

Constraining quark-hadron interface tension in the multi-messenger era

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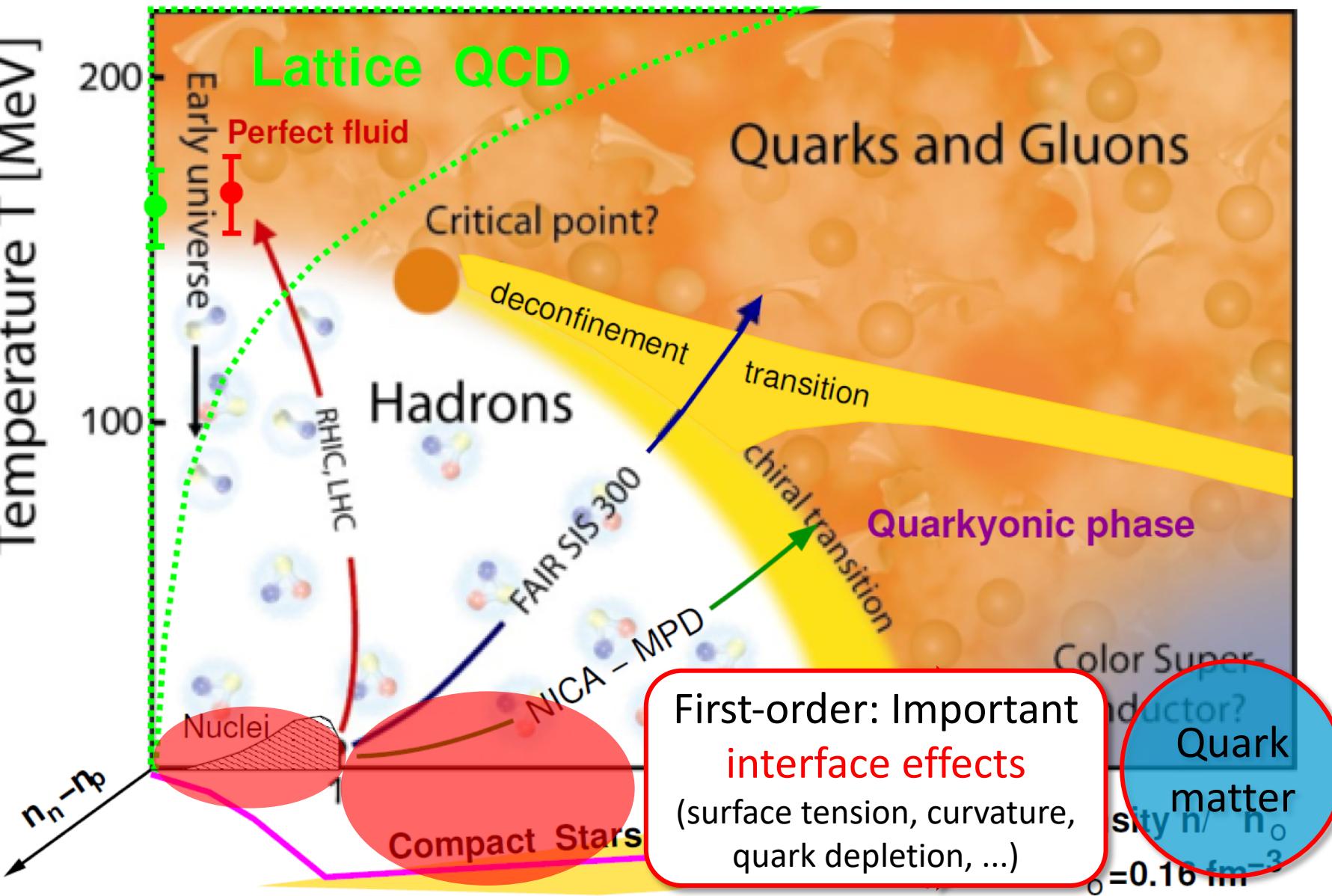
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QCD Phase Diagram [McLerran2009_NPB195-275]

Temperature T [MeV]



Estimations of surface tension

Lattice QCD: Huang, Potvin, Rebbi, Saniotis, ...
Lucini, Vettorazzo, et al. For vanishing chemical potentials!
de Forcrand, de Forcrand, Lucini, Vettorazzo, et al.

Effective models:

MIT bag model [Oertel_Urban2008_PRD77-074015], Linear sigma model
[Palhares_Fraga2010_PRD82-125018, Pinto_Koch_Randrup2012_PRC86-025203,
Kroff_Fraga2015_PRD91-025017], NJL model [Garcia_Pinto2013_PRC88-025207,
Ke_Liu2014_PRD89-074041], 3f Polyakov-quark-meson model [Mintz_Stiele_Ramos_Schaffner-
Bielich2013_PRD87-036004], Dyson-Schwinger equation approach [Gao_Liu2016_PRD94-
094030], Equivparticle model [Xia_Peng_Sun_Guo_Lu_Jaikumar2018_PRD98-034031], and
Nucleon-meson model [Fraga_Hippert_Schmitt2019_PRD99-014046]

$$\sigma < 30 \text{ MeV/fm}^2$$

Quasiparticle mode [Wen_Li_Liang_Peng2010_PRC82-025809]

$$\sigma = 30\text{--}70 \text{ MeV/fm}^2$$

NJL model adopting the MRE method [Lugones_Grunfeld_Ajmi2013_PRC88-045803]

$$\sigma = 145\text{--}165 \text{ MeV/fm}^2$$

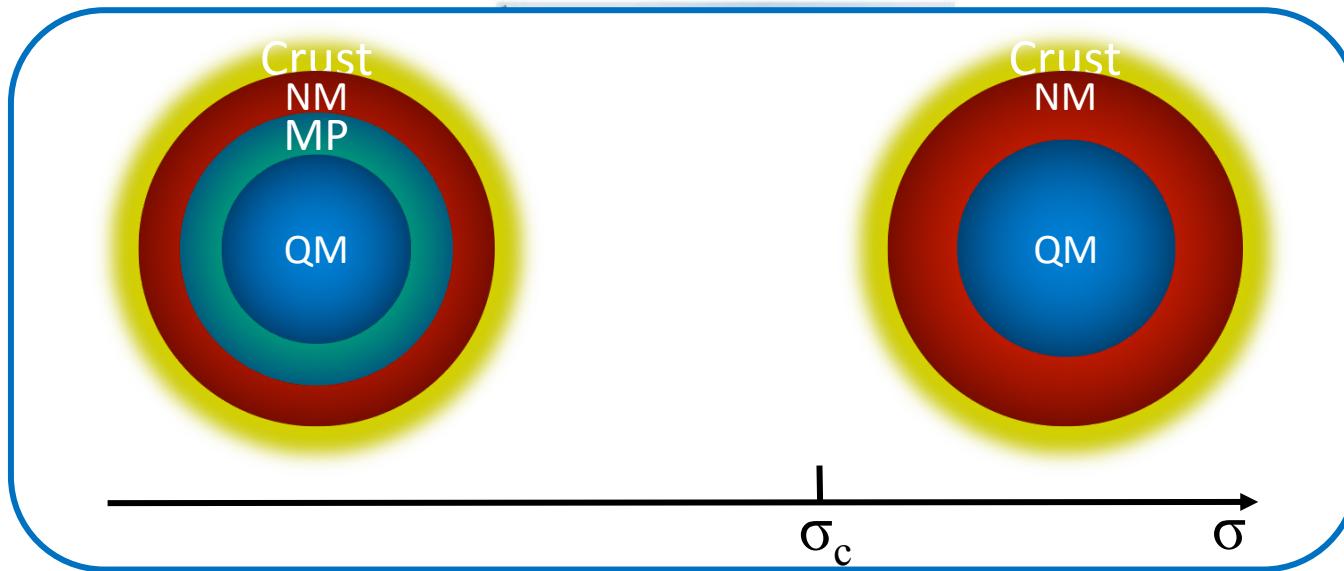
For color-flavor locked SQM, dimensional analysis suggests:

[Alford_Rajagopal_Reddy_Wilczek2001_PRD64-074017] $\sigma \approx 300 \text{ MeV/fm}^2$

For magnetized SQM, σ has a different value in the parallel and transverse directions
with respect to the magnetic field [Lugones_Grunfeld2017_PRC95-015804,
Lugones_Grunfeld2019_PRC99-035804]

Hadron-quark mixed phase inside compact stars

First-order phase transition [Masuda_Hatsuda_Takatsuka2016_EPJA52-65]



- $\sigma = 0$: point-like hadronic matter (HM) and quark matter (QM), i.e., Gibbs construction.
- Moderate σ : geometrical structures [Chiba, Endo, Heiselberg, Maruyama, Tatsumi, Voskresensky, Yasuhira, Yasutake, ...]
 - Droplet
 - Rod
 - Slab
 - Tube
 - Bubble
- $\sigma > \sigma_c$: bulk separation of HM and QM, i.e., Maxwell construction.

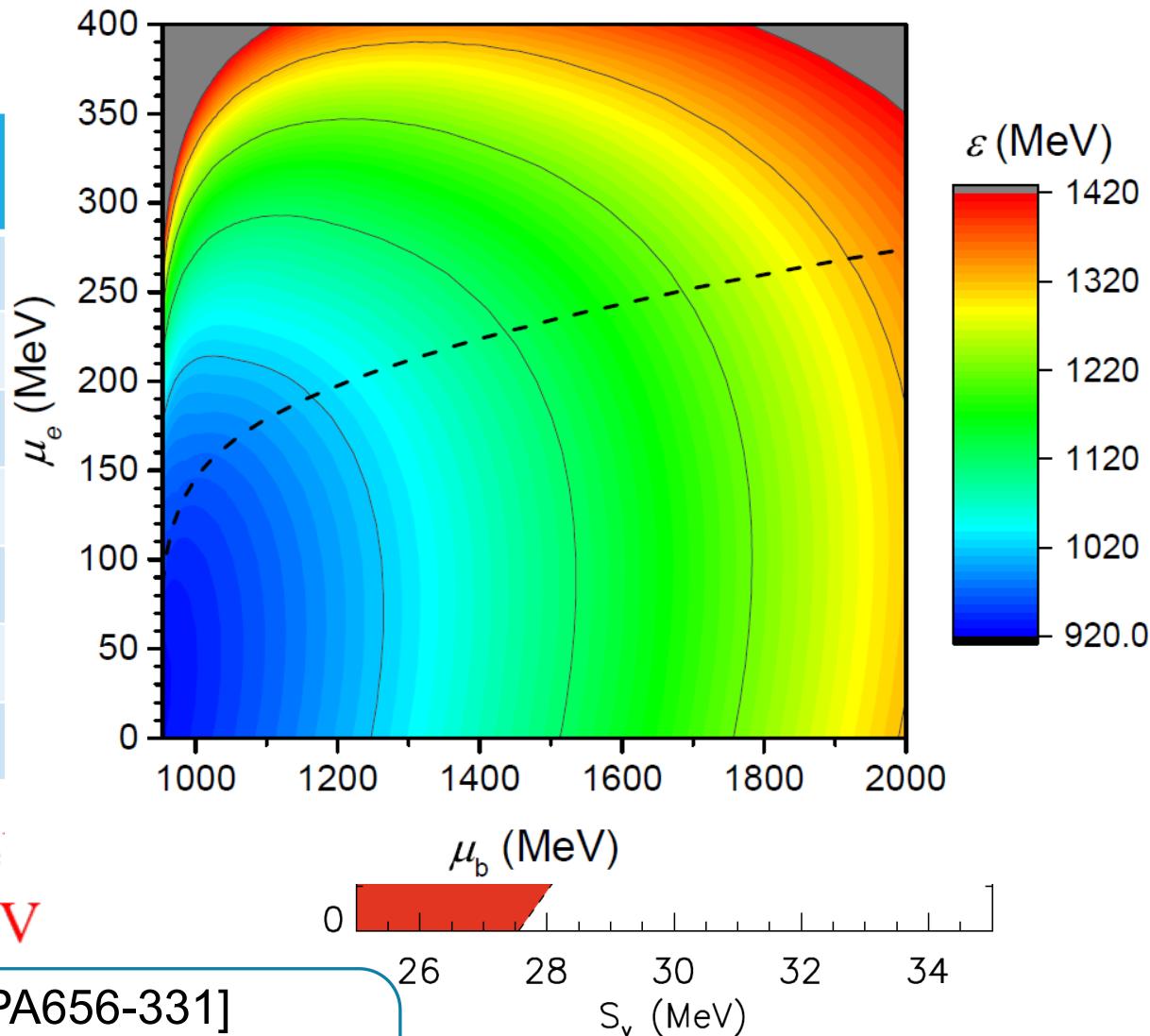
Nuclear matter (RMFT)

	E_{sym} (MeV)	L (MeV)
NL3	37.35	118.33
TM1	36.84	110.61
PK1	37.64	115.88
PK1r	37.83	116.51
TW99	32.77	55.31
DDME2	32.32	51.30
PKDD	36.80	90.24

Li_Han2013_PLB727-276:

$$E_{\text{sym}}(\rho_0) = 31.6 \pm 2.66 \text{ MeV}$$

$$L(\rho_0) = 58.9 \pm 16.0 \text{ MeV}$$



TW99 [Typel_Wolter1999_NPA656-331]

- | | |
|---|---------------|
| 1. the saturation density: ρ_0 (fm $^{-3}$) | 0.153 |
| 2. the energy: E/A (MeV) | -16.25 |
| 3. the incompressibility: k (MeV) | 240.2 |

Other choices (Δ 's):
Sun_Zhang_Zhang_Xia2019
_PRD99-023004

Quark matter [Fraga_Romatschke2005_PRD71-105014]

Perturbation model (pQCD): Expand the **thermodynamic potential density** up to the order of α_s in the MS scheme:

$$\Omega^{\text{pt}} = \sum_i^{N_f} (\omega_i^0 + \omega_i^1 \alpha_s)$$

$$\alpha_s(\bar{\Lambda}) = \frac{1}{\beta_0 L} \left(1 - \frac{\beta_1 \ln L}{\beta_0^2 L} \right), \quad \text{where } L = 2 \ln \left(\frac{\bar{\Lambda}}{\Lambda_{\overline{\text{MS}}}} \right) \text{ and } \Lambda_{\overline{\text{MS}}} \text{ is}$$

$$m_i(\bar{\Lambda}) = \hat{m}_i \alpha_s^{\frac{\gamma_0}{\beta_0}} \left[1 + \left(\frac{\gamma_1}{\beta_0} - \frac{\beta_1 \gamma_0}{\beta_0^2} \right) \alpha_s \right], \quad \text{the } \overline{\text{MS}} \text{ renormalization point.}$$

The **renormalization scale** $\bar{\Lambda} = \frac{C}{3} \sum_{i=u,d,s} \mu_i$ with $C = 1 \sim 4$.

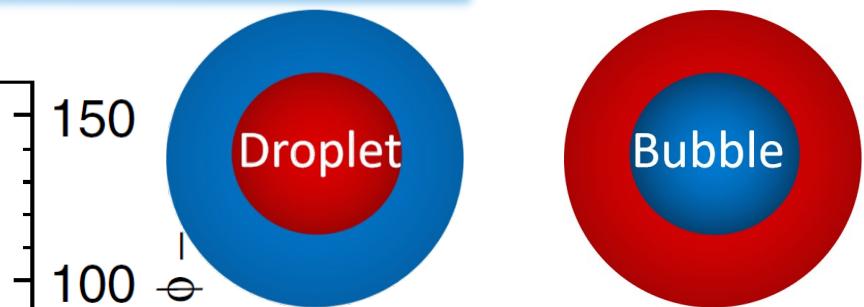
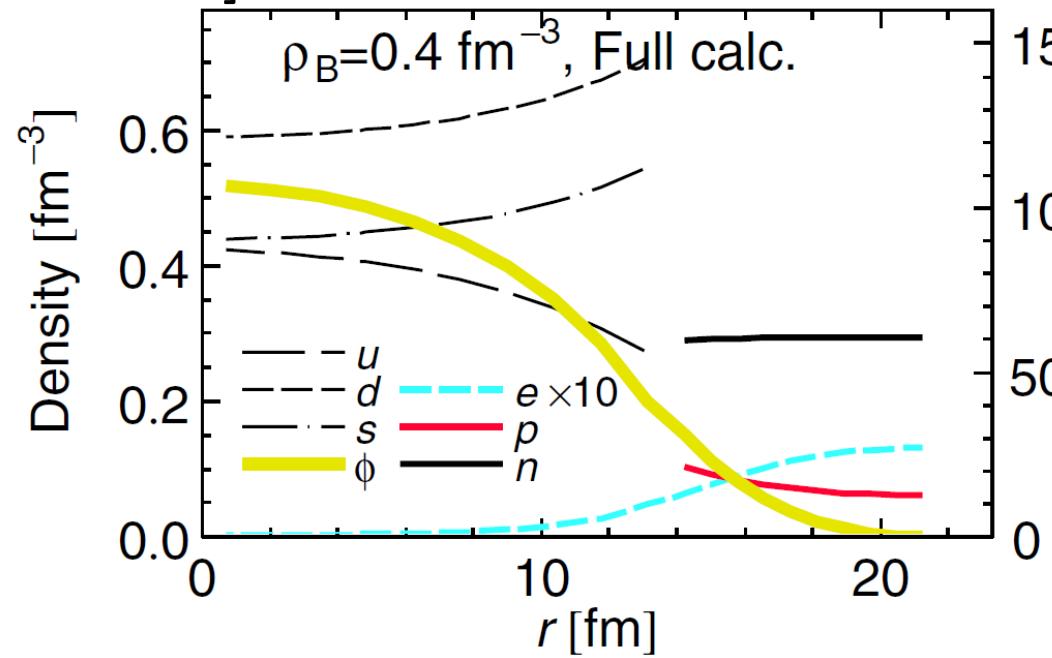
We introduce an extra **bag constant B** to take into account the **energy difference** between the **physical and perturbative vacua**:

$$\Omega = \Omega^{\text{pt}} + B$$

$$B = B_{\text{QCD}} + (B_0 - B_{\text{QCD}}) \exp \left[- \left(\frac{\sum_i \mu_i - 930}{\Delta \mu} \right)^4 \right]$$

where $B_0 = 50 \text{ MeV fm}^{-3}$, C , $\Delta \mu$ and B_{QCD} undetermined.

Mixed phase [Maruyama et al.2007_PRD76-123015]



$$\bar{\mu}_i = \mu_i(r) + q_i\varphi(r) = \text{constant}$$

$$r^2 \frac{d^2\varphi}{dr^2} + 2r \frac{d\varphi}{dr} + 4\pi\alpha r^2 n_{\text{ch}}(r) = 0$$

Coulomb potential obtained with linearization

$$\varphi^I = \frac{C^I}{r} \sinh\left(\frac{r}{\lambda_D^I}\right) + \varphi_0^I,$$

$$\varphi^O = \frac{C^O}{r(R_W + \lambda_D^O)} [\sinh(\tilde{r})\lambda_D^O + \cosh(\tilde{r})R_W] + \varphi_0^O$$

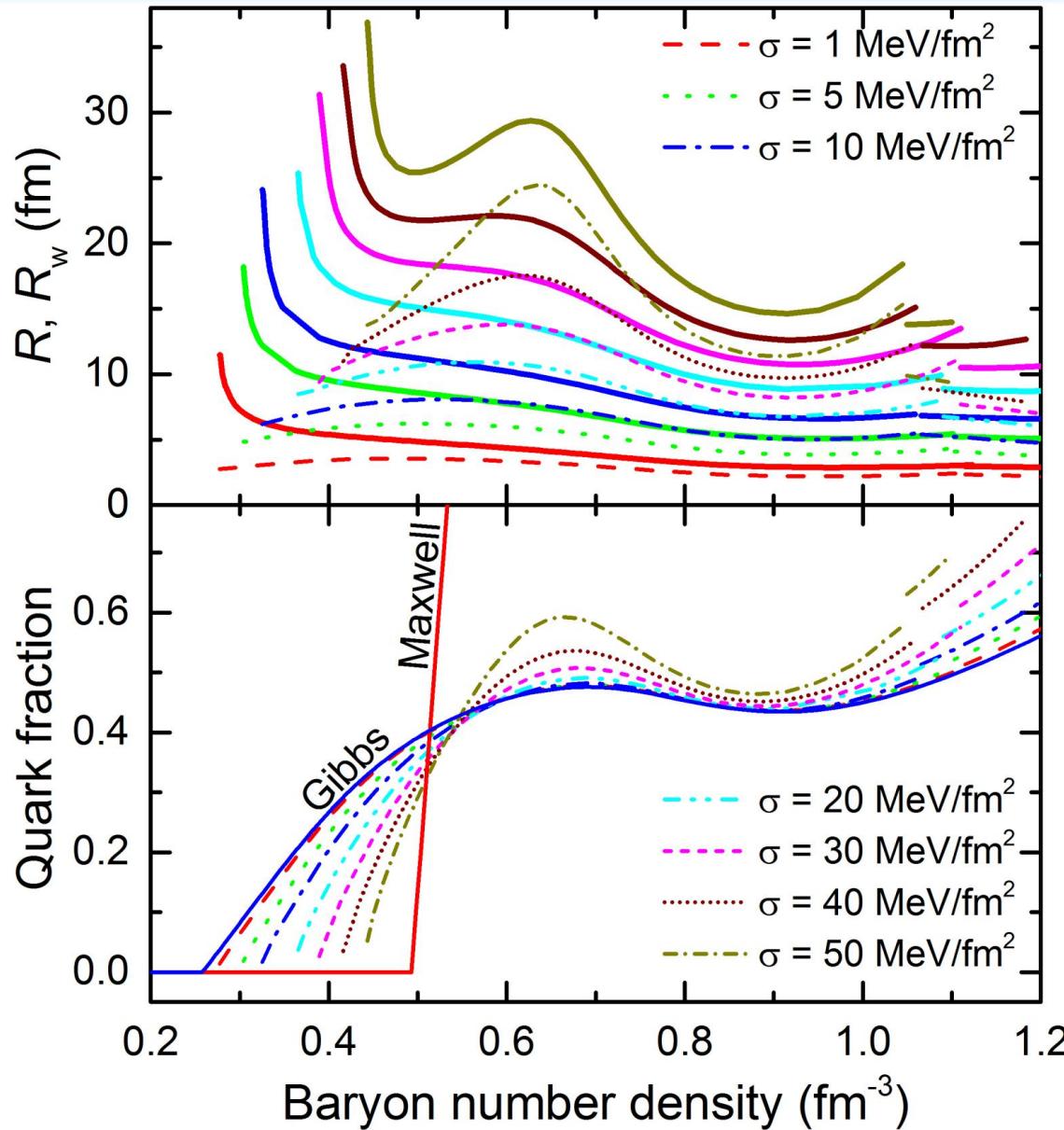
$$\text{with } \tilde{r} \equiv (r - R_W)/\lambda_D^O.$$

Dynamic stability of the quark-hadron interface:

$$P^I(R) - 2\frac{\sigma}{R} = P^O(R)$$

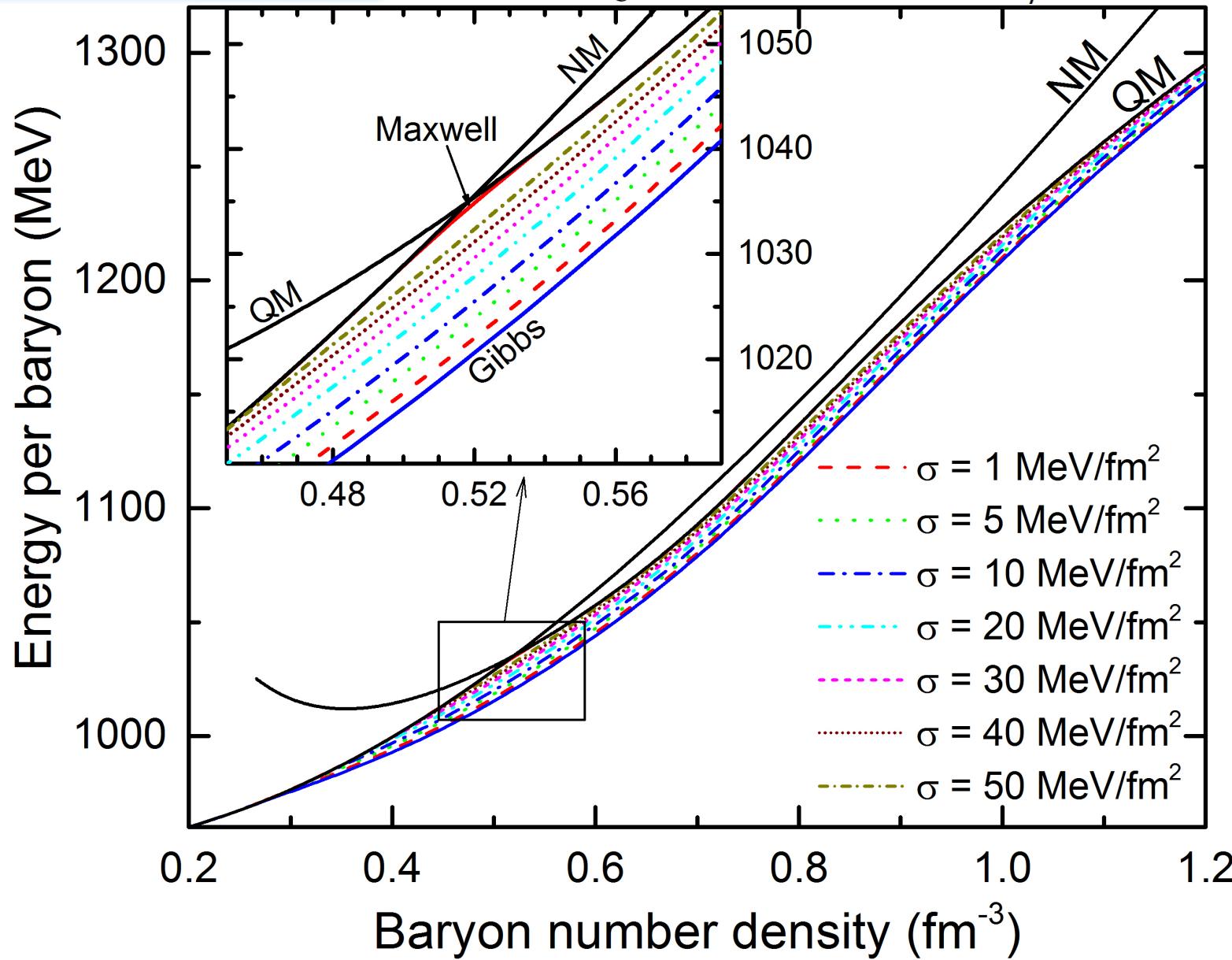
Minimize energy per baryon at fixed density

WS cell radius, droplet radius, and quark fraction

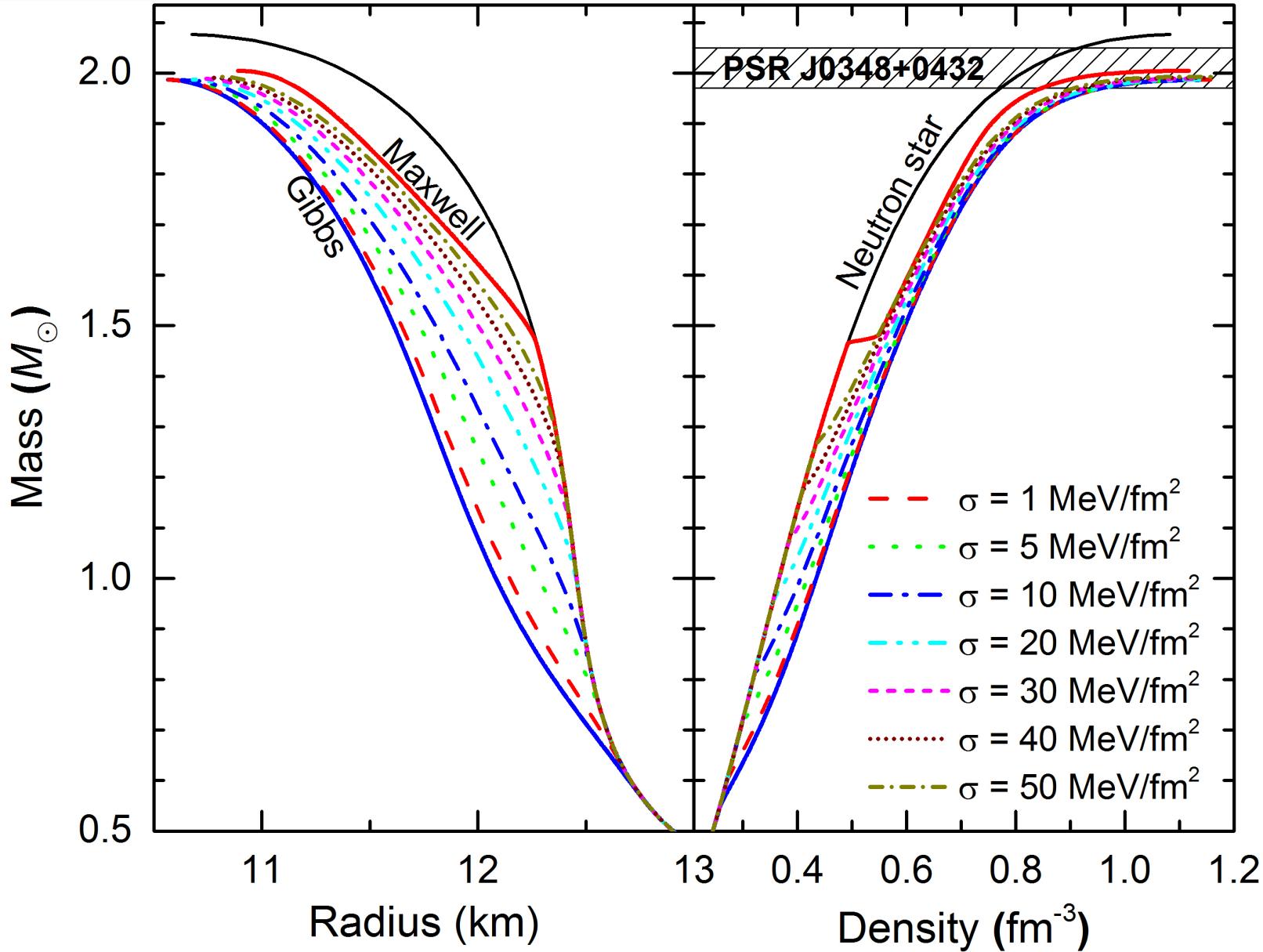


Equation of states

$$\sigma_c = 79.12 \text{ MeV/fm}^2$$



Mass, radius, and central density

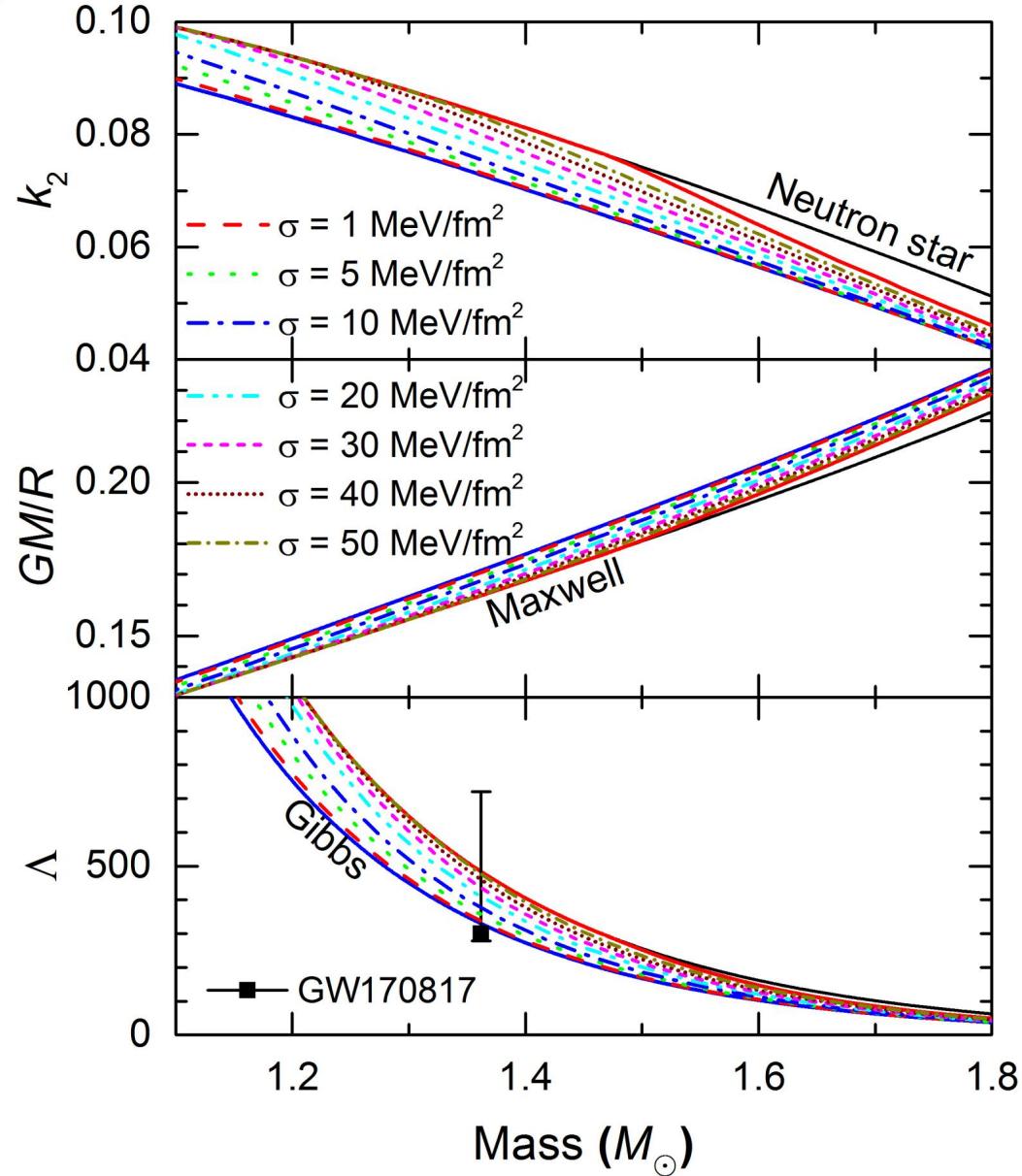


Love number, compactness, and tidal deformability

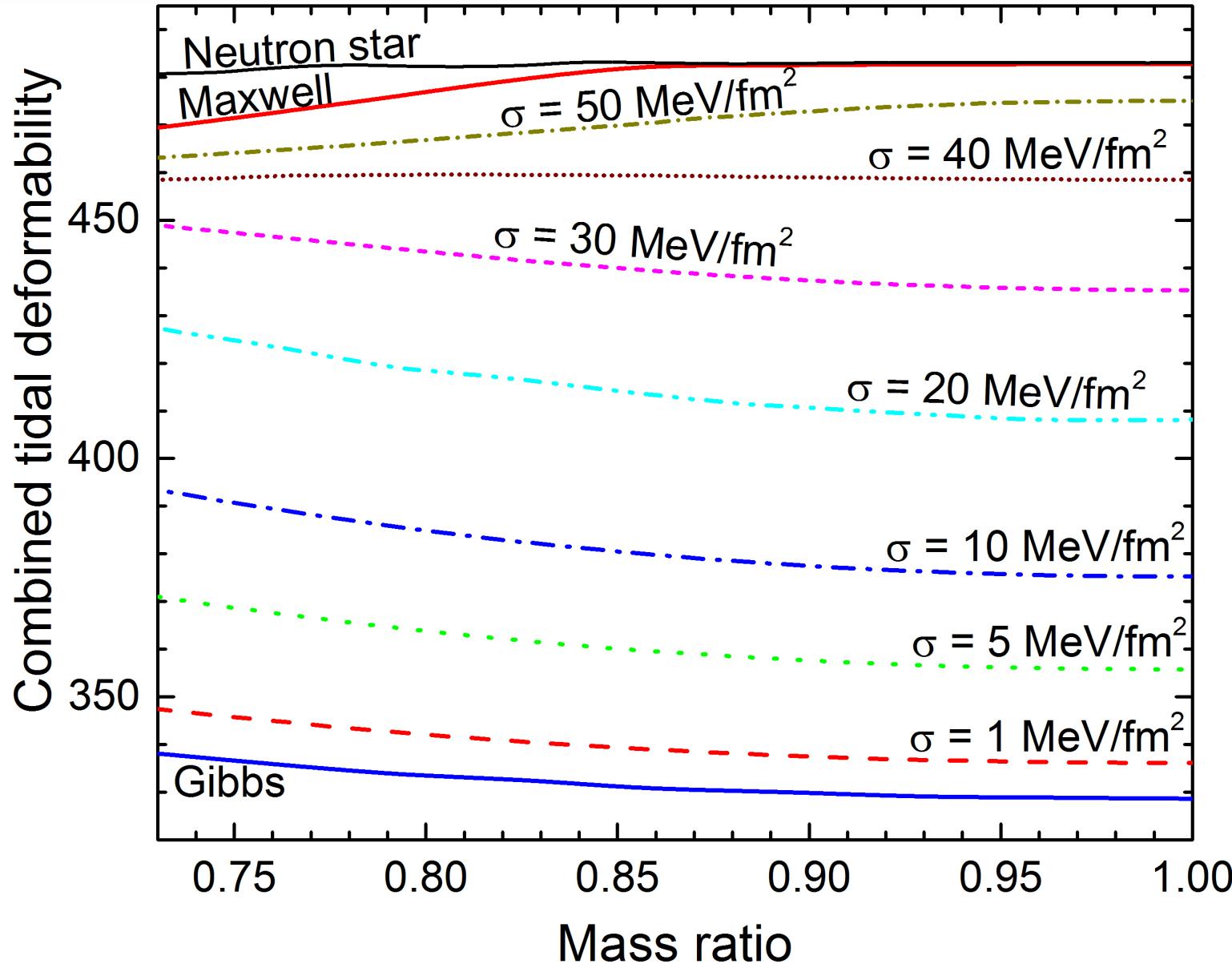
Tidal deformability:

$$\Lambda = \frac{2k_2}{3} \left(\frac{R}{GM} \right)^5$$

k_2 : the **second Love number**
[Damour_Nagar2009_PRD8
0-084035,
Hinderer_Lackey_Lang_Re
ad2010_PRD81-123016,
Postnikov_Prakash_Lattime
r2010_PRD82-024016]



Combined tidal deformability (GW170817)



Dependence on σ

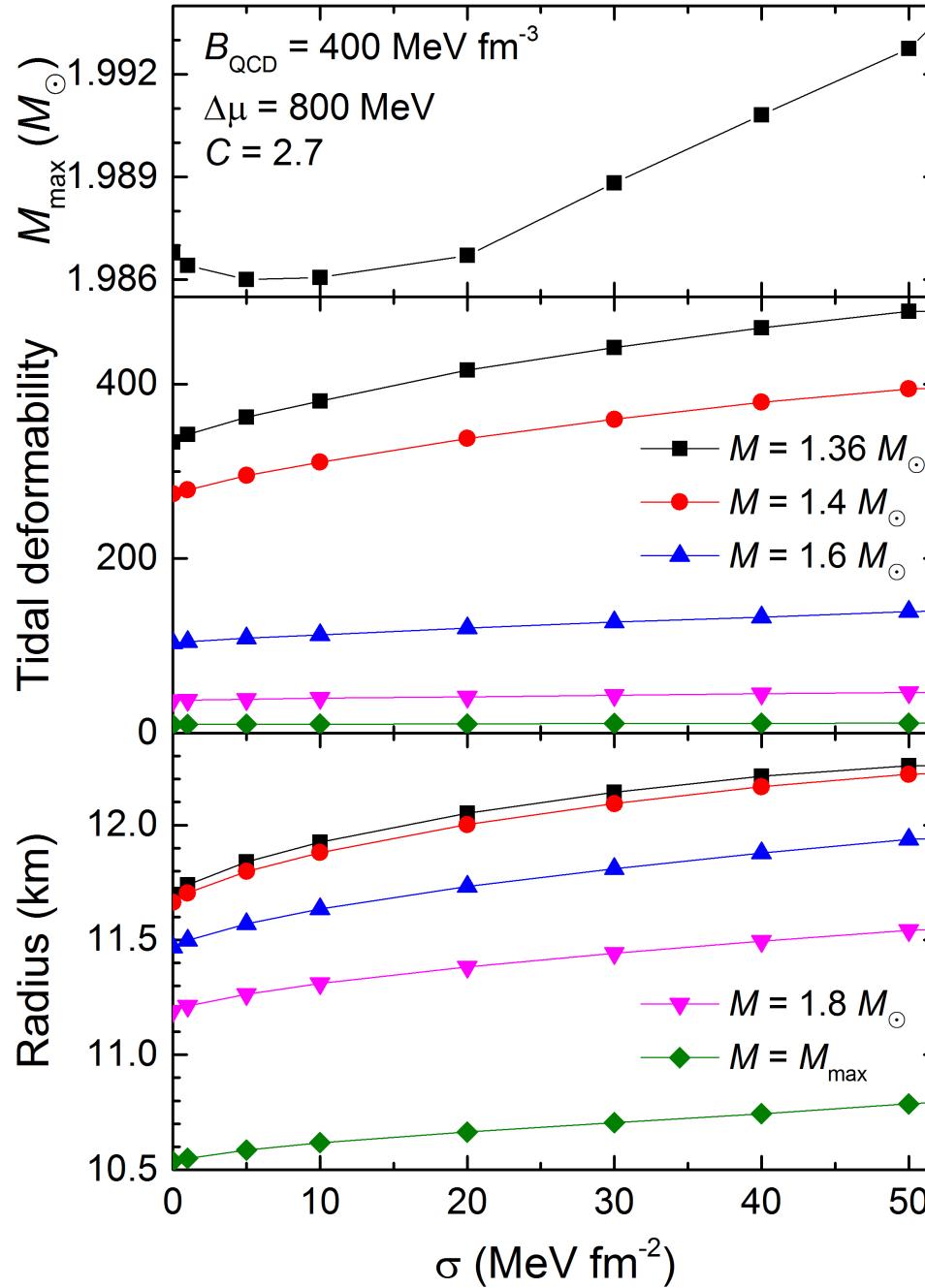
- $\sigma = 0$: Gibbs construction
- $\sigma > \sigma_c$: Maxwell construction

$$\sigma_c = 79.12 \text{ MeV/fm}^2$$

Moderate σ : interpolate
between two types of values

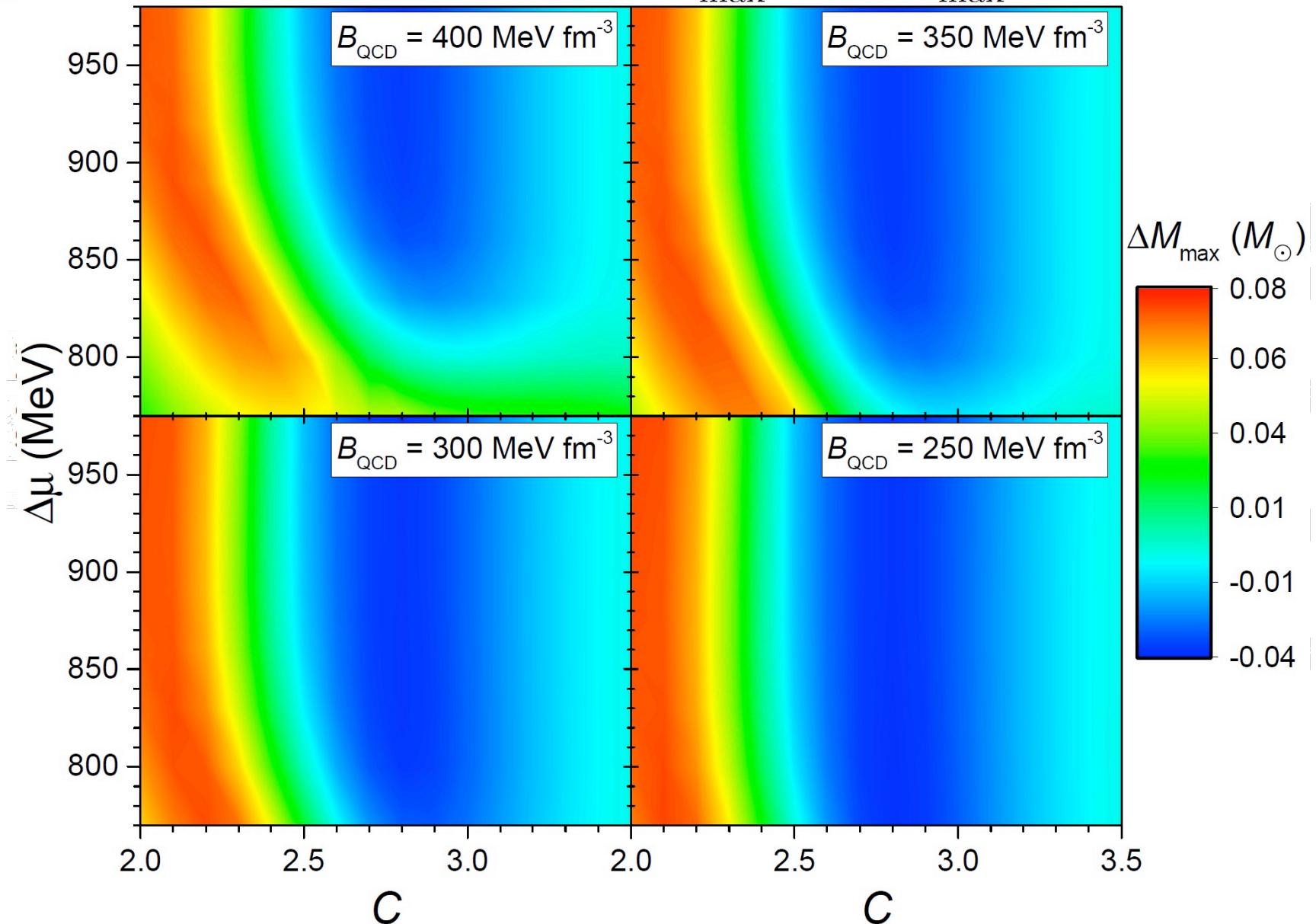
Our goal:

1. A universal function with respect to σ ;
2. Constrain σ with pulsar observations.

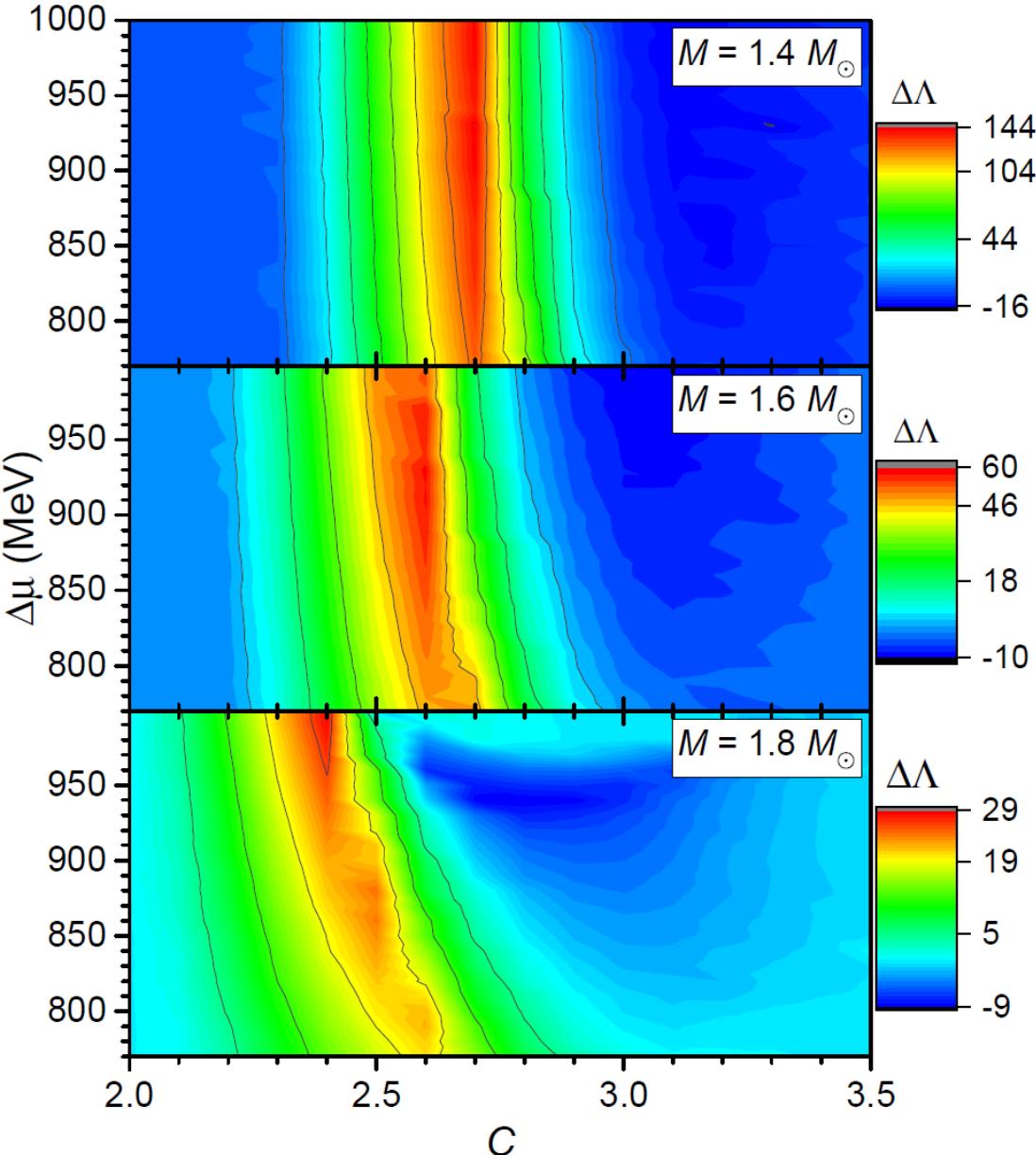
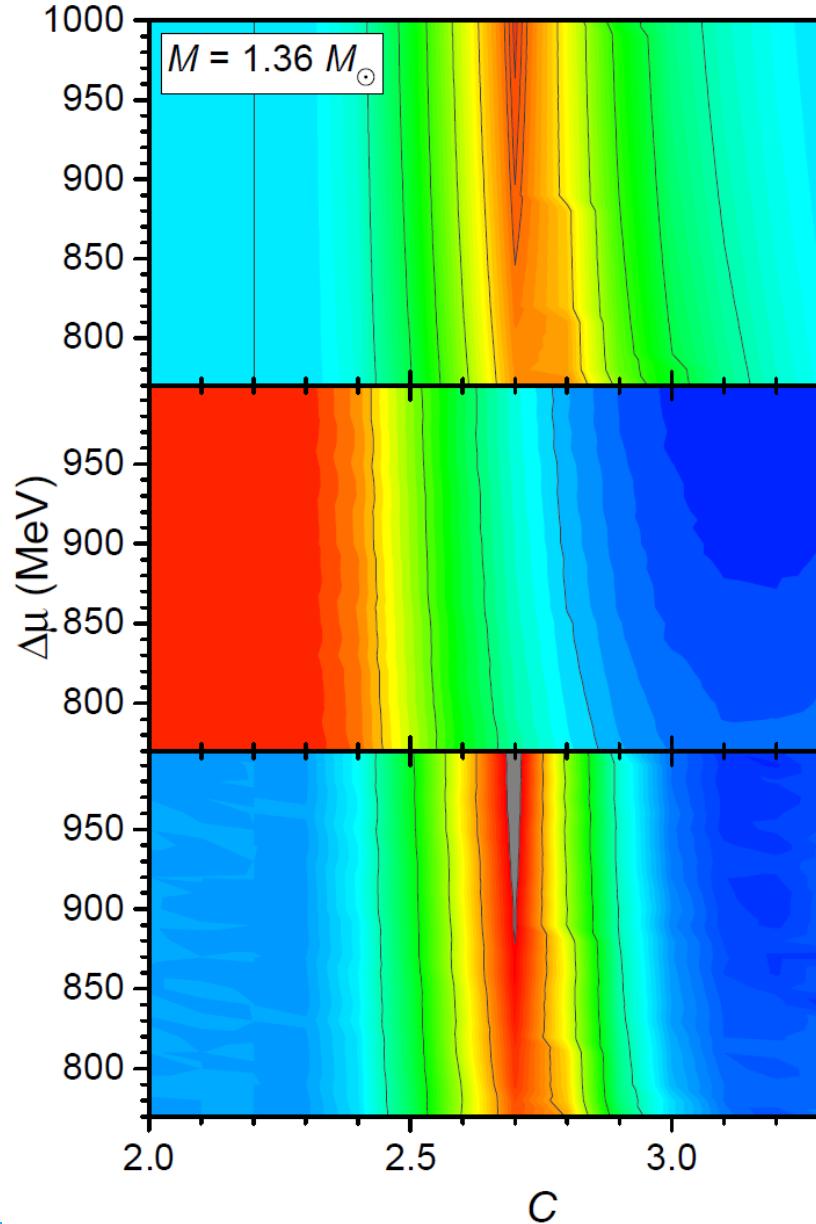


Maximum mass

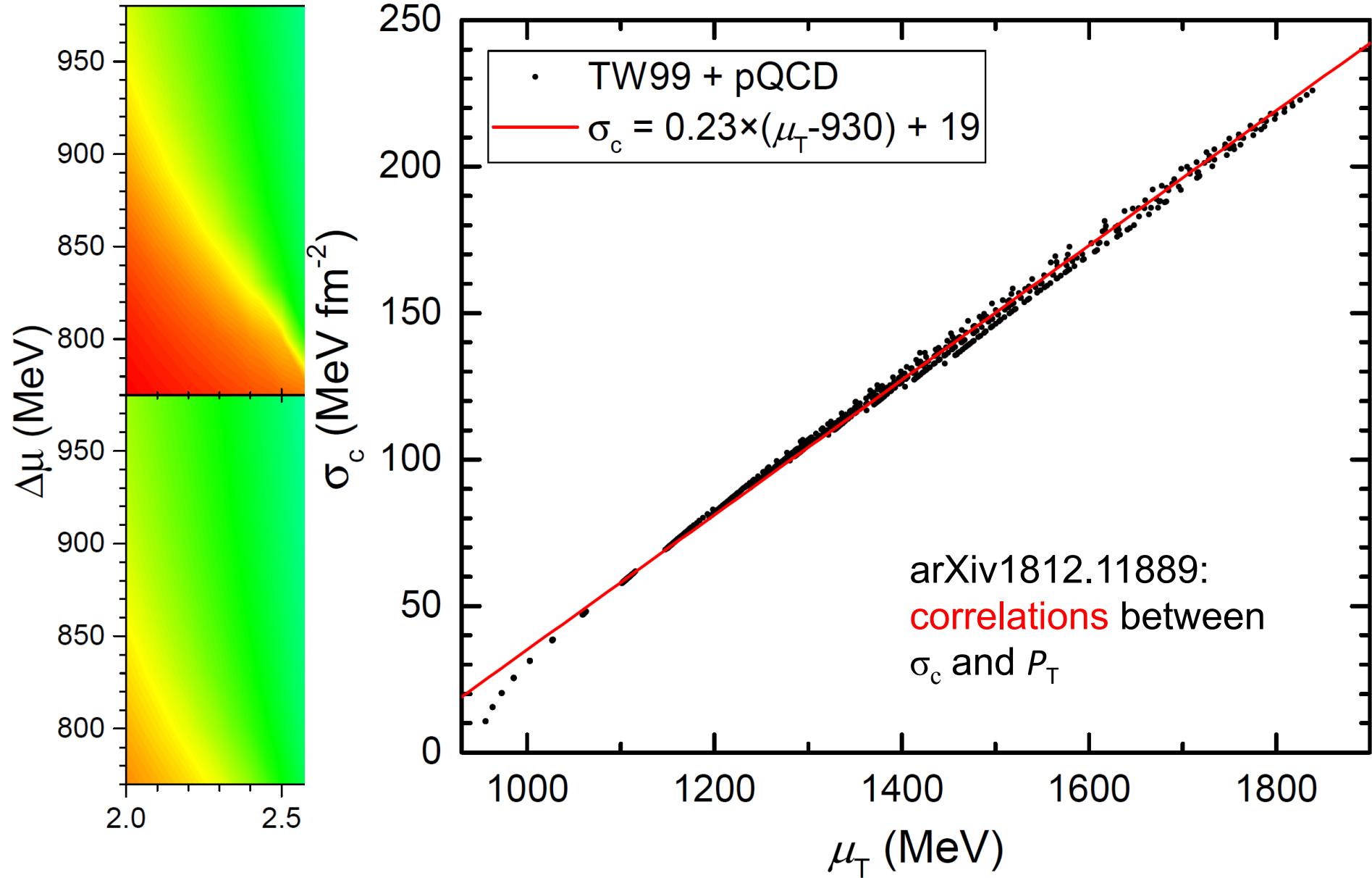
$$\Delta M_{\max} = M_{\max}^{\text{Maxwell}} - M_{\max}^{\text{Gibbs}}$$



Variations on radius and tidal deformability



Critical surface tension



Summary

The interface effects of quark-hadron mixed phase in compact stars were investigated. It was found that:

- ① The quark-hadron interface has sizable effects on the radii ($\Delta R \approx 600$ m) and tidal deformabilities ($\Delta \Lambda/\Lambda \approx 50\%$) of (1.36 solar mass) hybrid stars for certain choices of parameters;
- ② The maximum mass, tidal deformability, and radius increase with σ , where the maximum mass varies little;
- ③ A linear dependence of the critical surface tension σ_c on the chemical potential μ_T was obtained.

Possibilities in constraining the quark-hadron interface tension with GW observations and NICER mission.

Thank you!!!