Probing Neutron Star Equation of State with Deep Learning

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in collaboration with: Kenji Fukushima, Koichi Murase

YF, K. Fukushima, K. Murase, Phys. Rev. D **98**, 023019 (2018)
YF, K. Fukushima, K. Murase, arXiv:1903.03400 [nucl-th].

6 May 2019, The 8th Huada school

Neutron Star Equation of State

- The **Equation of State (EoS)** of dense matter: relation between pressure p and (mass) density ρ

$$p = p(\rho)$$

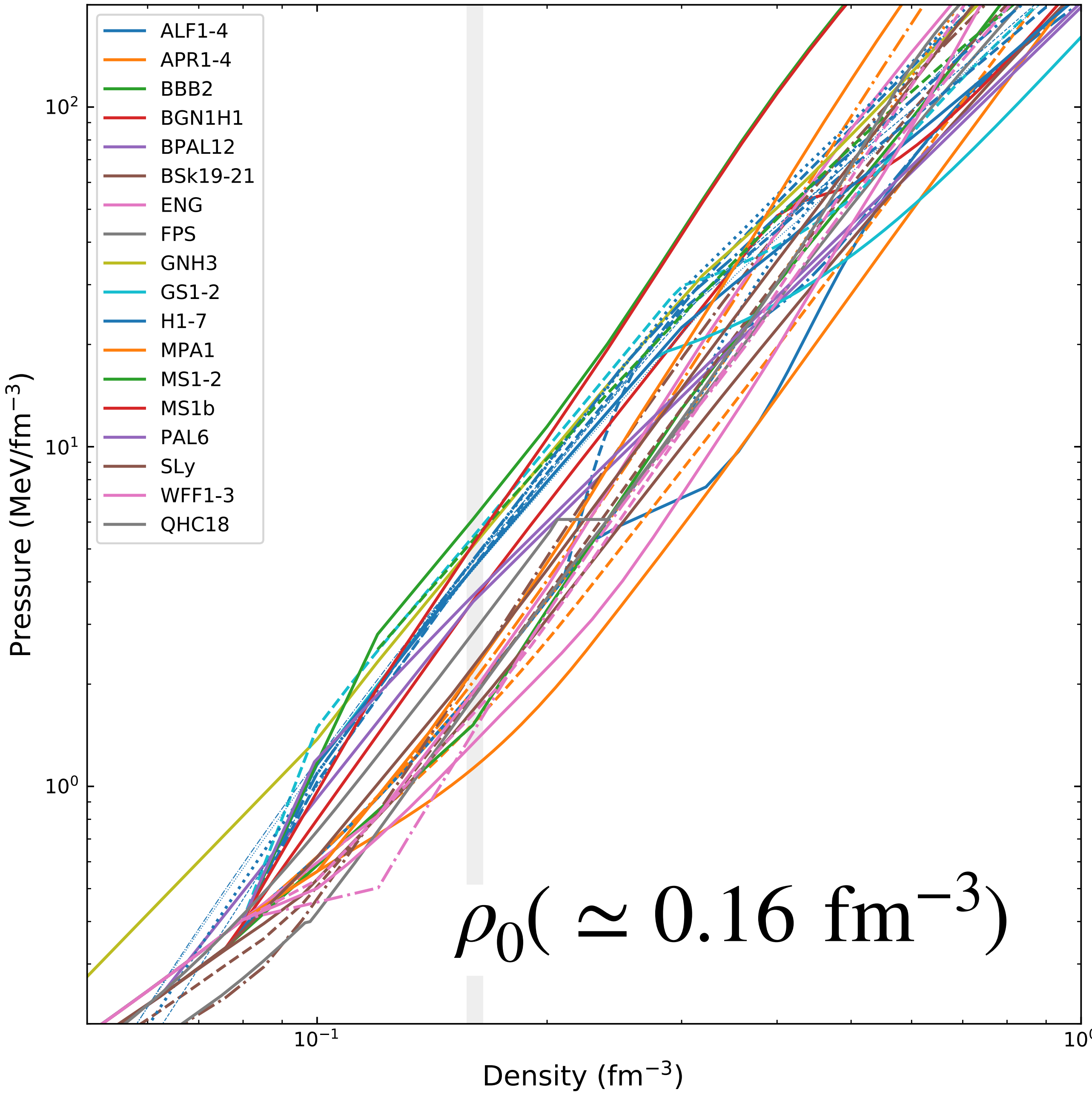
- Essential ingredients for neutron star theory; characterizes neutron star structure

- Should be derived from QCD in principle, but many difficulties such as…
    - sign problem; no lattice data
    - renormalization scale dependence in pQCD

Hester et al. (2002)

Current Status of the EoS

- Many nuclear theory calculations
  …but reliability of these models decline with growing ρ



Current Status of the EoS

- Many nuclear theory calculations
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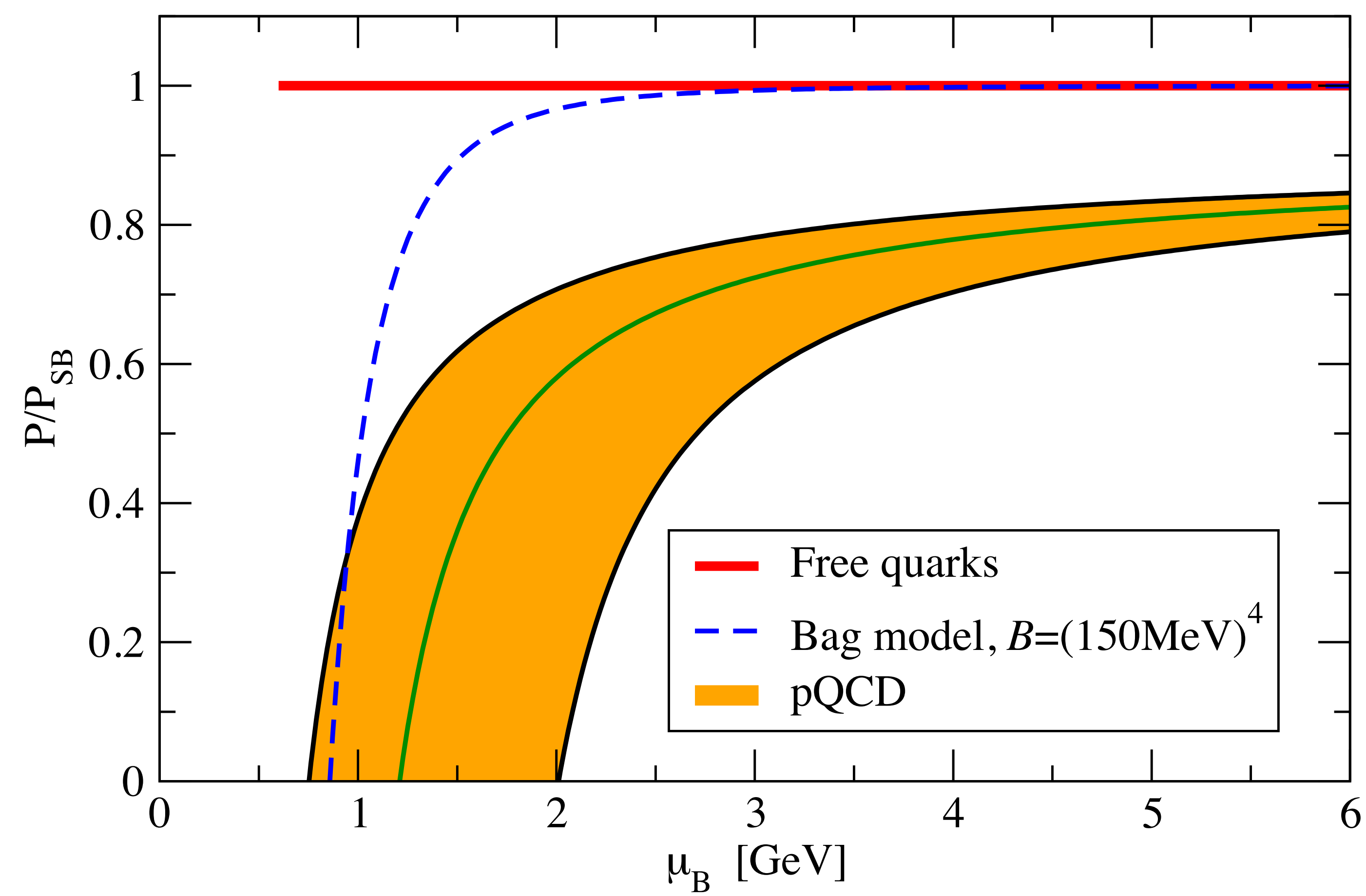
- Perturbative QCD calculation also has large uncertainty



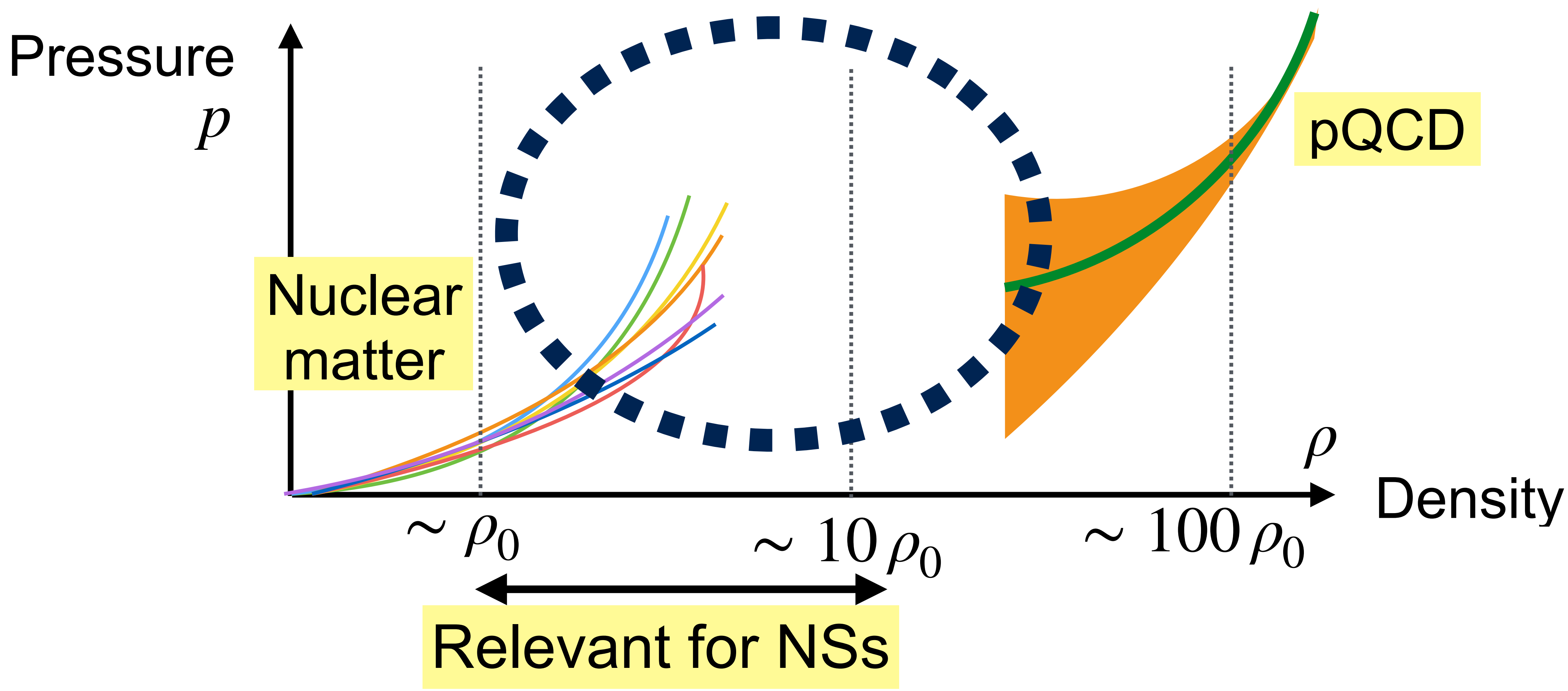
Freedman-McLerran (1977); Baluni (1978)
Fraga-Kurkela-Vuorinen (2014)

Current Status of the EoS

- Many nuclear theory calculations
  …but reliability of these models decline with growing ρ

- Perturbative QCD calculation also has large uncertainty

- Need systematic way to interpolate these two regions

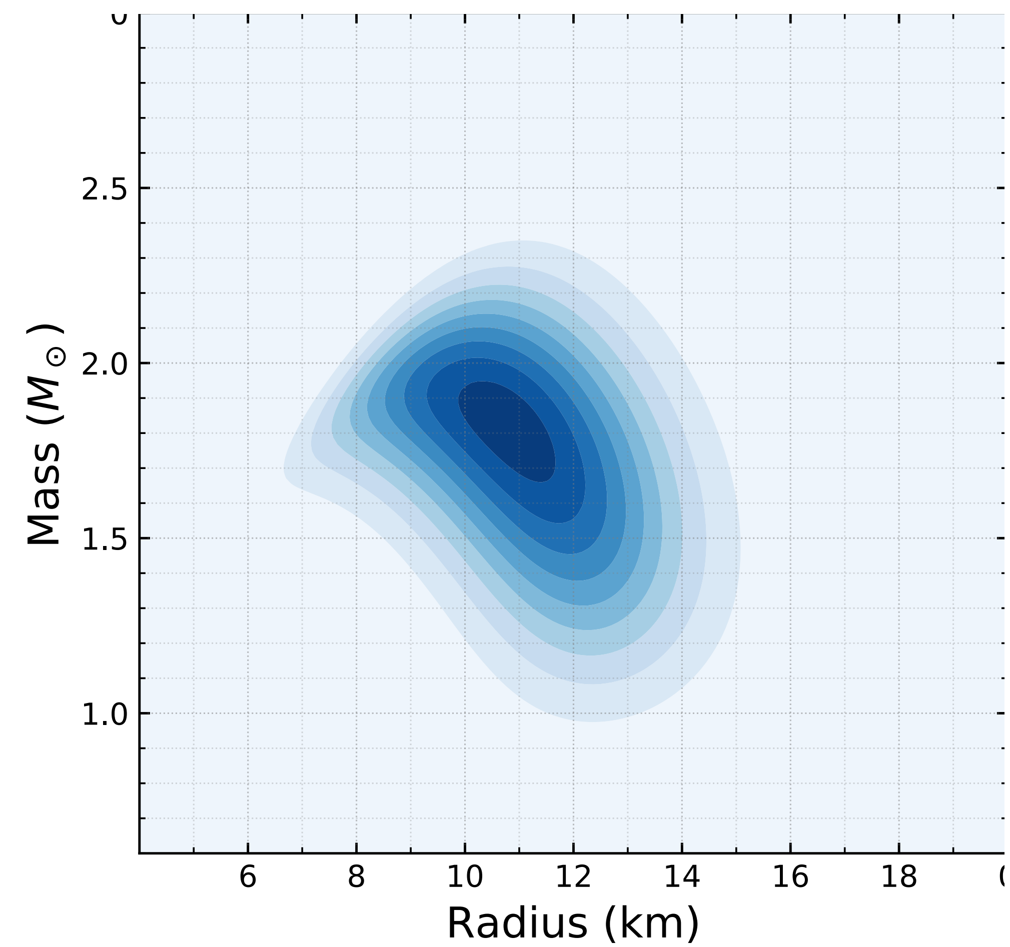


Systematic Way of EoS Inference

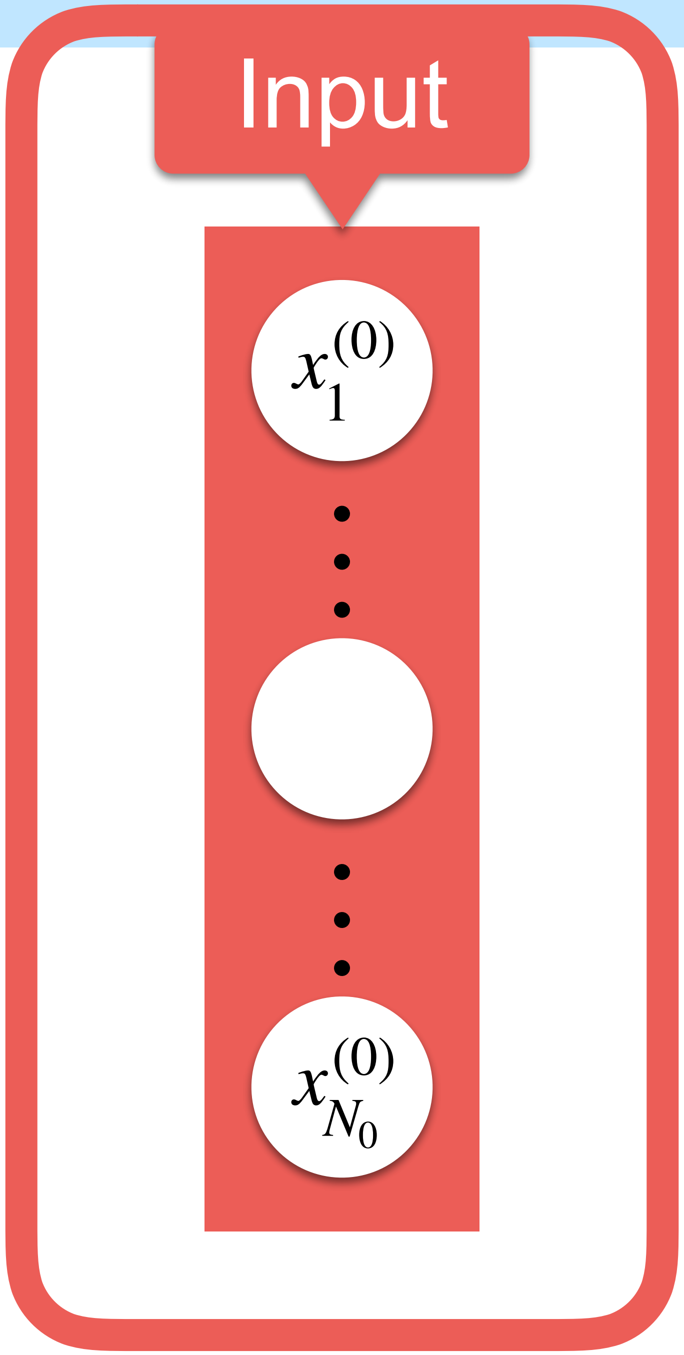
- Growing number of observables of NSs:
  e.g.) LIGO-Virgo, NICER experiment

- Utilize NS **masses** (***M***) and **radii** (***R***)
  To date 14 simultaneous measurement is provided;
  given in terms of likelihood distribution
  in (*R*, *M*) plane

- Infer EoS from *M*-*R*:
  Use **machine learning** (**ML**)
  with deep **neural network** (**NN**)



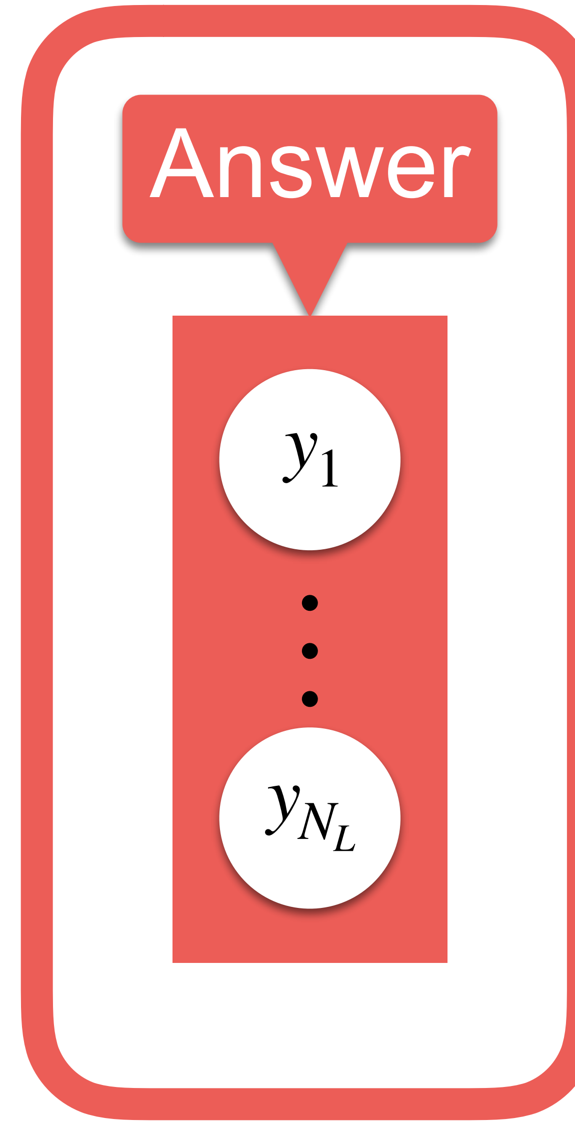
Method of Machine Learning



Training data

↓

we want to extract the relation between input and answer



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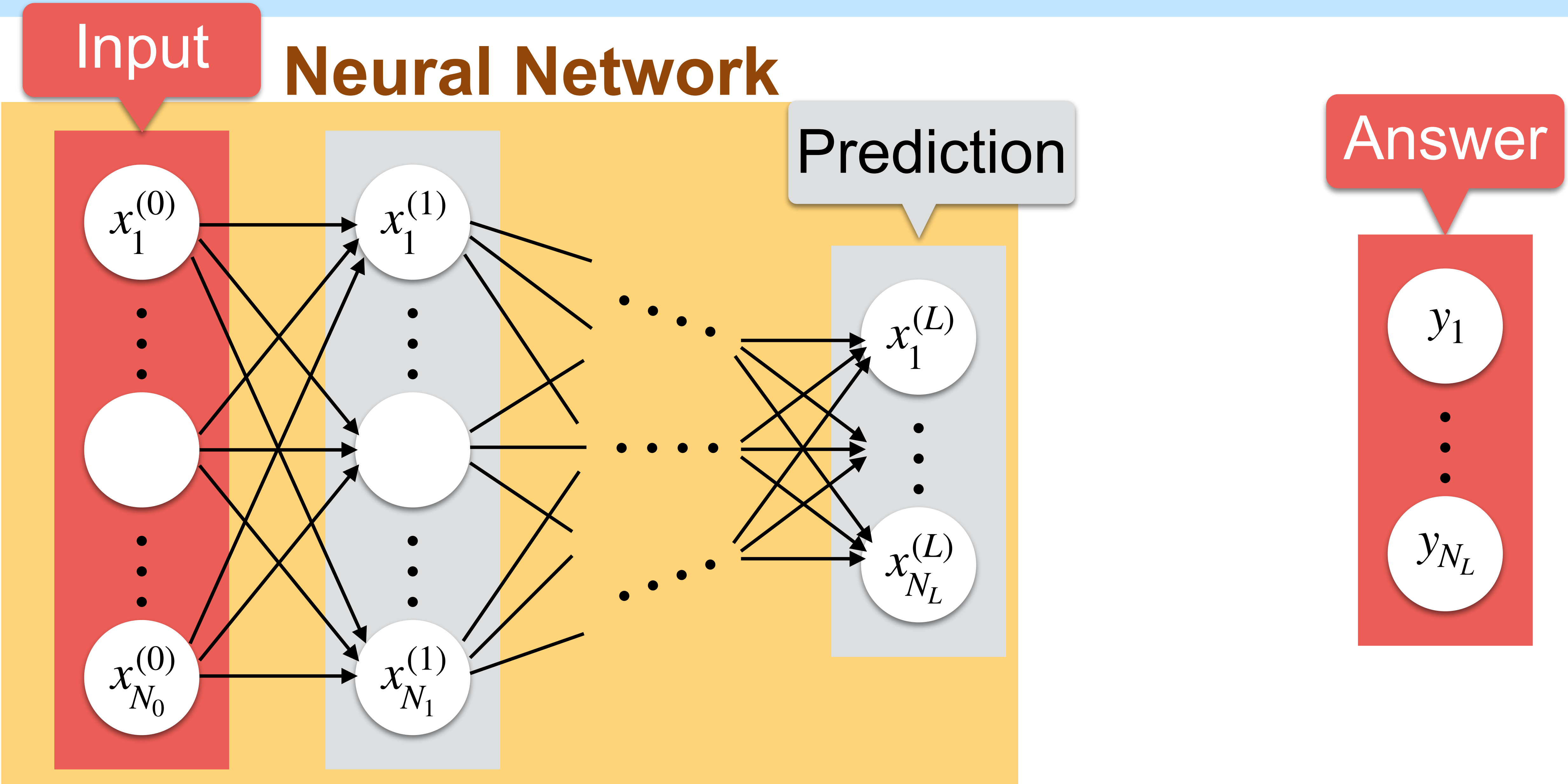
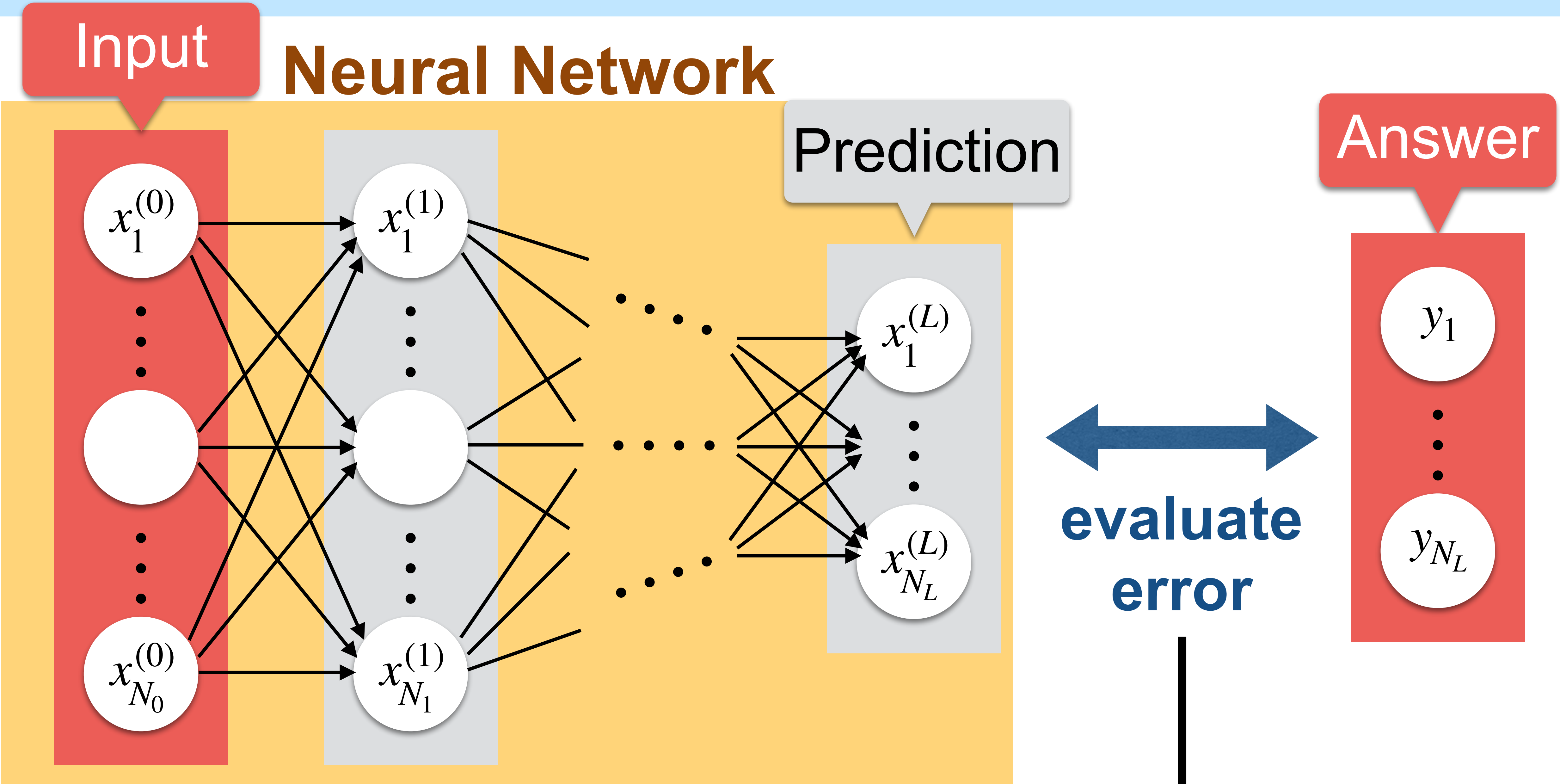


$$x_i^{(k+1)} = a^{(k+1)}\left(\sum_{i=1}^{N_k} W_{ij}^{(k+1)} x_j^{(k)} + b_i^{(k+1)}\right)$$

Method of Machine Learning



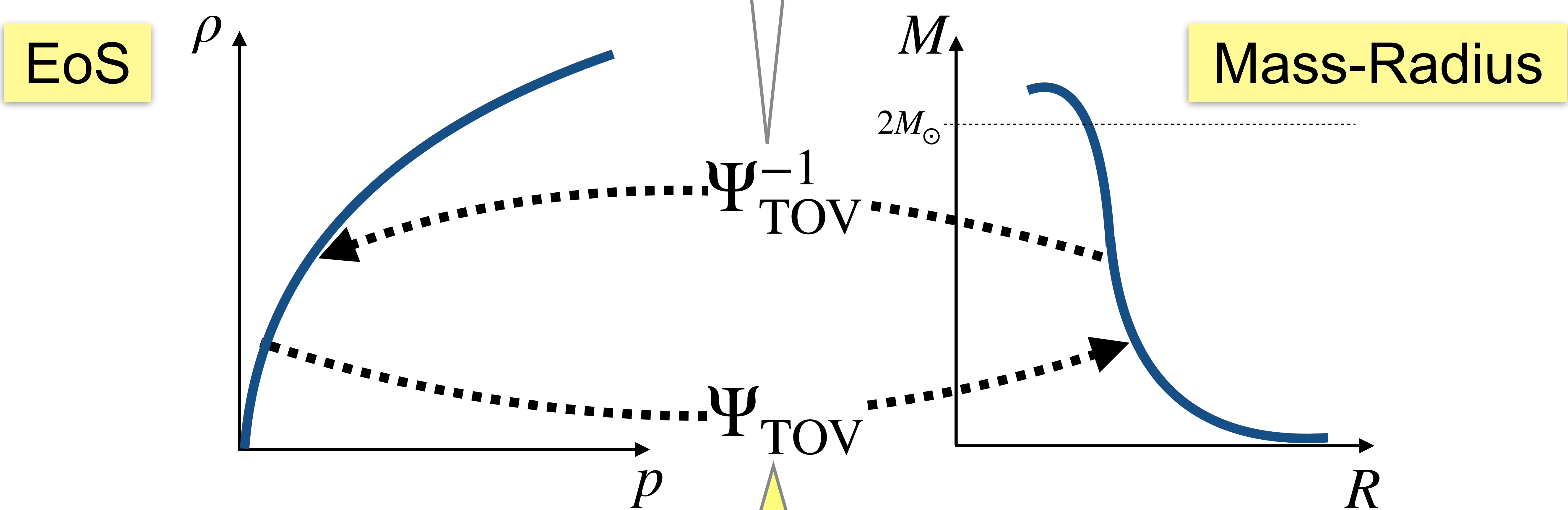
$$x_i^{(k+1)} = a^{(k+1)}\left(\sum_{i=1}^{N_k} \underline{W_{ij}^{(k+1)}} x_j^{(k)} + \underline{b_i^{(k+1)}}\right)$$

Tune these parameters inside NN to minimize the error

EoS & *M-R*: TOV Mapping Ψ_{TOV}

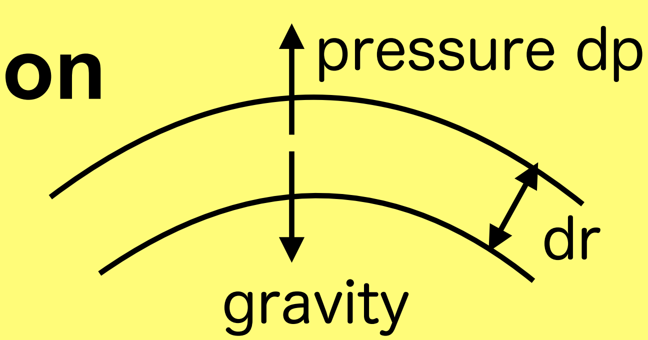
If *M-R* curve is given,
there is **one-to-one correspondence**

Lindblom (1992)



Tolman-Oppenheimer-Volkoff (TOV) equation

General relativistic structural equation
for stars at hydrostatic equilibrium



EoS & *M*-*R*: TOV Mapping $\Psi_{\rm TOV}$

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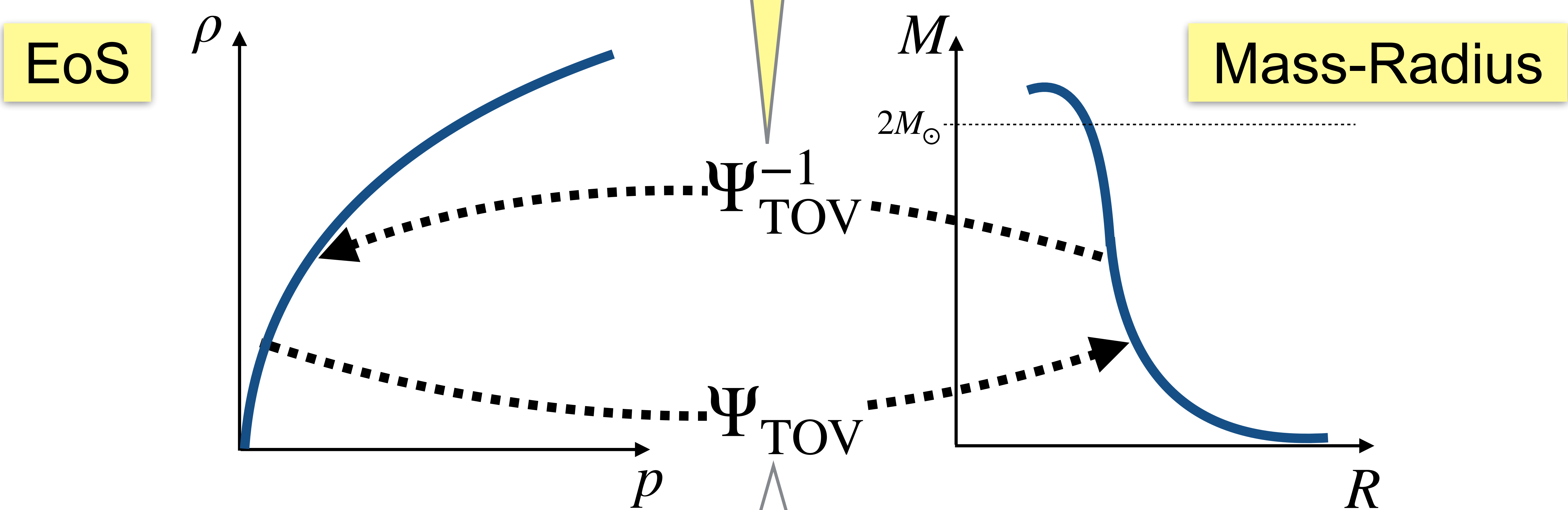

Tolman-Oppenheimer-Volkoff (TOV) equation

General relativistic equation for
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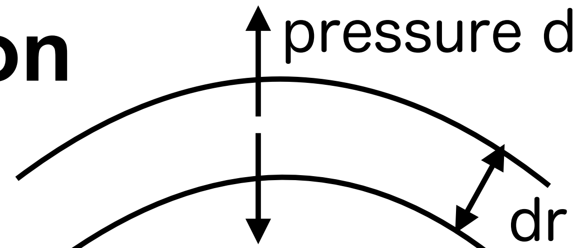
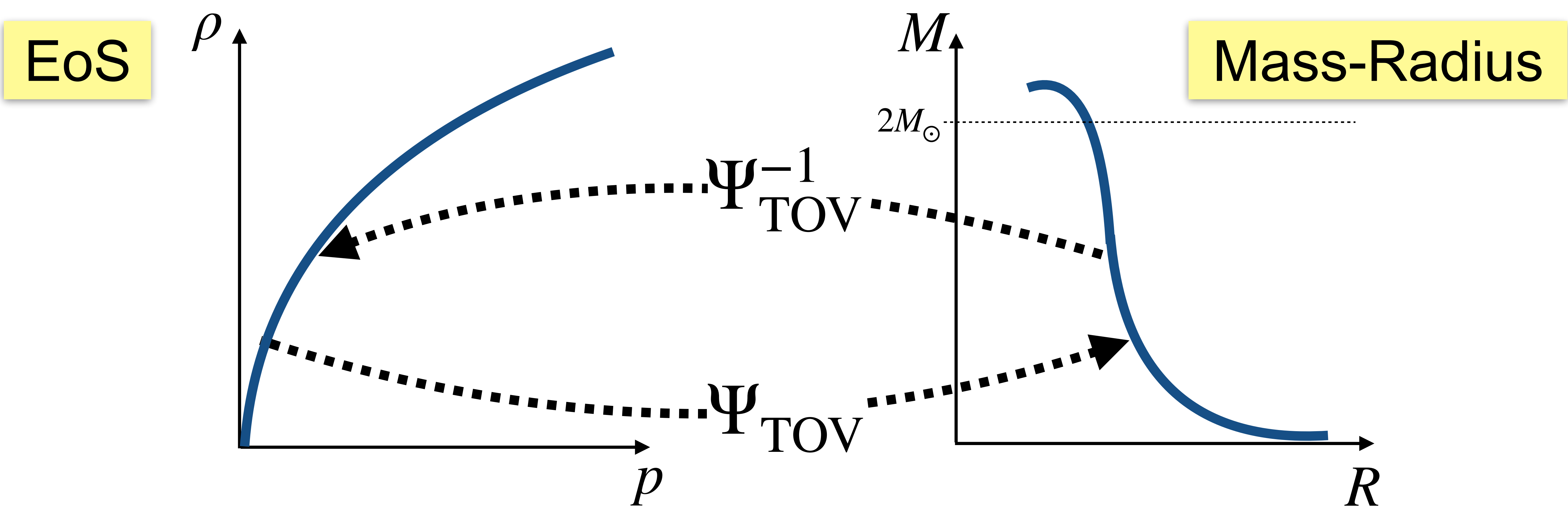


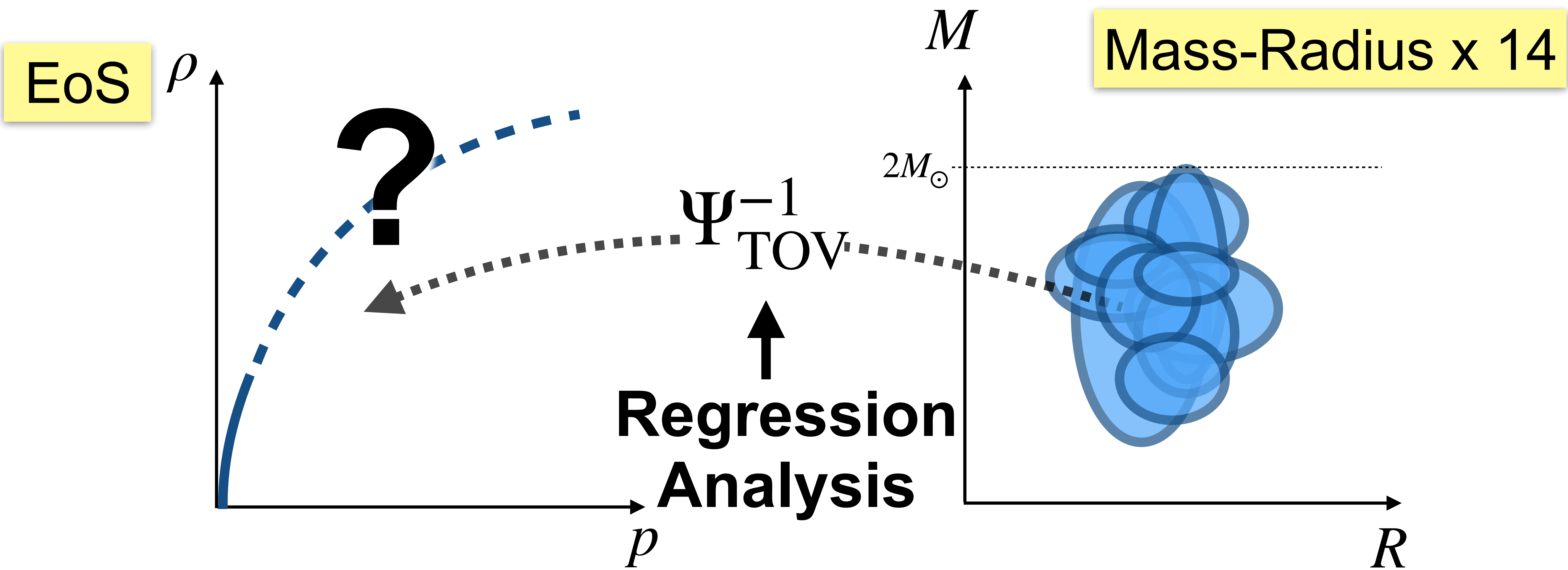
EoS & *M*-*R*: TOV Mapping Ψ_{TOV}

The operation of the TOV equation can be regarded as one-to-one mapping: **TOV mapping** Ψ_{TOV}



Regression Analysis of TOV Mapping

In reality… *M-R* **is point-like** and **has finite extent**
$\rightarrow$ finding Ψ^{-1}_{TOV} becomes non-trivial!

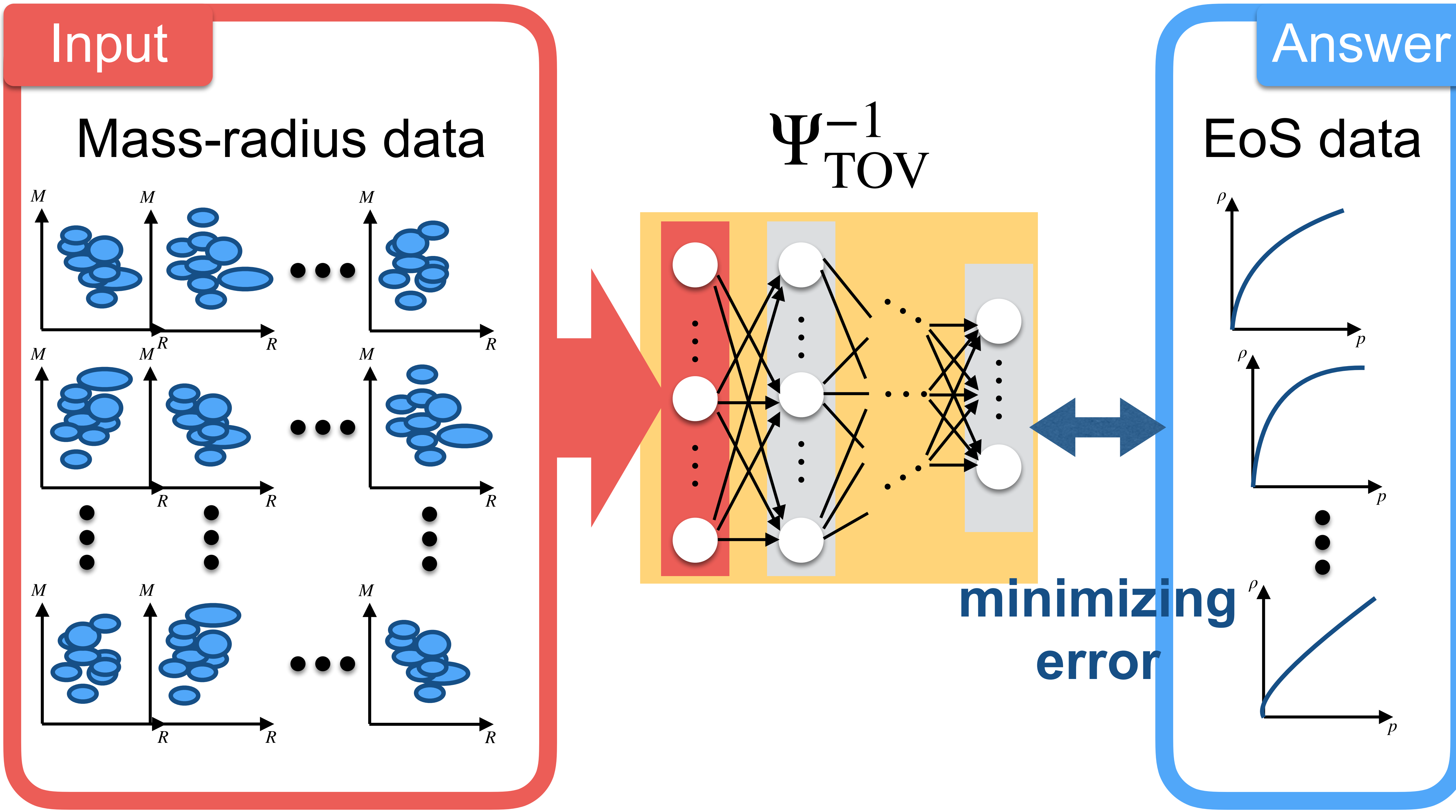


Express Ψ^{-1}_{TOV} in terms of NN
$\rightarrow$ problem becomes regression analysis finding

$$\text{EoS} = \Psi^{-1}_{\mathrm{TOV}}(M\text{-}R)$$

Schematic of TOV Regression

Finding the function EoS $= \Psi^{-1}_{\mathrm{TOV}}(M\text{-}R)$ using deep learning

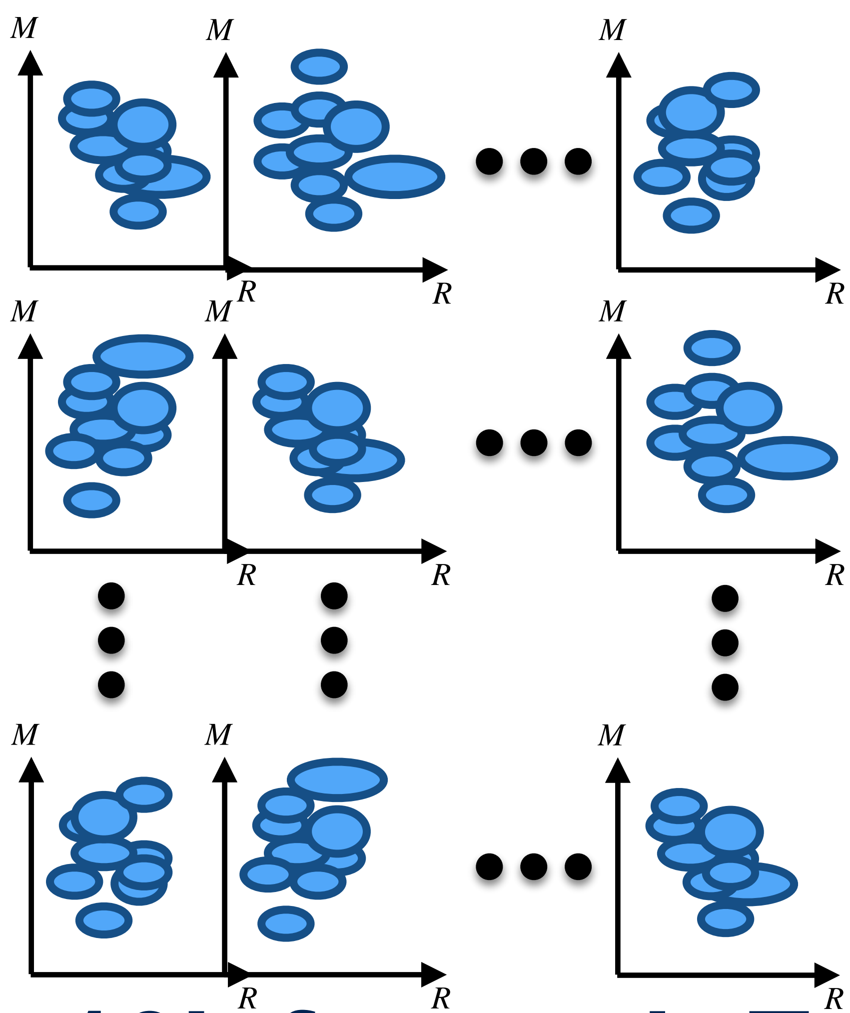


Training Data Generation

For regression, good training data sets are needed…

Input

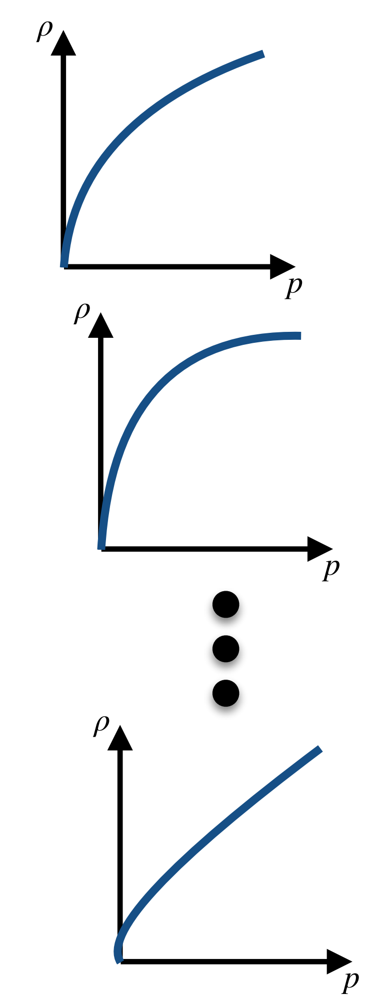
Mass-radius data



x 10k for each EoS

Answer

EoS data



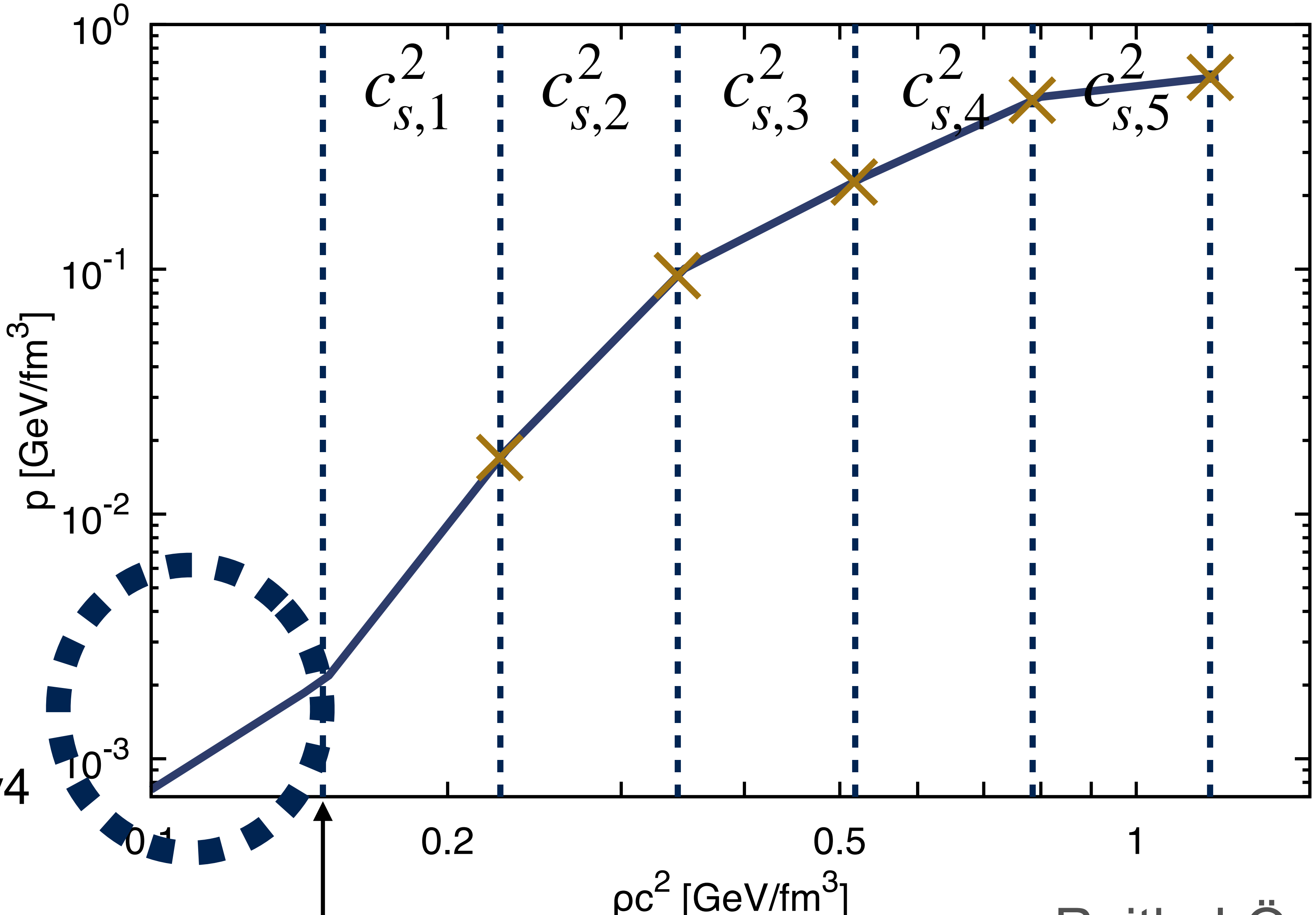
x 500

EoS parametrization

Here, we use **piecewise polytrope** with 5 parameters c_s^2

$$c_s^2 = dp/d\rho$$

EoS is interpolated by polytrope $p \propto \rho^\Gamma$



We assume SLy4 up to ρ0

saturation density ρ_0

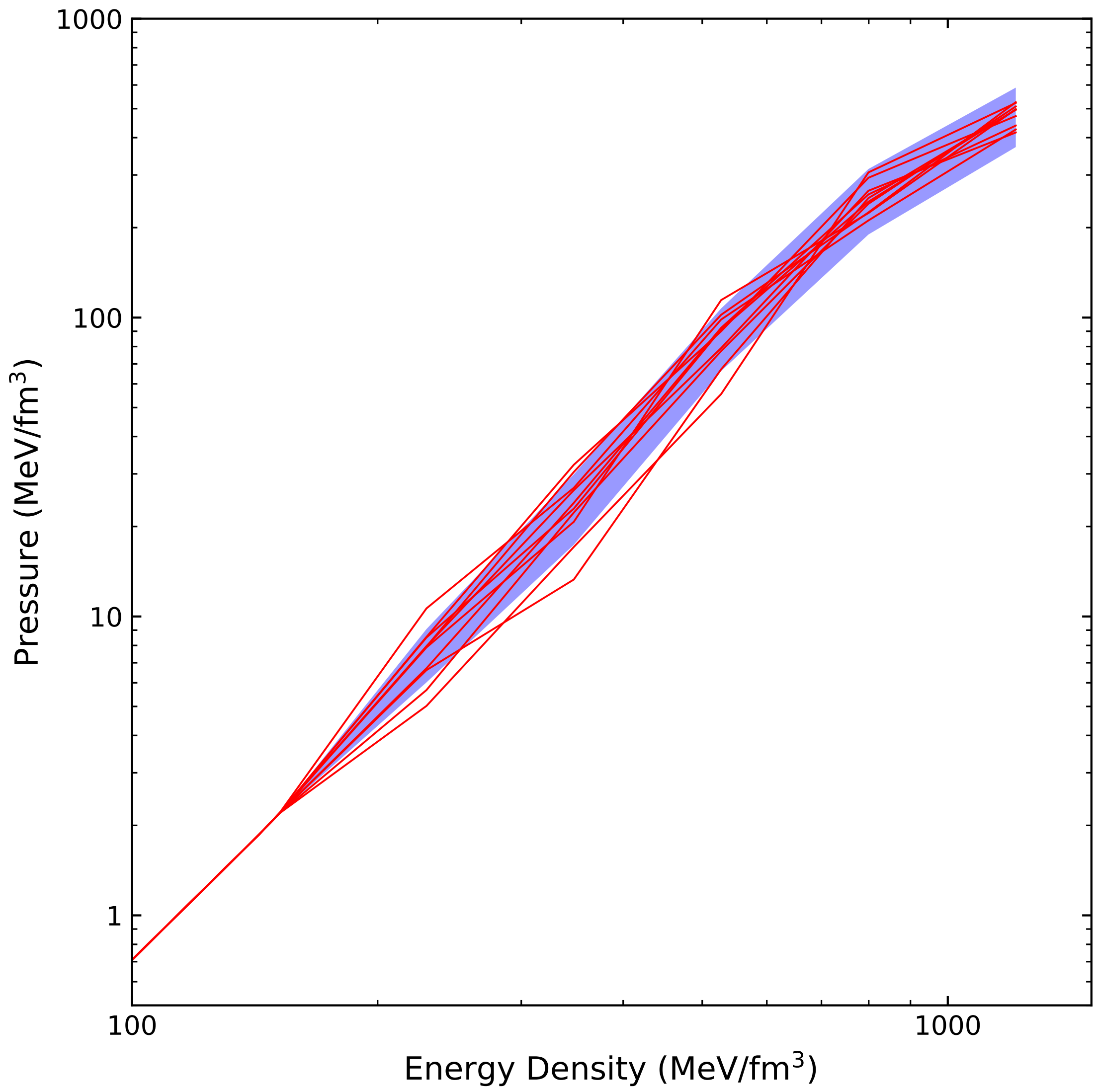
Raithel-Özel-Psaltis (2017)

Uncertainty Estimation

- Partial contribution to the uncertainty is estimated in the following way (bootstrapping):

- Set up 10 copies of NN independently but trained with the same procedure

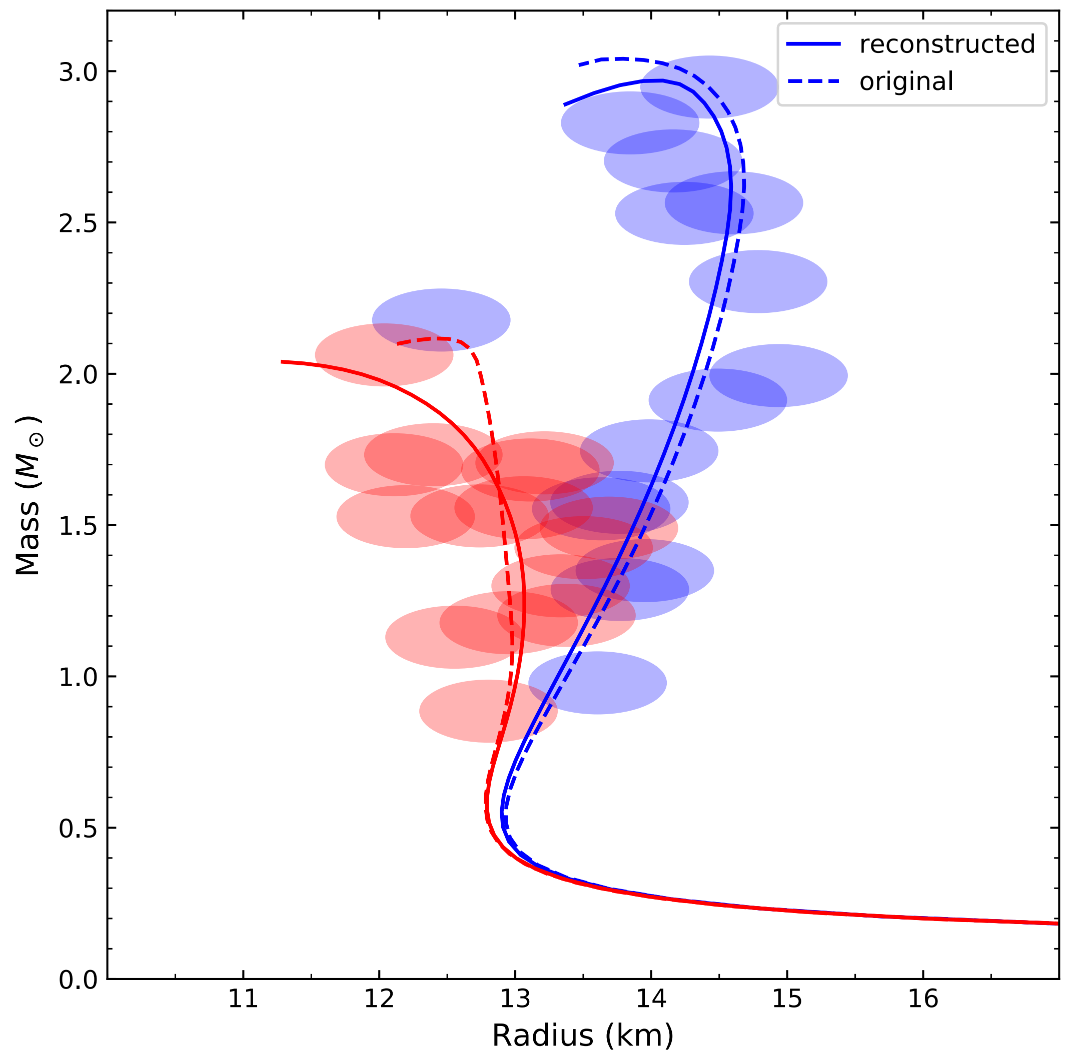
- We take the average and standard deviation of predicted result; identify them as most probable value and the prediction uncertainty

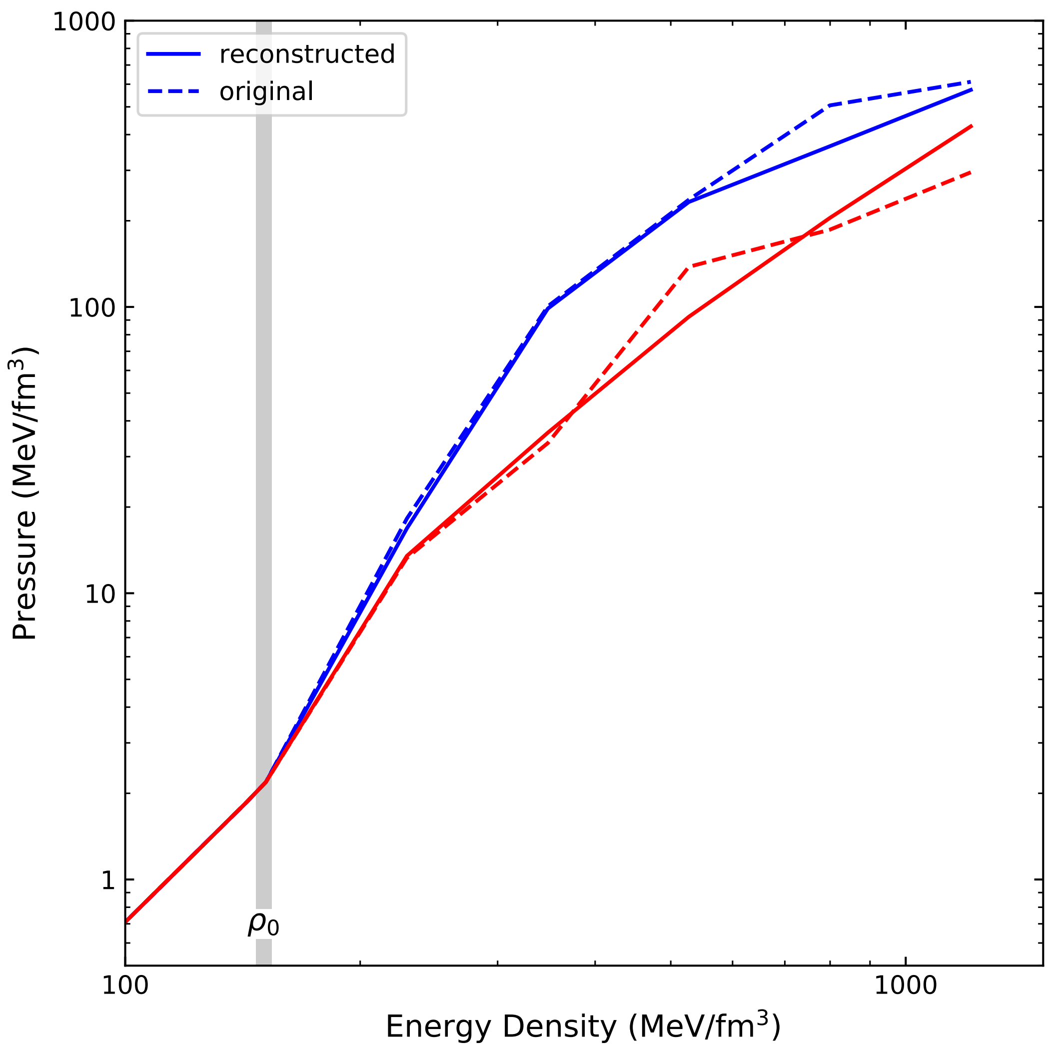


Performance Test with Mock Data

YF-Fukushima-Murase (2018)

Typical two data:





: Original EoSs and M-R

: Reconstructed EoSs and M-R

Performance Test with Mock Data

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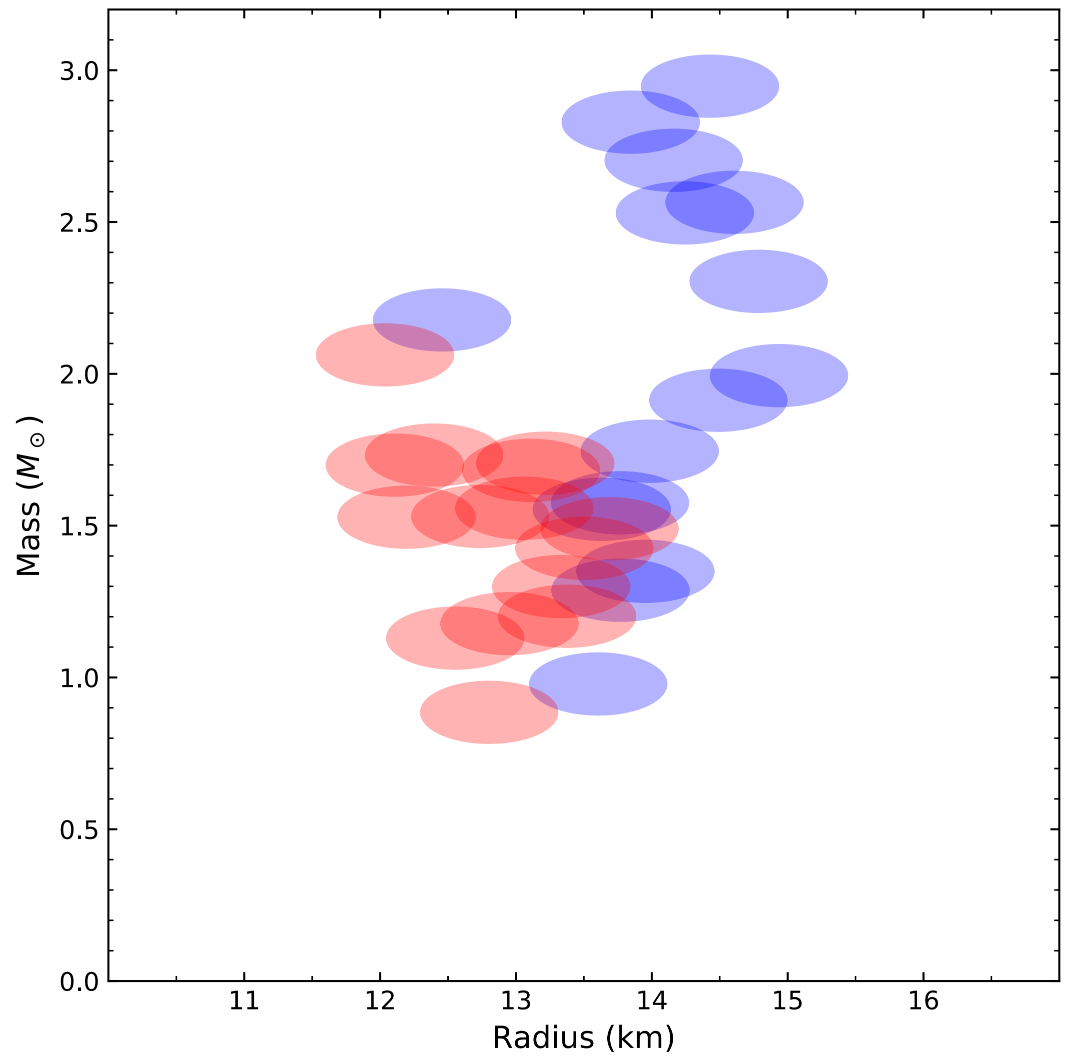
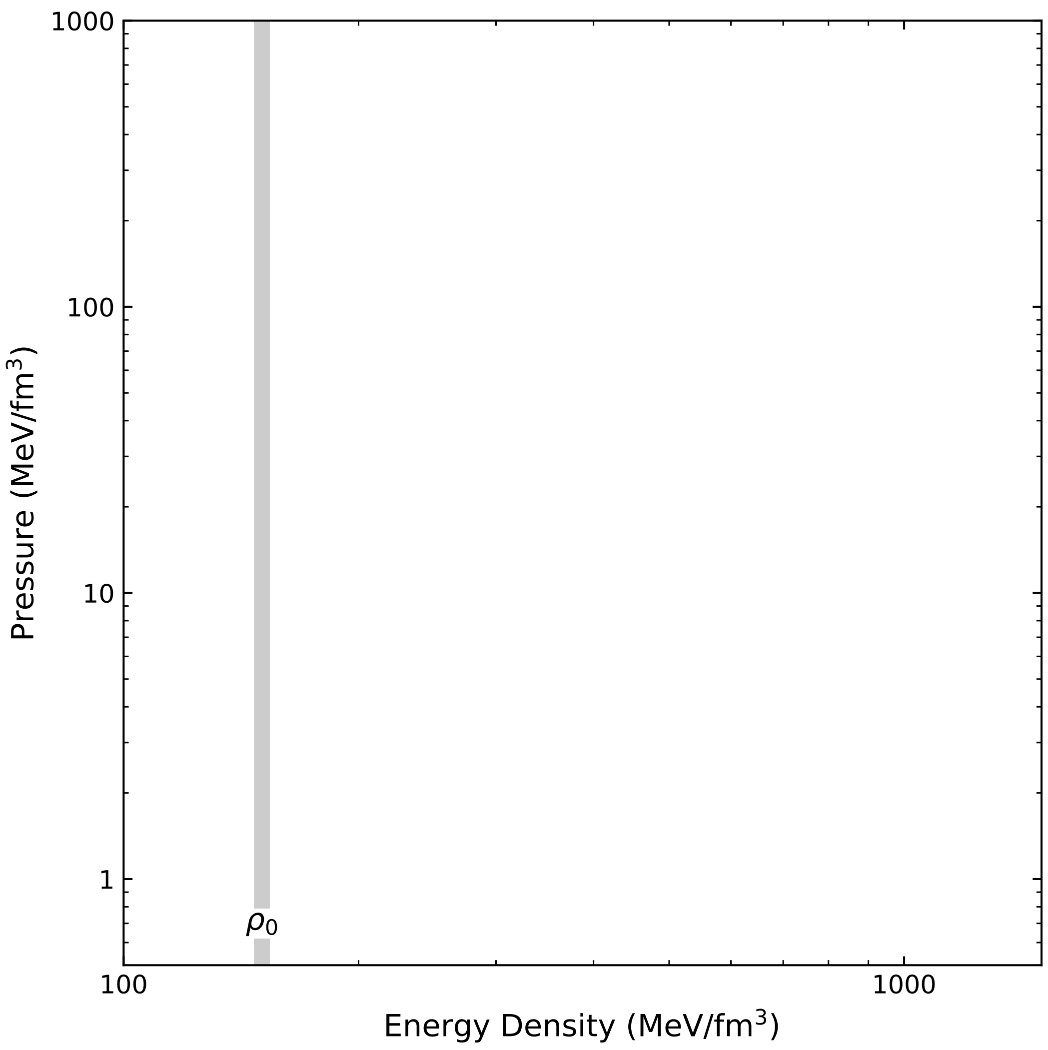
YF-Fukushima-Murase (2018)





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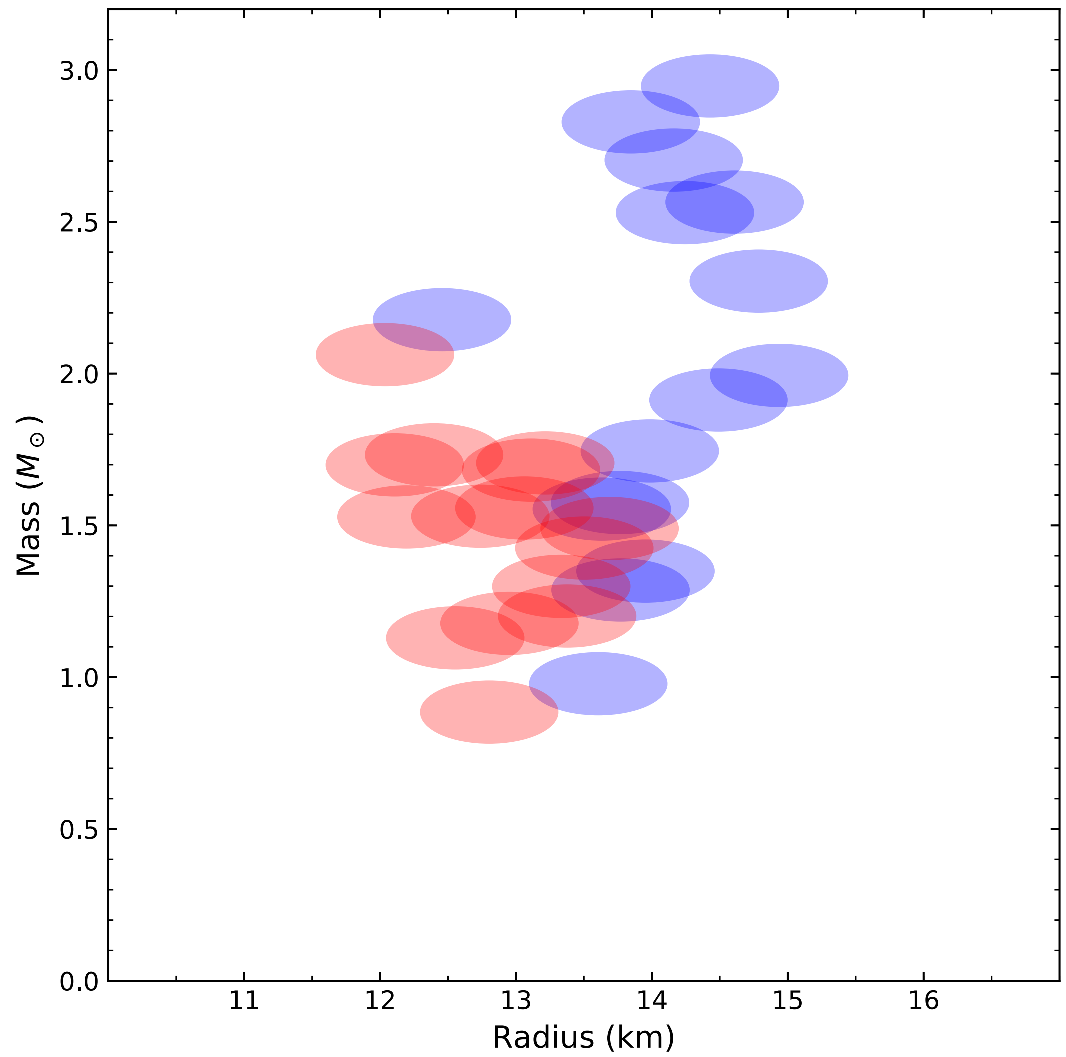

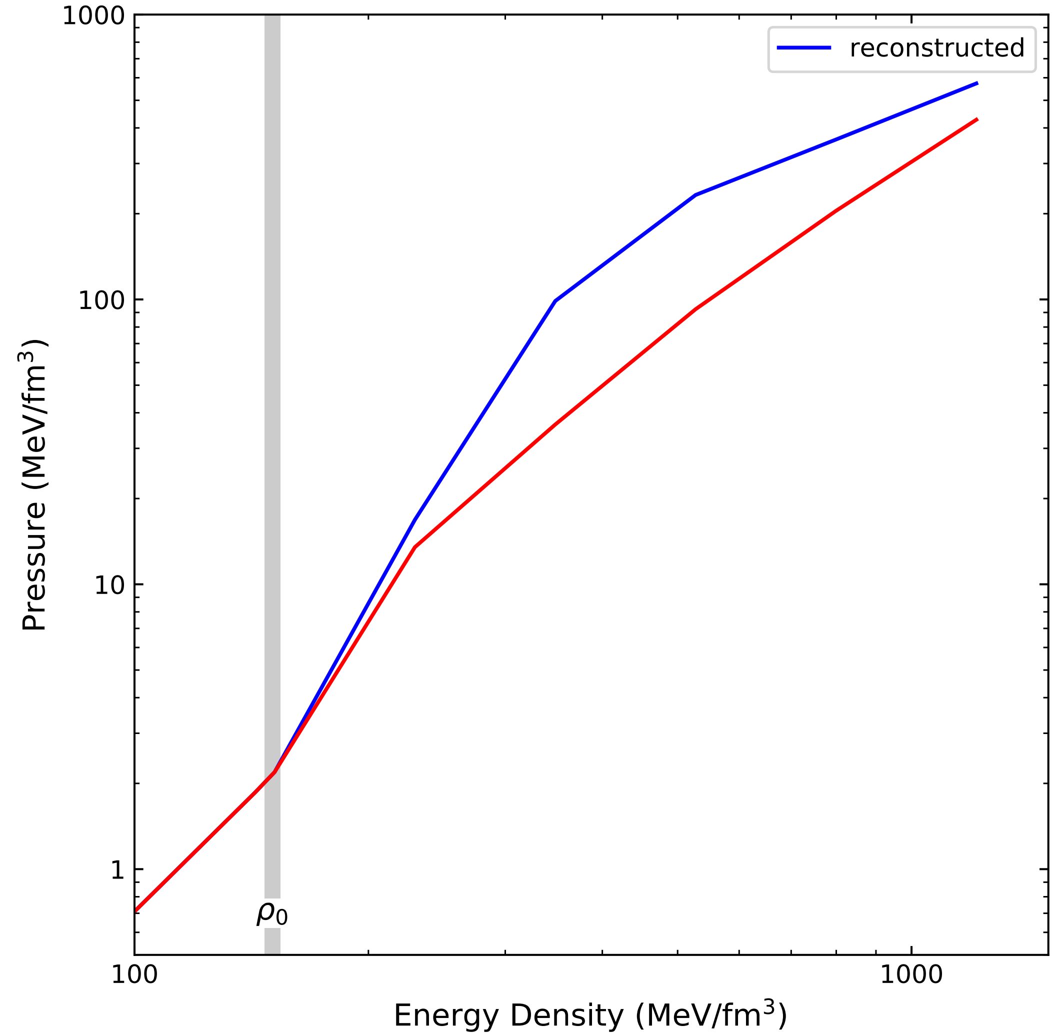

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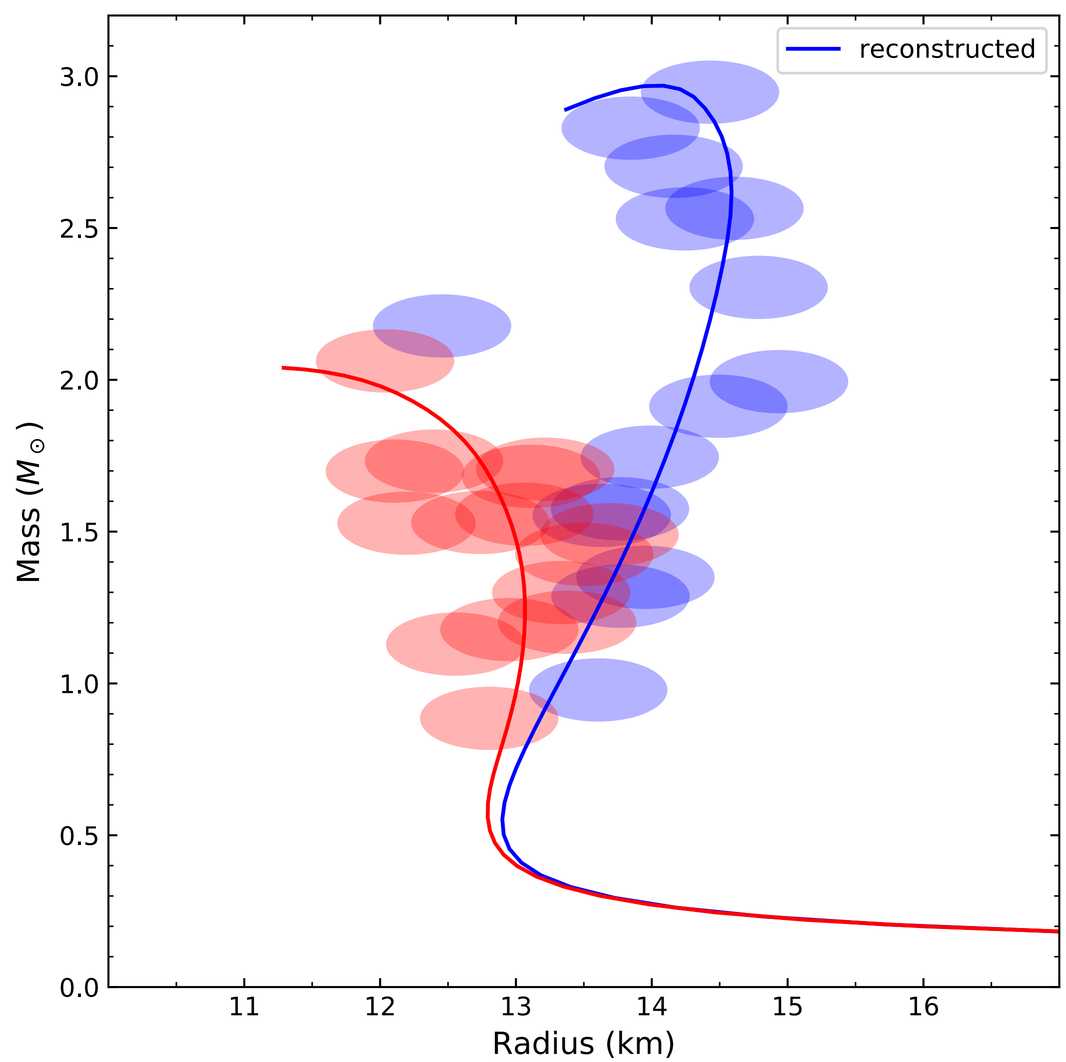

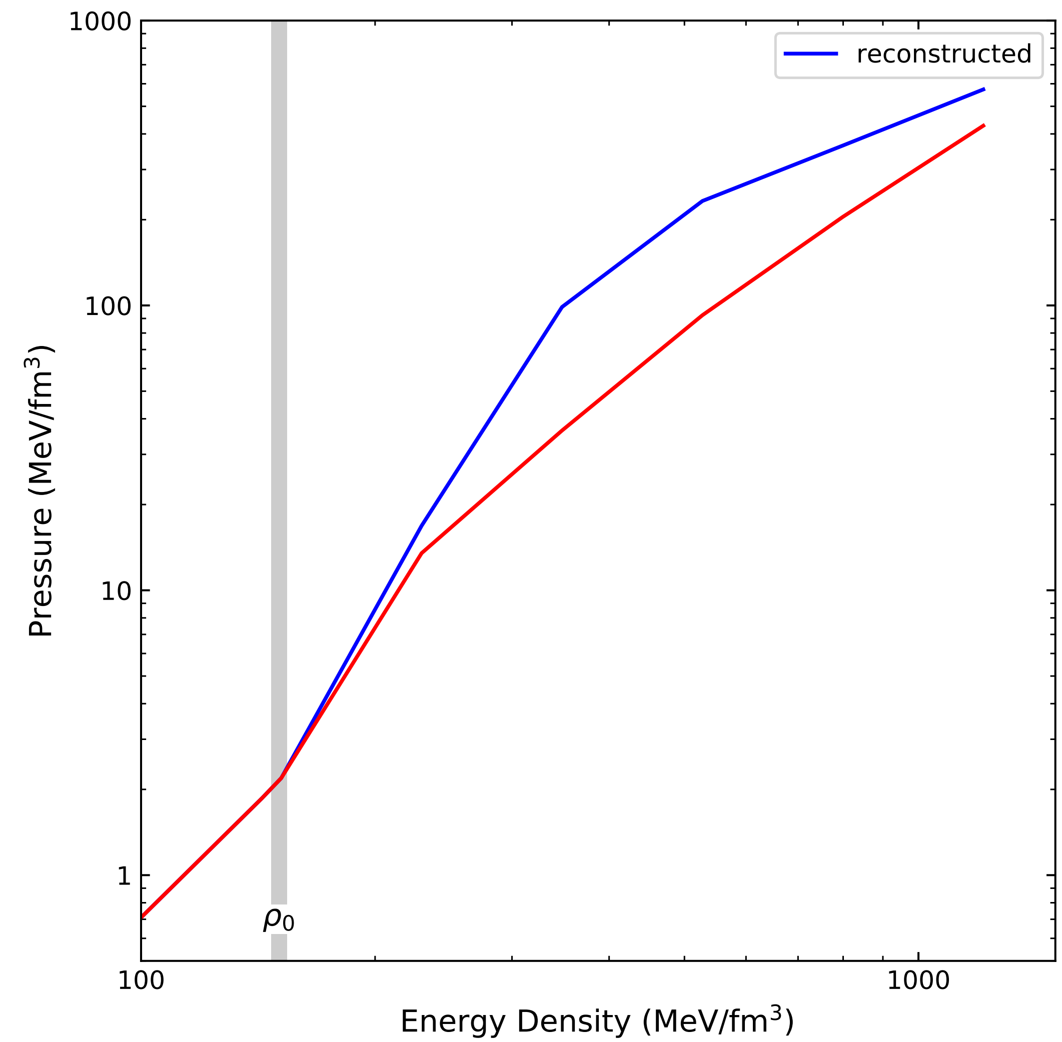

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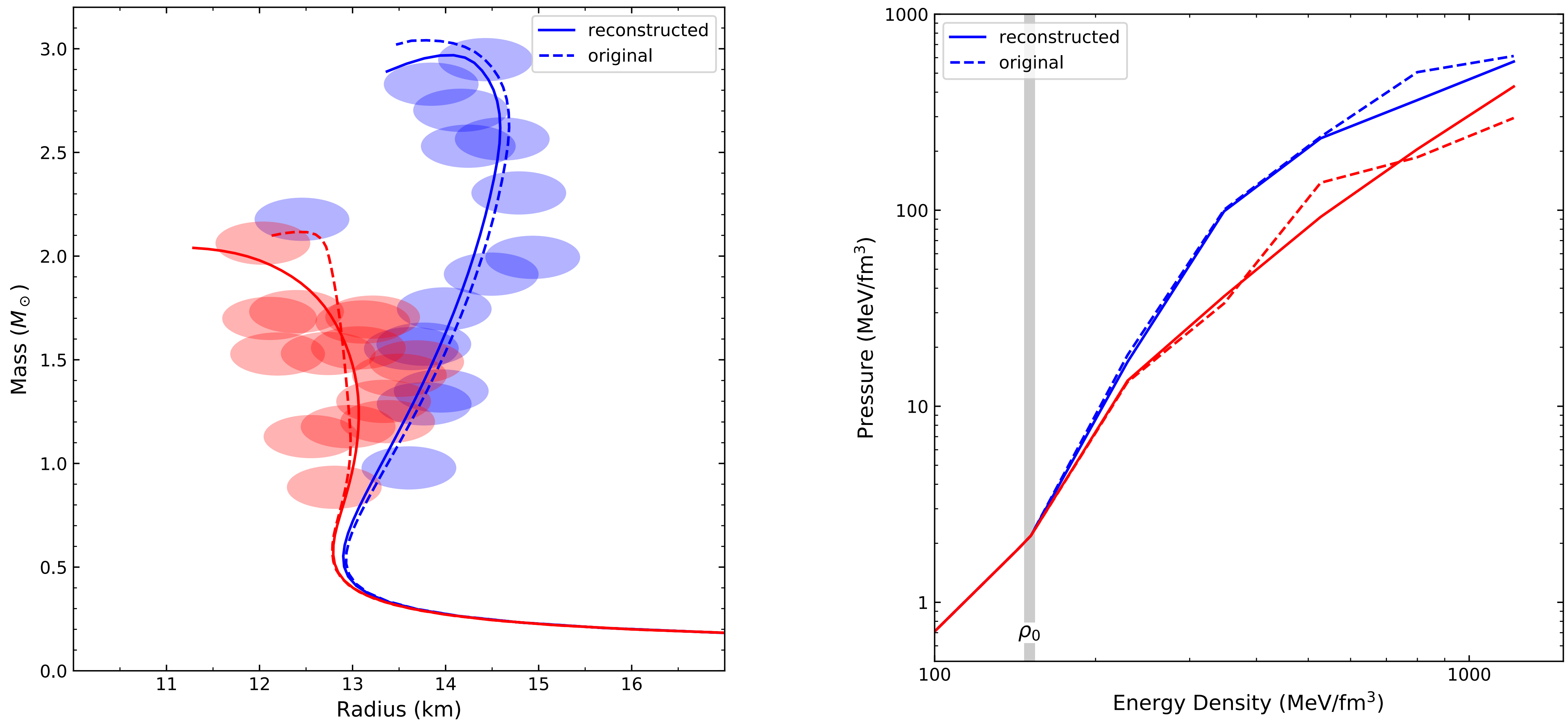
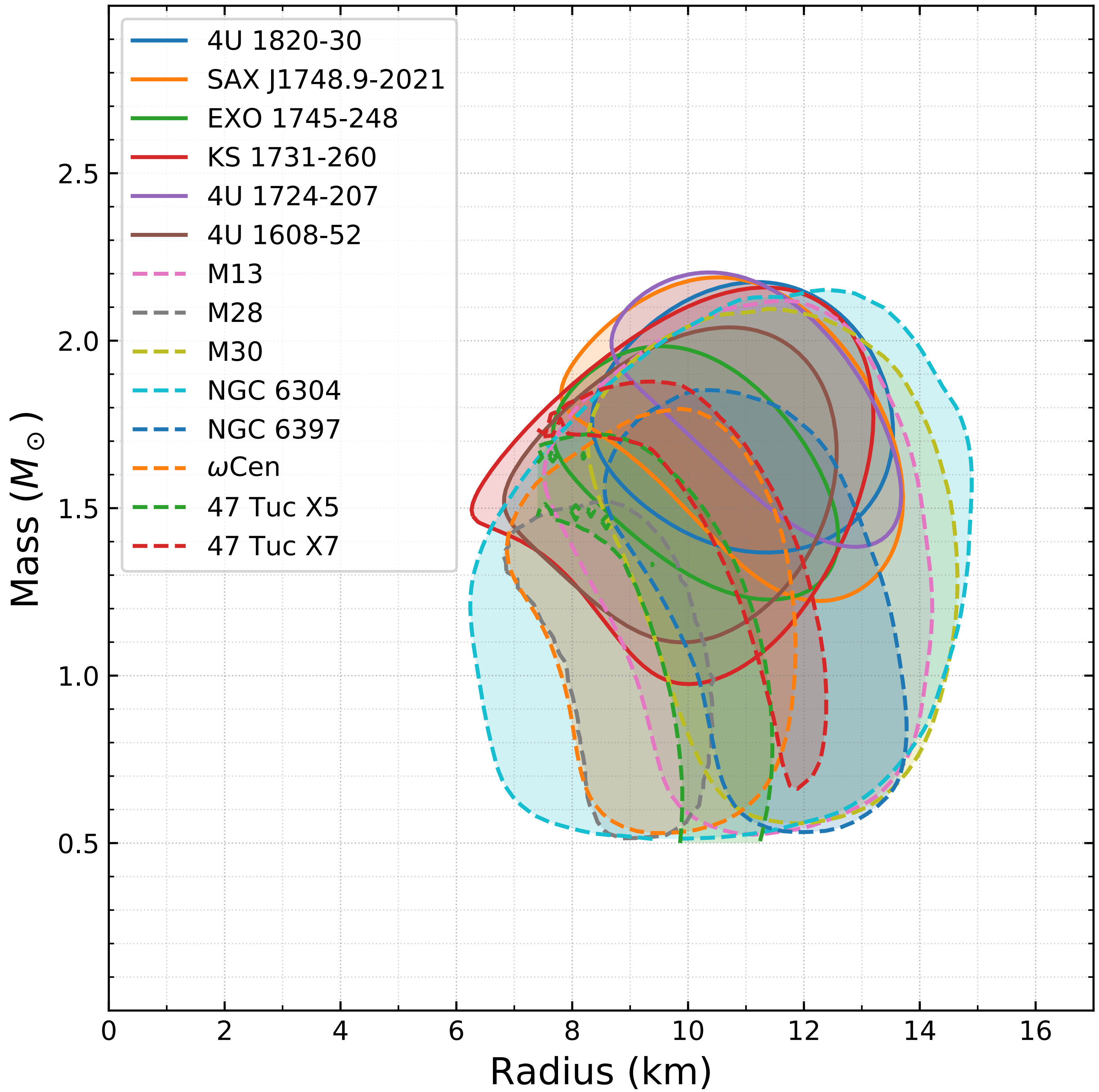


: Original EoSs and M-R

: Reconstructed EoSs and M-R

Real Data: 14 *M-R* Observation

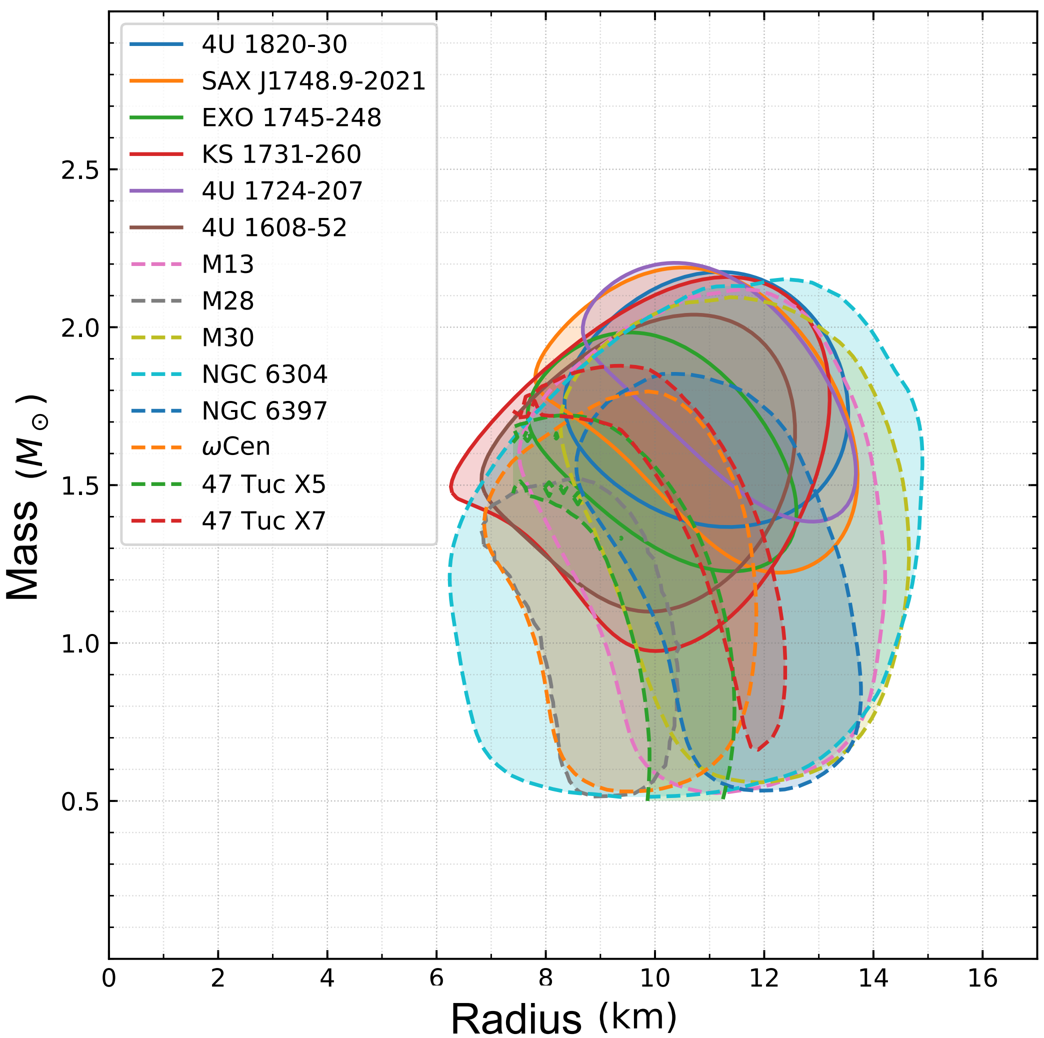


Shows the 68% credibility contour

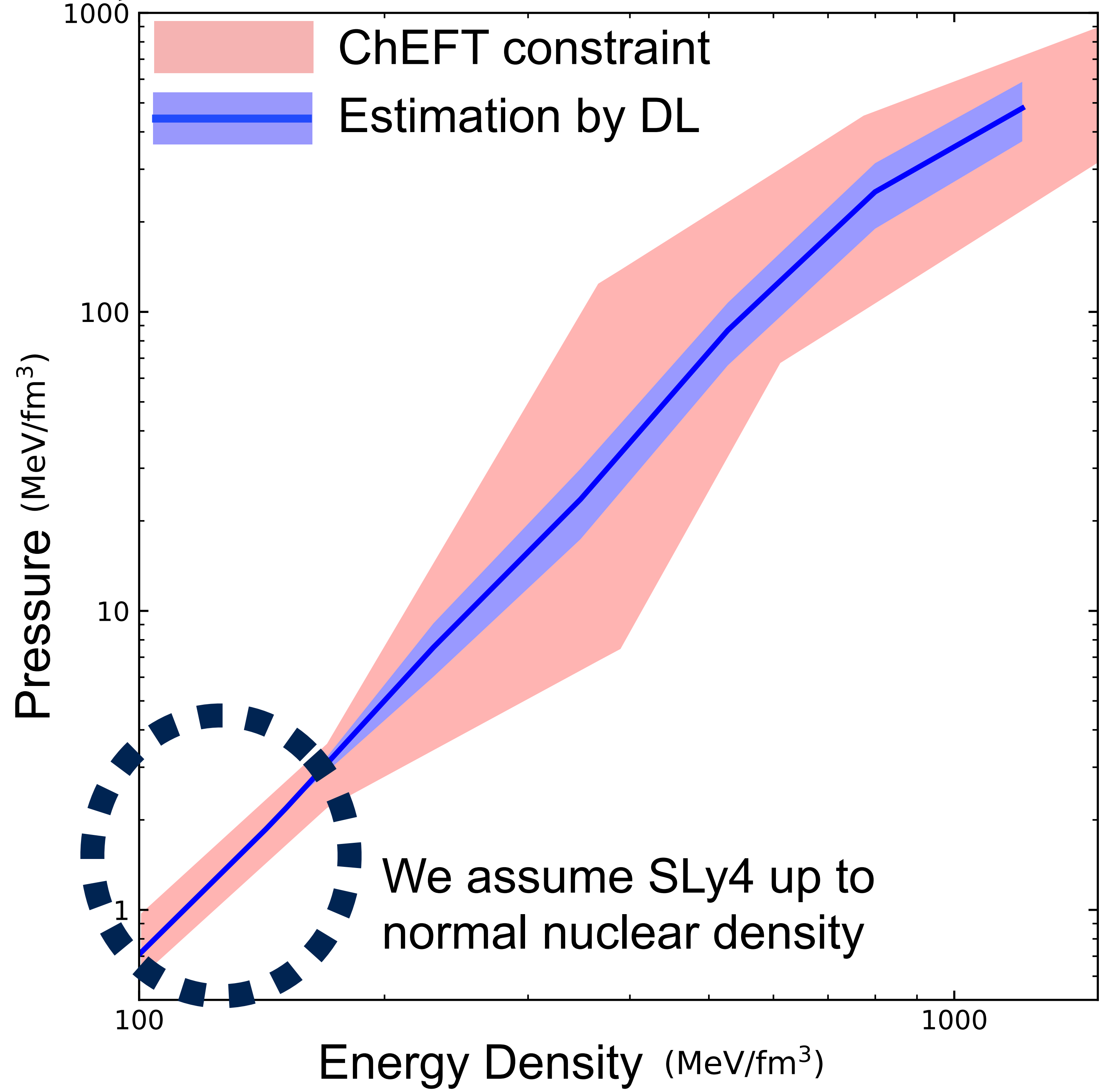
Result: EoS estimation

Estimation by deep learning

X-ray observation



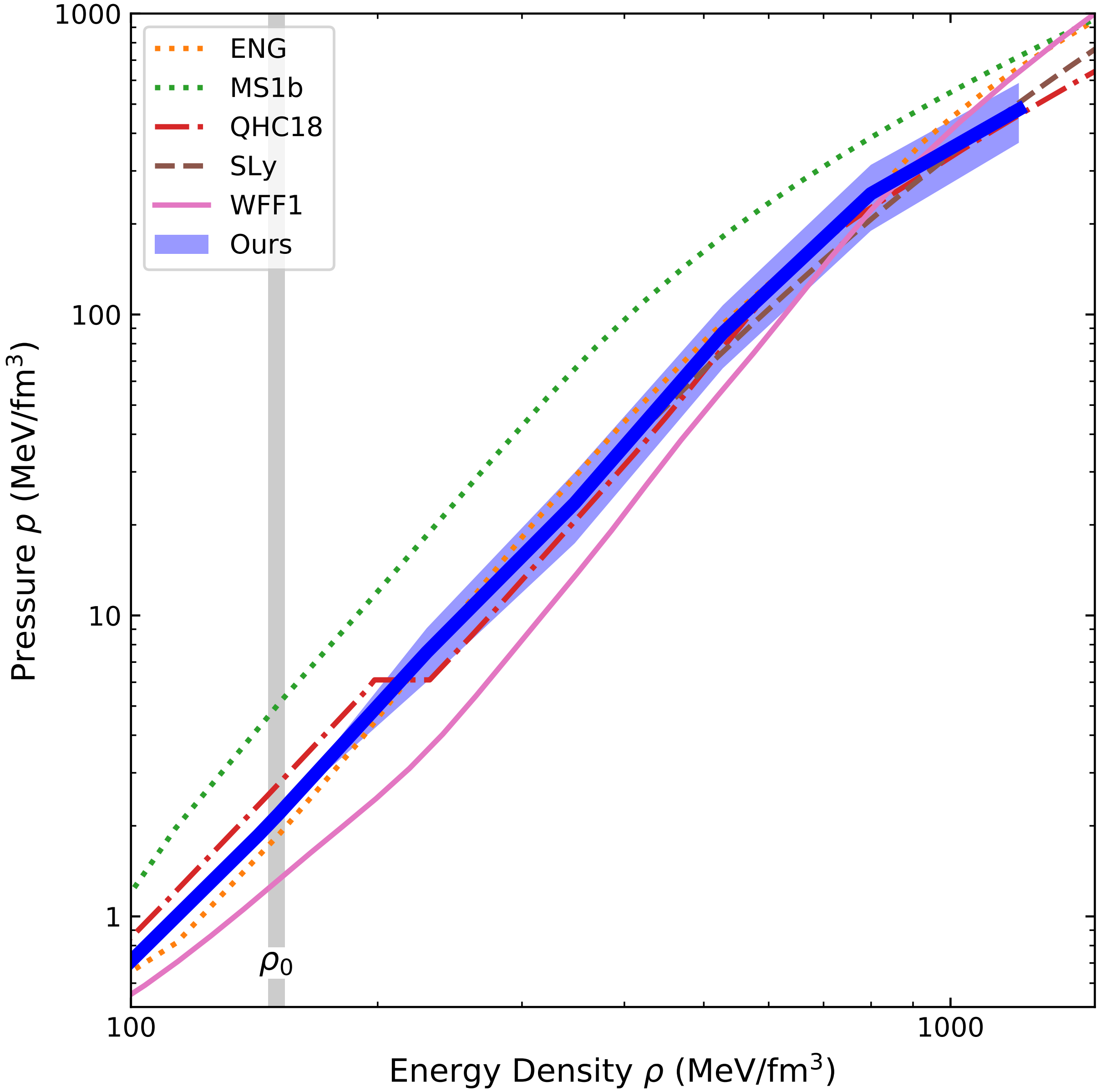
Estimated EoS

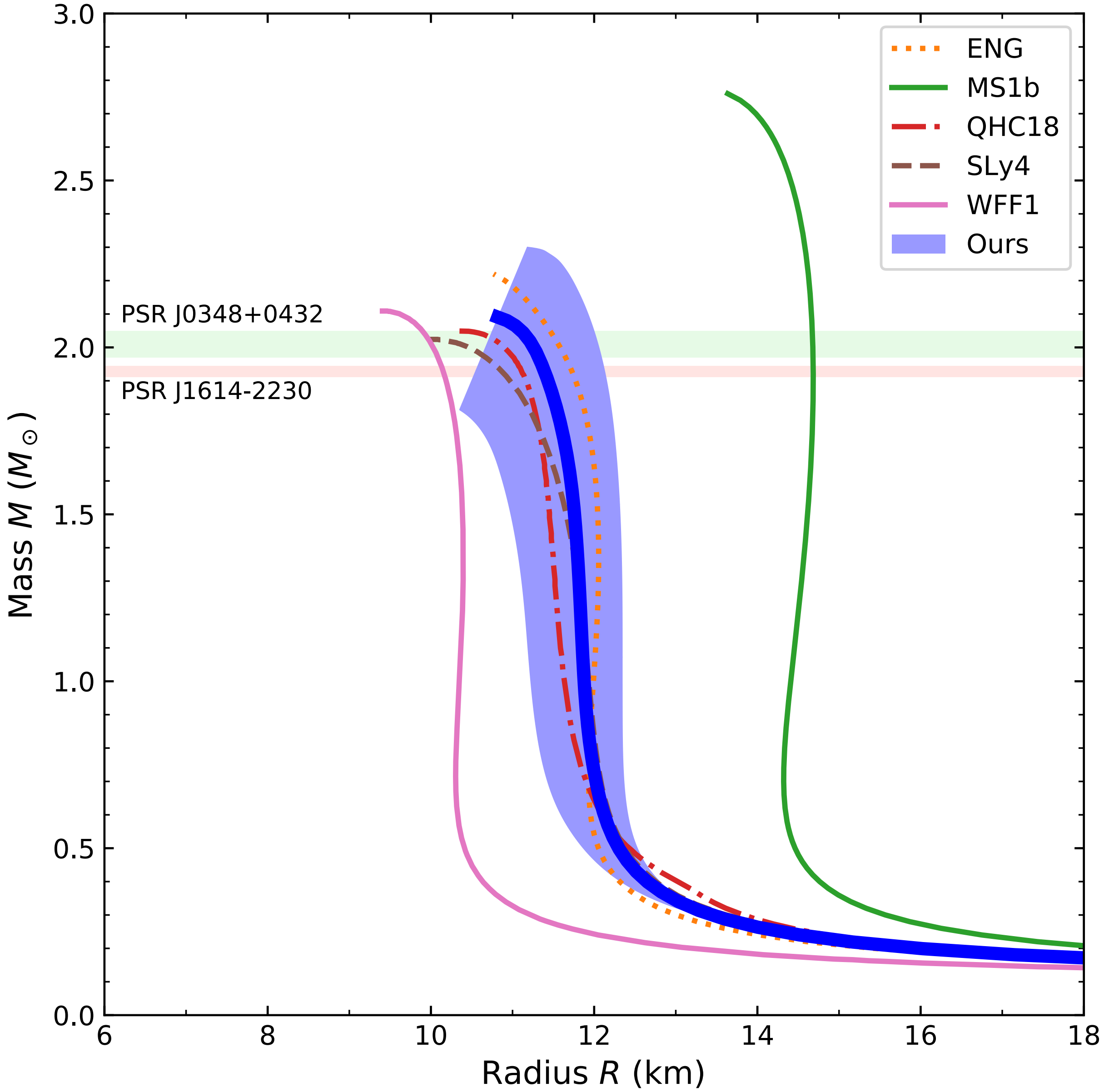


Hebeler-Lattimer-Pethick-Schwenk (2013)

Comparison with nuclear EoSs

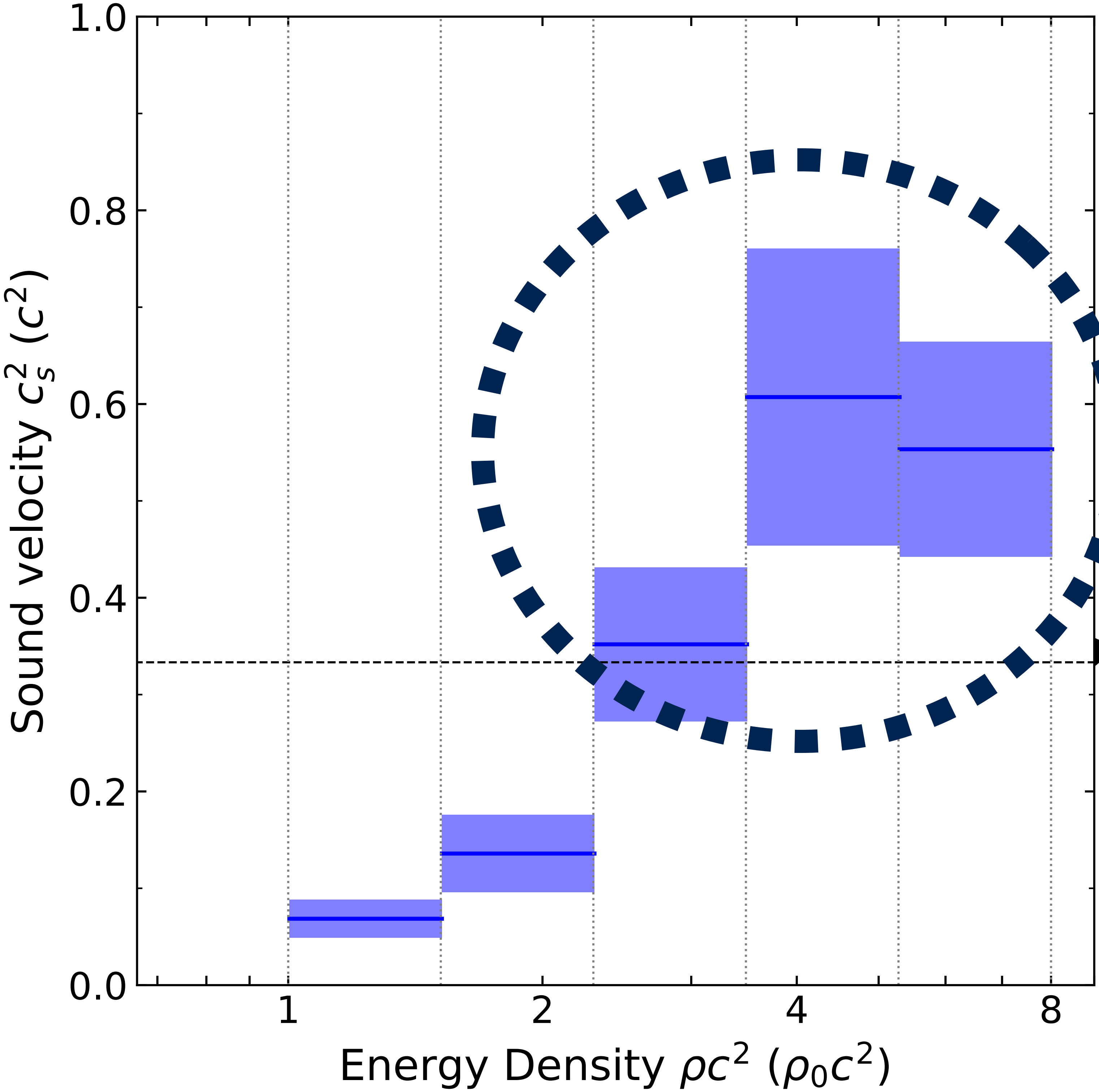
- seems consistent with APR, SLy, ENG, BSk20, etc…
- Our result does not have enough resolution to distinguish the first order phase transition





Result: Sound velocity

- **Average sound velocity:**



- **Sound velocity**:

$$c_s^2 = \partial p / \partial \rho$$

- Upper limit for ultra-relativistic particle:

$$c_s^2 = 1/3$$

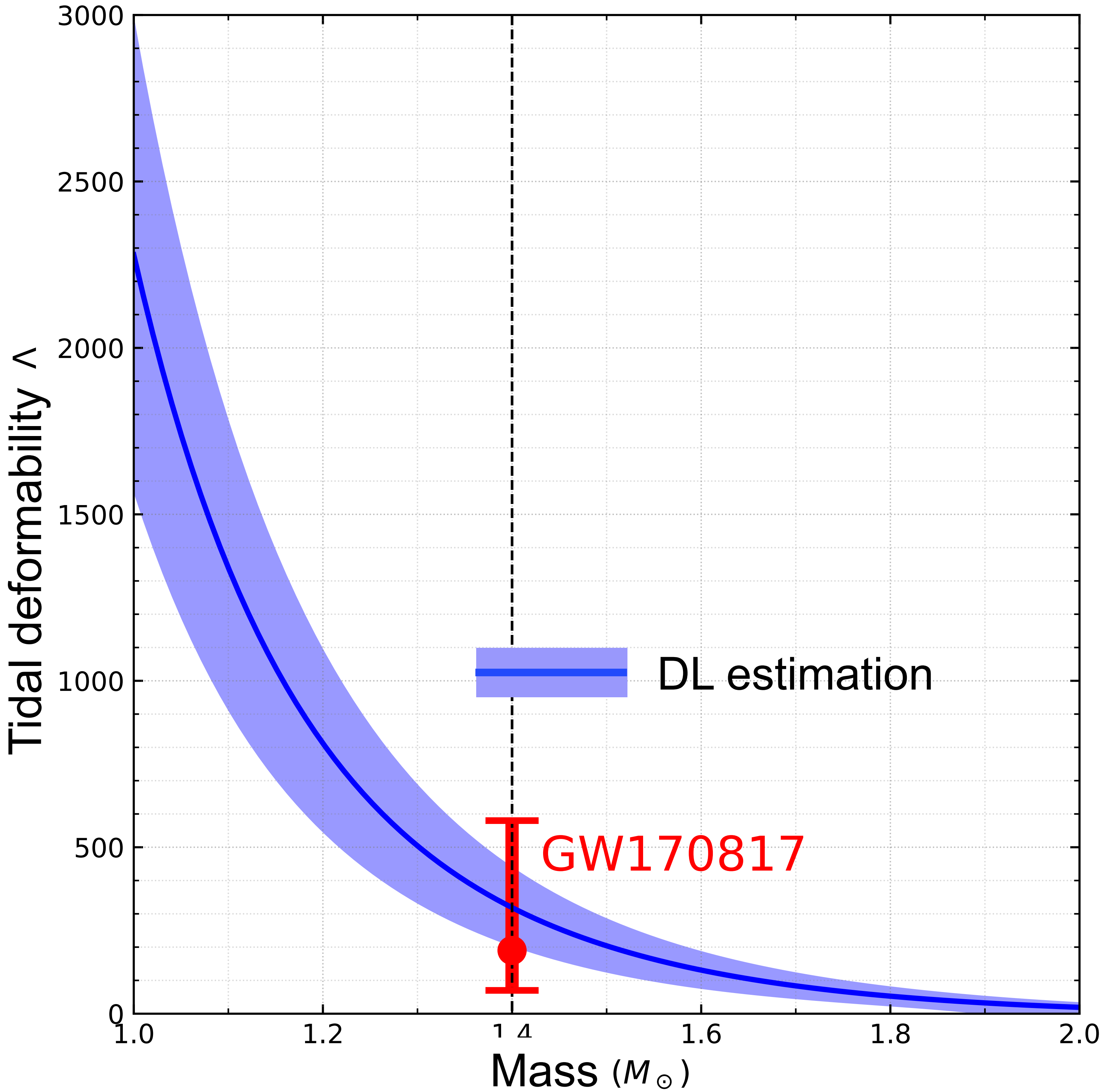
(conformal limit)

- **Sound velocity exceeds conformal limit!**

Bedaque-Steiner (2015)
Tews-Carlson-Gandolfi-Reddy (2018)

Comparison with GW170817

Tidal deformability Λ : calculated from the EoS



- Our result:

$$\Lambda(1.4M_\odot) = 320^{+120}_{-110}$$

- GW170817:

$$\Lambda(1.4M_\odot) = 190^{+390}_{-120}$$

Consistent with each other

Abbott et al. (2018)

Summary & Perspectives

- Established the method to estimate EoS from mass and radius observations

- Put significant constraint on EoS based on the real observations

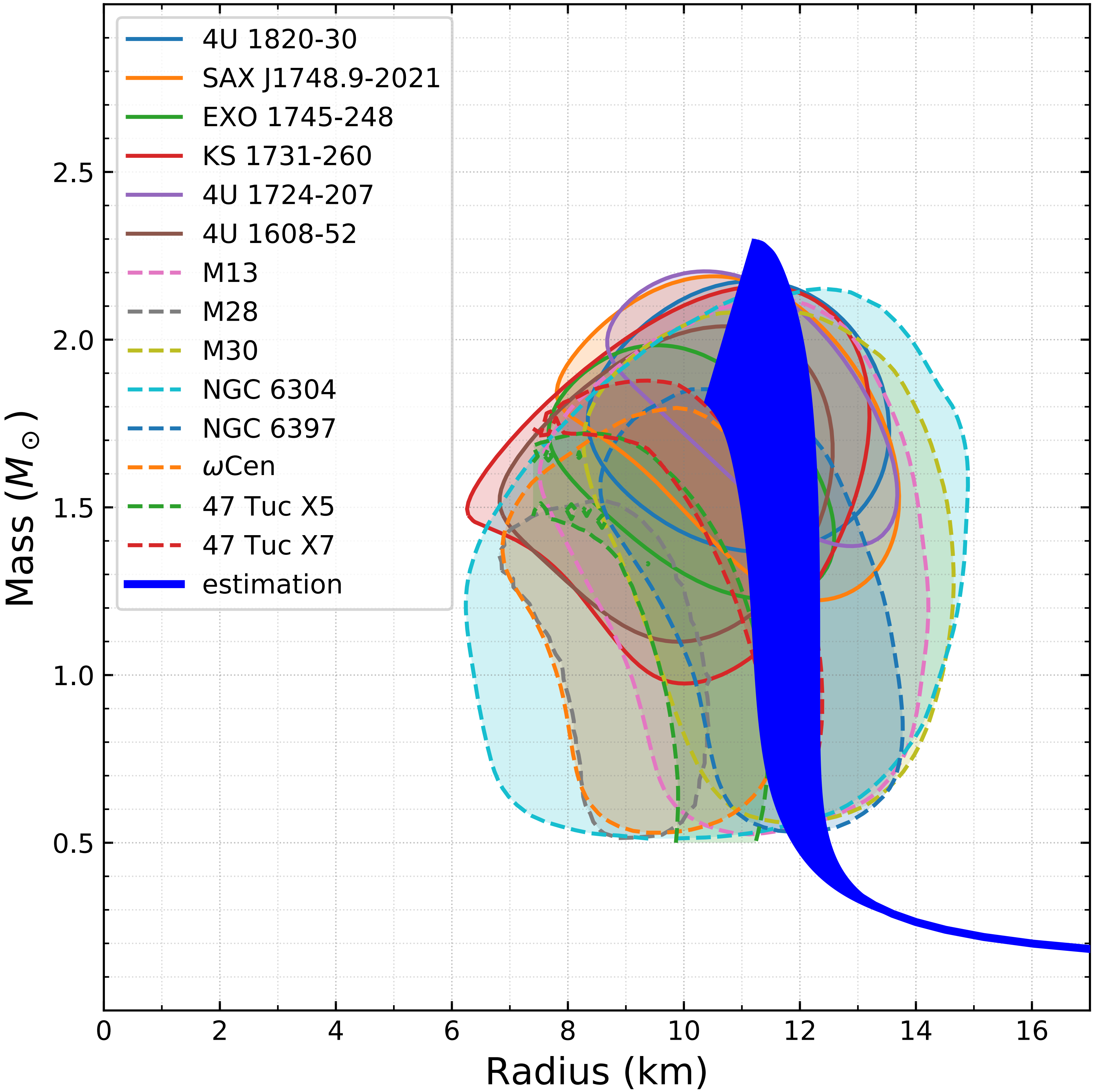
- Result seems to be consistent with independent study

- Need to rigorously quantify the uncertainty estimation; bootstrapping is optimistic about uncertainty

- Also study the bias effect of other contributions

Supplementary materials

観測データと予測結果 (*M*-*R*)



NN Structure

Layer index	Neurons	Activation Function
0	56	N/A
1	60	ReLU
2	40	ReLU
3	40	ReLU
4	5	tanh

Discussion with Bayesian Inference

Bayesian and neural network inference of EoS:

EoS $\boldsymbol{\theta} := \{c_{s,i}^2\}$ Observation $\mathscr{D} = \{(M_i, R_i)\}$

- **Bayesian** $f_{\mathrm{MAP}} = \underset{\boldsymbol{\theta}}{\mathrm{argmax}}[\underbrace{\Pr(\boldsymbol{\theta})\Pr(\mathscr{D}\,|\,\boldsymbol{\theta})}_{\Pr(\boldsymbol{\theta}\,|\,\mathscr{D})}]$

- **NN** minimizes $\langle \ell[f] \rangle = \int \underline{d\boldsymbol{\theta}} d\mathscr{D} \underline{\Pr(\boldsymbol{\theta})\Pr(\mathscr{D}\,|\,\boldsymbol{\theta})} \boxed{\ell(\boldsymbol{\theta}, f(\mathscr{D}))}$

approximated estimate → Bayesian

NN allows for more general choice of loss functions.
Bayesian assumes parametrized likelihood functions.

Stellar Radius Measurement

- We need three observables to measure the radius:

  Distance D

  Brightness → Total flux from star F

  Color → T_{eff} (Assuming black body radiation)

$$4\pi D^2 F = 4\pi R^2(\sigma_B T_{\text{eff}}^4) \;\; \rightarrow \;\; R = \sqrt{\frac{F}{\sigma_B T_{\text{eff}}^4}} D$$

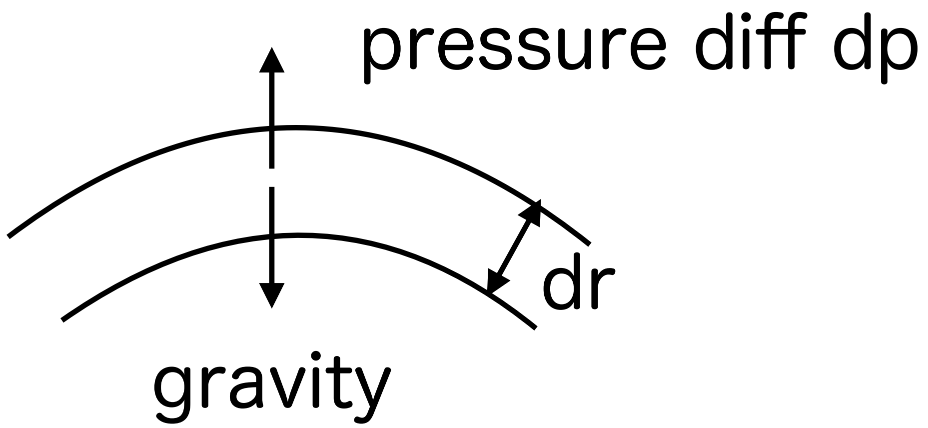
- General relativistic correction (gravitational lensing):

$$R_\infty = \left(1 - \frac{2GM}{Rc^2}\right)^{-1/2} R$$

TOV equation

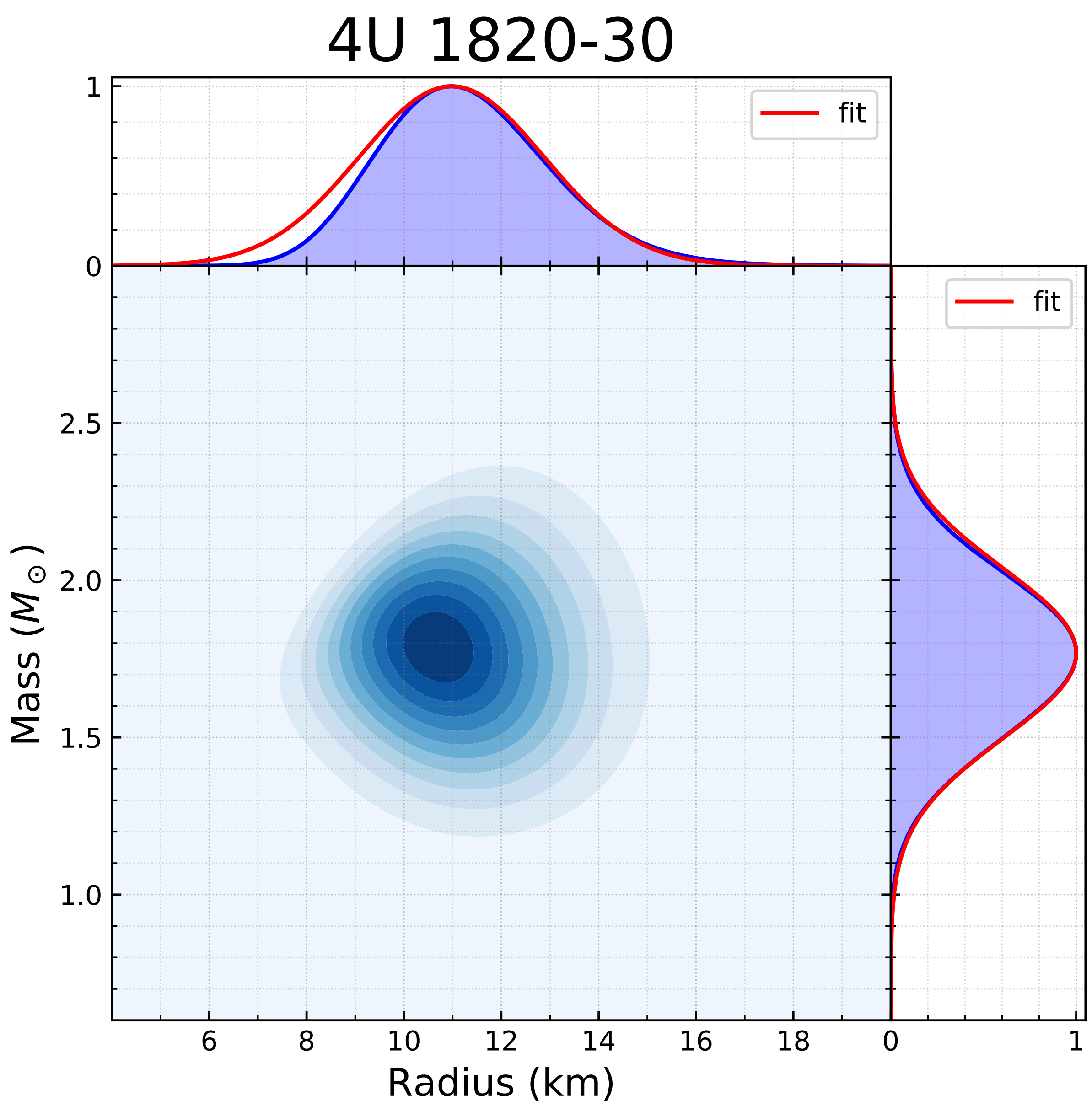
$$\frac{dp}{dr} = -(\rho + p)\,\frac{m + 4\pi r^3 p}{r(r - 2m)}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$



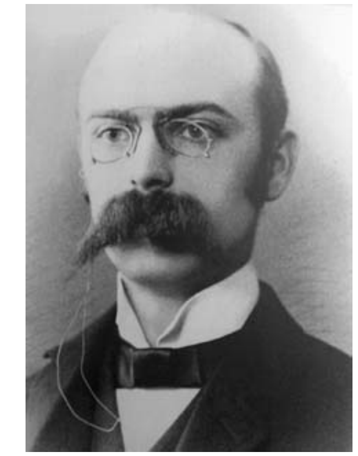
p(R) = 0 -> radius R

Marginalization



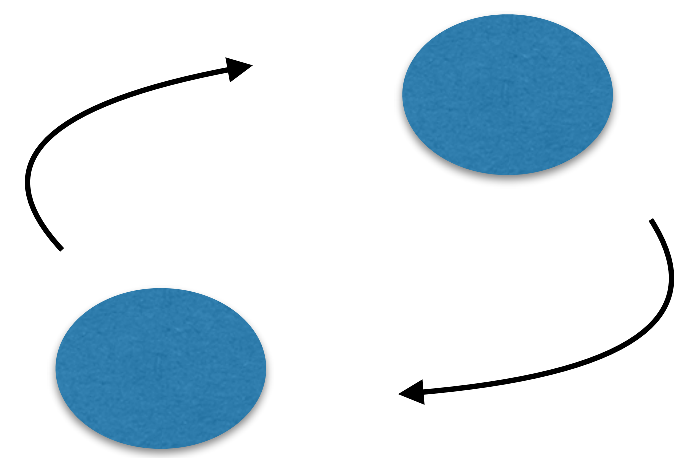
Tidal Deformability

$$Q^{(\mathrm{tid})} = -\lambda^{(\mathrm{tid})}\mathscr{E}^{(\mathrm{tid})}$$



A. E. H. Love

$$\Phi(r) = -\frac{M}{r} - Q\frac{P_2(\cos\theta)}{r^3} + \mathcal{O}(1/r^4)$$
$$+\frac{1}{3}\mathscr{E}r^2P_2(\cos\theta) + \mathcal{O}(r^3)$$



$Q^{(\mathrm{tid})}$: tidally induce quadrupole moment

$\mathscr{E}^{(\mathrm{tid})}$: tidal disturbing potential

$\lambda^{(\mathrm{tid})}$: tidal Love number (tidal deformability)

$$\lambda^{(\mathrm{tid})} \simeq R^5$$

dimensionless quantity: $\Lambda = \lambda^{(\mathrm{tid})}/M^5$

Performance Test with Mock Data

Overall performance test with 200 data

YF-Fukushima-Murase (2018)

Mass ($M_\odot$)	0.6	0.8	1.0	1.2	1.4	1.6	1.8
RMS (km)	0.16	0.12	0.10	0.099	0.11	0.11	0.12

- RMS: standard deviation of reconstructed R(M) from the original M-R

- (reconstruction accuracy) < (“observational error”) !
  (Now we choose as fixed: $\sigma_{M,i} = 0.1 M_\odot$, $\sigma_{R,i} = 0.5\,\mathrm{km}$)