Neutron stars and the properties of matter under extreme conditions:

Superfluidity -- vortices in neutron stars and quark matter

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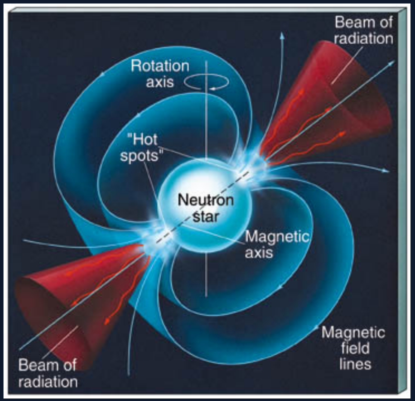
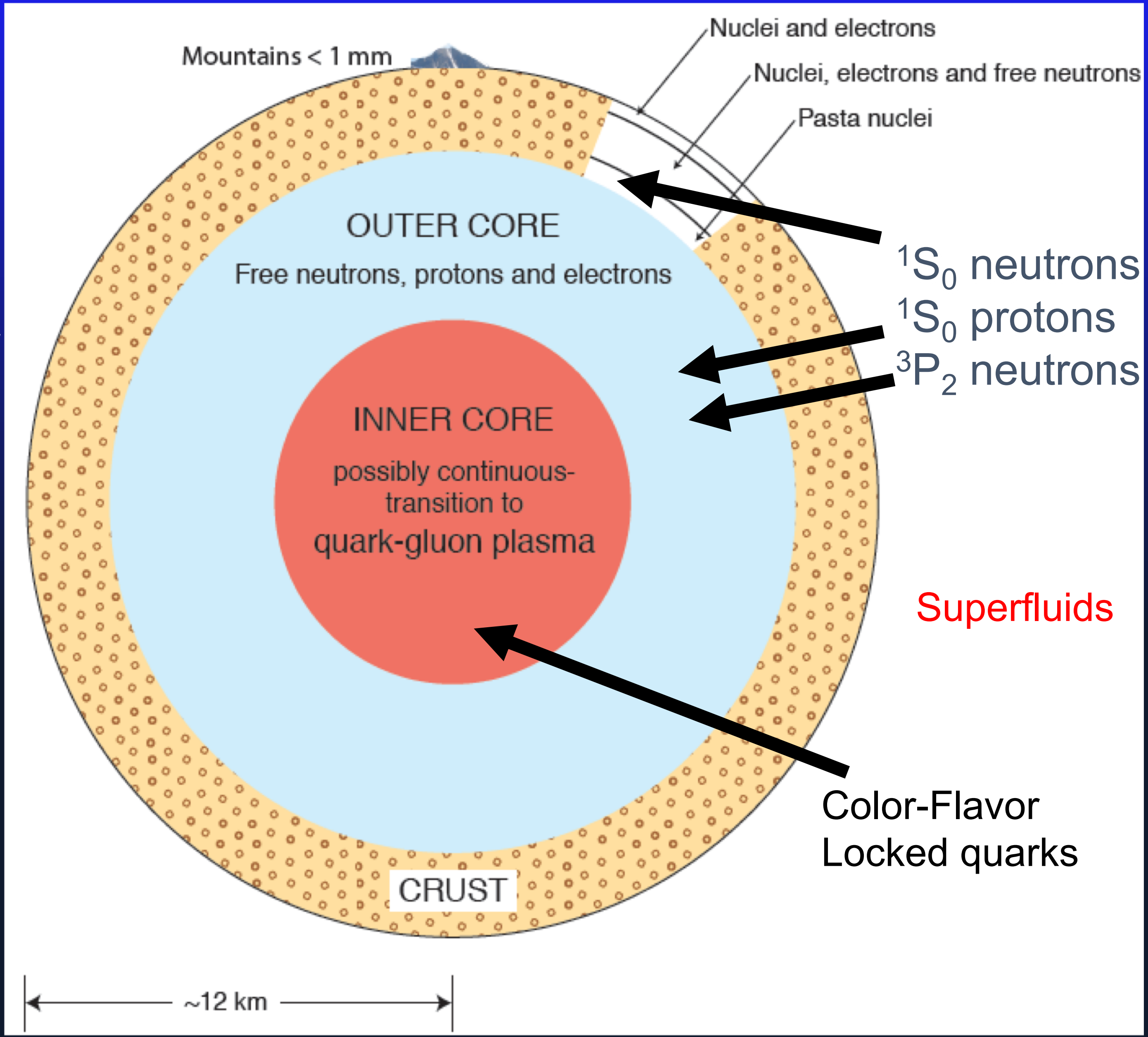
Neutron star interior

GB, T. Hatsuda,
T. Kojo, P. D. Powell,
Y. Song, and
T. Takatsuka,
From hadrons to quarks in neutron stars: a review.
Reports on Progress in Physics 81 (2018) 056902
arXiv:1707.04966





Superfluidity and Superconductivity

Basic superfluids in nature:

Condensed Bose atoms, e.g., ^{4}He liquid at T<2.17K,
atomic Bose condensates (^{23}Na, ^{87}Rb, ...)

Neutral BCS-paired Fermi atoms, e.g., ^{3}He liquid at T<1mK,
atomic fermions (^{6}Li,^{40}K), neutrons in neutron stars,
color-flavor locked paired quarks

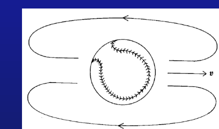
Charged BCS-paired fermions -- superconductors
e.g., electrons in metal, protons in neutron stars

The many faces of superfluidity

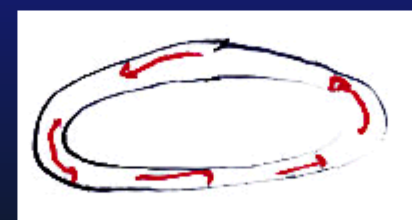
(A.J. Leggett, RMP 71, S318 (1999))

Flow through capillaries without friction
(viscosity $\eta < 0.0006\ \eta_{He\ I}$)

Frictionless flow of object (e.g., ion) through system



Superfluid flow: metastable
flow around a closed pipe “forever”



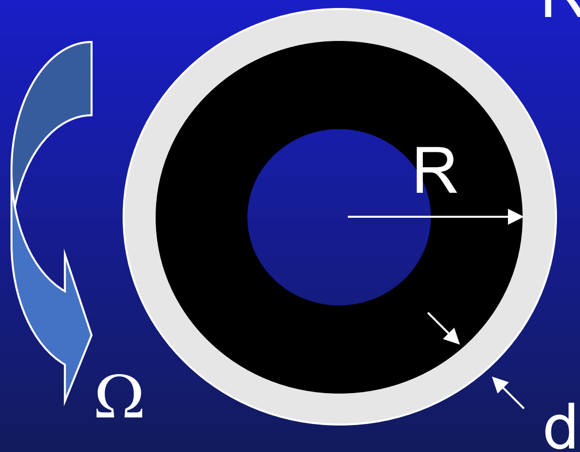
Vortices

Hess-Fairbank effect: equilibrium

Collective excitations, e.g., second sound

Josephson effect

Hess-Fairbank experiment (*Phys. Rev. Lett. 19, 216 (1967)*)



Rotate thin (d<<R) annulus of liquid ^{4}He at Ω

1) Rotate slowly at $T>T_\lambda$: $\Omega<\Omega_c \sim 1/mR^2$ liquid rotates classically with angular momentum $L=I_{classical}\Omega$.

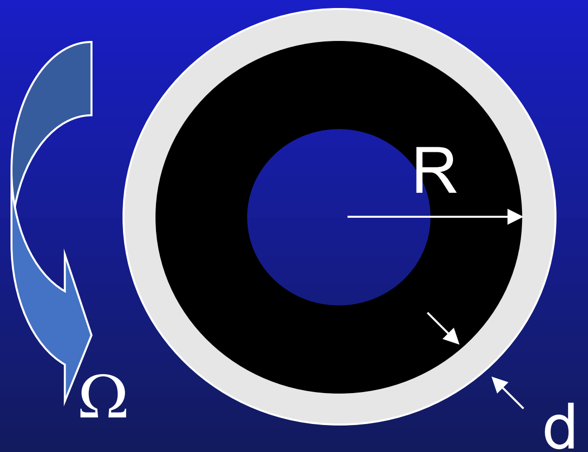
$$I_{classical} \sim NmR^2$$

2) Cool to $T<T_\lambda$: liquid rotates with reduced moment of inertia $I(T) < I_{classical}$. $I(T=0) = 0$.

Only the normal fluid rotates. $I(T) = (\rho_n/\rho)I_{classical}$
The superfluid component remains stationary in the lab.

Reduction of moment of inertia is an equilibrium phenomenon.

Superfluid flow



1) Rotate rapidly at $T>T_\lambda$: $\Omega > \Omega_c$
liquid rotates classically with angular momentum $L=I_{classical}\Omega$.

2) Continue rotating, cool to $T<T_\lambda$:
liquid rotates classically

3) Stop rotation of annulus. Liquid keeps rotating with $L = I_s\Omega$,
where $I_s = (\rho_s/\rho)\, I_{classical}$.

Only the superfluid rotates. The normal component is stationary.

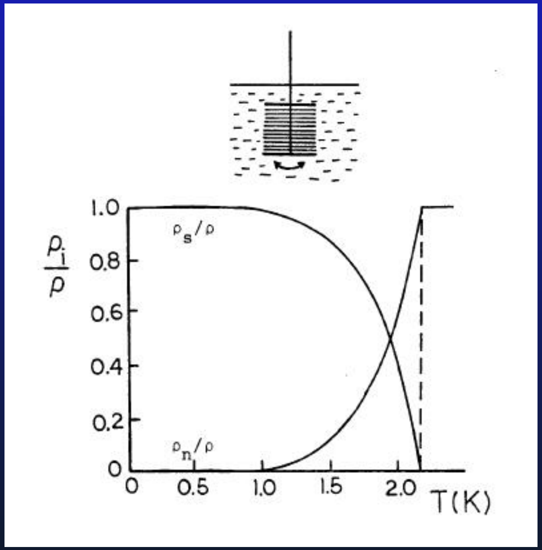
Superfluid flow is metastable (albeit with huge lifetime in macroscopic system)

Landau Two-Fluid Model

Can picture superfluid ^{4}He as two interpenetrating fluids:

Normal: density $\rho_n(T)$, velocity v_n

Superfluid: density $\rho_s(T)$, velocity v_s



Mass current = $\rho_s v_s + \rho_n v_n$

Entropy current = Tsv_n

:carried by normal fluid only

Second sound (collective mode) =

counter-oscillating normal and superfluids

Landau critical velocity ($v_{Landau} = \partial\omega_k/\partial k$) neither necessary nor sufficient to destroy superfluidity. When violated, $\rho_s < \rho$.

Meissner effect and reduced moment of inertia depend on existence of non-zero ρ_s

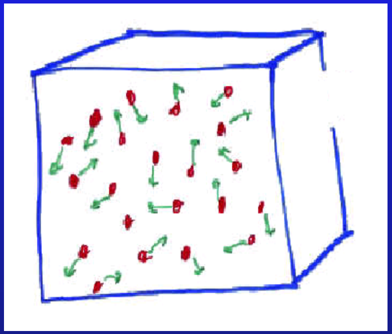
Critical defining property of superfluids and superconductors:

Reduced moment of inertia $I = (\rho_s / \rho) I_{classical}$

Meissner mass M ⇔ 1/ penetration depth of magnetic field

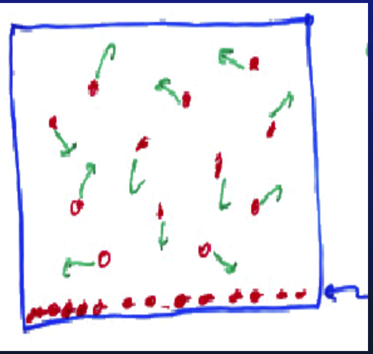
$$M^2 \sim g^2 \boldsymbol{\mu}^2 (\rho_s / \rho)$$

Bose-Einstein Condensation



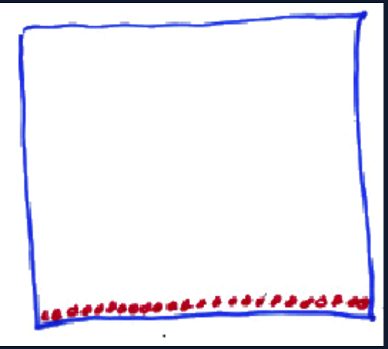
Hot atoms (bosons) in a box

Gravity



Cool below Bose-Einstein transition temperature

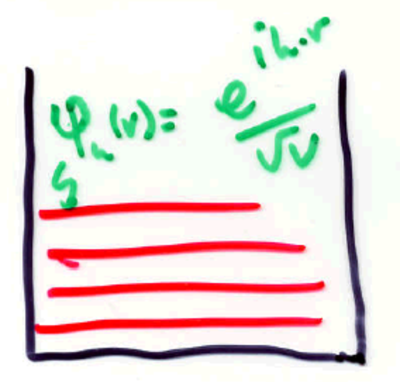
Bose-Einstein condensate



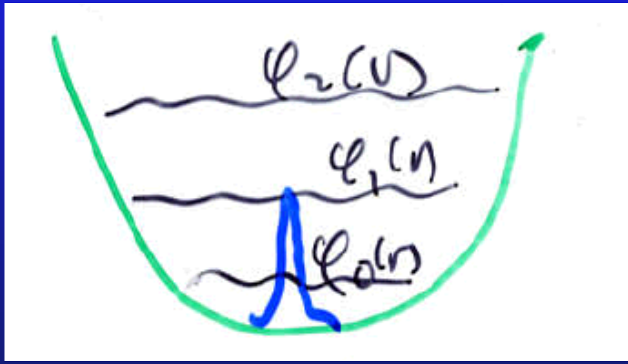
At absolute zero temperature motion “ceases”

Free Bose gas

$(\hbar = 1)$



Box



Potential well (trap)

In condensed system have macroscopic occupation of single (generally lowest) mode

$$\varphi_0(r) = \frac{e^{i\vec{0}\cdot\vec{r}}}{\sqrt{V}} = \frac{1}{\sqrt{V}}$$

: ground state

$$\varphi_m(r) = \frac{e^{im\varphi}}{\sqrt{V}}$$

: flow state (vortex)

Order parameter of condensate $\Psi(\vec{r}) = |\psi| e^{i\phi(\vec{r})}$

wave function of mode into which particles condense

Defined more rigorously by eigenfunction cf. largest eigenvalue of density matrix $\langle \psi(\vec{r}) \psi^\dagger(\vec{r}\,') \rangle \rightarrow \Psi(\vec{r}) \Psi(\vec{r}\,')^*$

Superfluid velocity: $\vec{v}_s(\vec{r}) = \frac{\hbar}{m} \nabla \phi$

Chemical potential: $\mu \doteq \partial\phi/\partial t$

Superfluid acceleration eqn.: $\frac{\partial \vec{v}_s}{\partial t} + \nabla \mu = 0$

Order parameter of BCS paired fermions

Pairing seen in amplitude to remove a pair of fermions (",+) then add pair back, and come back to same state:

$$\langle \psi^\dagger_\uparrow(1)\psi^\dagger_\downarrow(2)\psi_\downarrow(3)\psi_\uparrow(4)\rangle \simeq \langle \psi^\dagger_\uparrow(1)\psi^\dagger_\downarrow(2)\rangle \langle \psi_\downarrow(3)\psi_\uparrow(4)\rangle$$

[Cf., $\langle \psi(\vec{r})\psi^\dagger(\vec{r}\,')\rangle \rightarrow \Psi(\vec{r})\Psi(\vec{r}\,')^*$ in Bose system]

Order parameter. $\langle \psi_\downarrow(r)\psi_\uparrow(r)\rangle \rightarrow \Psi(r)$, as in Bose system

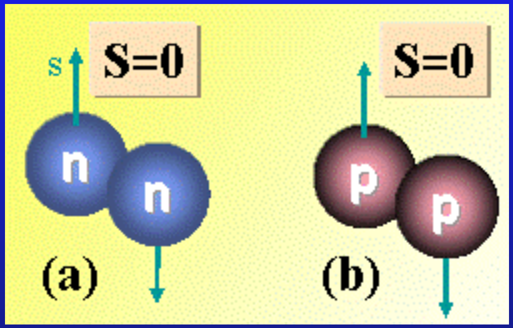
Similar physics as in Bose system

$$\Psi(\vec{r}) = |\psi| e^{i\phi(\vec{r})}$$

Supercurrent velocity: $\vec{v}_s(\vec{r}) = \frac{\hbar}{m}\nabla\phi$

Chemical potential: $\mu = -\partial\phi/\partial t$

BCS applied to nuclear systems - 1957



Pairing of even numbers of neutrons or Protons outside closed shells

*David Pines brings BCS to Niels Bohr's Institute in Copenhagen, Summer 1957, as BCS was being finished in Urbana.

*Aage Bohr, Ben Mottelson and Pines (57) suggest BCS pairing in nuclei to explain energy gap in single particle spectrum – odd-even mass differences

*Pairing gaps deduced from odd-even mass differences:

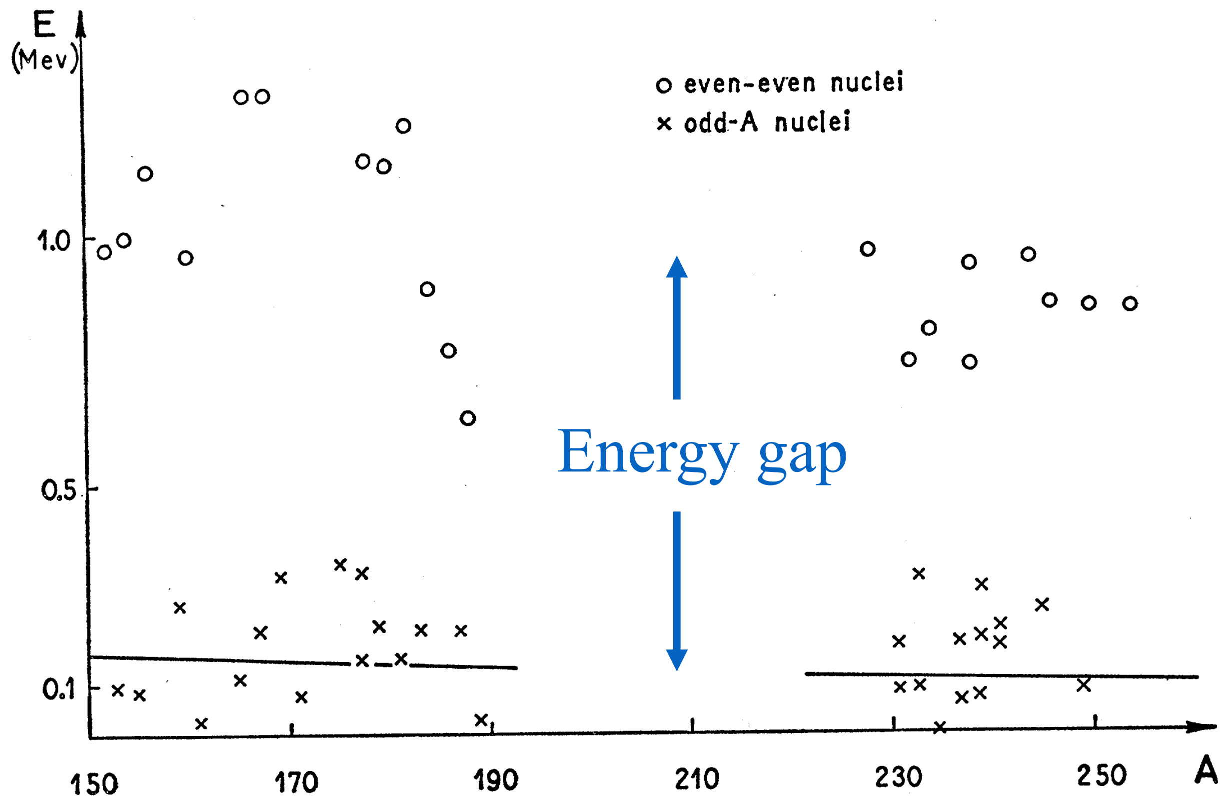
$\Delta \sim 12\ A^{-1/2}$ MeV for both protons and neutrons

Energies of first excited states:
even-even (BCS paired) vs. odd A (unpaired) nuclei

FIG. 1. Energies of first excited intrinsic states in deformed nuclei, as a function of the mass number. The experimental data may be found in *Nuclear Data Cards* [National Research Council, Washington, D. C.] and detailed references will be contained in reference 1 above. The solid line gives the energy $\delta/2$ given by Eq. (1), and represents the average distance between intrinsic levels in the odd-A nuclei (see reference 1).

The figure contains all the available data for nuclei with $150<A<190$ and $228<A$. In these regions the nuclei are known to possess nonspherical equilibrium shapes, as evidenced especially by the occurrence of rotational spectra (see, e.g., reference 2). One other such region has also been identified around $A=25$; in this latter region the available data on odd-A nuclei is still represented by Eq. (1), while the intrinsic excitations in the even-even nuclei in this region do not occur below 4 Mev.

We have not included in the figure the low lying $K=0$ states found in even-even nuclei around Ra and Th. These states appear to represent a collective odd-parity oscillation.



PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. Bohr, B. R. Mottelson, and D. Pines*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.

THE nuclear structure exhibits many similarities with the electron structure of metals. In both cases, we are dealing with systems of fermions which may be characterized in first approximation in terms of independent particle motion. For instance, the statistical level density, at not too low excitation energies, is expected to resemble that of a Fermi gas. Still, in both systems, important correlations in the particle motion arise from the action of the forces between the particles and, in the metallic case, from the interaction with the lattice vibrations. These correlations decisively influence various specific properties of the system. We here wish to suggest a possible analogy between the correlation effects responsible for the energy gaps found in the excitation spectra of certain types of nuclei and those responsible for the observed energy gaps in superconducting metals.

proximately[1]

$$\delta \approx 50A^{-1}\ \text{Mev}, \qquad (1)$$

where A is the number of particles in the nucleus.

If the intrinsic structure could be adequately described in terms of independent particle motion, we would expect, for even-even nuclei, the first intrinsic excitation to have on the average an energy $\frac{1}{2}\delta$, when we take into account the possibility of exciting neutrons as well as protons. Empirically, however, the first intrinsic excitation in heavy nuclei of the even-even type is usually observed at an energy of about 1 Mev (see Fig. 1). The only known examples of intrinsic excitations with appreciably smaller energy are the $K=0-$ bands which occur in special regions of nuclei, and which may possibly represent collective octupole vibrations.[2]

Rotational spectra of nuclei: $E = J^2/2I$, indicate moment of inertia, I, reduced from rigid body value, I_{cl} .

Reduction of moment of inertia due to BCS pairing = analog of Meissner effect. Detailed calculations by Migdal (1959).

Element	β [7]	x_p	x_n	$\left(\frac{\mathscr{I}}{\mathscr{I}_0}\right)_{\text{rect.}}$	$\left(\frac{\mathscr{I}}{\mathscr{I}_0}\right)_{\text{osc.}}$	$\left(\frac{\mathscr{I}}{\mathscr{I}_0}\right)_{\text{exper.}}$ [7]
Nd^{150}	0.26	0.54	0.94	0.15	0.38	0.35
Sm^{152}	0.24	0.65	1.02	0.17	0.43	0.38
Gd^{154}	0.26	0.52	0.88	0.13	0.35	0.36
Gd^{156}	0.33	0.87	1.37	0.22	0.57	0.48
Gd^{157}	0.29	0.93	1.60	0.22	0.64	0.60
Dy^{162}	0.30	0.84	1.43	0.23	0.57	0.50
Hf^{179}	0.20	0.99	1.75	0.27	0.66	0.52
Os^{186}	0.18	0.44	0.69	0.09	0.26	0.28
Th^{230}	0.22	0.63	0.95	0.15	0.40	0.43
Th^{232}	0.22	0.84	1.42	0.24	0.60	0.44
U^{238}	0.24	0.83	1.29	0.22	0.54	0.43

1.C:
1.E.6

Nuclear Physics **13** (1959) 655—674; © *North-Holland Publishing* (

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SUPERFLUIDITY AND THE MOMENTS OF INERTIA OF NUCLEI

A. B. MIGDAL

Atomic Energy Institute of USSR, Academy of Sciences, Moscow

Received 11 April 1959

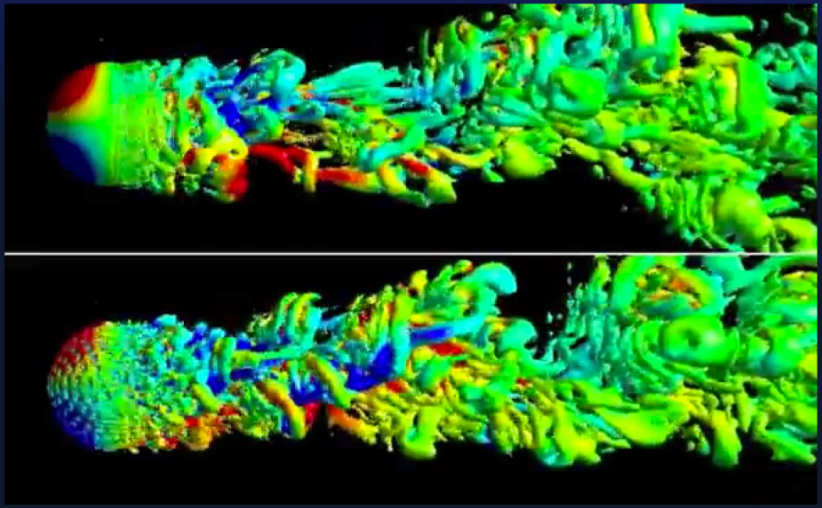
Abstract: A method is presented which permits one to study superfluidity in finite size systems. Moments of inertia are computed by this method in the quasi-classical approximation and satisfactory agreement with the observed values is obtained. The calculated increase of the moment of inertia upon transition from even to odd-mass nuclei and also the gyromagnetic ratio for rotating nuclei are in agreement with the experiments. These results thus confirm the assumption of superfluidity of nuclear matter.

Classical vortices

Sinks and bathtubs

Tornadoes

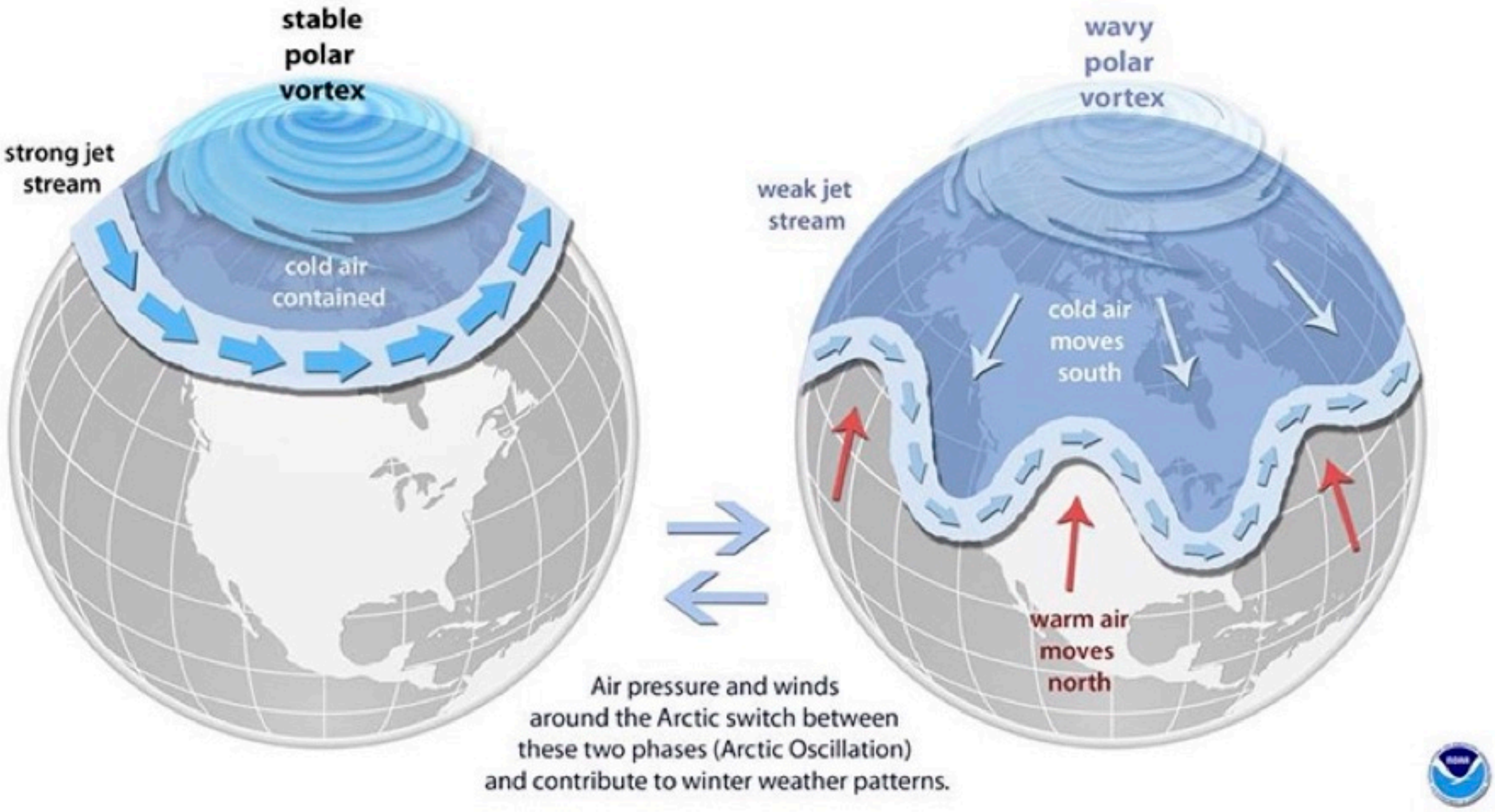
Airplane wingtips



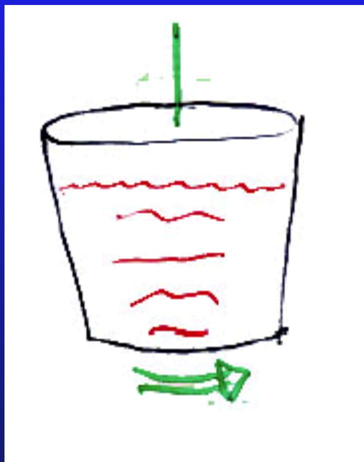
Turbulence: sphere vs. golf ball

The Science Behind the Polar Vortex

The polar vortex is a large area of low pressure and cold air surrounding the Earth's North and South poles. The term vortex refers to the counter-clockwise flow of air that helps keep the colder air close to the poles (left globe). Often during winter in the Northern Hemisphere, the polar vortex will become less stable and expand, sending cold Arctic air southward over the United States with the jet stream (right globe). The polar vortex is nothing new — in fact, it's thought that the term first appeared in an 1853 issue of E. Littell's *Living Age*.

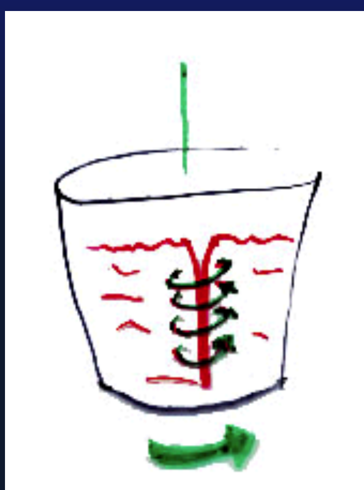


Quantized vortices in superfluids

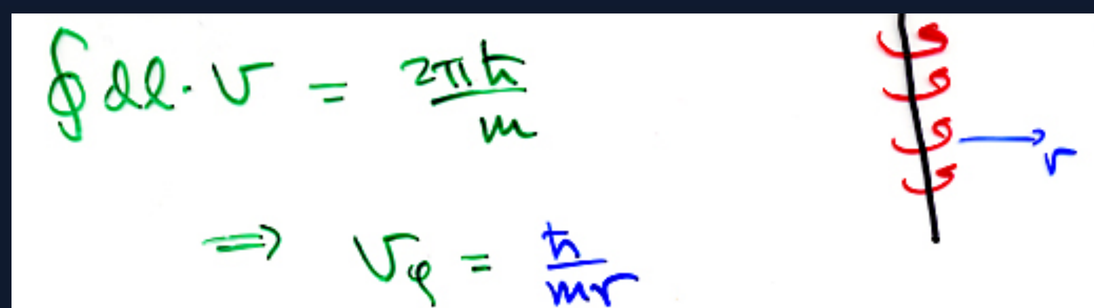


Spin glass of water – the water always turns with the glass.

But spin container of superfluid helium slowly. Helium liquid remains at rest!



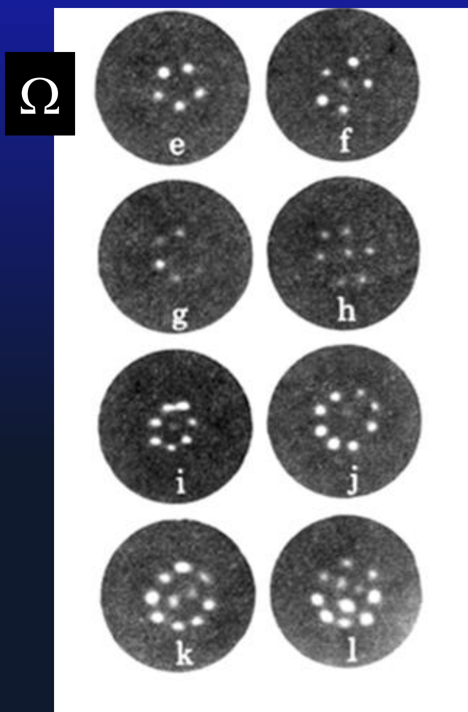
Spin fast enough. Form vortex in center of liquid!



Circulation around vortex quantized in units of $2\pi\hbar/m$!

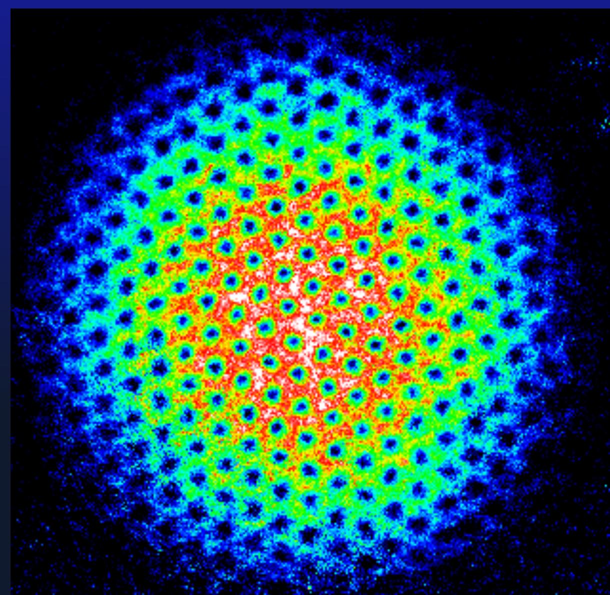
Quantized vortices in condensed matter physics

Superfluid ^{4}He, rotating bucket:



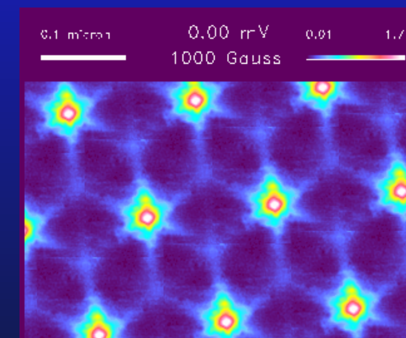
E. J. Yarmchuk, M. J. V. Gordon, R. E. Packard
Phys. Rev. Lett. **43**, 214 (1979)

Dilute atomic gas Bose-Einstein Condensate



JILA
MIT
ENS
Oxford

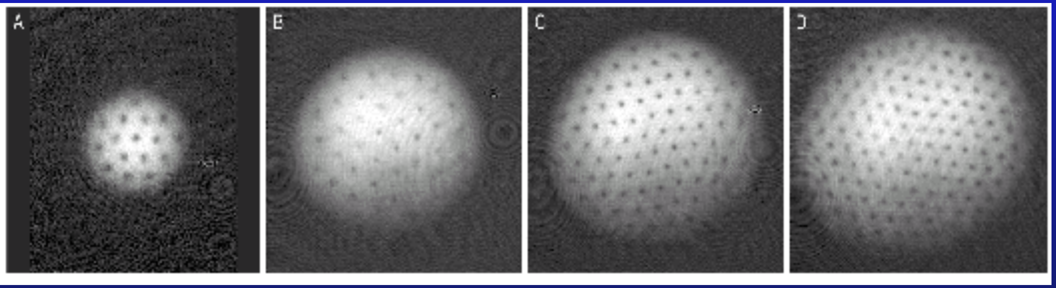
Type-II superconductors



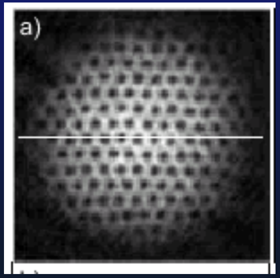
Bell Labs

Quantum Hall Systems

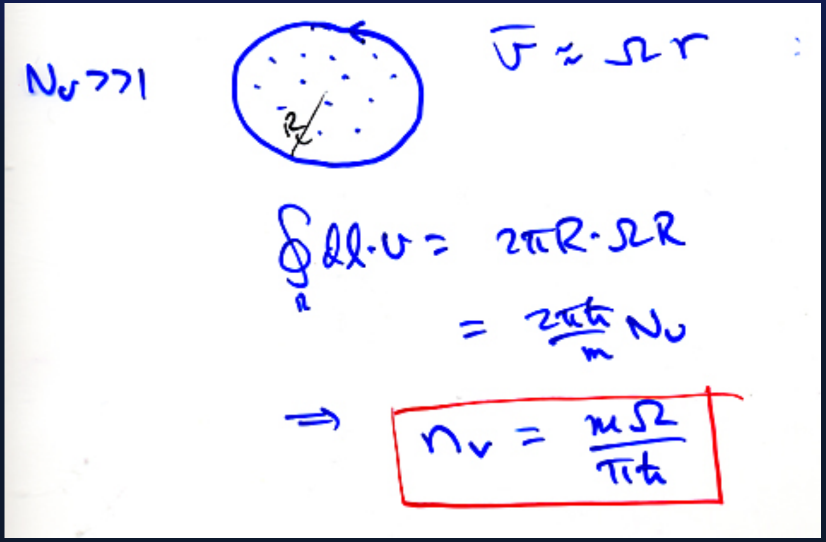
Rapidly rotating superfluid contains triangular lattice of vortices



Abo-Shaeer et al. (MIT) 2001



Engels et al. (JILA) 2002



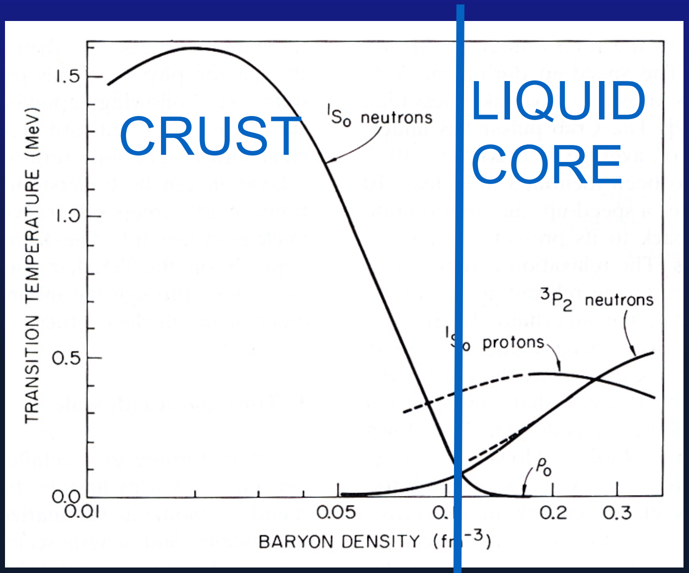
Solid body rotation on average

Superfluidity of nuclear matter in neutrons stars

Migdal 1959, Ginzburg & Kirshnits 1964; Ruderman 1967; GB, Pines & Pethick, 1969

Neutron stars (very big Dewars) have the preponderence of superfluids in the universe, and with the highest T_c's ~ 10^{10-11} K

Pairing gaps and T_c's estimated from scattering phase shifts



Neutron fluid in crust BCS-paired in relative $^1S_0$ states (singlet spin)

Neutron fluid in core $^3P_2$ paired (triplet spin)

Proton fluid $^1S_0$ paired

Quantum Monte Carlo $^1S_0$ nn gap in crust

Alex Gezerlis

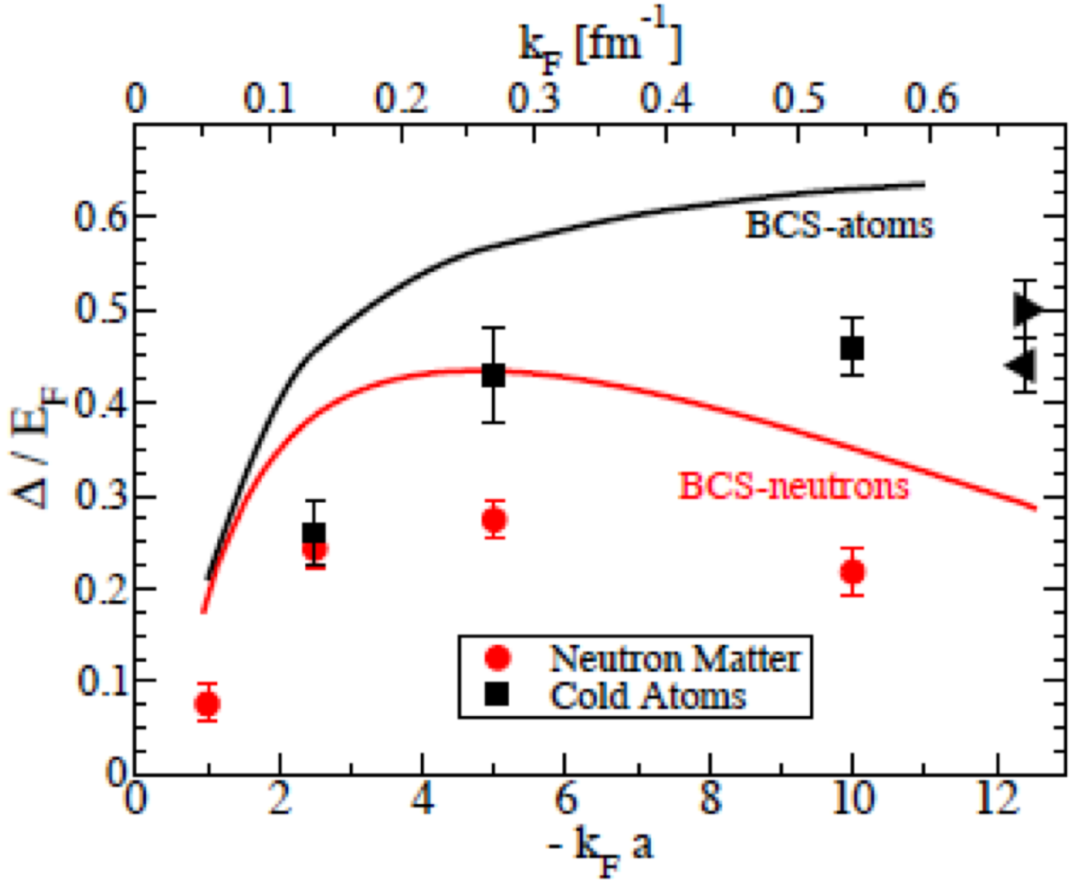
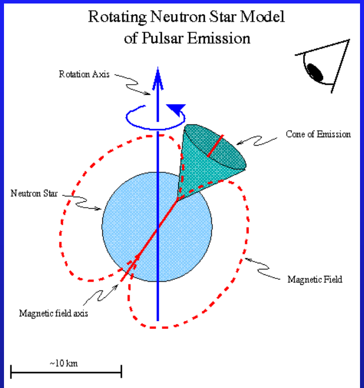
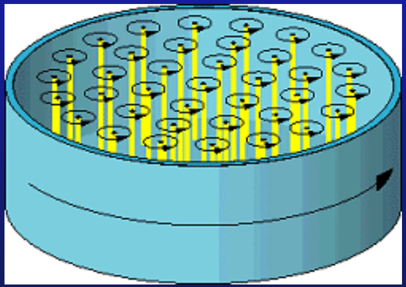


Figure 6.3: Superfluid pairing gap versus $k_F a$ for cold atoms ($r_e \approx 0$) and neutron matter ($|r_e/a| \approx 0.15$). BCS (solid lines) and QMC results (points) are shown. Also shown are QMC (right arrow) and experimental (left arrow) results at unitarity.

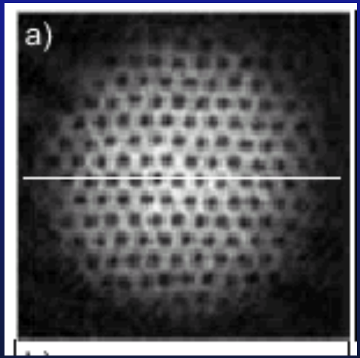
Rotating superfluid neutrons



Rotating superfluid threaded by triangular lattice of vortices parallel to stellar rotation axis



Bose-condensed ^{87}Rb atoms
Schweikhard et al., PRL 92 040404 (2004)



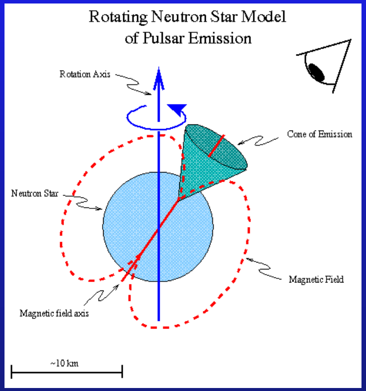
Quantized circulation of superfluid velocity about vortex:

$$\oint_C \mathbf{v_s} \cdot d\ell = \frac{2\pi\hbar}{2m_n}$$

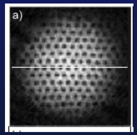
Vortex core ~ 10 fm
Vortex separation ~ $0.01P(s)^{1/2}$cm; Vela (PSR0833-45) ~ 10^{17} vortices

Superconducting protons in magnetic field

Even though superconductors expel magnetic flux, for magnetic field below critical value, flux diffusion times in neutron stars are >> age of universe. Proton superconductivity forms with field present.



Proton fluid threaded by triangular (Abrikosov) lattice of vortices parallel to magnetic field (for Type II superconductor)



Quantized magnetic flux per vortex: $\oint_C \mathbf{A} \cdot d\ell = \frac{2\pi\hbar c}{2e}$ $= \phi_0 = 2 \text{ X } 10^{-7}$ G.

Vortex core ~ 10 fm,

$n_{vort} = B/\phi_0$ => spacing ~ 5×10^{-10} cm $(B/10^{12}G)^{-1/2}$

Pulsar glitches

Sudden speedups in rotation period, relaxing back in days to years, with no significant change in pulsed electromagnetic emission: to date 536 glitches detected in 189 pulsars

Vela (PSR0833-45)   Period=1/Ω =0.089sec

20 glitches since discovery in 1969

$\Delta\Omega/\Omega \sim 10^{-6}$ Largest = 3.14×10^{-6} on Jan. 16, 2000

Moment of inertia ~ 10^{45} g-cm^2 => $\Delta E_{rot} \sim 10^{43}$ erg

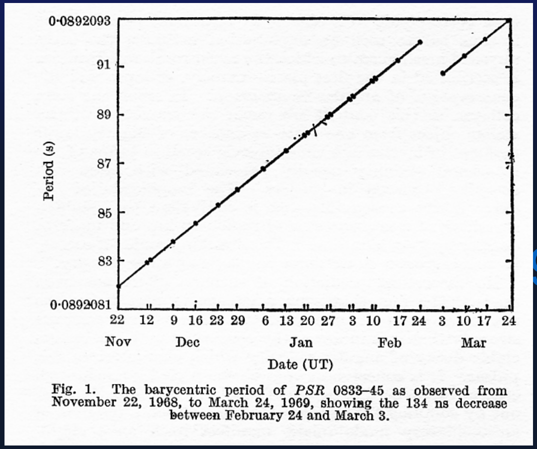


Fig. 1. The barycentric period of *PSR* 0833–45 as observed from November 22, 1968, to March 24, 1969, showing the 134 ns decrease between February 24 and March 3.

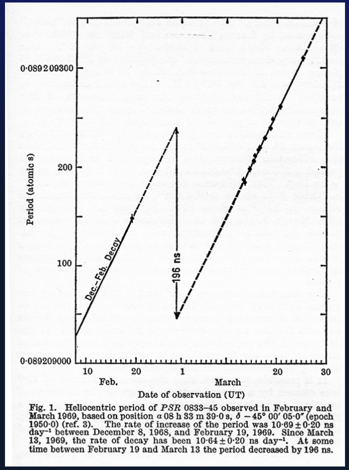
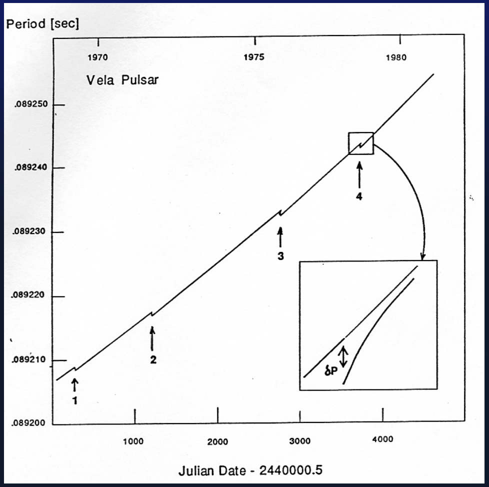


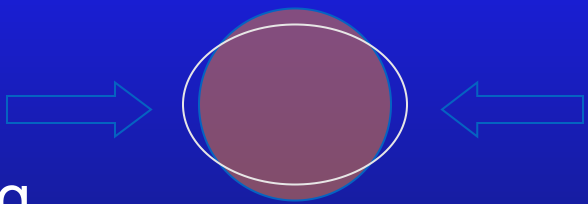
Fig. 1. Heliocentric period of *PSR* 0833–45 observed in February and March 1969, based on position α 08 h 33 m 39·0 s, δ −45° 00′ 05·0″ (epoch 1950·0) (ref. 3). The rate of increase of the period was 10·69 ± 0·20 ns day^{-1} between December 8, 1968, and February 19, 1969. Since March 13, 1969, the rate of decay has been 10·64 ± 0·20 ns day^{-1}. At some time between February 19 and March 13 the period decreased by 196 ns.



Reichley and Downs, Nature 1969   *Radhakrishnan and Manchester, Nature 1969*

Crab (PSR0531+21) P = 0.033sec   28 glitches since 1969, $\Delta\Omega/\Omega \sim 10^{-8}$ -10^{-9}

Starquake?



As star slows down, mechanical stresses increase in crust -- possibly past the breaking point of matter. Cracking = starquake tends to make crust more spherical (*Ruderman 1968, GB et al. 1969, GB & Pines 1971*).

Conservation of angular momentum => $\Delta\Omega/\Omega = -\Delta I / I$

Surface motion of ~ 1 cm would give $\Delta\Omega/\Omega \sim 10^{-6}$

BUT

$\Delta E \sim 10^{43}$ erg/glitch too much energy to store in crust to enable ~ 4-5 glitches per decade.

Sendai earthquake (or gravitational wave?)

Tokyo, March 11, 2011

Time scales

Need intermediate time scale (~ months) to understand glitches.

Slowing down: P/(dP/dt) ~ age of pulsar ~ 10^3-10^6 y

Spin down of charged particles: $\tau \sim \tau_{Alfven} \sim R(4\pi\rho)^{1/2}/B \sim 10$ s

Normal quasiparticle scattering: $\tau_{np} \sim E_f/T^2 \sim 10^{-11}$s

Superfluid q.p. scattering: $\tau_{np} \sim e^{\Delta/T}/E_f$ $(\Delta/T \sim 10^2 - 10^3)$

Vortex dynamics only promising way to get required time scales

Neutron vortex-charged particle scatterings:

$\tau_{e^- \text{- vortex core exc.}} \sim \tau_{em} e^{\Delta_n^2/E_f T} \sim 10^{20}$ s ($^1S_0$ vortices)

$\tau_{e^- \text{-} ^3P_2} \sim 10^8 P(\text{sec})/\Delta_n(\text{MeV}) \sim 2$ mos. (magnetized $^3P_2$ vortices)

(*Sauls, Stein, & Serene, Muzikar, Sauls, & Serene*)

Length scales

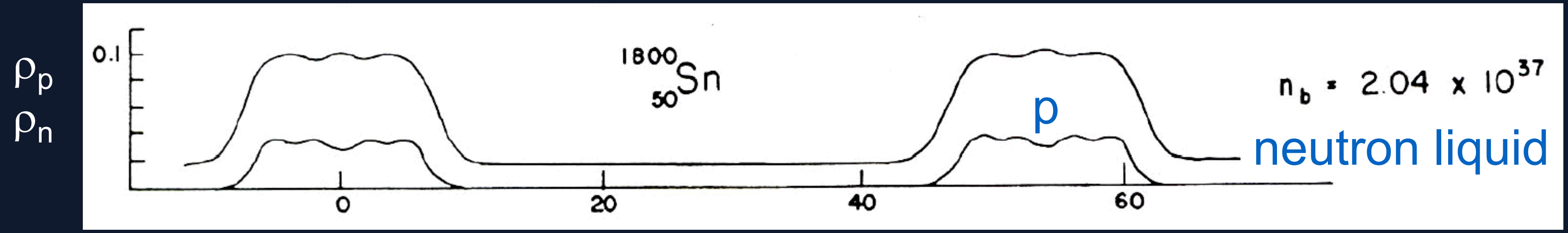
Spacing of n vortices ~ 10^{-2} cm

Spacing of p vortices ~ 5 X 10^{-10} cm

Spacing of nuclei ~ 2 X 10^{-12} $(\rho/\rho_{nm})^{1/3}$ cm

Nuclear size, R_A ~ 10^{-12} cm

Neutron superfluid coherence length, ξ_n ~ 10^{-12} cm ~ R_A



N pairing inside and outside nuclei connected

Physical picture of glitches

Since pulse structure not notably affected by glitch, must be internal phenomenon in the neutron star. Long time scales for response indicate well-oiled machinery -- superfluidity! [Metastable superfluid flow (Packard 1972).]

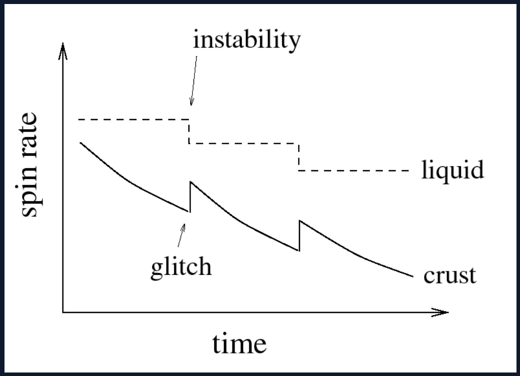
Pulses connected - via magnetic field - to the crust.

Neutron superfluids in interior act as a reservoir of angular momentum. Transfer of angular momentum to crust speeds it up => glitch

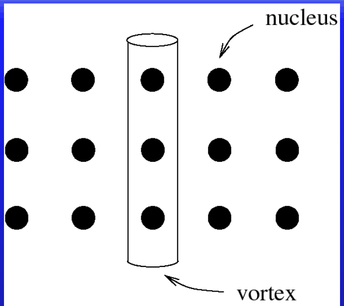
Where in neutron star is the reservoir?

How is the differential velocity between the crust and liquid maintained?

How is the reservoir tapped?

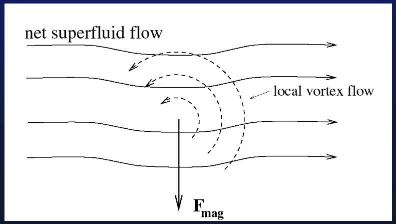


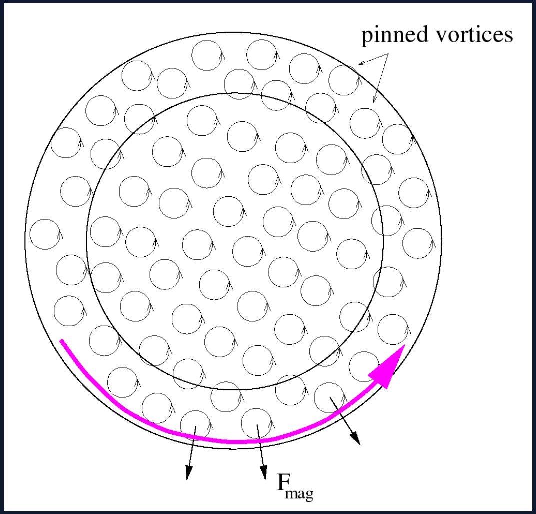
Vortex model of glitches



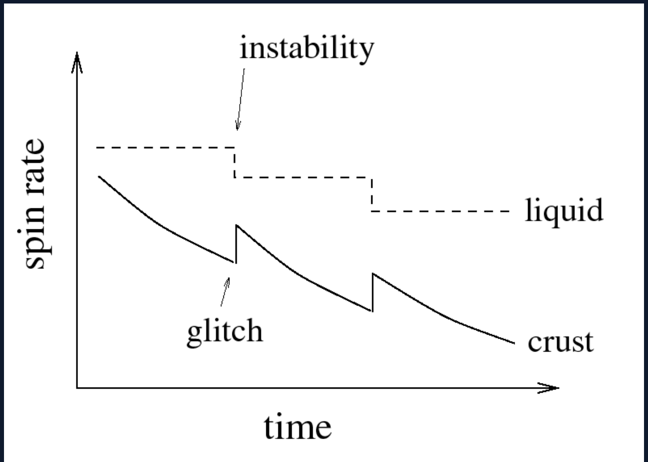
Pin vortices on nuclei in inner crust.
E ~ few Mev/nucleus.
(Bogoliubov- de Gennes calculations suggest pinning between nuclei)

$n_{vortices}$ fixed => $\Omega_{superfluid}$ fixed; Ω_{crust} decreases as star radiates.
As $\Omega_{sf} - \Omega_{crust}$ grows, Magnus force = $\rho_s\ \Omega \times (\mathbf{v}_{vortex} - \mathbf{v}_{superfl})$ drives unpinning (glitch) and outward relaxation.





Collective outward motion of many ($\sim 10^{14}$) Vortices produces large glitch



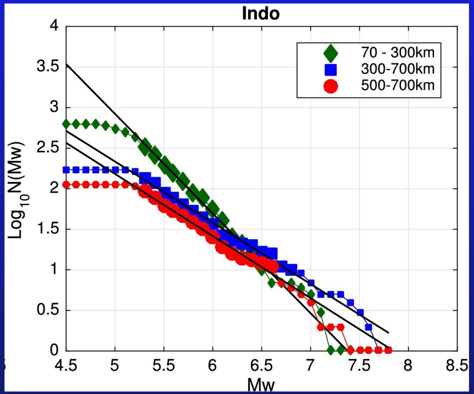
Glitches vs. earthquakes?

Distribution of earthquakes of amplitude A ~ 10^M

(M=Richter mag.)

$$\frac{dN}{d\ln A} \sim \frac{1}{A^b}$$

b ~ 1

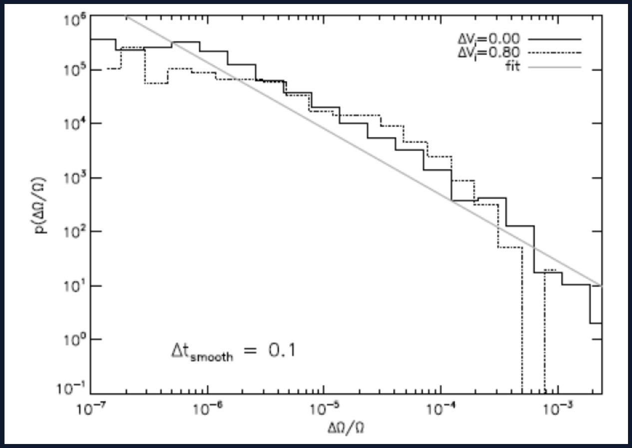


Energy release E ~ $A^{3/2}$

$$\frac{dN}{d\log E} \sim \frac{1}{E^{2/3}}$$

(Sendai 2011, M = 9.0, E ~ 480 Megatons TNT)

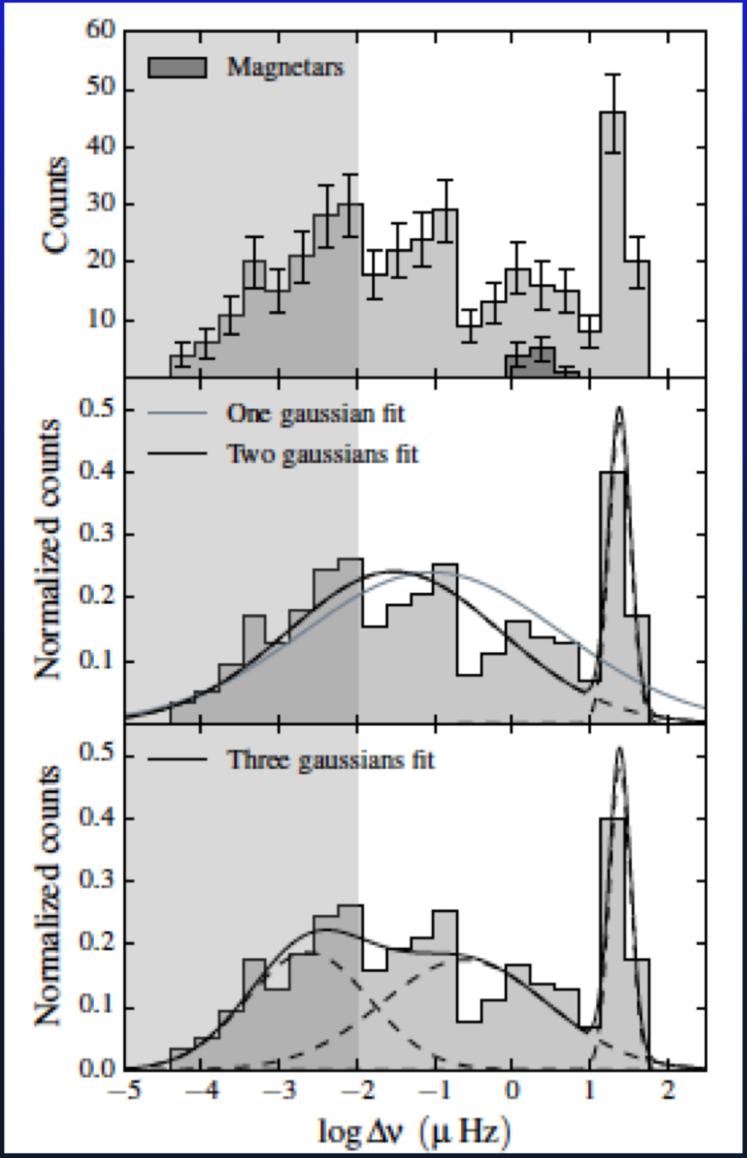
Simulation of neutron star glitches via Gross-Pitaevskii eqn., with pinning sites (*Warszawski and Melatos, MNRAS 415, 1611 (2011)*):



similar power law falloff
vs. amplitude $\Delta\,\Omega/\Omega$

Glitch distribution vs. relative amplitude is bimodal

Fuentes et al., Espinoza et al., A&A 608, A131 (2017)

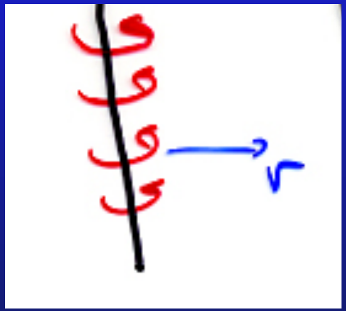


Vortices in superfluids: quantized circulation

Order parameter $\Psi(\vec{r}) = |\psi| e^{i\phi(\vec{r})}$

Momentum $p = \hbar \nabla \phi$

Superfluid velocity $\vec{v}(\vec{r}) = (\hbar/m)\nabla\phi$



Quantized circulation $\oint_C \vec{v} \cdot \vec{d\ell} = \frac{2\pi\hbar n}{m}$ n = integer

Singly quantized (n=1) vortex flow: $v_\varphi(r) = \hbar/mr$

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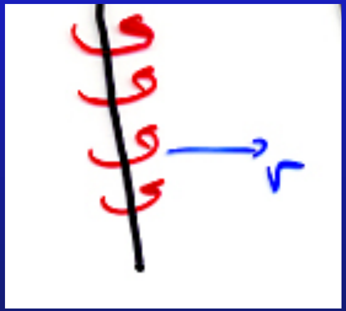

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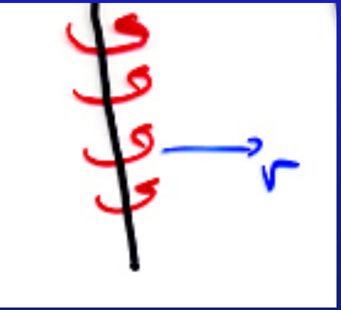

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How is the superfluid velocity related to the momentum?

$$\phi = \vec{p} \cdot \vec{r} - \mu t \Rightarrow \vec{v} = \vec{p}/\mu$$

μ = chemical potential including rest mass

In circulation, replace m by μ more correctly

$$\oint_C \vec{v} \cdot d\vec{\ell} = \frac{2\pi\hbar n}{\mu}$$

Vortices in superfluids: quantized circulation

Order parameter $\Psi(\vec{r}) = |\psi| e^{i\phi(\vec{r})}$ Momentum $p = \hbar \nabla \phi$

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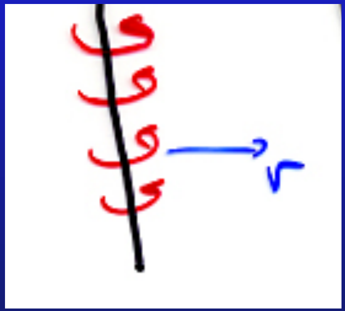

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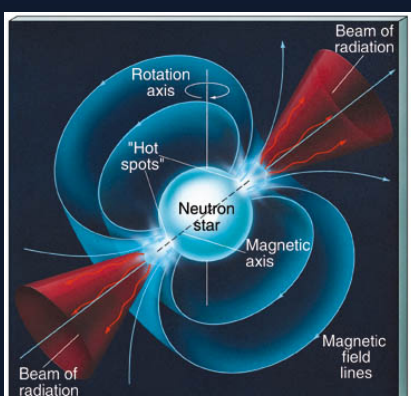
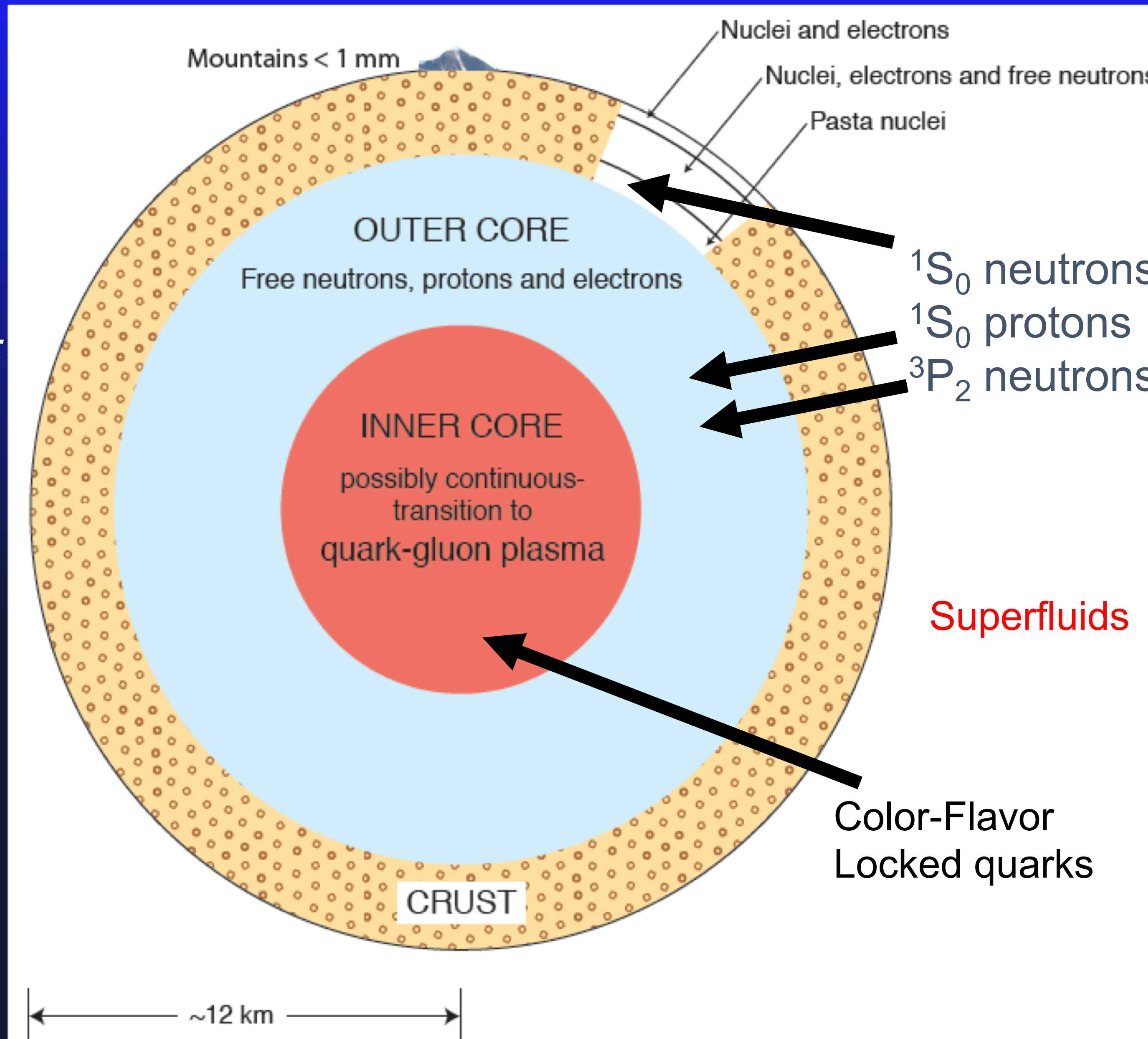
$$\oint_C \vec{v} \cdot d\vec{\ell} = \frac{2\pi\hbar n}{\mu}$$

In superfluid ^{4}He, -7.17K correction to m_4 is only ~ 1 : 6 X10^{12}

Neutron star interior

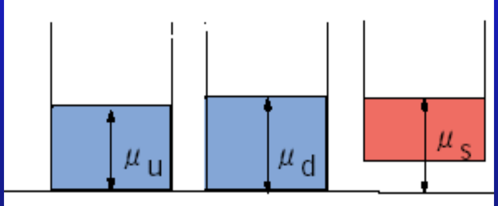
GB, T. Hatsuda,
T. Kojo, P. D. Powell,
Y. Song, and
T. Takatsuka,
From hadrons to quarks in neutron stars: a review.
Reports on Progress in Physics 81 (2018) 056902
arXiv:1707.04966





Color pairing in quark matter

In quark matter have "free quarks" = spin ½ with *flavor* u,d,s and *color* = internal degree of freedom for SU(3) gauge symmetry.



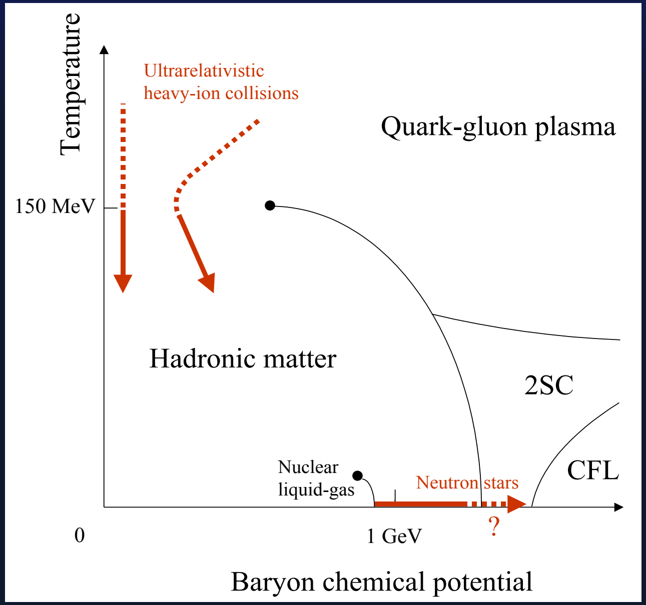
Two interesting pairing states:

2SC (u,d)

$\langle u_R d_B \rangle$

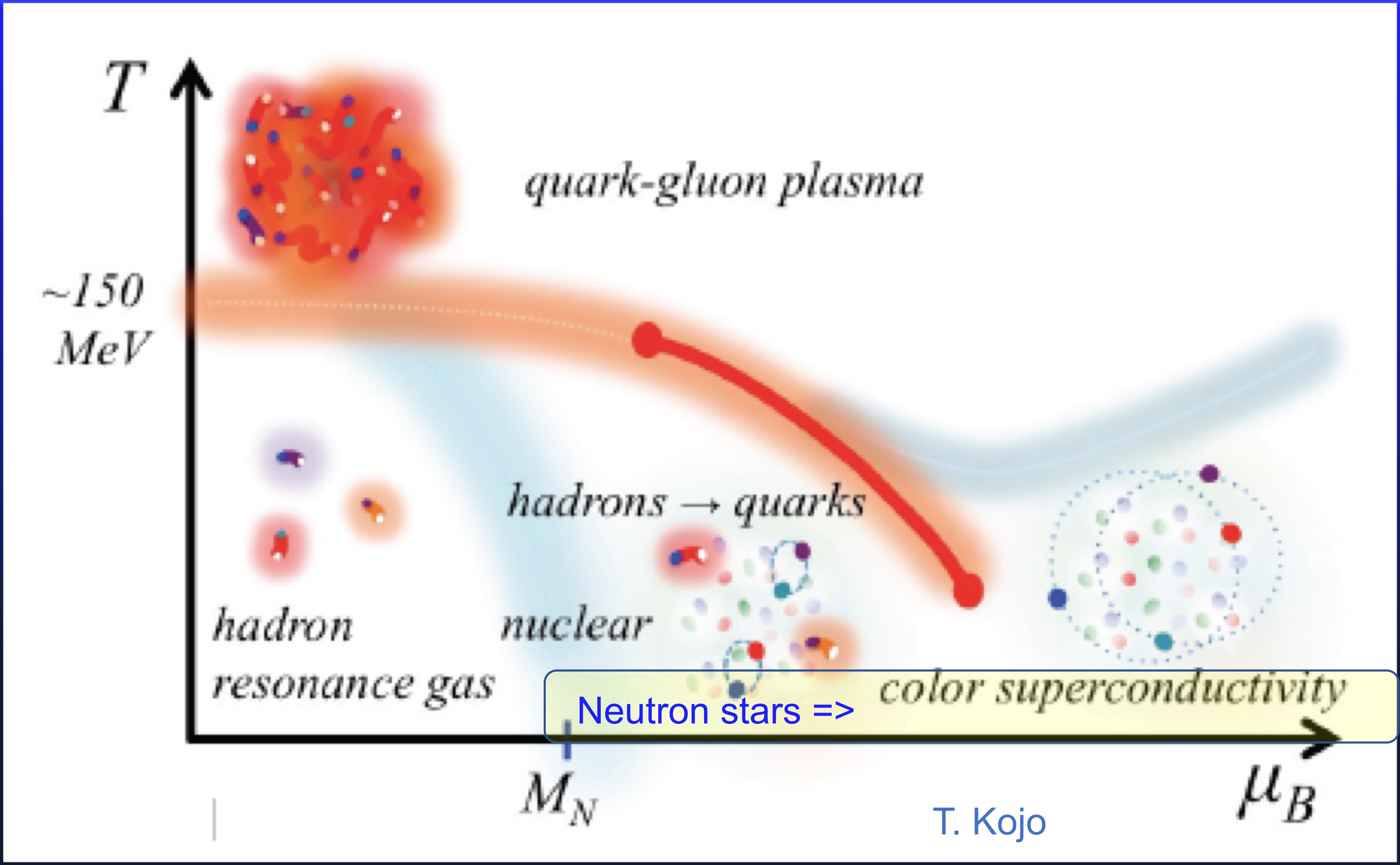
Color-flavor locked (CFL) (m_u=m_d=m_s)

$\langle u d \rangle = \langle d s \rangle = \langle s u \rangle$



2SC similar to superconducting protons:
e.m. vortices in magnetic field.
London moment under rotation

CFL similar to superfluid neutrons:
$U(1)_B$ vortices under rotation
(Partial screening of magnetic fields.)

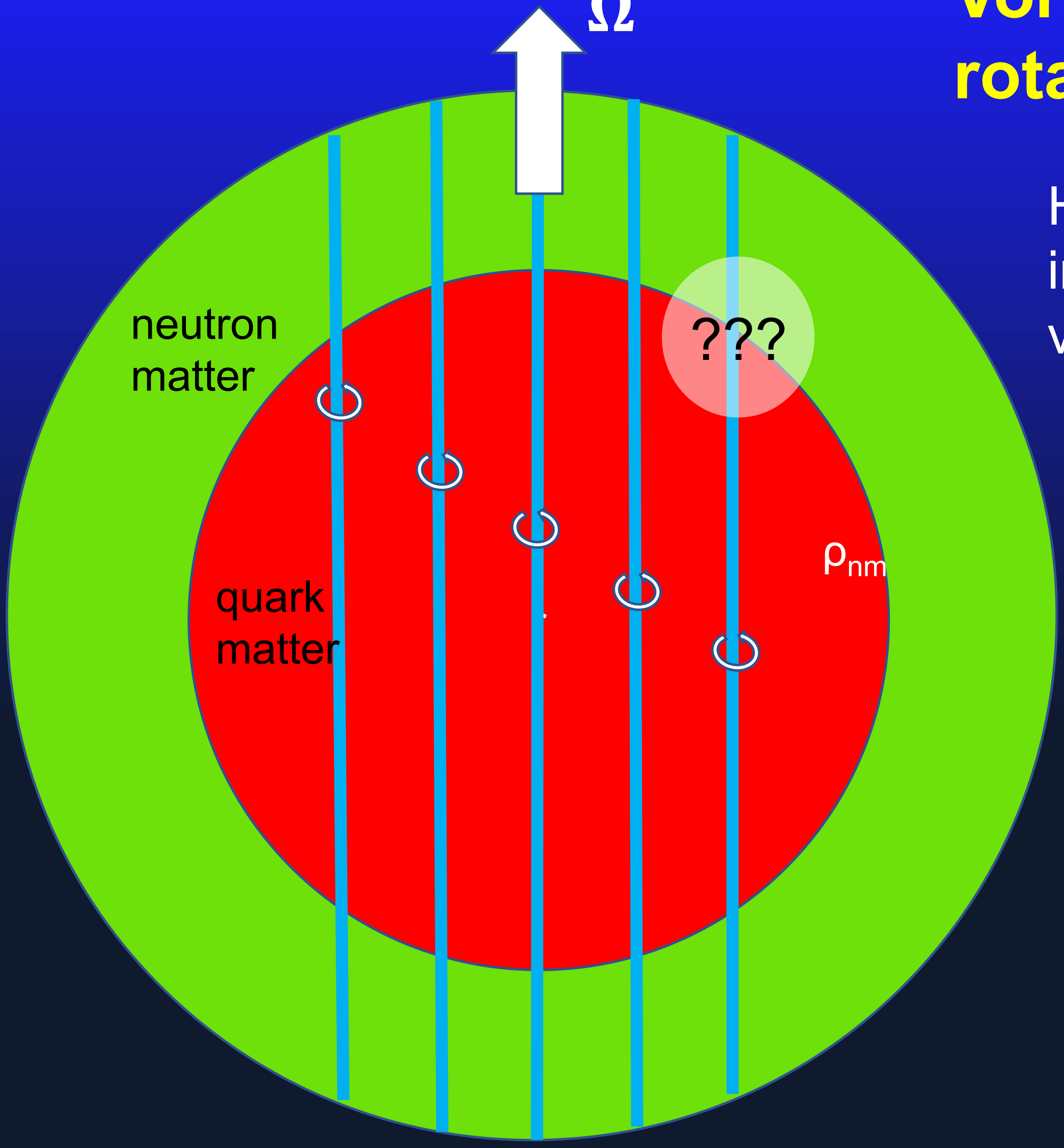


Possible quark-hadron continuity:

E. Fradkin & S. Shenker, PRD19, 3682 (1979)

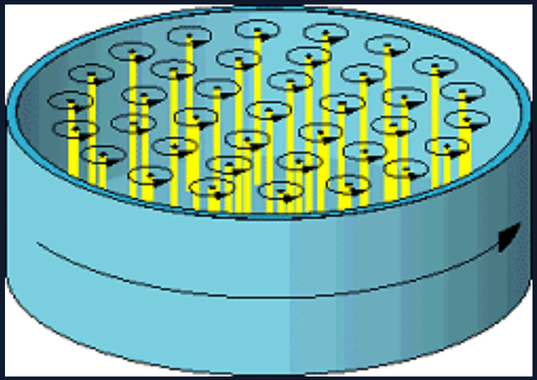
T. Schaefer & F. Wilczek, PRL 82, 3956 (1999)

Vortices threading rotating neutron star



How do neutron vortices interface with quark (CFL) vortices??

M. Alford, GB, K. Fukushima, T. Hatsuda, & M. Tachibana, Phys. Rev. D 99, 036004 (2019); arXiv:1803-05115



Try to match circulations

Circulation: $$C = \oint_C \vec{v} \cdot \vec{d\ell} = \frac{2\pi\hbar n}{\mu}$$ v = superfluid velocity p/μ

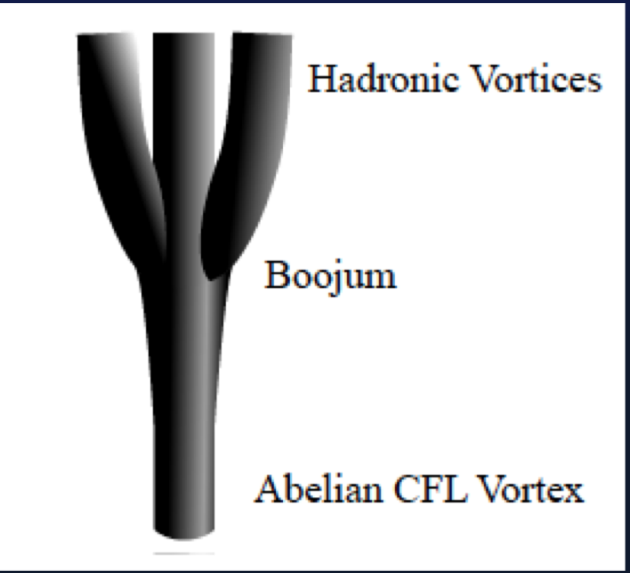
In paired hadronic phase $\mu = 2\mu_n$ (μ_n=neutron chemical potential).

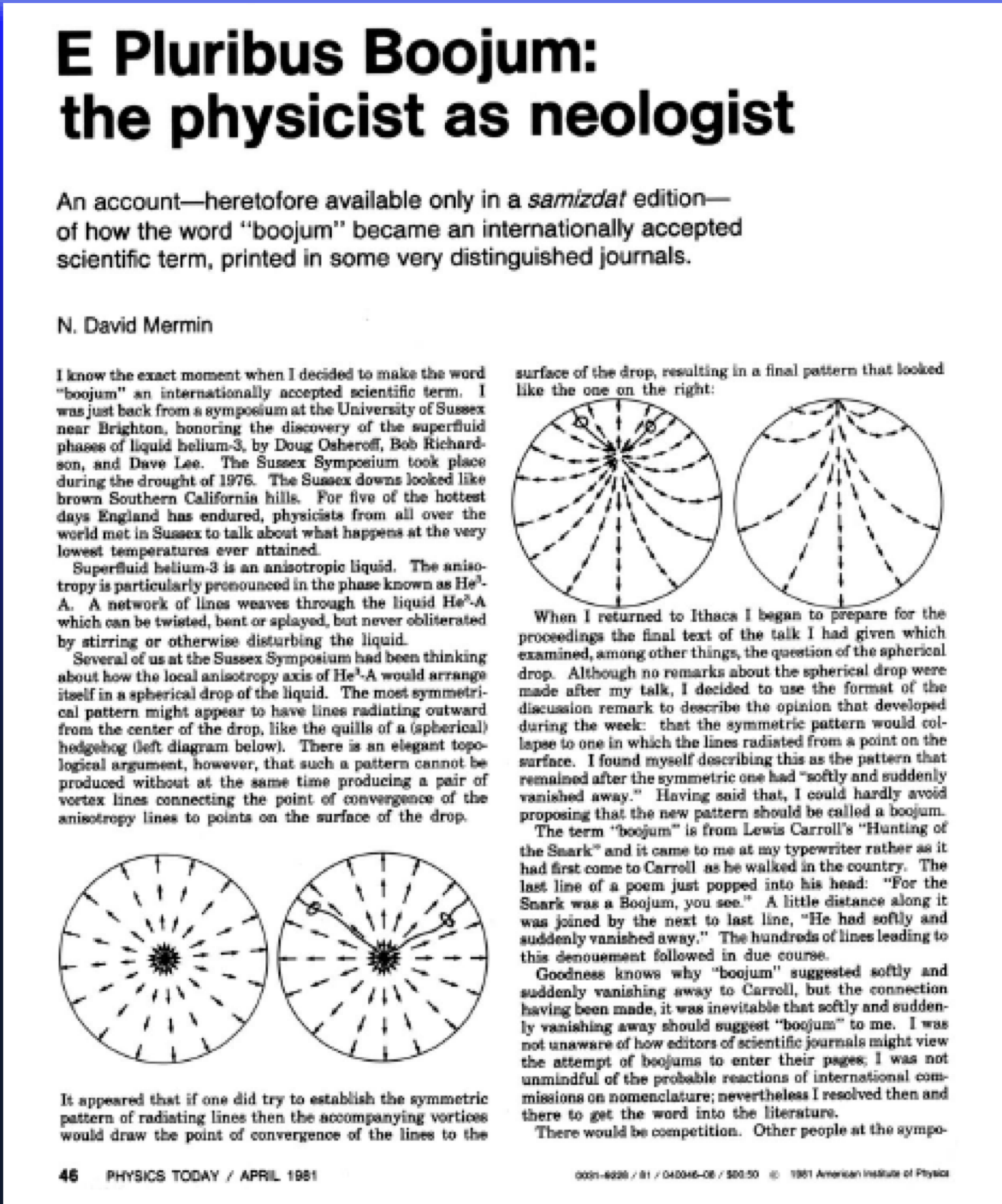
In paired quark phase $\mu = 2\mu_q = 2\mu_n/3$ (μ_q= quark chemical potential), since nucleon is made of 3 quarks, $\mu_n = 3\mu_q$

=> quark phase superfluid velocity = 3 X velocity in hadronic phase.

Continuity in flow states in neutron star, would require 3 hadron vortices merging into a single quark vortex.

A boojum!





E Pluribus Boojum: the physicist as neologist

An account—heretofore available only in a *samizdat* edition—of how the word "boojum" became an internationally accepted scientific term, printed in some very distinguished journals.

N. David Mermin

I know the exact moment when I decided to make the word "boojum" an internationally accepted scientific term. I was just back from a symposium at the University of Sussex near Brighton, honoring the discovery of the superfluid phases of liquid helium-3, by Doug Osheroff, Bob Richardson, and Dave Lee. The Sussex Symposium took place during the drought of 1976. The Sussex downs looked like brown Southern California hills. For five of the hottest days England has endured, physicists from all over the world met in Sussex to talk about what happens at the very lowest temperatures ever attained.

Superfluid helium-3 is an anisotropic liquid. The anisotropy is particularly pronounced in the phase known as He^3-A. A network of lines weaves through the liquid He^3-A which can be twisted, bent or splayed, but never obliterated by stirring or otherwise disturbing the liquid.

Several of us at the Sussex Symposium had been thinking about how the local anisotropy axis of He^3-A would arrange itself in a spherical drop of the liquid. The most symmetrical pattern might appear to have lines radiating outward from the center of the drop, like the quills of a (spherical) hedgehog (left diagram below). There is an elegant topological argument, however, that such a pattern cannot be produced without at the same time producing a pair of vortex lines connecting the point of convergence of the anisotropy lines to points on the surface of the drop.

It appeared that if one did try to establish the symmetric pattern of radiating lines then the accompanying vortices would draw the point of convergence of the lines to the surface of the drop, resulting in a final pattern that looked like the one on the right:

When I returned to Ithaca I began to prepare for the proceedings the final text of the talk I had given which examined, among other things, the question of the spherical drop. Although no remarks about the spherical drop were made after my talk, I decided to use the format of the discussion remark to describe the opinion that developed during the week: that the symmetric pattern would collapse to one in which the lines radiated from a point on the surface. I found myself describing this as the pattern that remained after the symmetric one had "softly and suddenly vanished away." Having said that, I could hardly avoid proposing that the new pattern should be called a boojum.

The term "boojum" is from Lewis Carroll's "Hunting of the Snark" and it came to me at my typewriter rather as it had first come to Carroll as he walked in the country. The last line of a poem just popped into his head: "For the Snark was a Boojum, you see." A little distance along it was joined by the next to last line, "He had softly and suddenly vanished away." The hundreds of lines leading to this denouement followed in due course.

Goodness knows why "boojum" suggested softly and suddenly vanishing away to Carroll, but the connection having been made, it was inevitable that softly and suddenly vanishing away should suggest "boojum" to me. I was not unaware of how editors of scientific journals might view the attempt of boojums to enter their pages; I was not unmindful of the probable reactions of international commissions on nomenclature; nevertheless I resolved then and there to get the word into the literature.

There would be competition. Other people at the sympo-

0031-9228 / 81 / 040046-08 / $00.50 © 1981 American Institute of Physics

David Mermin
Physics Today
April 1981

A boojum tree

BCS pairing in Color Flavor Locked (CFL) phase

In free equally populated up, down, and strange quark matter have $SU(3)_F$ symmetry in flavor (uds) and $SU(3)_C$ symmetry in color (rgb)

Most favored BCS pairing state is anti-symmetric in spin, flavor (i), and color (α):

$$\Phi_{\alpha i} \propto \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\langle q_{\beta j} C \gamma_5 q_{\gamma k}\rangle \chi_{\mathrm{spin-singlet}}$$

$$\Phi = \begin{pmatrix} \Phi^{\bar{r}\bar{u}} & 0 & 0 \\ 0 & \Phi^{\bar{g}\bar{d}} & 0 \\ 0 & 0 & \Phi^{\bar{b}\bar{s}} \end{pmatrix} \chi \rightarrow \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{pmatrix} \chi$$

flavor ->

color ->

CFL order parameter in ground state

Pairing with correlation of color and flavor reduces symmetry from $SU(3)_C$ X $SU(3)_F$ X $U(1)_B$ to $SU(3)_{C+F}$

Abelian vortex in CFL phase

Order parameter matrix of Abelian CFL vortex.

$$\Phi(r,\phi) = \Delta \cdot f(r) e^{i\varphi} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Circulation = $2\pi\hbar/2\mu_q = 3(2\pi\hbar/2\mu_n)$

But this vortex is unstable against decay into three color flux tubes with lower kinetic energy *(A. P. Balachandran et al. PR D 73 (2006); E. Nakano et al., PR D 78, 045002 (2008). Phys. Lett. B 672 (2009)):*

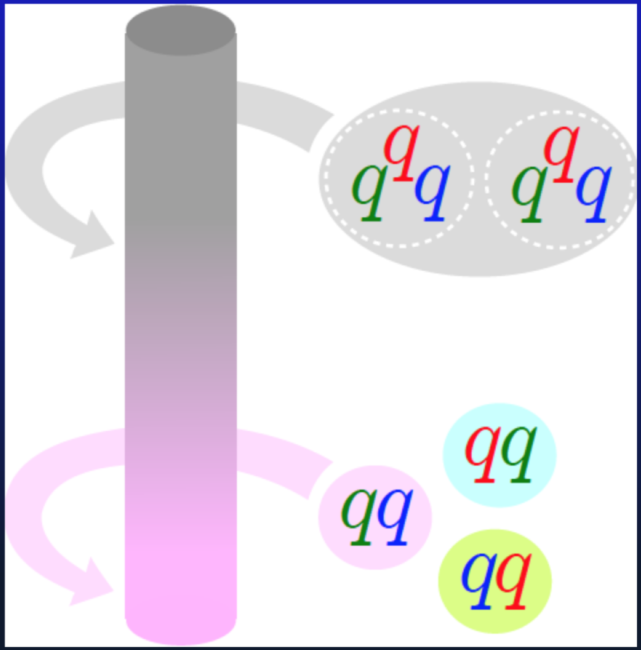
Color flux tube

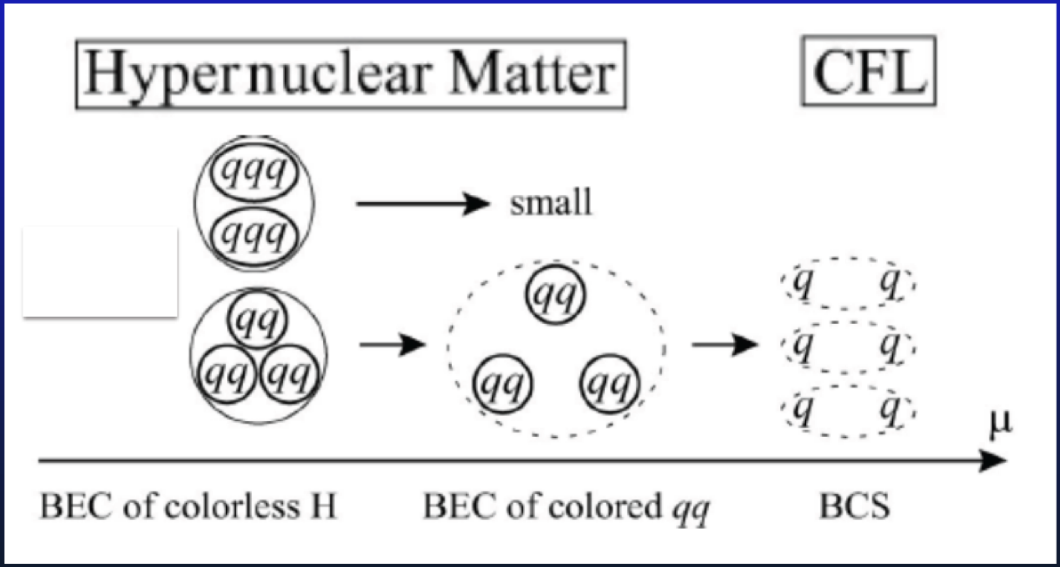
$$\Phi^R_{\alpha i} = \Delta \begin{pmatrix} e^{i\varphi} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix} = \Delta e^{\frac{i}{3}\varphi} \begin{pmatrix} e^{\frac{2i}{3}\varphi} f(r) & 0 & 0 \\ 0 & e^{-\frac{i}{3}\varphi} g(r) & 0 \\ 0 & 0 & e^{-\frac{i}{3}\varphi} g(r) \end{pmatrix}$$

"red" flux tube order parameter

Leading phase (1/3) is $U(1)_B$. Phases within => color rotation, and do not contribute to circulation = $\frac{1}{3} 2\pi\hbar/2\mu_q = 2\pi\hbar/2\mu_n$

Single color flux tube has circulation 1/3 that of initial (unstable) Abelian CFL vortex – same as a single original hadronic vortex.





Pairing continuity
K. Fukushima, PRD (2004)

Conclude that three hadronic vortices can turn into three non-Abelian CFL vortices, with no discontinuity in circulation. But: gauge invariance???

Gauge invariant description of flux tubes

$$\Phi^R_{\alpha i} = \Delta \begin{pmatrix} e^{i\varphi} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix} = \Delta e^{\frac{i}{3}\varphi} \begin{pmatrix} e^{\frac{2i}{3}\varphi} f(r) & 0 & 0 \\ 0 & e^{-\frac{i}{3}\varphi} g(r) & 0 \\ 0 & 0 & e^{-\frac{i}{3}\varphi} g(r) \end{pmatrix}$$

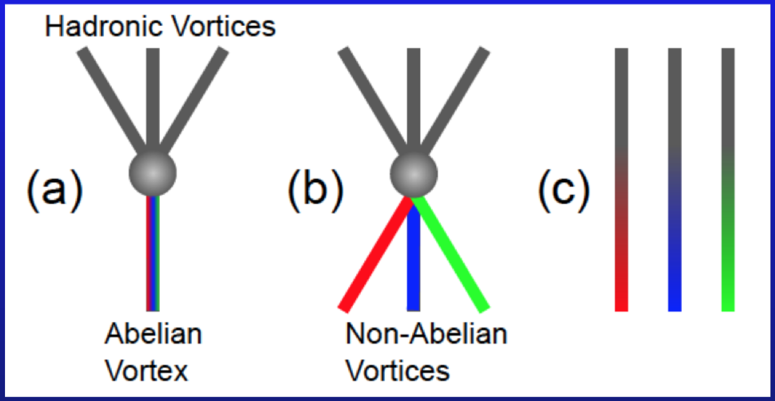
red flux tube order parameter

Then $\Upsilon(\vec{r}) = \frac{1}{6}\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\Phi_{\alpha i}\Phi_{\beta j}\Phi_{\gamma k}$ $= e^{i\varphi}\Delta^3 f(r) g^2(r)$

is gauge invariant order parameter, independent of choice of color of the gauge fixed $\Phi^R_{\alpha i}$. Only one gauge invariant physical object.

Determinant of $\Phi^R_{\alpha i}$

Quark-hadron continuity?



Can envision continuous evolution of vortices from nuclear (hadronic) phase to quark phase provided order parameter in hadronic phase is anti-symmetric in flavor.

BCS pairs in neutron gas have 6 quarks: ddu + ddu.

$$\langle nn \rangle \rightarrow \langle ud \rangle \langle ud \rangle \langle dd \rangle$$

Cannot arrange into flavor anti-symmetric quark pairing.
But in $SU(3)_{flavor}$ invariant hadronic matter with equal mass n, p, Λ, Σ, and Ξ baryons can have flavor antisymmetric pairings

$$\langle -\sqrt{\frac{1}{8}}[\Lambda\Lambda] + \sqrt{\frac{3}{8}}[\Sigma\Sigma] + \sqrt{\frac{4}{8}}[N\Xi] \rangle$$

Connecting neutron matter to usual CFL quarks requires transition.
Other quark matter pairings, e.g., $^3P_2$ pairing, could work.

Further technical issues

Stability of core of gauge invariant flux tube against (1D) spontaneous flavor-symmetry breaking:
M. Eto, Y. Hirono, M. Nitta, and S. Yasui, PTEP 2014, 012D01 (2014)

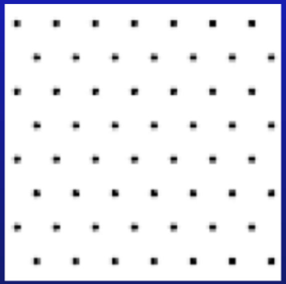
Fate at interface with hadronic matter of color flux in cores of gauge invariant flux tubes? Should not penetrate into hadronic phase

Our understanding of (de)confinement is challenged by vortex continuity problem!!!

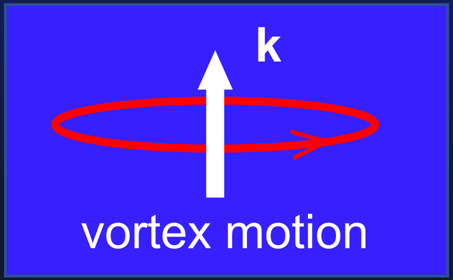
A few open physics issues

Pairing states for $m_s > m_u$ and m_d? Spatially inhomogeneous Fulde-Ferrel-Larkin-Ovchinnikov states.

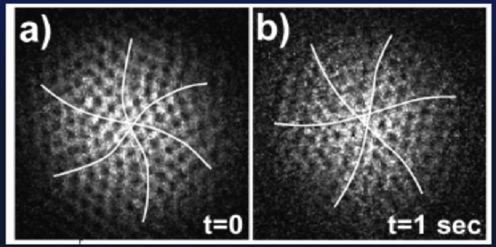
Geometry of vortex arrays in rotating CFL matter. Triangular?



Modes of quark matter vortex arrays?



Tkachenko modes



Schweikhard et al., PRL92 040404 (2004)

Effect of vortices in quark matter on glitches?

谢谢

Early history of superfluidity

(Superfluidity and superconductivity developed on separate tracks)

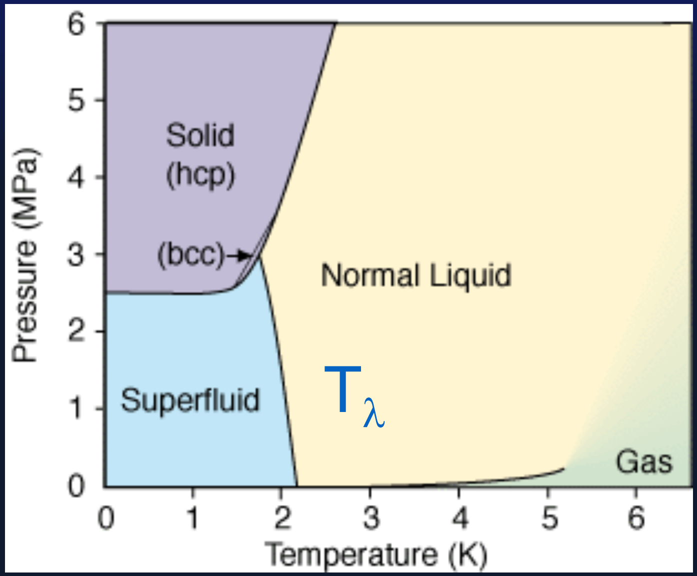
1908: liquefaction of ^{4}He by *Kamerlingh Onnes*, Leiden (T » 1.2 K)

1911: discovery of superconductivity by *Kamerlingh Onnes*, Leiden

1933: Meissner effect -- superconductors expel magnetic fields

1937-38: discovery of superfluidity of ^{4}He: *Kapitza, Allen and Misen*

$T < T_\lambda$=2.17K = lambda point. Called "superfluid" by Kapitza.



^{4}He phase diagram

Theory landmarks

1938: Connection of superfluidity and Bose-Einstein condensation
(F. London)

1941: Landau two-fluid picture : superfluid (ρ_s) + normal (ρ_n)

1949: Quantization of vorticity: Onsager

1957: BCS theory of superconductivity

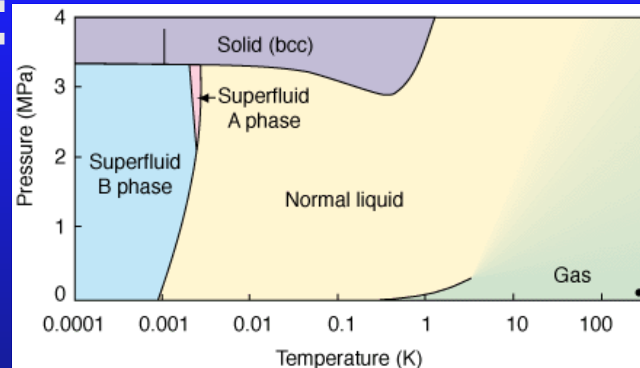
Experimental landmarks

1967: Hess-Fairbank experiment -- reduction of moment of inertia
(analog of Meissner effect)

1973: superfluid ^{3}He

1995-2000: superfluidity of trapped atomic Bose-Einstein

New superfluids:

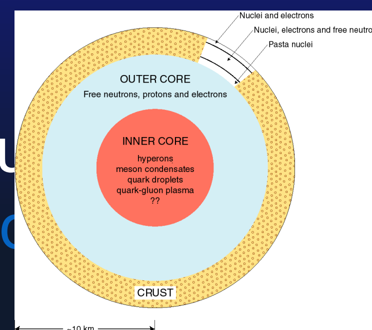


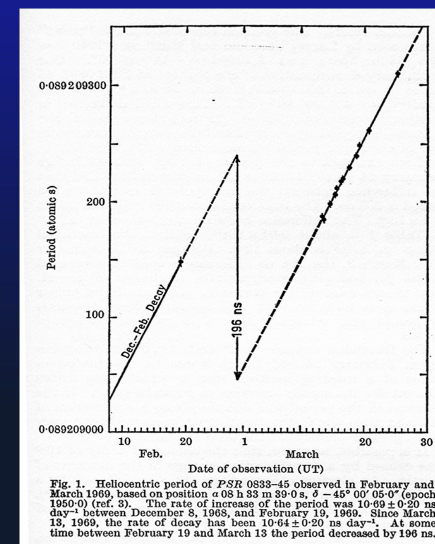
^3He: A and B phases

Dilute solutions of ^3He
in superfluid ^4He (T_3» few μ K)

Neutron and proton fluids in
neutron stars: pulsar speedu

Color superconductors in qc





Vela pulsar: Radhakrishnan &
Manchester, Nature 1969

Trapped atomic boson
& fermionic gases