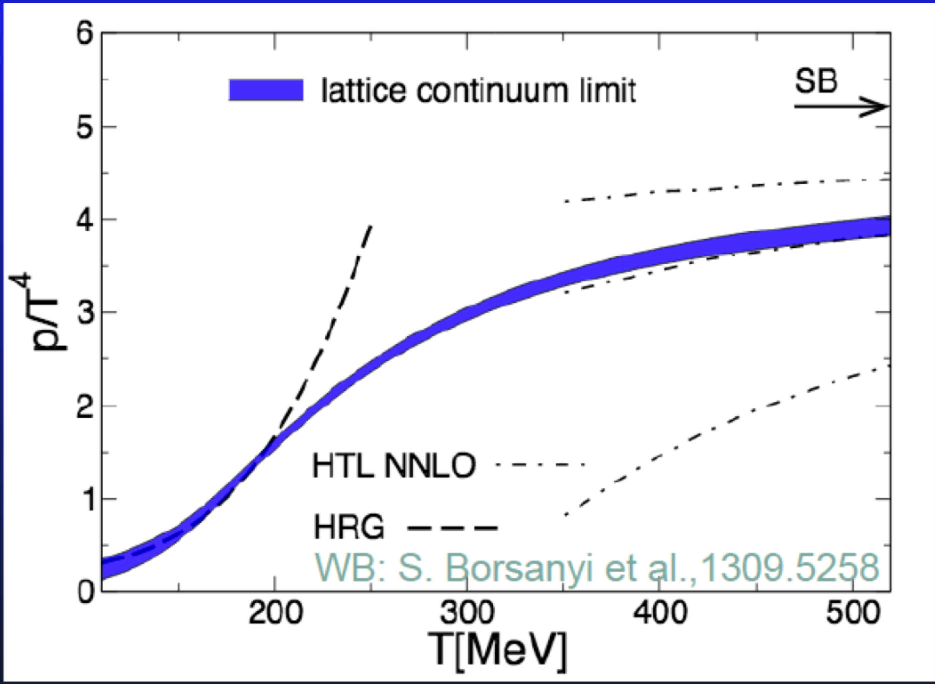
Asakawa-Yazaki critical point (1989)

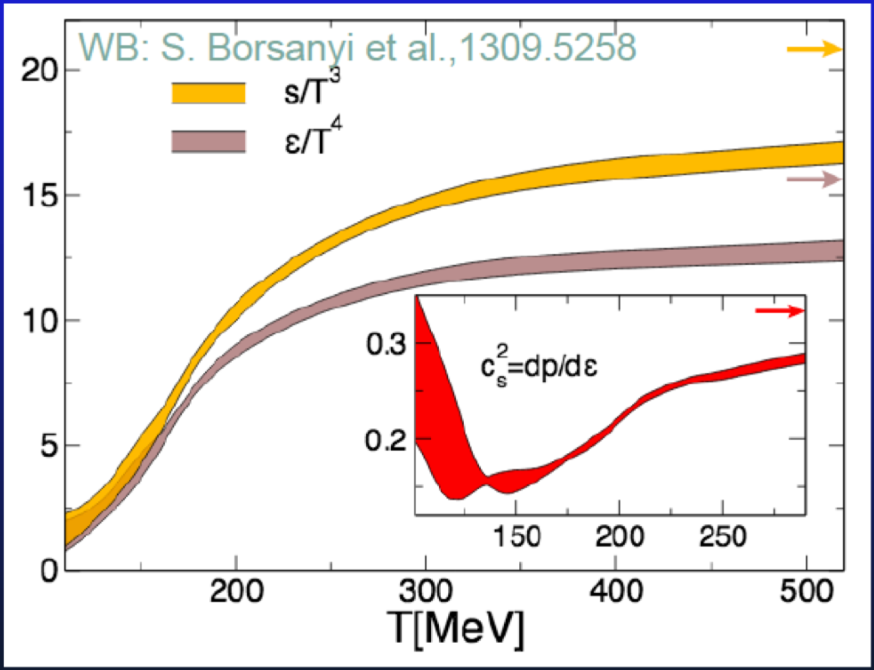
Search in RHIC & SPS energy scans.

States of color superconductivity – diquark BCS pairing

2SC / Color flavor locked
(Alford, Rajagopal, Wilczek, ...)

Crossover at zero net density: see no evidence of phase transition in pressure, entropy, or energy density.

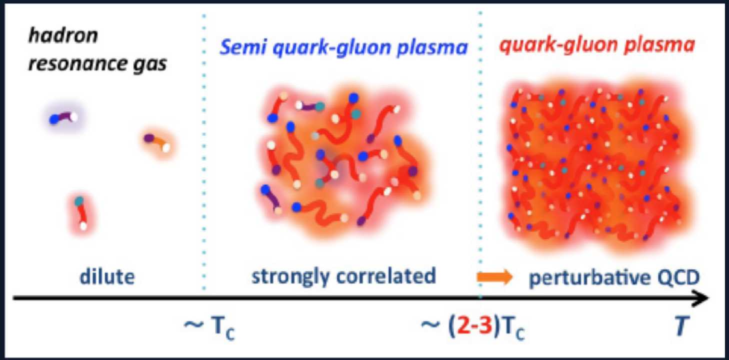




Wuppertal-Budapest lattice collaboration

WB: S. Borsanyi et al., PLB (2014)

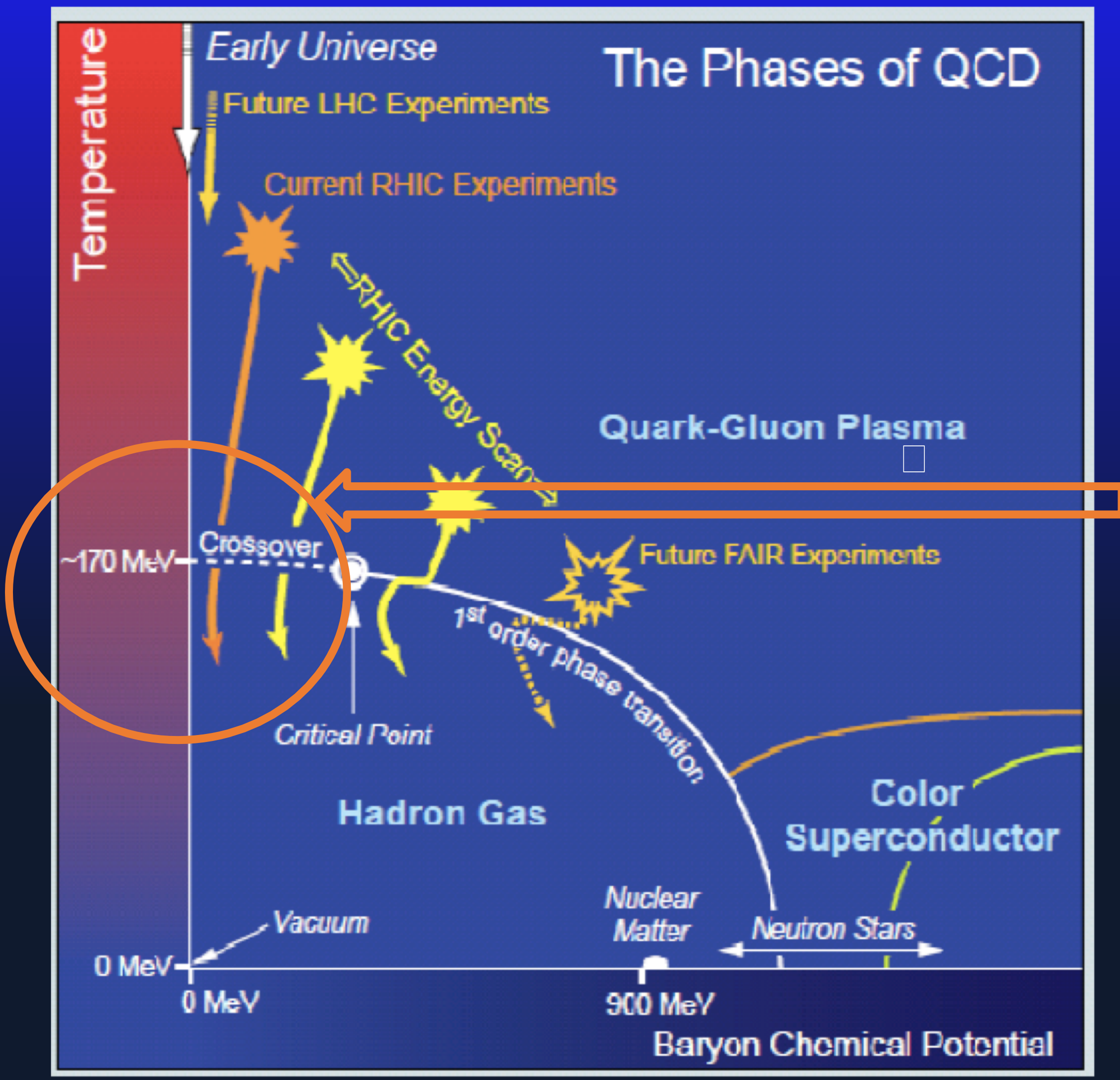
HotQCD: A. Bazavov et al., PRD (2014)



Lattice gauge theory not yet well implemented for finite baryon density!!

Fermion sign problem

Crossover at zero net baryon density



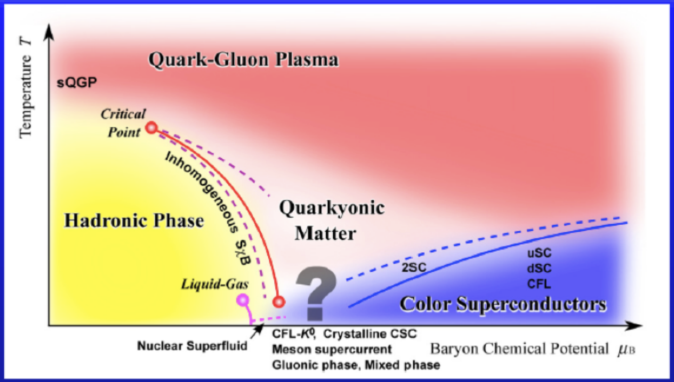
QCD lattice gauge theory -- for finite light quark masses -- predicts crossover from confined phase at lower T to deconfined phase at higher T.

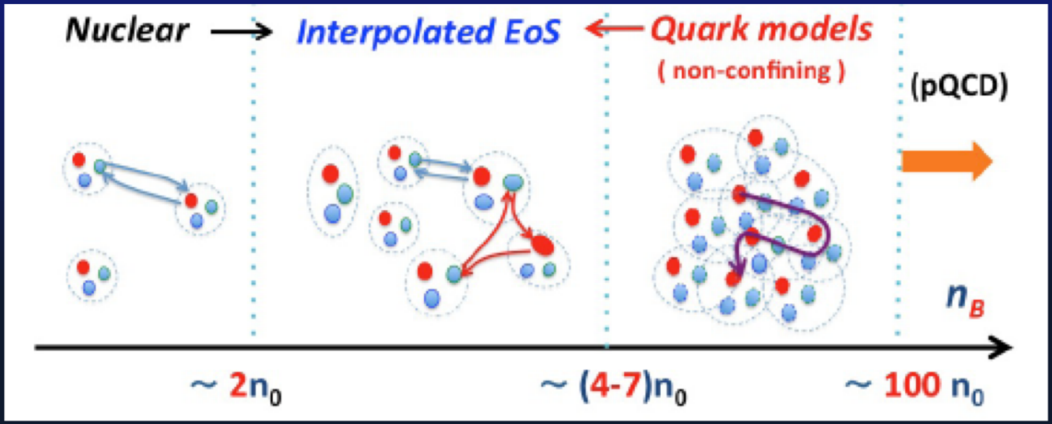
Do quarks roam freely in the deconfined phase? If so, they must also roam freely at lower T.

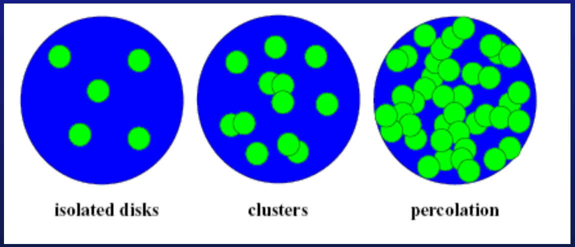
Are there really quarks running about freely in this room?

No free quarks even above the crossover!

In confined region quarks are inside hadrons. Also have quarks and antiquarks in the QCD forces between hadrons. With higher density or temperature, form larger clusters, which percolate at the crossover. In deconfined regime clusters extending across all of space.







Percolation of clusters along the density axis, at zero temperature.

Quarks can still be bound even if deconfined.

$n_{perc} \sim 0.34\ (3/4\pi\ r_n^3)\ \ \mathrm{fm}^{-3}$

r_n = nucleon radius

n_0 = density of matter inside large nucleus.

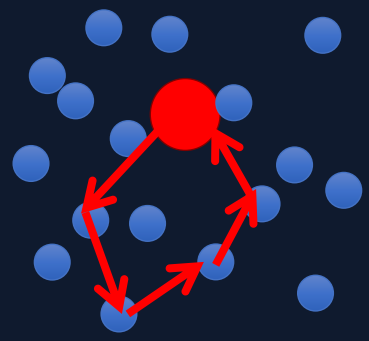
Classical vs. quantum percolation

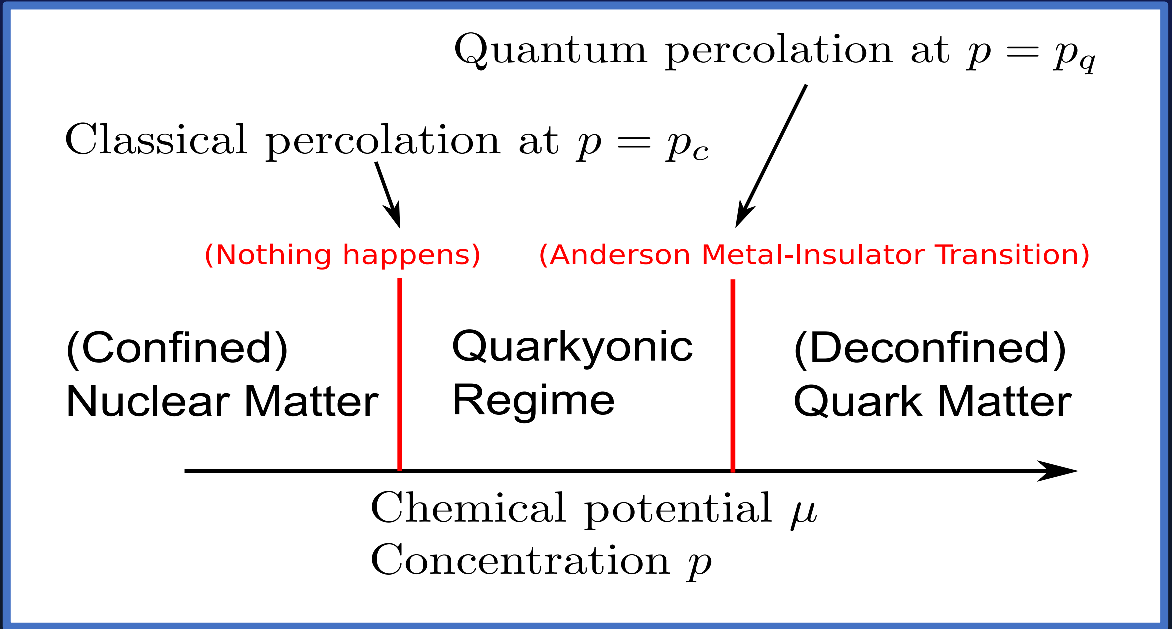
But aren't nucleons, with long distance cloud of mesons always overlapping?

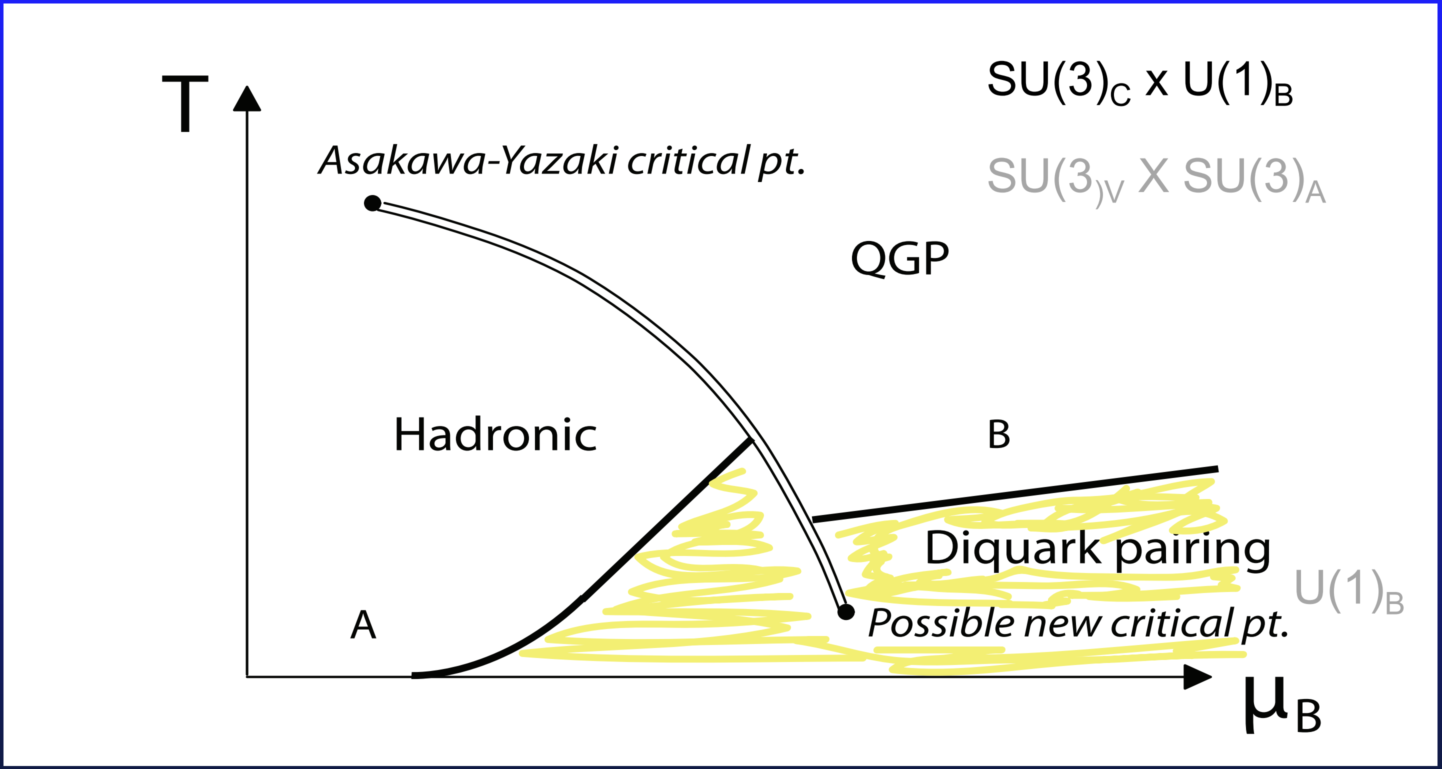
Does anything actually happen at classical percolation transition? No obvious lattice calculation to do!

Distinguish classical (geometric) percolation
from quantum percolation in terms of wave functions

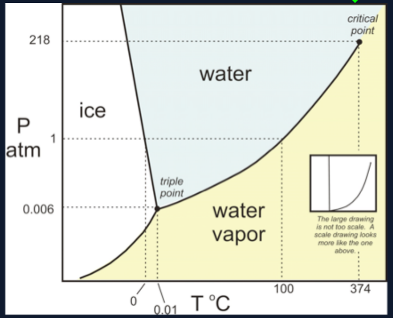
Deconfinement as (inverse) Anderson localization
(K. Fukushima):







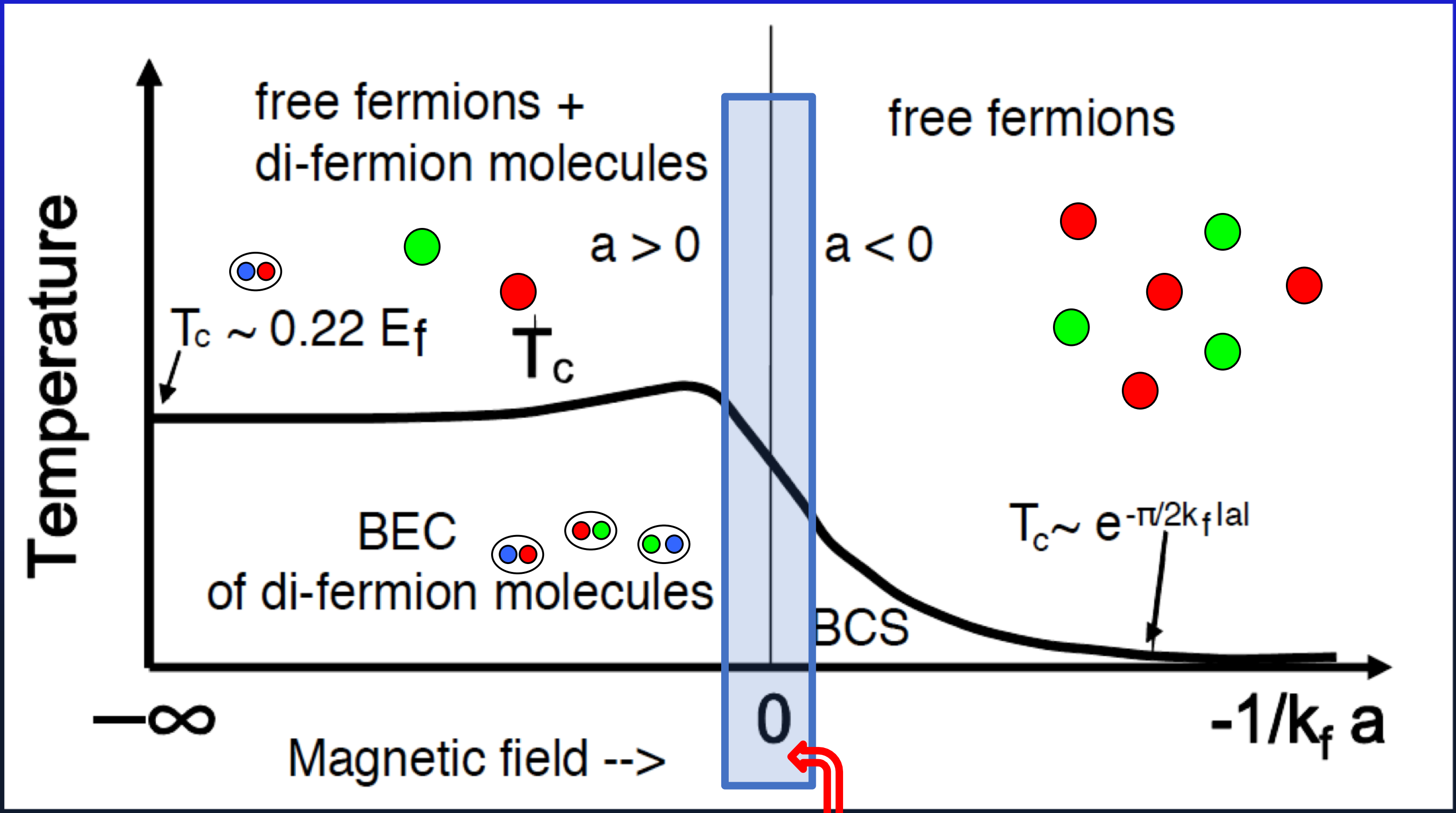
Critical points similar to those in liquid-gas phase diagram (H_2O). Neither critical point necessary!!



Can go continuously from A to B around the upper critical point. Liquid-gas phase transition.

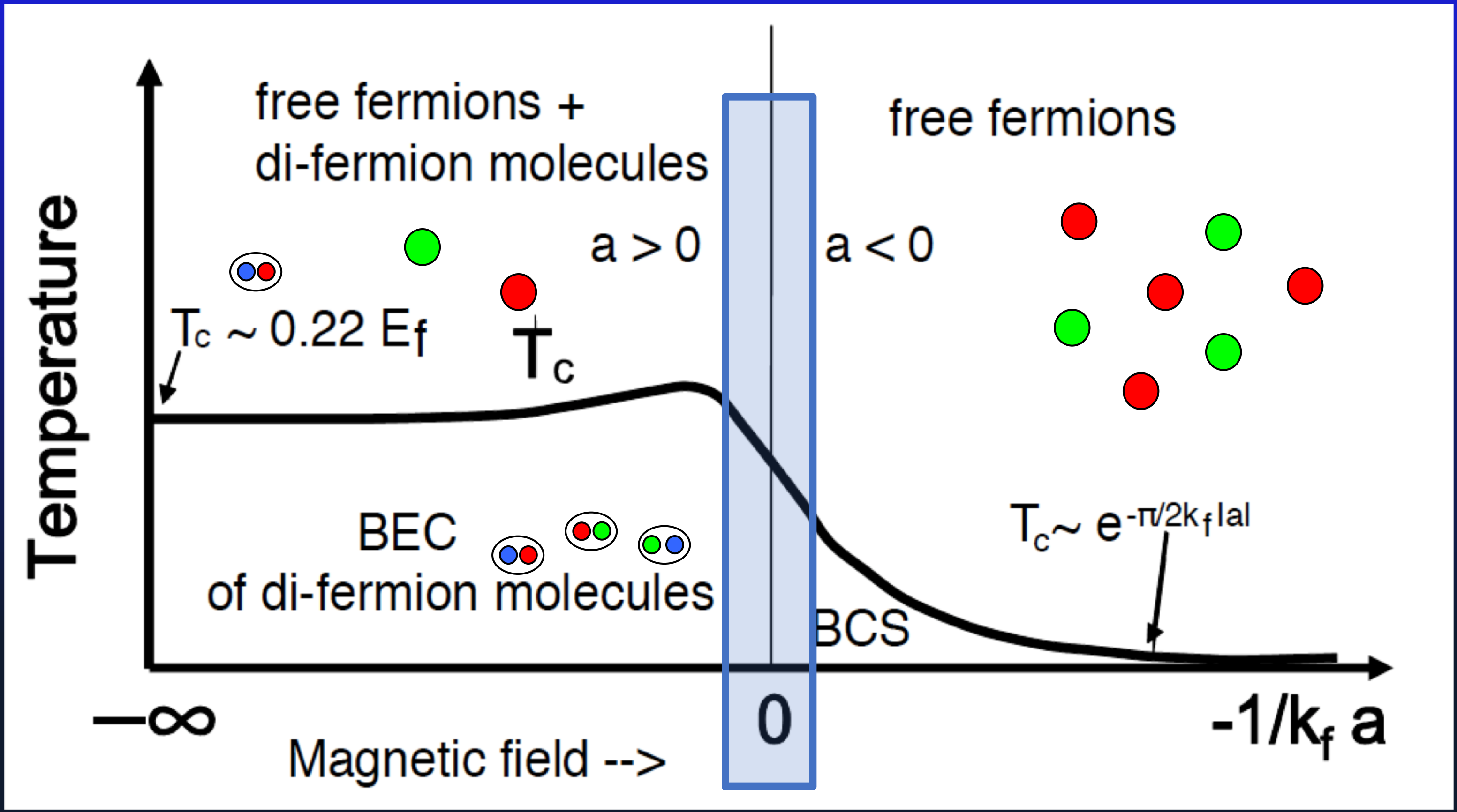
In lower shaded region have BCS pairing of nucleons, of quarks, and possibly other states (meson condensates). Different symmetry structure than at higher T.

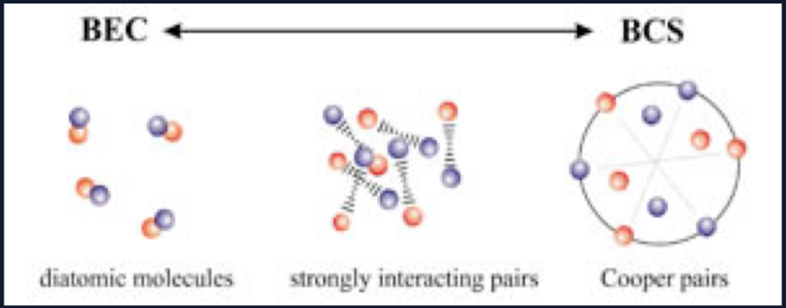
Phase diagram of ultracold atomic fermion gases: in T and strength of the particle interactions



Unitary regime (Feshbach resonance) – BEC-BCS crossover. No phase transition through crossover

Phase diagram of ultracold atomic fermion gases: in T and strength of the particle interactions

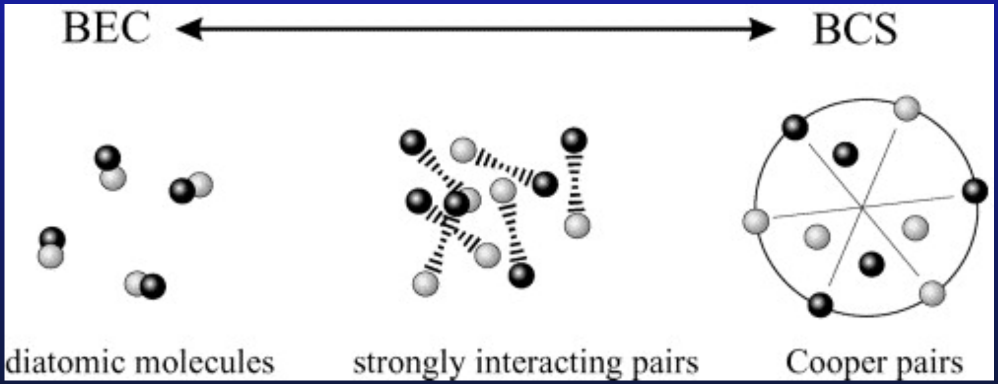




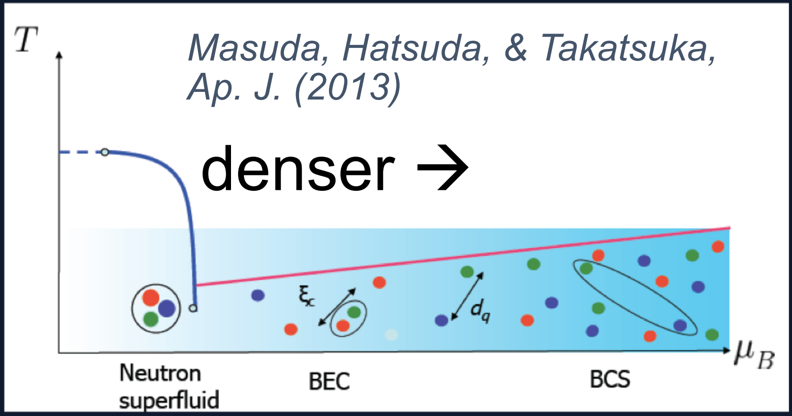
Smooth evolution of states in atomic clouds -- and nuclear matter (?)

GB, T.Hatsuda, M.Tachibana, & N.Yamamoto. J. Phys. G: Nucl. Part. 35, 10402 (2008)

Evolution of Fermi atoms with weakening attraction between atoms:

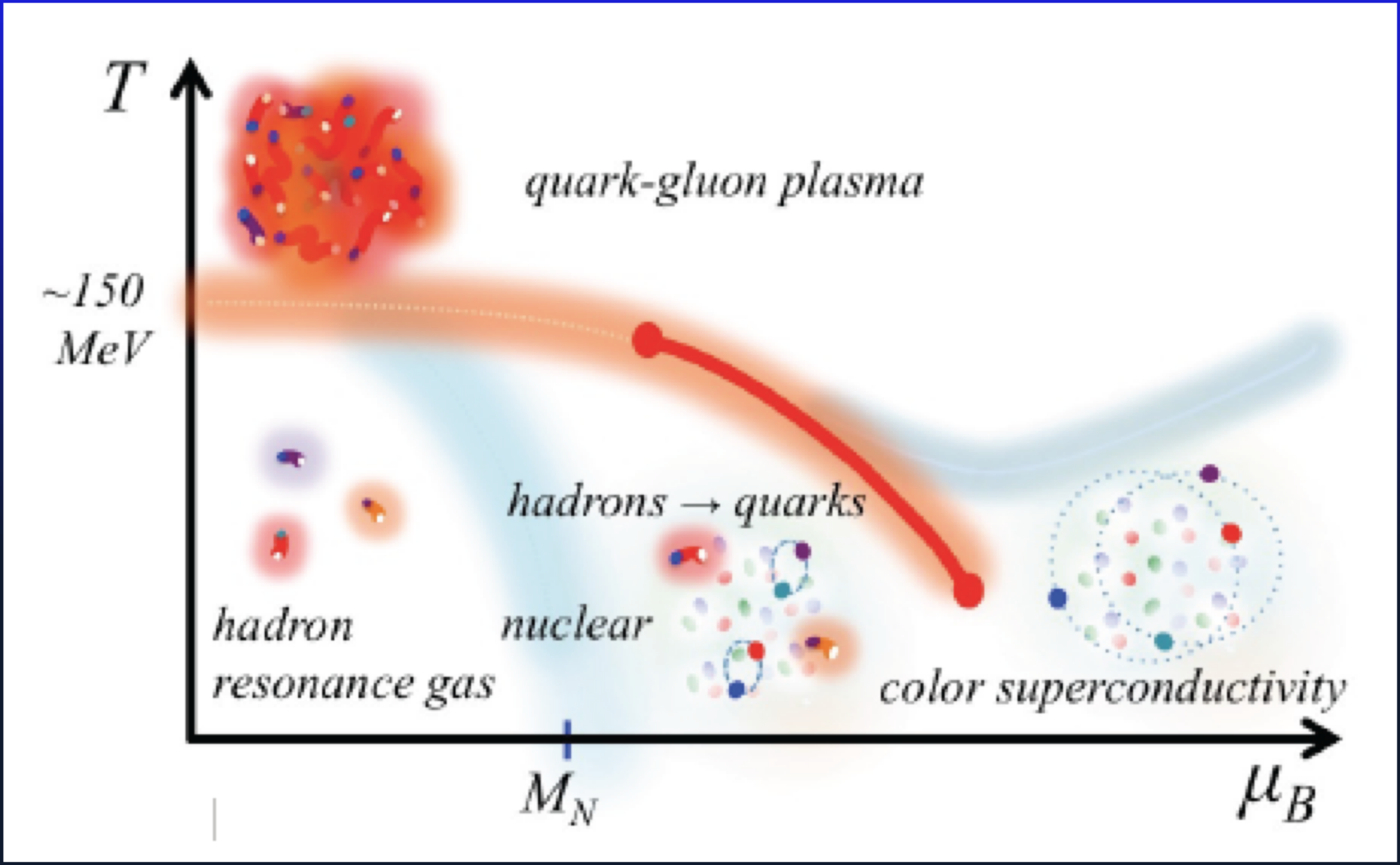


Similarly, as nuclear matter becomes denser can one expect "continuous" evolution from hadrons (nucleons) to quark pairs (diquarks)?



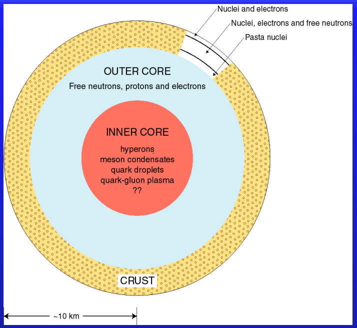
Quark hadron continuity

(Schäfer-Wilczek 1999)



T
~150
MeV
quark-gluon plasma
hadrons → quarks
hadron
resonance gas
nuclear
color superconductivity
M_N
μ_B

Quark matter cores in neutron stars

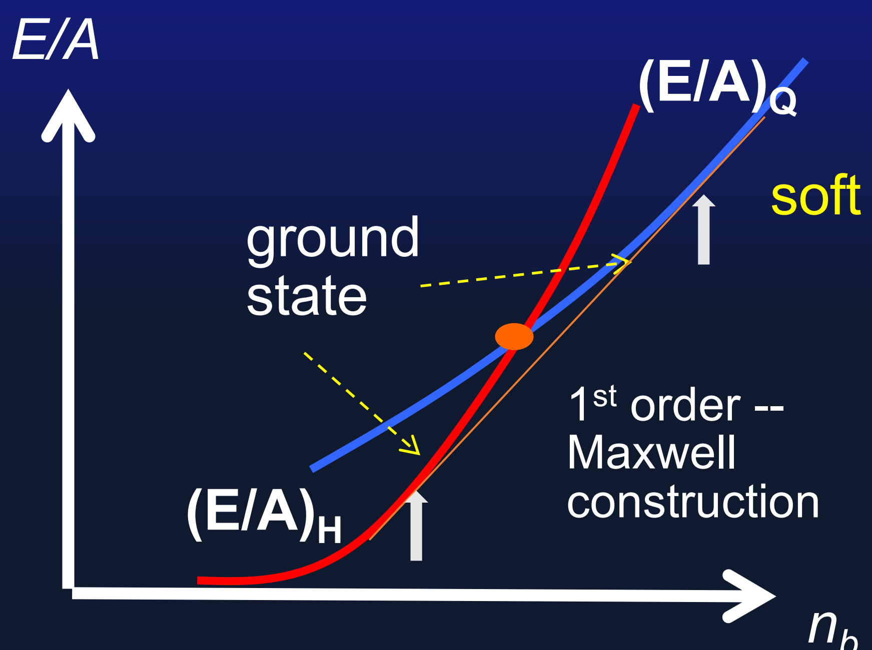


Canonical picture: compare calculations of eqs. of state of hadronic matter and quark matter.

GB & S.A. Chin (1976)

Crossing of thermodynamic potentials => first order phase transition.

ex. nuclear matter using 2 & 3 body interactions, vs. perturbative expansion or bag models.

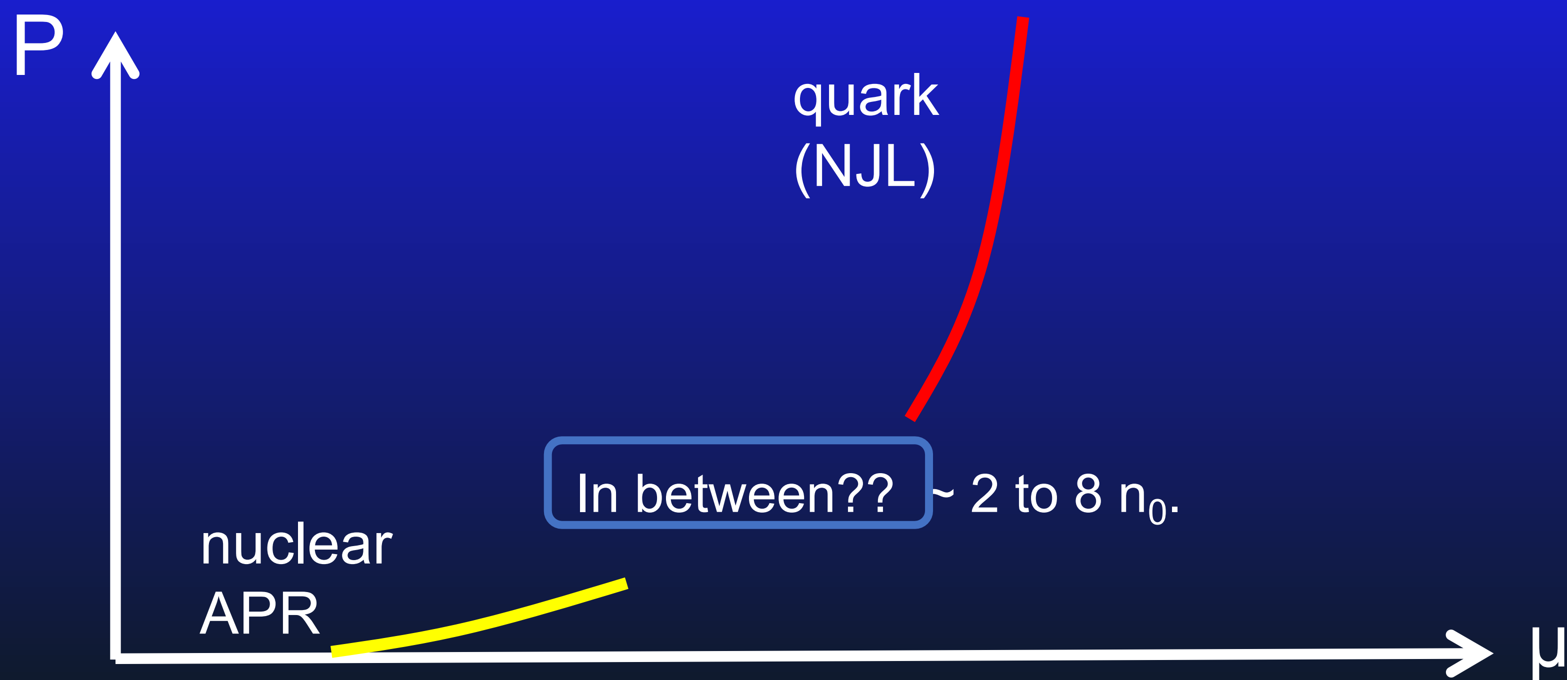


Assumes hadronic state at high densities – not possible when hadrons substantially overlap

Allows only quark equations of state lying under hadronic at high density. Soft only and therefore can't support two solar mass stars.

Typically conclude transition at $n \sim 10 n_{nm}$ -- would not be reached even in high mass neutron stars => at most small quark matter cores

Have good idea of equation of state at nuclear densities and at high densities. Look at pressure vs. baryon chemical potential



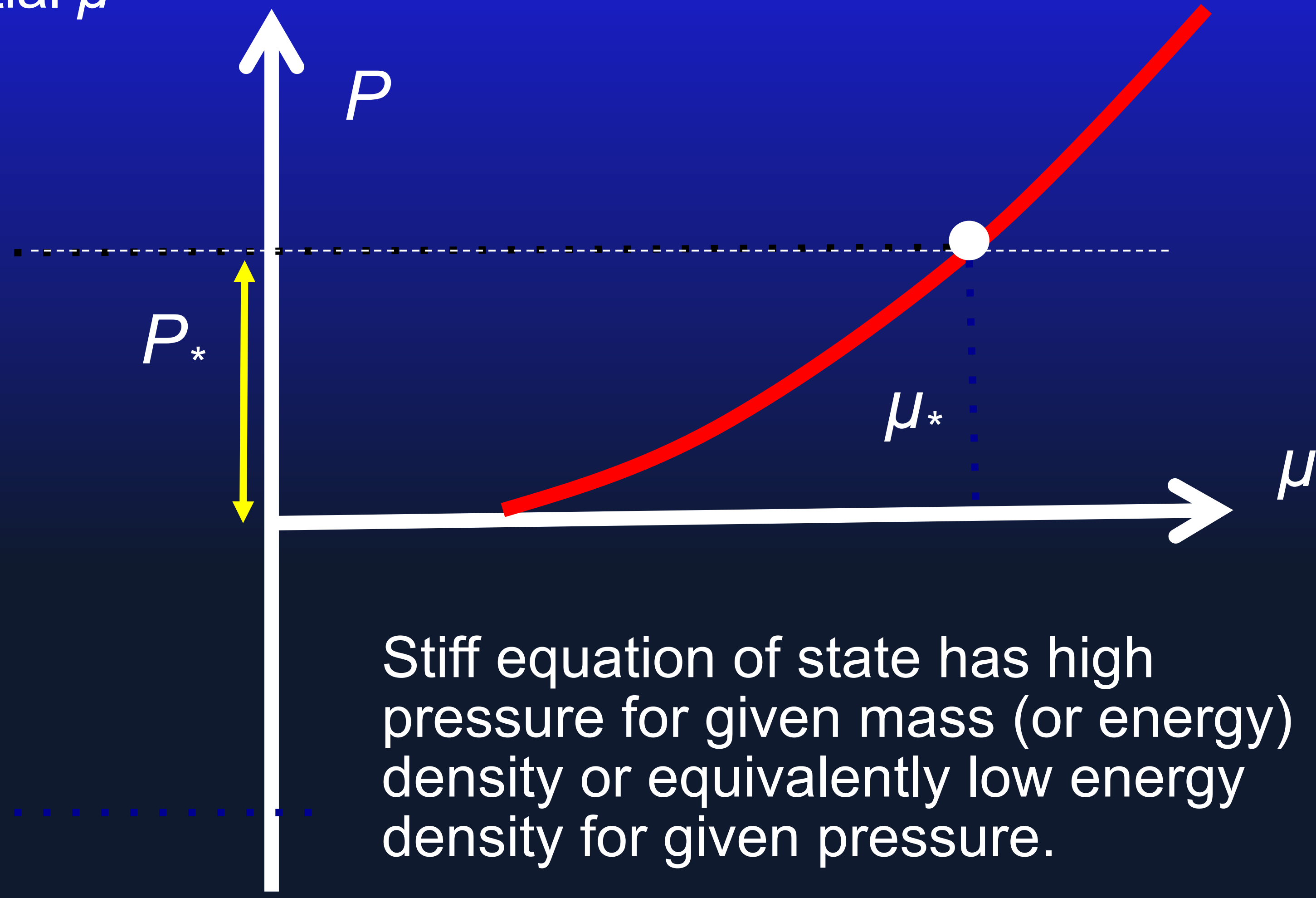
Quarks in Nambu-Jona-Lasinio (NJL) model with universal repulsive short-range qq coupling (*Kunihiro*)

$$\mathcal{L}_V^{(4)} = -g_V \left(\overline{q}\gamma^\mu q\right)^2$$

APR = Akmal, Pandharipande, Ravenhall nucleonic equation of state with nucleonic potentials (2 and 3 body) fit to NN scattering and light nuclei

How can QCD give large mass neutron stars?

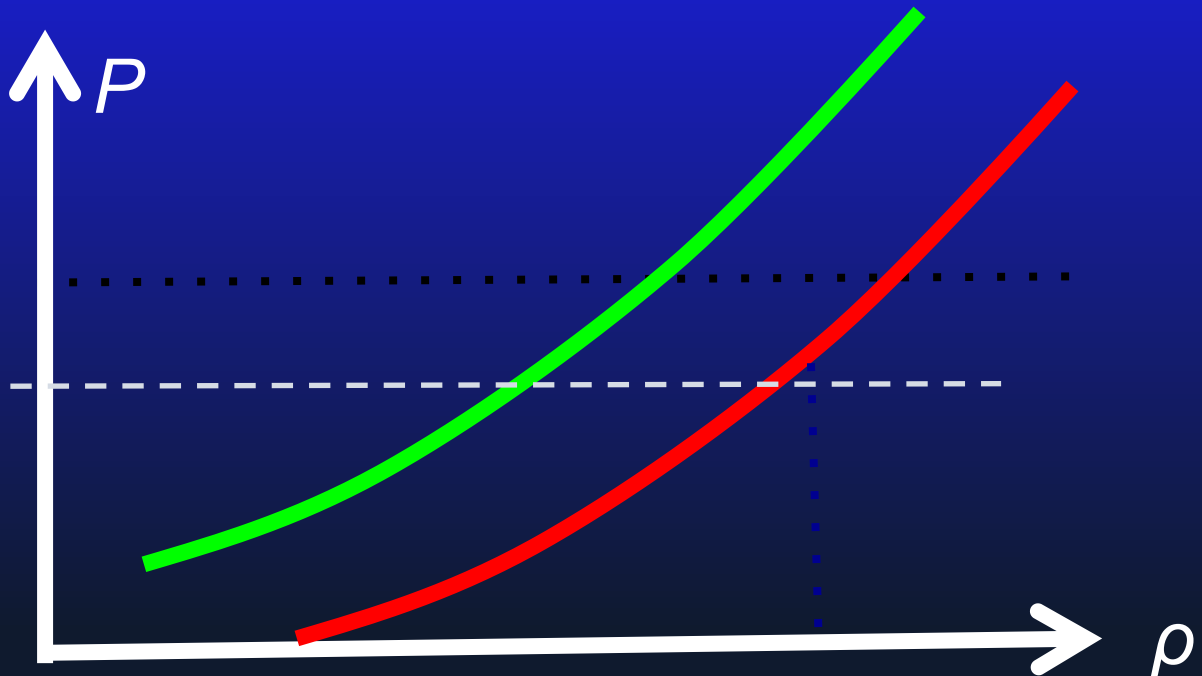
Pressure P is a continuous function of baryon chemical potential μ



Stiff equation of state has high pressure for given mass (or energy) density or equivalently low energy density for given pressure.

T. Kojo, P. D. Powell, Y. Song, & GB, PR D 91, 045003 (2015)

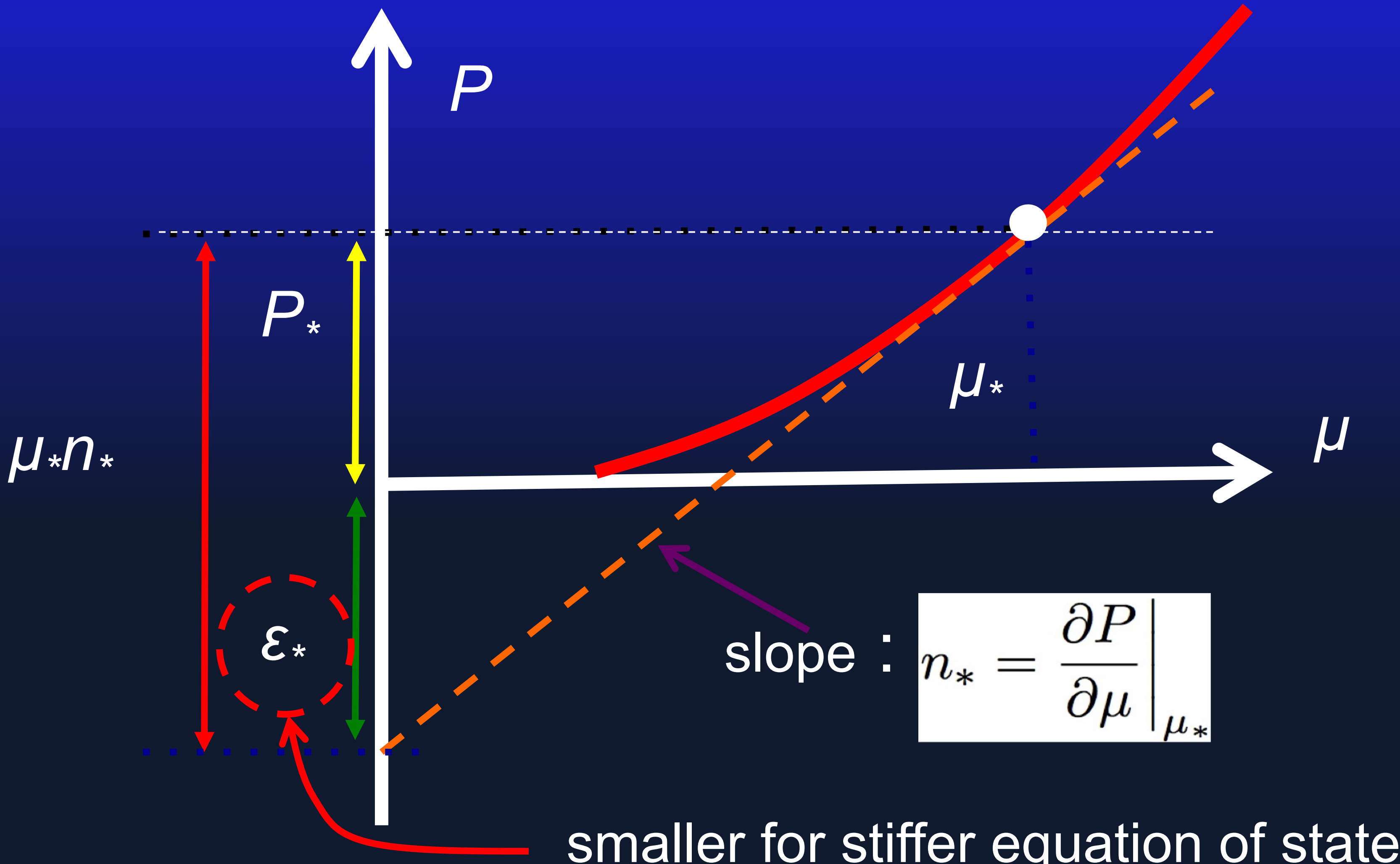
Stiffer equations of state given more massive neutron stars, with lower central densities



Green equation of state is stiffer than red.
Has larger pressure for given mass density *ρ*,
and has smaller *ρ* for given pressure P

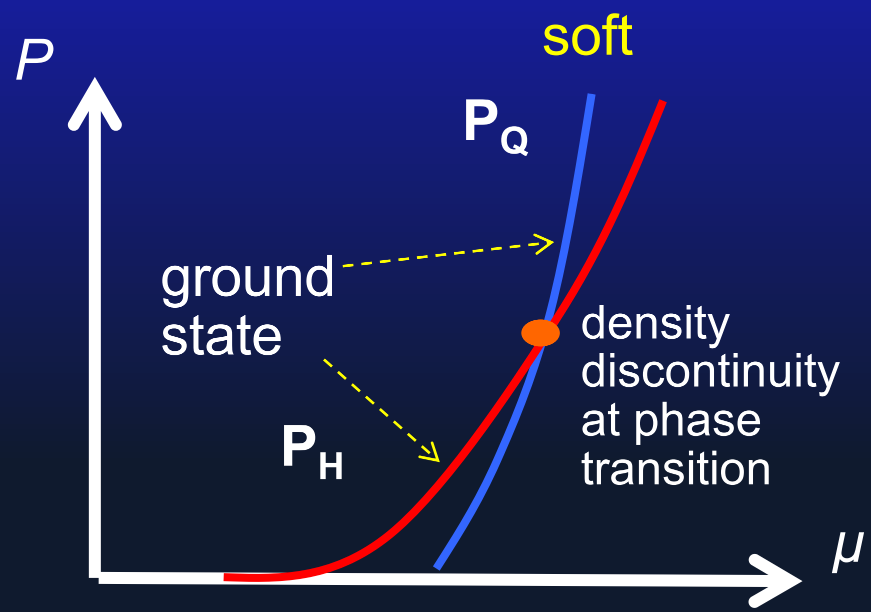
How can QCD give large mass neutron stars?

Energy or mass density $\varepsilon = \rho c^2 = \mu n - P$



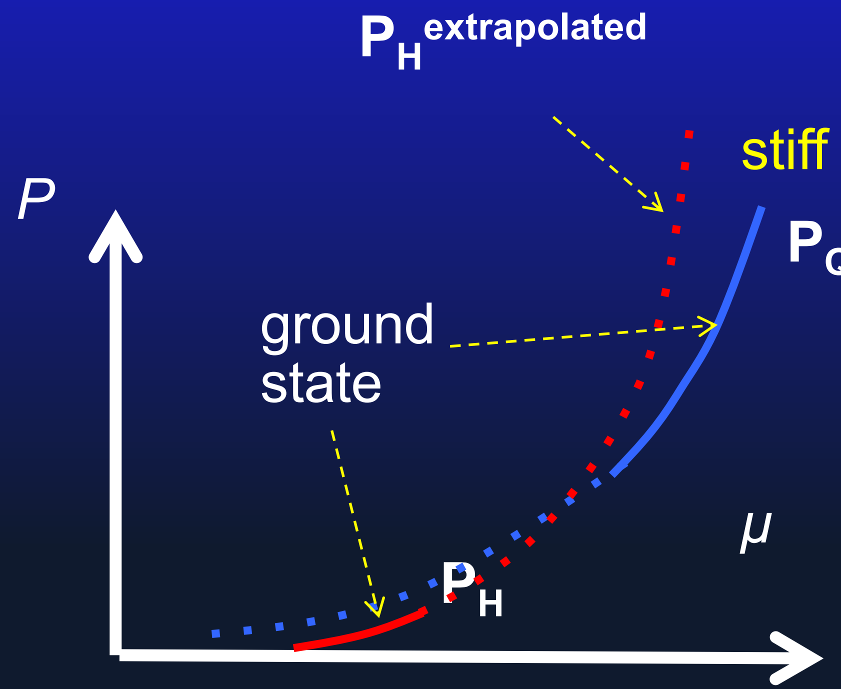
Hybrid eqs. of state are intrinsically softer

Phase with larger P at given μ thermodynamically preferred



Assumes hadronic state at high densities – not possible when hadrons substantially overlap

Continuous eqs. of state can be much stiffer



Hadrons only at low density and quark matter at high density. In between???

Model calculations of neutron star matter within NJL model

NJL Lagrangian $\mathcal{L} = \bar{q}(i\gamma_\mu\partial^\mu - m_q + \mu\gamma_0)q + \mathcal{L}^{(4)} + \mathcal{L}^{(6)}$

$$\mathcal{L}_\chi^{(4)} = G\sum_{a=0}^{8}[(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma_5\tau_a q)^2]$$

chiral interactions

$$\mathcal{L}_d^{(4)} = H\sum_{A,A'=2,5,7}[(\bar{q}i\gamma_5\tau_A\lambda_{A'}C\bar{q}^T)(q^T Ci\gamma_5\tau_A\lambda_{A'}q)$$

BCS pairing interactions

$\mathcal{L}^{(6)}$ = Kobayashi-Maskawa-'t Hooft six quark axial anomaly

plus universal repulsive quark-quark vector coupling

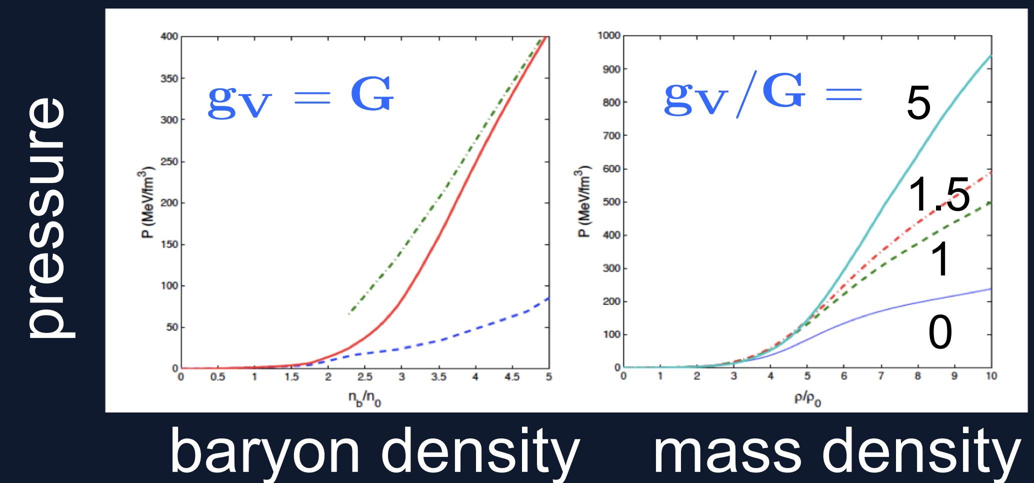
$$\mathcal{L}_V^{(4)} = -g_V\left(\overline{q}\gamma^\mu q\right)^2$$

T. Kunihiro

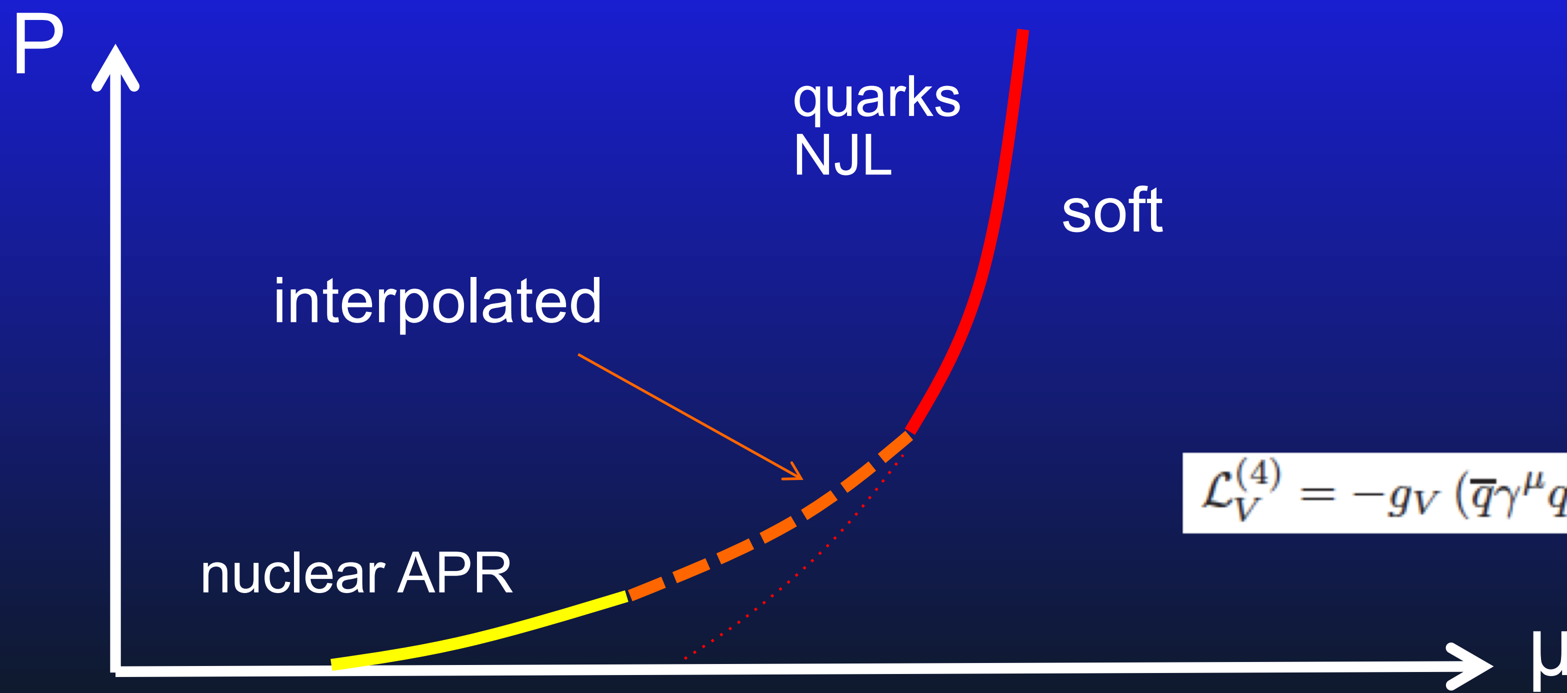
Include u,d, and s quarks

K. Masuda, T. Hatsuda, & T. Takatsuka, Ap. J.764, 12 (2013)

GB, T. Kojo, T. Hatsuda, T. Takatsuka, & Y. Song ROPP 81 (2018) 056902



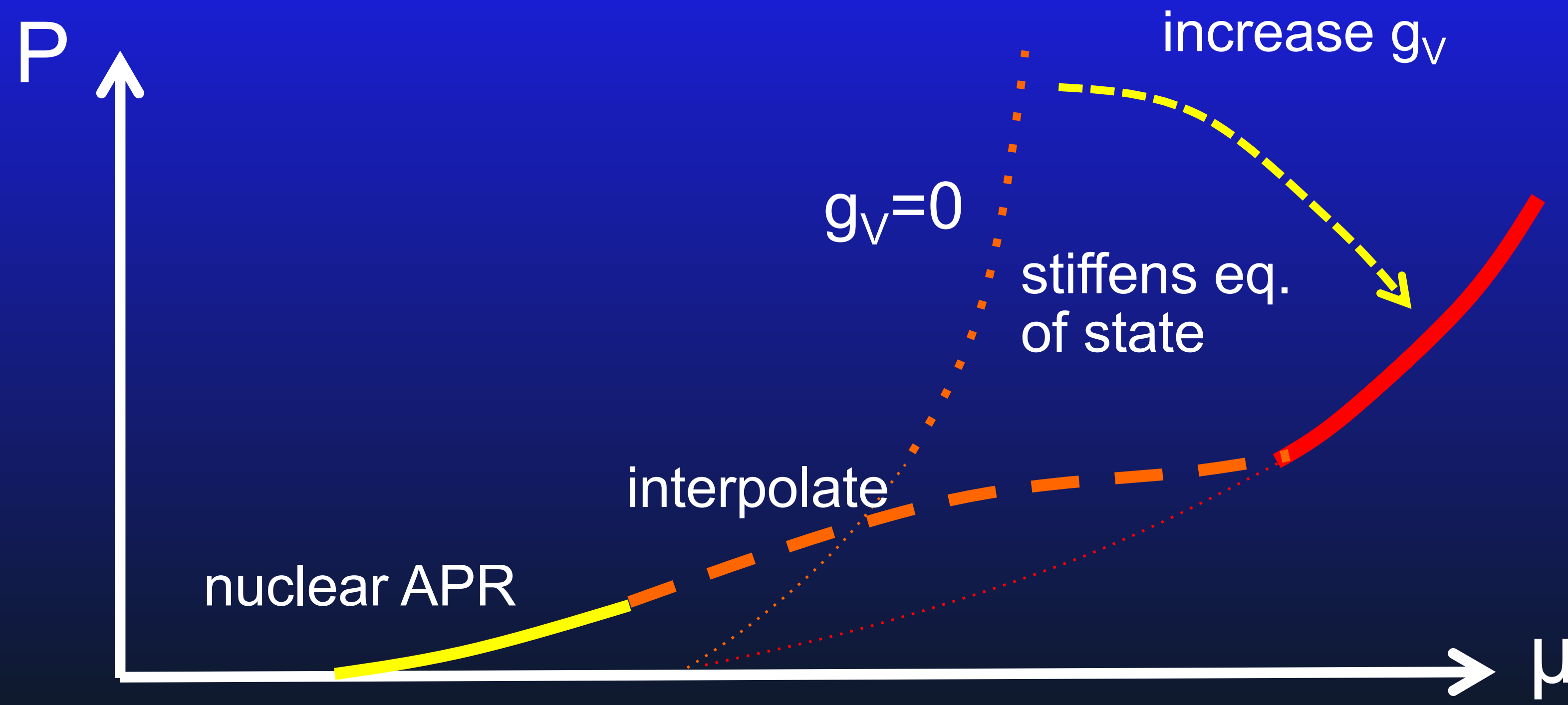
Minimal model: $g_V = 0$



$$\mathcal{L}_V^{(4)} = -g_V\left(\overline{q}\gamma^\mu q\right)^2$$

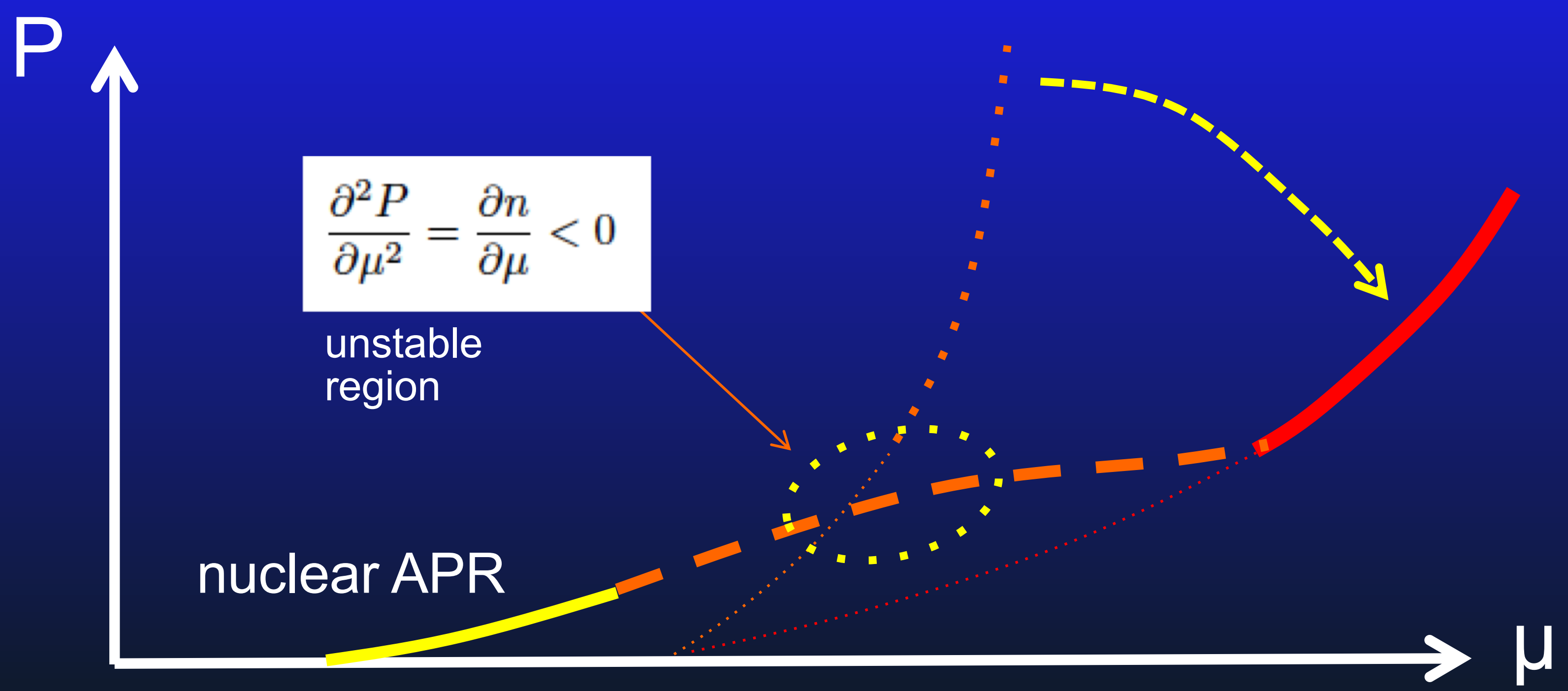
Soft quark equation of state does not allow high mass neutron stars

Vector interaction stiffens eq. of state



Shift of pressure in quark phase towards higher μ

Vector interaction stiffens eq. of state

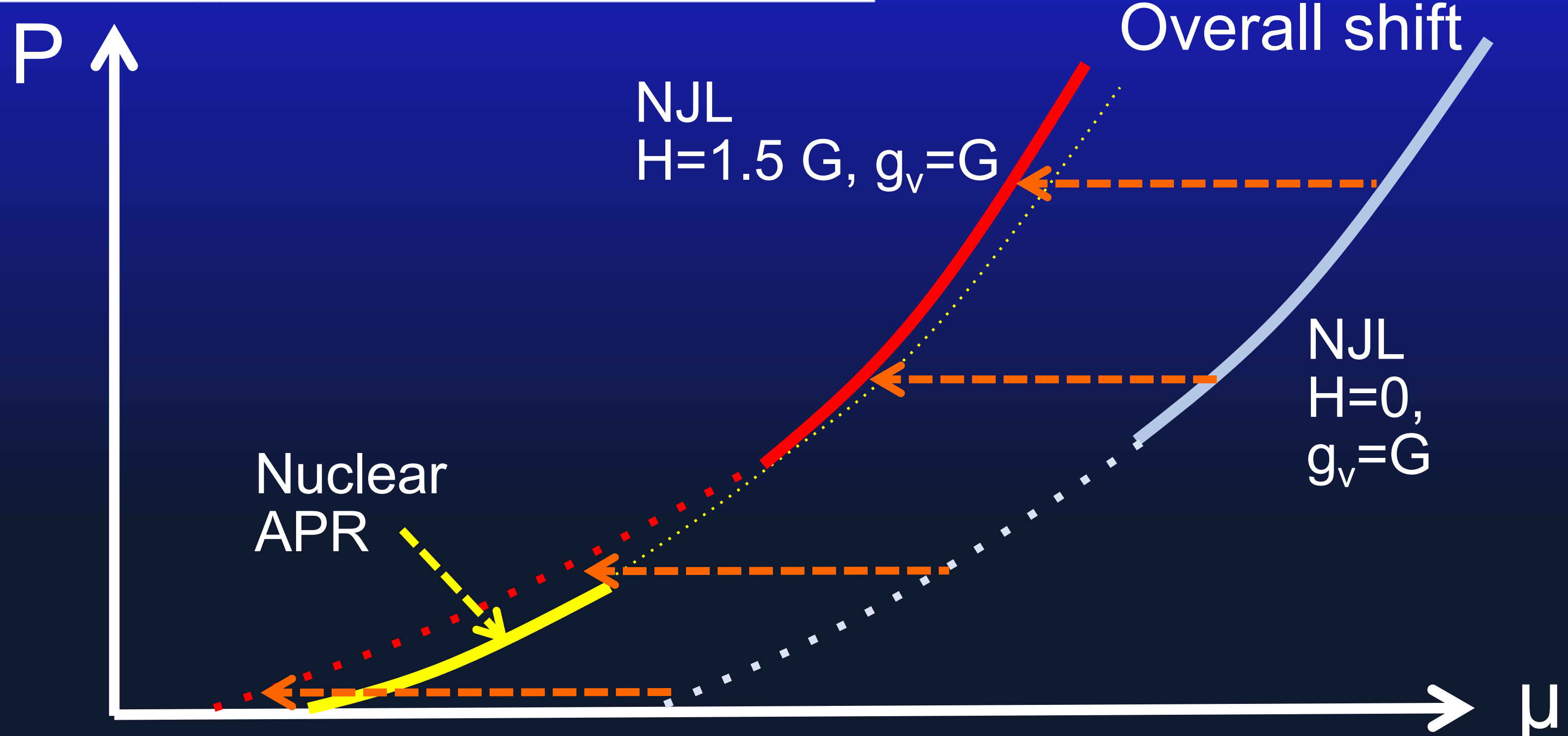


Larger g_V leads to unphysical thermodynamic instability


In this house, we obey
the laws of thermodynamics!

Restore stability with increased BCS (diquark) pairing interaction, H

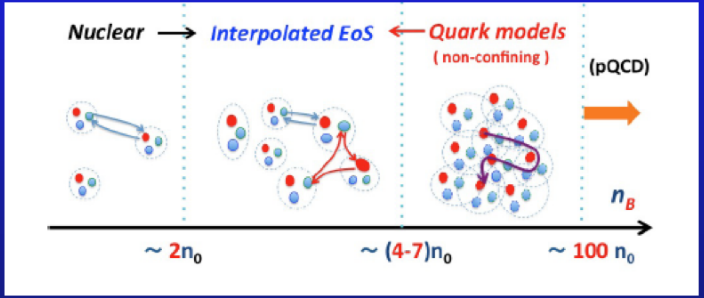
$$\mathcal{L}_d^{(4)} = H \sum_{A,A'=2,5,7} [(\bar{q} i\gamma_5 \tau_A \lambda_{A'} C \bar{q}^T)(q^T C i\gamma_5 \tau_A \lambda_{A'} q)$$



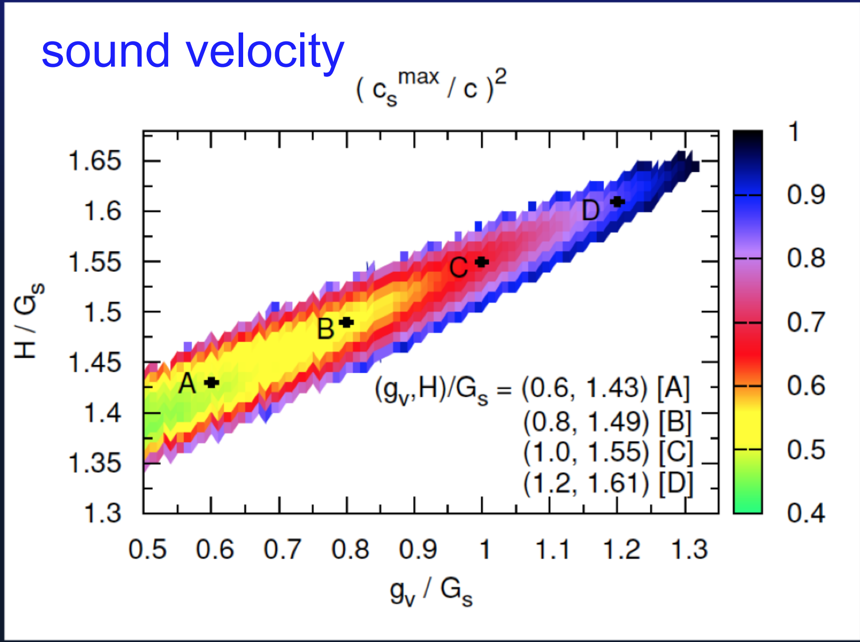
Increased BCS pairing (onset of stronger 2-body correlations) as quark matter comes nearer to becoming confined

QHC19 (quark-hadron crossover) equation of state:

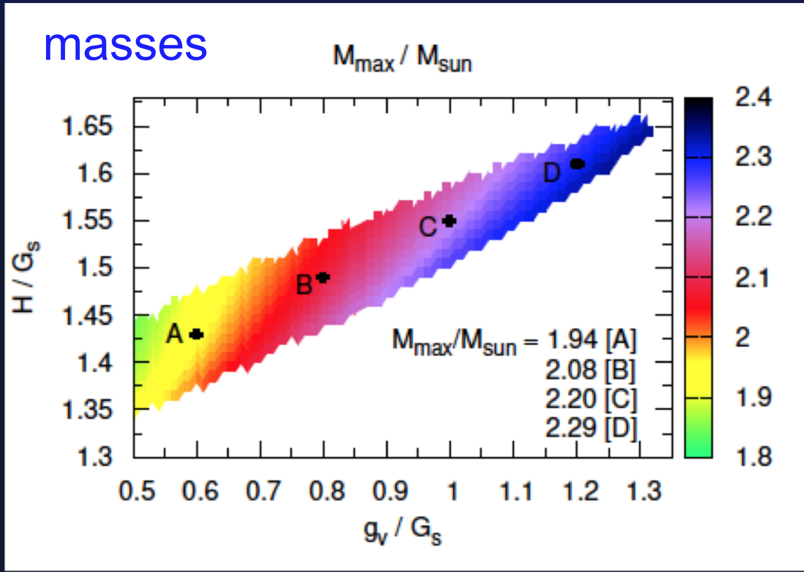
GB, S. Furusawa, T. Hatsuda, T. Kojo & H. Togashi, arXiv:1903:08963



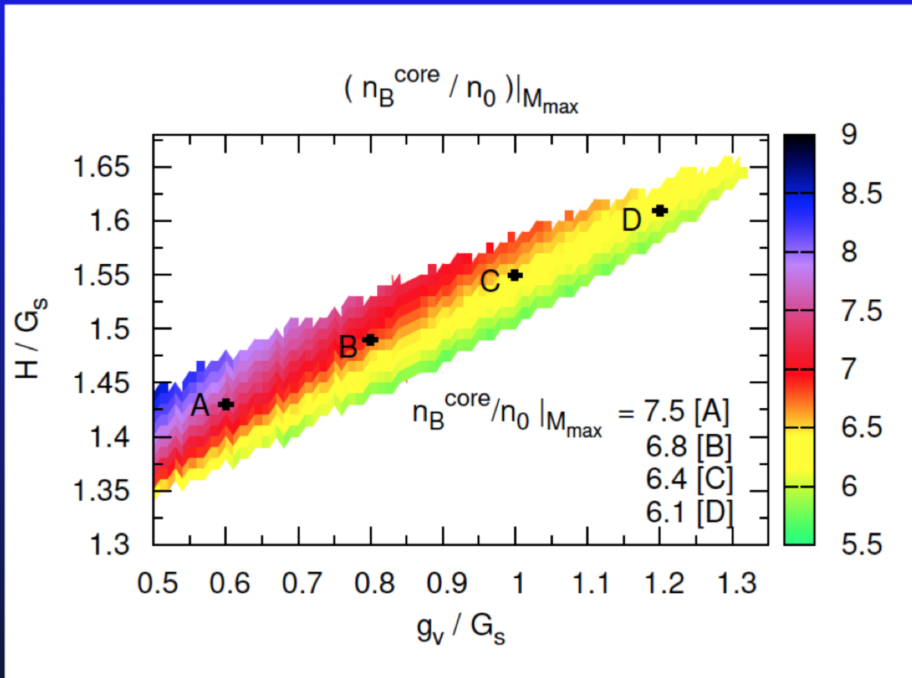
Parameters g_v and H severely restricted by requirement that sound velocity does not exceed speed of light. Must be in colored region.

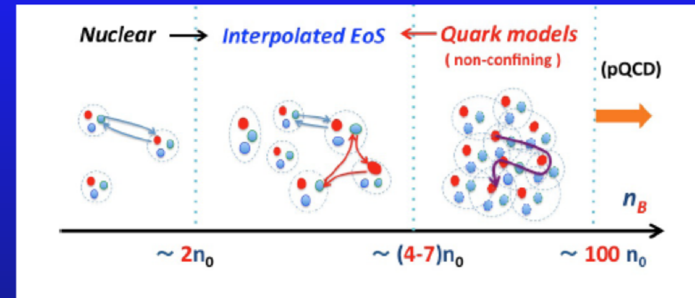


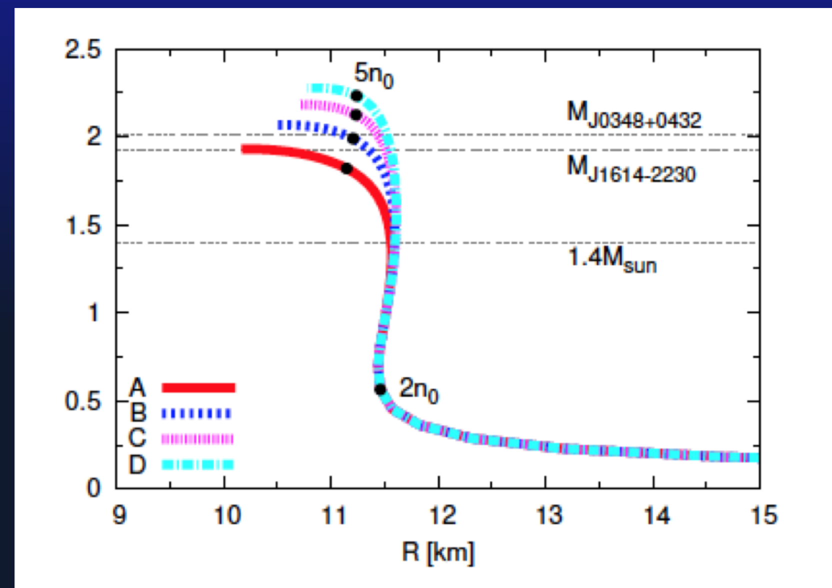
Further restriction that maximum neutron star mass > 2 solar masses

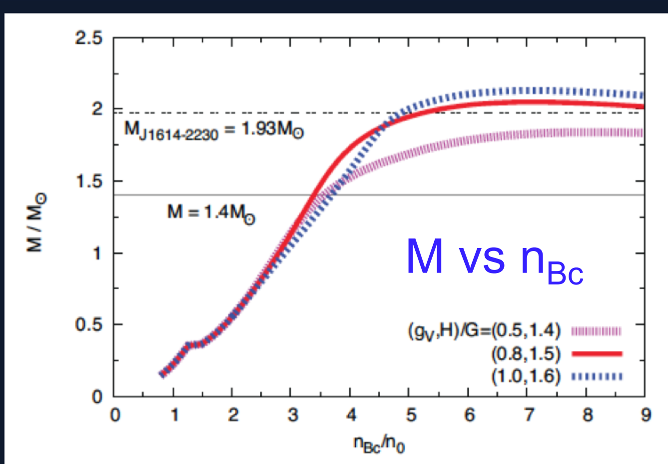


Maximum central density in star





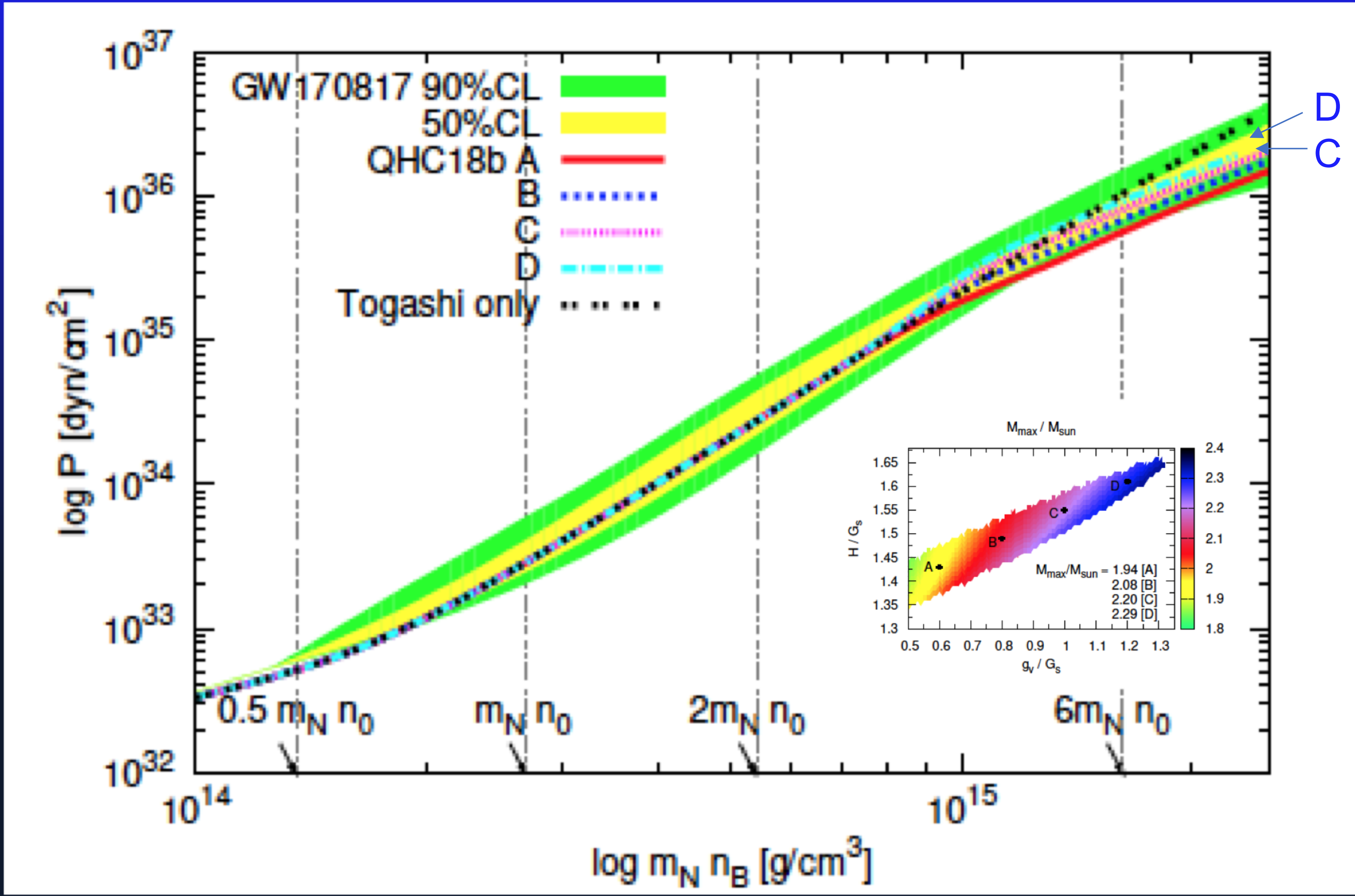




Neutron star radius vs. mass
(tune better with NICER data)

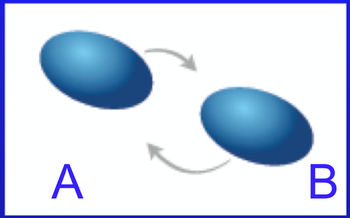
Compare QHC18 with LIGO inference of pressure vs. rest mass density,

B. P. Abbott et al. (LIGO & Virgo) PRL 121, 161101 (2018)



Also consistent with eq. of state inferred from M vs. R observations

Neutron star tidal deformability



Orbiting neutron stars each deform each other, inducing quadrupolar distortion in the other

$$Q_{ij} = \int d^3r\ \rho(r) \left(r_i r_j - \frac{1}{3}\delta_{ij} r^2 \right)$$

quadrupole moment of star; measure r from center of A

$$\Phi_{\rm tidal}(\vec{r}) = \frac{G_N M_{\rm B}}{2R^3} \left(r^2 - 3(\vec{r}\cdot\hat{R})^2 \right)$$

tIdal force of B felt by A; R = separation of two stars

$$\boxed{g_{00} \sim -G\left(\frac{M}{r} - \frac{3Q_{ij}}{2r^3}\hat{n}_i\hat{n}_j + \ldots \right) + \frac{1}{2}\mathcal{E}_{ij}\hat{n}_i\hat{n}_j}$$

metric perturbation. $\hat{n}$ = unit vector from A to B

$$\mathcal{E}_{ij} = \frac{\partial^2 \Phi_{\rm tidal}}{\partial r_i \partial r_j}$$

tidal tensor from B as felt by A.

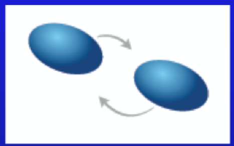
$$Q_{ij} = -\lambda_{\rm A}\mathcal{E}_{ij}$$

defines tidal deformability of A

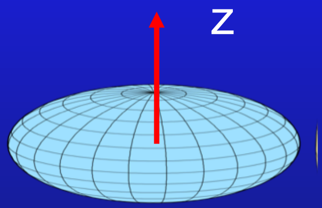
$$\Lambda = 32\frac{\lambda G}{R_s^5}$$

dimensionless tidal deformability, where $R_S = \dfrac{2MG}{c^2}$ is the Schwarzschild radius

Model calculation for self-gravitating classical incompressible star



R = distance between the two stars
R_0 = radius of unperturbed star



$\epsilon < 0$

Quadrupolar deformation of star

$$\delta R_A = P_2(\cos\theta)\epsilon R_0$$

Produces quadrupole moment $Q_{zz} = (2/5)M_A R_0^2 \epsilon$

Decreases gravitation energy of star by $\Delta E_{\mathrm{def}} = (3/25)GM_{\mathrm{A}}^2\epsilon^2/R_0$

Interaction energy with tidal force $\Delta E_{\mathrm{tide}} = -(3/5)G_N M_{\mathrm{A}} M_{\mathrm{B}} R_0^2 \epsilon/R^3$

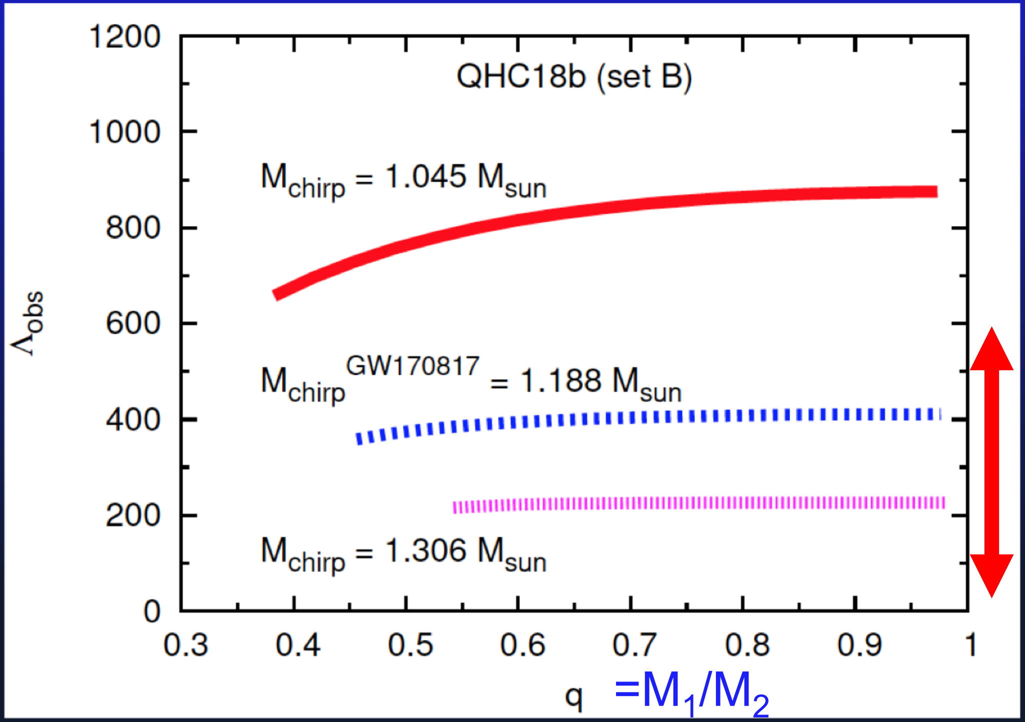
$$\frac{\partial}{\partial\epsilon}(\Delta E_{\mathrm{def}} + \Delta E_{\mathrm{tide}}) = 0 \implies \epsilon = (5/2)(M_{\mathrm{B}}/M_{\mathrm{A}})(R_0/R)^3$$

Induced quadrupole moment in A $\quad Q_{zz} = M_{\mathrm{B}} R_0^5/R^3$

$$\lambda_A = R_0^5/G$$

$$\boxed{\Lambda = 16(R_0/R_S)^5}$$

Neutron star tidal deformability in QHC19



$\Lambda_{M=1.4}$ = 190 (+390 -120)
from GW70817

For 2 $n_0 < n_B <$ 7-8 n_0 matter is intermediate between purely hadronic and purely quark

Quark model eqs. of state can be stiffer than previously thought, allowing for neutron star masses > 2 $M_\odot$

Use QHC19 in neutron star merger simulations!!!
https://compose.obspm.fr/eos/140/.

Much more to do:

Uncertainties in nuclear matter equation of state (APR, Togashi, etc.)

Uncertainties in interpolating from nuclear matter to quark matter lead to errors in maximum neutron star masses and radii

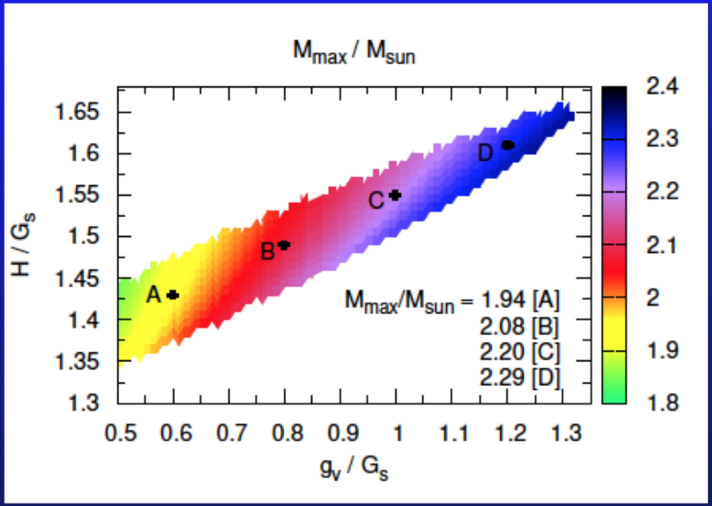
Uncertainties in the vector coupling and pairing forces;

Going beyond the NJL model -- running g_v

Finite temperature equation of state (≤ 100 MeV) for modelling neutron star -- neutron star (or black hole) mergers as sources of gravitational radiation (GB, S. Furusawa, T. Hatsuda, T. Kojo, H. Togashi)

Cooling and transport properties

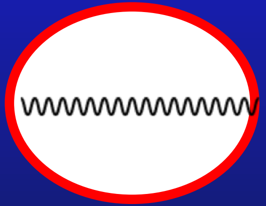
The parameters g_V and H more microscopically



Y. Song, GB, T. Hatsuda, T. Kojo

$$L \sim G(\bar{q}q)^2 + H(\bar{q}\bar{q})(qq) - g_V(\bar{q}\gamma^\mu q)^2$$

Perturbative QCD (one loop)



=>

energy contribution from quark vector channel

$$E_{\rm QCD}^{\rm vec} = \frac{4\pi\alpha_s}{27}\int_{p,p'} \mathrm{Tr}[S(p)\gamma^\mu]\mathrm{Tr}[S(p')\gamma_\mu]D(p-p') = \frac{3\alpha_s p_F^4}{2\pi^3} = g_V n_q^2$$

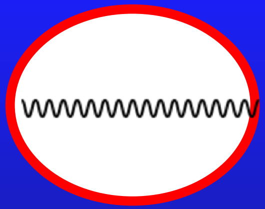
S(p) = free quark Green's function

D = $1/(p-p')^2$ = massless gluon Green's function

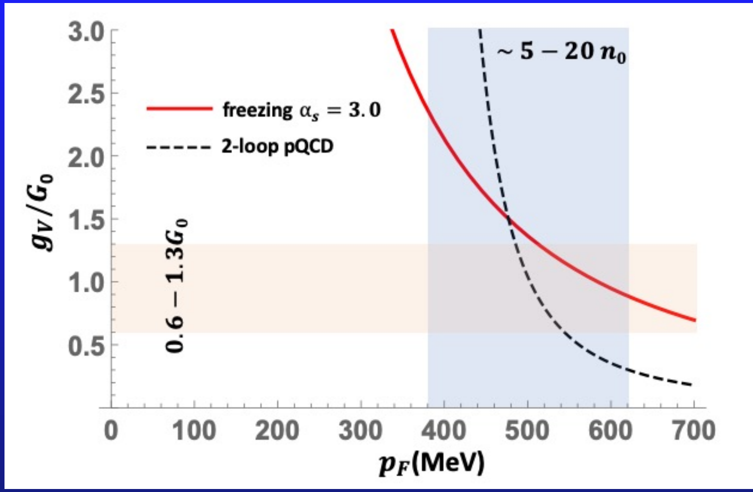
(tensor decorations in numerator drop out)

$n_q = 3\, n_B$ => $n_B = p_F^3/\pi$

$$\boxed{g_V = \frac{\pi\alpha_s}{6p_F^2}}$$



=> $g_V = \frac{\pi \alpha_s}{6 p_F^2}$



One loop running

$$\alpha_s(\mu) = \frac{6\pi}{(33 - 2N_f)\ln(\mu/\Lambda_{QCD})}$$

Two loop running coupling:

Diverges as p_F -> 0. Wrong.

$$\alpha_s(\mu_q) = \frac{4\pi}{9 \ln \tilde{\mu}^2}\left(1 - \frac{64 \ln \ln \tilde{\mu}^2}{81 \ln \tilde{\mu}^2}\right)$$

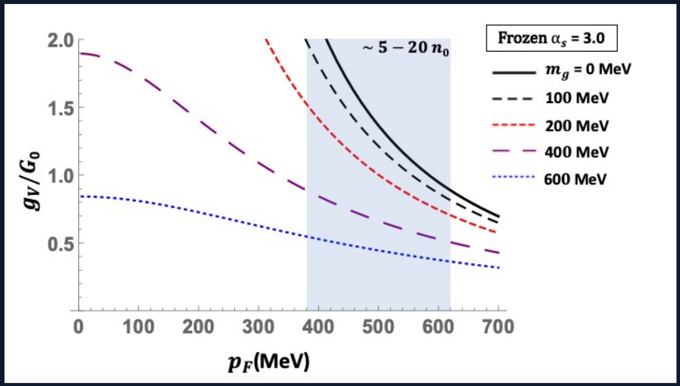
$\tilde{\mu} \equiv \mu_q/\Lambda_{\mathrm{QCD}}$

$\Lambda_{\mathrm{QCD}} = 340 \ \mathrm{MeV}$

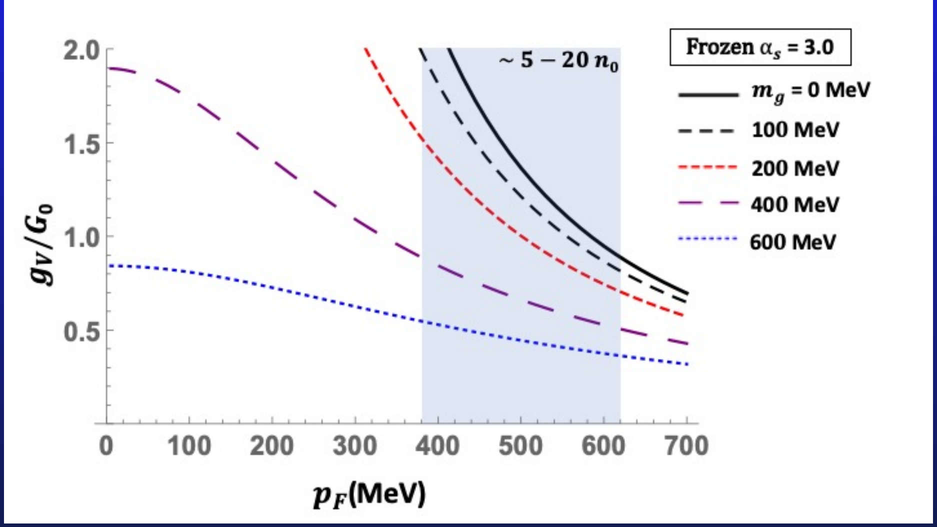
Running coupling without Landau pole. Dimensionally $g_v \sim \frac{1}{\Lambda_{QCD}^2}$

Include gluon mass m_g :

D(q) -> $1/(q^2 - m_g^2)$



Include gluon mass m_g : $D(q) \to 1/(q^2 - m_g^2)$



Regularizes behavior at low density

Approximate parametrization:

$$g_V(p_F) \simeq \frac{4\pi\alpha_s}{27m_g^2 + 24p_F^2}$$

Expect further corrections from chiral quark masses and BCS (diquark) pairing.
These tend to be suppressed by finite gluon mass.

In future QHC include density dependence of g_V, as well as isovector repulsion

$$g_v(n_u + n_d + n_s)^2 + g_\tau^{(3)}(n_u - n_d)^2 + g_\tau^{(8)}(n_u + n_d - 2n_s)^2$$

Estimating H from N-Δ mass splitting

T. Kojo, to be published

N (spin ½, isospin=1/2) = nucleon, m = 940 MeV
Δ (spin 3/2, isospin = 3/2) = excited nucleon, m = 1232 MeV

$$\mathcal{L}_S = H \sum_{A=2,5,7} \left(\bar{\psi} i\gamma_5 \tau_2 \lambda_A \psi_C\right)\left(\bar{\psi}_C i\gamma_5 \tau_2 \lambda_A \psi\right),$$

Isoscalar scalar diquark pairing

$$\mathcal{L}_A = H' \sum_{A=2,5,7} \left(\bar{\psi} \gamma_\mu \tau_2 \vec{\tau} \lambda_A \psi_C\right)\left(\bar{\psi}_C \gamma^\mu \tau_2 \vec{\tau} \lambda_A \psi\right).$$

Isovector axial vector diquark

Calculation of mass splitting from NJL-Fadeev
(Ishii, Bentz, & Yazaki, Nucl. Phys. A 587 (1995))
$r_H = H/G_0$; $r_H' = H'/G_0 > 0$

$$M_N \simeq 1.70 - 0.21 r_H' - 0.33 r_H \quad [\mathrm{GeV}],$$
$$M_\Delta \simeq 1.52 - 0.22 r_H' \quad [\mathrm{GeV}].$$

$\Rightarrow m_\Delta - m_N =$ $$-0.01 r_H' + 0.33 r_H \simeq 0.47 \quad [\mathrm{GeV}]$$

$\Rightarrow$ $\boxed{H/G \gtrsim 1.4}$:remarkably consistent
with H/G ~ 1.5-1.6 from QHC19,
vs. H/G = 0.5(0.75) from Fierz transformation in NJL.

Expect enhancement because of ever so strong correlations as quark matter approaches confinement