The Kavli Institute for Astronomy and Astrophysics at Peking University

北京大学科维理天文与天体物理研究所

第八届华大QCD讲习班 [The 8th Huada (CCNU) School on QCD]

6-10 May 2019, CCNU (Wuhan, China)

Degeneracy in Studying the Supra-nuclear Equation of State and Modified Gravity

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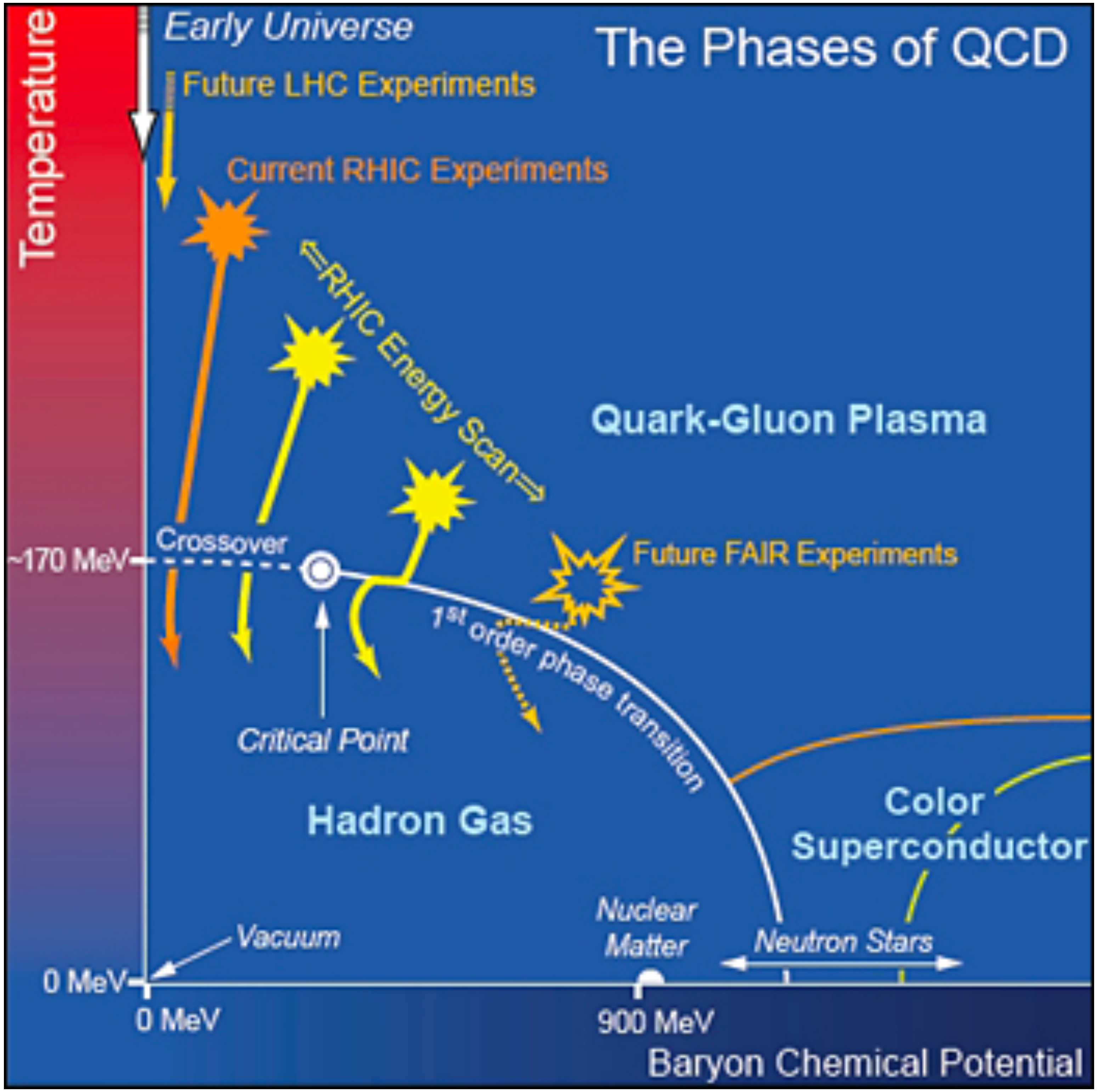
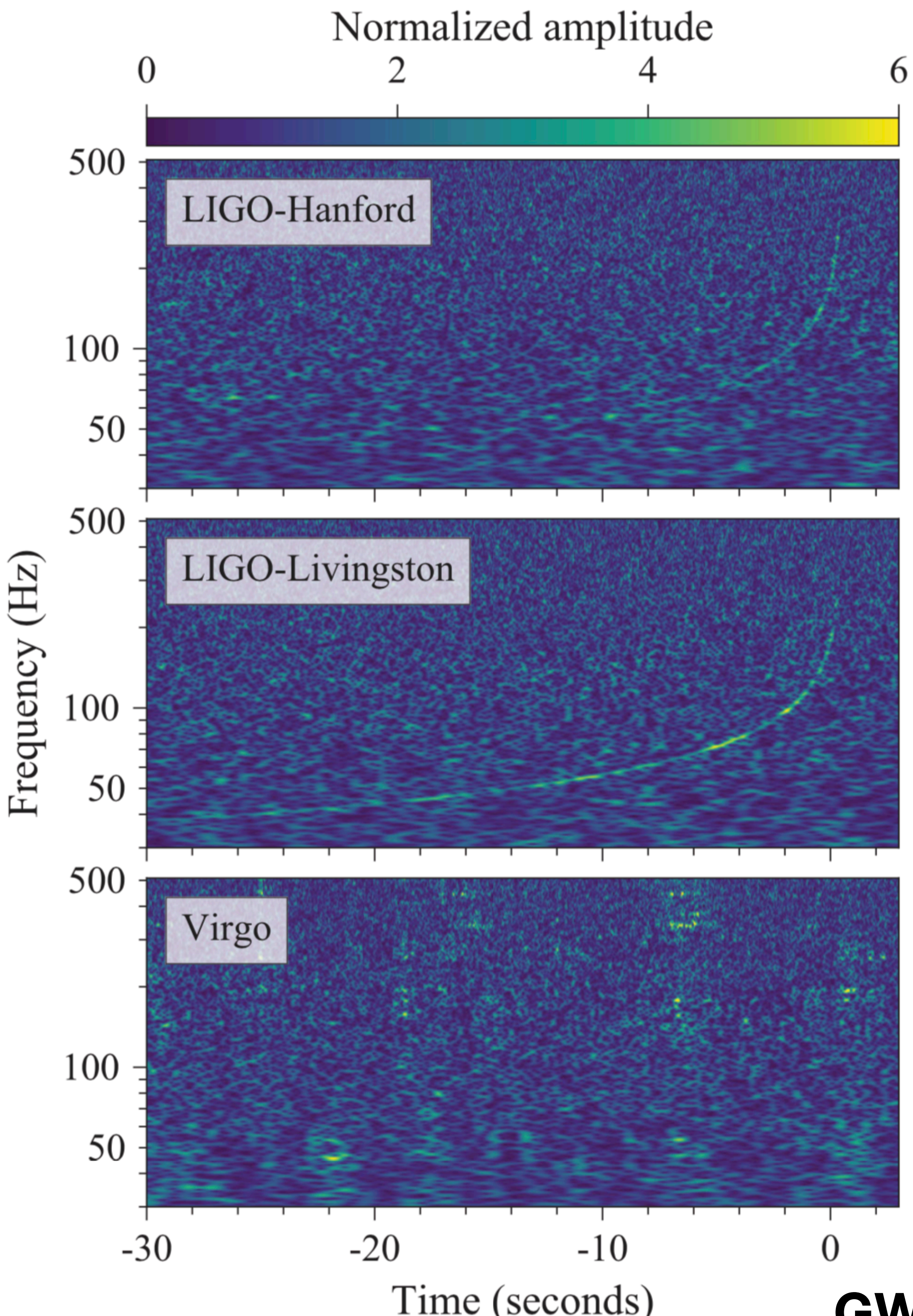
QCD diagram

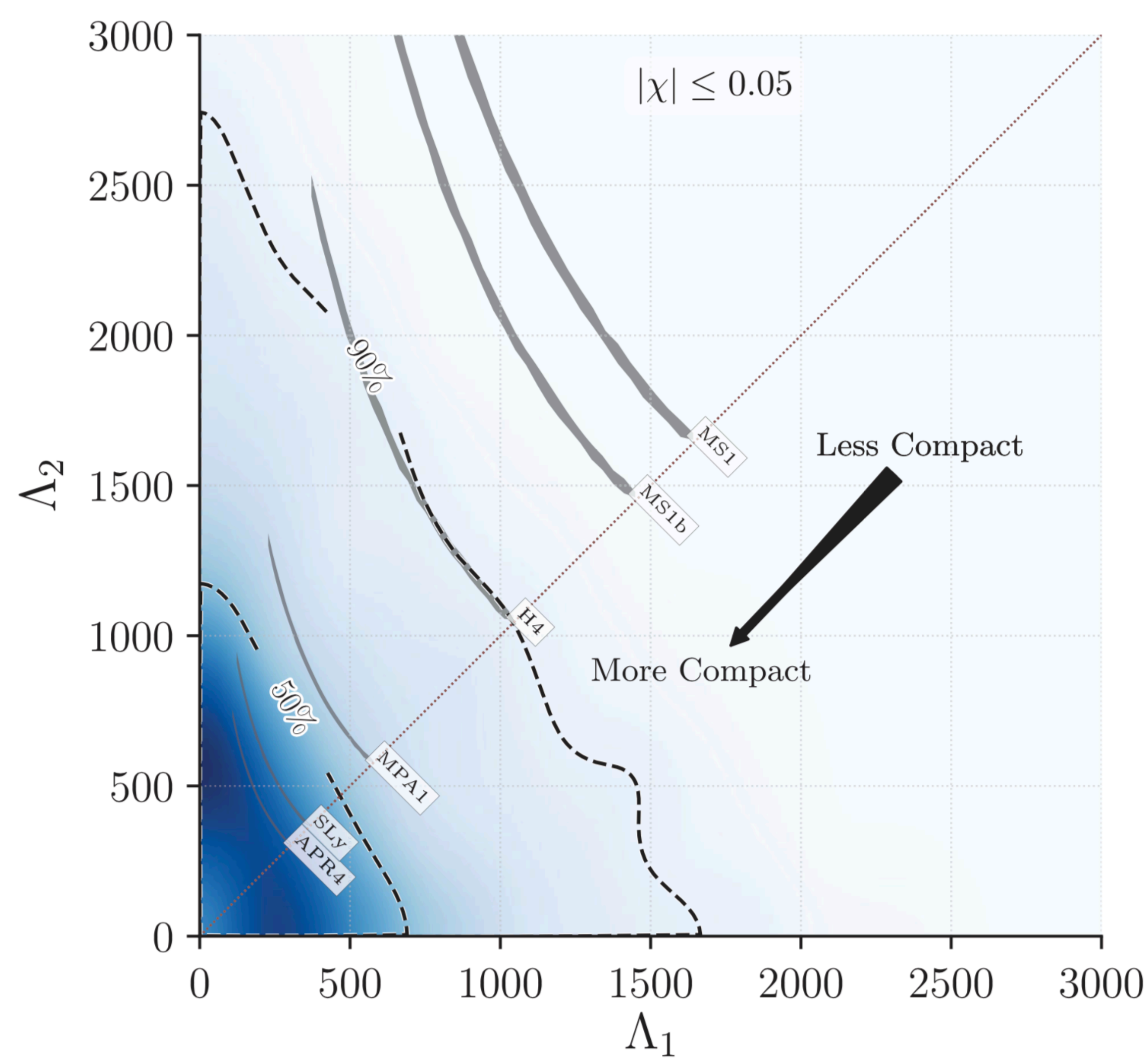

The Phases of QCD
Temperature
Early Universe
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
Future FAIR Experiments
~170 MeV
Crossover
Critical Point
1st order phase transition
Hadron Gas
Color Superconductor
Nuclear Matter
Neutron Stars
Vacuum
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

Why we need EOS?

Abbott et al. 2017



Tidal deformability



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Extrapolation is an enemy

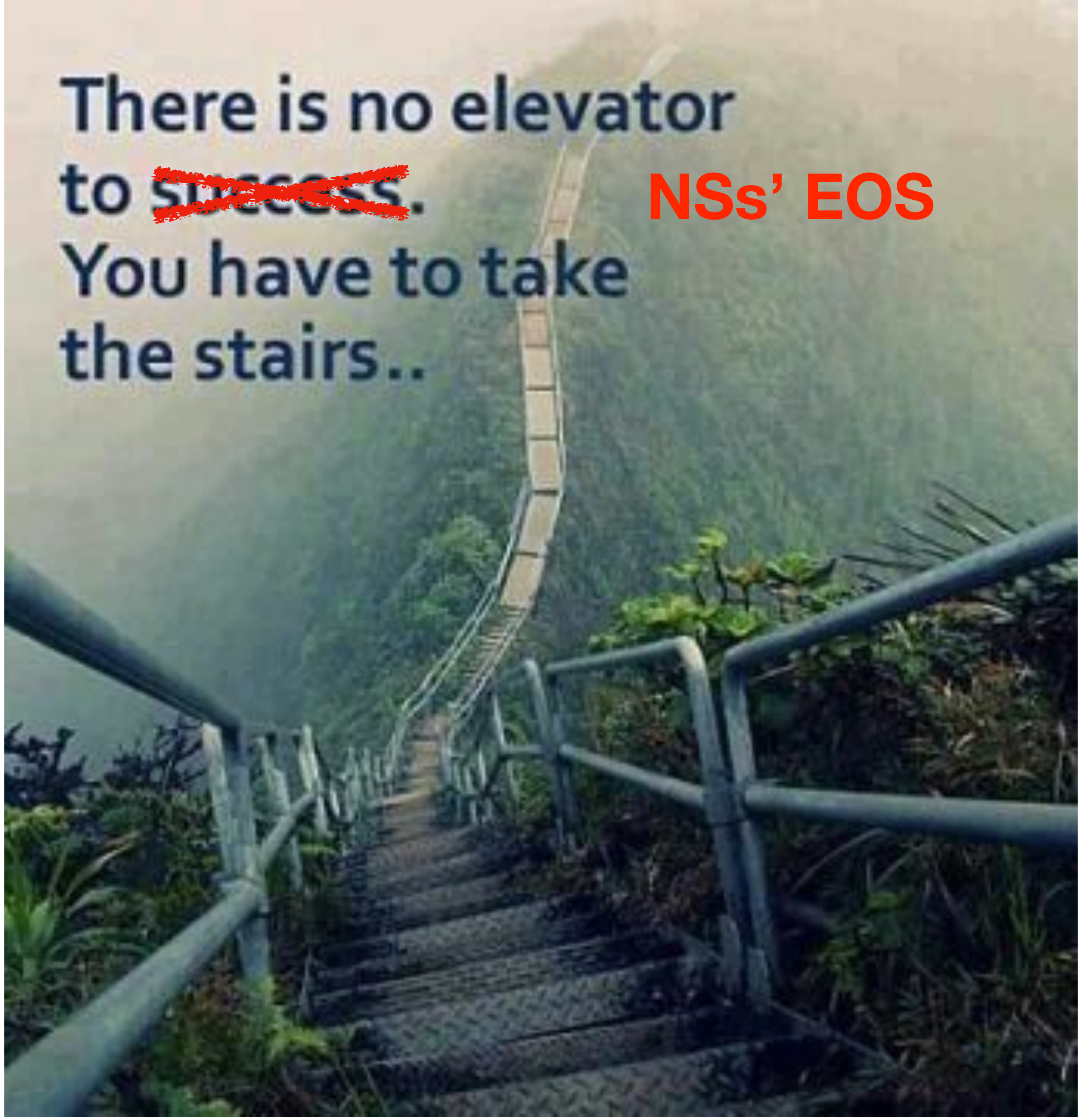
- **Dense nuclear matters**

$$\rho_s \to 2\rho_s \;/\; 3\rho_s \;/\; 4\rho_s$$

- **General relativity**

$$\left.\begin{array}{l}\textbf{Laboratory exp.}\\[1em] \textbf{Solar system}\\[1em] \textbf{Binary pulsars}\end{array}\right\} \to \textbf{highly dynamical spacetime}$$

NSs’ EOS is not given, but earned



Tolman-Oppenheimer-Volkoff (TOV)

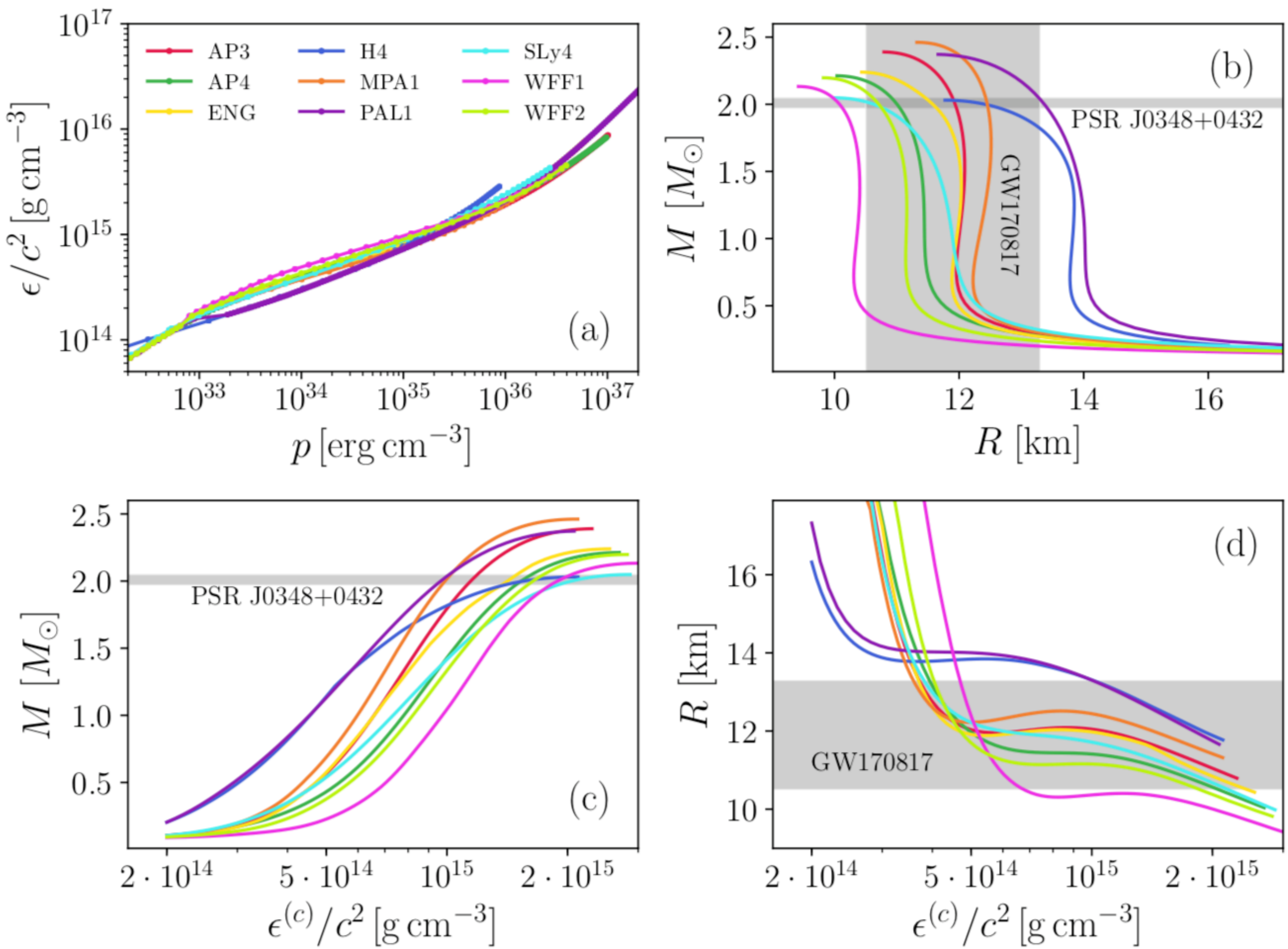
$$\frac{\mathrm{d}p}{\mathrm{d}r} = -G\frac{\epsilon + p}{r^2}\frac{m + 4\pi r^3 p}{1 - 2Gm/r},$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \epsilon,$$

Key ingredient: EOS $\epsilon = \epsilon(p)$

Tolman 1939
Oppenheimer & Volkoff 1939

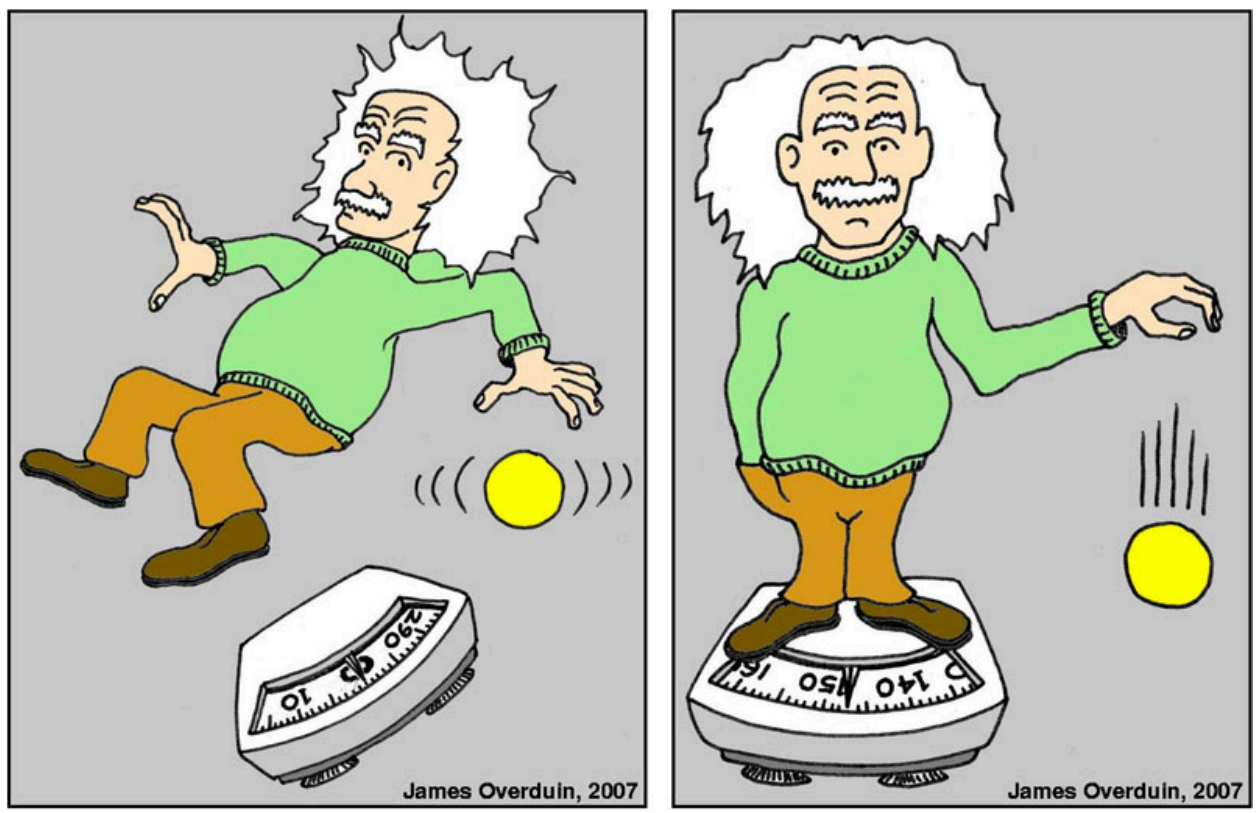
EOSs and NSs



Lattimer & Prakash 2001

General relativity assumed

$$S = \frac{1}{16\pi G}\int \mathrm{d}x^4 \sqrt{-g}\, R + \boxed{S_{\mathrm{matter}}\left[\psi; g_{\mu\nu}\right]}$$



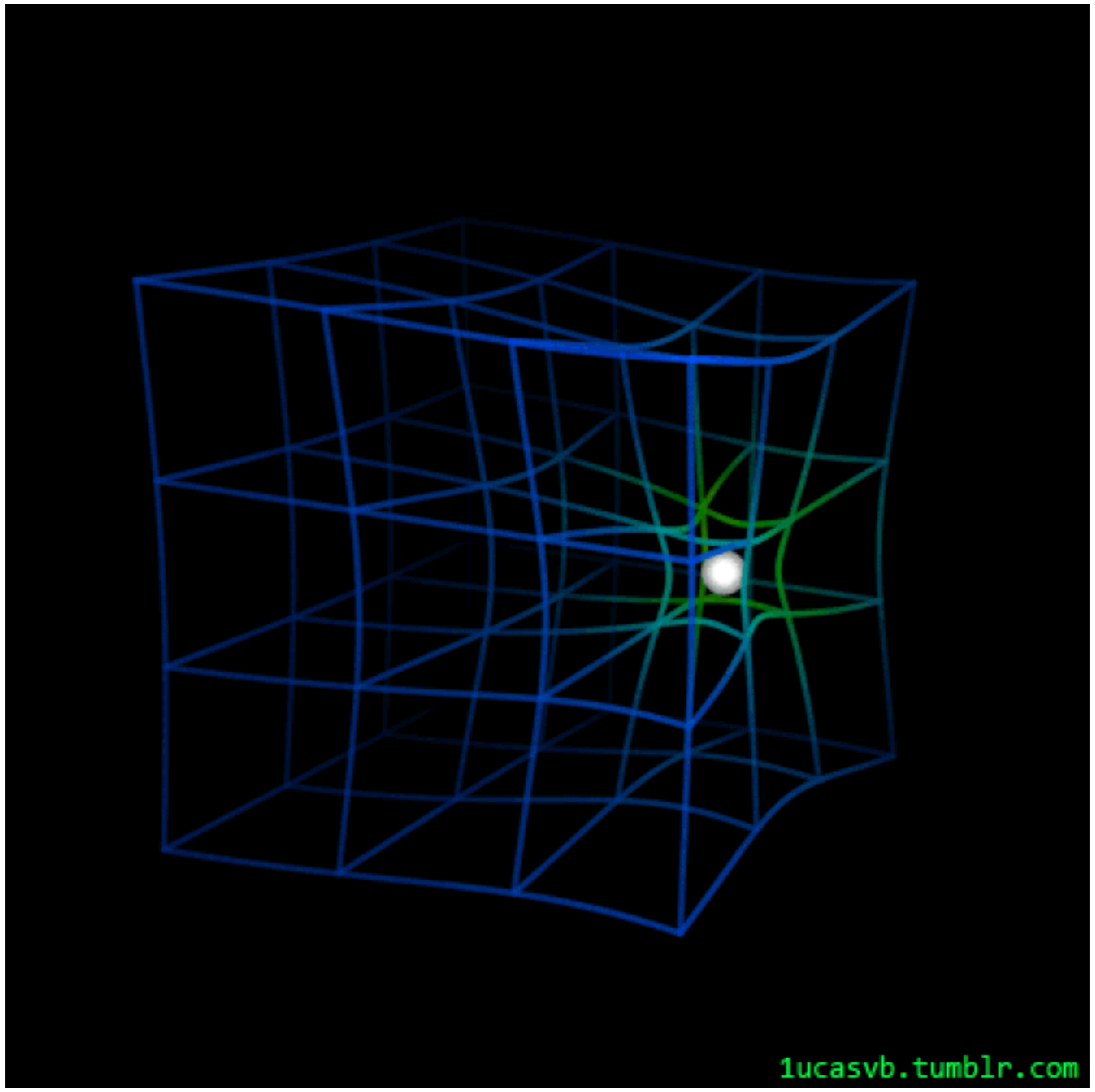
The gravitational "force" as experienced locally while standing on a massive body (such as the Earth) is the same as the pseudo-force experienced by an observer in a non-inertial (accelerated) frame of reference.

General relativity

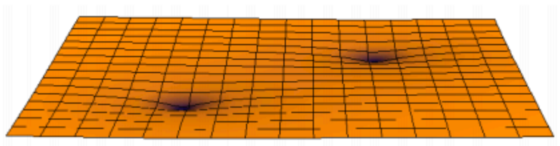
$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

EOS goes here

$$T_{\mu\nu} = \left(\epsilon + p\right) u_\mu u_\nu + p\, g_{\mu\nu}$$

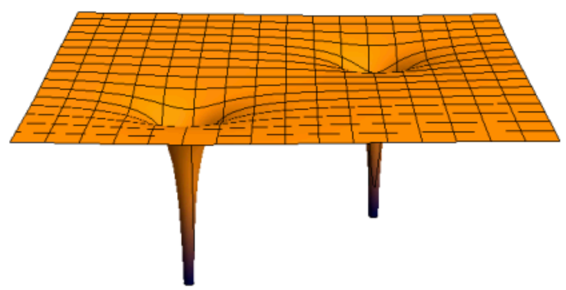


John A. Wheeler: *"Matter tells spacetime how to curve, and spacetime tells matter how to move."*



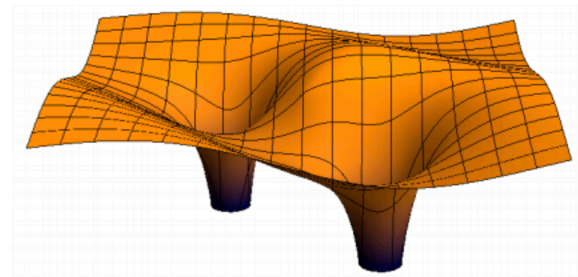
Solar System

G1



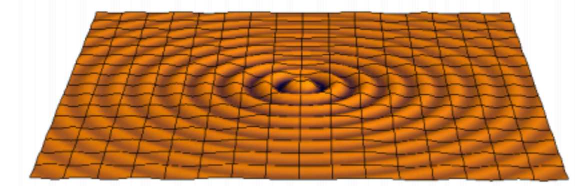
Binary Pulsar

G2



BBH Merger

G3



LIGO/Virgo Sites

GW

Strong-field gravity

G1 — Quasi-stationary weak-field regime

G2 — Quasi-stationary strong-field regime

G3 — Highly dynamical strong-field regime

GW — Radiation regime

Wex 2014

Example: scalar-tensor gravity

$$\boxed{S = \frac{c^4}{16\pi G_*}\int \frac{\mathrm{d}^4 x}{c}\sqrt{-g_*}\left[R_* - 2g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right] + S_m\left[\psi_m; A^2(\varphi)g^*_{\mu\nu}\right]}$$

non-minimal coupling

$$V(\varphi) = 0$$

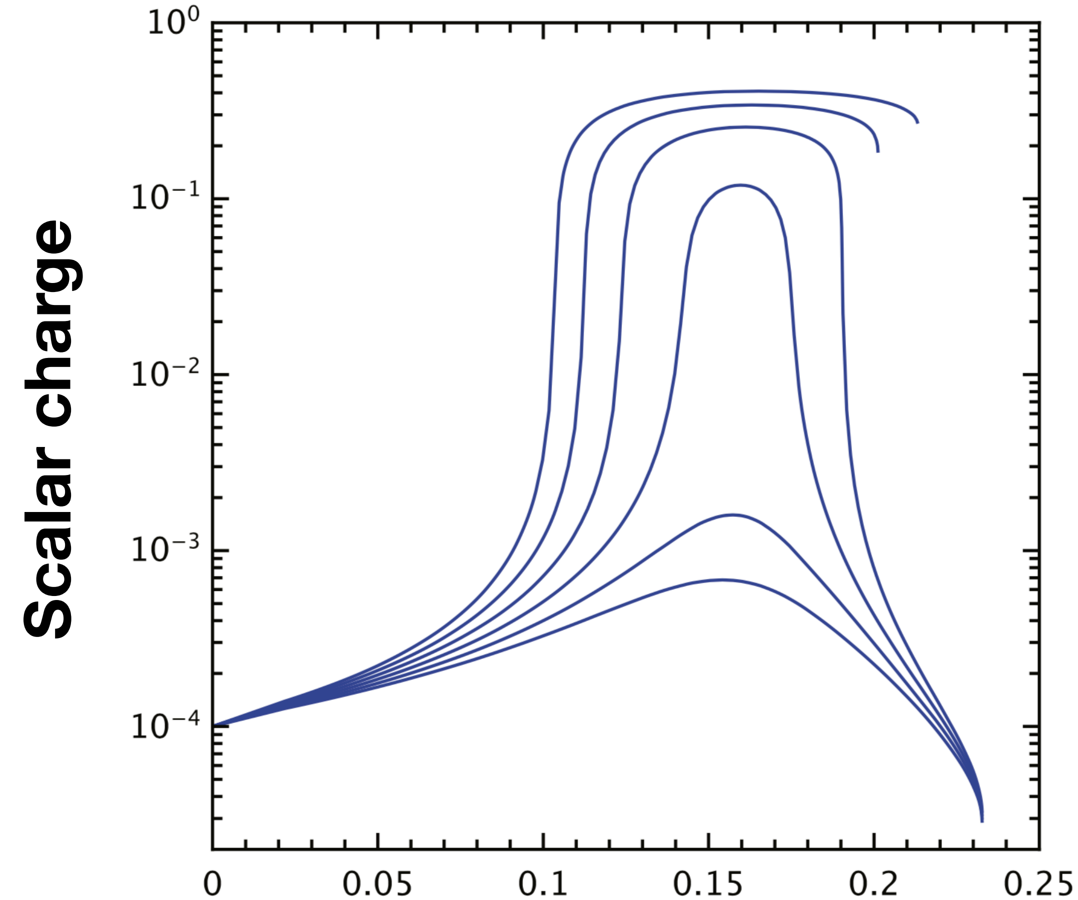
$$A(\varphi) = \exp\left(\beta_0\varphi^2/2\right)$$

$$\alpha_0 = \beta_0\varphi_0$$

As example, we consider a class of cosmologically well-motivated scalar-tensor theories $T(\alpha_0,\beta_0)$, that are solely described by two theory parameters: α_0 & β_0

Damour & Esposito-Farèse 1993
Damour & Esposito-Farèse 1996

Example: scalar-tensor gravity

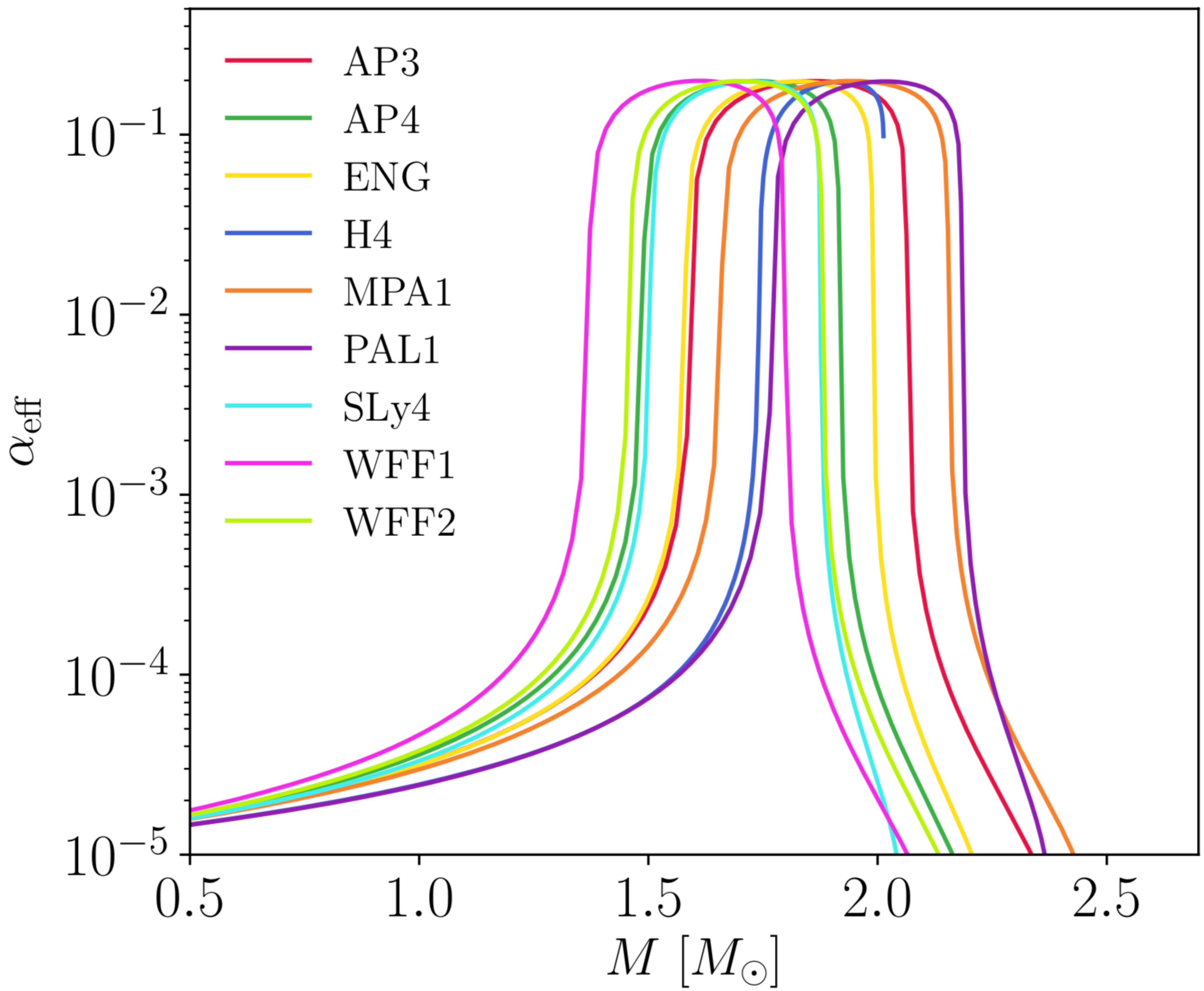


Fractional grav. energy $\sim \dfrac{GM}{Rc^2}$

Nonperturbative *spontaneous scalarization* could happen for isolated NSs

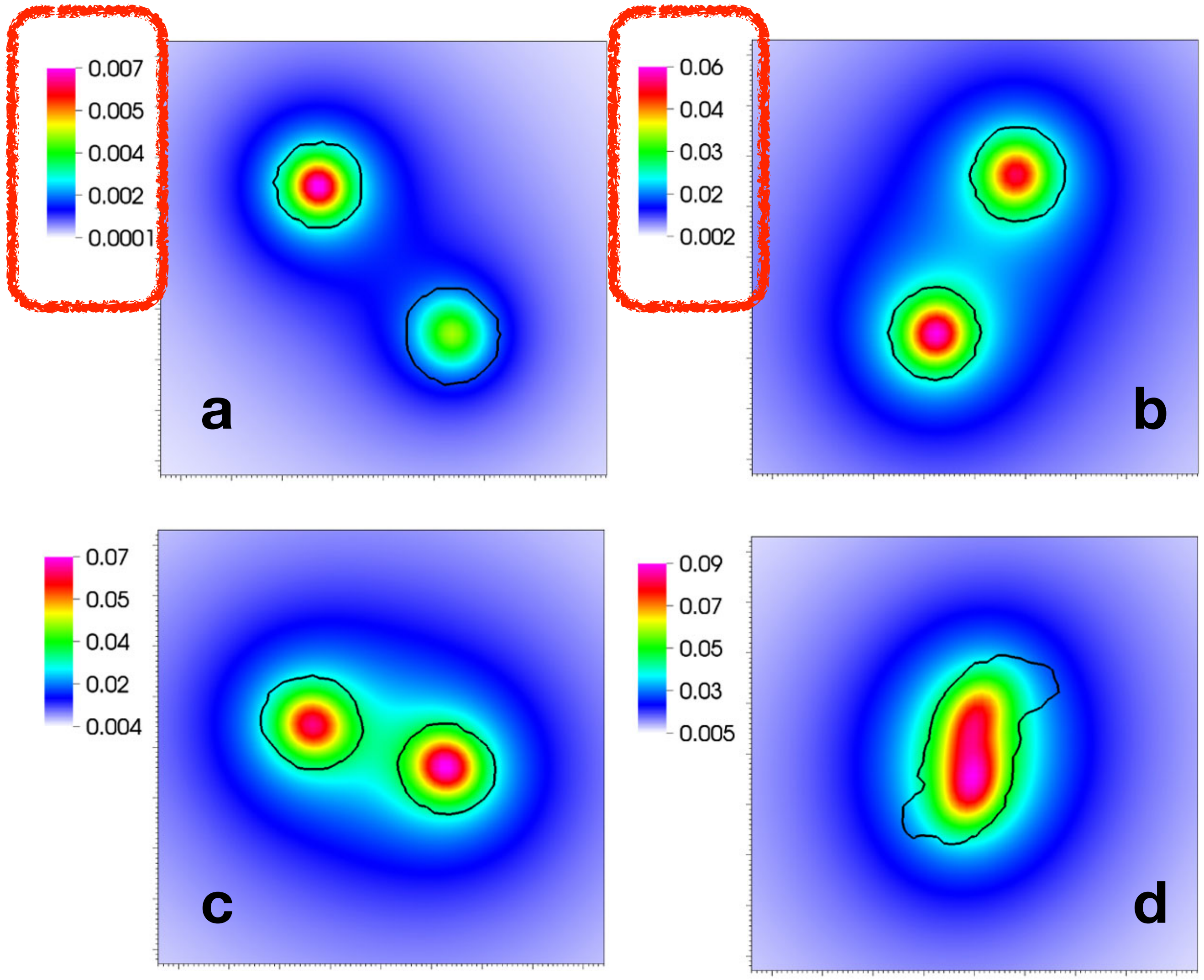
Damour & Esposito-Farèse 1993
Damour & Esposito-Farèse 1996
Shao & Wex 2016

Example: scalar-tensor gravity



Shao [arXiv:1901.07546]

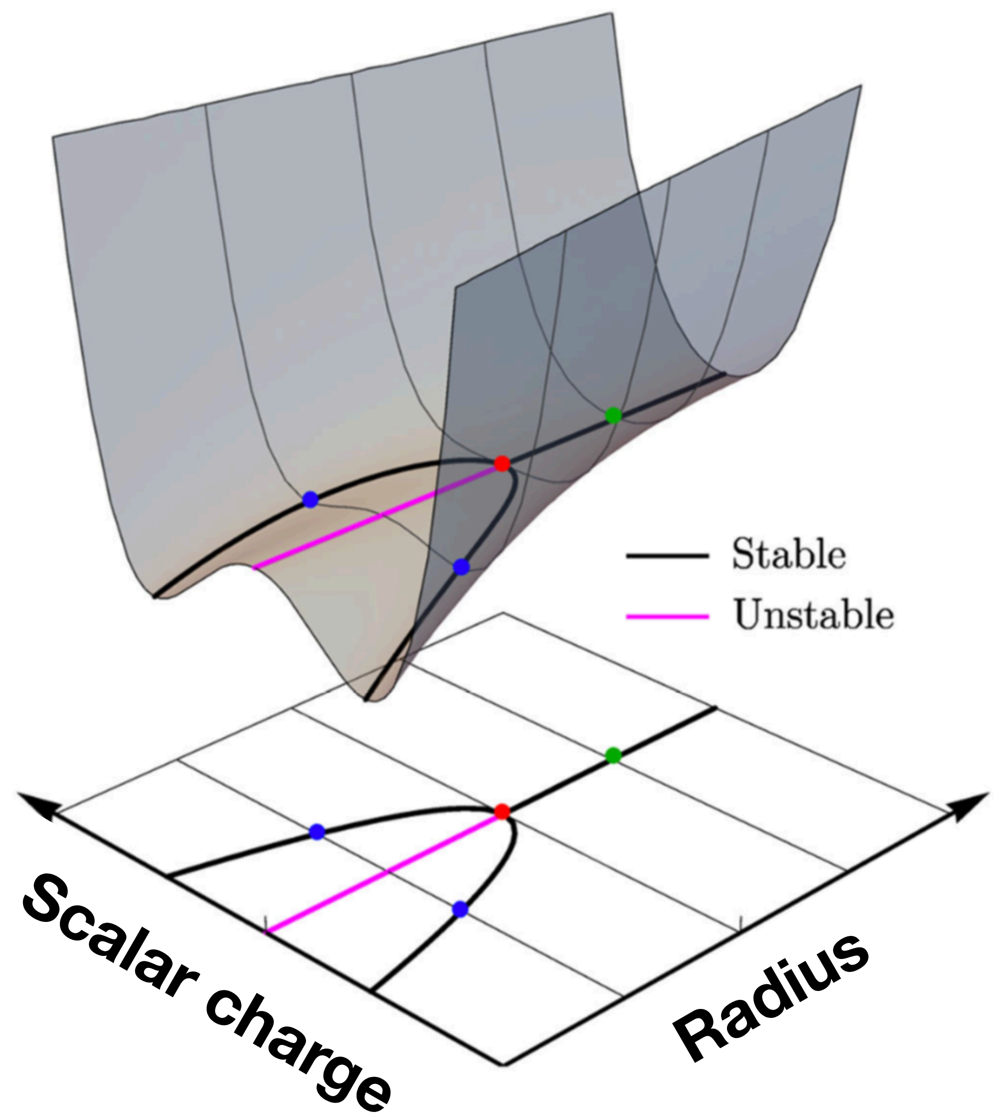
Example: scalar-tensor gravity



In the numerical simulation of BNS mergers, *dynamical scalarization* happens when two NSs reach a critical point

Barausse et al. 2013
Sennett & Buonanno 2016
Shao et al. 2017

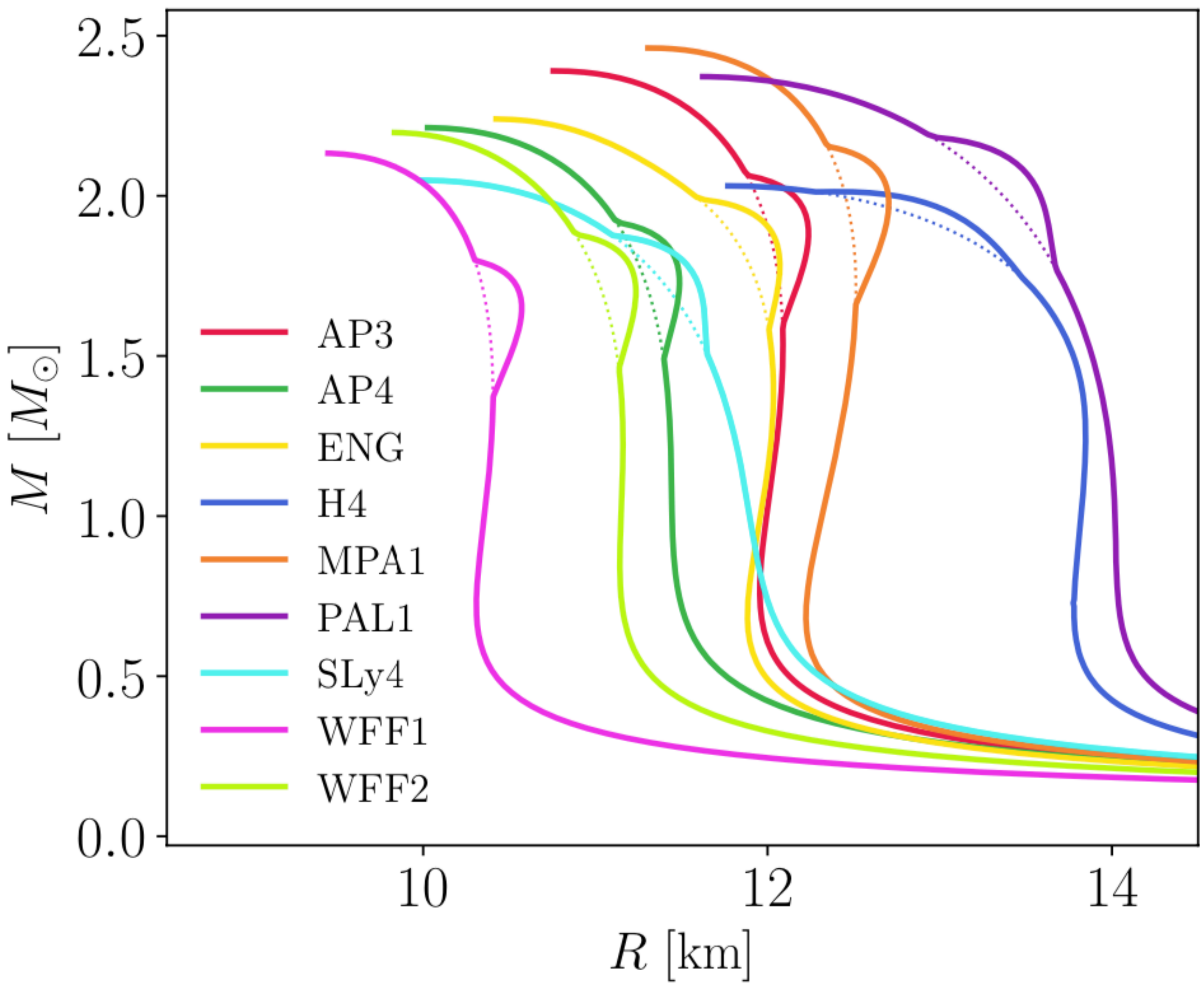
Example: scalar-tensor gravity



Strong-field behavior is analogous to Landau's *phase transition* after a critical point

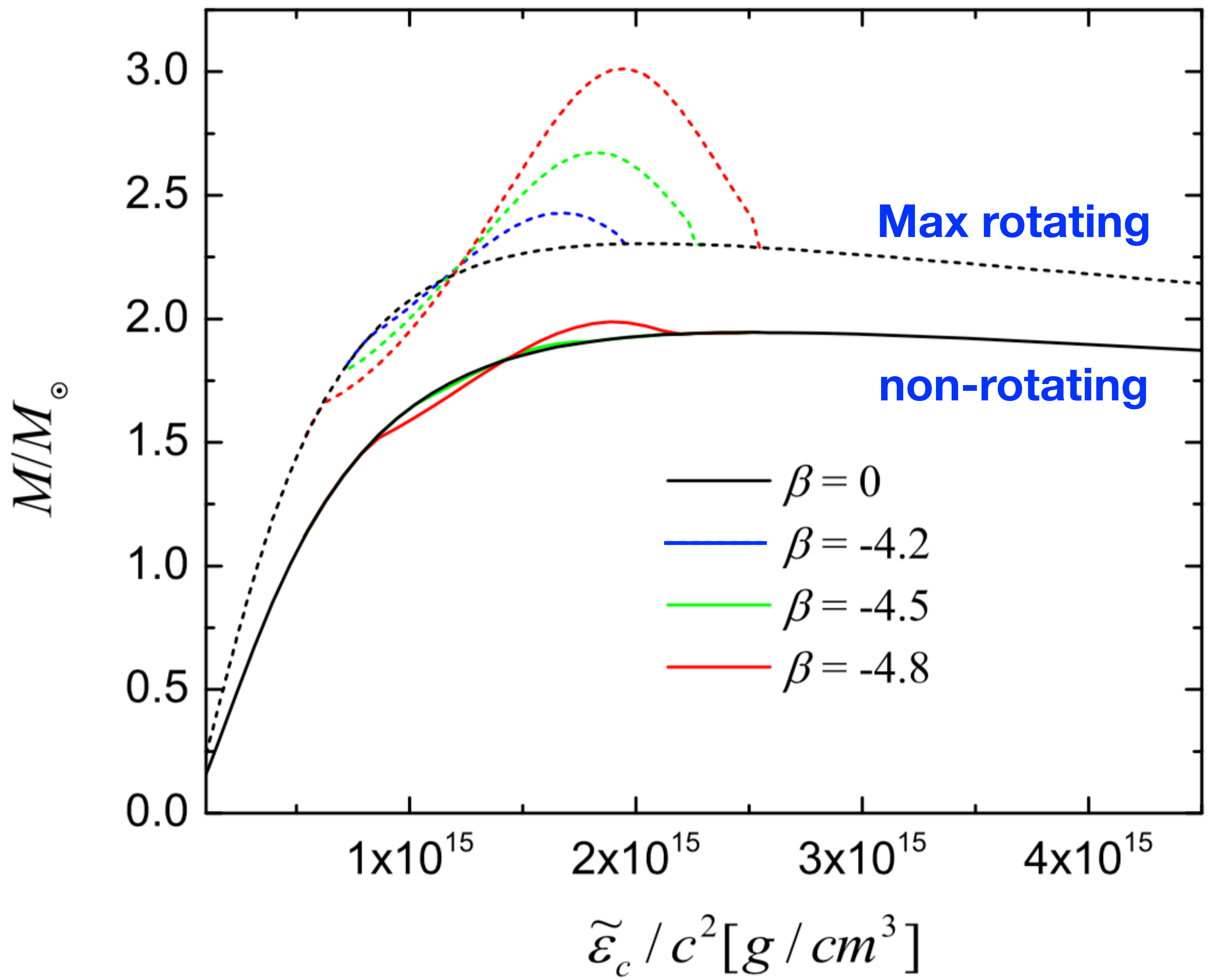
Damour & Esposito-Farèse 1996
Esposito-Farèse 2004
Sennett, Shao, Steinhoff 2017

Scalarized NSs



Shao [arXiv:1901.07546]

Scalarized NSs



Doneva et al. 2013

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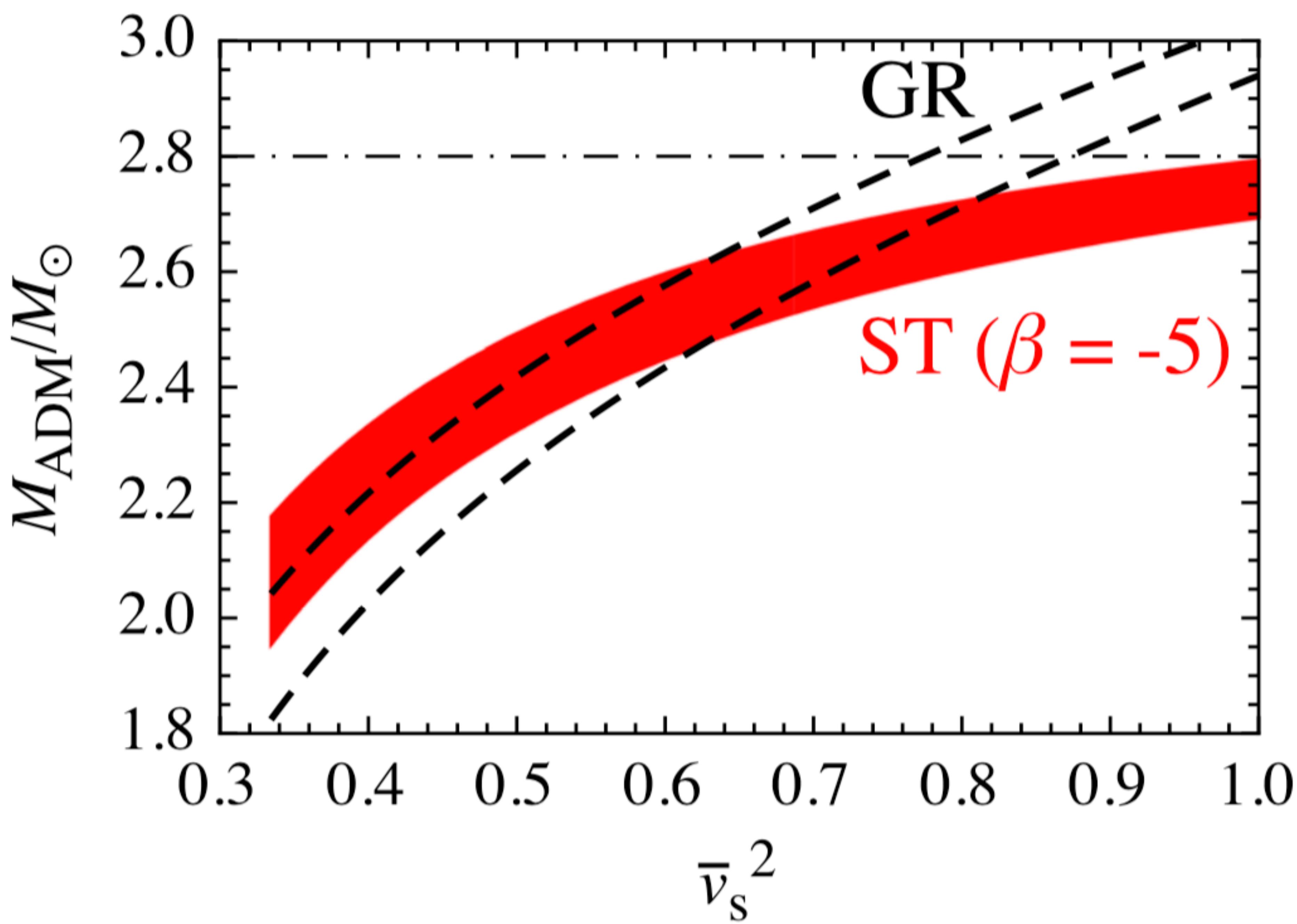
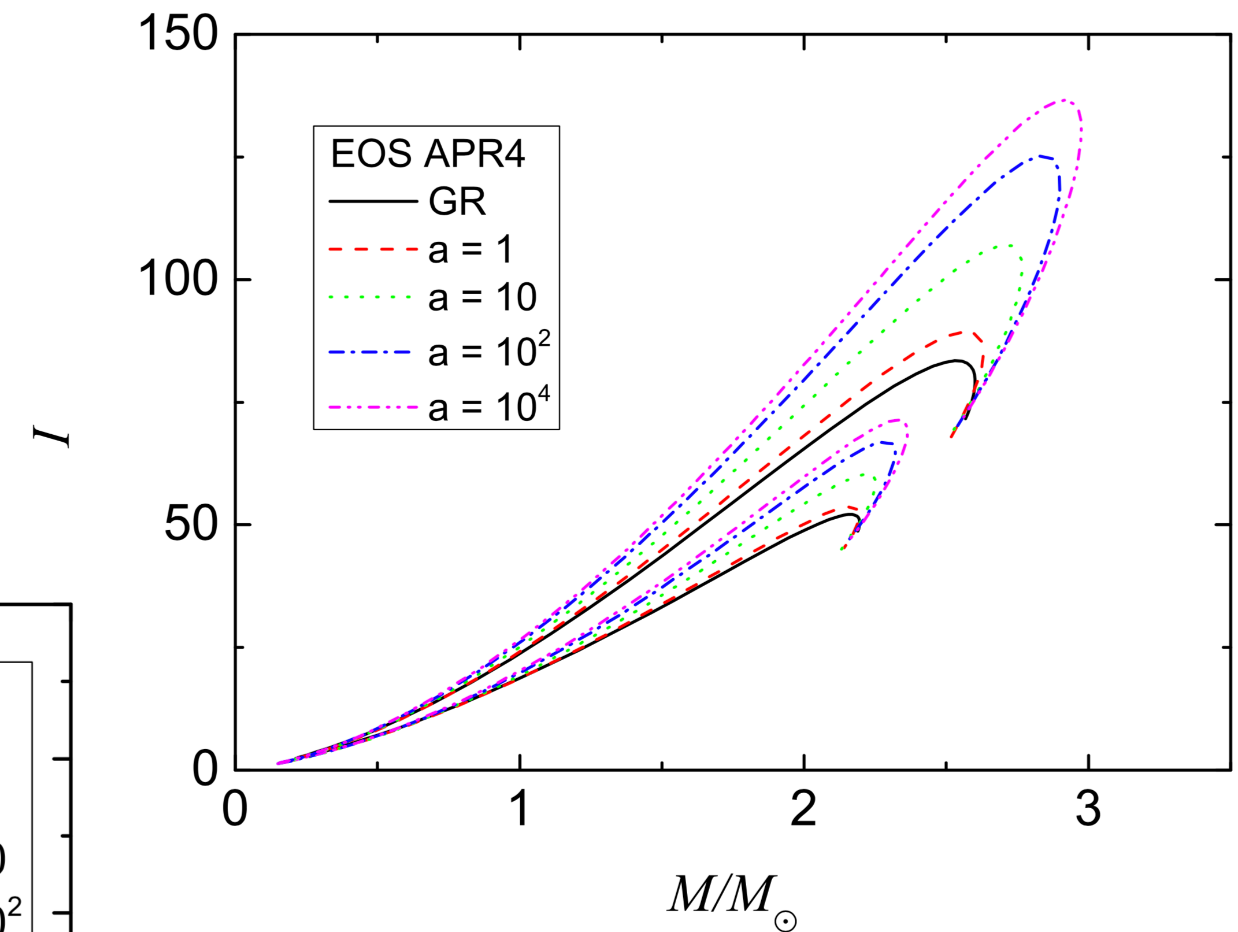
NS mass in scalar-tensor gravity

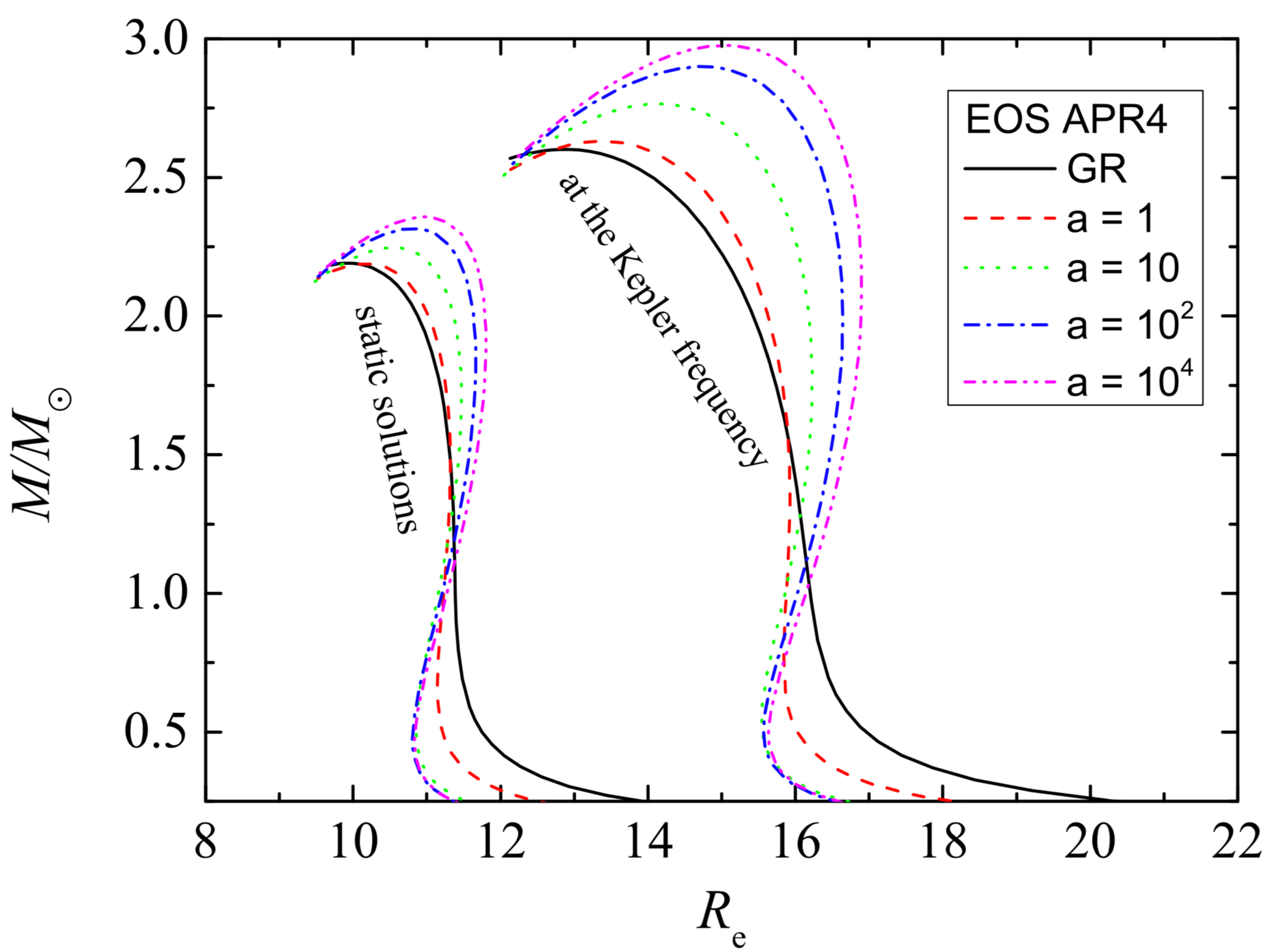


Sotani & Kokkotas 2017

f(R) theory

Explaining the accelerated Universe

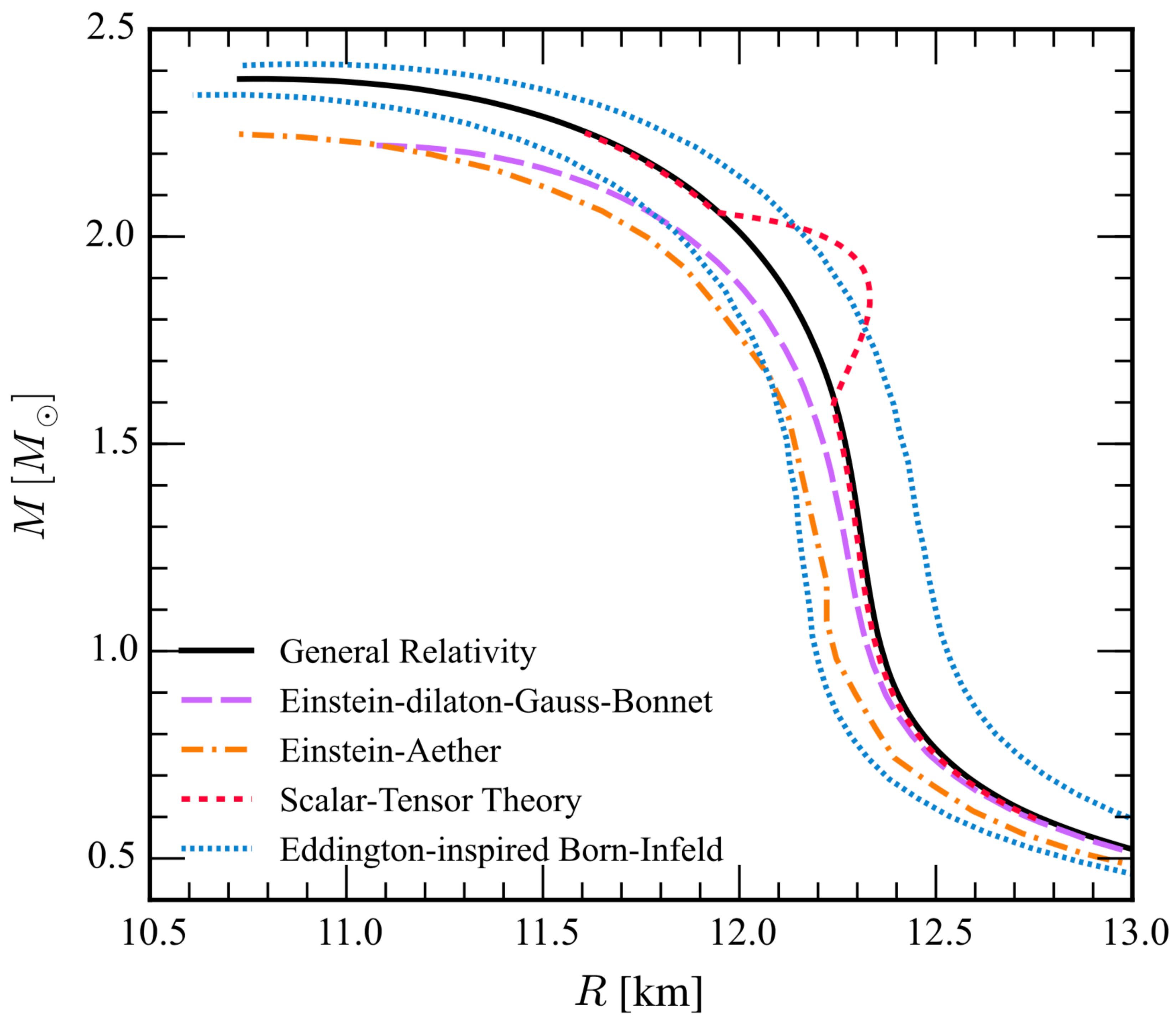




$$f(R) = R + aR^2$$

Yazadjiev et al. 2015

More examples



Glampedakis et al. 2015

Remarks

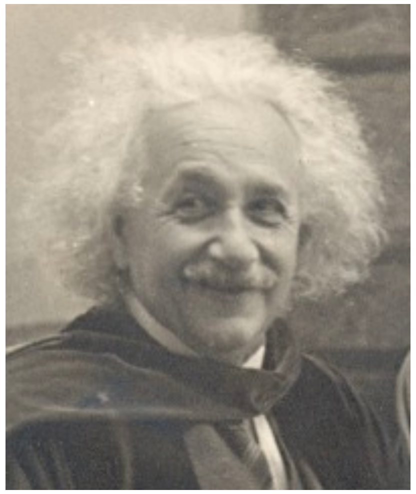
- There could still be a noticeable deviation in the strong field, though

  1. GR was tested to a very high precision

  2. We all love GR

- There could be degeneracy between (uncertain) EOS and (uncertain) gravity in the strong-field regime

- One should not take GR as given everywhere, instead she has to earn it!



Parameterized post-Newtonian (PPN)

Metric

$$
\begin{aligned}
g_{00} = & -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 \\
& + 2(1+\zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} \\
& + (2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3), \\
g_{0i} = & -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U \\
& - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}), \\
g_{ij} = & (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).
\end{aligned}
$$

PPN parameter

Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories
γ	How much space-curvature produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" in the superposition law for gravity?	1	β	β
ξ	Preferred-location effects?	0	ξ	ξ
α_1	Preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
α_3	Violation of conservation of total momentum?	0	0	0
ζ_1		0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

Potential

$$
\begin{aligned}
U &= \int \frac{\rho'}{|\mathbf{x}-\mathbf{x}'|} d^3x', \\
U_{ij} &= \int \frac{\rho'(x-x')_i(x-x')_j}{|\mathbf{x}-\mathbf{x}'|^3} d^3x', \\
\Phi_W &= \int \frac{\rho'\rho''(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3} \cdot \left(\frac{\mathbf{x}'-\mathbf{x}''}{|\mathbf{x}-\mathbf{x}''|} - \frac{\mathbf{x}-\mathbf{x}''}{|\mathbf{x}'-\mathbf{x}''|}\right) d^3x'\, d^3x'', \\
\mathcal{A} &= \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')]^2}{|\mathbf{x}-\mathbf{x}'|^3} d^3x', \\
\Phi_1 &= \int \frac{\rho' v'^2}{|\mathbf{x}-\mathbf{x}'|} d^3x', \\
\Phi_2 &= \int \frac{\rho' U'}{|\mathbf{x}-\mathbf{x}'|} d^3x', \\
\Phi_3 &= \int \frac{\rho'\Pi'}{|\mathbf{x}-\mathbf{x}'|} d^3x', \\
\Phi_4 &= \int \frac{p'}{|\mathbf{x}-\mathbf{x}'|} d^3x', \\
V_i &= \int \frac{\rho' v_i'}{|\mathbf{x}-\mathbf{x}'|} d^3x', \\
W_i &= \int \frac{\rho'[\mathbf{v}'\cdot(\mathbf{x}-\mathbf{x}')](x-x')_i}{|\mathbf{x}-\mathbf{x}'|^3} d^3x'.
\end{aligned}
$$

Matter

$$
\begin{aligned}
T^{00} &= \rho(1 + \Pi + v^2 + 2U), \\
T^{0i} &= \rho v^i \left(1 + \Pi + v^2 + 2U + \frac{p}{\rho}\right), \\
T^{ij} &= \rho v^i v^j \left(1 + \Pi + v^2 + 2U + \frac{p}{\rho}\right) + p\delta^{ij}(1 - 2\gamma U).
\end{aligned}
$$

Will & Nordtvedt 1972
Will 1993

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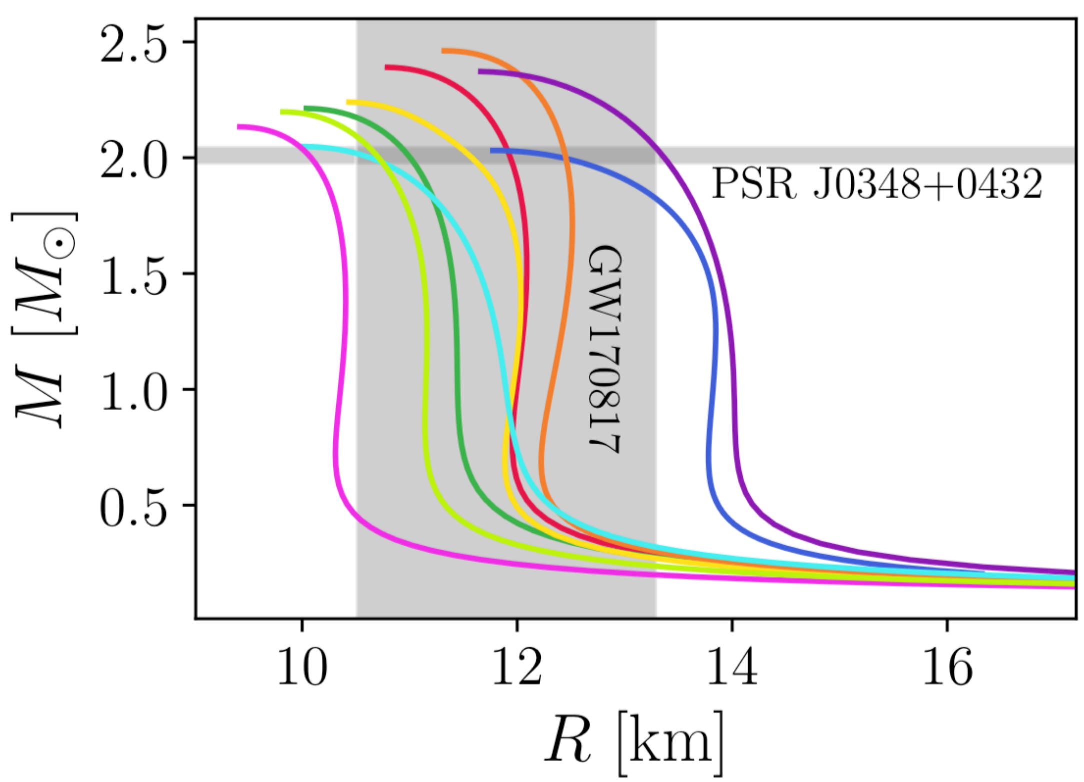
TOV with PPN

$$
\begin{aligned}
\frac{\mathrm{d}p}{\mathrm{d}r} &= -\frac{Gm\rho}{r^2}\left[1+\Pi+\frac{p}{\rho}+(5+3\gamma-6\beta+\zeta_2)\,\frac{Gm}{r}+(\gamma+\zeta_4)\,4\pi\frac{r^3 p}{m}\right], \\
\frac{\mathrm{d}m}{\mathrm{d}r} &= 4\pi r^2\rho\left[1+(1+\zeta_3)\,\Pi-\frac{1}{2}\,(11+\gamma-12\beta+\zeta_2-2\zeta_4)\,\frac{Gm}{r}\right].
\end{aligned}
$$

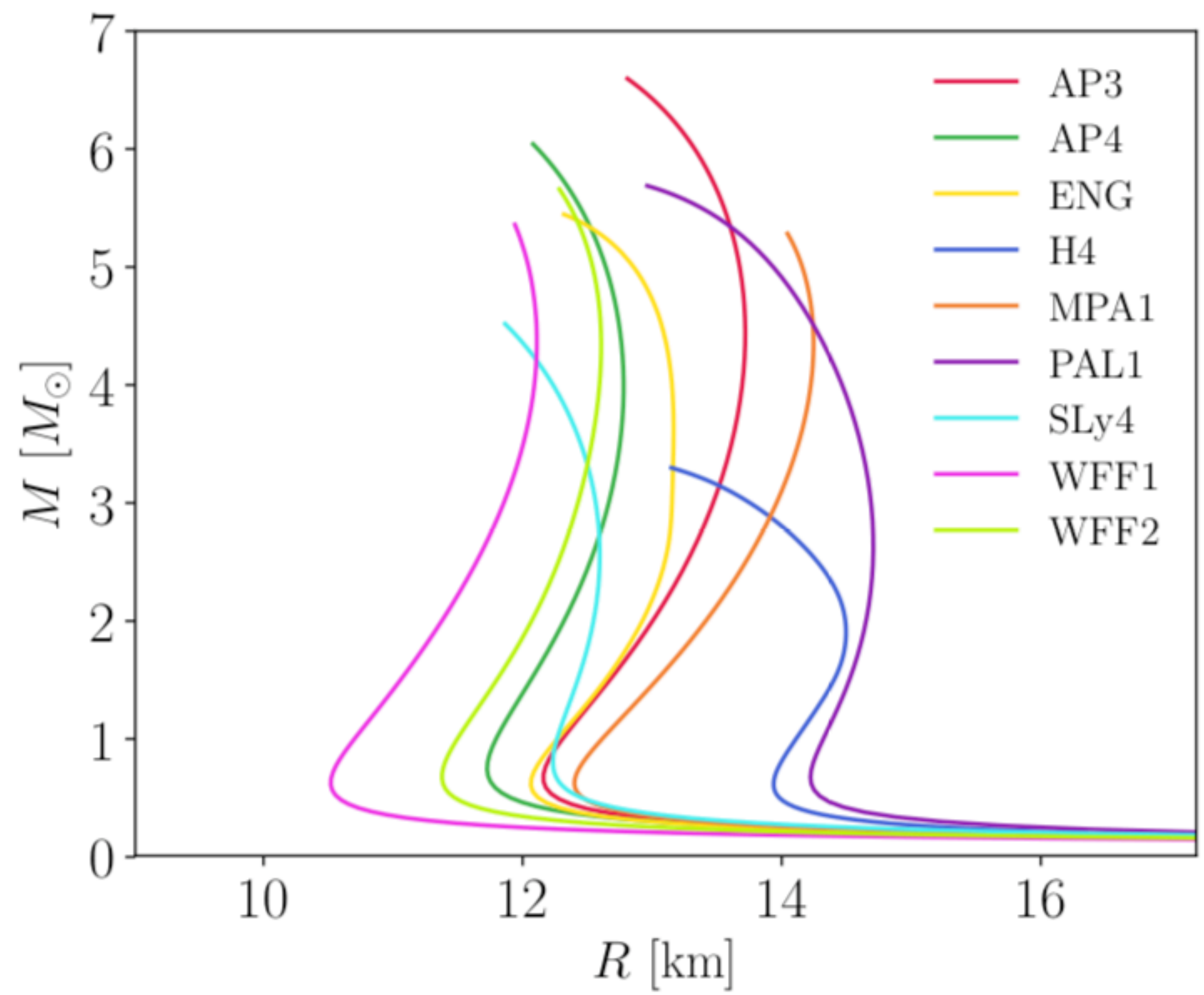
Wagoner & Malone 1974
Ciufolini & Ruffini 1983
Glampedakis et al. 2015

M(R) relation at 1 PN (GR)

full GR



GR@1PN



Shao, unpublished

Therefore, naive post-Newtonian expansion for TOV eqs will NOT be useful…

post-TOV formalism

$$\frac{\mathrm{d}p}{\mathrm{d}r} = \boxed{-G\frac{\epsilon + p}{r^2}\frac{m + 4\pi r^3 p}{1 - 2Gm/r}} - \frac{Gm\rho}{r^2}\left(\mathcal{P}_1 + \mathcal{P}_2\right) ,$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = \boxed{4\pi r^2 \epsilon} + 4\pi r^2 \rho \left(\mathcal{M}_1 + \mathcal{M}_2\right) ,$$

Use post-Newtonian expansion for corrections, but keep GR *resummed*

Glampedakis et al. 2015
Glampedakis et al. 2016

$$\left.\begin{aligned} \mathcal{P}_1 &= \delta_1 \frac{Gm}{r} + 4\pi\delta_2 \frac{r^3 p}{m} , \\ \mathcal{M}_1 &= \delta_3 \frac{Gm}{r} + \delta_4 \Pi , \end{aligned}\right\} \text{@1PN}$$

$$\left.\begin{aligned} \mathcal{P}_2 &= \pi_1 \frac{G^2 m^3}{r^5 \rho} + \pi_2 \frac{G^2 m^2}{r^2} + \pi_3 G r^2 p + \pi_4 \frac{\Pi p}{\rho} , \\ \mathcal{M}_2 &= \mu_1 \frac{G^2 m^3}{r^5 \rho} + \mu_2 \frac{G^2 m^2}{r^2} + \mu_3 G r^2 p + \mu_4 \frac{\Pi p}{\rho} + \mu_5 \Pi^3 \frac{r}{Gm} , \end{aligned}\right\} \text{@2PN}$$

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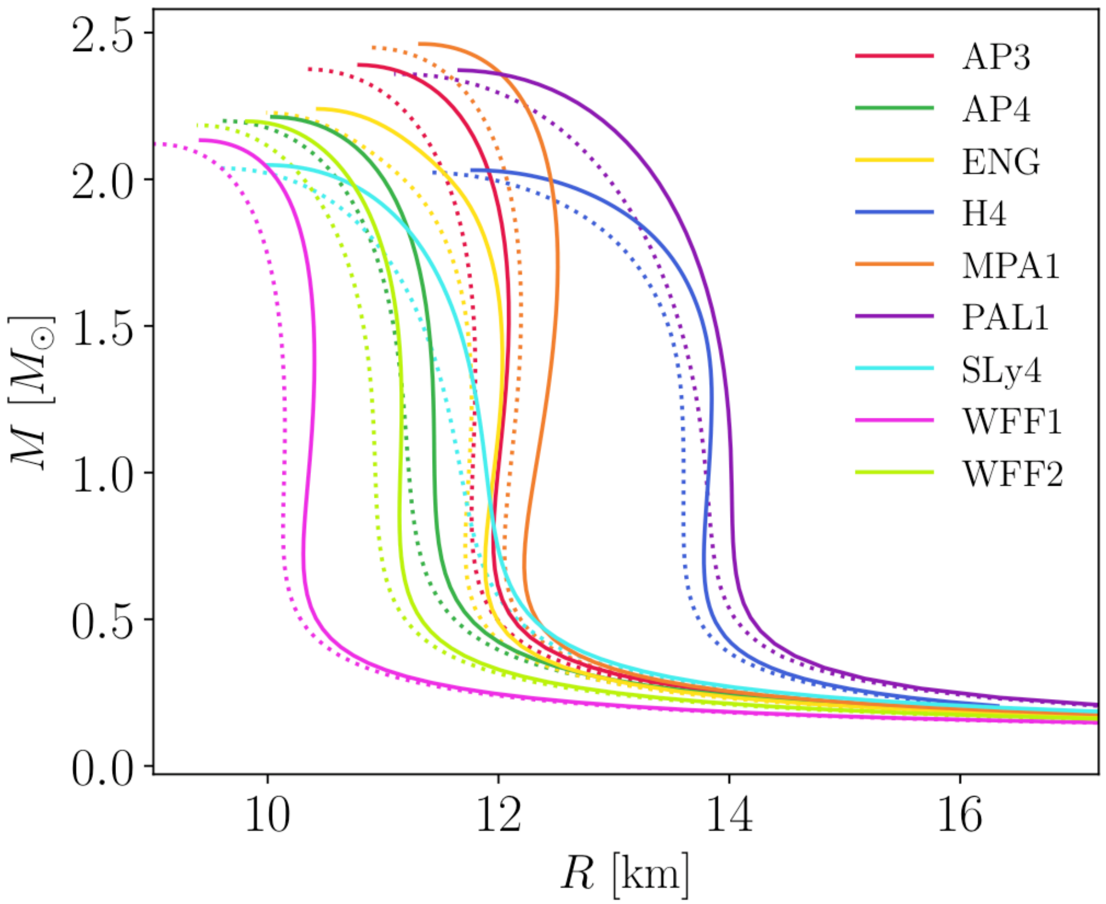
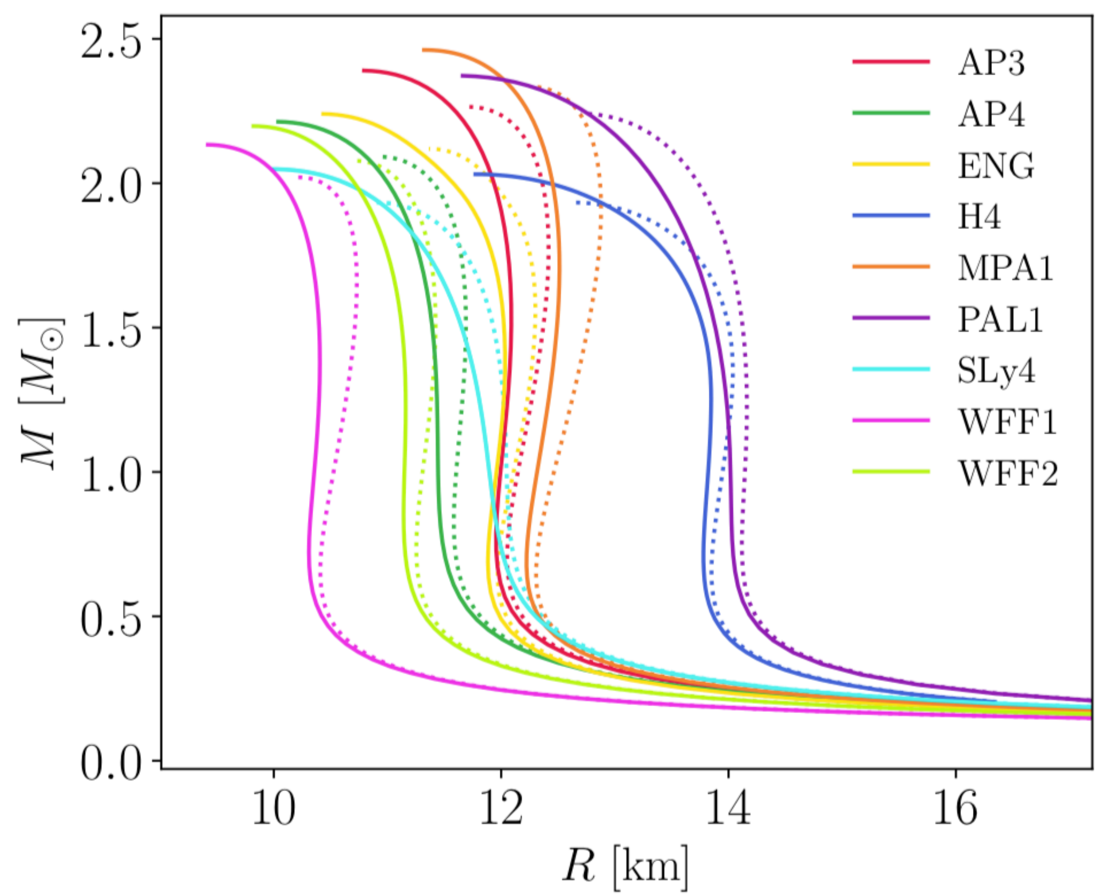
M(R) relation with post-TOV

Two random examples…



$\delta_3 = 0.5$



$\mu_1 = -0.5$

Shao [arXiv:1901.07546]

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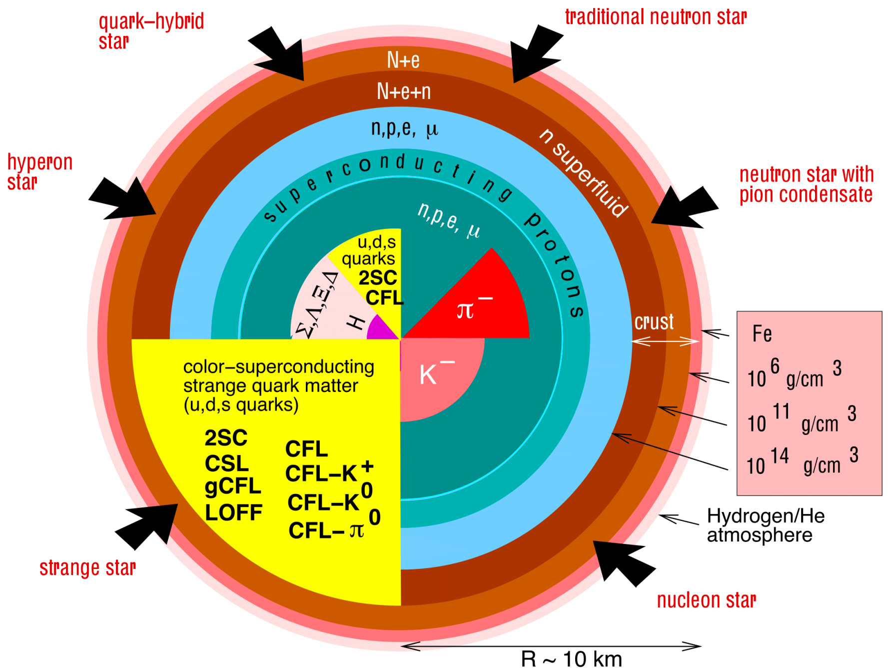
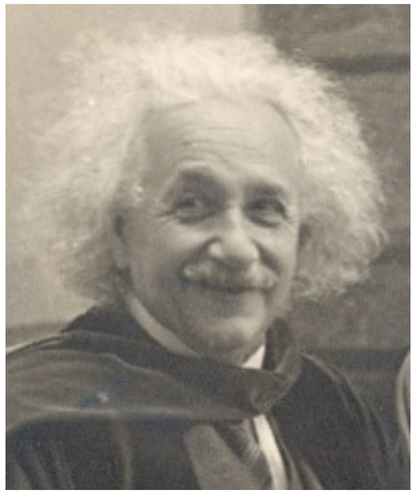
Possible approaches

1. Assume the gravity, then probe the EOS
2. Assume the EOS, then probe the gravity
3. Pin down the gravity (from elsewhere), then probe the EOS
4. Pin down the EOS (from elsewhere), then probe the gravity
5. Investigate the gravity and EOS together



entangles with



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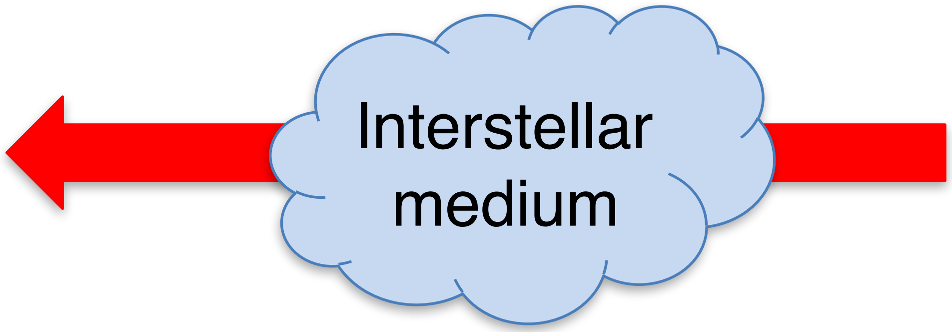
Pulsar timing



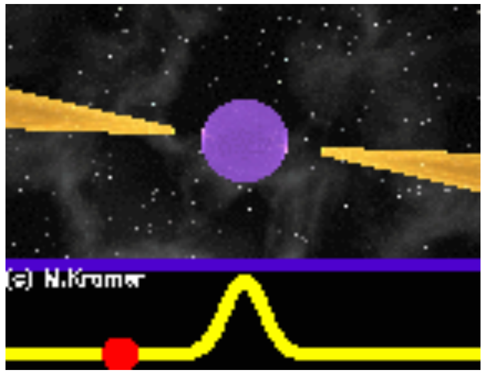
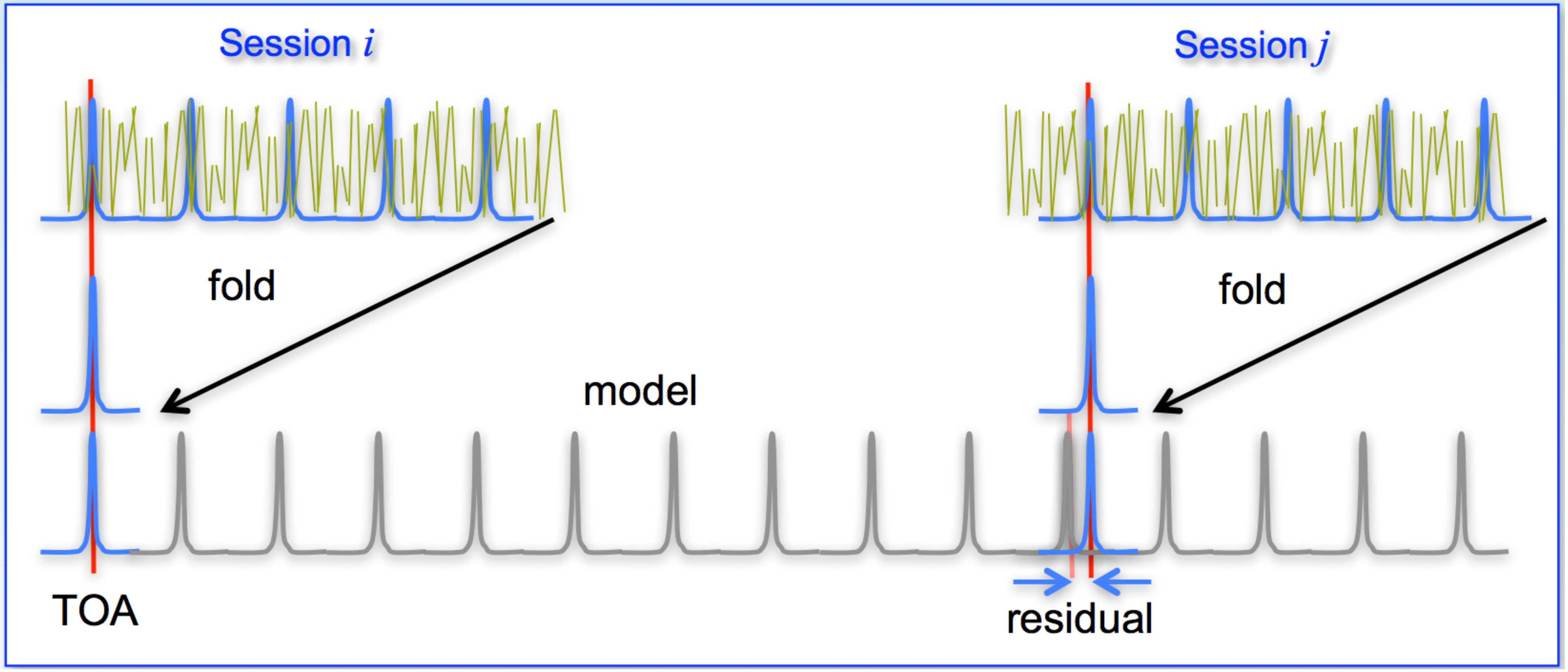


Figure Credit: M. Kramer

Phase-connected timing solution:

Credit: N. Wex



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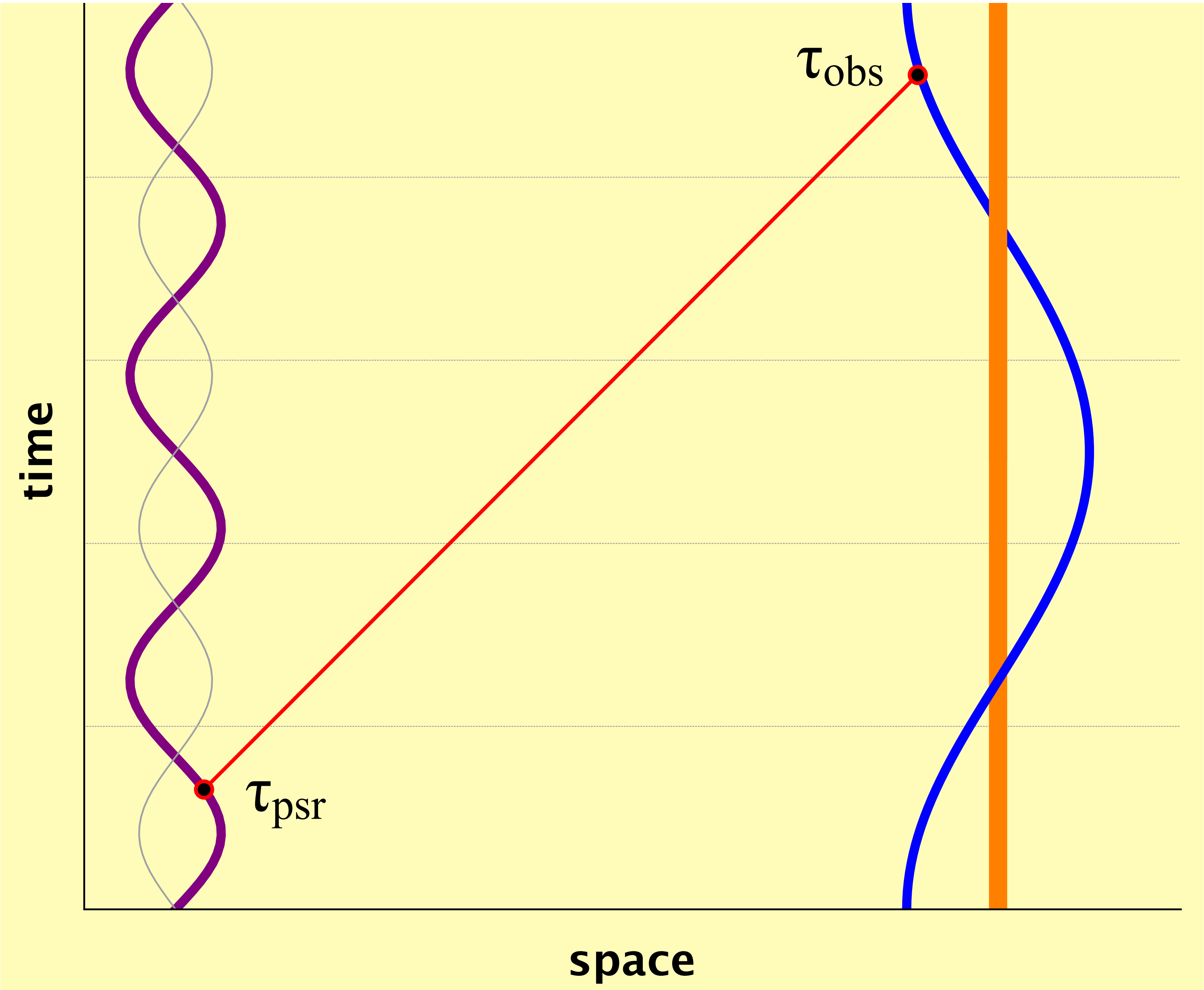
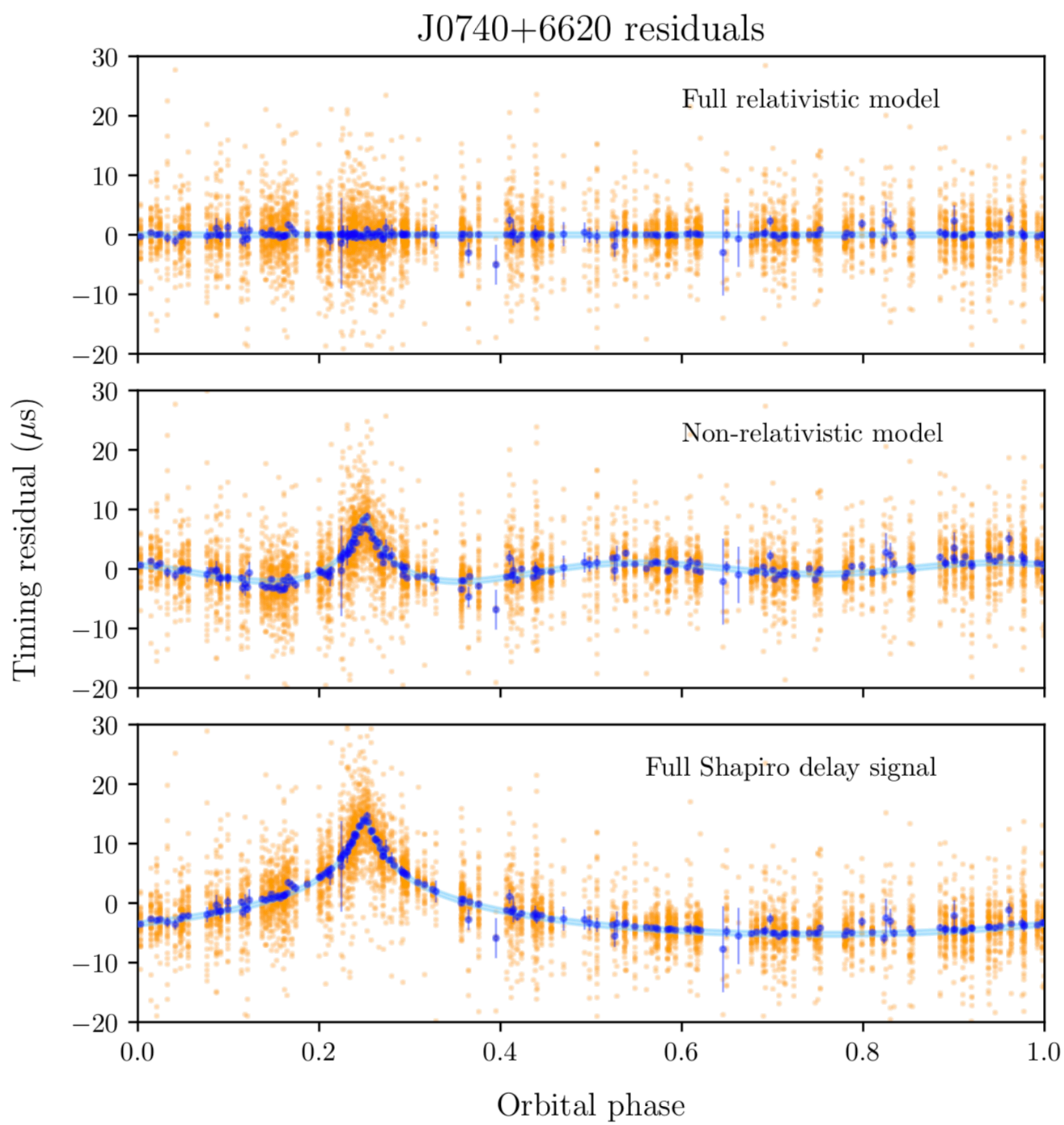
Timing model



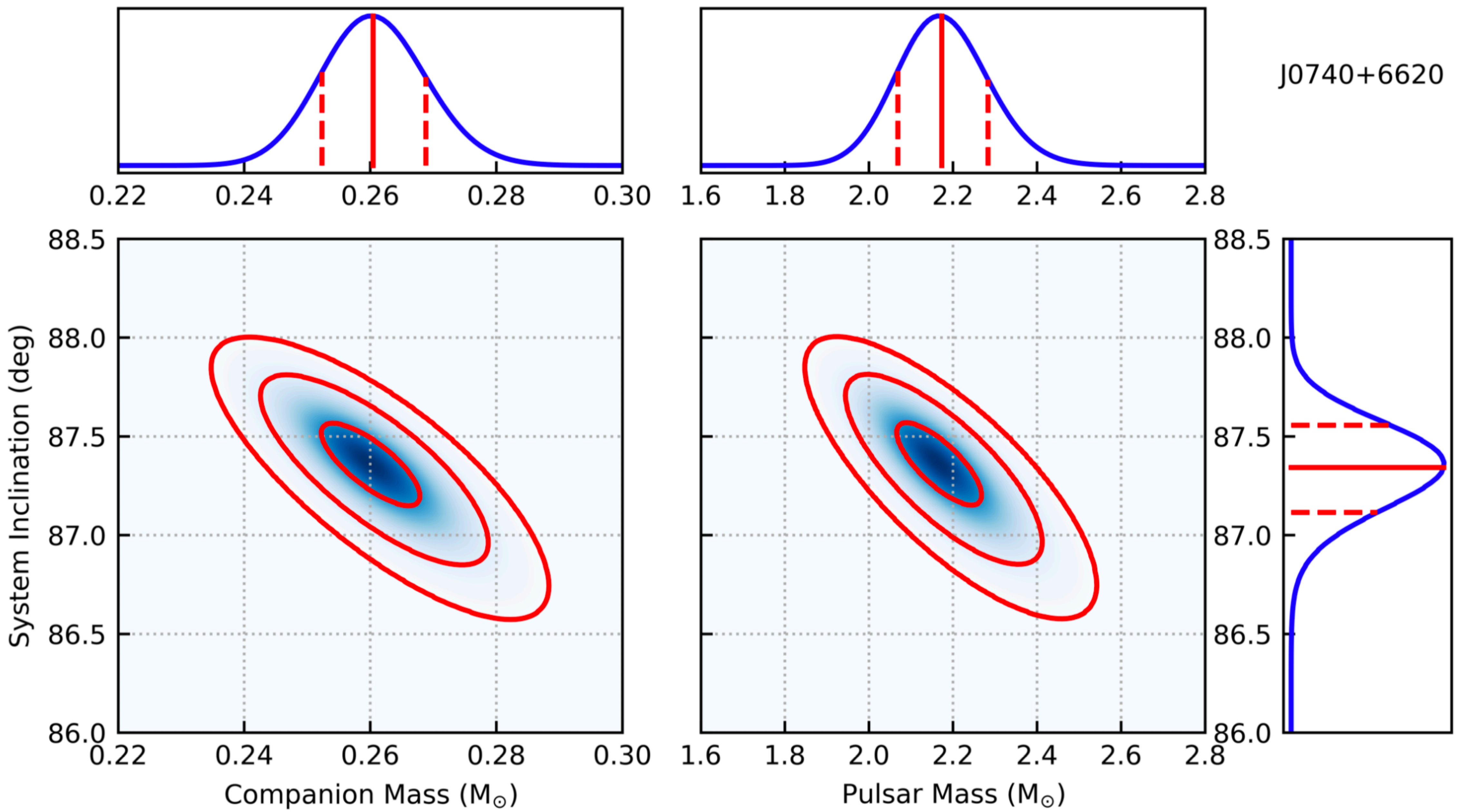
©N. Wex

Example: Shapiro delay



Cromartie et al. 2019

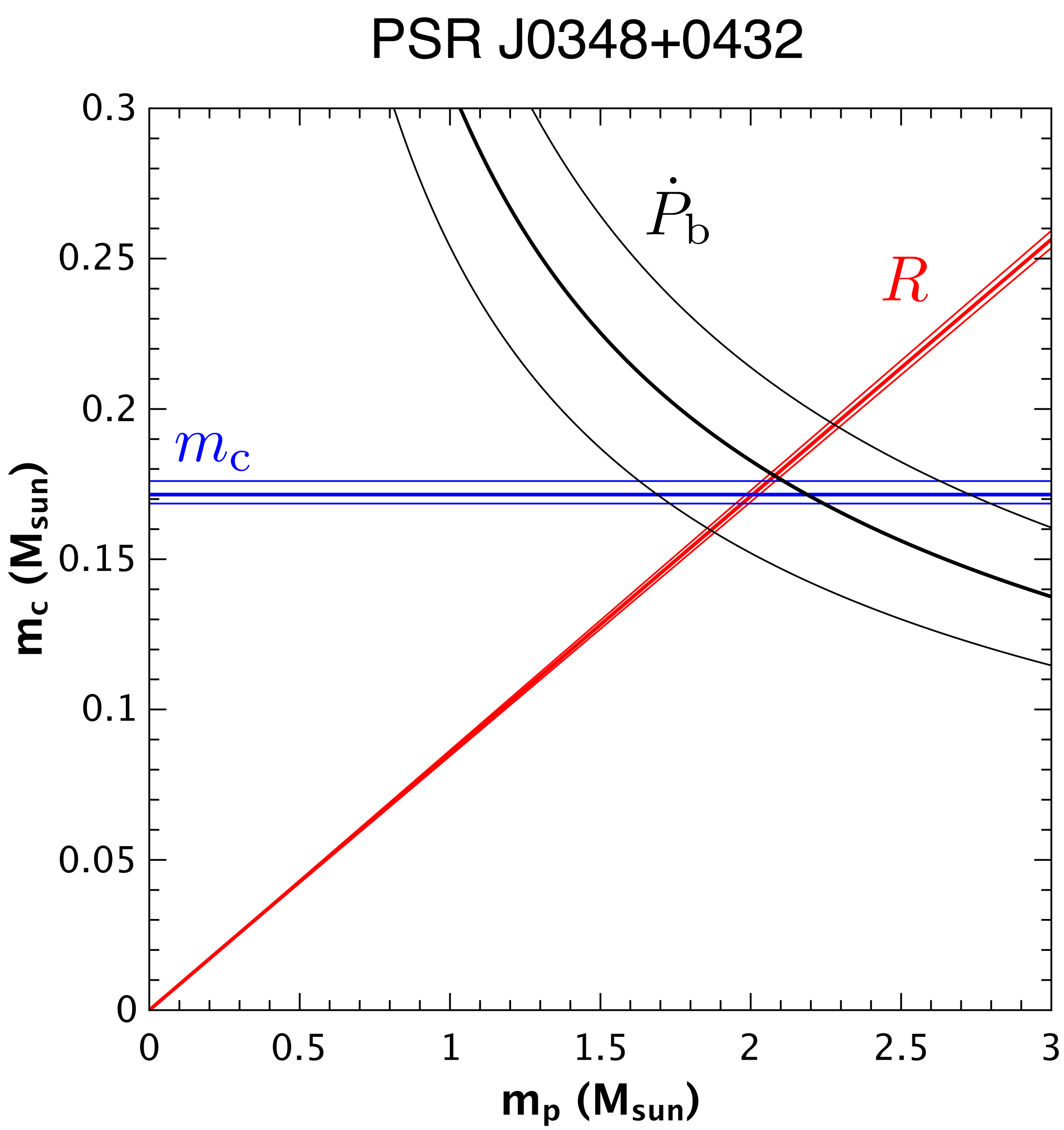
Example: Shapiro delay



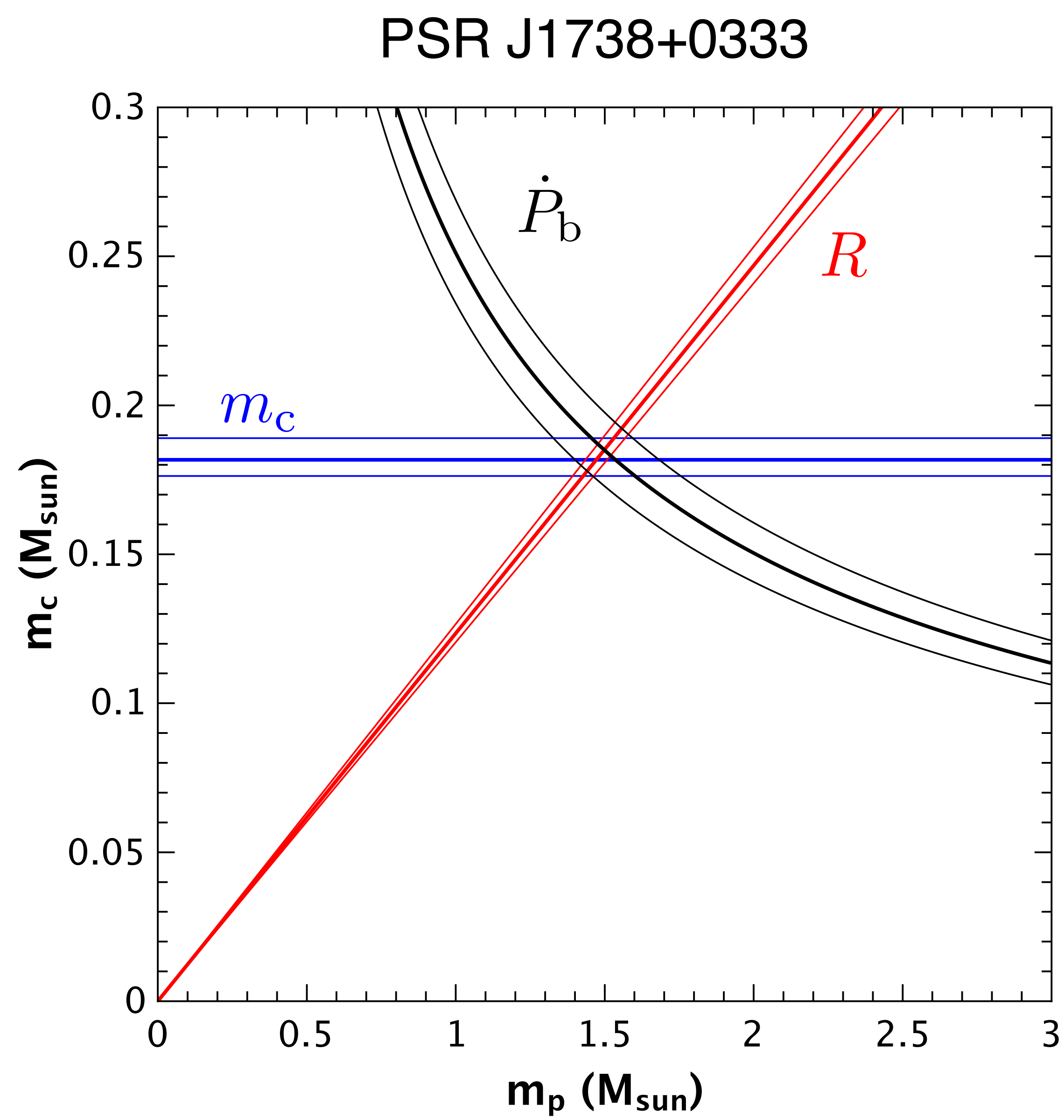
Cromartie et al. 2019

$$m_{\rm PSR} = 2.17^{+0.11}_{-0.10}\, M_\odot$$

PSRs J0348+0432 and J1738+0333



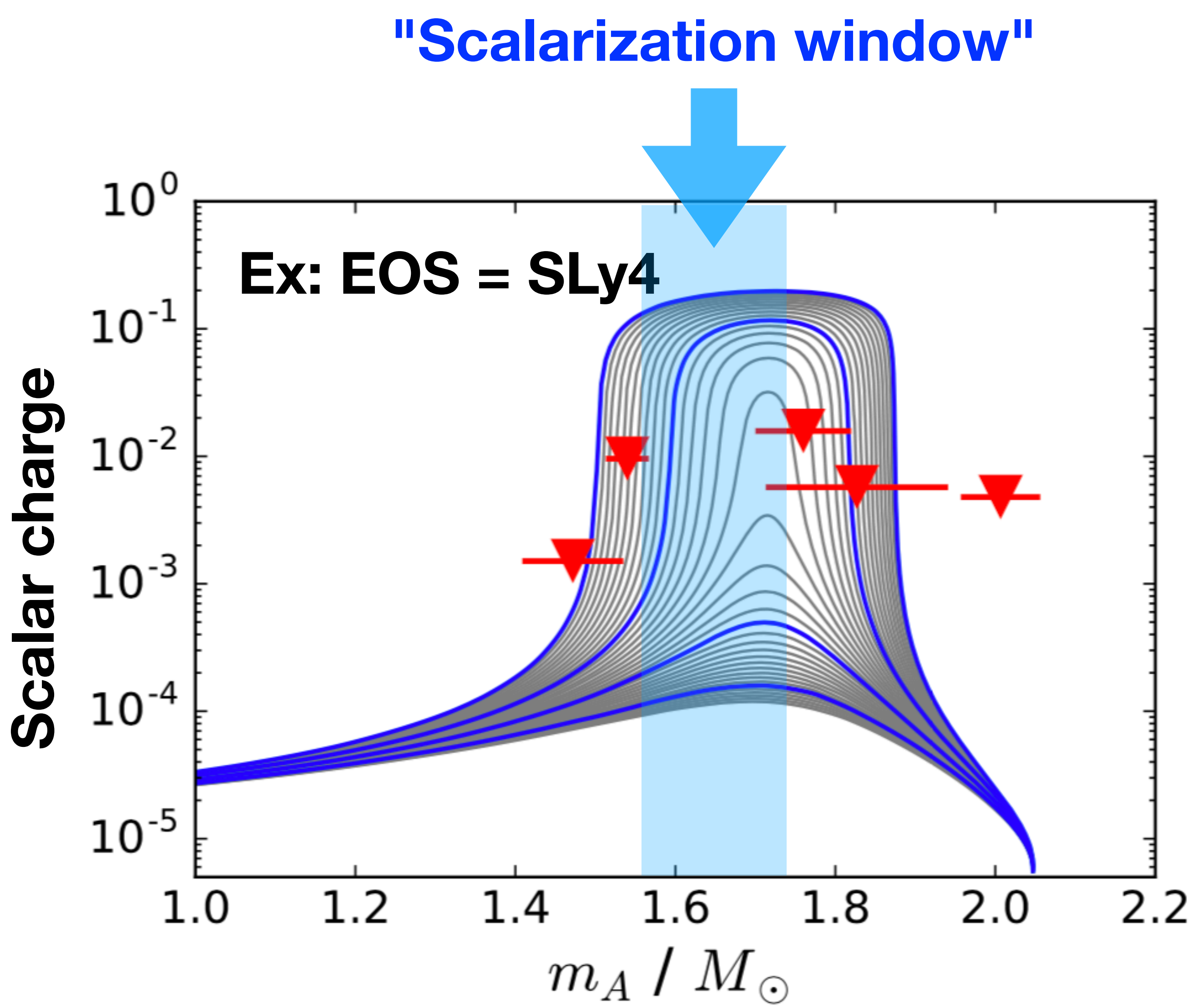
Antoniadis et al. 2013

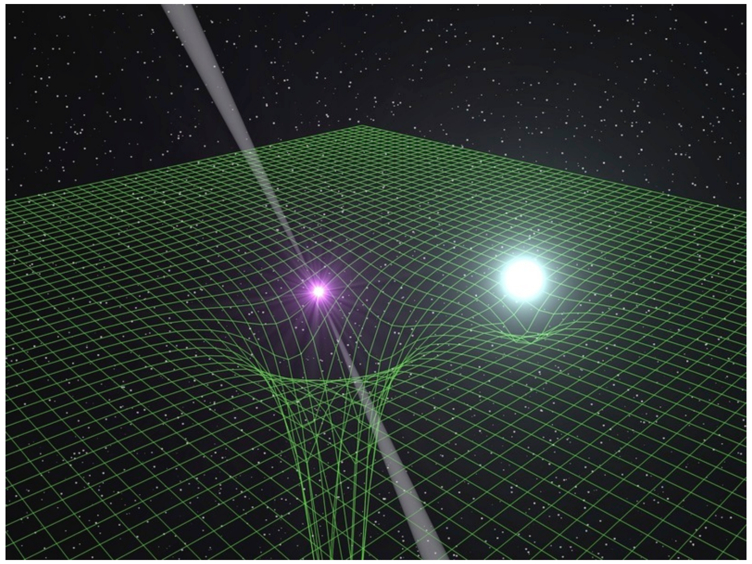


Freire et al. 2012

Due to their asymmetry, neutron-star white-dwarf systems provide stringent limits on *dipole radiation* $\dot{P}_b^{\rm dipole} \propto (\alpha_{\rm NS} - \alpha_0)^2$

Combination of five NS-WDs





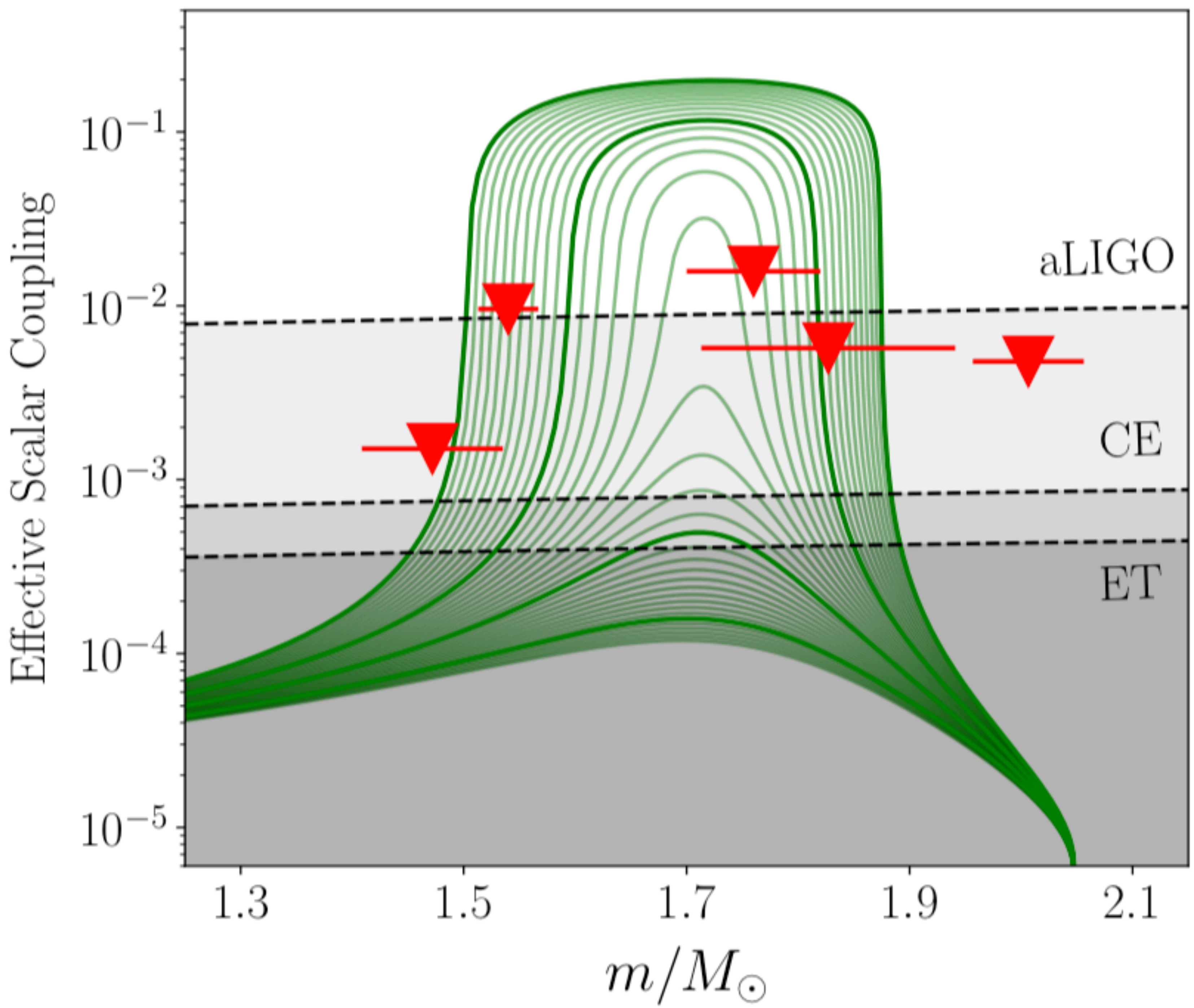
Strong-field effects could happen at different NS masses for different EOSs

Shibata et al. 2014

Combining five best-timed NS-WD binaries put the best limits on a class of scalar-tensor theories for different EOSs

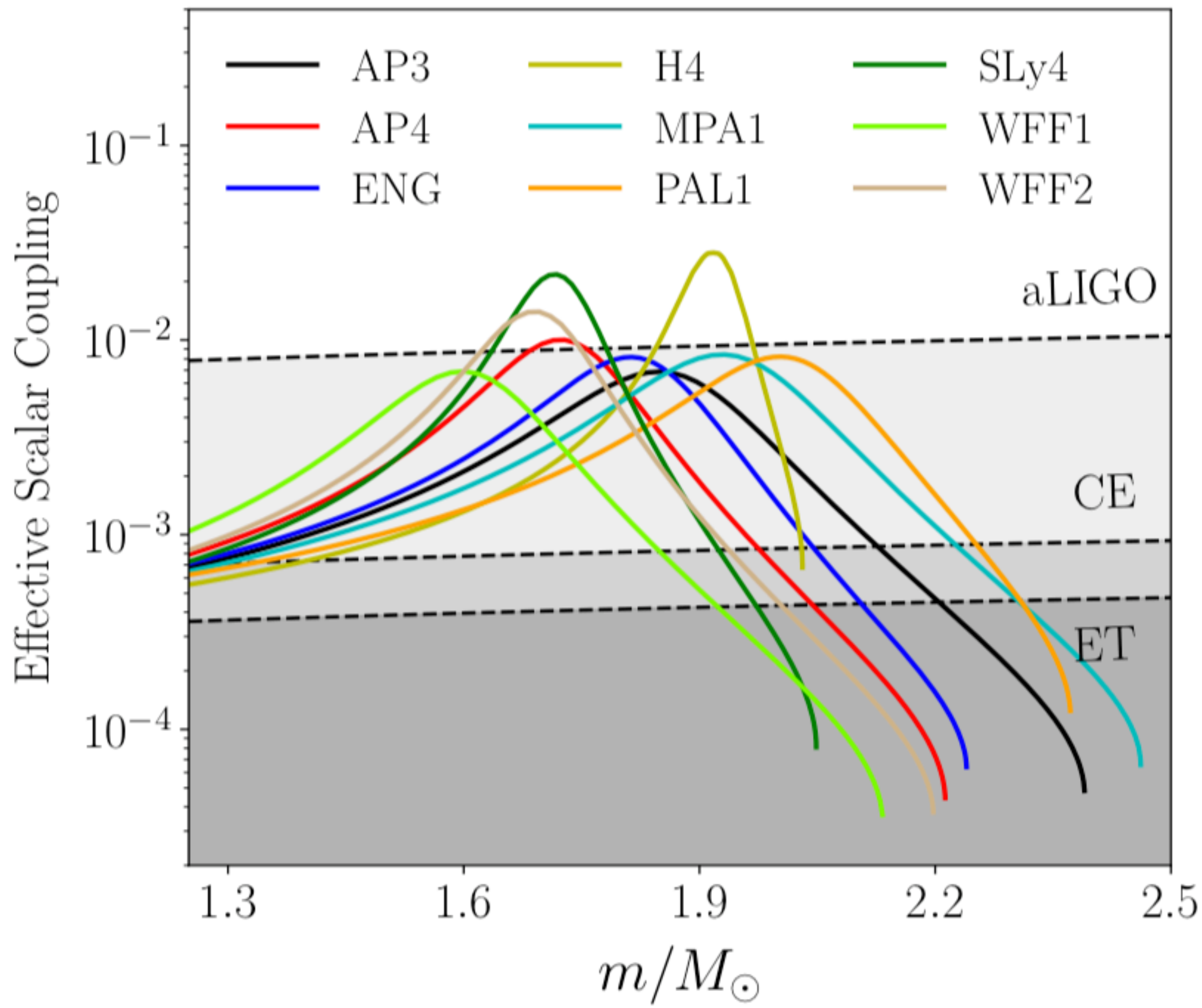
Shao et al. 2017

Gravitational waves (GWs)



Shao et al. 2017

Can NSs still be scalarized?

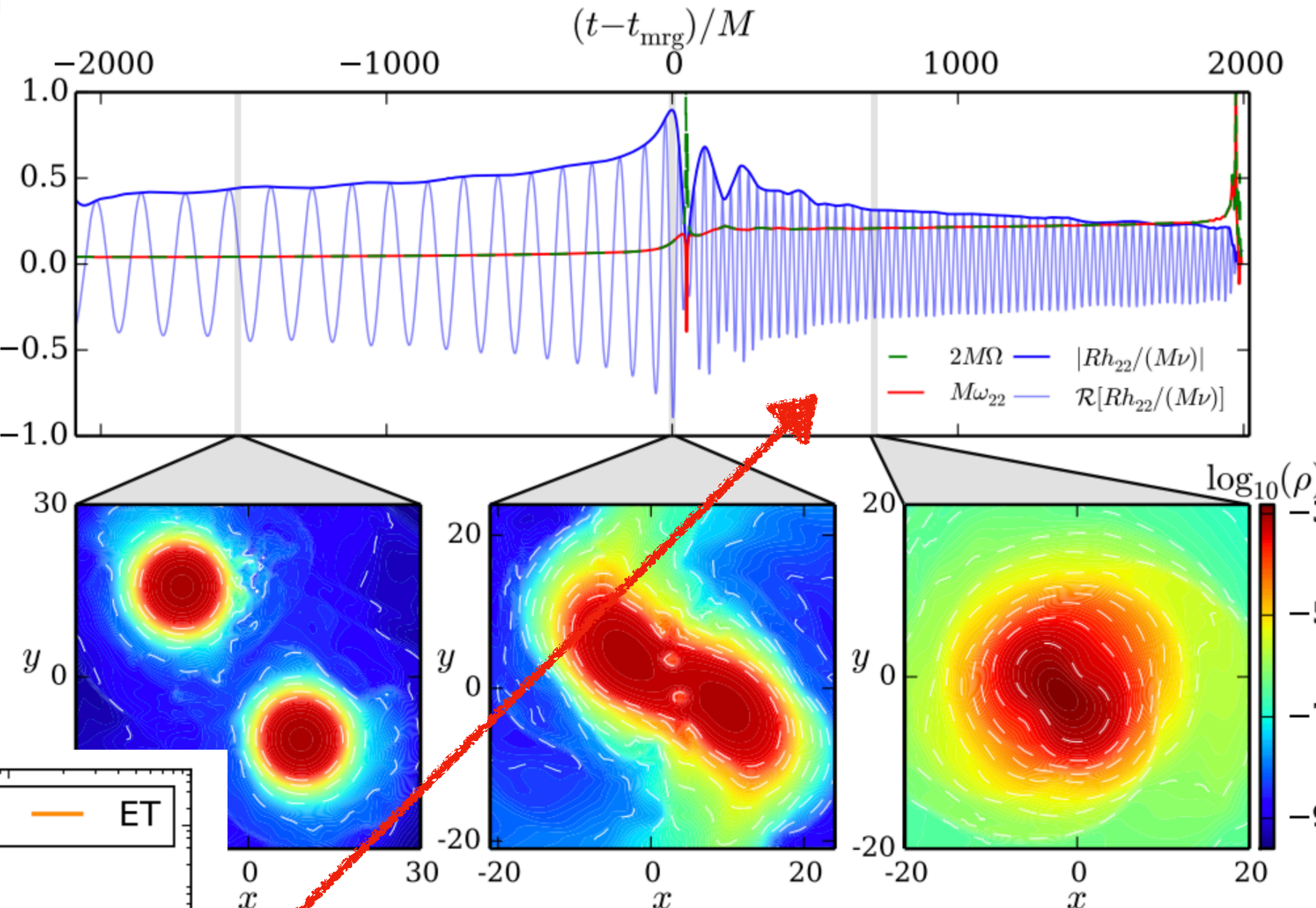


The maximum scalar charges of NSs that are still compatible with *all* binary pulsar observations

Shao et al. 2017

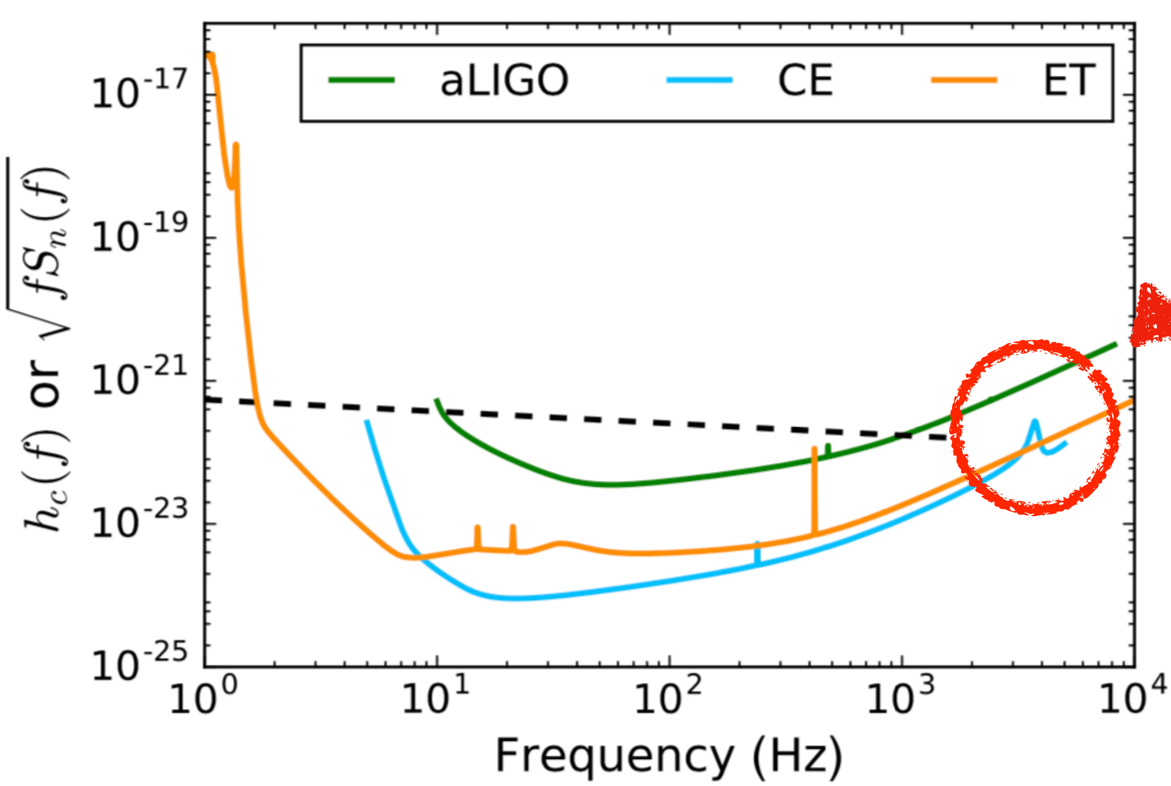
GWs: post-merger signal

Merger-ringdown signal encodes vital information for the end product of merger and the EOS of NSs



Bernuzzi et al. 2015

Shao et al. 2017, PRX 7:041025



Useful for probing dynamical scalarization!

aLIGO: Advanced LIGO
ET: Einstein Telescope
CE: Cosmic Explorer

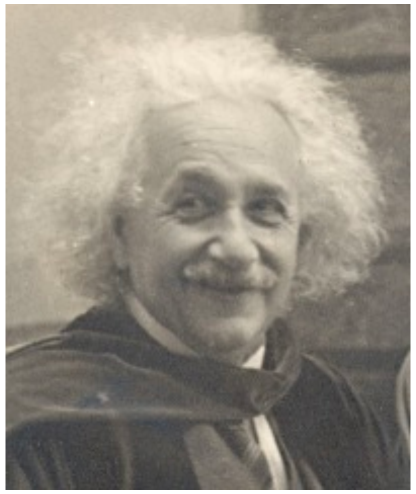
Remarks

- We should view the degeneracy as an opportunity, instead of a trouble

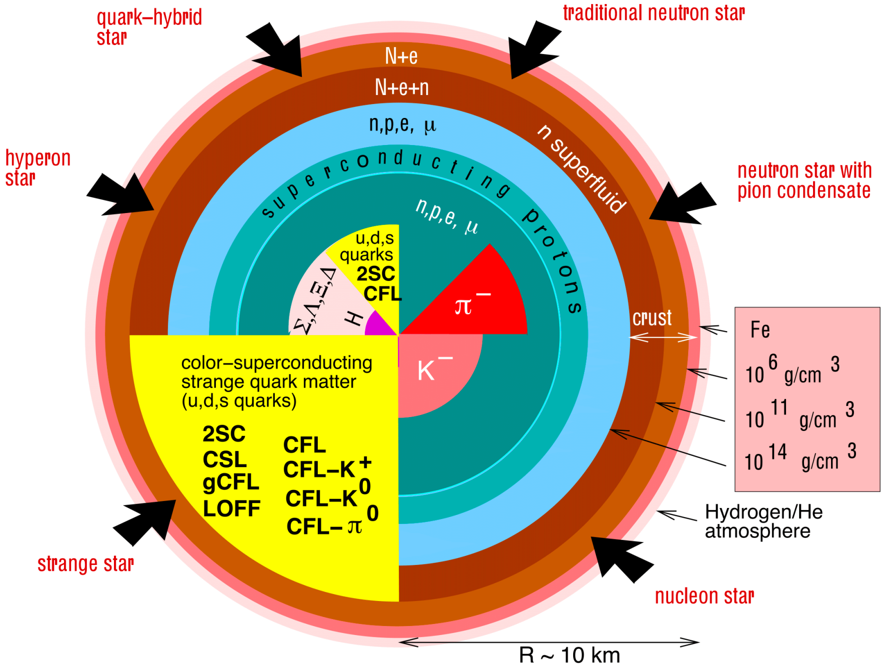
- Think about universal relations: I-Love-Q, et cetera

Yagi & Yunes 2013
Doneva & Pappas 2017

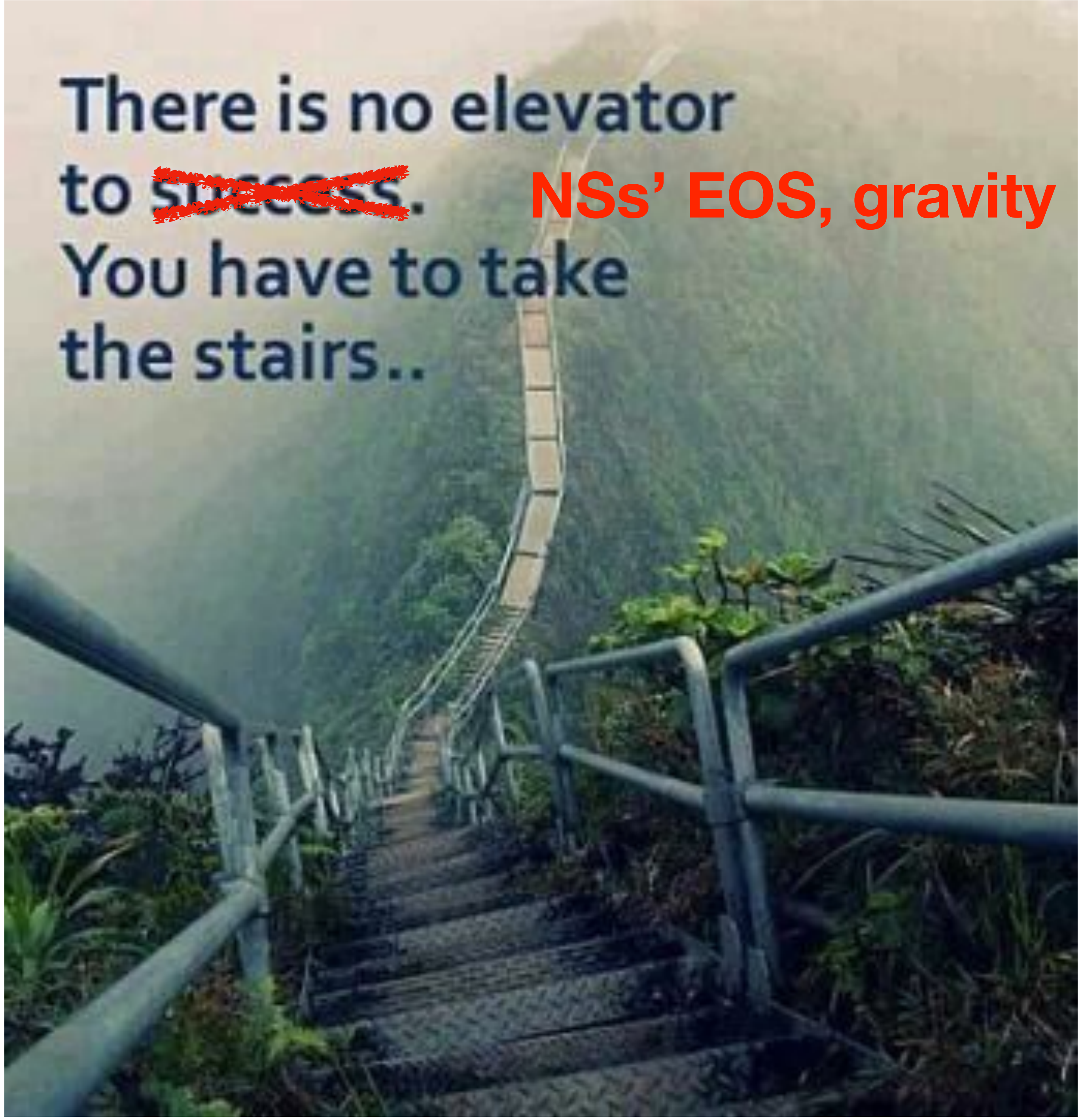
- It provides us a wider window to look at EOSs, to think about strong-field gravity, as well as to probe unknowns with new precision experiments



***entangles* with**



NSs' EOS is not given, but earned



At least, there is a way!

Probably many ways, as we are learning from the new GW window!


Thank you!