Neutron stars and the properties of matter under extreme conditions: nuclear physics of the interior – and the equation of state

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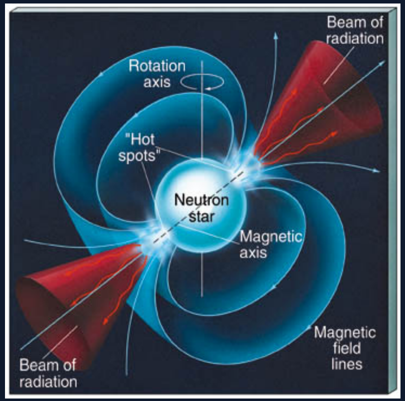
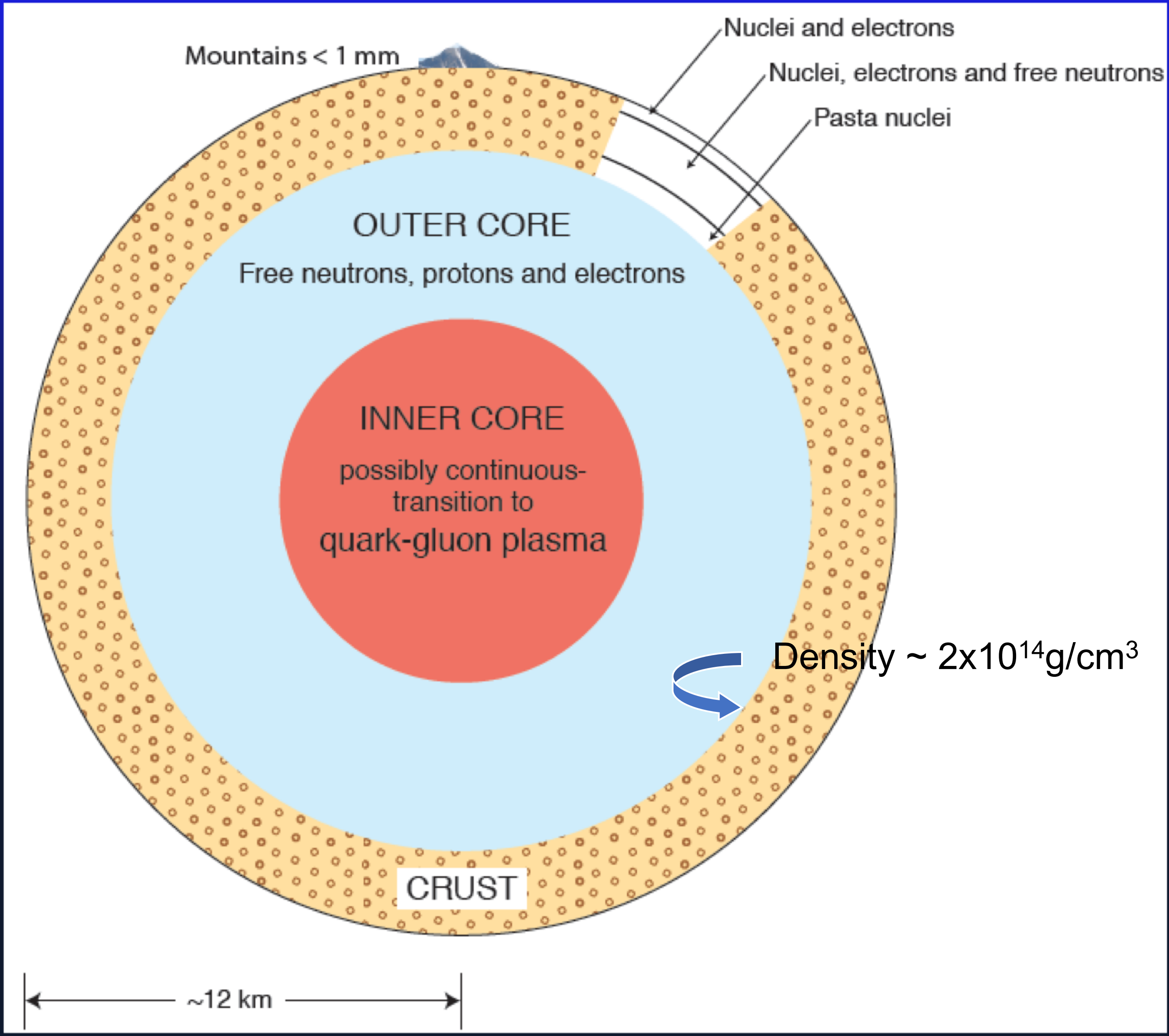
Neutron star interior

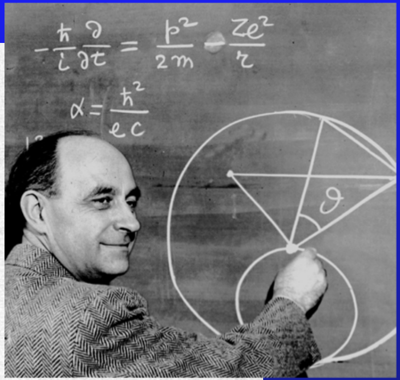
Mass ~ 1.4-2+ M_{sun}
Radius ~ 10-12 km
Temperature
~ 10^6-10^9 K

Surface gravity
~10^{11} that of Earth
Surface binding
~ 1/10 mc^2

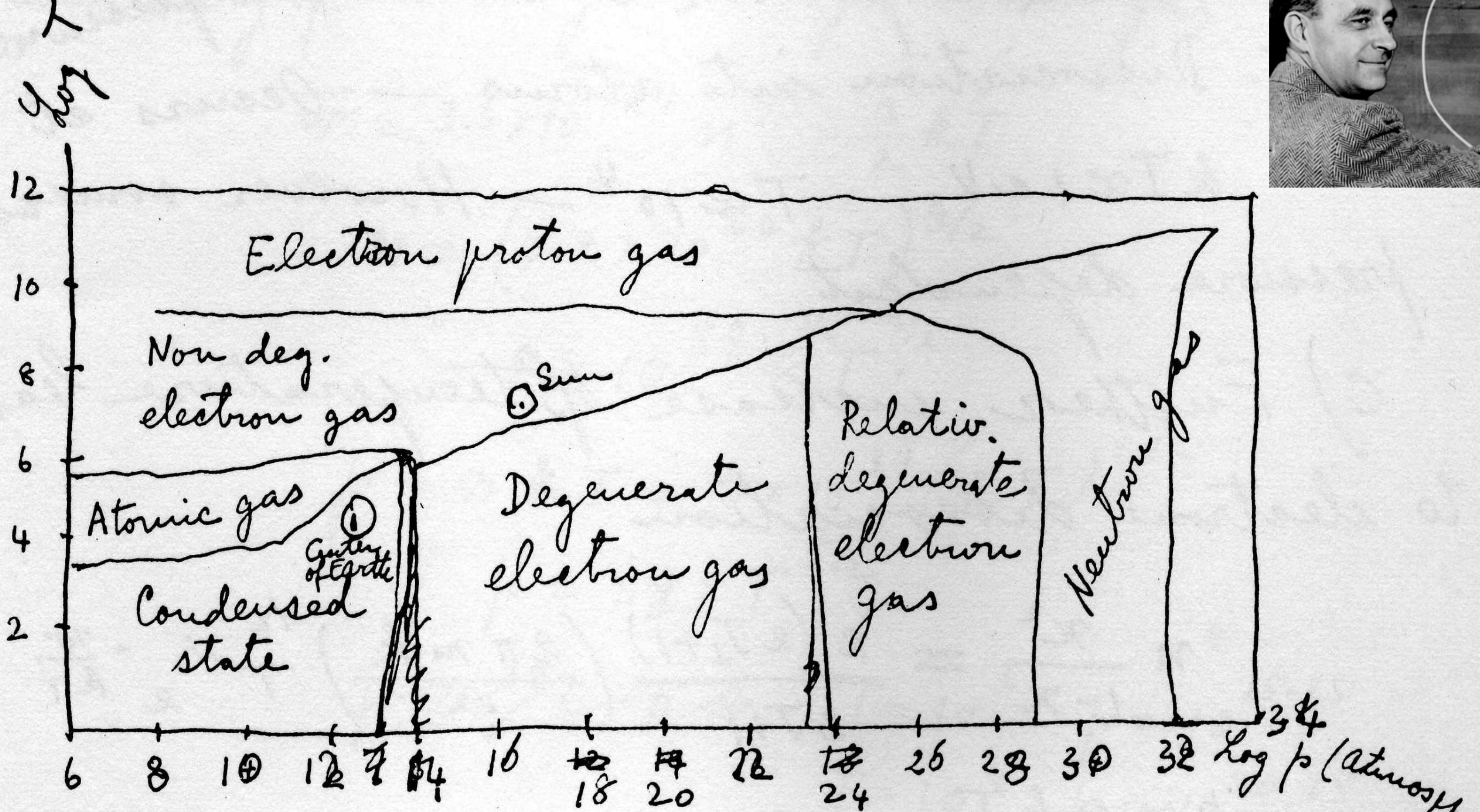




E. Fermi: *Notes on Thermodynamics and Statistics* (1953))



70 — Matter in unusual conditions



Start from ordinary condensed matter with ~~[illegible]~~ equation of state controlled by ordinary chemical forces.

Neutron stars below the surface

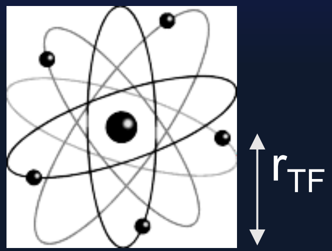
Surface composition is ideally ^{56}Fe, endpoint of thermonuclear burning. Possible "impurities" (^{4}He, etc.), especially in accreting neutron stars in binaries.

T (surface) ~ 10^{6-7} K = 0.1-1 KeV >> $T_{melting}(^{56}Fe)$ ~ 1800 K

=> surface is liquid

Ionization:

Atomic radius: $r_{Thomas\text{-}Fermi} = 0.8853\, a_0/Z^{1/3}$



Z = nuclear charge, $a_0 = \frac{\hbar^2}{m_e e^2}$ = Bohr radius

Matter begins to ionize for interatomic spacing

$r_c = (3/4\pi\, n_{atoms})^{1/3} \lesssim r_{TF}$ => $\rho > m\, AZ/\, a_0^3 \sim 10\, AZ$ g/cm^3

~ 10^4 g/cm^3 for ^{56}Fe; A = atomic number

Electron degeneracy

Electrons become degenerate for $T << T_e$

T_e = electron degeneracy temperature
$= p_e^2/2m_e = 2.5 \times 10^9 K (\rho/\rho_s)^{2/3}$

p_e = electron Fermi momentum;

$\rho_s = ((m_e c)^3/3\pi^2\hbar^3)\, m_n A/Z \sim 3 \times 10^5$ g/cm^3

At $T = 10^8$ K, degeneracy sets in at $\rho > 3 \times 10^4$ g/cm^3

For $\rho >> \rho_s$ => electrons are relativistic

Neutron stars are dark inside: no photons

Photon dispersion relation in matter

$$\omega_{photon}(k) = (c^2k^2 + \omega^2_{plasma})^{1/2}$$

plasma frequency:

$$\omega^2_{pl} = \frac{4\pi n_e e^2}{m_e}$$

(non-relativistic)
($m_e \to \mu_e$ in general)

or

$$\left(\frac{\omega_{pl}}{T}\right)^2 = \frac{16}{3\pi}\frac{e^2}{\hbar v_F}\left(\frac{\epsilon_F}{T}\right)^2$$

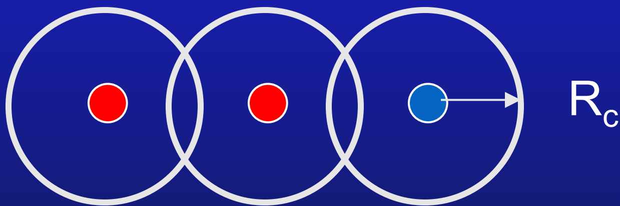
ϵ_F = Fermi energy
v_F = Fermi velocity

For degenerate electrons, ω_{pl} >> T, and thus

Number of photons $\sim e^{-\omega_{pl}/T}$ greatly suppressed

Matter solidifies

$T_{melting} \sim E_{binding}/\Gamma_m$ where $\Gamma_m \sim 10^2$



Wigner-Seitz cell containing one atom

$$4\pi R_c^3/3 = 1/n_{atoms} = m_n A/\rho$$

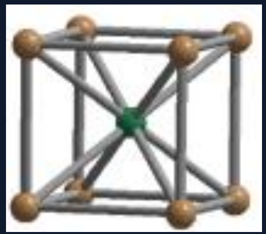
$$-E_b \simeq -\int_{cell} d^3r \frac{Ze^2}{r} n_e + \frac{3}{5}\frac{Z^2e^2}{R_c} = -\frac{9}{10}\frac{Z^2e^2}{R_c}$$

$$T_m \simeq \frac{9}{10}\frac{Z^2e^2}{\Gamma_m R_c} \simeq \frac{Z^{5/3}}{\Gamma_m}\frac{e^2}{\hbar c} m_e c^2 \left(\frac{\rho}{\rho_s}\right)^{1/3}$$

$\rho_s = ((m_e c)^3/3\pi^2\hbar^3)\, m_n A/Z \sim 3 \times 10^5$ g/cm^3

For Z = 26, $\Gamma_m = 10^2$, $T_m \sim 10^8$ K
Melt at $\rho \sim 5 \times 10^7$ g/cm^3,
about 10m below surface

Form BCC lattice

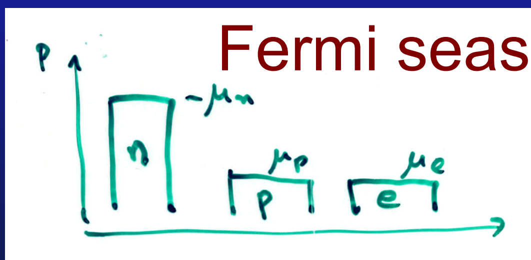


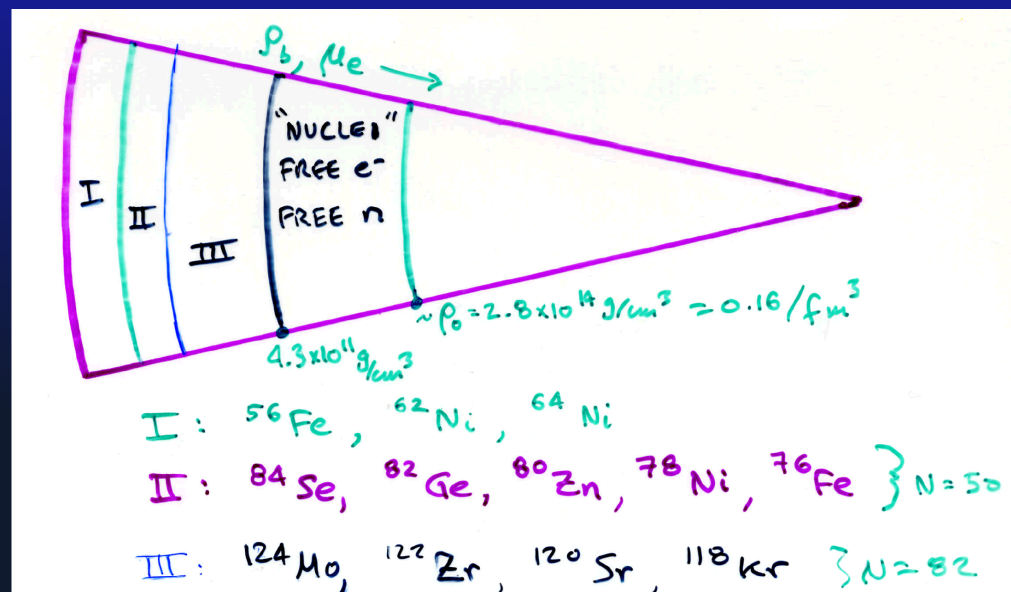
Nuclei before neutron drip

$e^- + p \longrightarrow n + \nu$: makes nuclei neutron rich as electron Fermi energy increases with depth

$n \longrightarrow p + e^- + \bar{\nu}$: not allowed if e^- state already occupied

Beta equilibrium: $\mu_n = \mu_p + \mu_e$



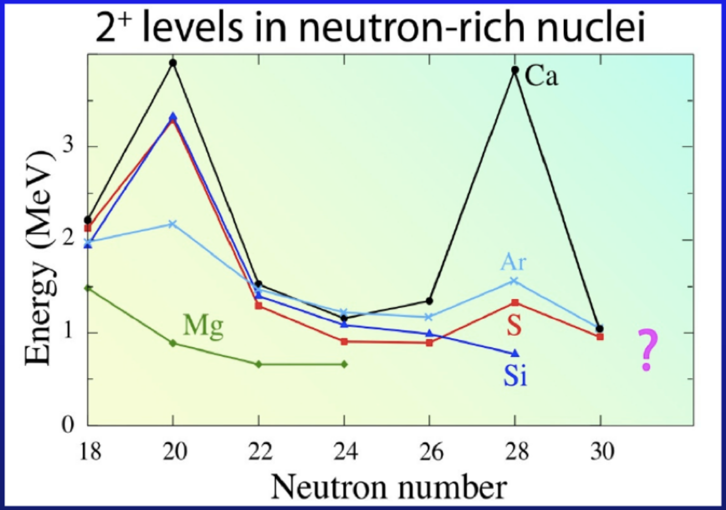


Shell structure (spin-orbit forces) for very neutron rich nuclei?

Do N=50, 82 remain neutron magic numbers? Proton shell structure?

Being explored at rare isotope accelerators: RIKEN Rare Ion Beam Facility, and later GSI (MINOS), FRIB, RAON (KoRIA)

Modification of shell structure for N >> Z



Usual shell closings
(N ~ Z) at 20, 28, 50, 82, 126

Spin-orbit forces and hence shell structure modified by tensor and 3-body forces in neutron rich nuclei

even

No shell effect for Mg(Z=12), Si(14), S(16), Ar(18) at N=20 and 28. Bastin et al. PRL (2007)

Oxygen: shell closure at N=16 Otsuka et al PRL (2005)

Calcium: shell closure at N=34

D. Steppenbeck et al. Nature (2013)

Binding of ^{47}P, ^{49}S, ^{52}Cl, ^{54}Ar, ^{57}K, $^{59;60}$Ca, and ^{62}Sc

O.B. Tarasov et al., PRL 121, 022501 (2018)

Nuclear sizes: minimize energy

$E_{interaction}(Z,A) = E_{bulk} + E_{surface} + E_{Coulomb} + E_{symmetry} + \ldots$

$E_{surface} = a_s A^{2/3} \sim R_n^2$ R_n = nuclear radius, $a_s \sim 18$MeV

$E_{Coulomb} = a_C Z^2/A^{1/3} \sim Z^2/R_n = \sim x^2 A^{5/3}$, $Z/A = x$, $a_C \sim 0.7$MeV

1) At fixed x, balance nuclear Coulomb vs. surface energies per nucleon:

$$\frac{\partial}{\partial A}\left(\ldots A^{-1/3} + \ldots x^2 A^{2/3}\right) = 0$$

=> $E_{surface} = 2\,E_{Coul}$ $A \sim 12/x^2$ (cf. ^{56}Fe at x = 1/2)

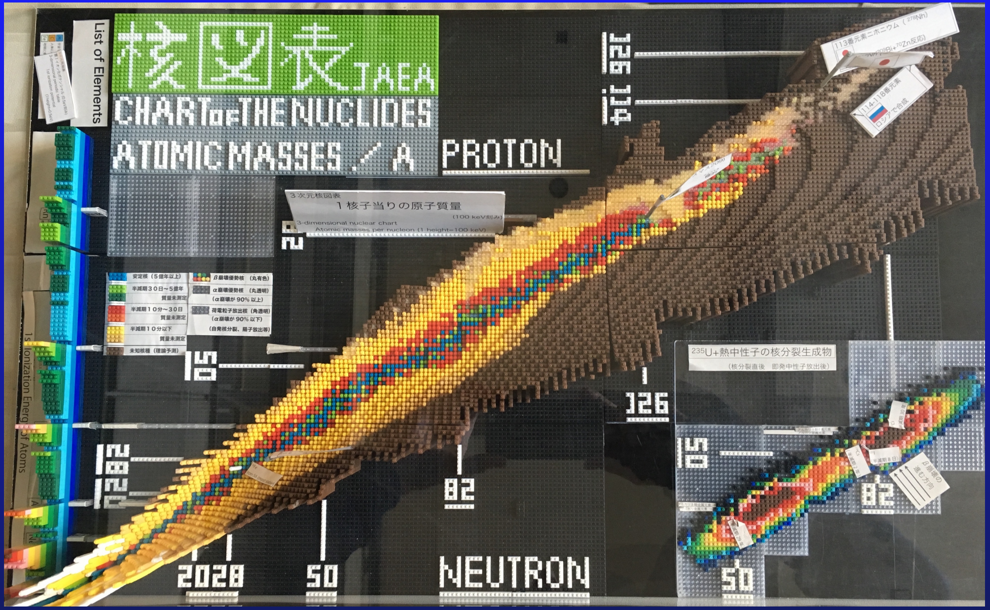
2) Best Z/A: No energy cost to convert n to $p+e^-$ (+neutrino)

=> beta equilibrium condition on chemical potential in nuclei:

$\mu_n - \mu_p = \mu_e$ μ_e = electron chemical potential (w. m_e) determined by density, driving x

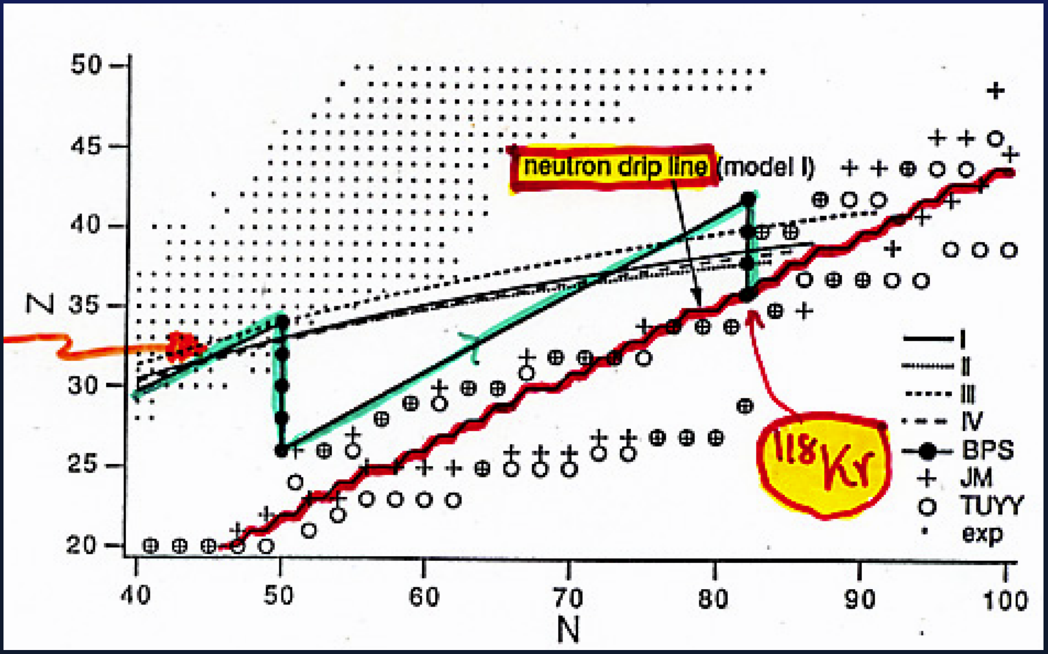
Valley of β stability

PROTON



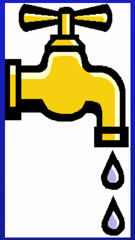
JPARC, Tokai

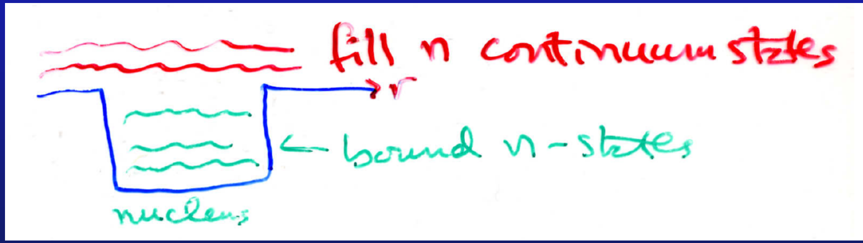
nuclei
before drip



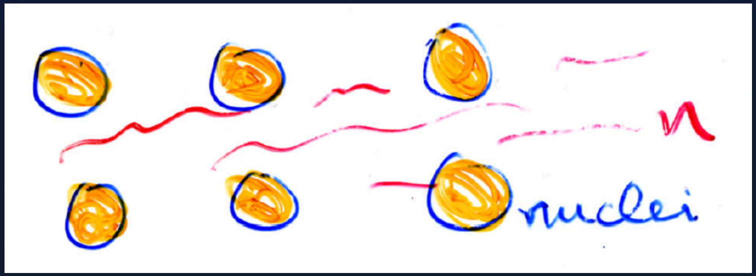
Neutron drip

Beyond density $\rho_{drip} \sim 4.3 \times 10^{11}$ g/cm^3 neutron bound states in nuclei become filled. Further neutrons must go into continuum states. Form degenerate neutron Fermi sea.





Neutrons in neutron sea are in equilibrium with those inside nucleus (common μ_n)



Protons appear not to drip, but remain in bound states until nuclei merge in interior liquid.

Cross section of nuclei before and after drip

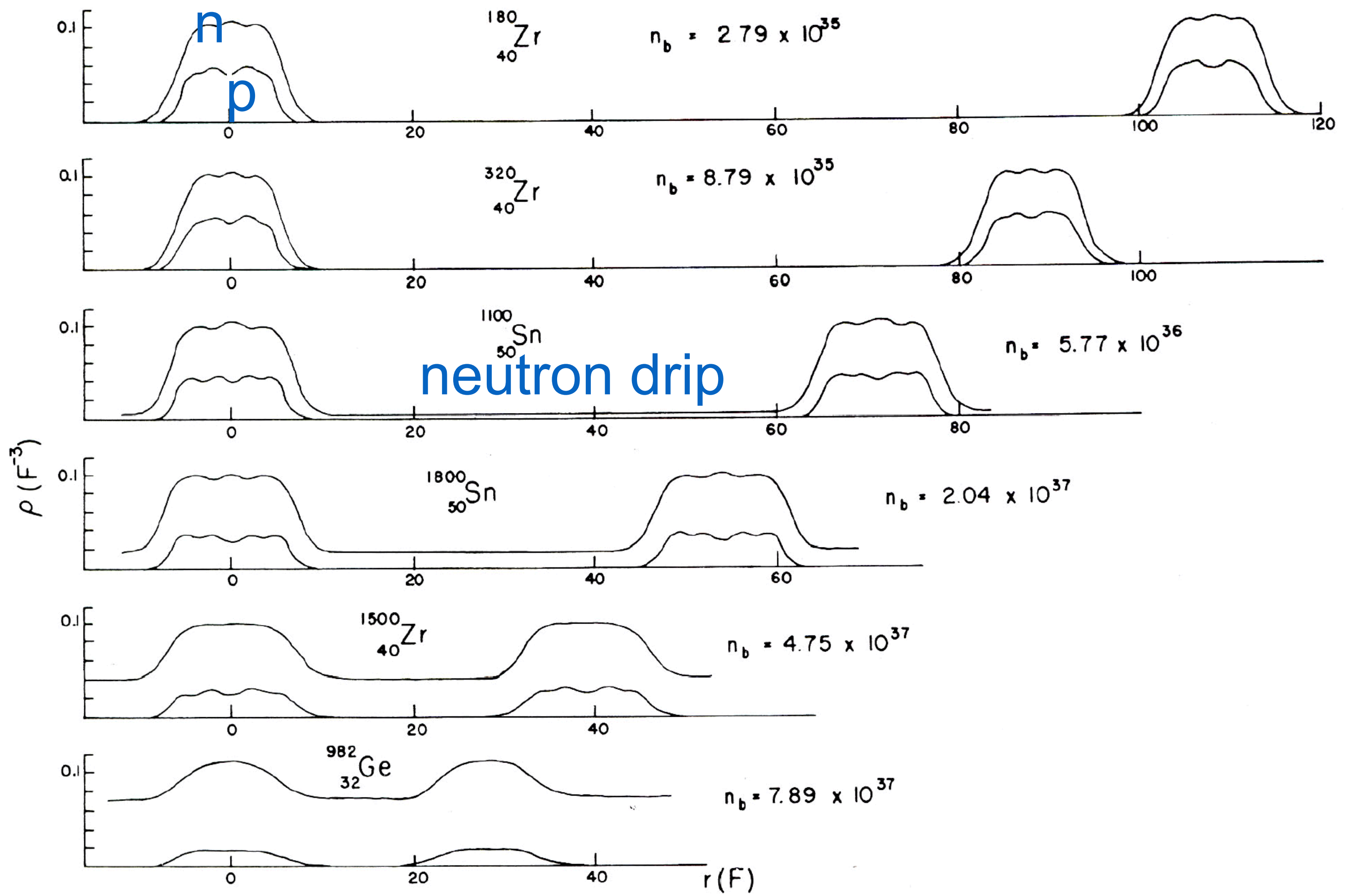
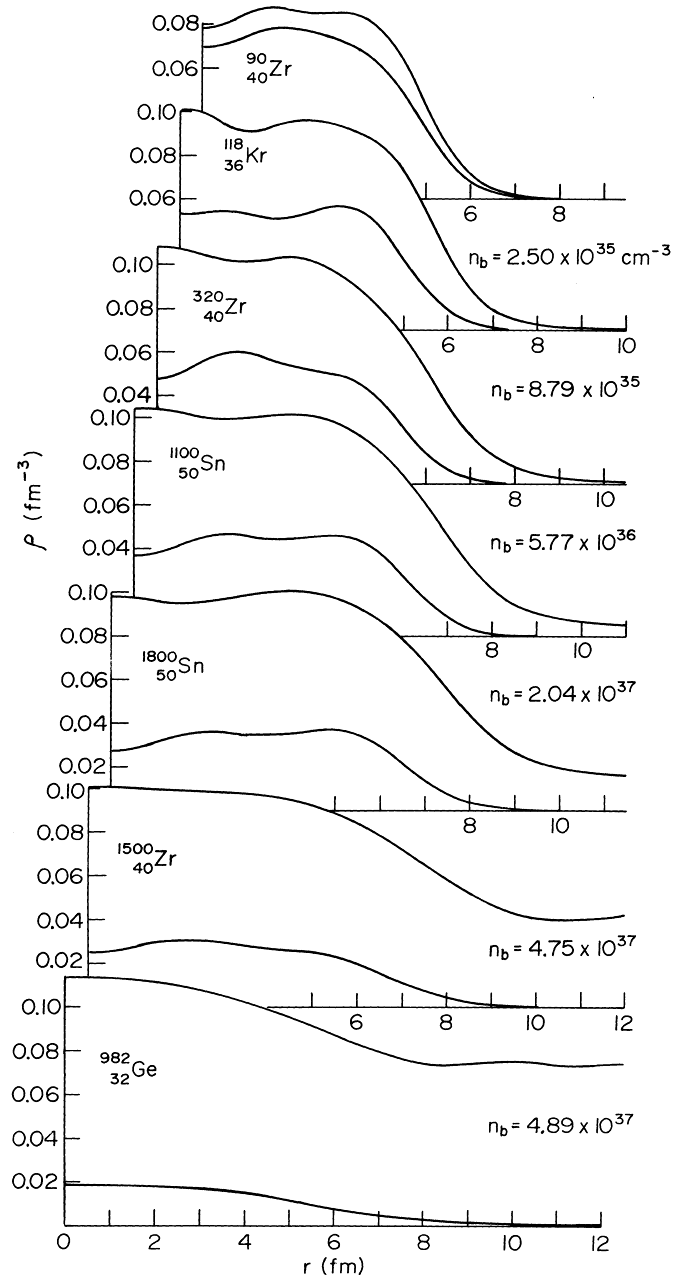


Figure 5 Density profiles of lattice unit cells in the crust for various average densities. The upper curve is the neutron number density; the lower is the proton number density; n_b denotes the average nucleon density, measured in nucleons/cm^3. The horizontal axis is distance in fm. From (27). (The density of nuclear matter is 1.7×10^{39} nucleons/cm^3.)

J. Negele and D. Vautherin, Nucl. Phys. A207 (1973) 298

Cross sections of nuclei in crust

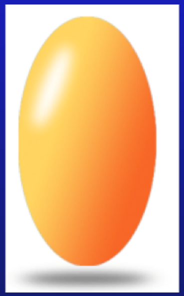


neutron drip

How stable are spherical nuclei?

Take incompressible liquid drop with small quadrupolar deformation:

$$P_2(\Omega) = \frac{3\cos^2\theta - 1}{2}$$

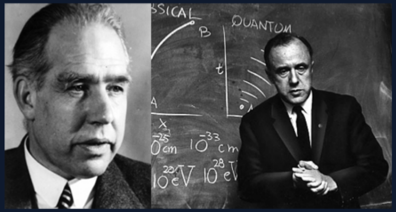


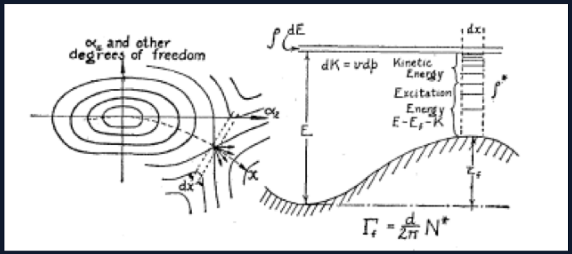
Radius $R \rightarrow R(1 + \epsilon P_2(\Omega))$

Area increases by factor $(1+3\epsilon^2/5)$.
Coulomb energy decreases by factor $(1- \epsilon^2/5)$.

$$\delta(E_{surf} + E_{coul}) = \frac{\epsilon^2}{5}(3E^0_{surf} - E^0_{coul})$$

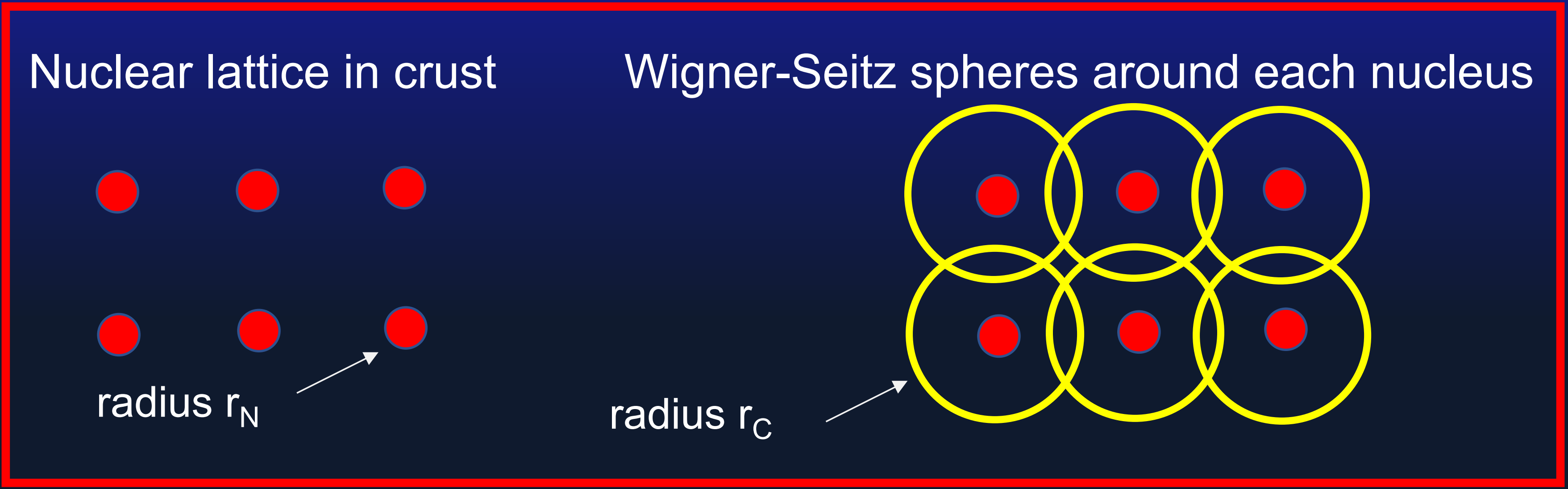
For $E_{coul} > 3E_{surf}$ have spontaneous deformation. Can have first order jump to lower energy configuration before that though. More accurate is $E_{coul} > 2E_{surf}$ or $Z^2/A > 2\ a_s/a_c \sim 50$ (Bohr-Wheeler fission criterion)





New states of nuclei deep in neutron star crust

Nuclei deform at high density in crust. Calculate Coulomb energy
of nuclei including the electron background.
Wigner-Seitz method: draw sphere around each nucleus of radius r_C where $(4\pi/3) r_C^3 \, n_{nuclei} = 1$



Coulomb energy of lattice $$E_{coul} \simeq E^0_{coul} \left(1 - \frac{3}{2}\frac{r_C}{r_N} + \frac{1}{2}\left(\frac{r_C}{r_N}\right)^3 \right)$$

vanishes for $r_N = r_C$

Bohr-Wheeler instability of nuclei

Z/A of nuclei for given A obey $E_{surface} = 2E_{coul}$

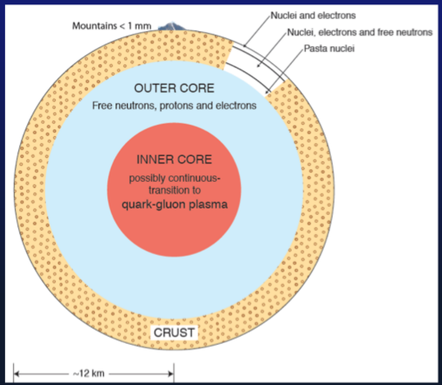
Bohr-Wheeler => instability for $E^0_{coul} > 2E_{surface}$

or $E_{coul} < E^0_{coul}/4$

$$E_{coul} \simeq E^0_{coul}\left(1 - \frac{3}{2}\frac{r_C}{r_N} + \frac{1}{2}\left(\frac{r_C}{r_N}\right)^3\right)$$

Instability for
$$1 - \frac{3}{2}\frac{r_C}{r_N} + \frac{1}{2}\left(\frac{r_C}{r_N}\right)^3 < \frac{1}{4}$$

or $r_N/r_C \sim$ ½.



When nuclei fill ~1/8 of space

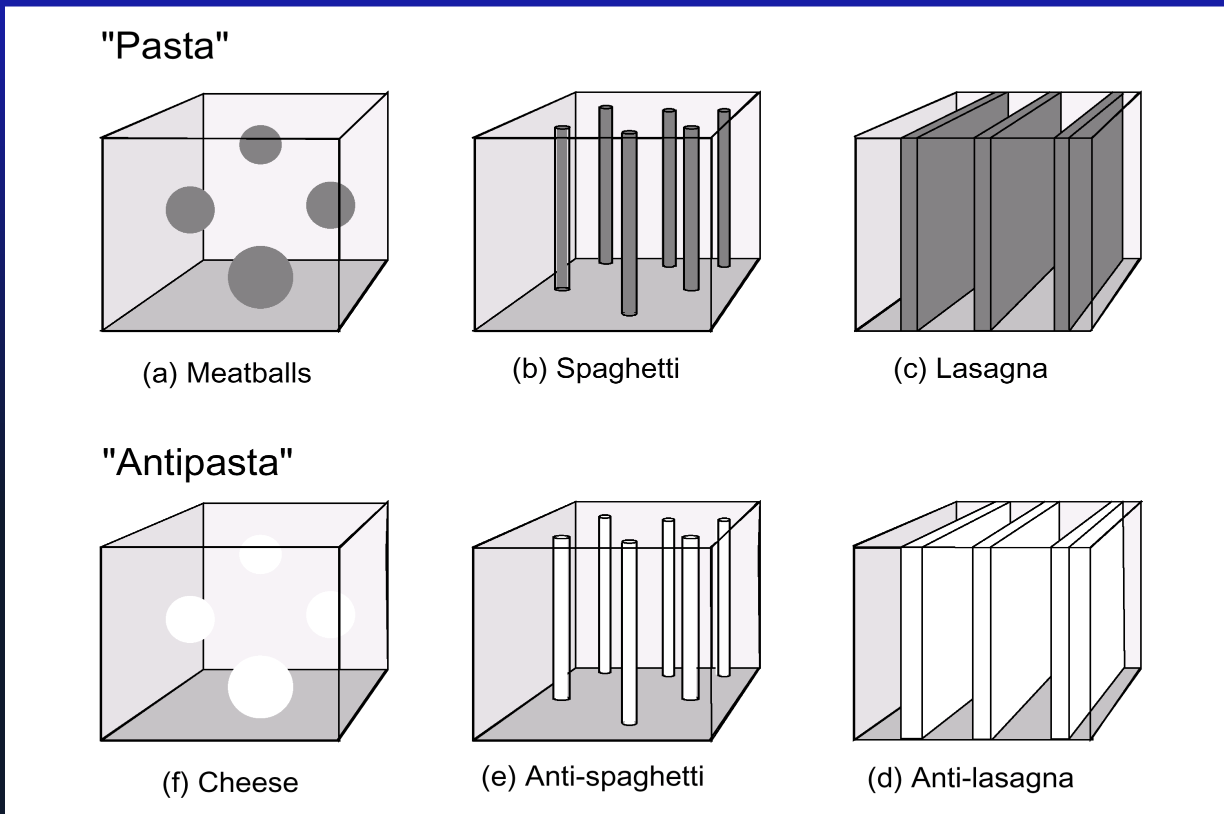
→ fission and onset of non-spherical shapes

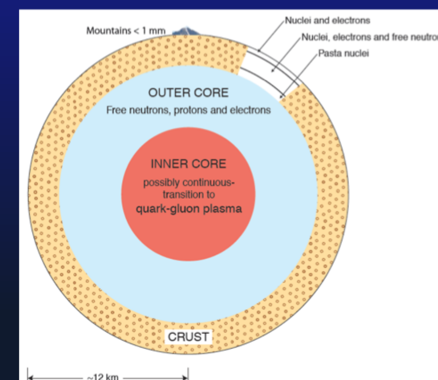
Transition to liquid interior at $n/n_0 \sim 0.5$ (10% uncertain)

Pasta Nuclei in inner crust

D. G. Ravenhall, C. J. Pethick, and J. R. Wilson, PRL 50, 2066 (1983)

When Coulomb wins over surface energies:
as in Bohr-Wheeler criterion for nuclear fission ($Z^2/A > 50$)





Involves over half the mass of the crust !! Effects on crust
bremsstrahlung of neutrinos, pinning of n vortices, modes of crust, ... ??

Lorentz, Pethick and Ravenhall
PRL 70 (1993) 379

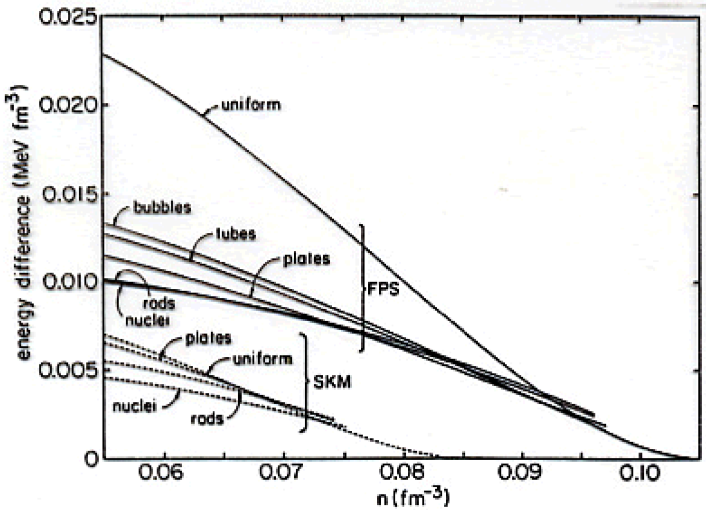


FIG. 1. Energies per unit volume as a function of density for the one-fluid phase, and the three-, two-, and one-dimensional nucleus phases, with (for FPS) the bubble (inverted structure) versions of the first two, after subtraction of the energy of the two-fluid phase, neglecting Coulomb and interface effects. The two nuclear interactions illustrated are SKM [6] and the version of FPS [8, 9] described in the text.

Energy densities

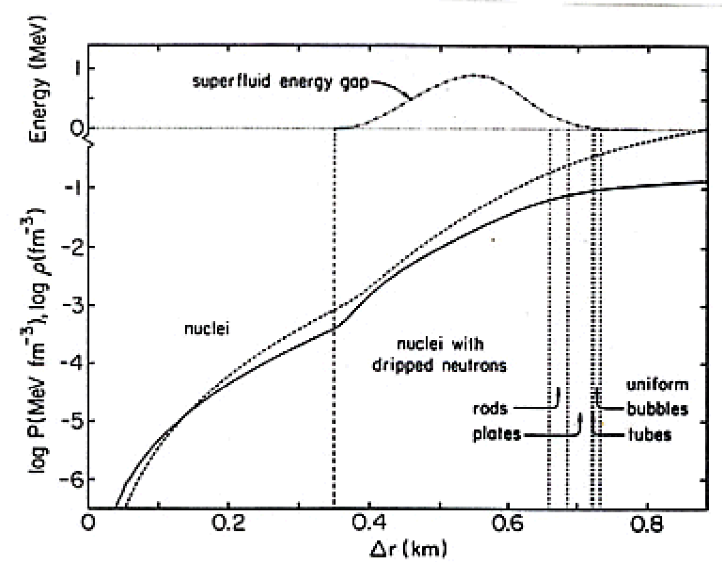
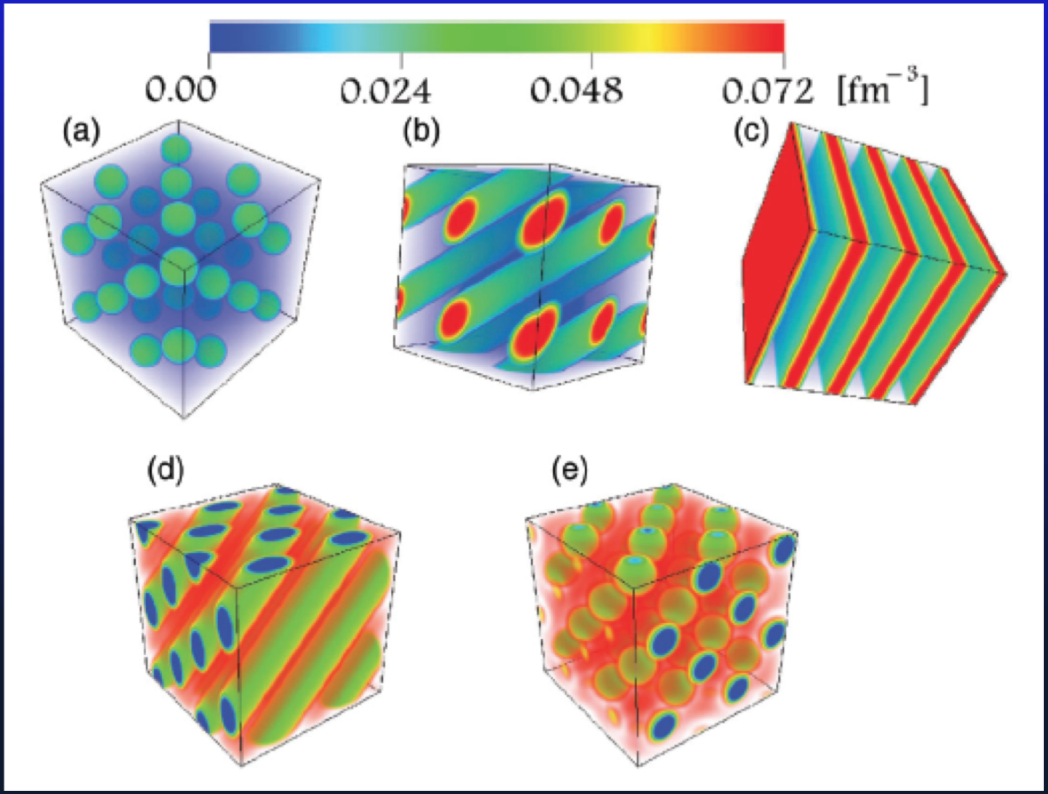


FIG. 2. Profile of a neutron star crust as given by FPS [8, 9]. The distance (in km) is measured from the surface. The solid line is density ρ/m_n, in fm^{-3}, and the dashed line is pressure, in MeV fm^{-3}, plotted logarithmically. Vertical lines indicate the phase boundaries described in the text. At the top is shown the superfluid energy gap [22].

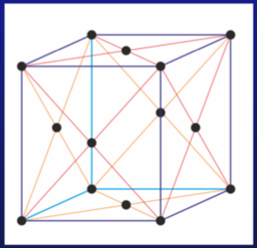
Profile of neutron star

Pasta phases of symmetric nuclear matter ($n_n = n_p$)

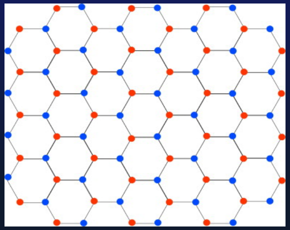
M. Okamoto, T. Maruyama, K. Yabana, & T. Tatsumi , PRC 88, 025801 (2013)



Proton density distributions



face-centered cubic lattice



honeycomb lattice

(a) n_B = 0.01 fm^{-3} fcc lattice
(b) n_B= 0.024 fm^{-3} cylinders in honeycomb lattice
(c) n_B= 0.05 fm^{-3} slabs
(d) n_B=0.08 fm^{-3} cylinders in honeycomb lattice
(e) n_B=0.09 fm^{-3} spherical bubbles in fcc lattice

The liquid interior

Transition to liquid interior at $n/n_0 \sim 0.5$ (10% uncertain)

Neutrons (likely superfluid) ~ 95% Non-relativistic
Protons (likely superconducting) ~ 5% Non-relativistic
Electrons (normal, $T_c \sim T_f e^{-137}$) ~ 5% Fully relativistic

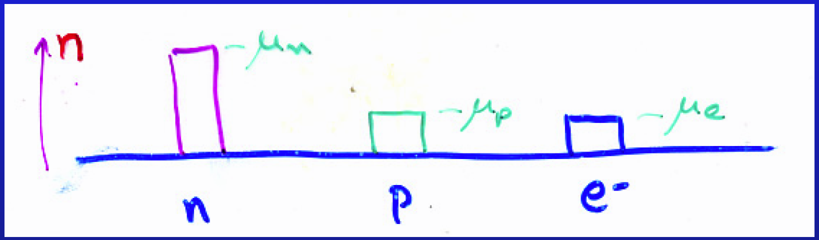
Eventually muons, hyperons??, quark matter and possible exotica:

- pion condensation
- kaon condensation
- quark droplets

Uncertainties in nuclear matter liquid: interpolations between pure neutron matter and symmetric nuclear matter.

Why "neutron" star?

β equilibrium: $\mu_n = \mu_p + \mu_e$
Charge neutrality: $n_p = n_e$



Non-interacting matter:

$\mu_n = p_n^2/2m_n, \quad \mu_p = p_p^2/2m_p, \quad \mu_e = cp_e = cp_p$

$\Rightarrow p_e/p_n \approx p_n/2m_nc \Rightarrow n_p/n_n \approx (p_n/2m_nc)^3 \sim 0.03$

Mean field effects: $(p_n^2/2m_n) + V_n = (p_p^2/2m_p) + V_p + p_pc$

$V_p < V_n$ (favors fewer neutrons) $\Rightarrow n_p/n_n \approx 0.05$

Matter is primarily neutron liquid

Estimate the value of V_n-V_p to get $n_p/n_n = 0.05$

Neutron Star Models

Equation of state: E = energy density = $\rho\, c^2$

n_b = baryon density

$P(\rho)$ = pressure = $n_b^2\, \partial(E/n_b)/\partial\, n_b$

Tolman-Oppenheimer-Volkoff equation of hydrostatic balance:

$$\frac{\partial P(r)}{\partial r} = -G\frac{\rho(r) + P(r)/c^2}{r\,(r - 2Gm(r)/c^2)}[m(r) + 4\pi r^3 P(r)/c^2]$$

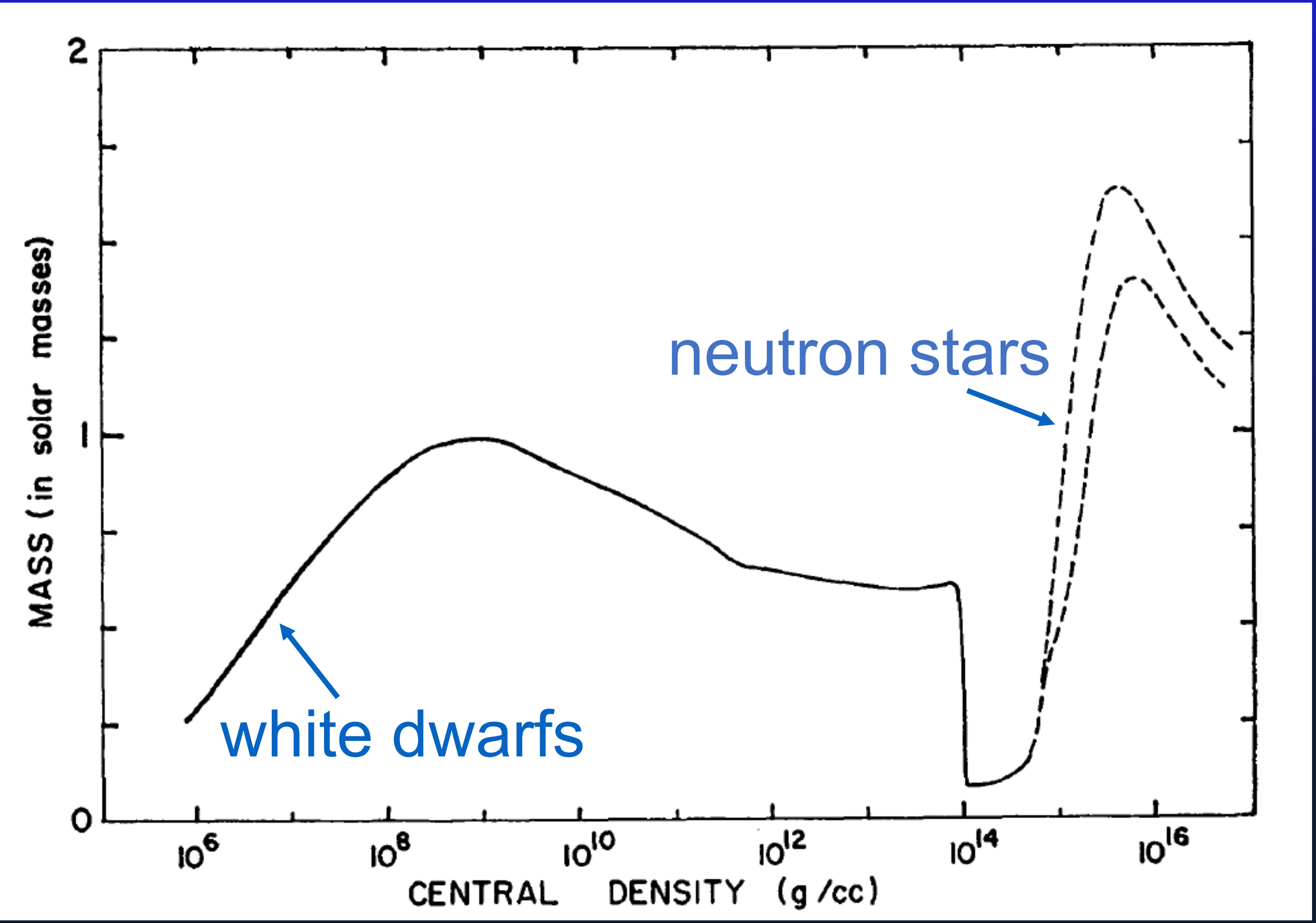
general relativistic corrections

$$m(r) = \int_0^r 4\pi r'^2 dr' \rho(r')$$ = mass within radius r

1) Choose central density: $\rho(r=0) = \rho_c$
2) Integrate outwards until P=0 (at radius R)
3) Mass of star

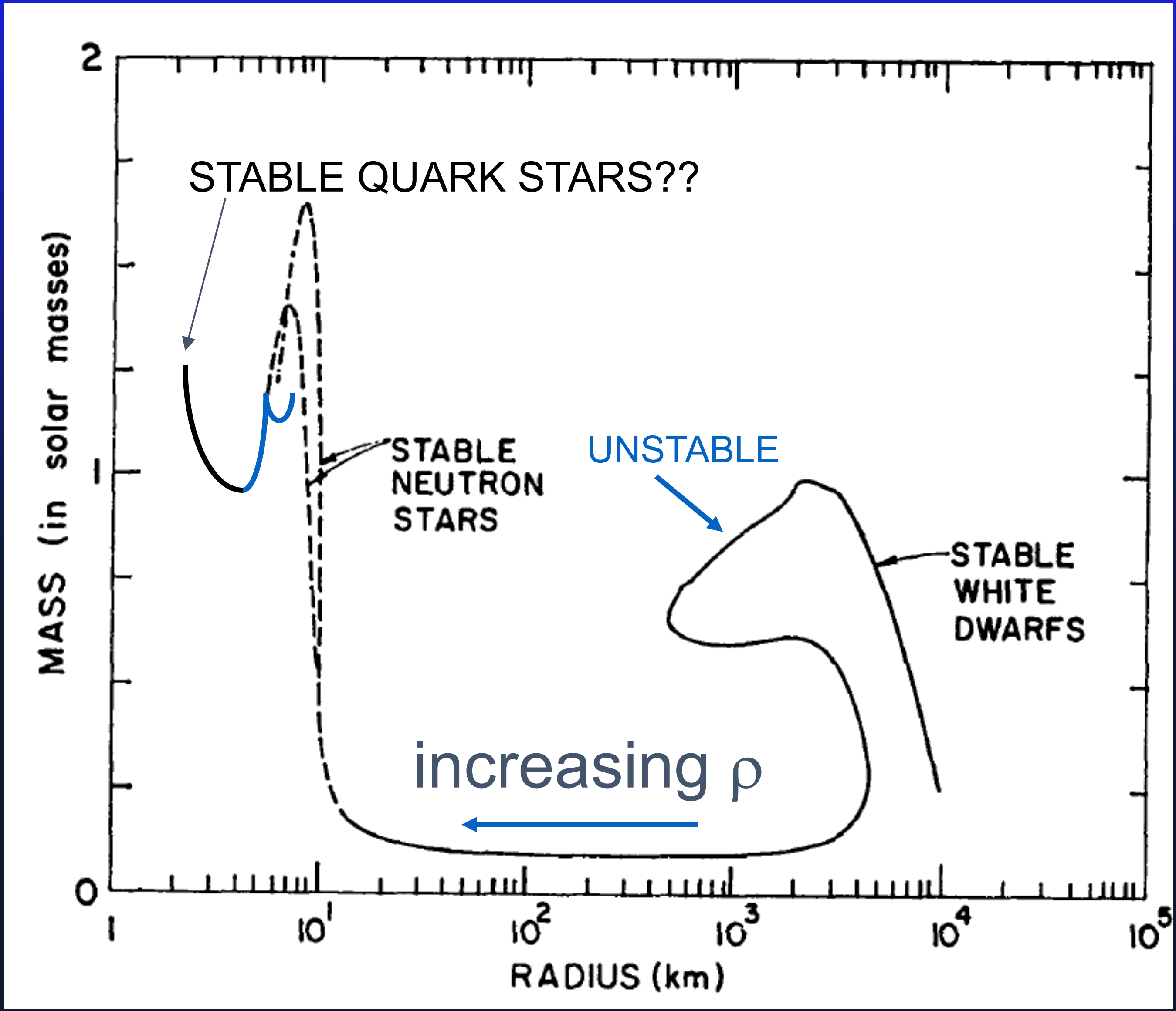
$$M = \int_0^R 4\pi r^2 dr\, \rho(r)$$

Families of cold condensed objects: mass vs. central density



BPS, Ap.J 170, 299 (1971)

Mass vs. radius, and stability



Problem: Solve the TOV equation analytically for constant mass density, ρ

Scaling TOV equation

$$\frac{\partial P(r)}{\partial r} = -G\frac{\rho(r) + P(r)/c^2}{r\left(r - 2Gm(r)/c^2\right)}\left[m(r) + 4\pi r^3 P(r)/c^2\right]$$

Dimensionless variables $\rho = \epsilon_0^4\,\tilde{\rho}$, $P = \epsilon_0^4\,\tilde{P}$, $r = \zeta\,\tilde{r}$
with $\zeta = 1/\epsilon_0^2\sqrt{G}$

Scaled TOV equation: $\frac{\partial \tilde{P}(r)}{\partial \tilde{r}} = \frac{1}{\tilde{r}^2}\frac{(\tilde{\rho} + \tilde{P})(\tilde{m}(\tilde{r}) + 4\pi\tilde{r}^3\tilde{P})}{1 - 2\tilde{m}(\tilde{r})/\tilde{r}}$

with $\tilde{m}(\tilde{r}) = \int_0^{\tilde{r}} 4\pi\tilde{r}^2\tilde{\rho}(\tilde{r})d\tilde{r}$

$$\zeta = \frac{\hbar^{3/2}c^{7/2}}{\epsilon_0^2 G^{1/2}} = \left(\frac{m_p c^2}{\epsilon_0}\right)^2 \frac{\hbar}{m_p c\,\alpha_G^{1/2}}$$

$$M \propto \frac{m_p}{\alpha_G^{3/2}}\left(\frac{m_p c^2}{\epsilon_0}\right)^2 = 1.86\left(\frac{m_p c^2}{\epsilon_0}\right)^2 M_\odot$$

$$R \propto \zeta = 17.2\left(\frac{m_p c^2}{\epsilon_0}\right)^2 \text{ km}$$

Scale of masses and radii

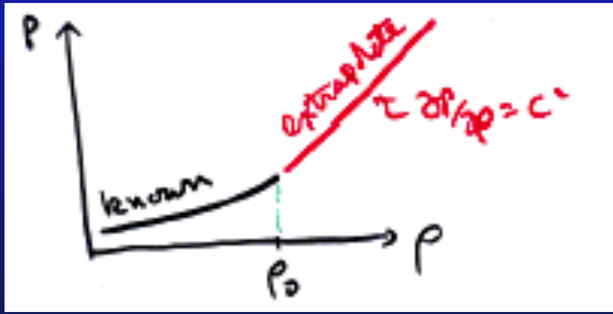
$$\frac{R_{schwarzschild}}{R} = \frac{2MG}{c^2 R} \sim 1$$

Upper bound to neutron star mass:

require speed of sound, c_s, in matter in core not to exceed speed of light: $c_s^2 = \partial P/\partial \rho \leq c^2$

Maximum core mass when $c_s = c$

Rhodes and Ruffini (PRL 1974)



$\rho_0 = 4\rho_{nm}$ => $M_{max} = 2.2\ M_\odot$

$2\rho_{nm}$ => $2.9\ M_\odot$

V. Kalogera and G.B., Ap. J. 469 (1996) L61

Properties of liquid interior near nuclear matter density

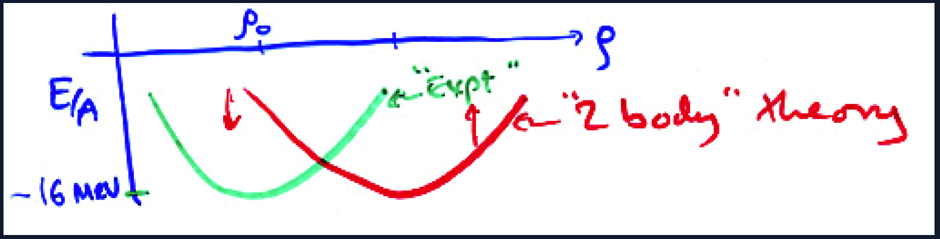
Determine N-N potentials from
- scattering experiments E<300 MeV
- deuteron, 3 body nuclei (^3He, ^3H)

ex., Paris, Argonne, Urbana 2 body potentials

Solve Schrödinger equation by variational techniques

Large theoretical extrapolation from low energy laboratory nuclear physics at near nuclear matter density

Two body potential alone:



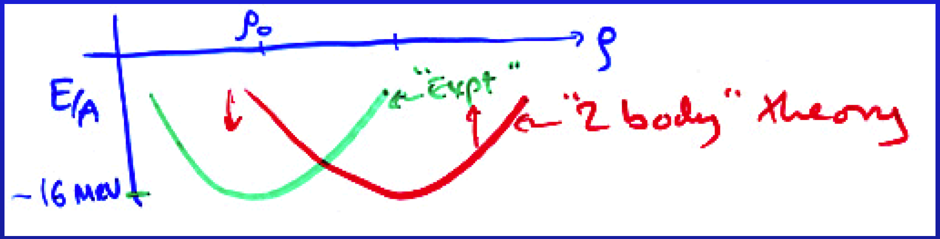
Underbind ^3H: Exp = -8.48 MeV, Theory = -7.5 MeV

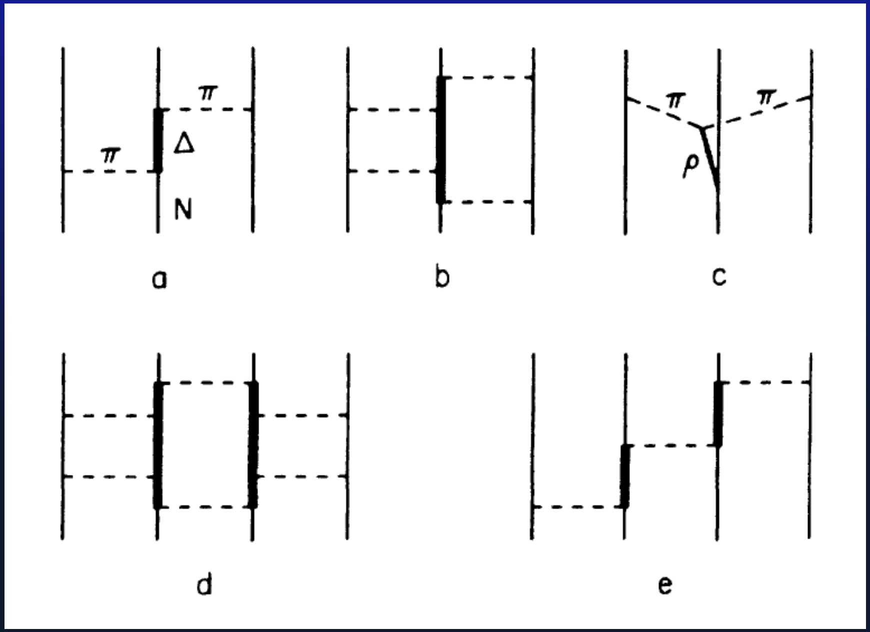
^4He: Exp = -28.3 MeV, Theory = -24.5 MeV

Importance of 3 body interactions

Attractive at low density

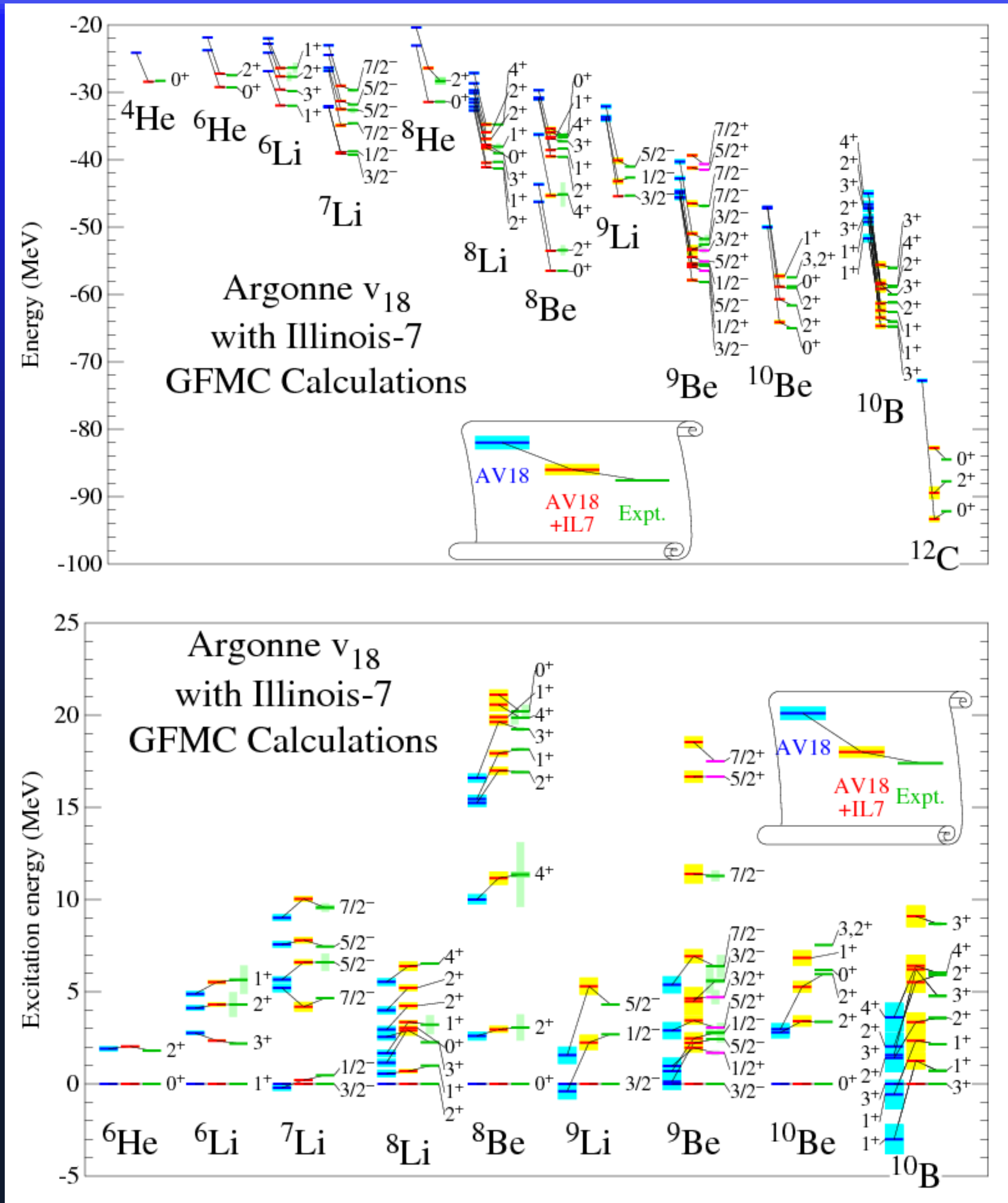
Repulsive at high density





Various processes that lead to three and higher body intrinsic interactions (not described by iterated nucleon-nucleon interactions).

Stiffens equation of state at high density
Large uncertainties!

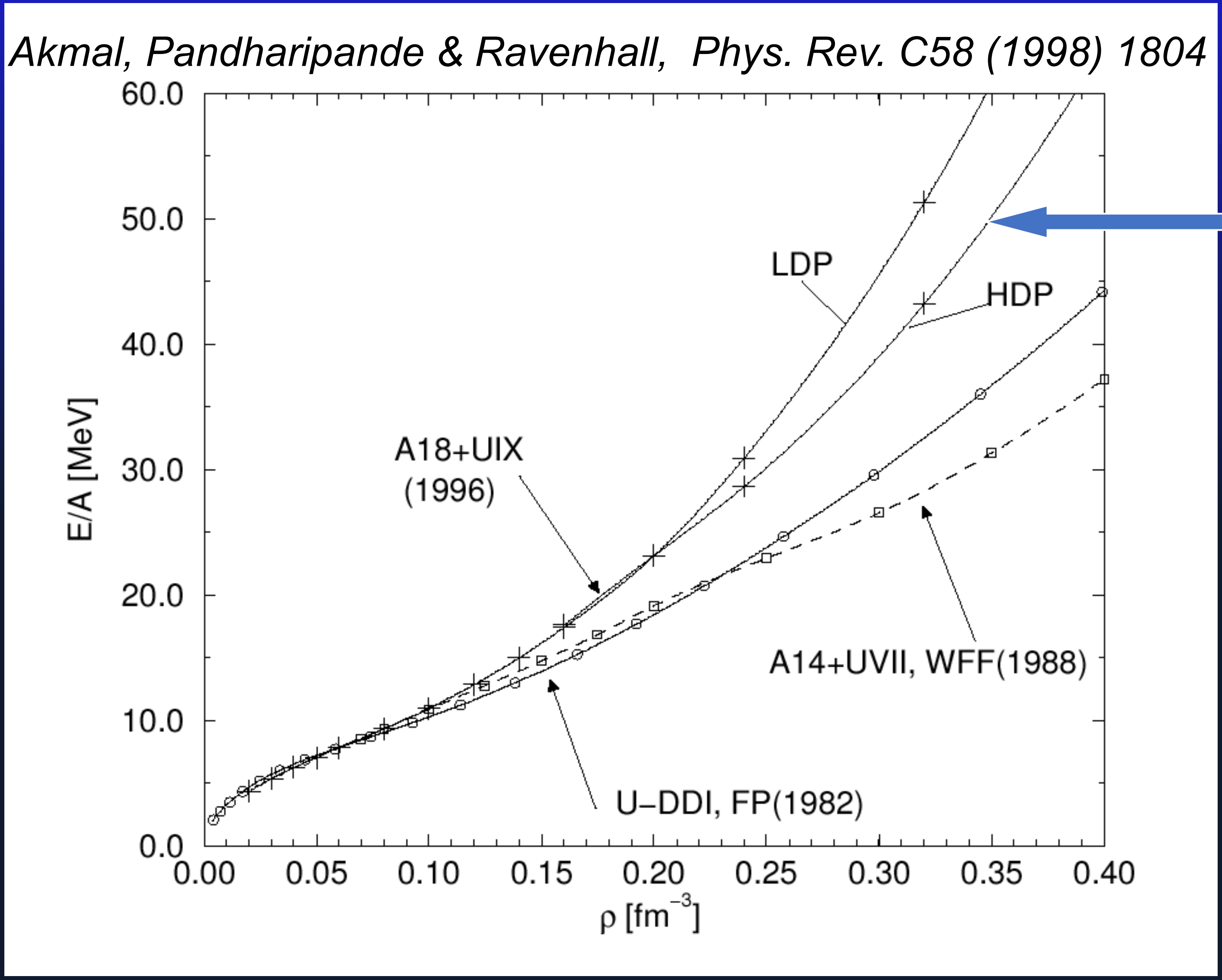


Calculation in Green's functionsMonte Carlo of energy levels of light nuclei. AV18+ Illinois 7 three body interaction.

Quantum Monte Carlo methods for nuclear physics
Carlson, J. *et al.* Rev.Mod.Phys. 1067 arXiv:1412.3081 [nuclth]

Standard construction of neutron star models

1) Compute energy per nucleon in neutron matter (pure or in beta equilibrium: $\mu_n = \mu_p + \mu_e$). Include 2 and 3 body forces between nucleons



π^0 condensate

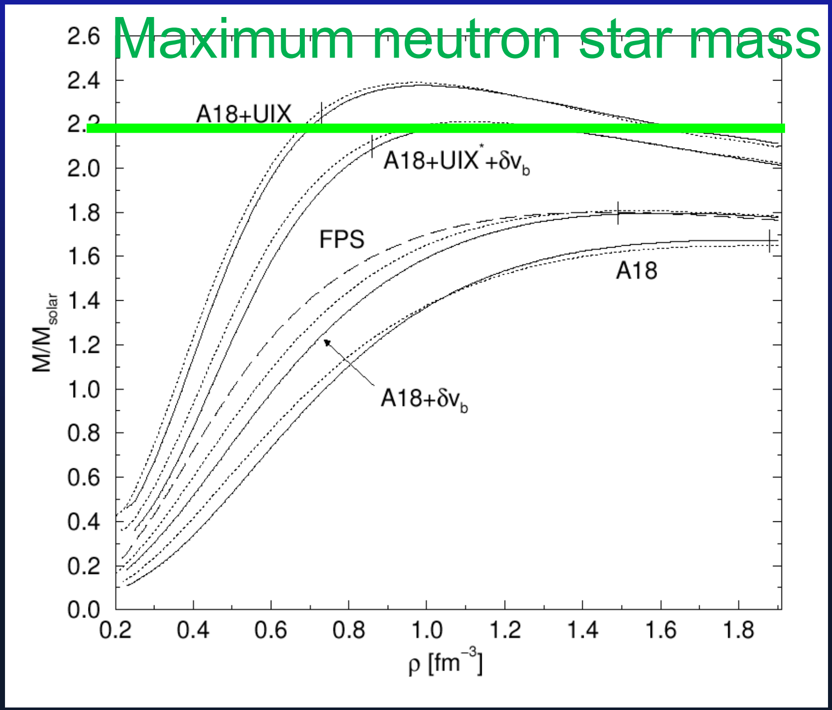
Neutron star models using *static interactions* between *nucleons*

E = energy density = ρc^2
n_b = baryon density
P(r) = pressure = $n_b^2\, d(E/n_b)/dn_b$

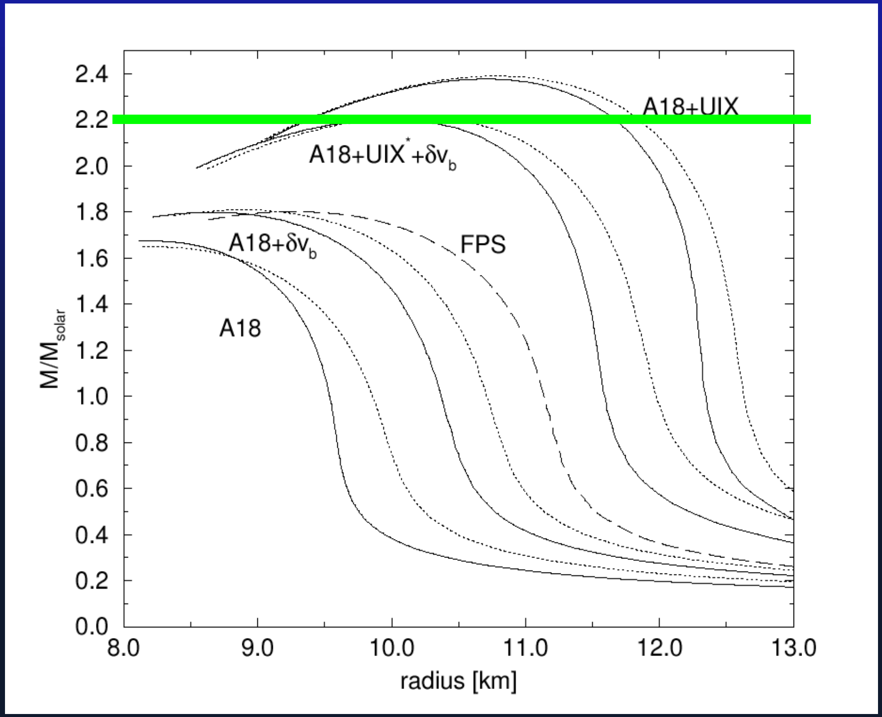
$$\frac{\partial P(r)}{\partial r} = -G\frac{\rho(r) + P(r)/c^2}{r\,(r - 2Gm(r)/c^2)}[m(r) + 4\pi r^3 P(r)/c^2]$$

TOV equation

$$M = \int_0^R 4\pi r^2 dr \rho(r)$$



Mass vs. central density



Mass vs. radius

Akmal, Pandharipande and Ravenhall, 1998

APR equation of state