Neutron Stars in the Multi-Messenger Era

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Lecture 1: Basic notions of dense matter. Nuclear interactions and nuclear matter, effective field theory. Mass and radius.

Lecture 2: Phase transitions, linear response, proto-neutron star evolution, supernova neutrino emission and detection.

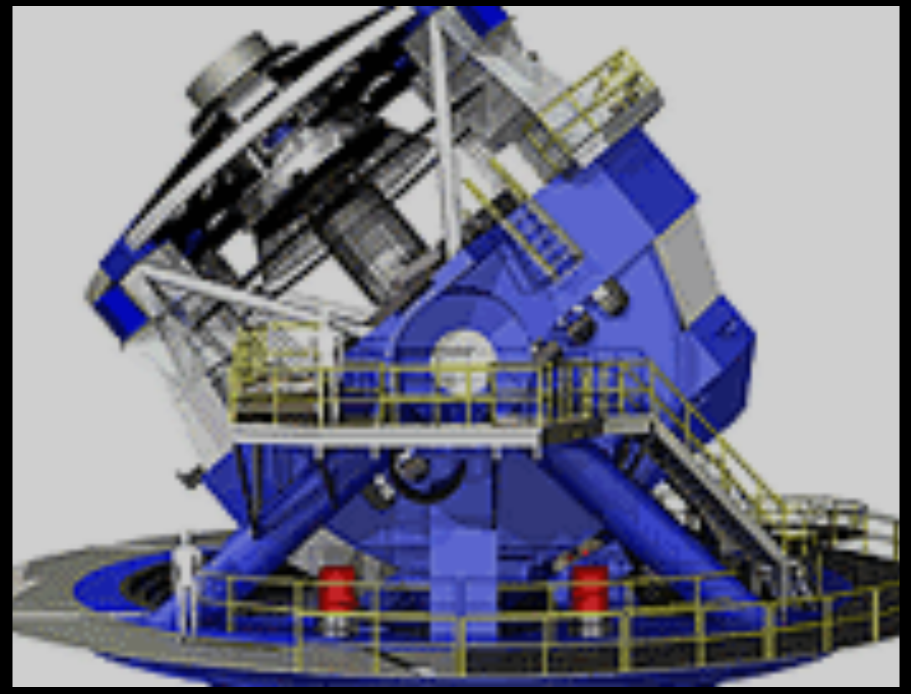
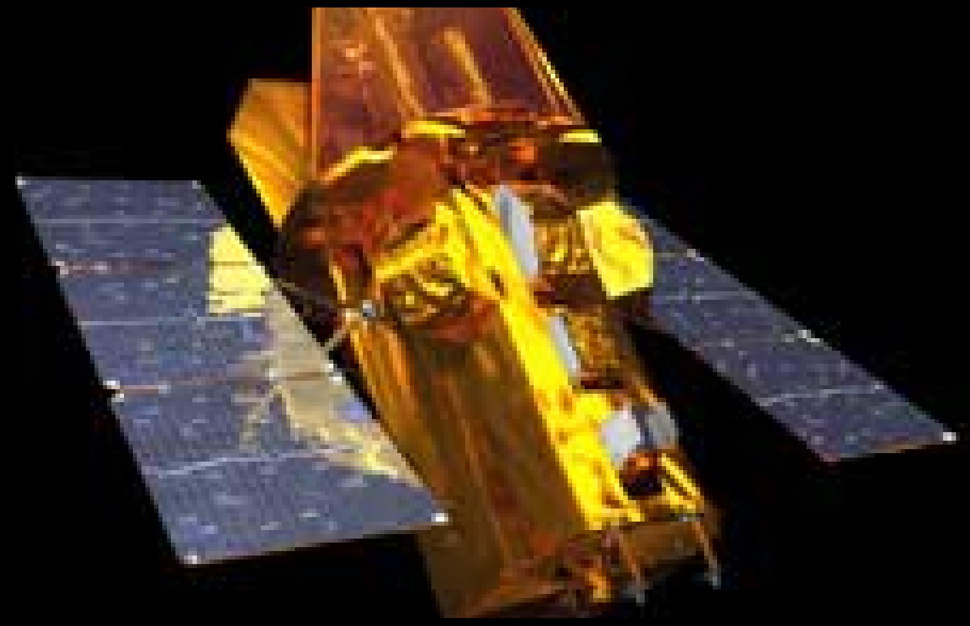
Lecture 3: Late neutron star cooling: Thermal and transport properties of degenerate matter, cooling of isolated neutron stars, heating and cooling in accreting neutron stars. Observational constraints.

Lecture 4: Neutron stars as laboratories for particle physics:Dark matter candidates (axions and other light weakly interacting particles, WIMPs, compact dark objects). Constraints from observations of neutron star masses, radii and cooling.

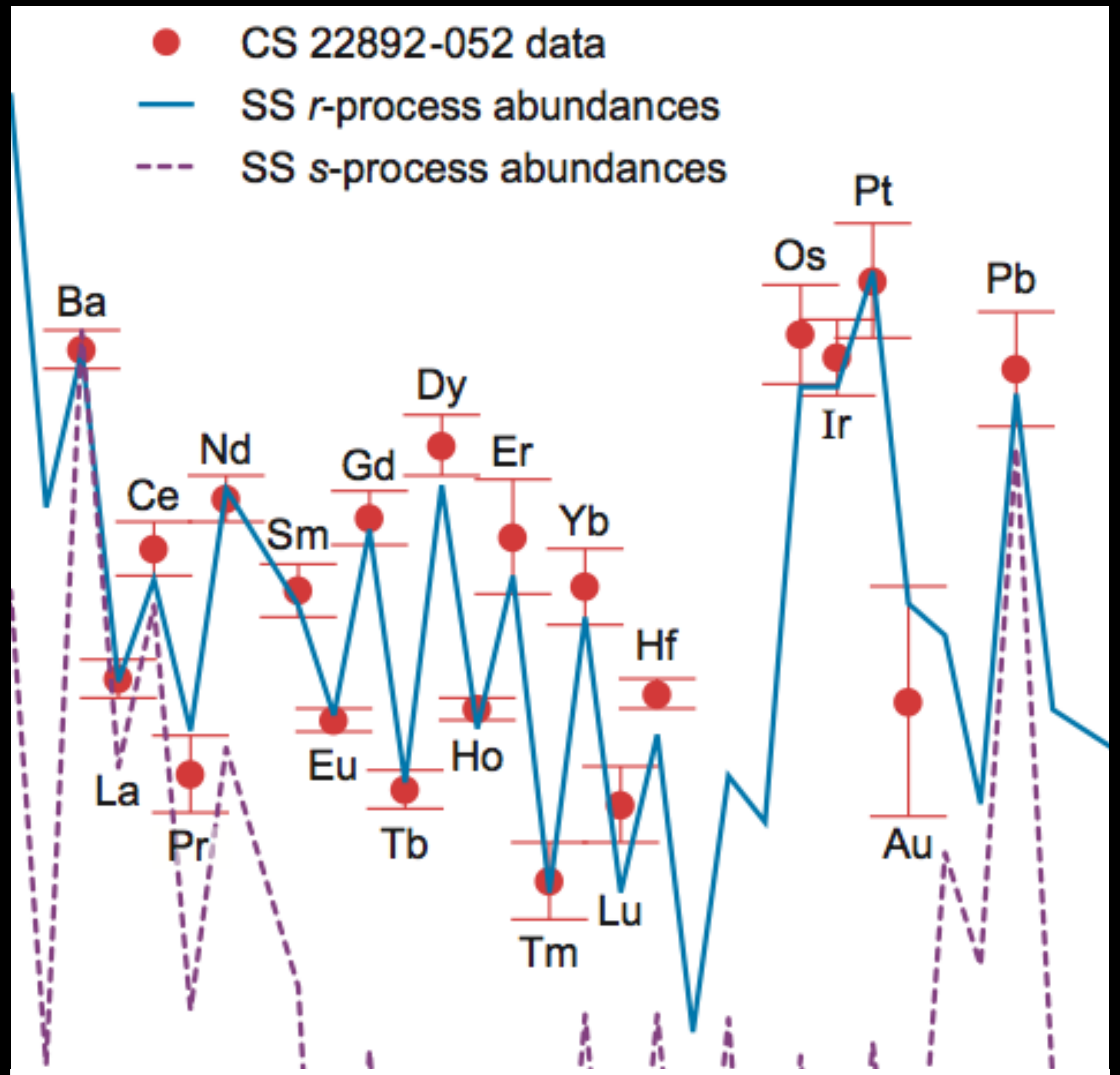
INSTITUTE for
NUCLEAR THEORY

Best of Times: The Multi-Messenger Era

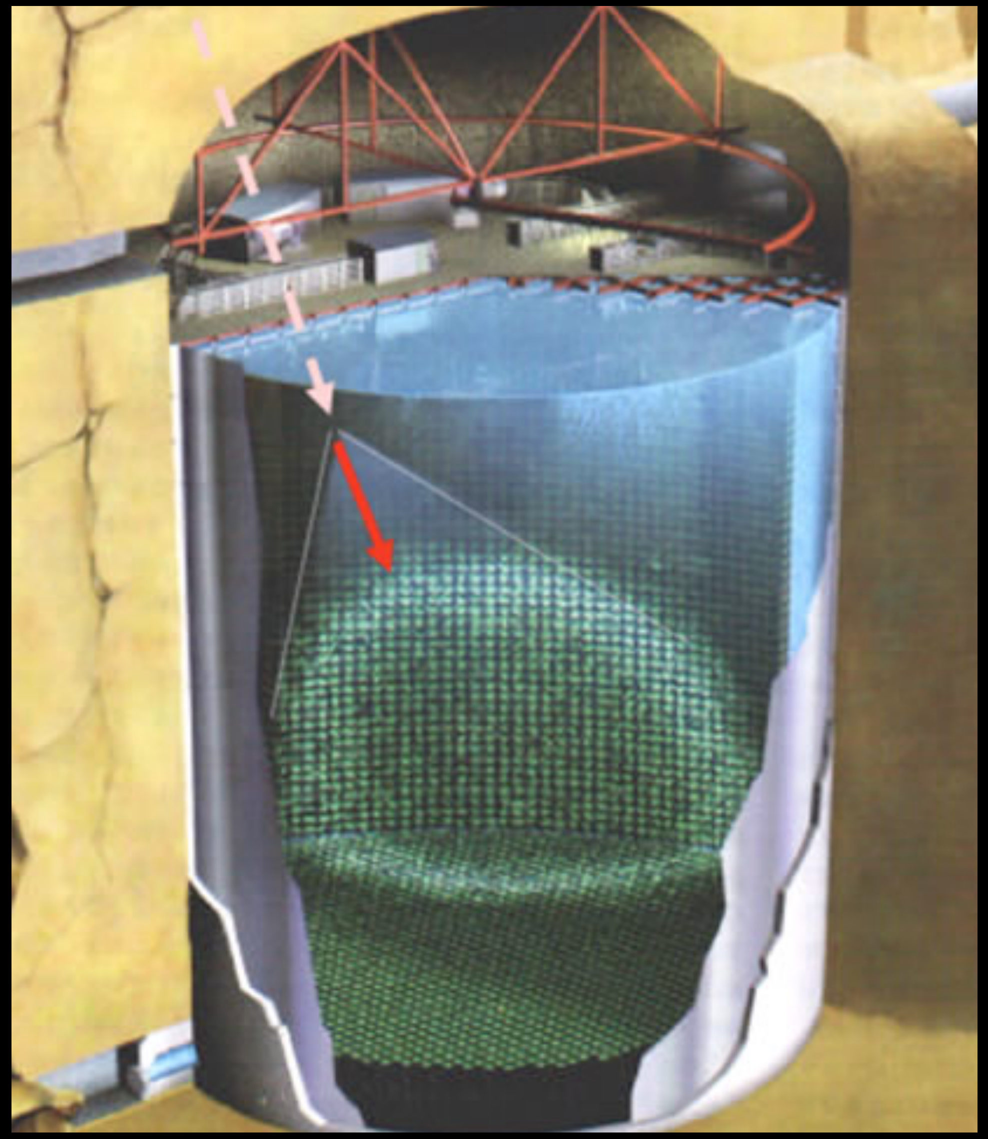


Photons



Abundances



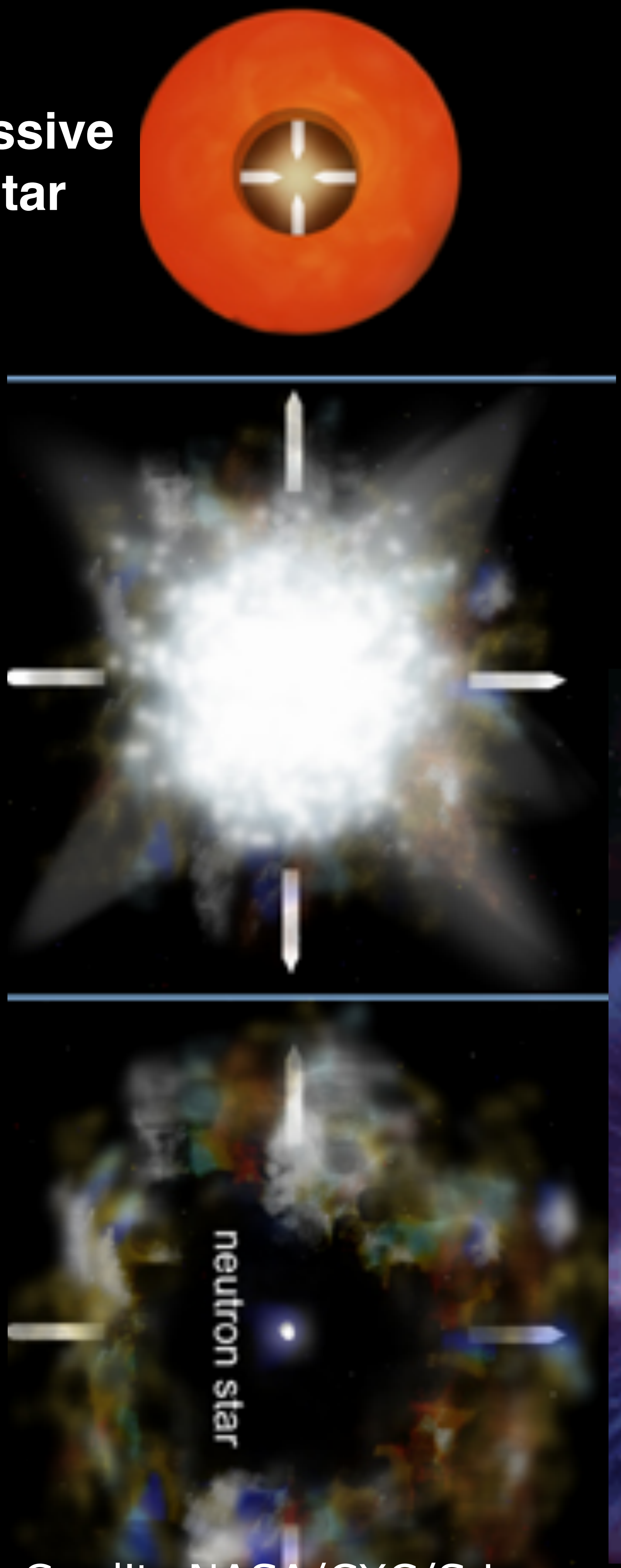
Neutrinos

Gravitational Waves

Sources: Neutron Stars are Central

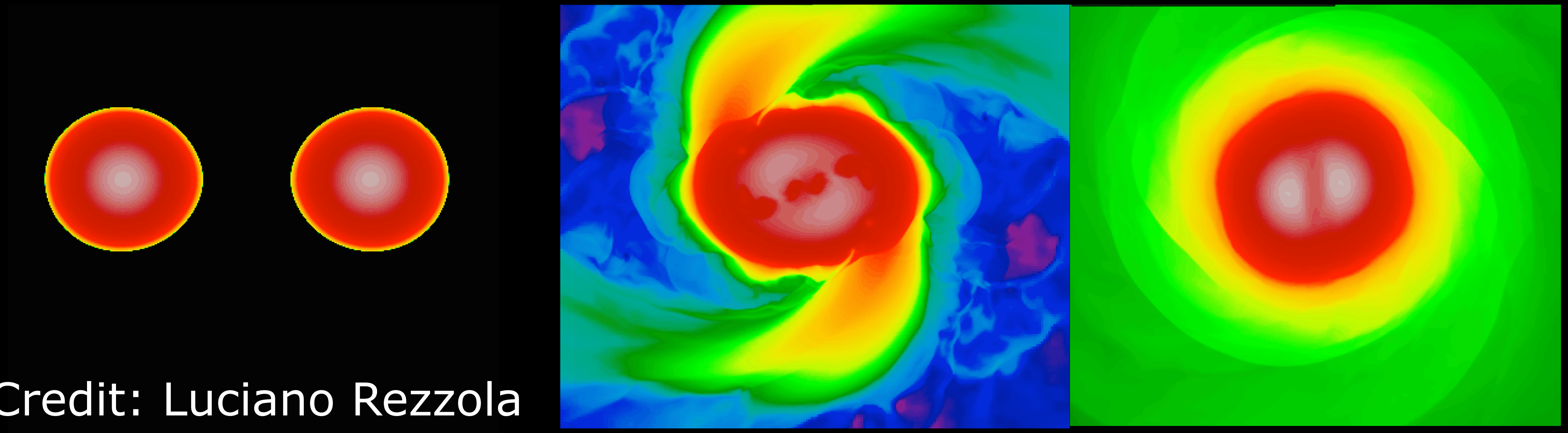
Supernova

Massive Star



Credit: NASA/CXC/S Lee

Binary Neutron Star Mergers

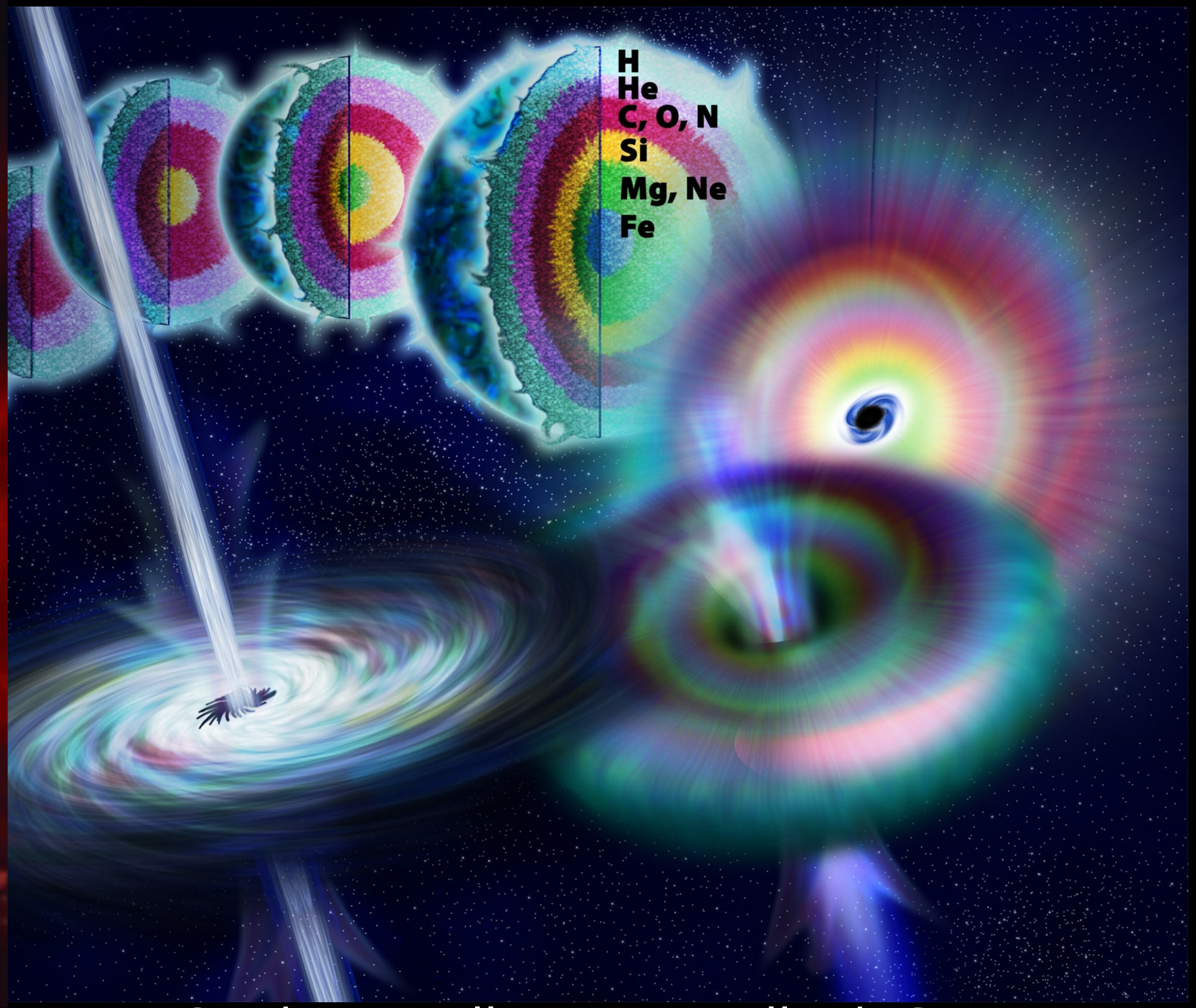


Credit: Luciano Rezzola

Accreting Neutron Stars

Credit: David Hardy & PPARC

Gamma-Ray Bursts



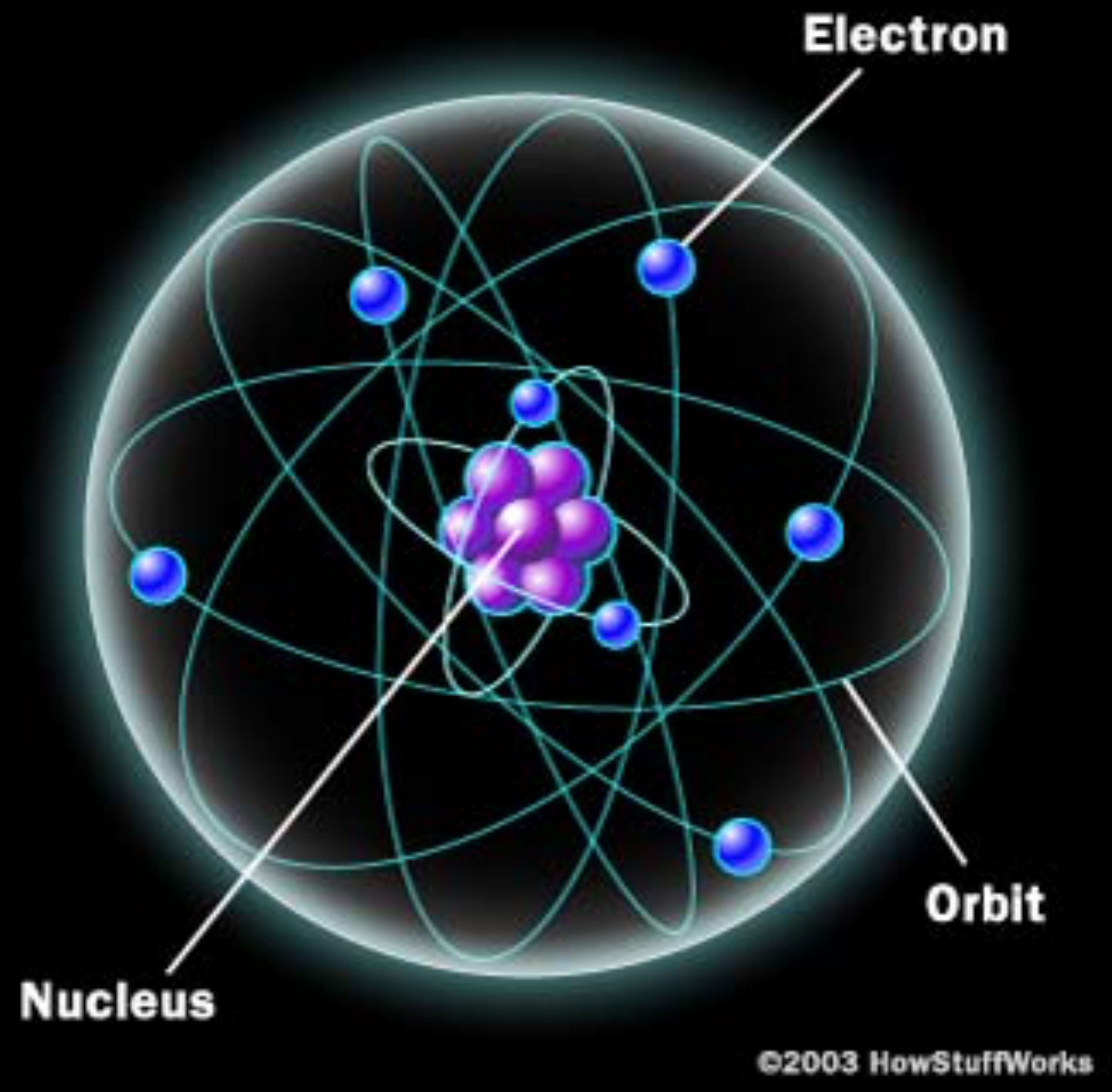
Credit: Nicolle Rager Fuller/NSF

Dense Matter

Density of iron at atmospheric pressure

$$\rho \simeq 8 \text{ g/cm}^3$$

Density of an Fe atom: $\rho \simeq 8 \text{ g/cm}^3$



Density of an Fe nucleus: $\rho \simeq 2.5 \times 10^{14} \text{g/cm}^3$

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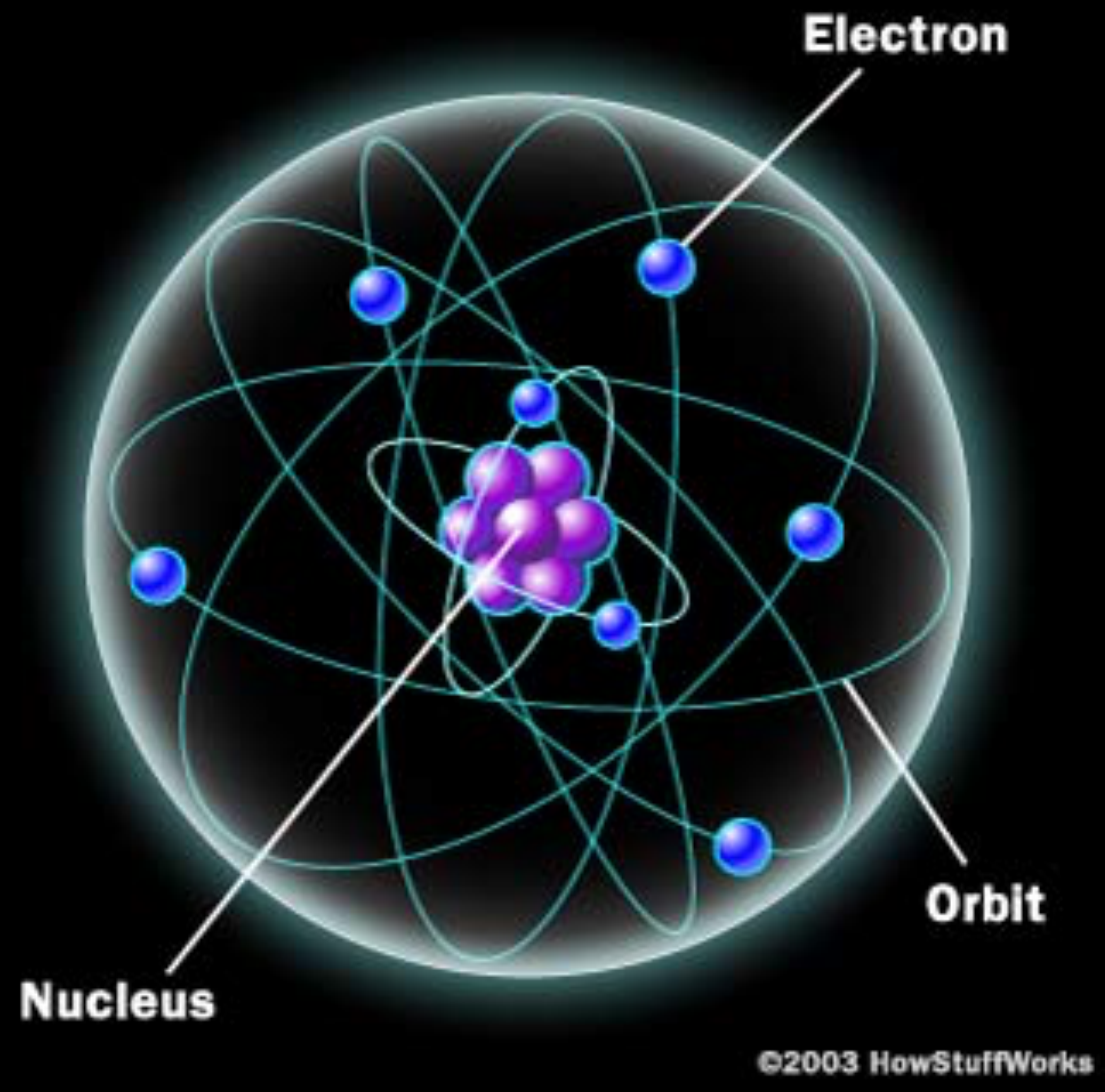

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We live in an empty world !

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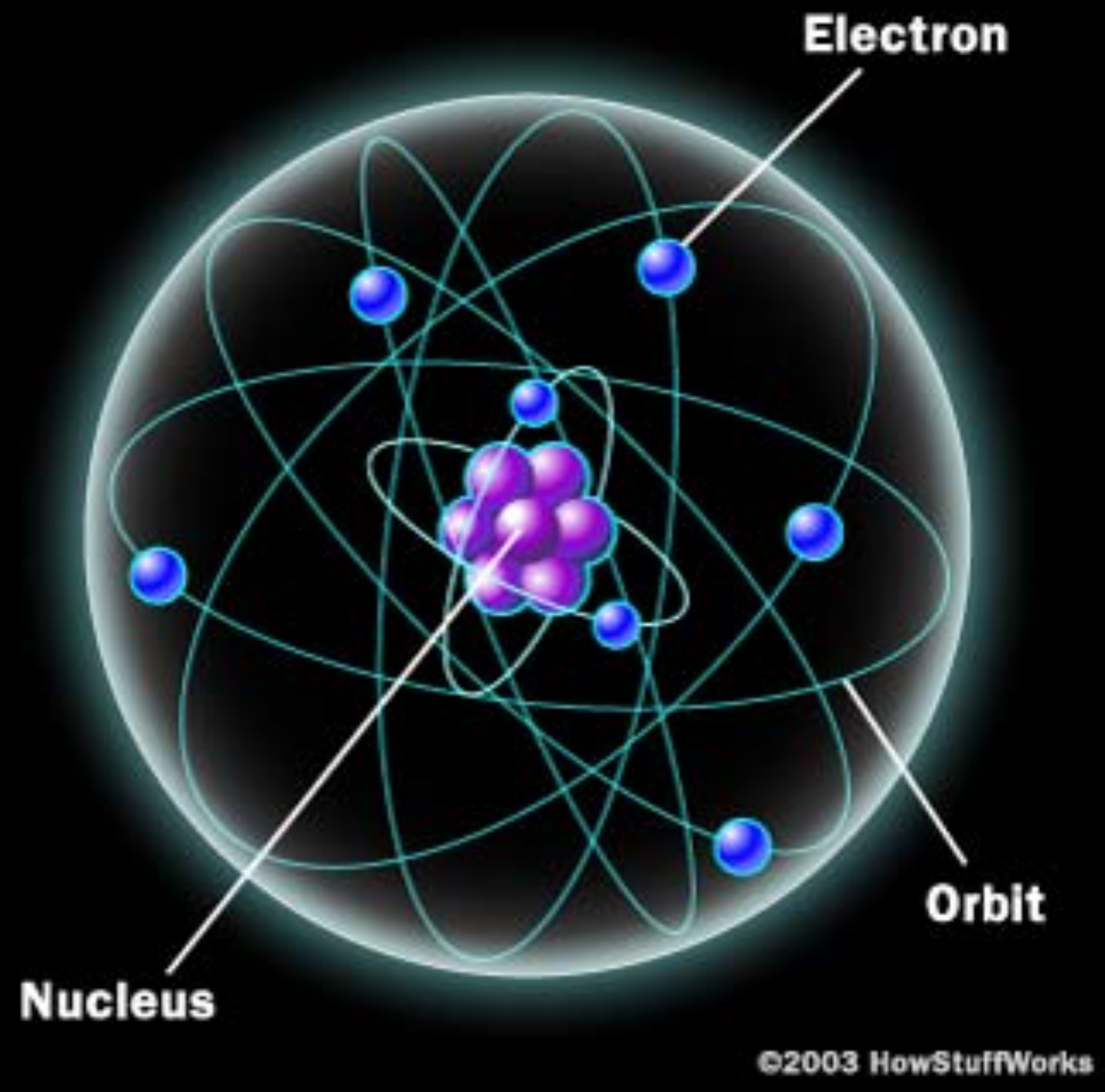

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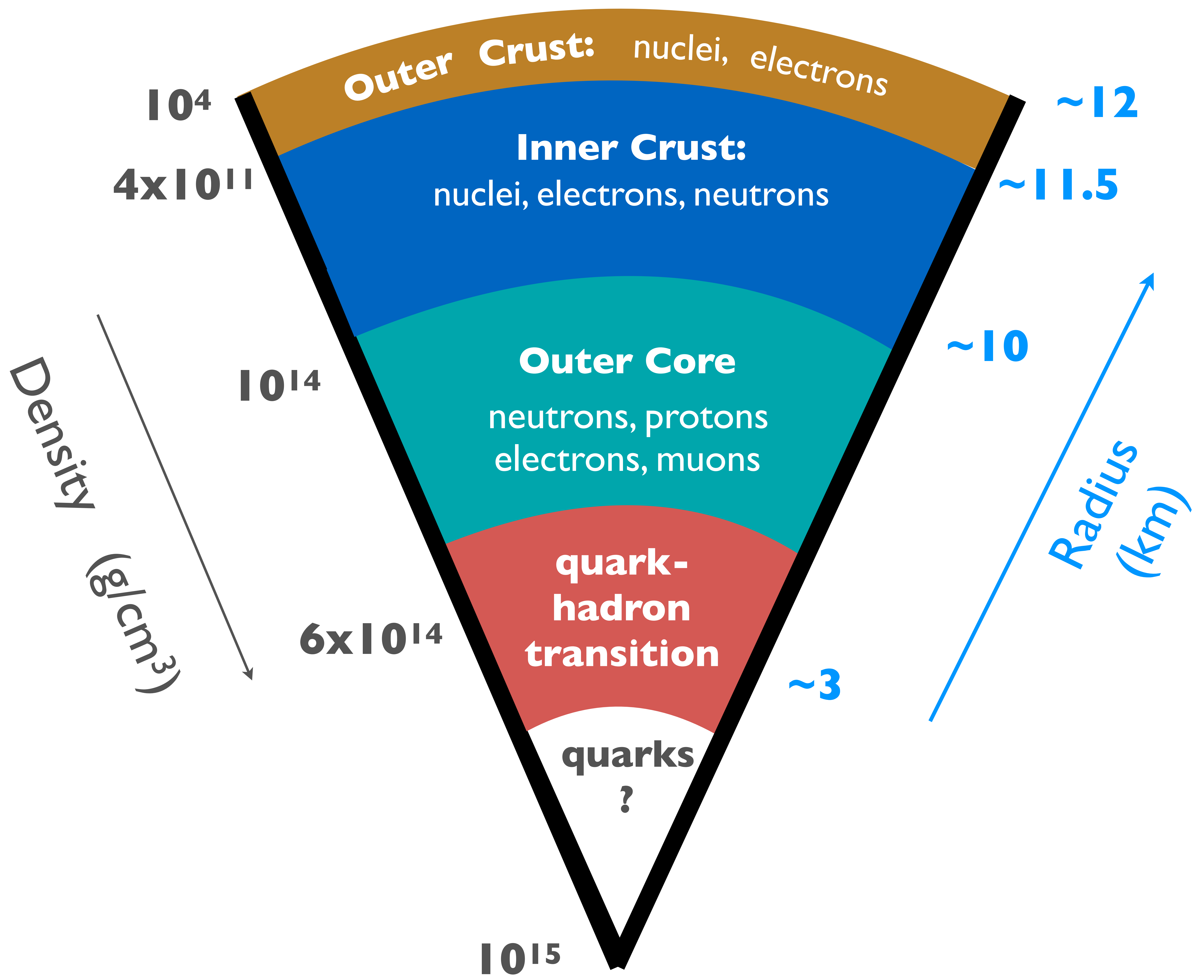
We live in an empty world !

Compressing matter begins with the compression of electrons.

Compressing Matter:
A tale of frustration and liberation

Density	Fermi Energy (Frustration)	Phenomena (Liberation)
10^3 - 10^6 g/cm^3	Electron Fermi Energy $\mu_e = 10$ keV– MeV	Ionization
10^6 - 10^{11} g/cm^3	Electron Fermi Energy $\mu_e = 1-25$ MeV	Neutron-rich Nuclei $e+p \rightarrow n+\nu_e$
10^{11} - 10^{14} g/cm^3	Neutron Fermi Energy $\mu_n = 1-30$ MeV	Neutron-drip superfluidity
10^{14} - 10^{15} g/cm^3	Neutron Fermi Energy $\mu_n = 30-1000$ MeV	Nuclear matter Quarks ?

Composition and Phases of Dense Matter in Neutron Stars



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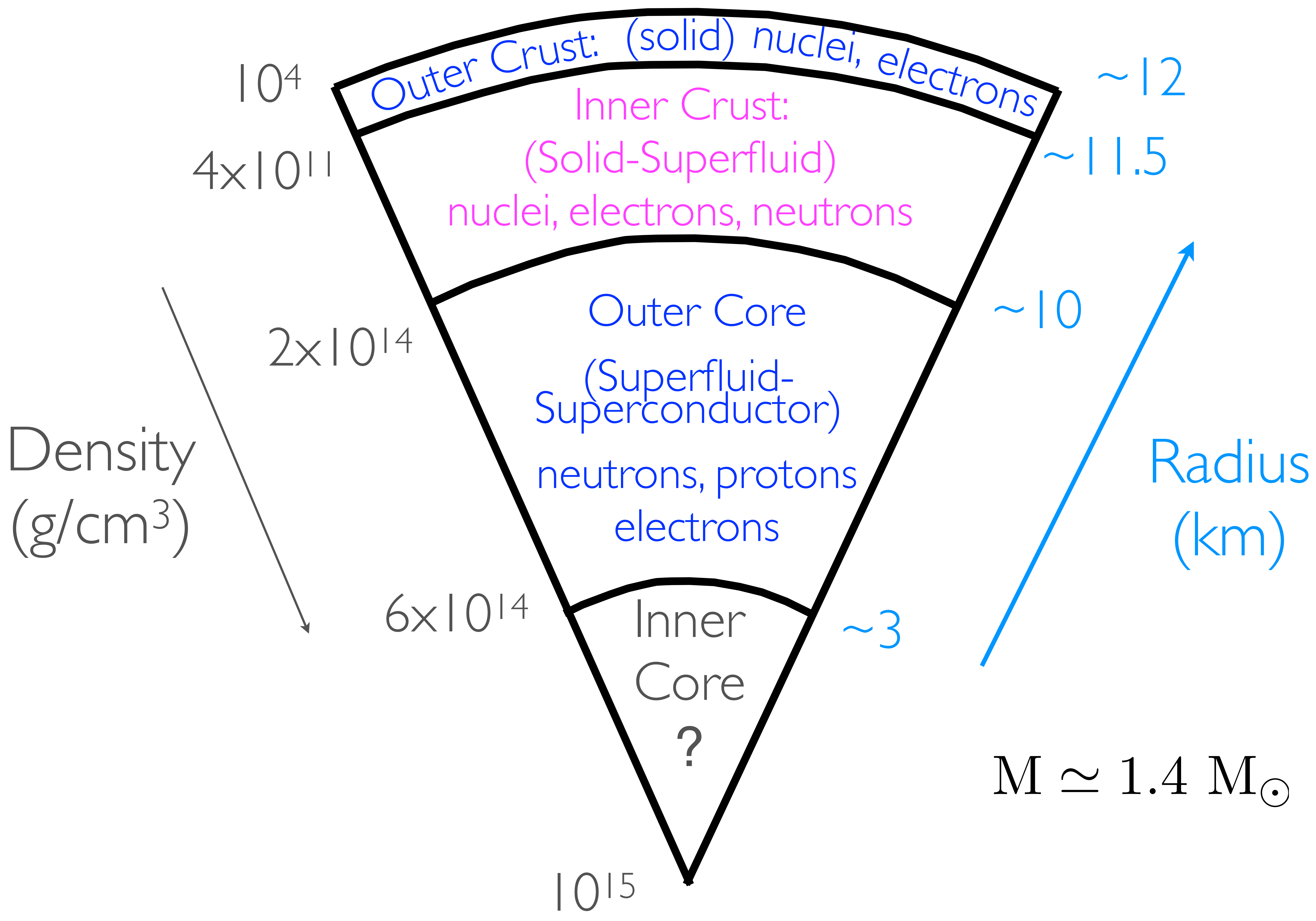


Lecture 1: Basic Notions

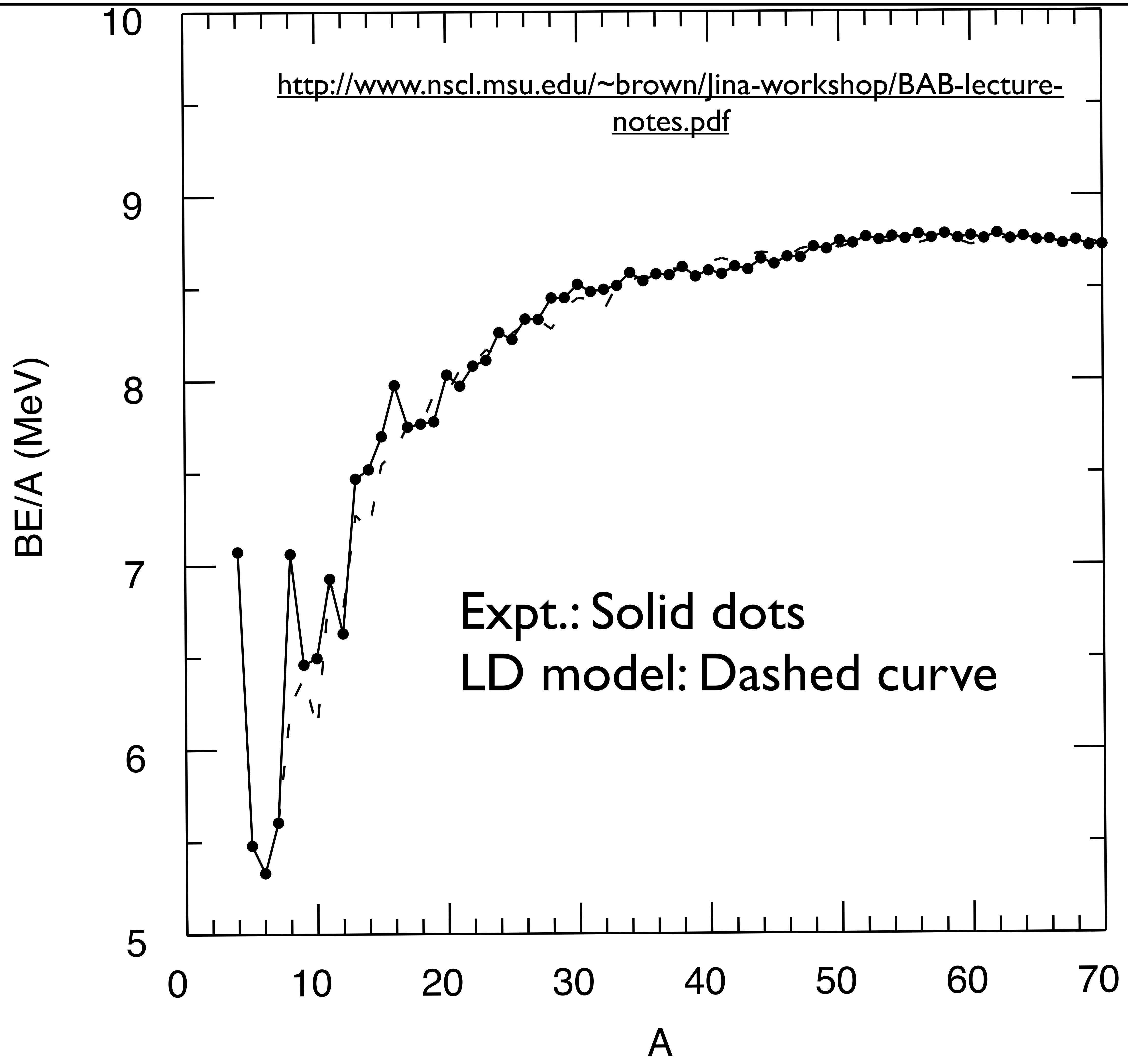
- Nuclei as drops of nuclear matter.
- Nuclear interactions and Effective Field Theory.
- First-order phase transitions and heterogeneous phases.
- Neutron star structure

Liquid drop model:

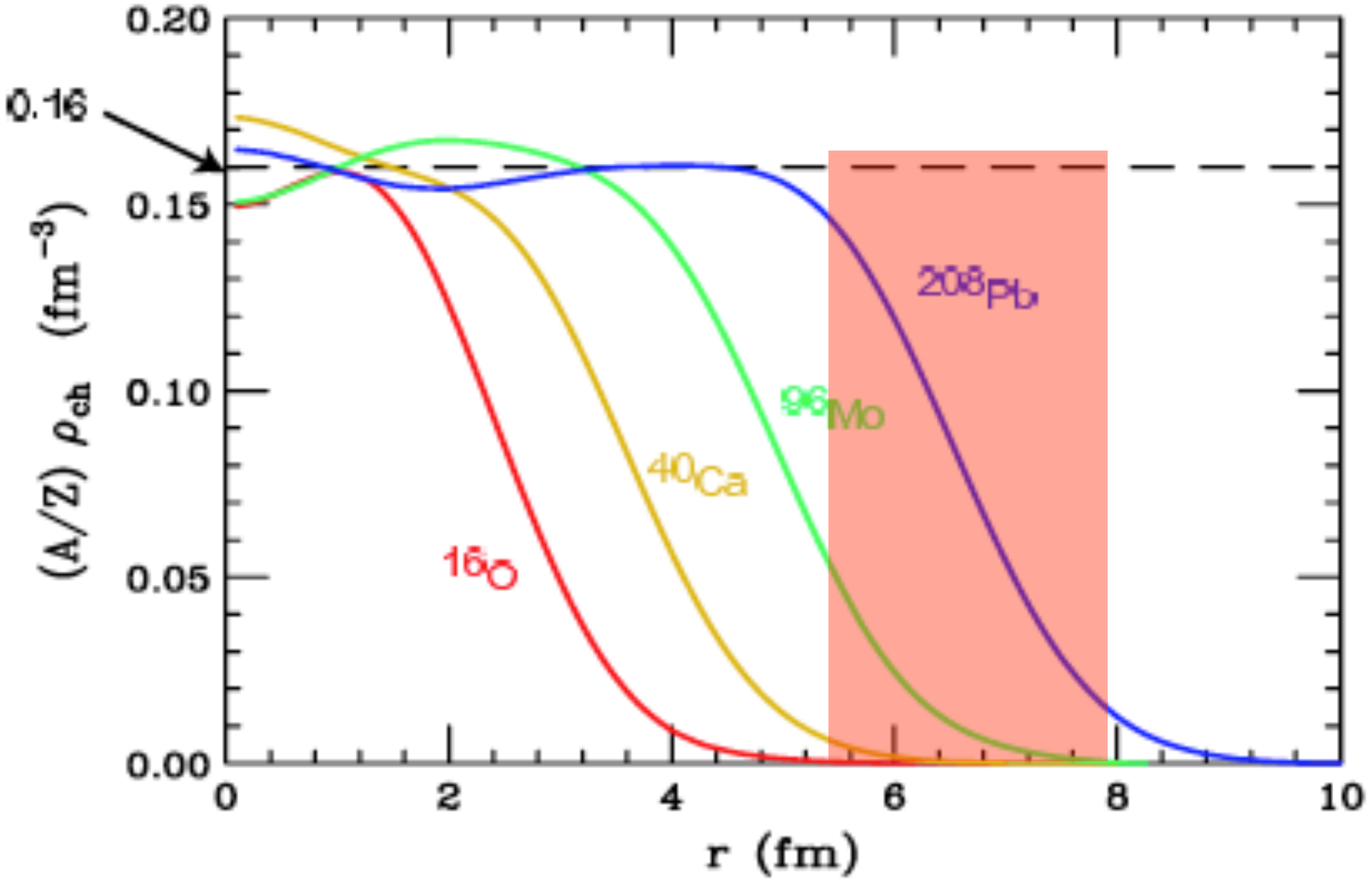
Binding Energy:

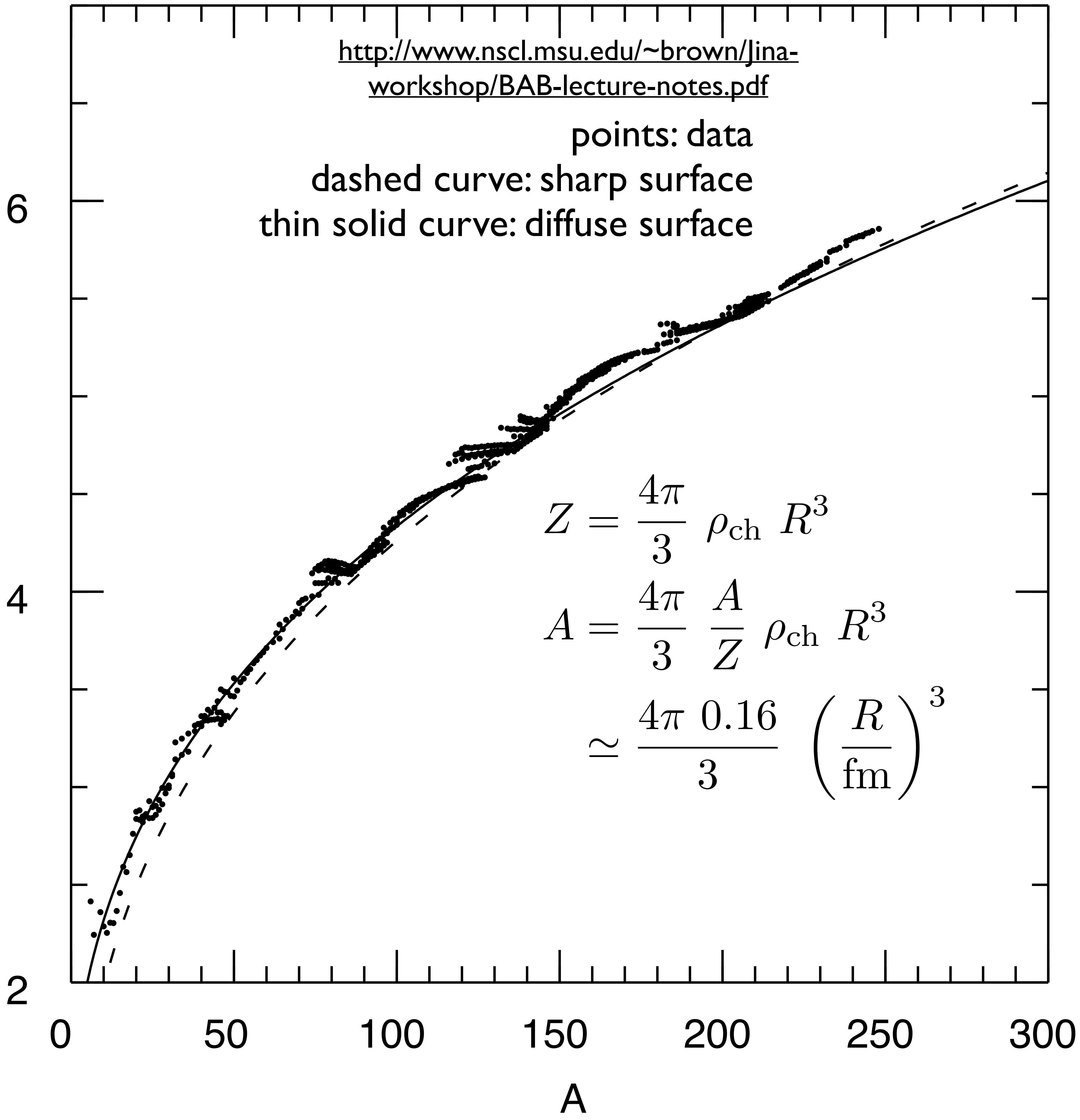
$$\boxed{\mathrm{BE}(A,Z) = \alpha_{\mathrm{bulk}}\ A - \alpha_{\mathrm{sym}}\ \frac{(N-Z)^2}{A} - \alpha_S\ A^{2/3} - \alpha_C\ \frac{Z^2}{A^{1/3}}}$$

$$\alpha_{\mathrm{bulk}} = 15.49\ \mathrm{MeV}$$
$$\alpha_{\mathrm{sym}} = 22.6\ \mathrm{MeV}$$
$$\alpha_{\mathrm{S}} = 17.23\ \mathrm{MeV}$$
$$\alpha_C = 0.697\ \mathrm{MeV}$$



Nuclei as drops of nuclear matter





Nuclear saturation density $n_0 \approx 0.16$ fm^{-3}

Volume energy/nucleon $\approx$ -16 MeV

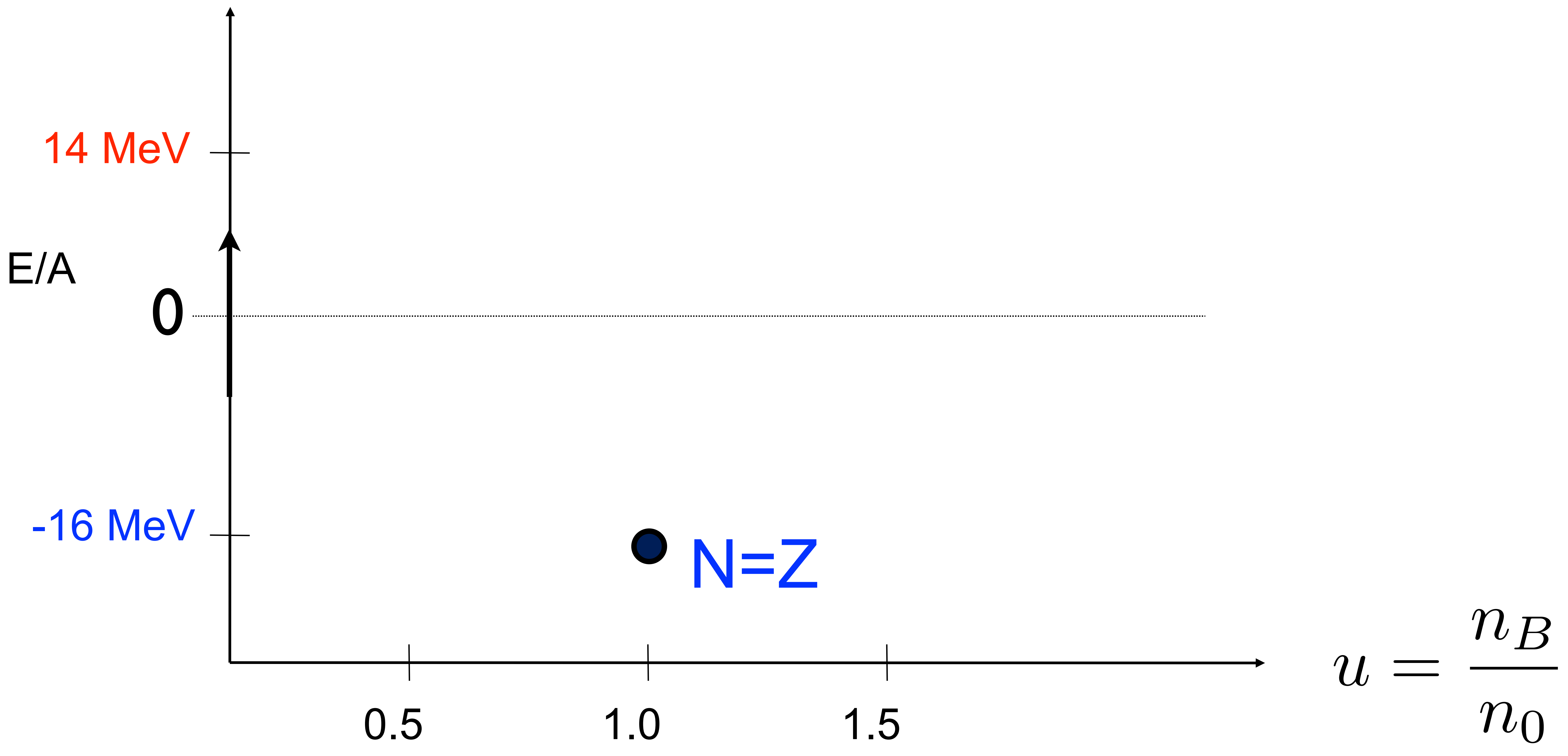
Symmetry energy/nucleon $\approx$ 28-32 MeV

Surface tension $\approx$ 1 MeV/fm^2

Coulomb energy $\approx$ 0.86 $(Z^2/A^{1/3})$ MeV

Energy of uniform nucleonic matter

$$E(n_n, n_p) = E_{\mathrm{SNM}}(n_B) + S(n_B) \left(\frac{n_n - n_p}{n_B}\right)^2 + \cdots$$



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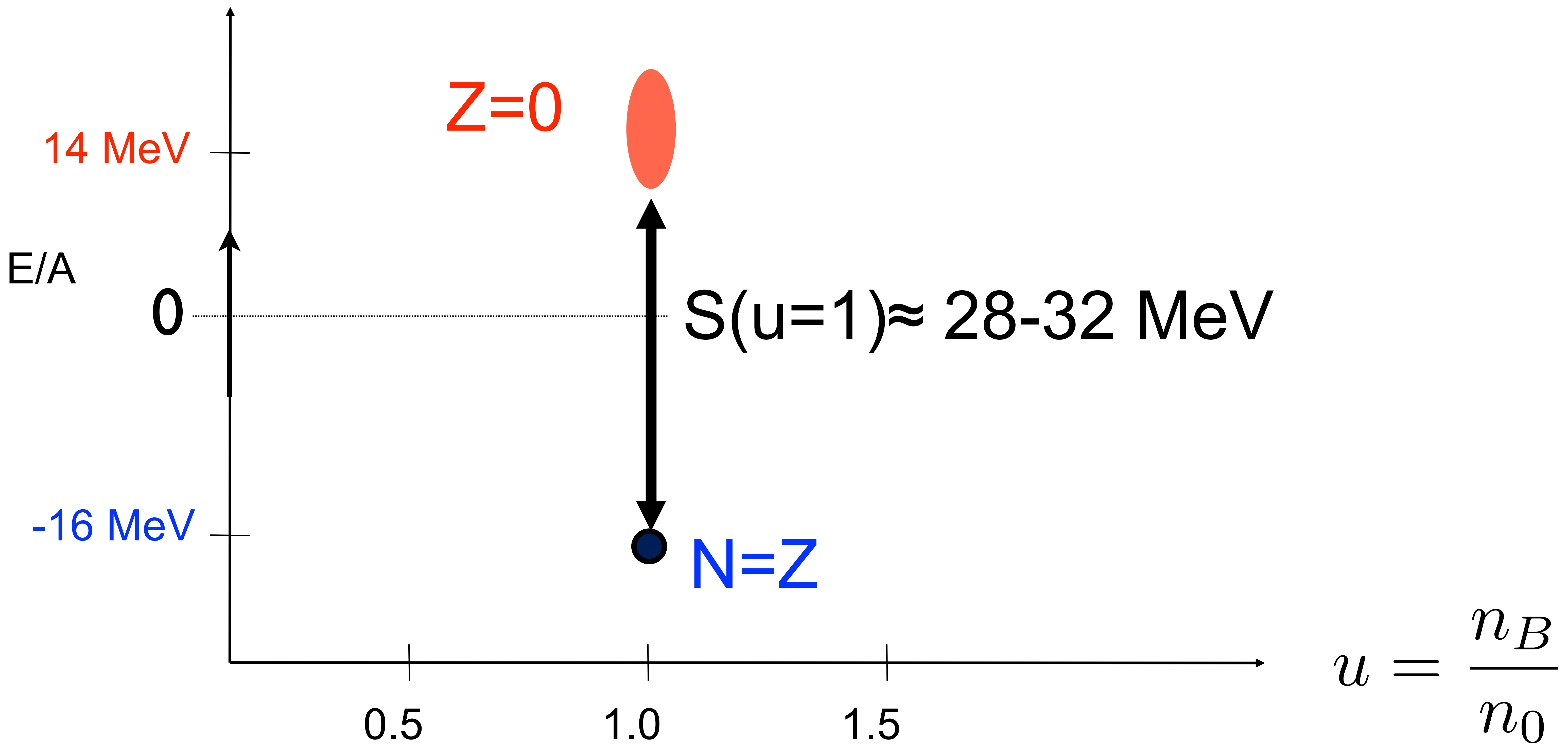

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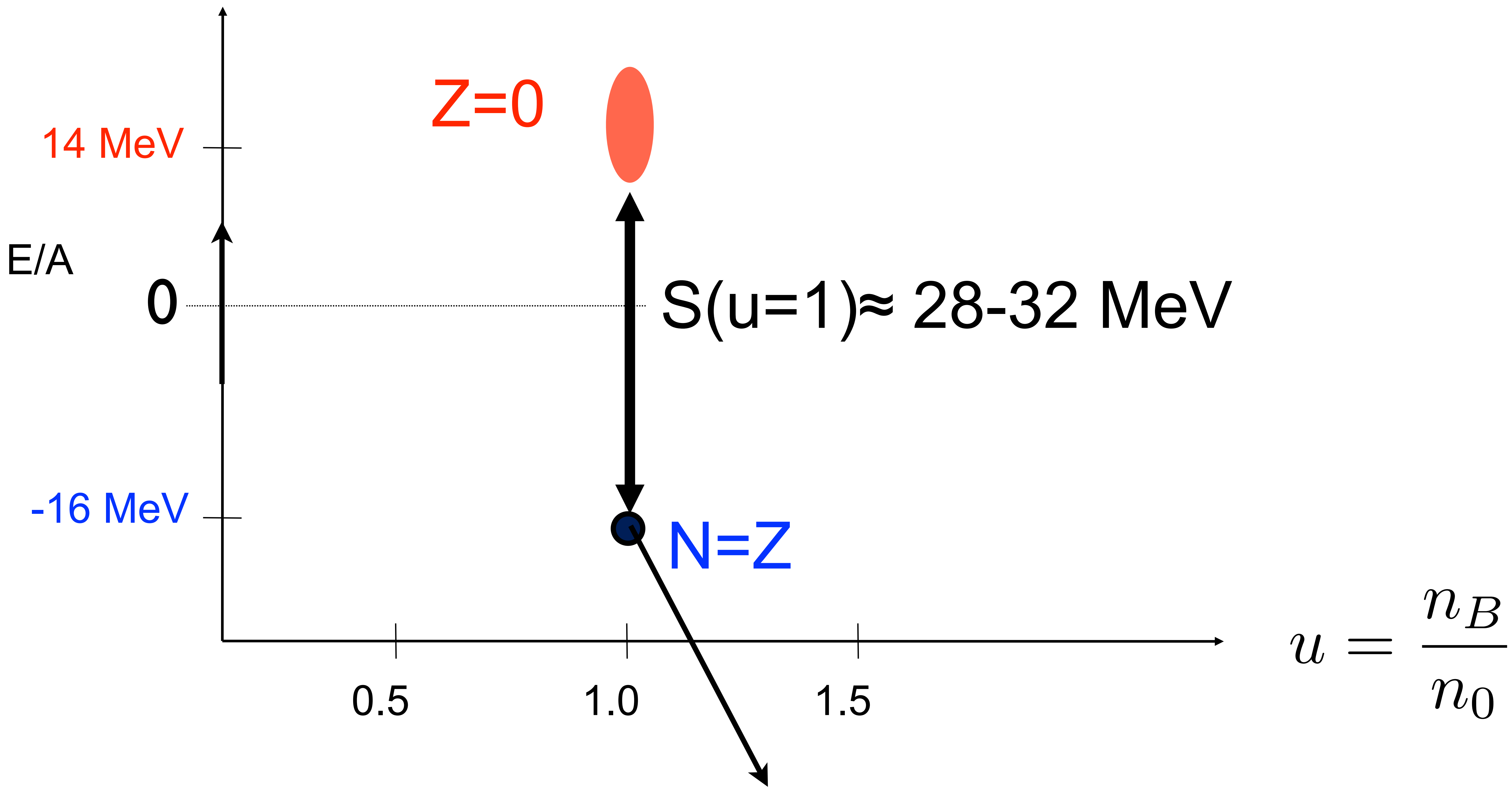

$$P(n) = n^2 \frac{\partial(E/A)}{\partial n} \quad \Rightarrow \quad P(n_0) = 0$$

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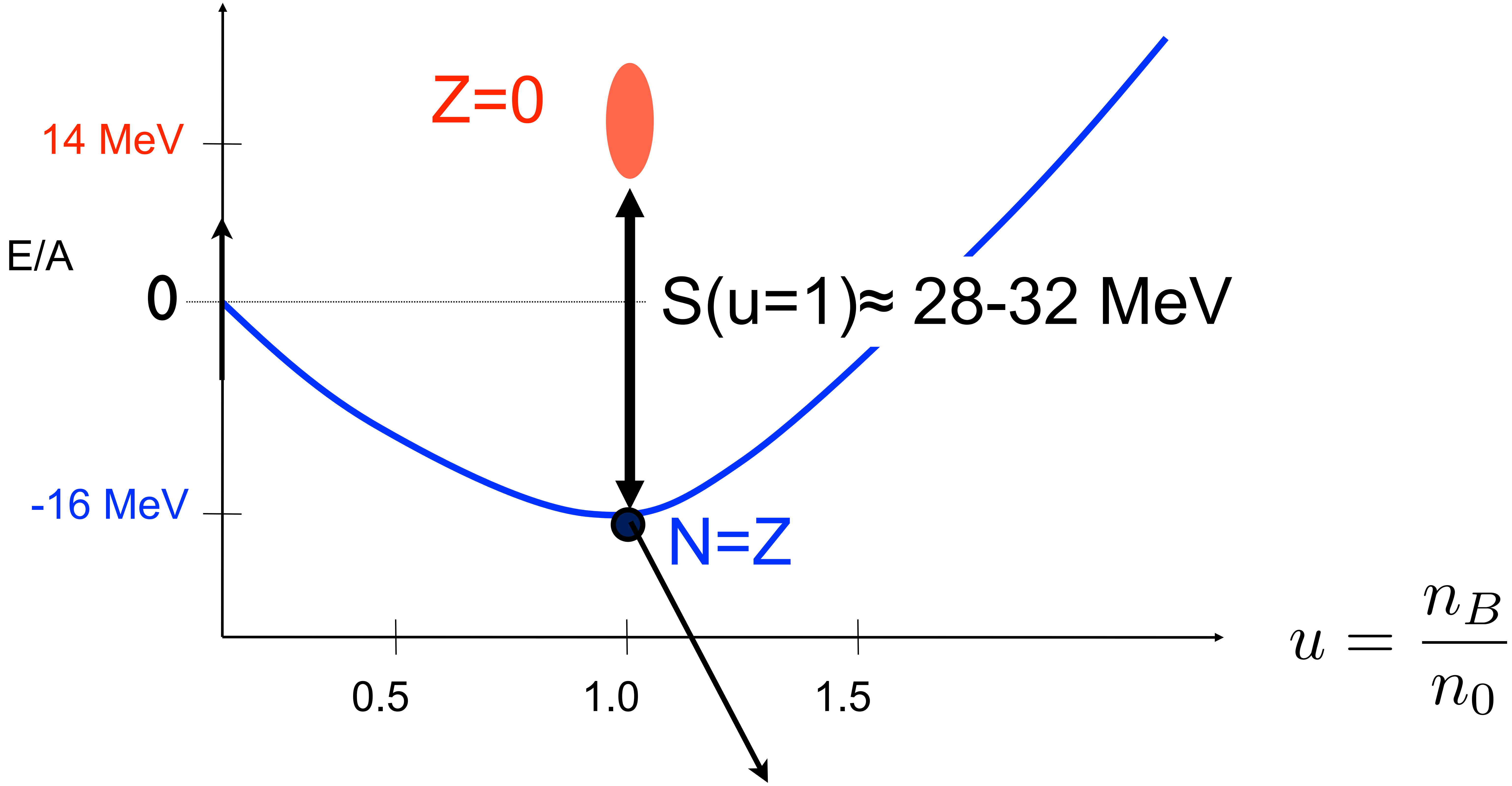

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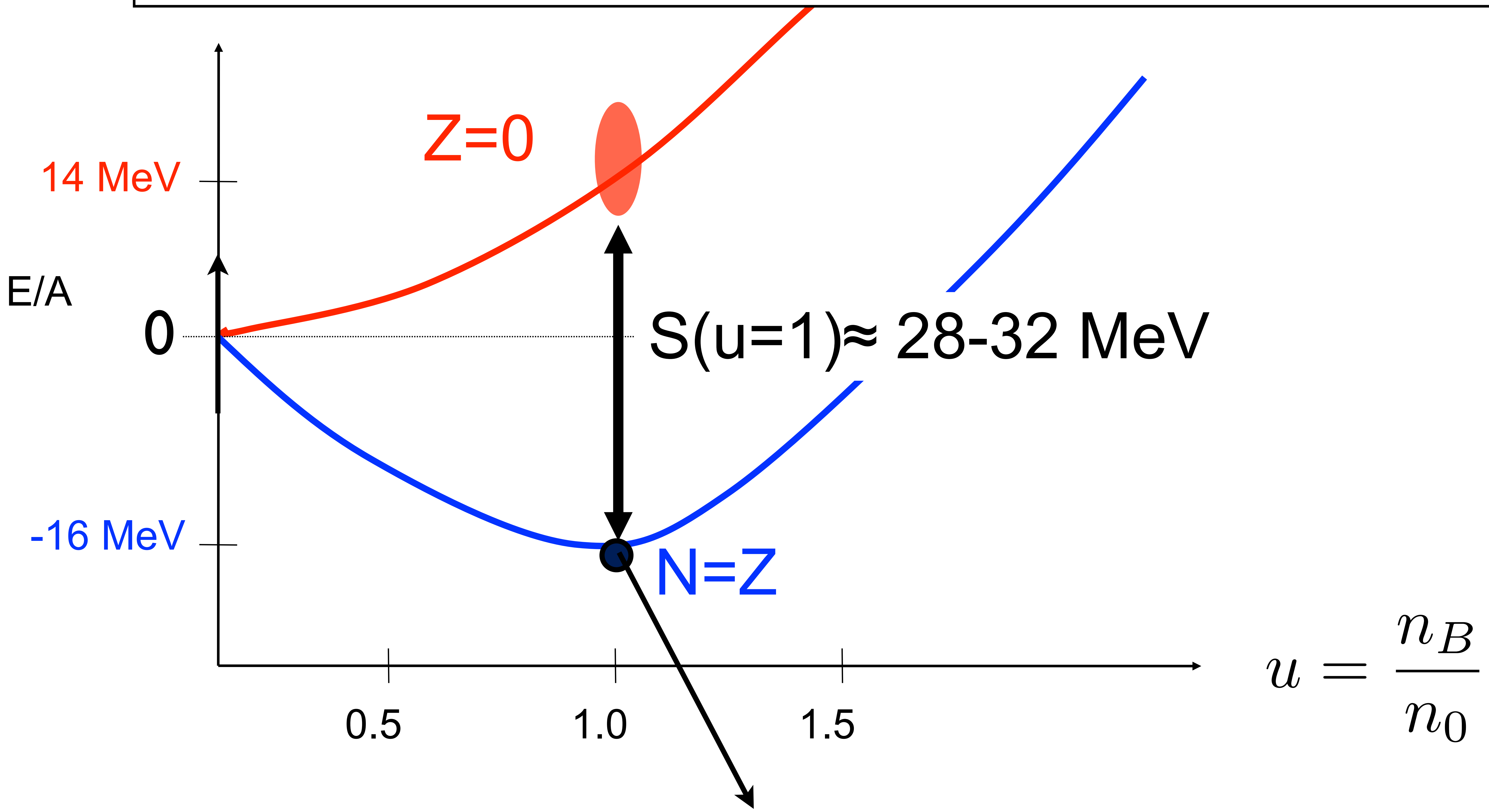

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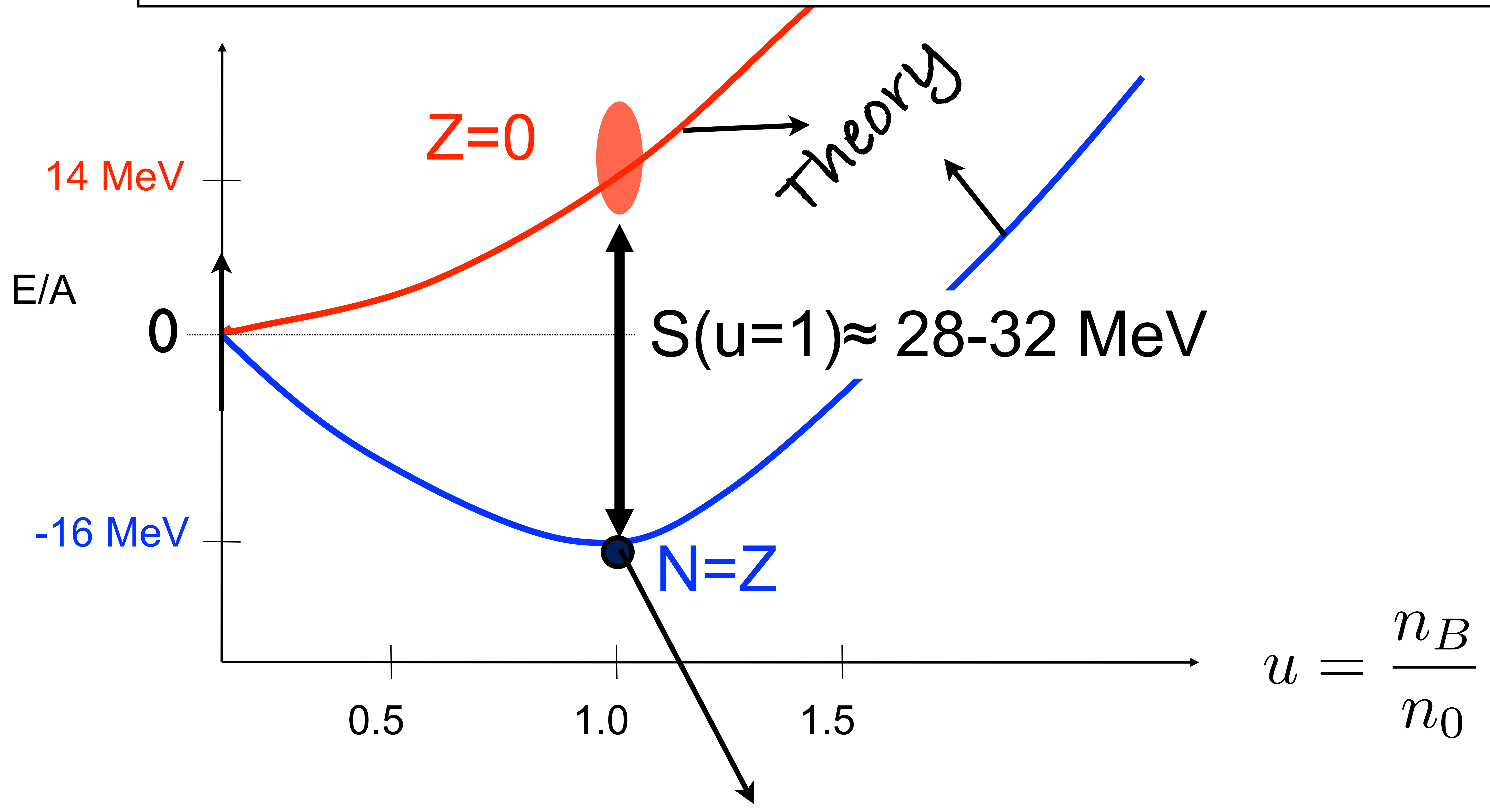

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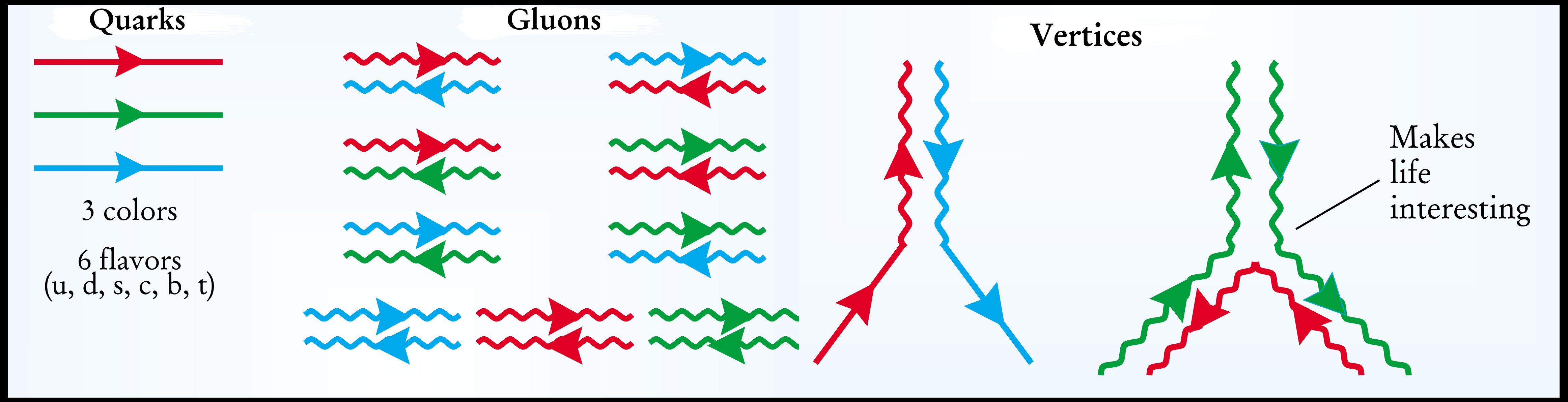
Nuclear Interactions

QCD (Lagrangian) is simple is write down

$$\mathcal{L} = \frac{1}{4g^2} G^{\alpha}_{\mu\nu} G^{\alpha}_{\mu\nu} + \sum_j \bar{q}_j (i\gamma^{\mu} D_{\mu} + m_j) q_j$$



F. Wilczek, Physics Today (2000)

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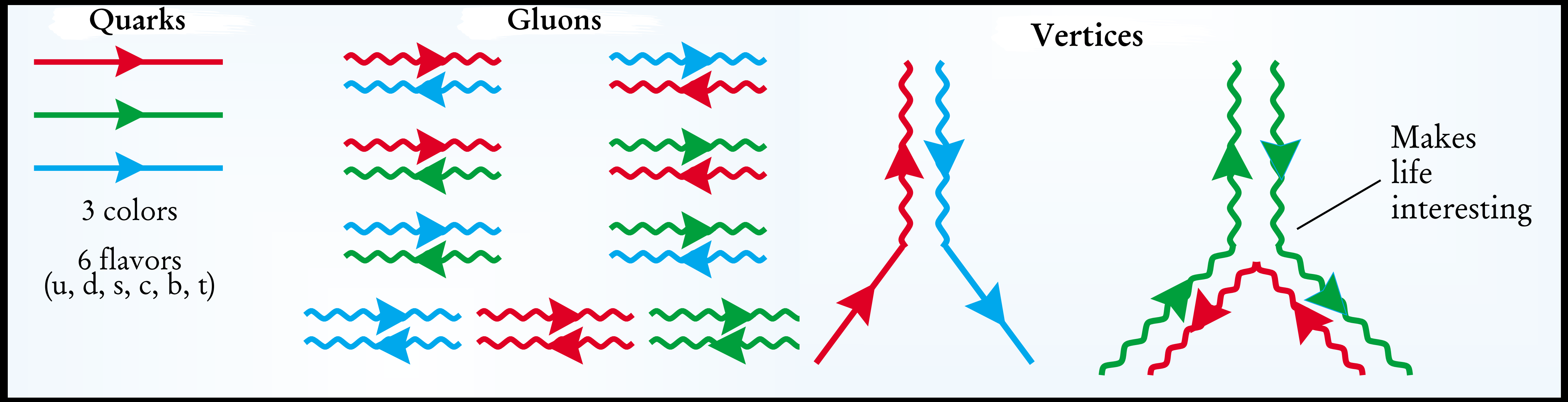

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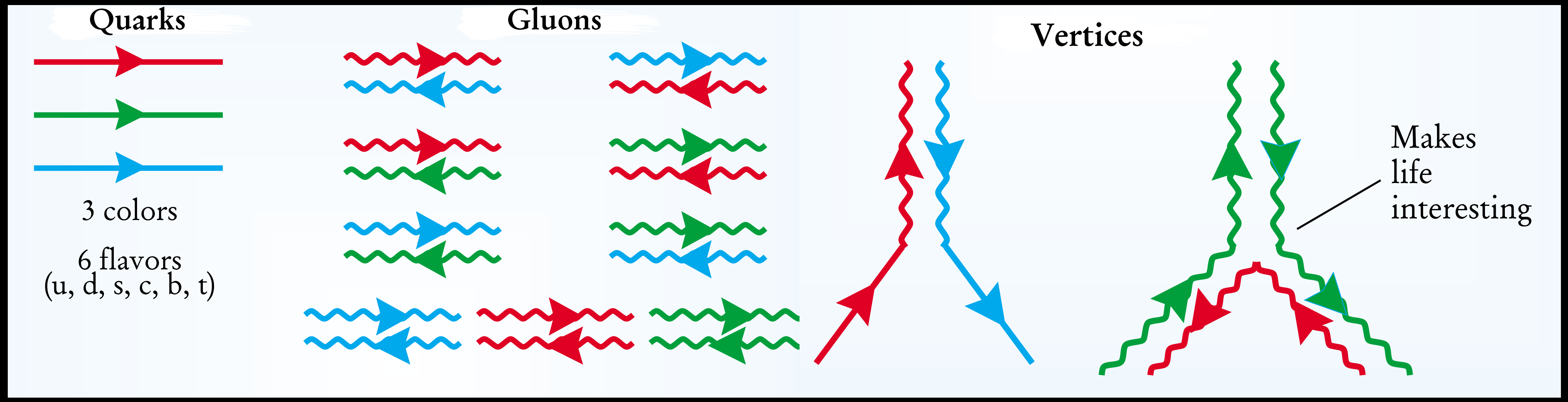

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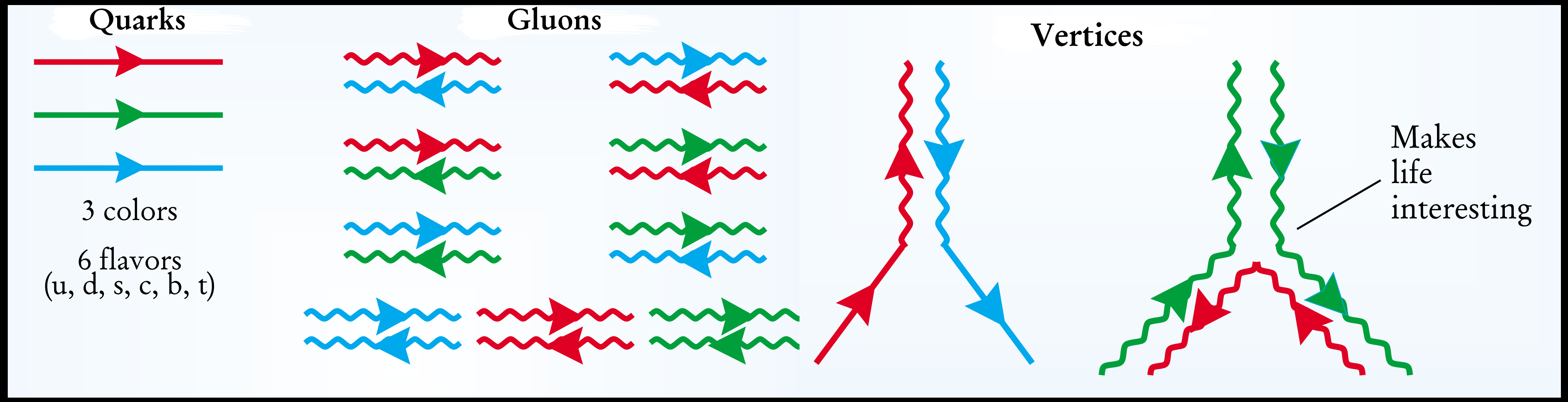

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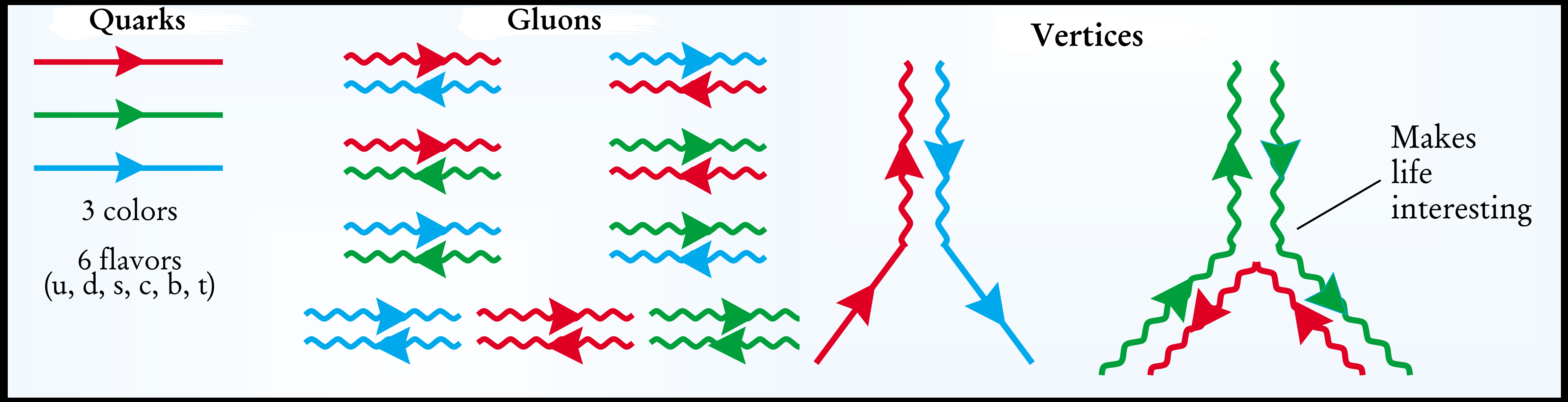

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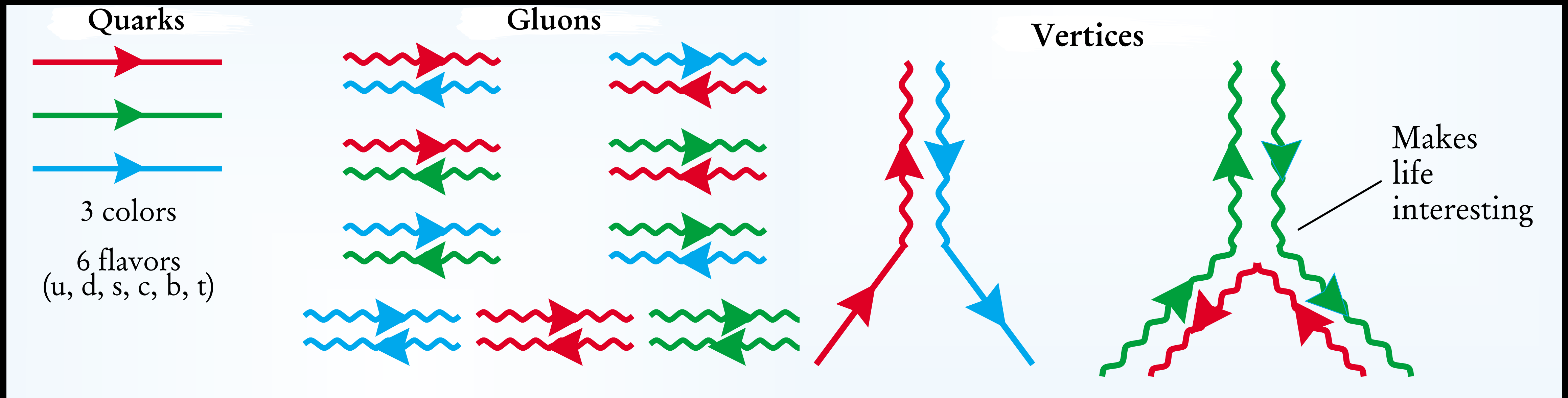

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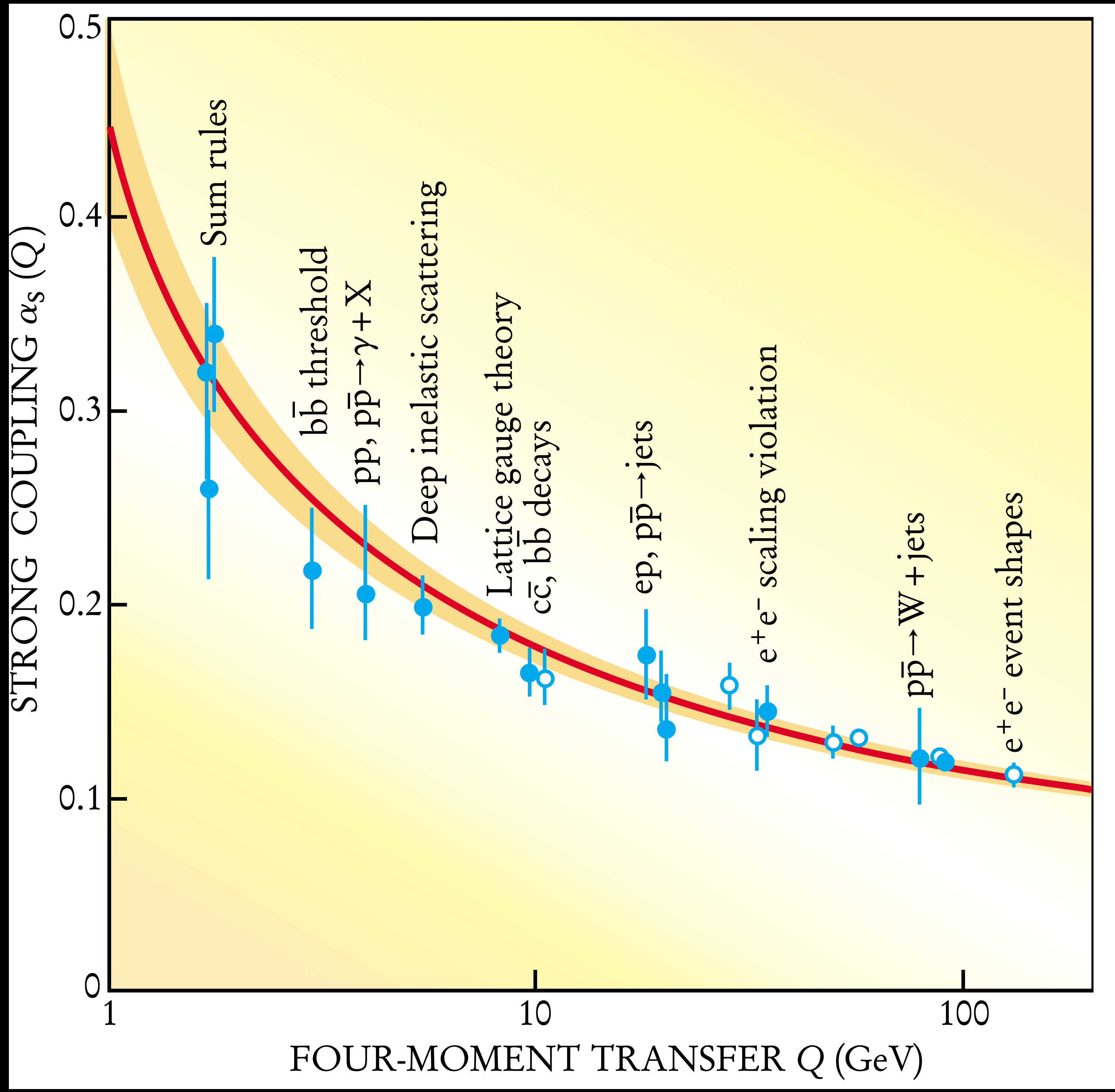

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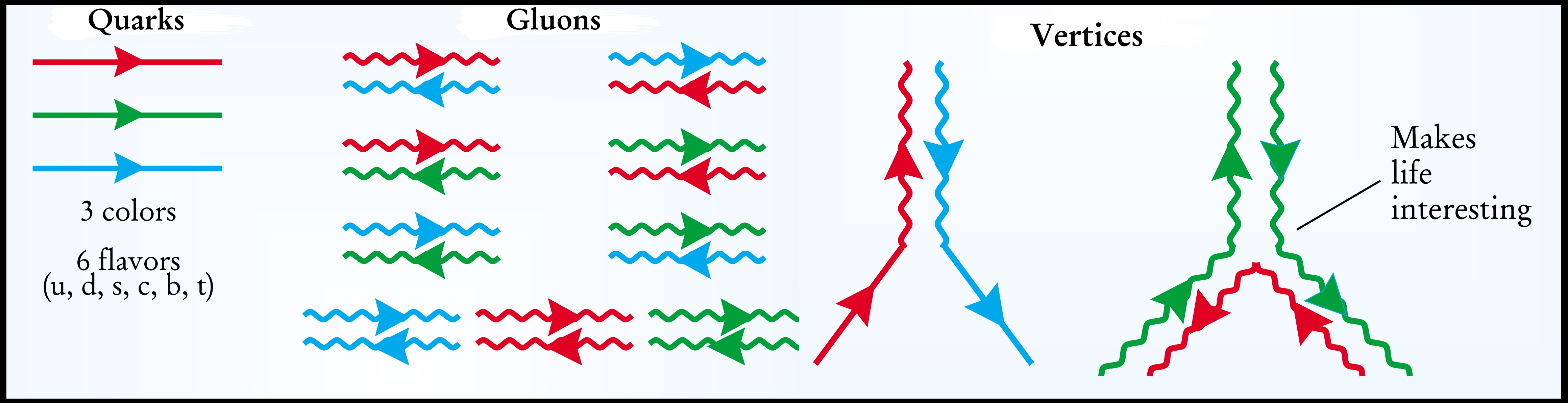

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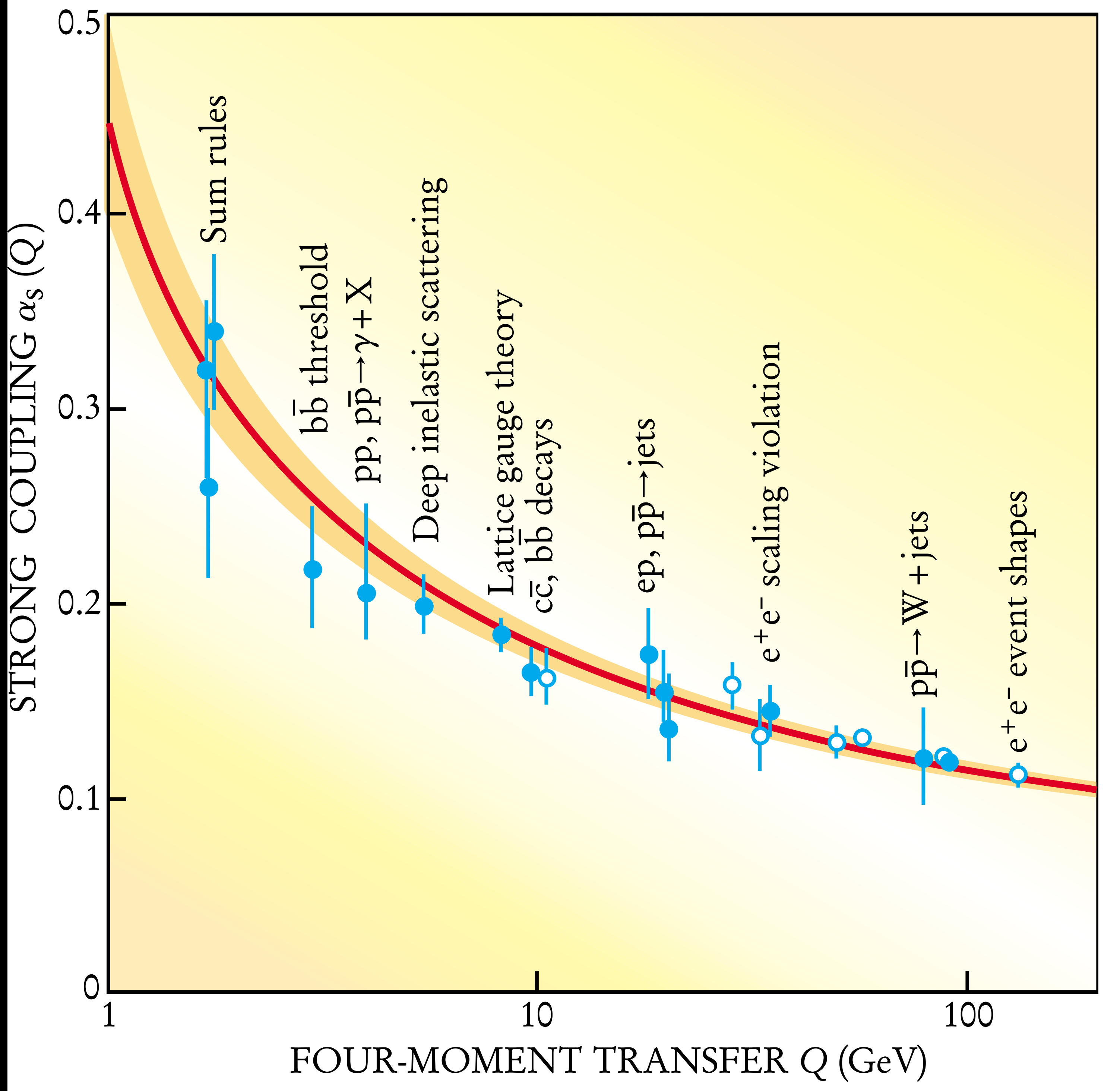


but is difficult to solve at low energy.

It gets simpler at high energy (asymptotic freedom).

The low energy QCD vacuum is non-perturbative:

- It confines quarks to color singlet states.
- Spontaneously breaks chiral symmetry.

Nuclear Interactions

- Baryons and mesons are the relevant low energy degrees of freedom at low energy. Interactions between them are strong, complex, and short-range.
- Pions are special. They are the Goldstone bosons associated with chiral symmetry breaking and provide the longest range force between nucleons.
- Other mesons are significantly heavier. It is not very useful to single them out as mediators of the strong interaction between composite color singlet states.
- How then can we write down a theory of strong interactions between nucleons at low energy ?

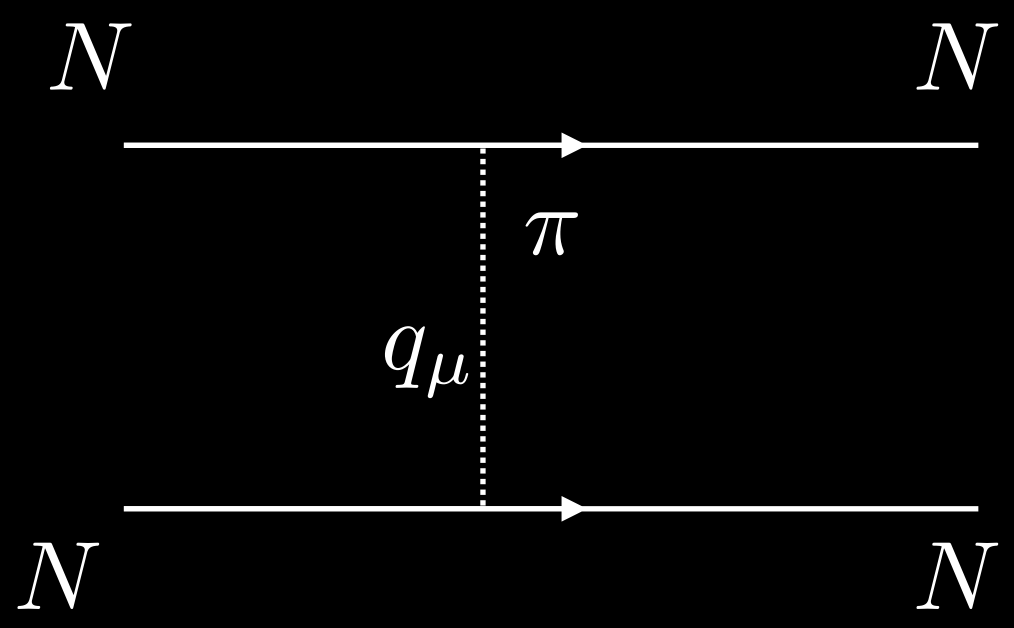
Potential Models

Effective Field Theories (EFT)

Nucleon-Nucleon Potentials

One-pion exchange:

$$\mathcal{L}_{NN\pi} = \psi_N^\dagger \left(i\partial_t - \frac{\nabla^2}{2M_N} \right) \psi_N \; - \frac{g_a}{f_\pi} \; \psi_N^\dagger \tau^a \sigma \cdot \nabla \pi^a \psi_N$$



$$V_\pi(q) = -\left(\frac{g_A}{\sqrt{2} f_\pi} \right)^2 \; \tau_1 \cdot \tau_2 \; \frac{\sigma_1 \cdot \mathbf{q} \; \sigma_2 \cdot \mathbf{q}}{q^2 + m_\pi^2}$$

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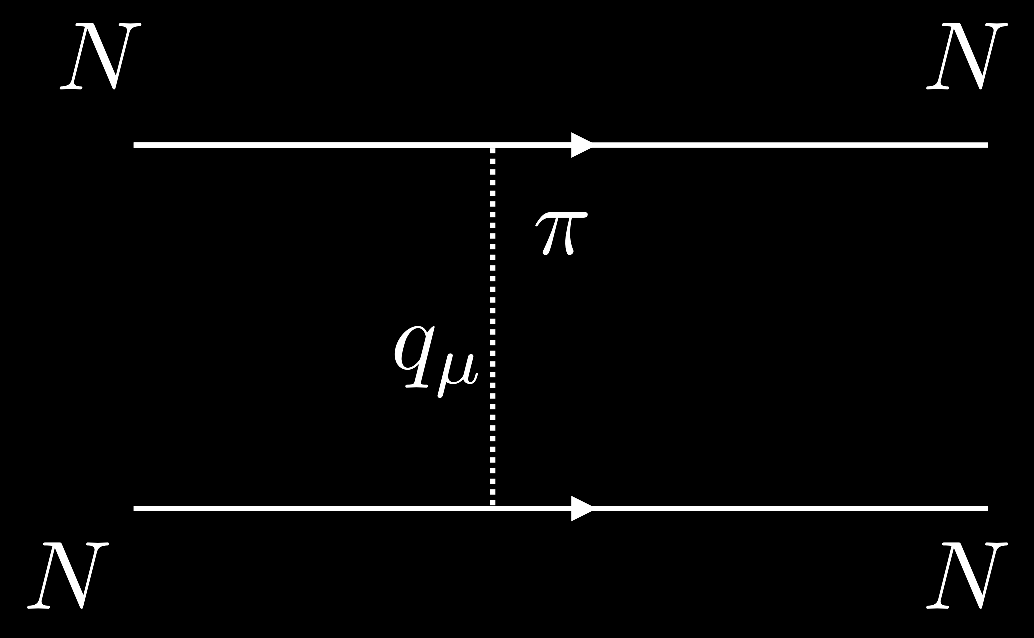

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In coordinate space the potential is

$$V_\pi(q) = -\left(\frac{g_A}{\sqrt{2} f_\pi} \right)^2 \; \tau_1 \cdot \tau_2 \; \left[S_{12} \; \left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) \frac{e^{-m_\pi r}}{r} - \frac{4\pi}{3} \; \sigma_1 \cdot \sigma_2 \; \delta^3(r) \right]$$

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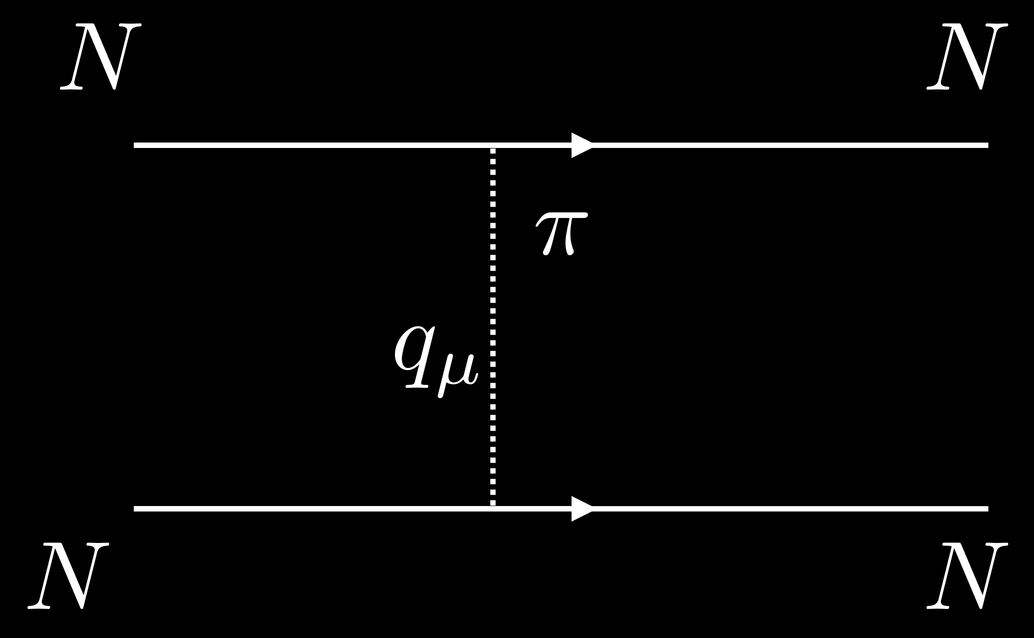

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Potential depends on spin and iso-spin.

It has a tensor component: $S_{12} = 3(\sigma_1 \cdot \hat{r}_1) \; (\sigma_2 \cdot \hat{r}_2) - \sigma_1 \cdot \sigma_2$

It is singular: $V(r \to 0) \approx \dfrac{1}{r^3}$

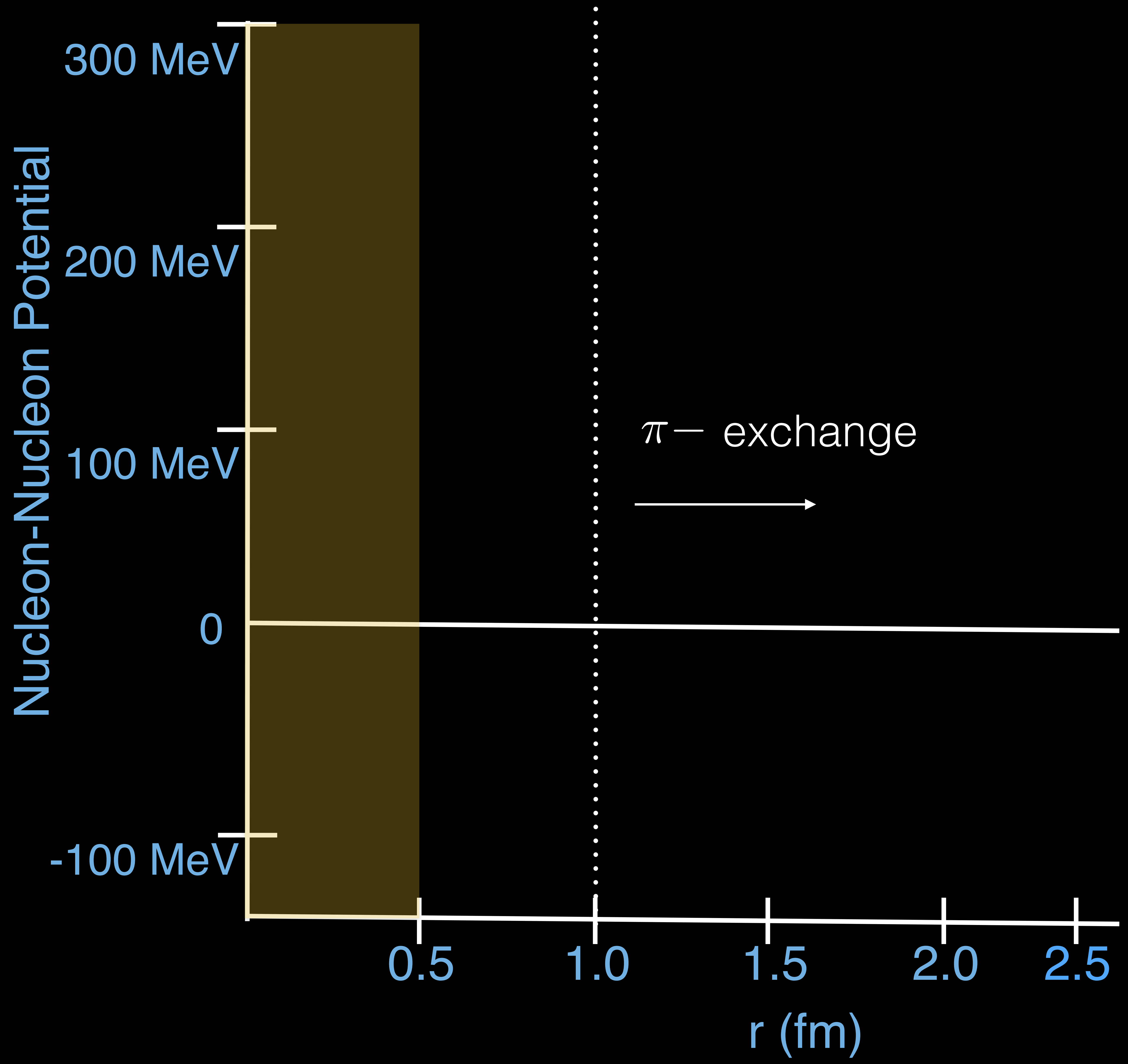
Nuclear Forces at Short Distances

They are essential even at low energy.

Are constrained by nucleon-nucleon scattering data (phase shifts).

Models favor strong repulsion. (hard-core)

Range of these forces is comparable to the intrinsic size of the nucleon.



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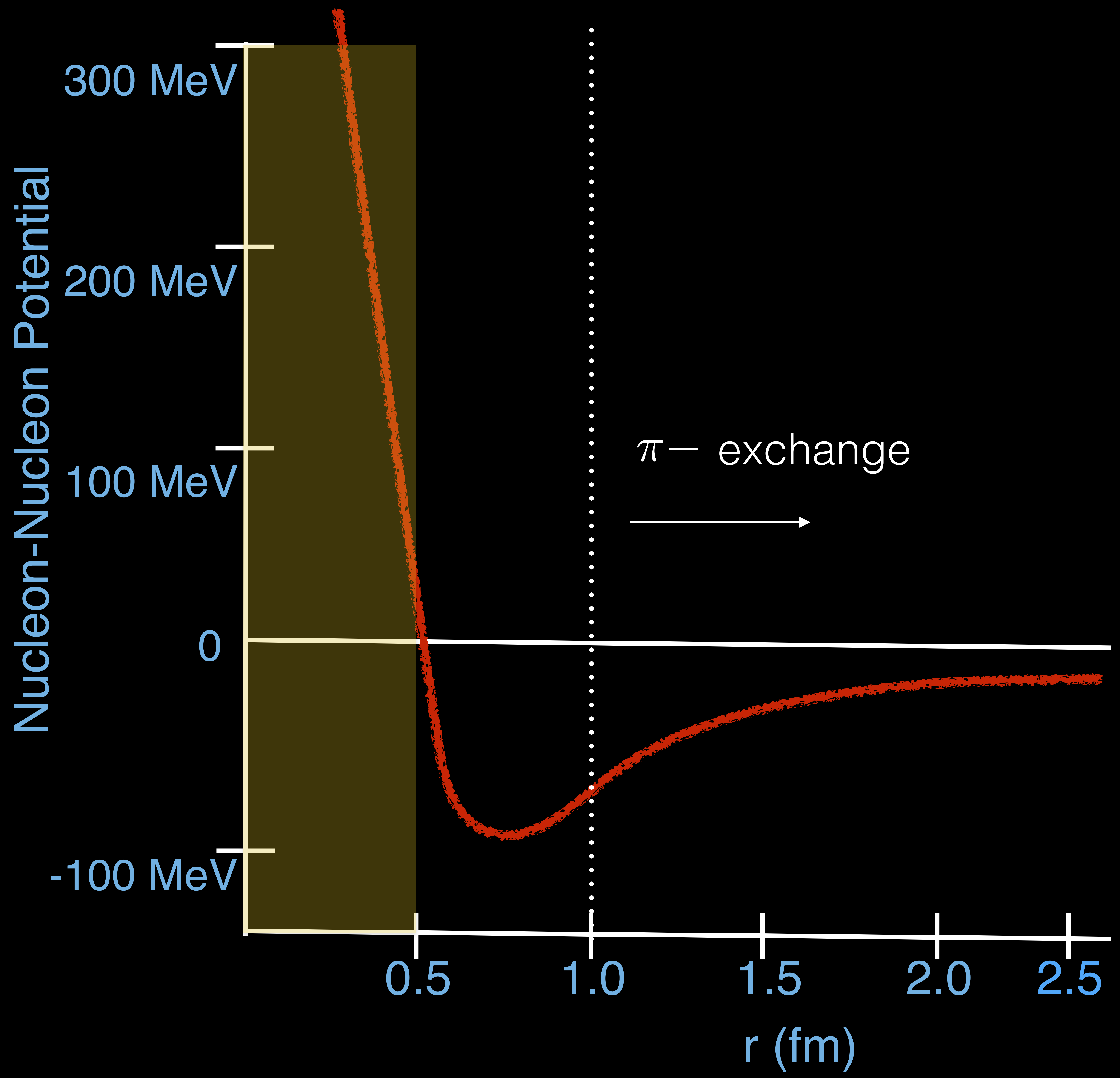

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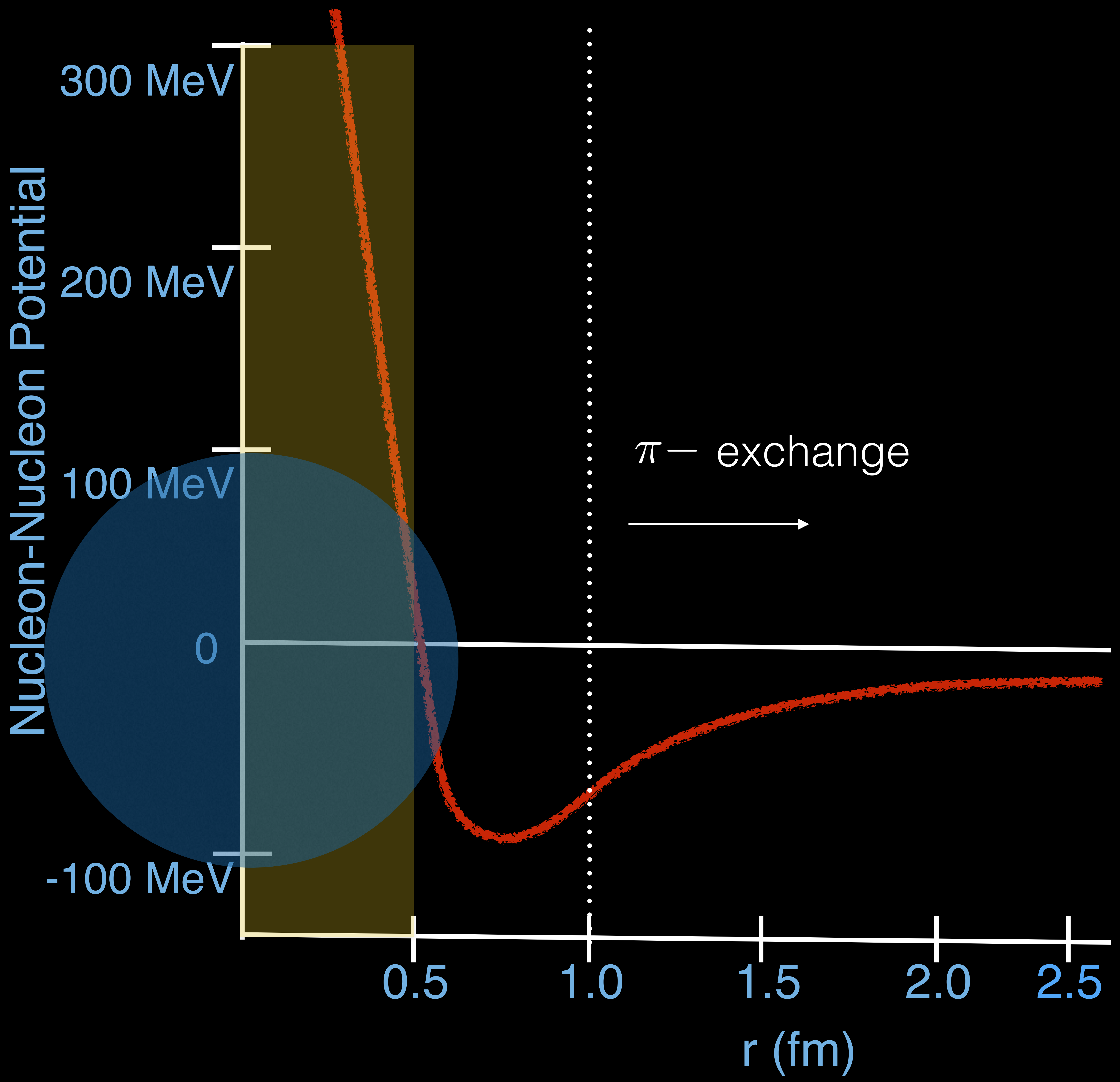

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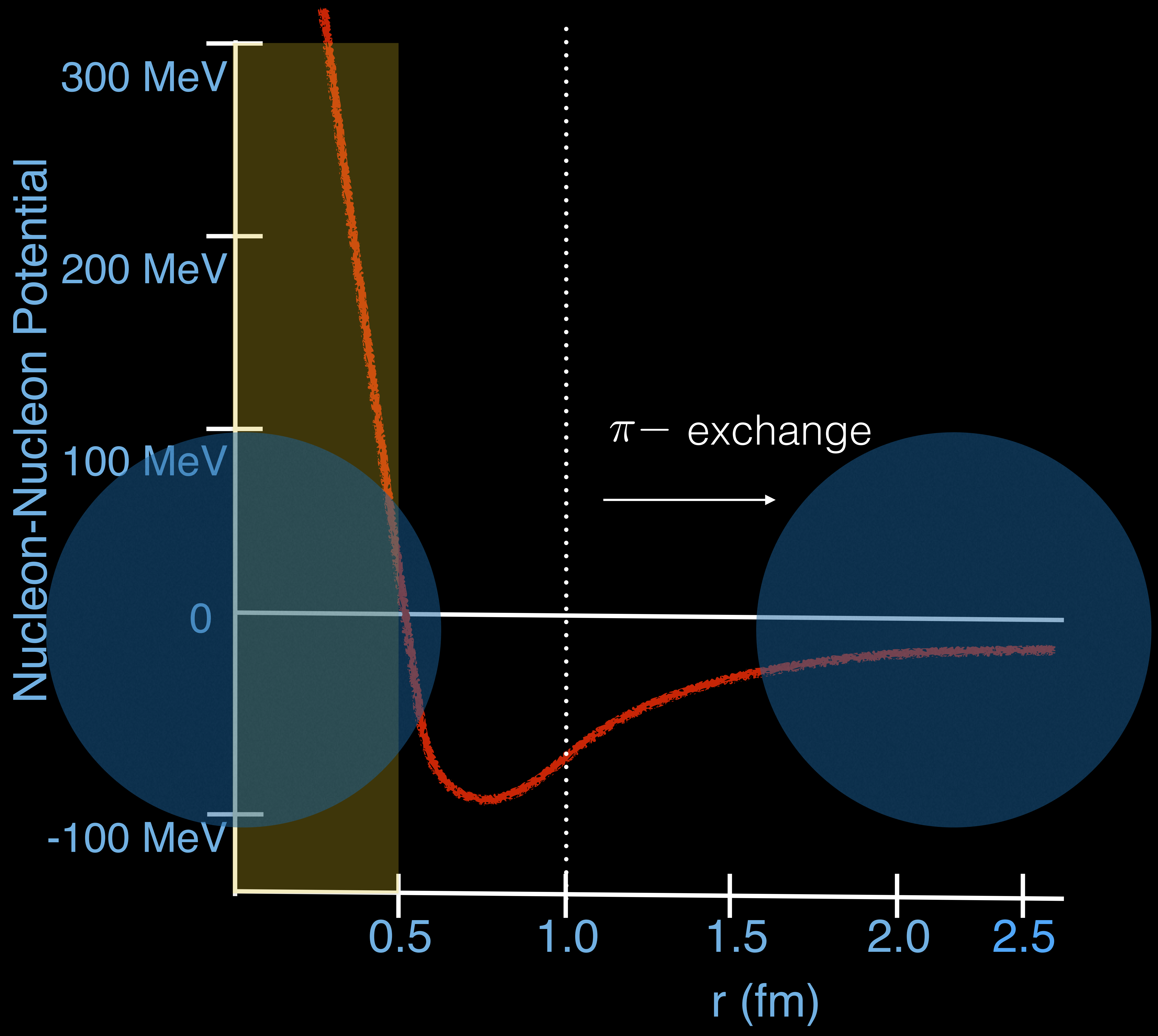

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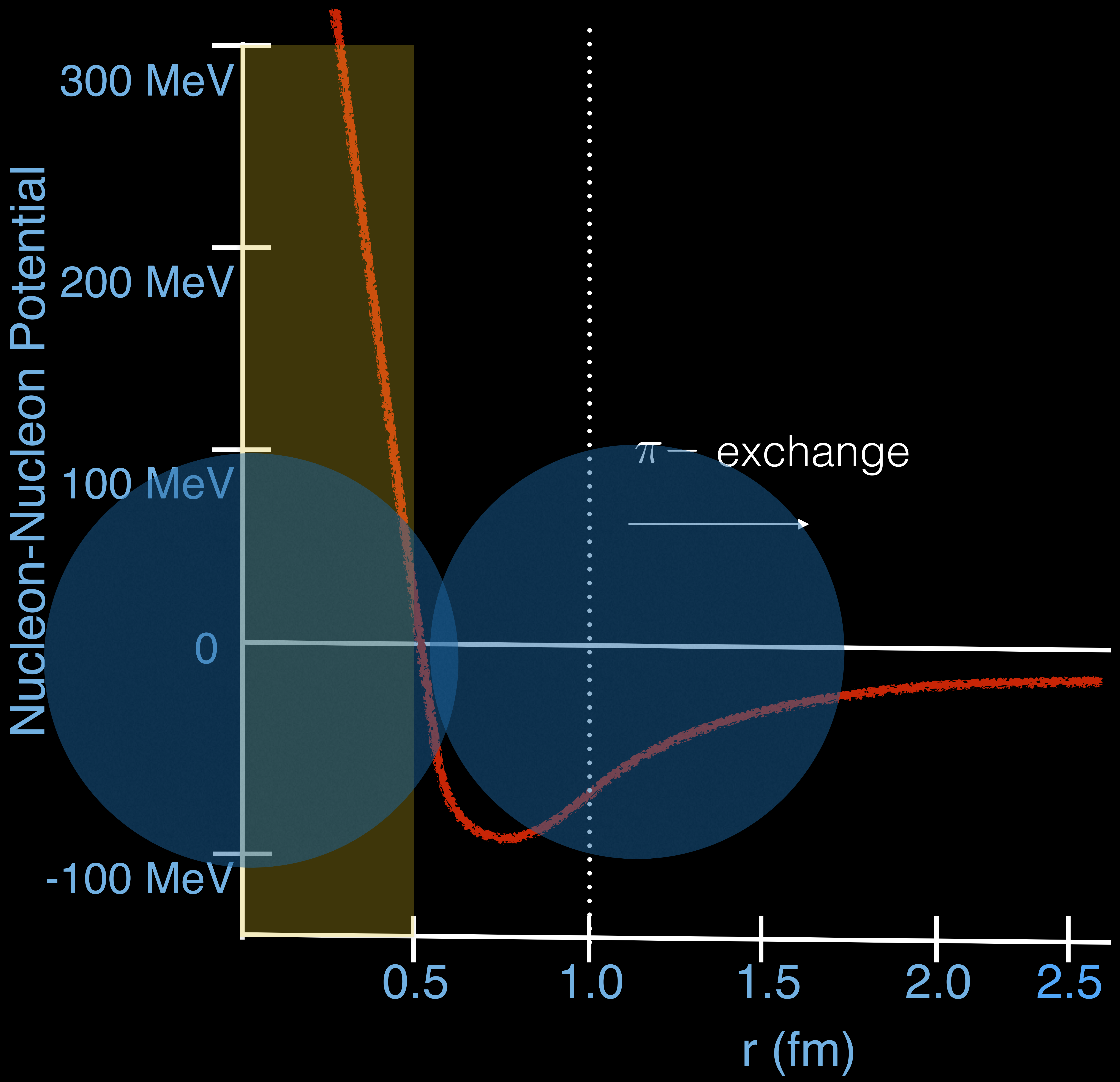


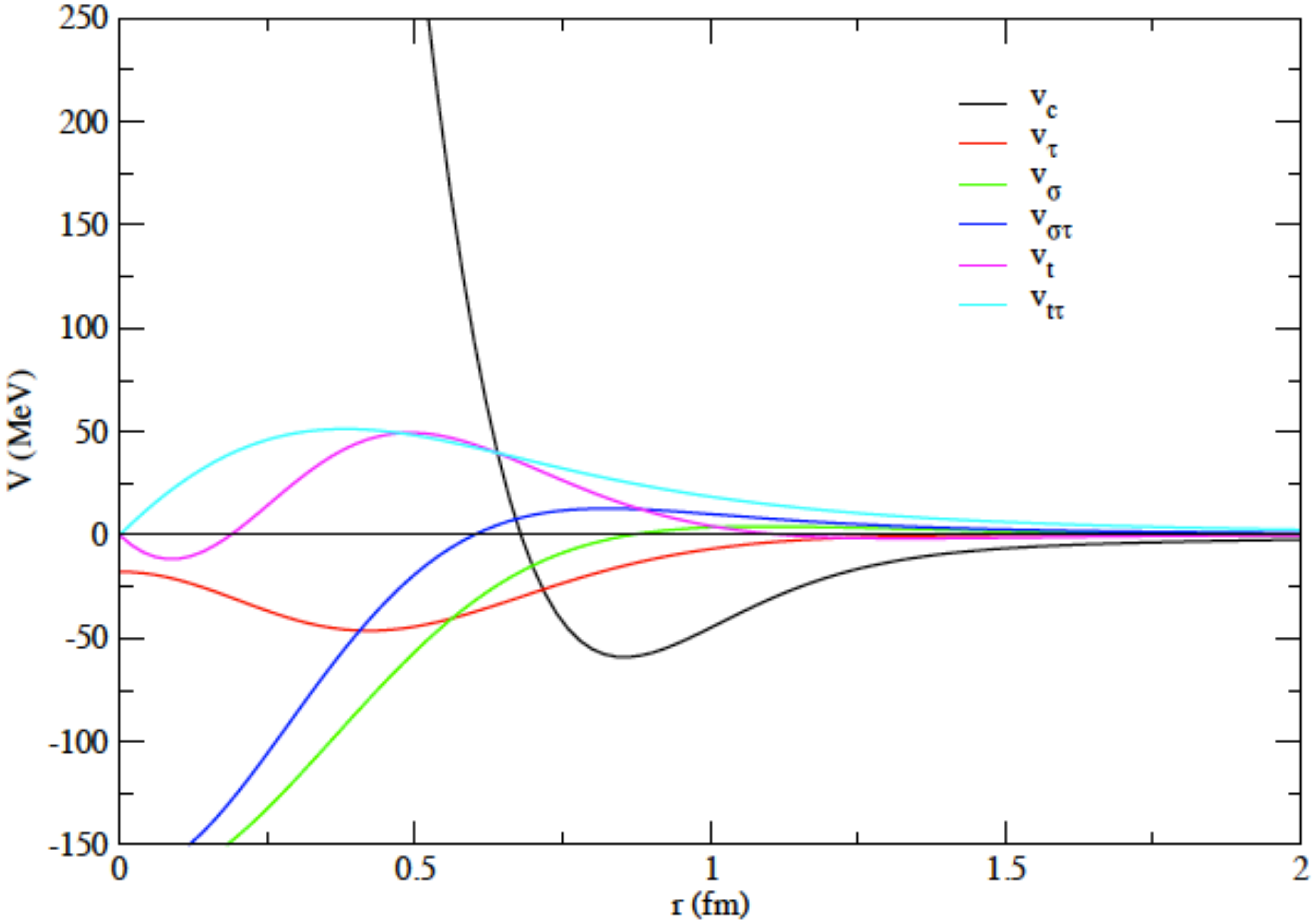
A Realistic Potential Model

$$V_{ij} = \sum_p v_p(r_{ij})\ O^p_{ij}$$

Intricate spin, isospin and tensor structure.

$$\begin{aligned} O^p_{ij} = & [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i \cdot \sigma_j), (\mathbf{L}\cdot\mathbf{S})^2] \\ & + [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\sigma_i \cdot \sigma_j), (\mathbf{L}\cdot\mathbf{S})^2] \otimes \tau_i \cdot \tau_j \\ & + [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}] \otimes T_{ij} \\ & + [1, \sigma_i \cdot \sigma_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}] \otimes (\tau_i + \tau_j)_z \end{aligned}$$

$$S_{ij} = 3\sigma_i \cdot \hat{r}_{ij}\sigma_j \cdot \hat{r}_{ij} - \sigma_i \cdot \sigma_j \qquad T_{ij} = 3\tau_{iz}\tau_{jz} - \tau_i \cdot \tau_j$$



Potential is Neither Unique Nor Observable (in QM)

Potential Models: Relies on a set of (reasonable) assumptions about the short distance behavior to solve the Schrödinger equation and fit observables.

Effective Field Theory: Relies on a separation of scales to Taylor expand potential in powers of momenta or inverse radial separation. Coefficients of the expansion are determined by fitting to observables.

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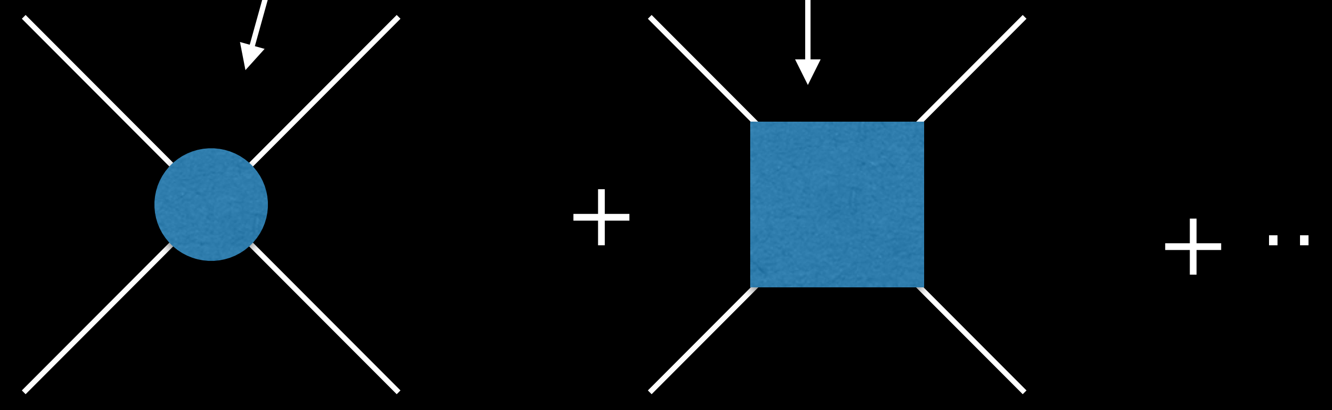
A simple (heuristic) EFT example:

$$V_\omega(q) = \frac{g_\omega^2}{q^2 + m_\omega^2} \approx \frac{g_\omega^2}{m_\omega^2} - \frac{g_\omega^2}{m_\omega^2}\frac{q^2}{m_\omega^2} + \ldots$$

Exchange of heavy bosons at low energy cannot be resolved.



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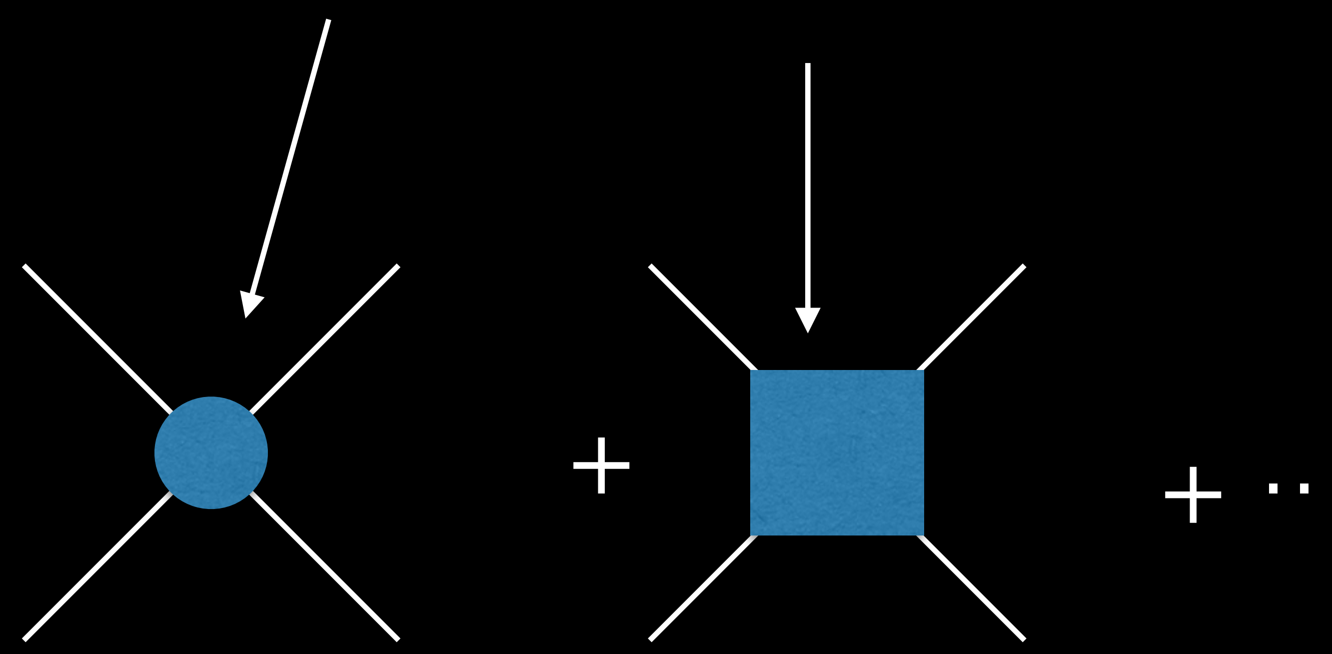
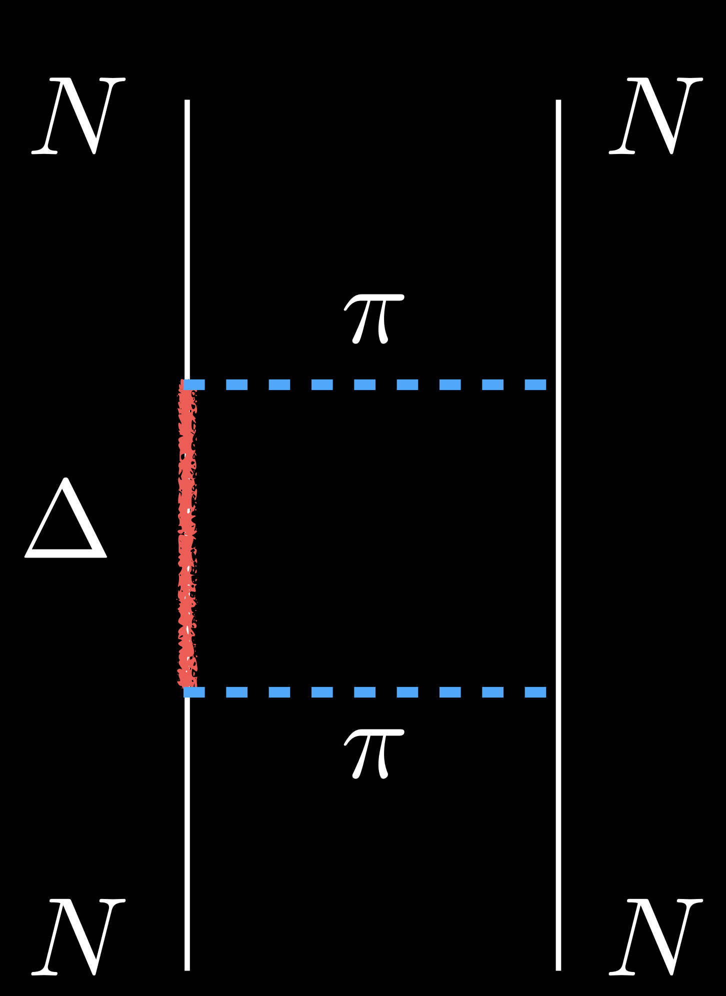


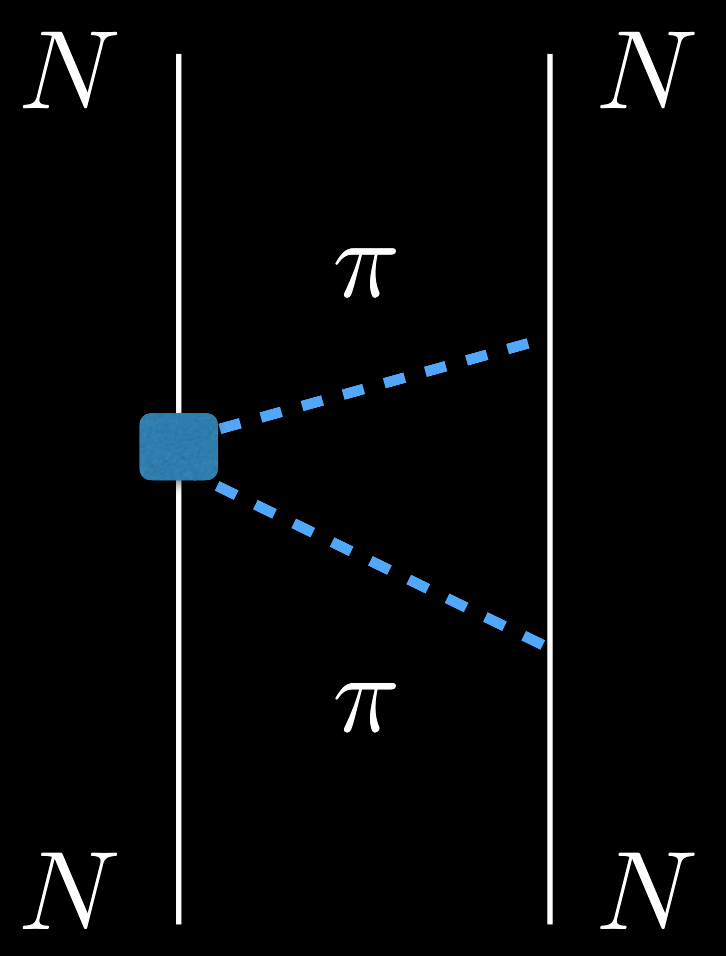
When several heavy particles may be exchanged, or when the underlying mechanism is unknown, the general expansion is

$$V_{\text{short}}(q) = C_0 + C_2\,\frac{q^2}{\Lambda^2} + \ldots$$

Nucleons are composite with internal excitations

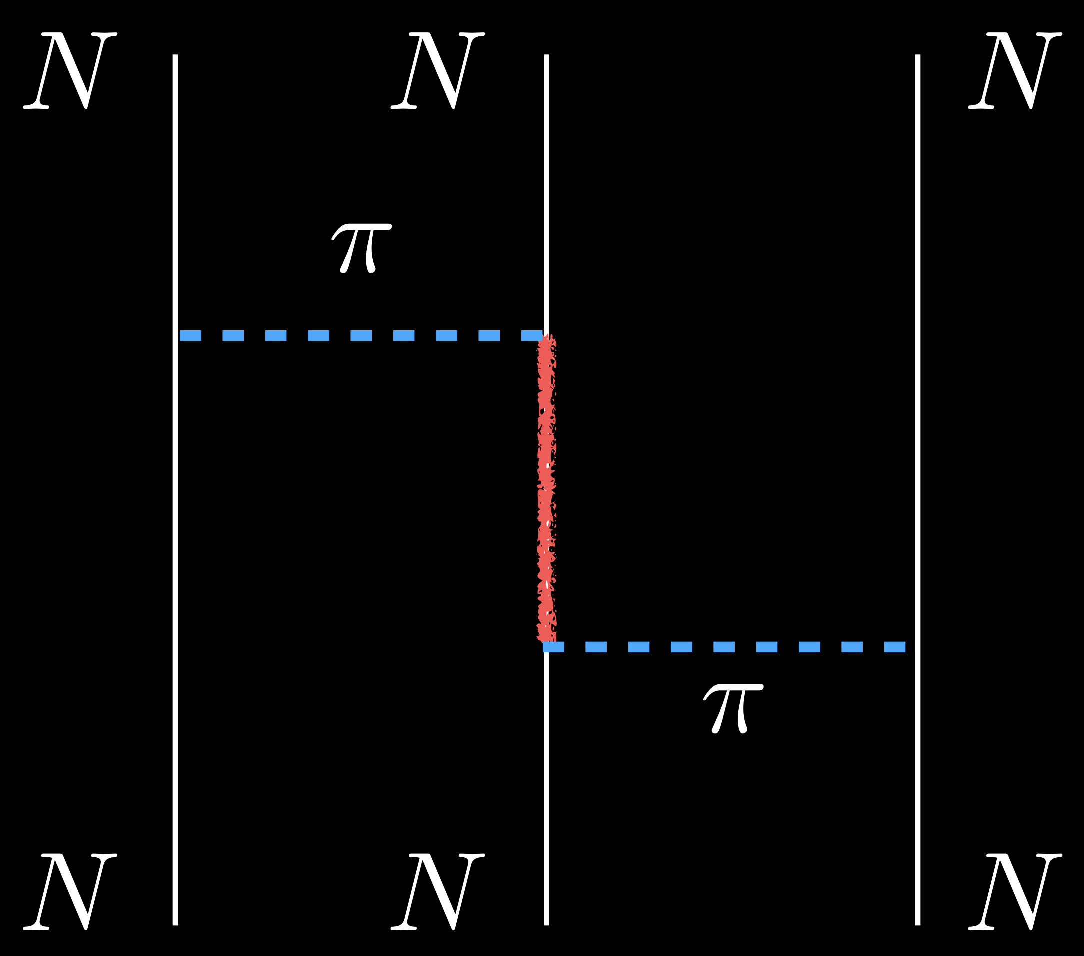


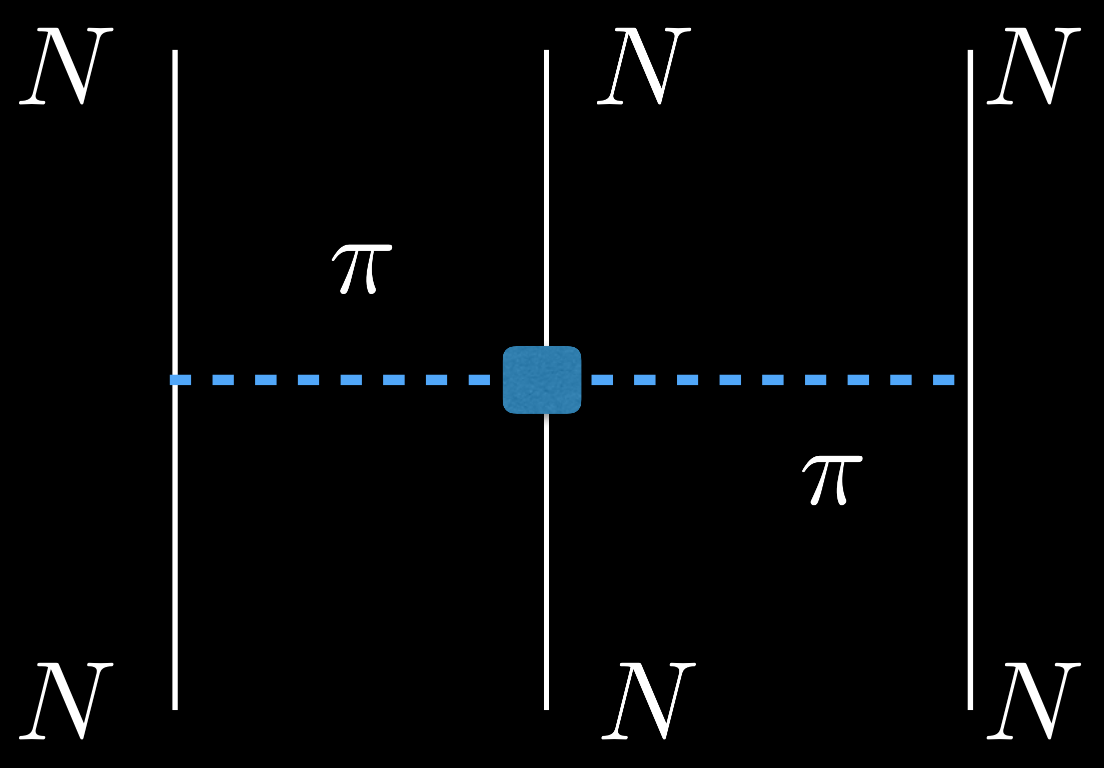
At low energy



$$m_\Delta - m_N \simeq 1232 - 939 \text{ MeV} \approx 300 \text{ MeV}$$

There are three and many-body forces:





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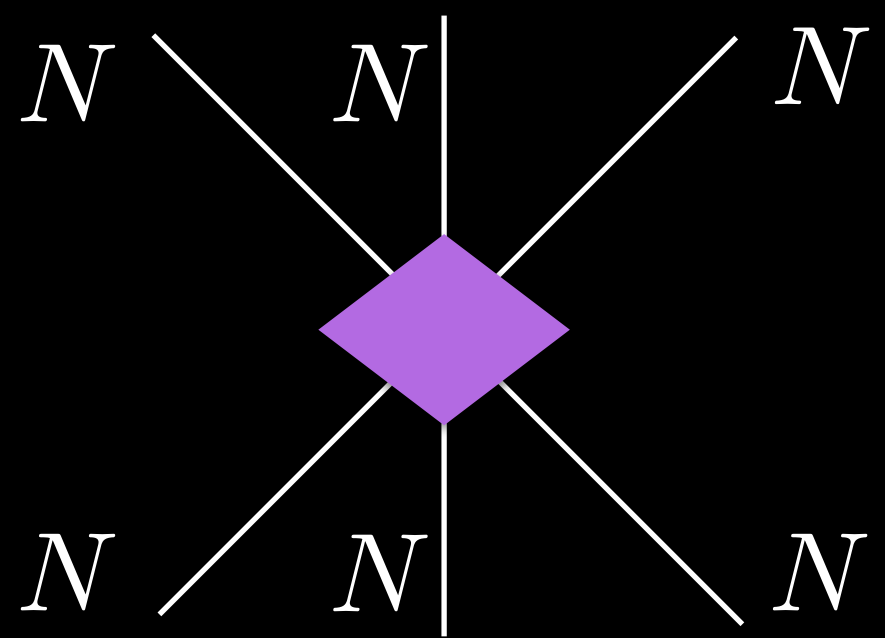
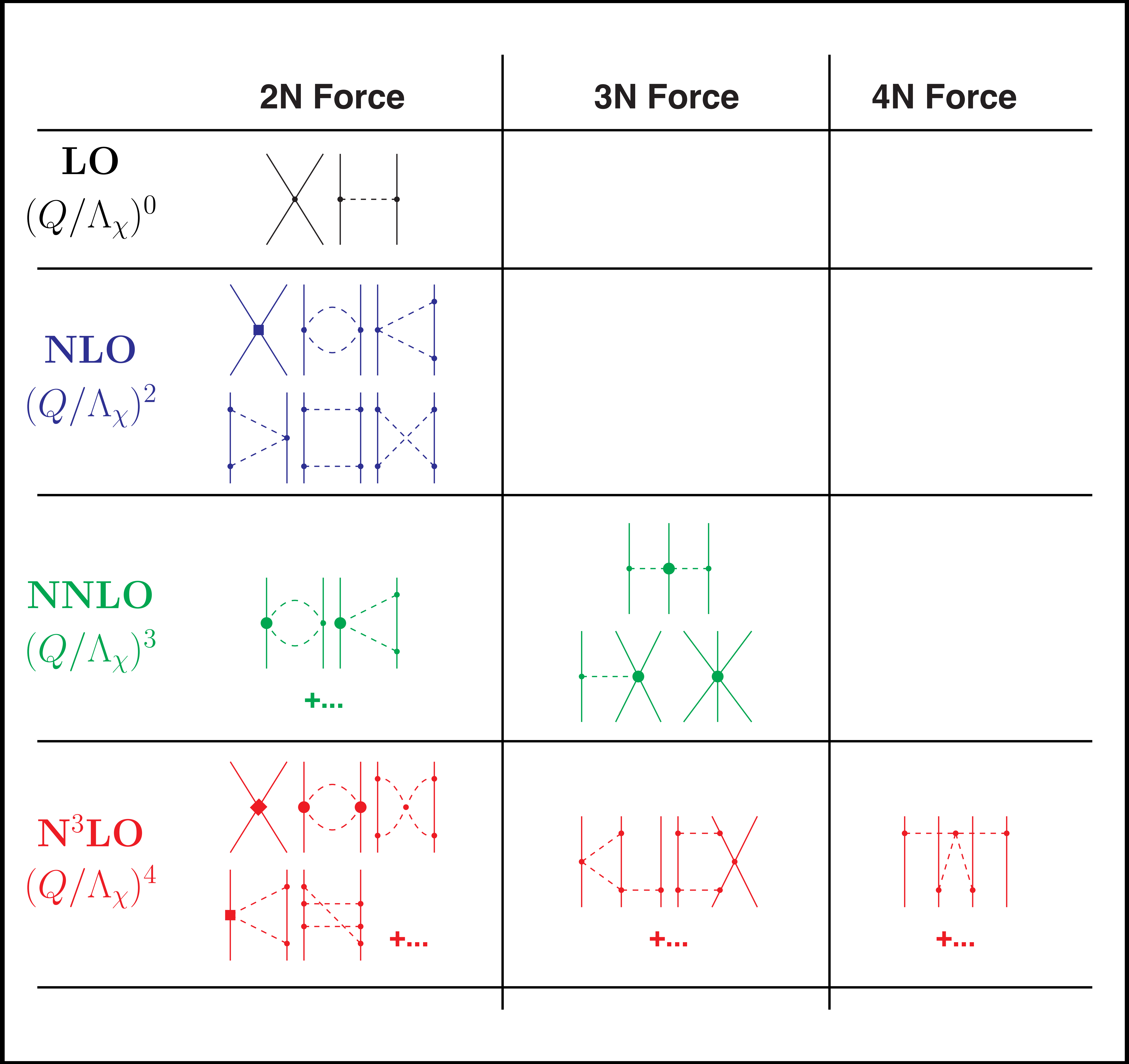


Chiral EFT

Systematic approach to low energy nuclear interactions.

Expectation is that the expansion will remain valid up to nuclear density.

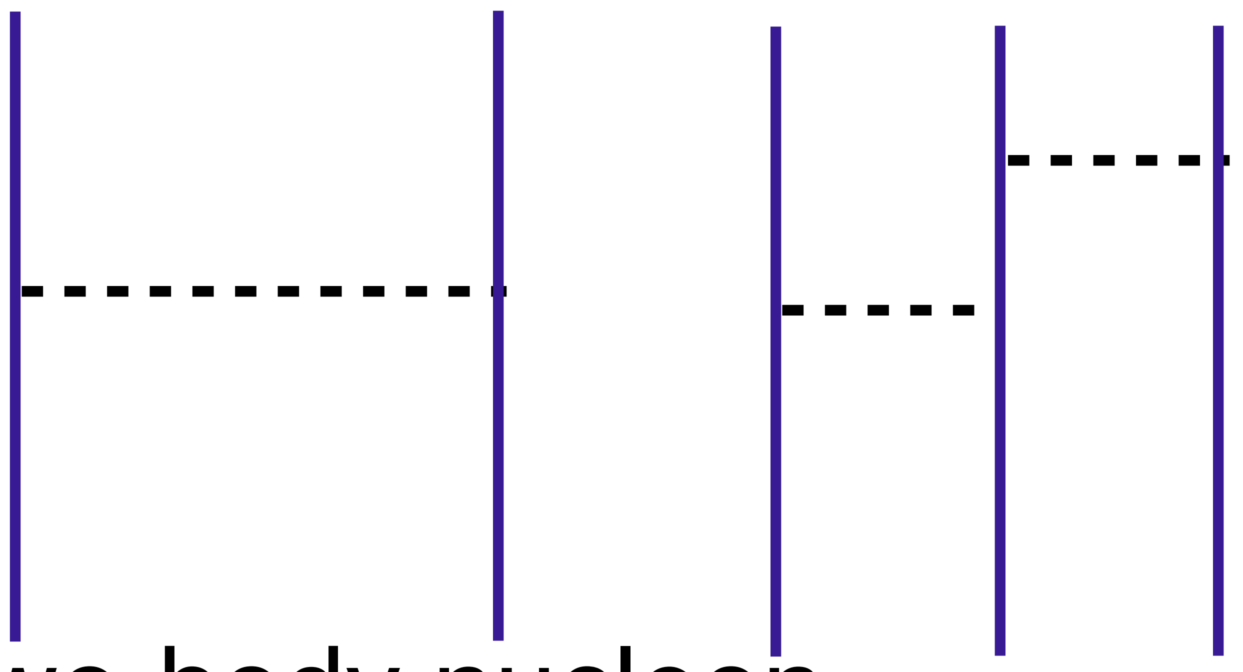
Consistent treatment of two, three and many-body forces.



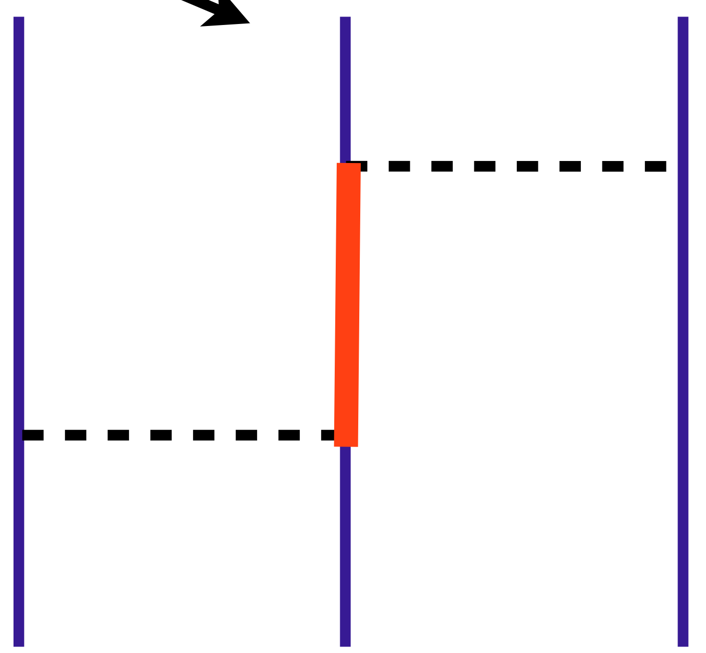
Weinberg (1990), Ordonez, Ray, van Kolck (1996), Kaplan, Savage, Wise (1996), Epelbaum, Meissner, Gloeckle (1999), Machleidt (2001) …

Ground State Energy

$$H_{\text{nuclear}} = \frac{\nabla^2}{2M} + V_{\text{NN}} + V_{\text{NNN}} + \cdots$$



two-body nucleon-nucleon potential is well constrained by scattering data.

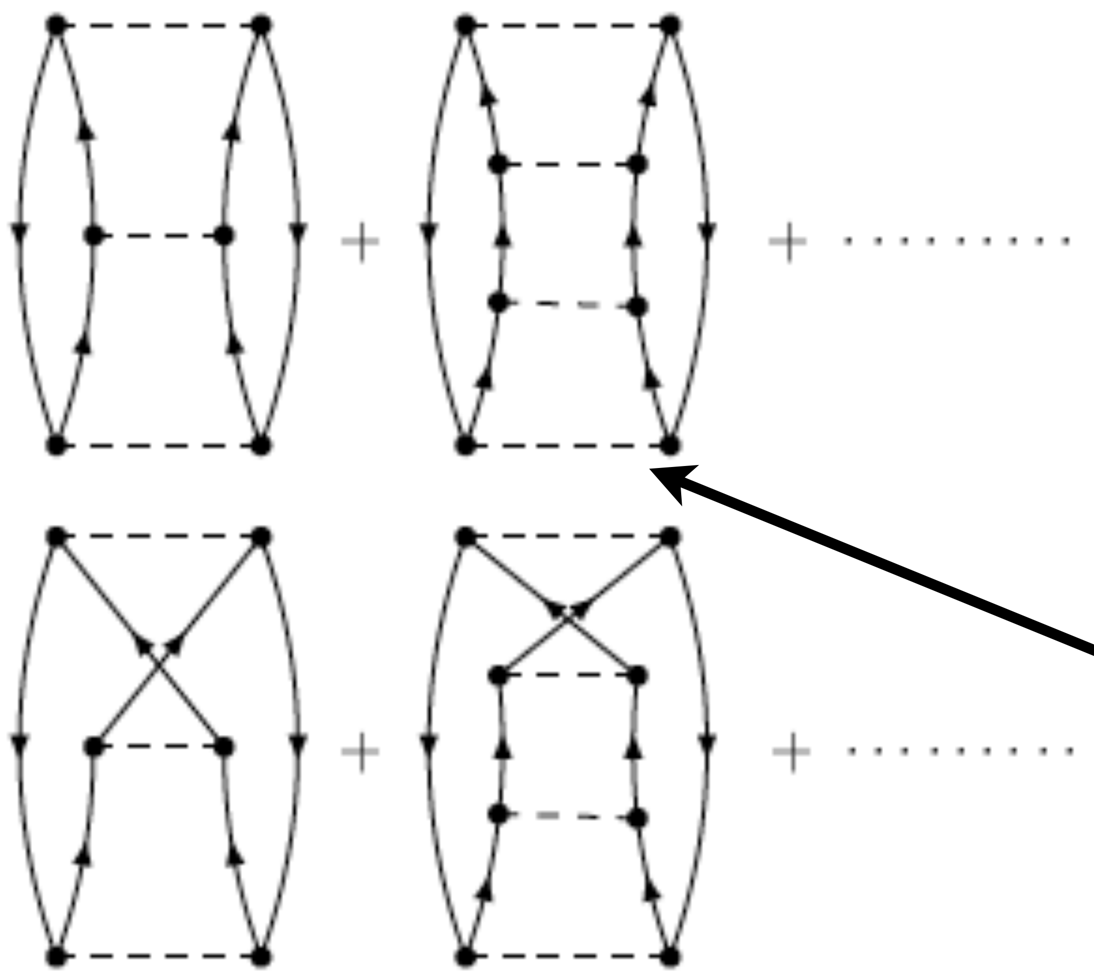


three-neutron potential is constrained by light nuclei.

Quantum Many-Body Theory:
Quantum Monte Carlo
Diagrammatic Methods
(perturbation theory)

$E(\rho_n, \rho_p)$: Energy per particle

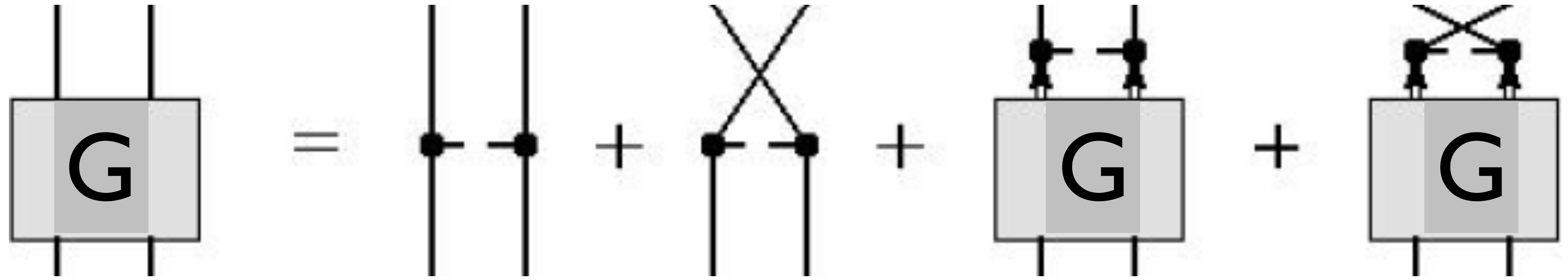
Diagrammatic Methods



Sum certain classes of Feynman diagrams to capture non-perturbative aspects.

nucleon-nucleon interaction

Eg. Bruckner or G-matrix Theory:



$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | v | k_3 k_4 \rangle +$$

$$+ \sum_{k_3' k_4'} \langle k_1 k_2 | v | k_3' k_4' \rangle \frac{\left(1-\Theta_F(k_3')\right)\left(1-\Theta_F(k_4')\right)}{\omega - e_{k_3'} - e_{k_4'}} \langle k_3' k_4' | G(\omega) | k_3 k_4 \rangle$$

Quantum Monte Carlo

Variational Monte Carlo: $$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

Greens Function Monte Carlo:

$$\begin{aligned} \Psi(\tau) = \exp[-(H - E_0)\tau]\Psi_V &= \sum_n \exp[-(E_n - E_0)\tau] a_n \psi_n \\ \Psi(\tau \to \infty) &= a_0 \psi_0 \end{aligned}$$

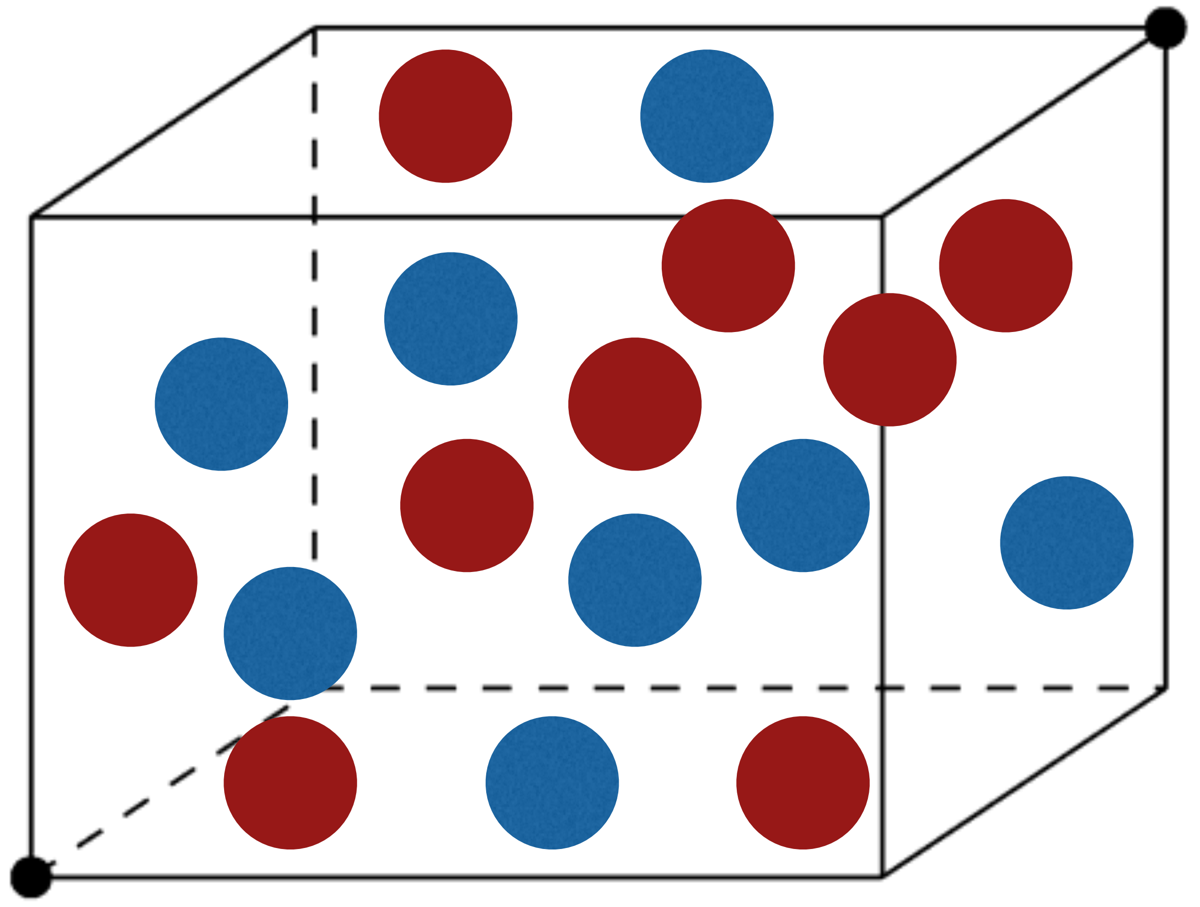
- Evolve particle coordinates.
- MC kinetic terms.
- Explicitly compute potential.

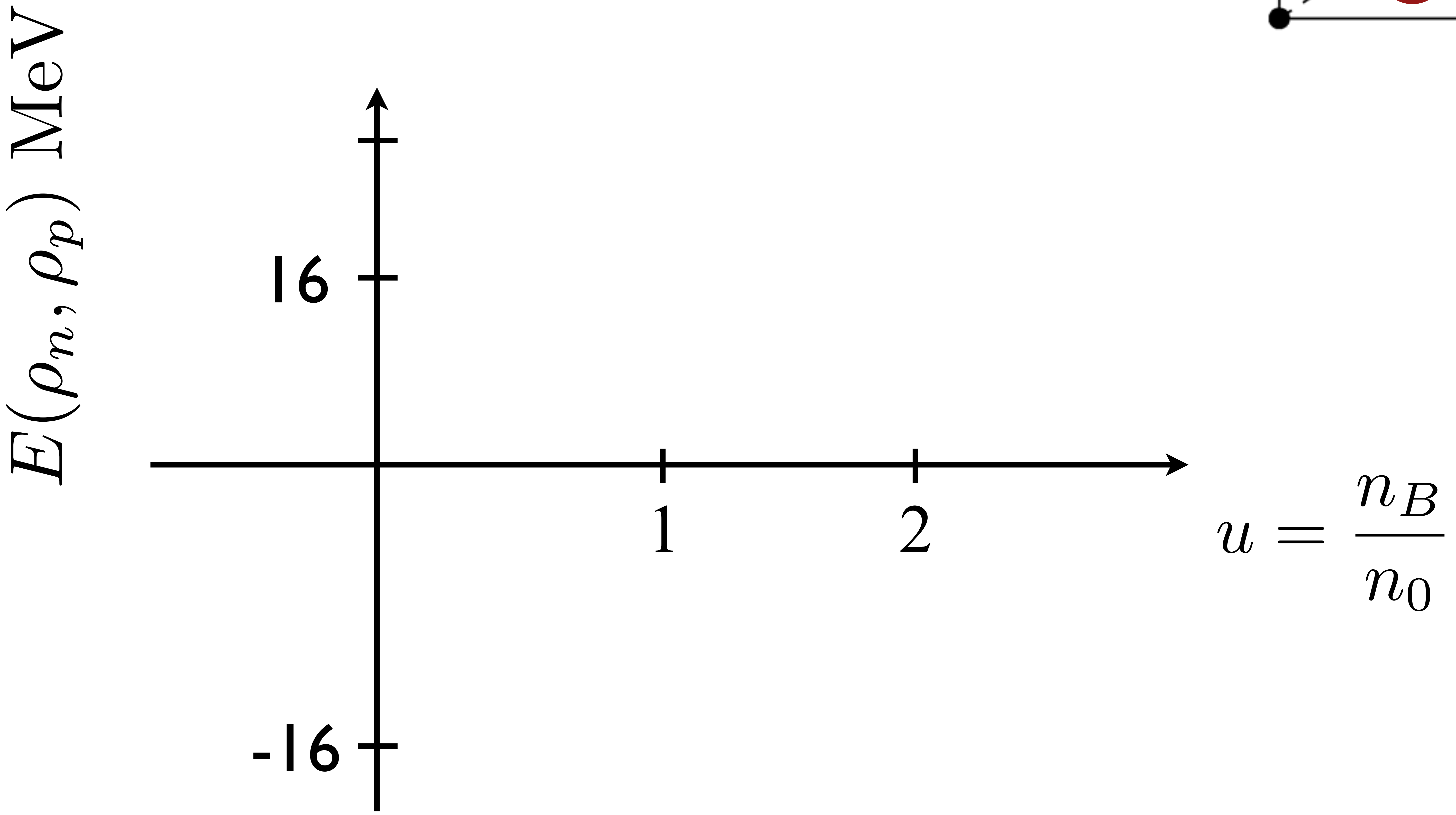
$$\Psi(\mathbf{R}_n, \tau) = \int G(\mathbf{R}_n, \mathbf{R}_{n-1}) \cdots G(\mathbf{R}_1, \mathbf{R}_0) \Psi_V(\mathbf{R}_0) d\mathbf{R}_{n-1} \cdots d\mathbf{R}_0$$

Fermion sign problem - limits GFMC

Energy of Uniform Matter: Nucleons in a Large Box

Given a Hamiltonian and a many-body approximation we can calculate the energy of N neutrons + M protons in a box. EFT potentials allow us to estimate the error due to unknown short-distance dynamics.





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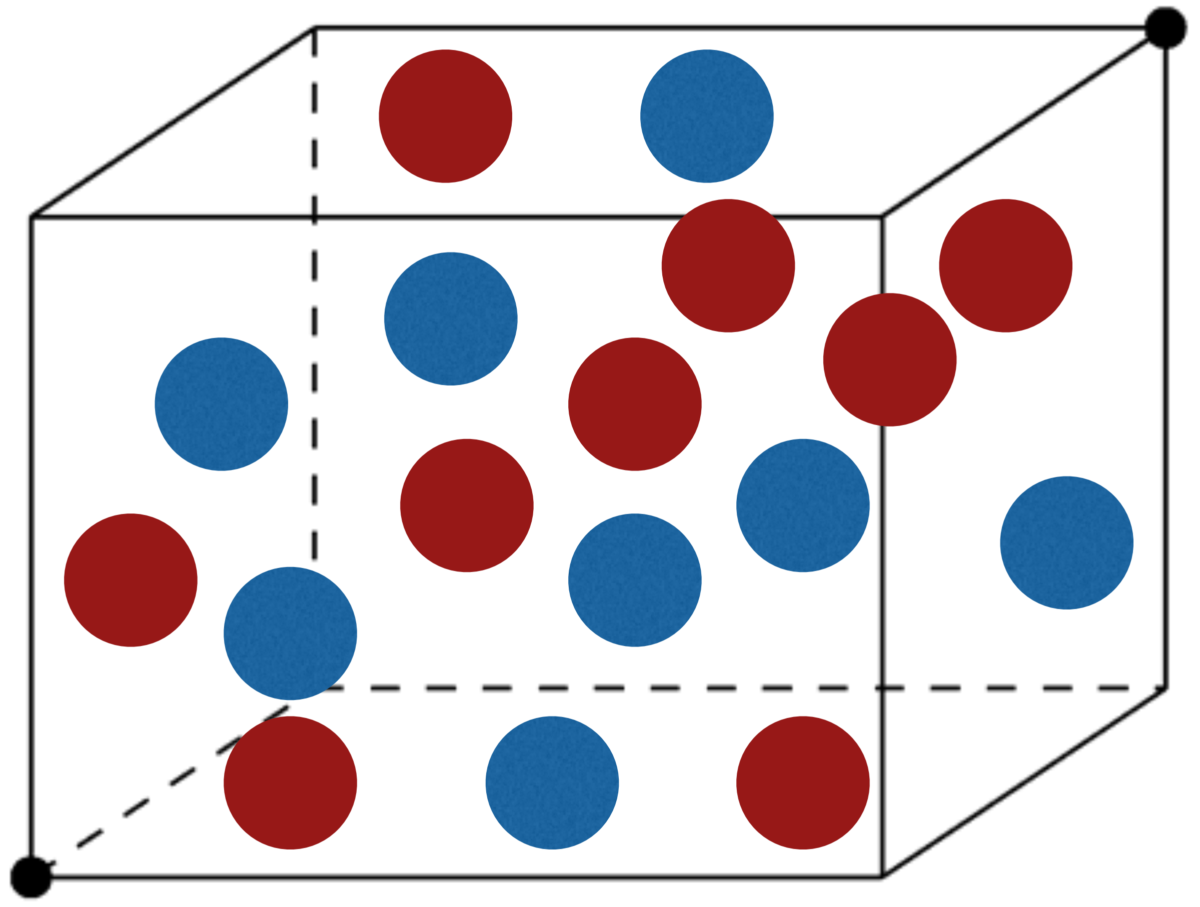

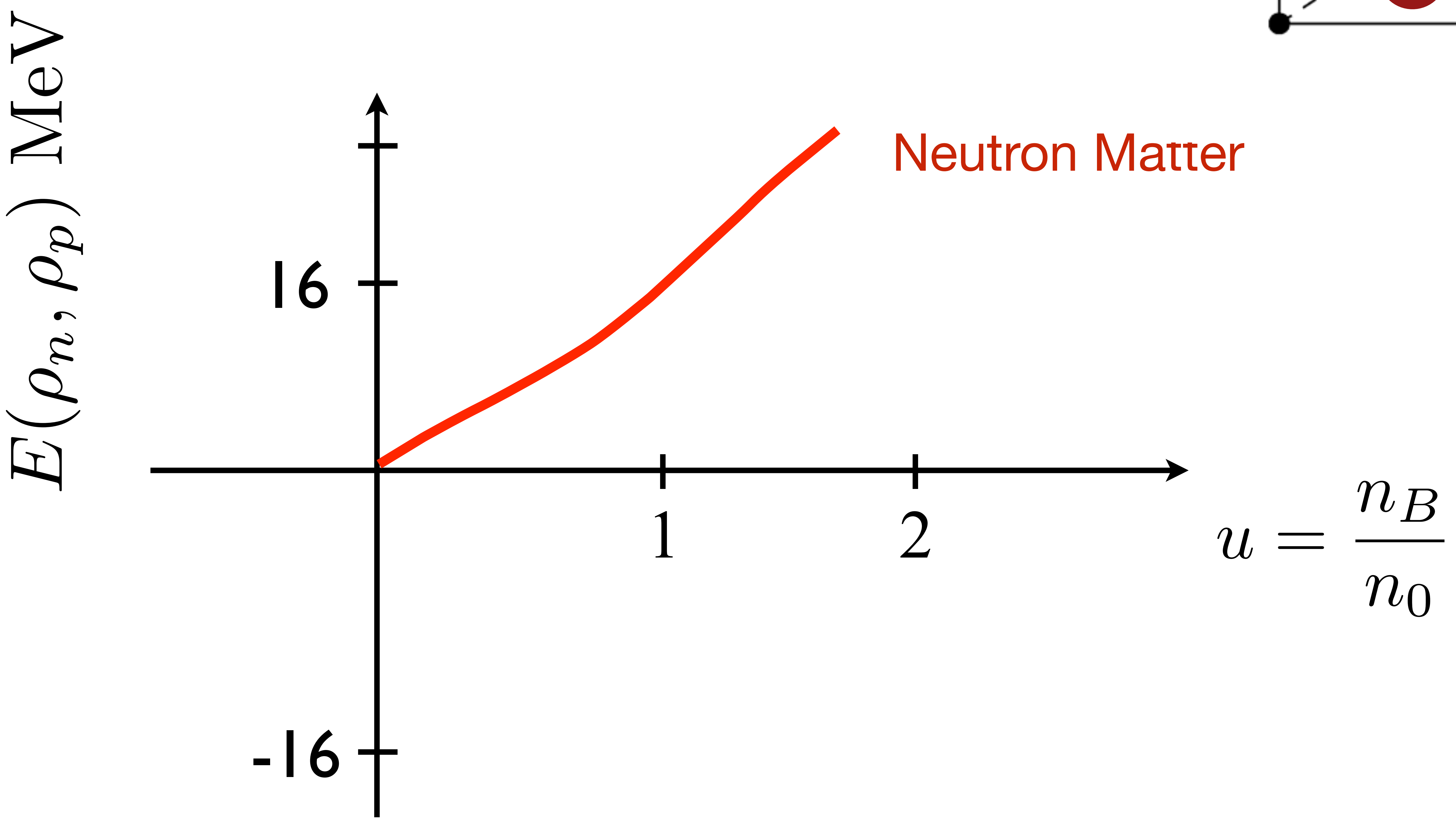

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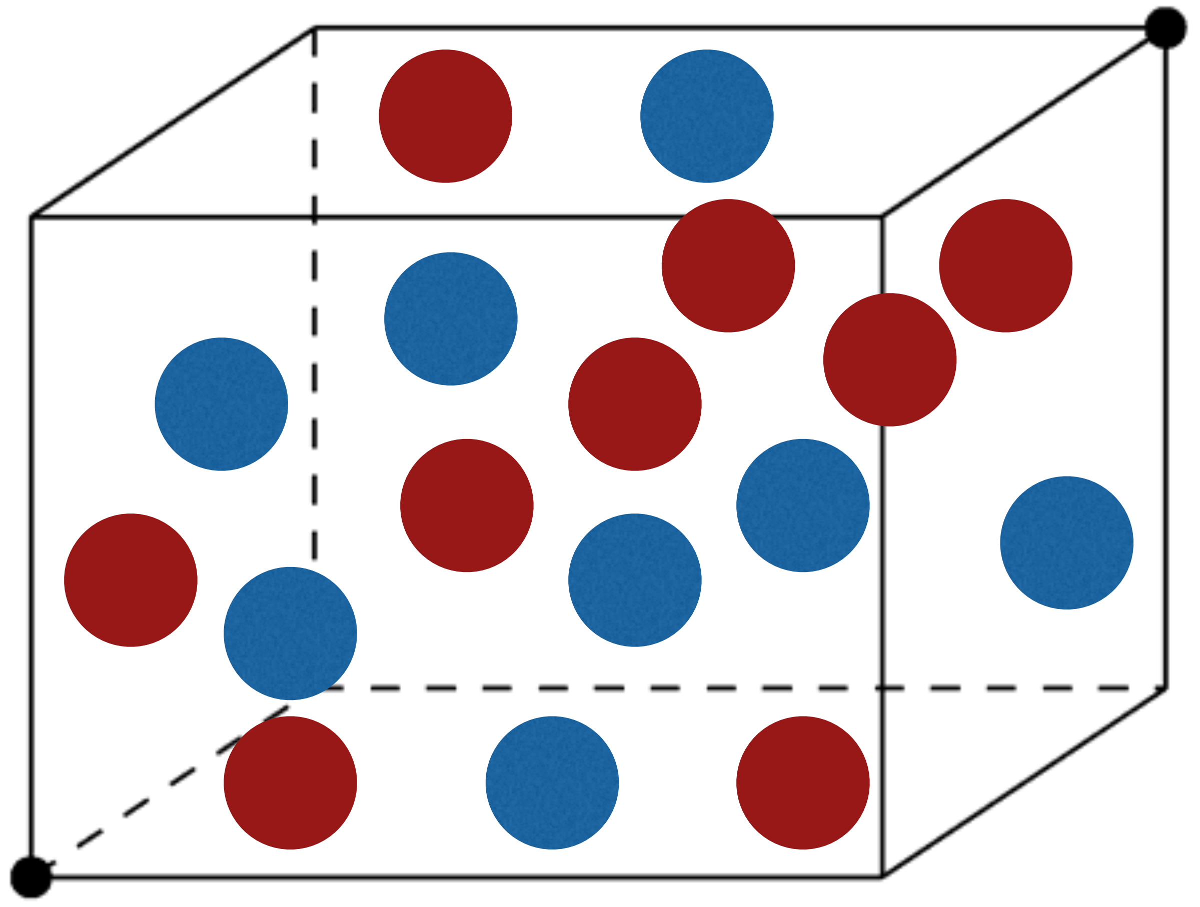

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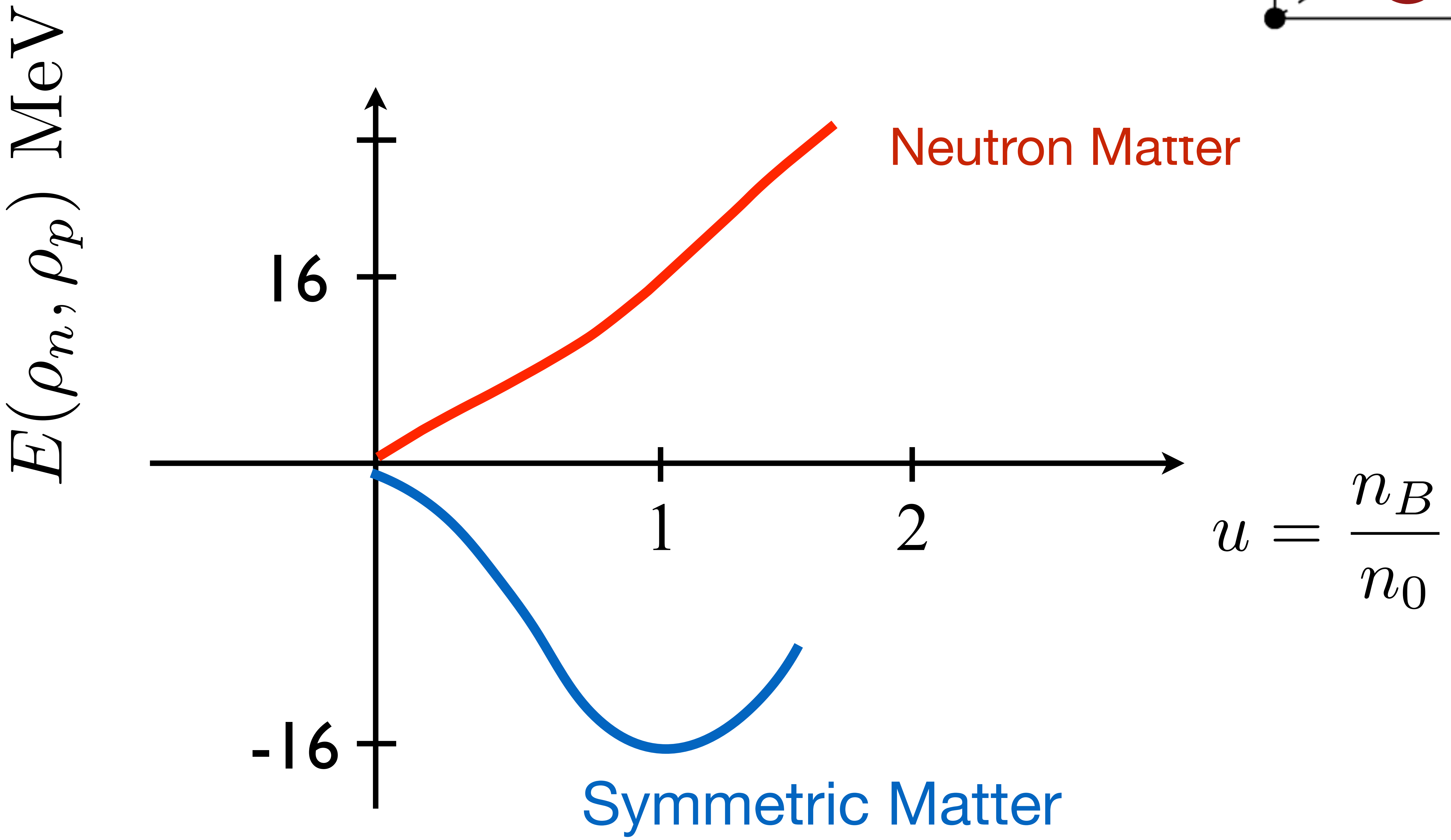

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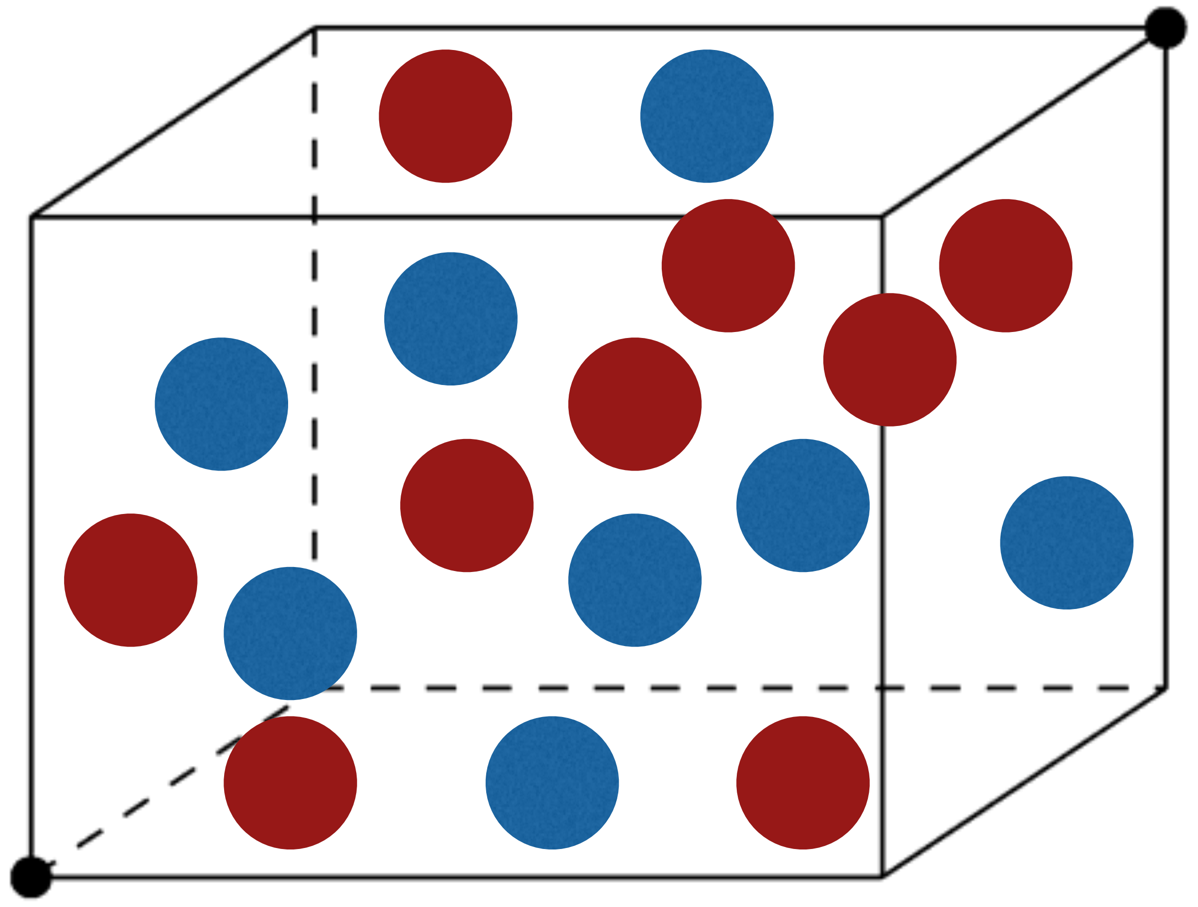

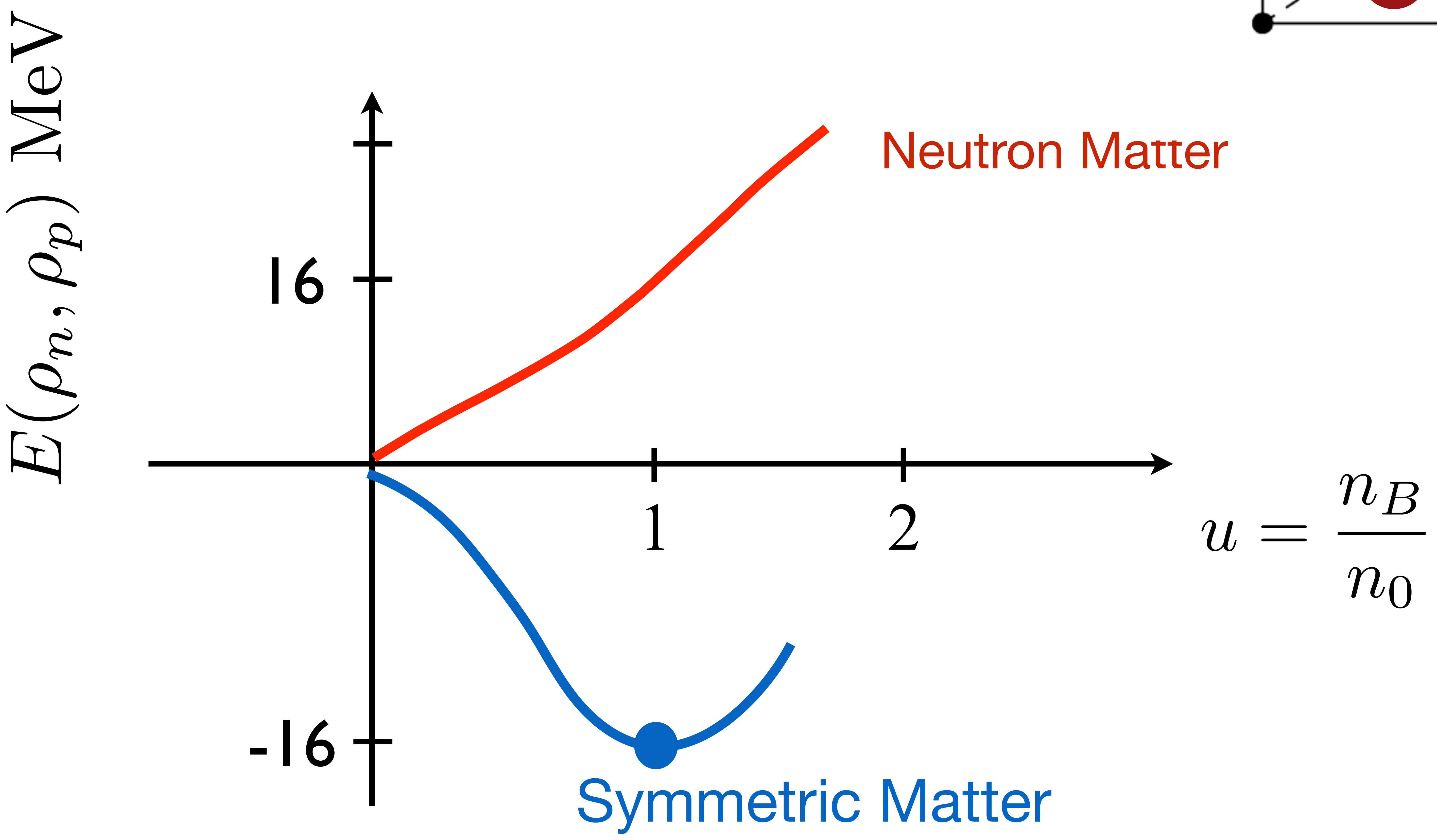

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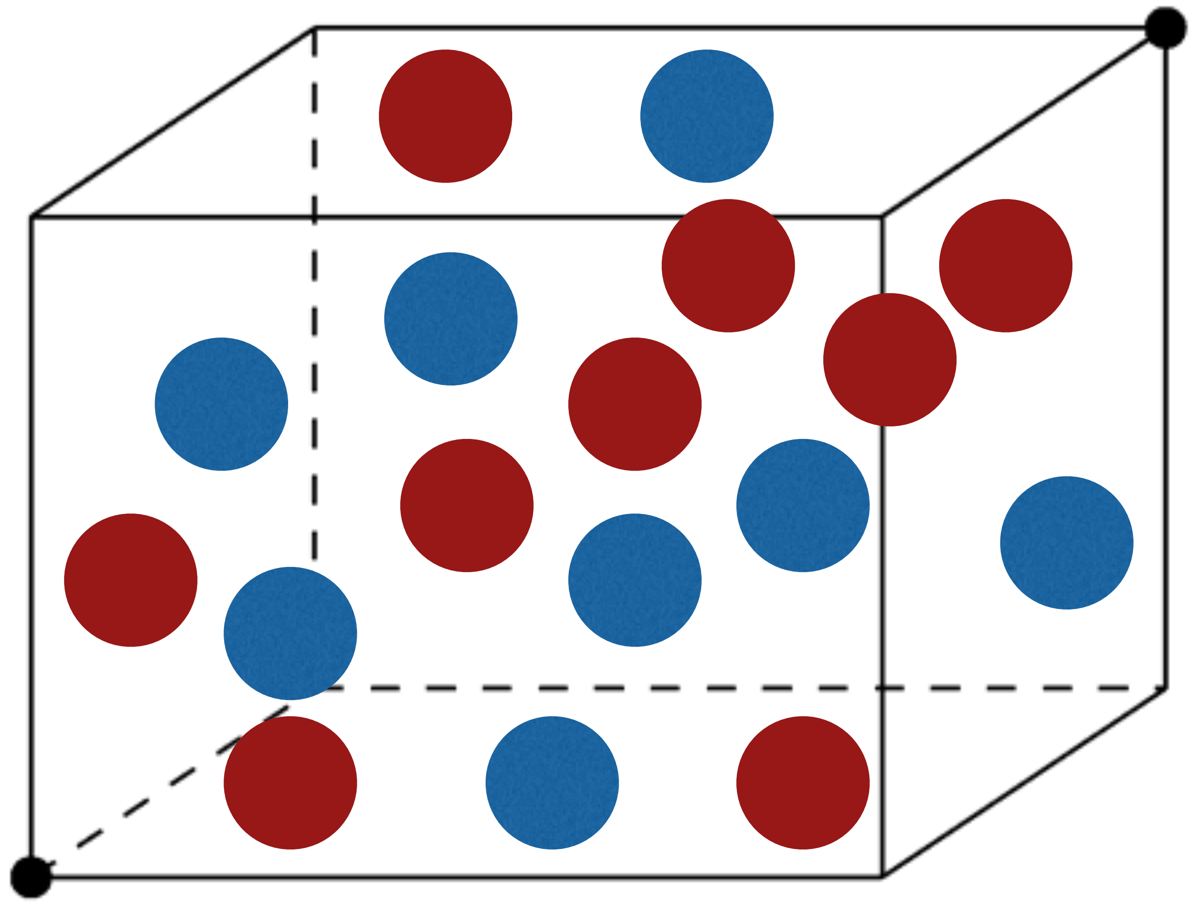

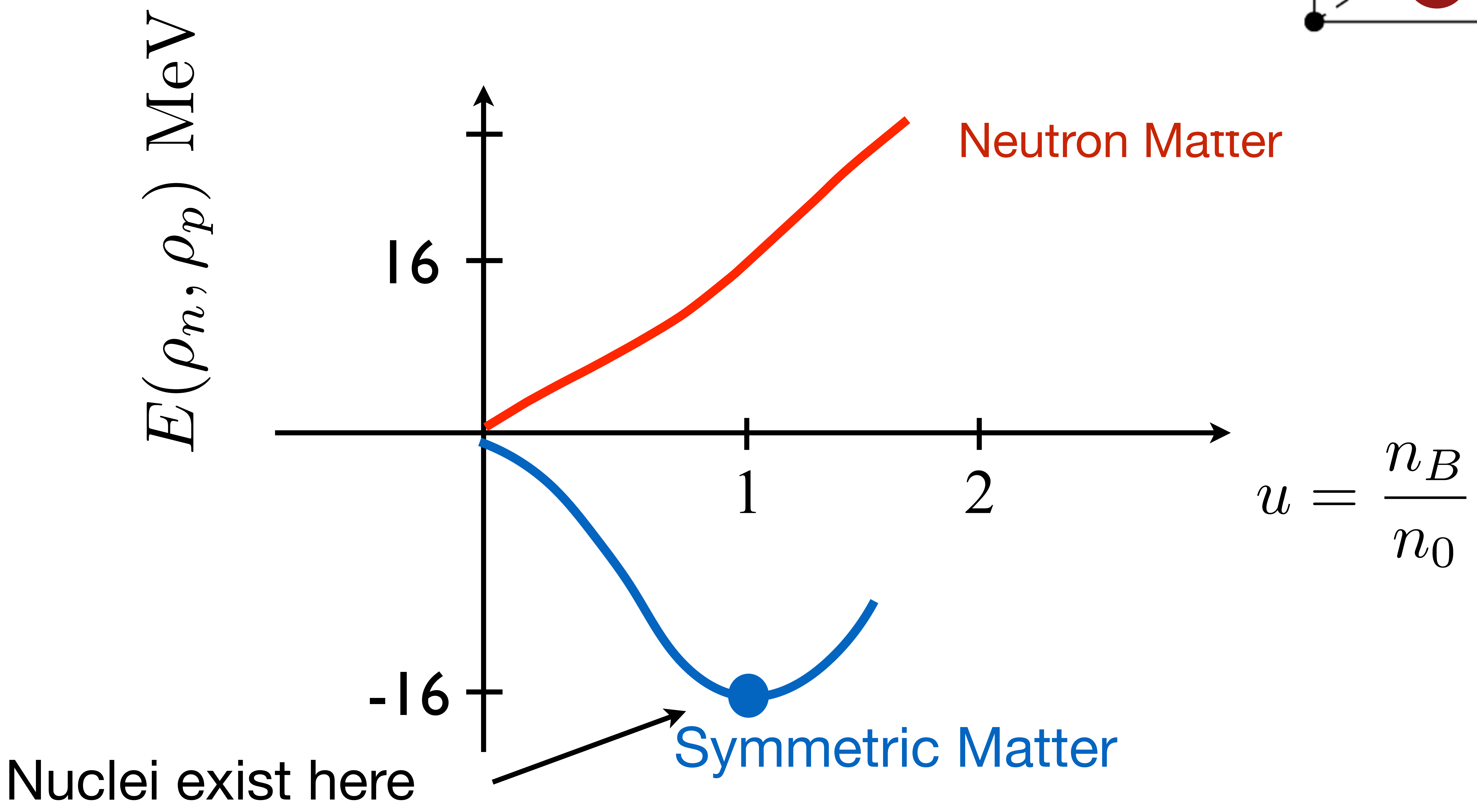

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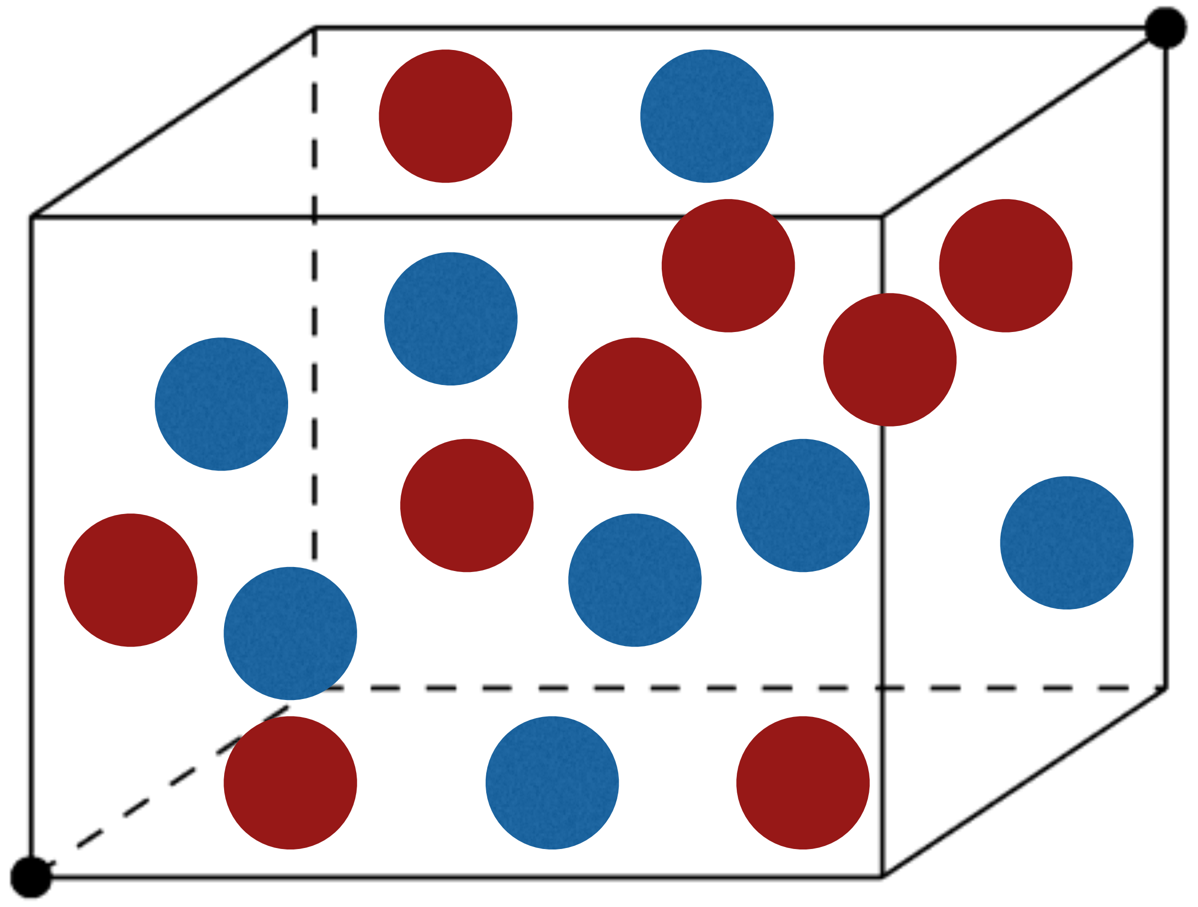

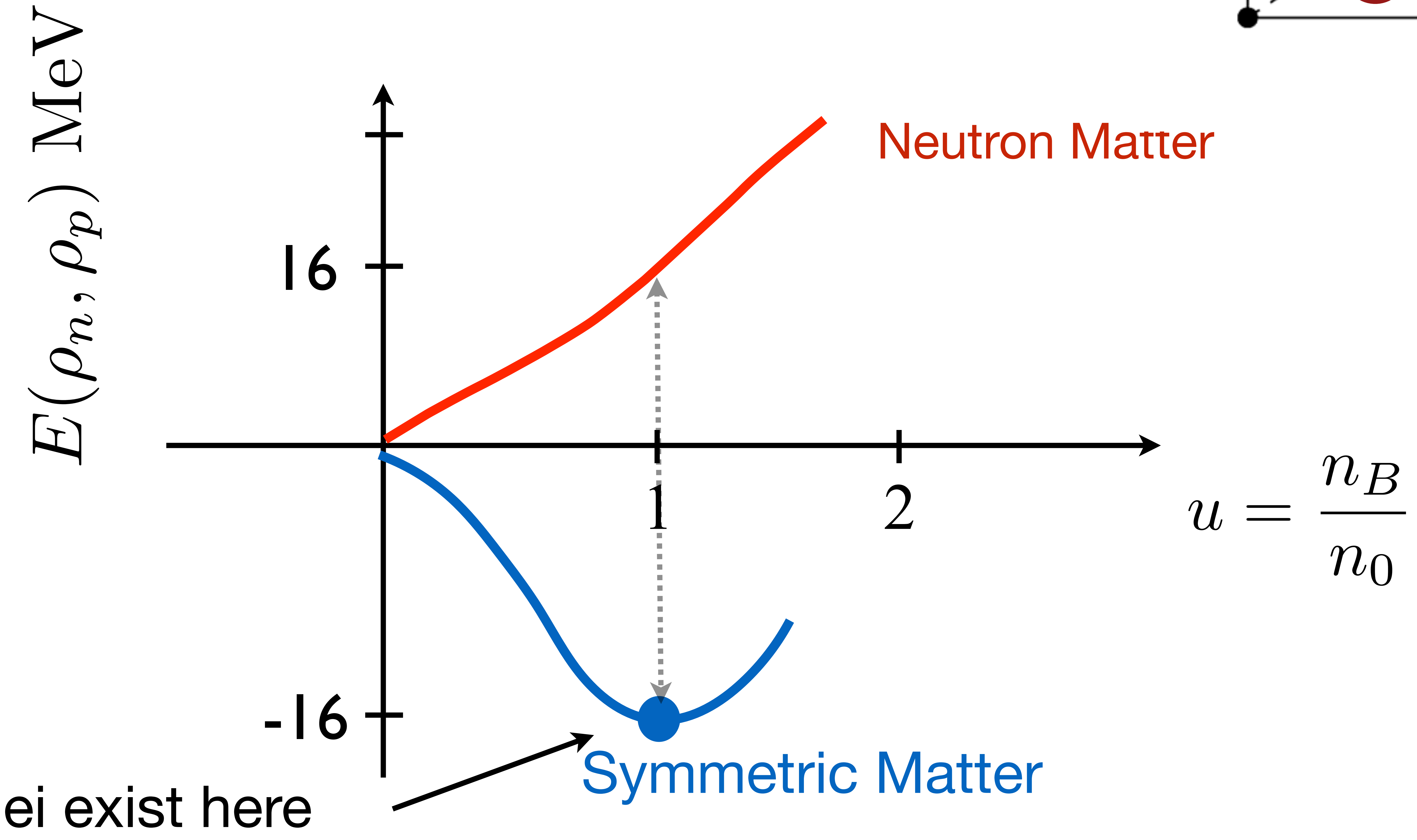

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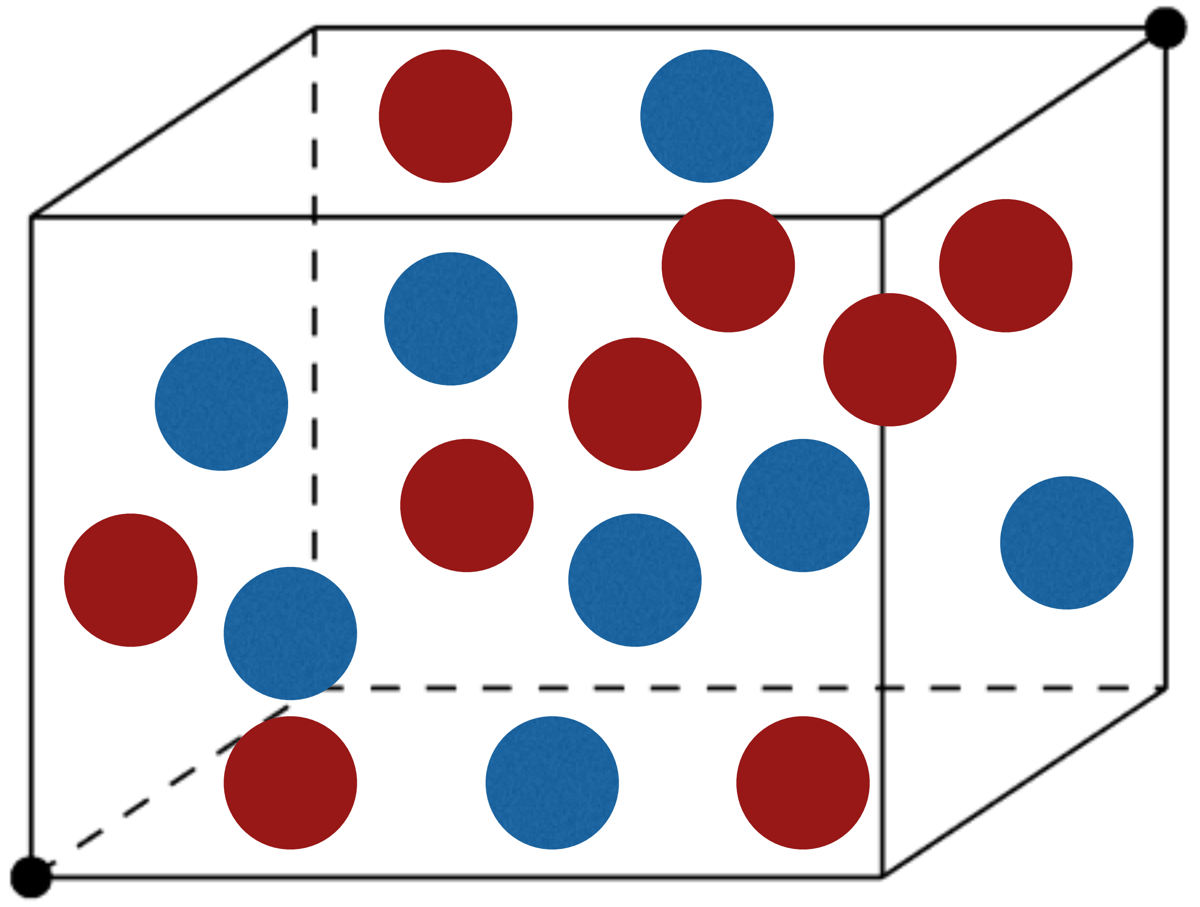

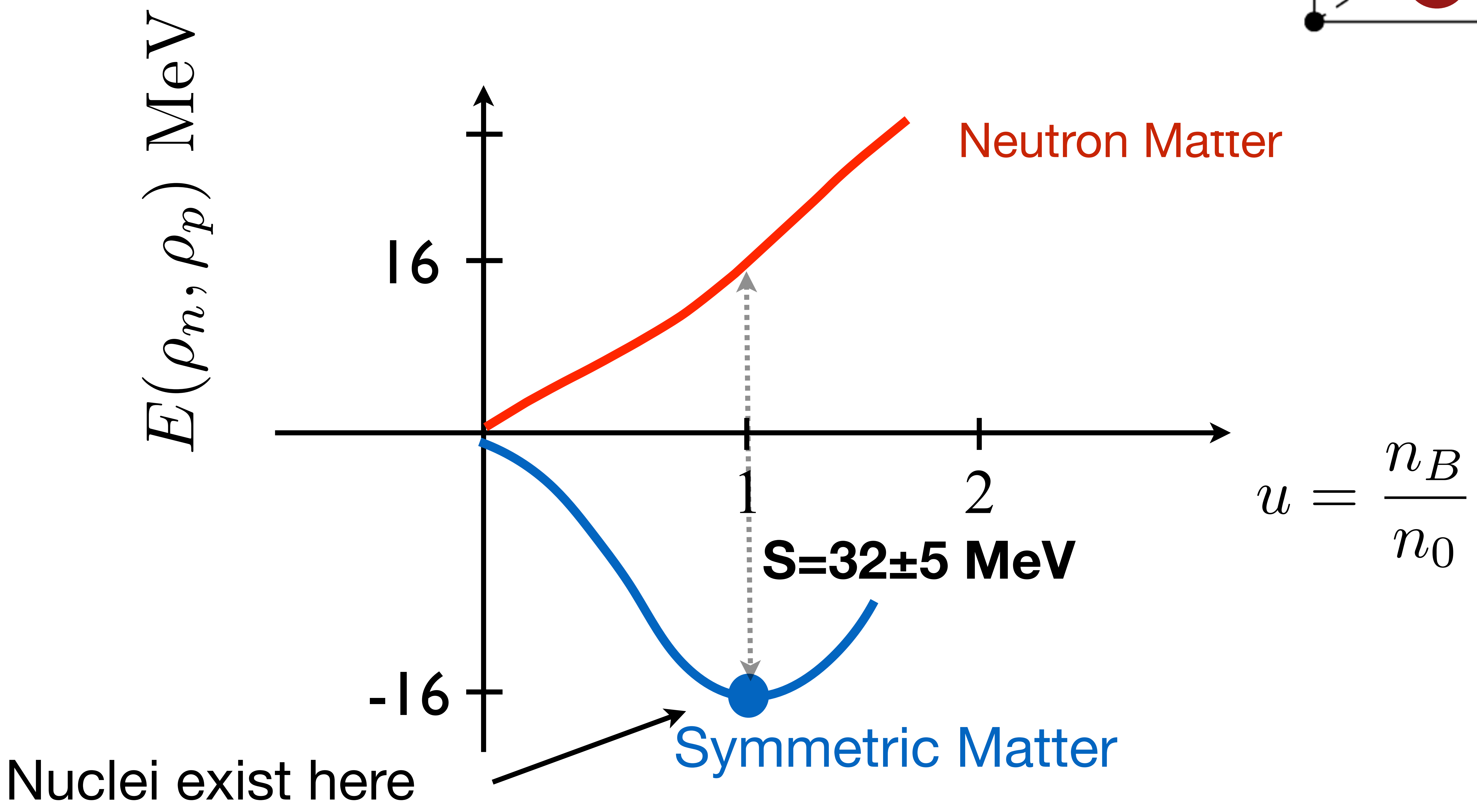

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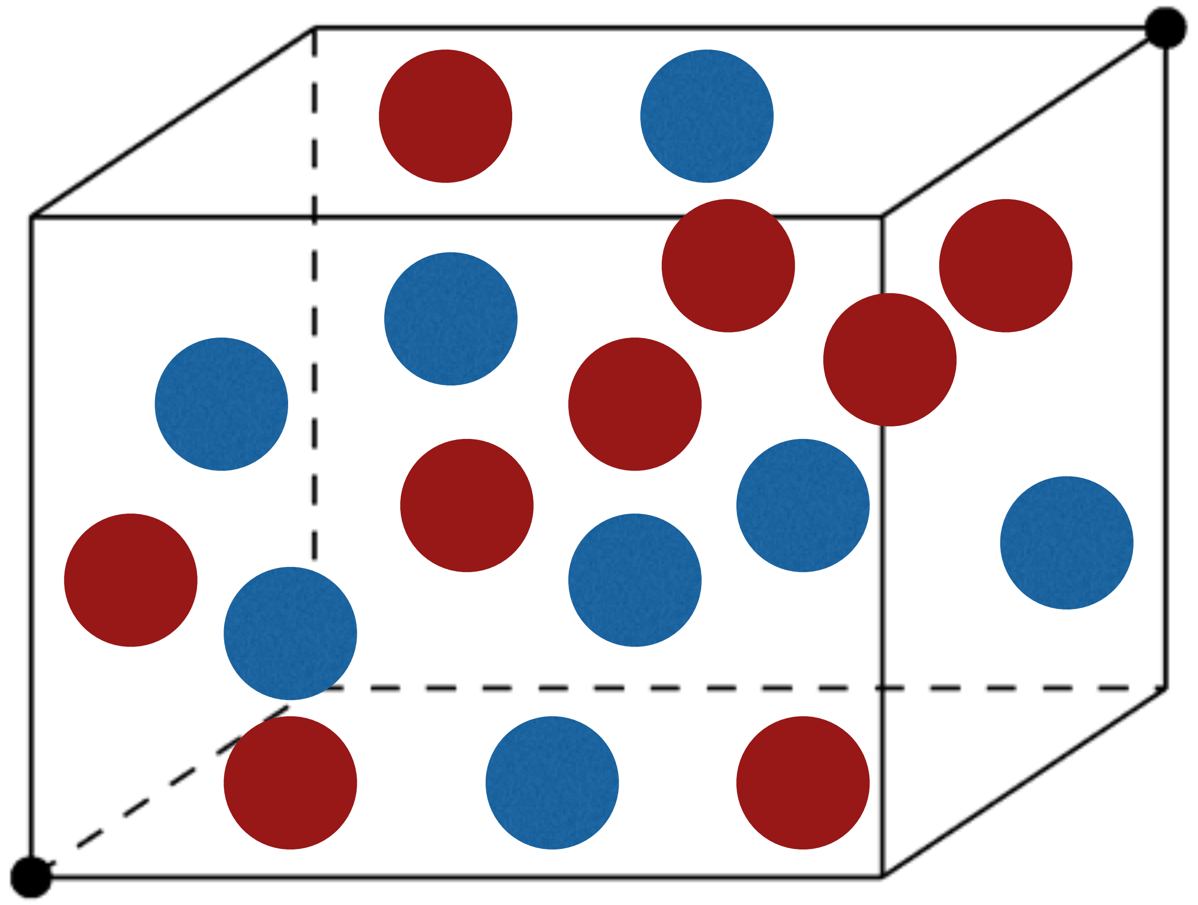

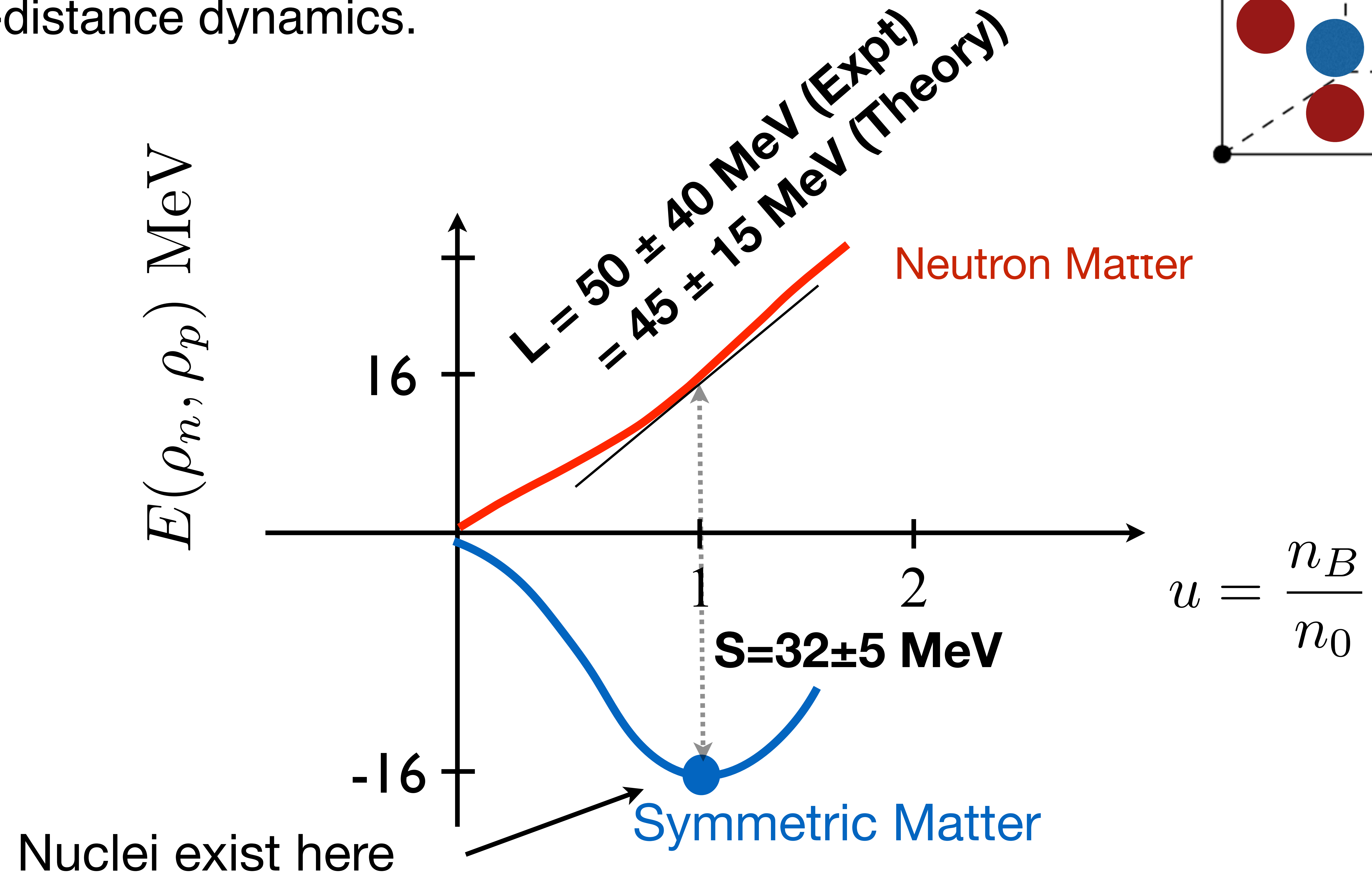

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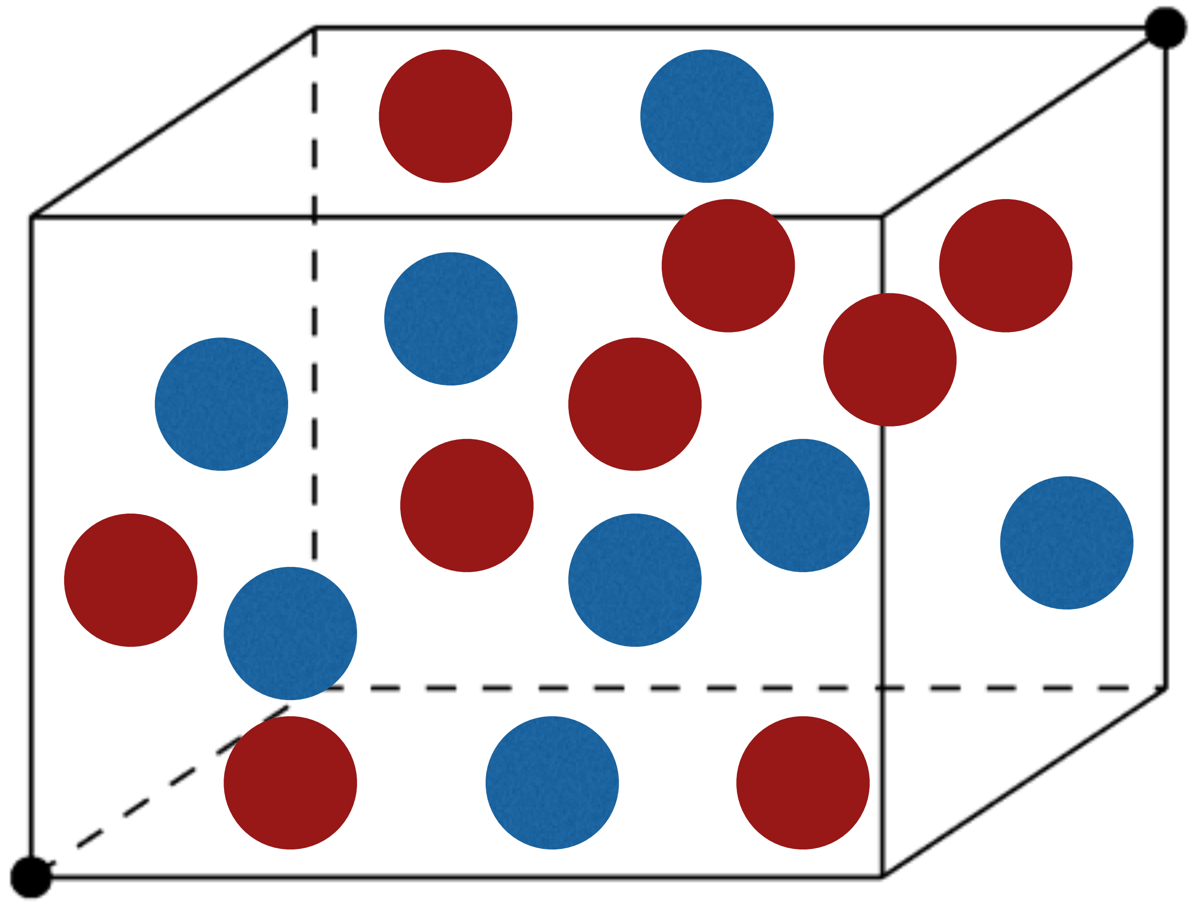
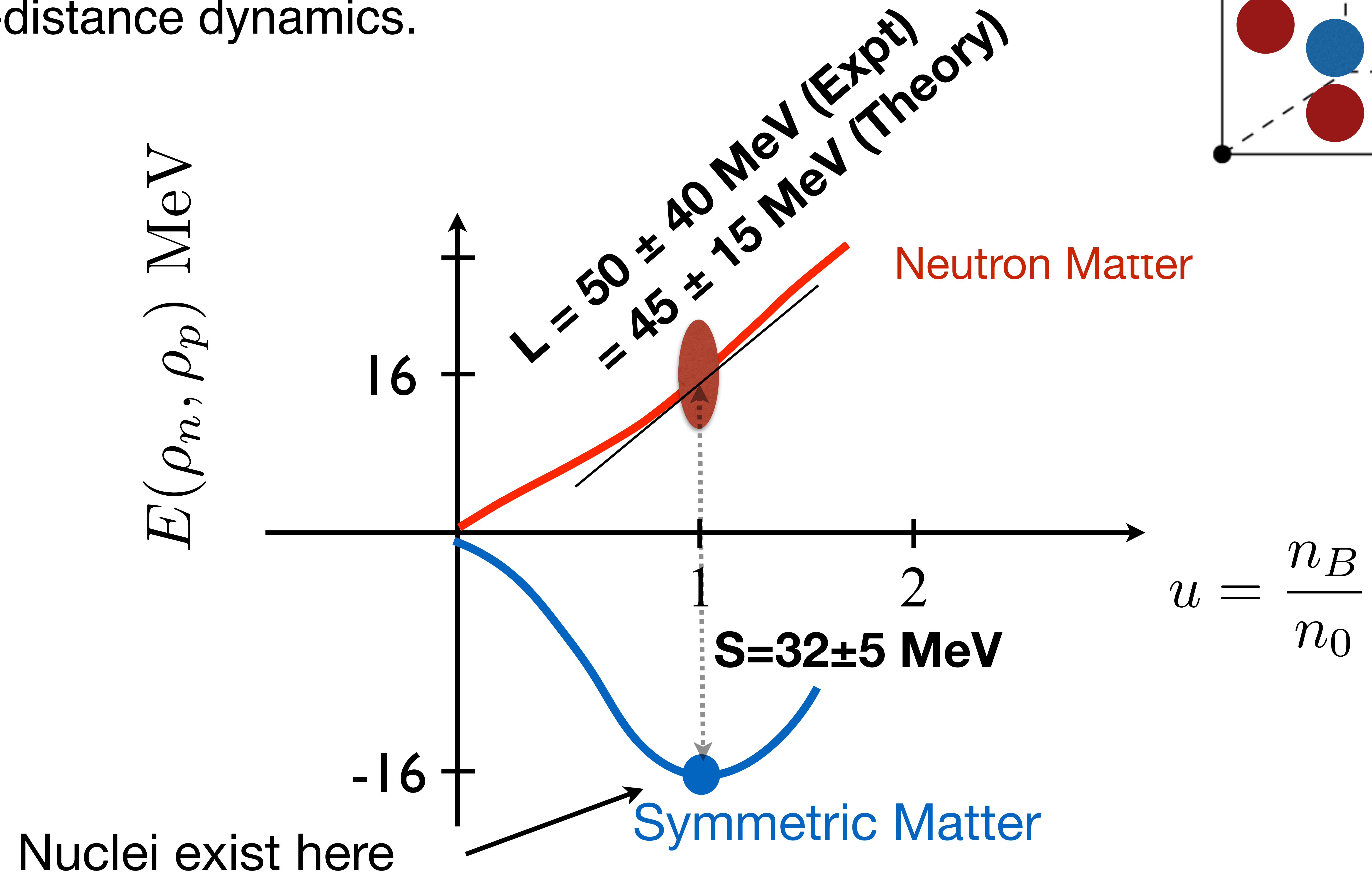


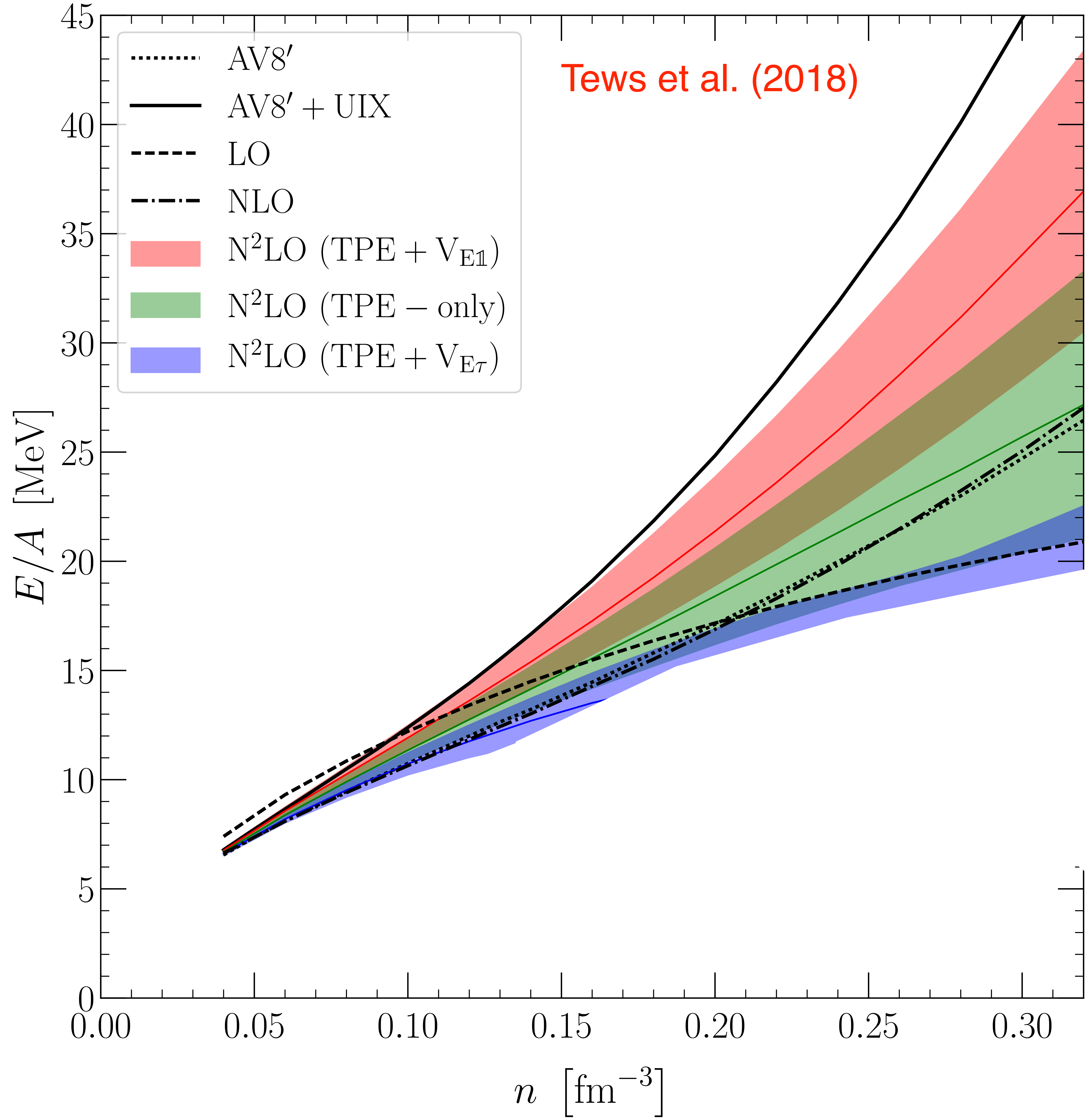


Equation of State of Neutron Matter

Reliable calculations of neutron matter are now possible using QMC and EFT inspired Hamiltonians.

Order-by-order convergence is good at n=0.16 fm^{-3} and reasonable at n=0.32 fm^{-3}.

	n=0.16 fm^{-3}	n=0.32 fm^{-3}
Energy (MeV)	15 ± 3	30 ± 15
Pressure (MeV/fm^{-3})	2.5 ± 1	13 ± 5



Akmal & Pandharipande 1998, Hebeler and Schwenk 2009, Gandolfi, Carlson, Reddy 2010, Tews, Kruger, Hebeler, Schwenk (2013), Holt Kaiser, Weise (2013), Roggero, Mukherjee, Pederiva (2014), Wlazlowski, Holt, Moroz, Bulgac, Roche (2014), Tews et al. (2018)

Nuclear Saturation, (A)symmetry Energy & Neutron Matter

Symmetric matter has zero pressure and is self-bound at a characteristic density $n_0 \approx 0.16\ \mathrm{fm}^{-3}$

Energy per particle of symmetric matter is about -16 MeV.

Its costs energy to make matter asymmetric.

Kinetic (Fermi) energy and potential energy costs are comparable. Total cost at saturation is about 30 MeV.

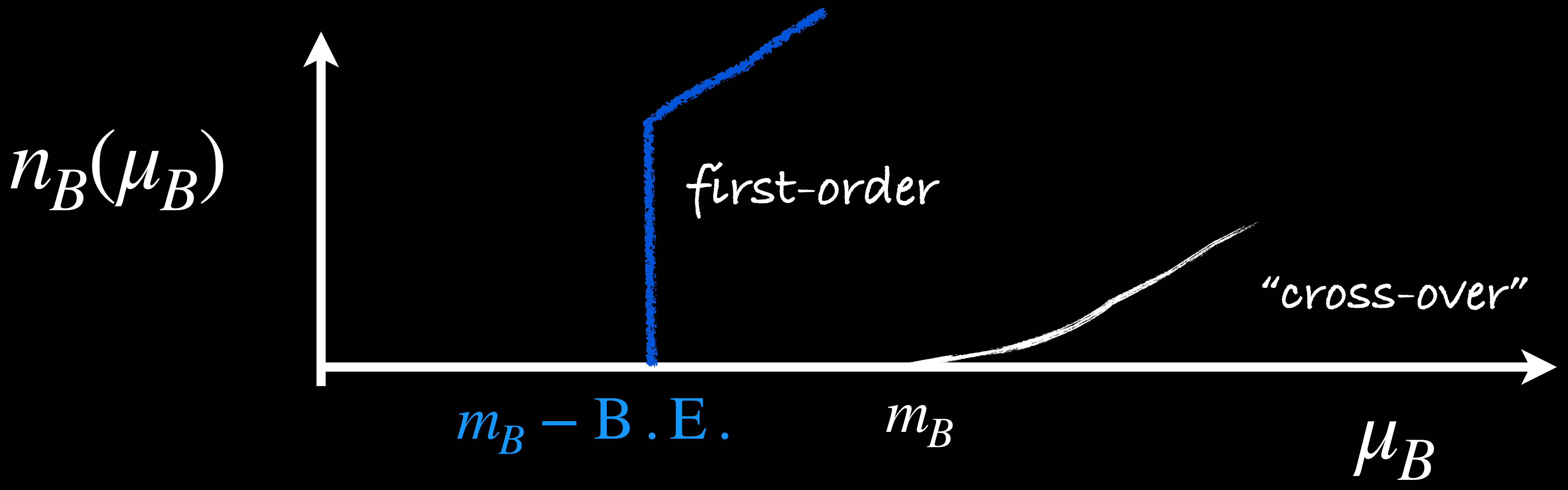
It is possible to calculate the energy of pure neutron matter up to about twice nuclear saturation density. Errors due to uncertainties in nuclear Hamiltonian (especially three-body forces) grows rapidly with density.

Why is matter heterogeneous at sub-nuclear density?

Thinking Grand

$$\underset{\text{pressure}}{P(\mu,T)} = -\underset{\substack{\text{energy}\\\text{density}}}{\epsilon} + \mu\, \underset{\substack{\text{number}\\\text{density}}}{n} + \underset{\substack{\text{entropy}\\\text{density}}}{s}\; T = -\frac{\overset{\text{free energy}}{\Omega}}{V}$$

The vacuum responds to a chemical potential and finite temperature and by producing a finite density of particles with the lowest free energy.



Thinking Grand Canonically

$$\underset{\text{pressure}}{P(\mu,T)} = -\underset{\text{energy density}}{\epsilon} + \mu \underset{\text{number density}}{n} + \underset{\text{entropy density}}{s}\, T = -\overset{\text{free energy}}{\frac{\Omega}{V}}$$

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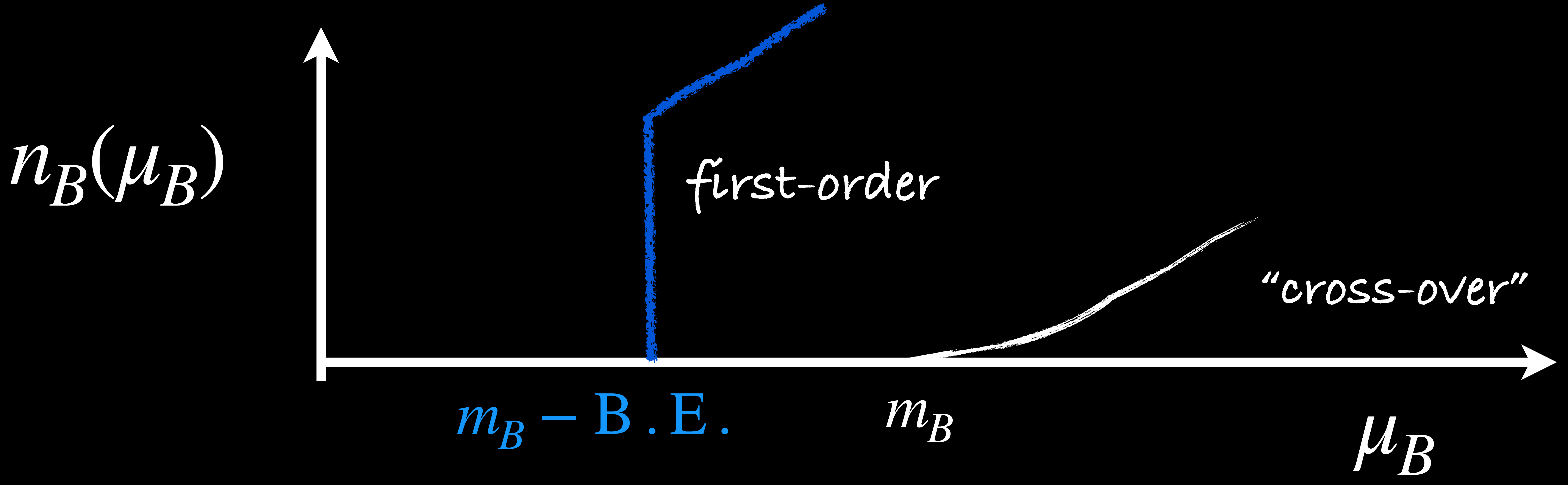
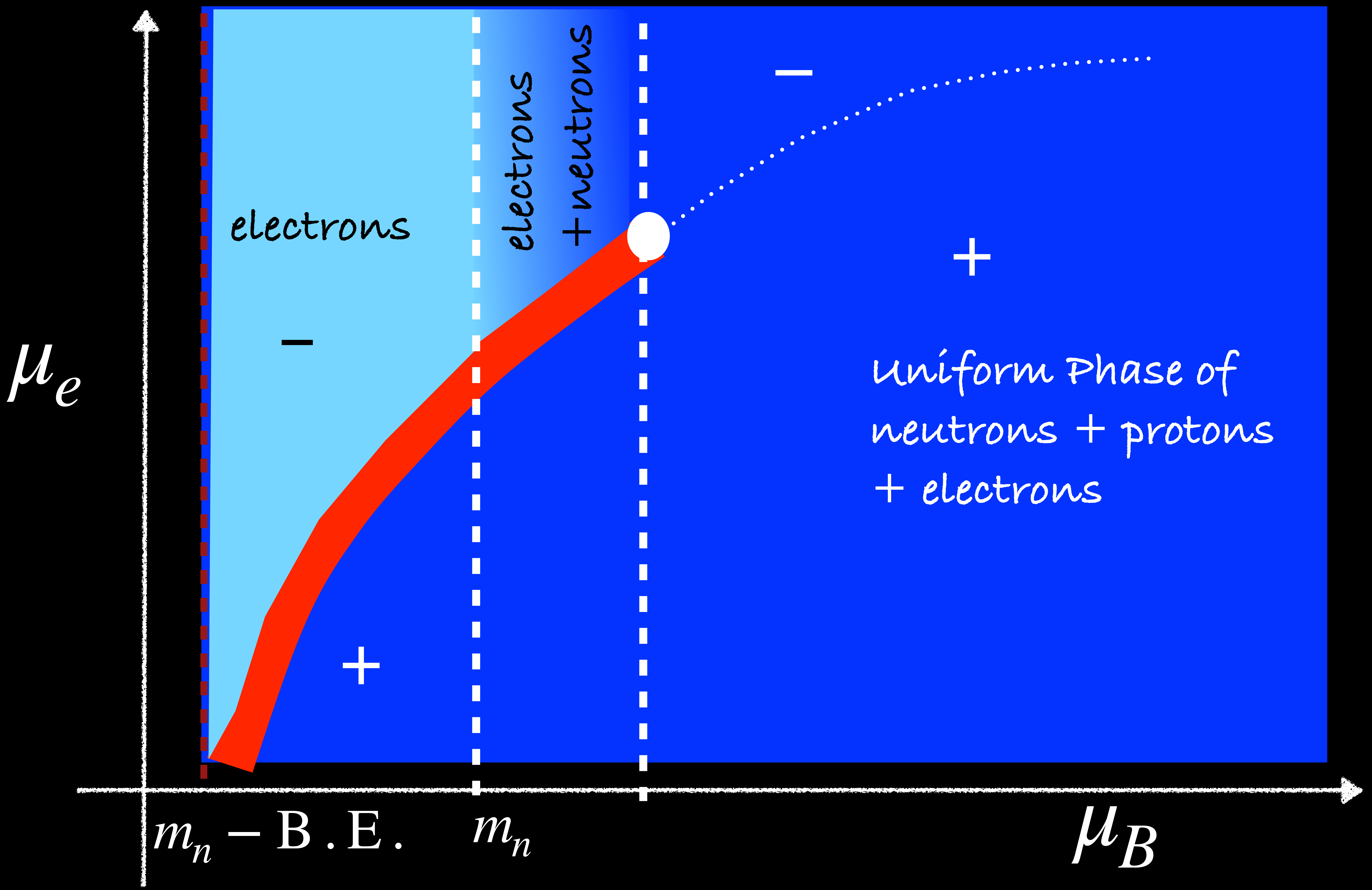


First-order transitions with 2 conserved Charges



Global charge neutrality

Local charge neutrality

Energy cost due to Coulomb and surface energies.

Surface and Coulomb Energies

$$E_S = 4\pi\sigma\ R^2 \qquad E_C = \frac{3}{5}\alpha_{\rm em}\ \frac{Z^2}{R} = \frac{3}{5}x_p^2\alpha_{\rm em}\ \frac{A^2}{R}$$

Constant density : $R = r_0 A^{1/3}$ $\quad r_0 = \left(\frac{3}{4\pi\ n_0}\right)^{1/3} \simeq 1.14\ \text{fm}$

$$\boxed{\frac{E_S}{A} = 4\pi\ r_0^2\ \sigma\ \ A^{-1/3}} \qquad \boxed{\frac{E_C}{A} = \frac{3\ \alpha_{\rm em}\ x_p^2}{5\ r_0}\ A^{2/3}}$$

At the minimum : $\frac{2}{3}E_C - \frac{1}{3}E_S = 0$

or : $E_S = 2E_C$

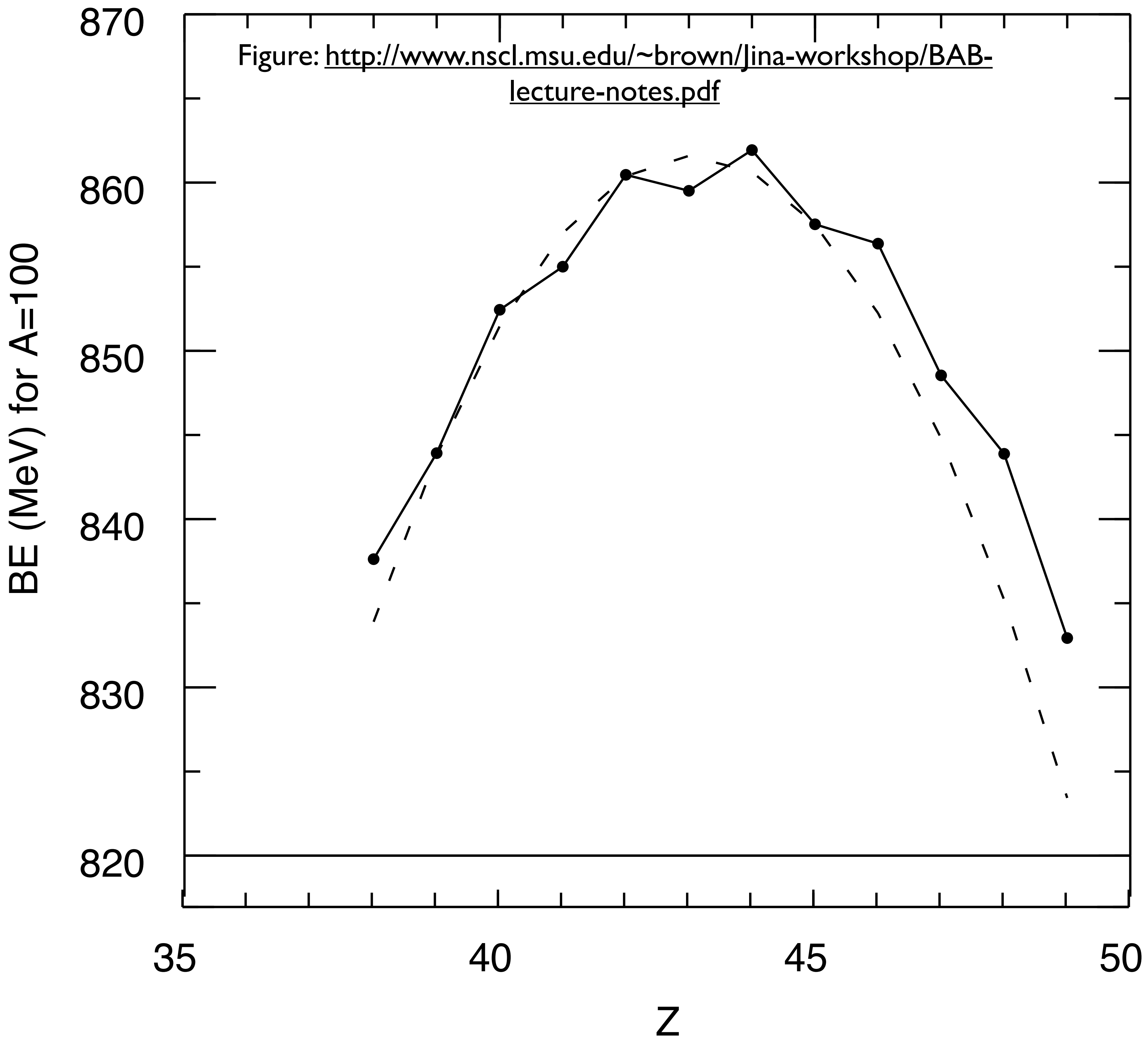
Most favored nucleus has $A \simeq \left(\frac{r_0}{1.2\ \text{fm}}\right)^3 \left(\frac{\sigma}{1\ \text{MeV/fm}^2}\right)\ \frac{4\pi}{x_p^2}$

Neutron-rich nuclei

At fixed A:

(i) The nuclear symmetry energy favors small (N-Z).

(ii) Coulomb energy favors small Z.



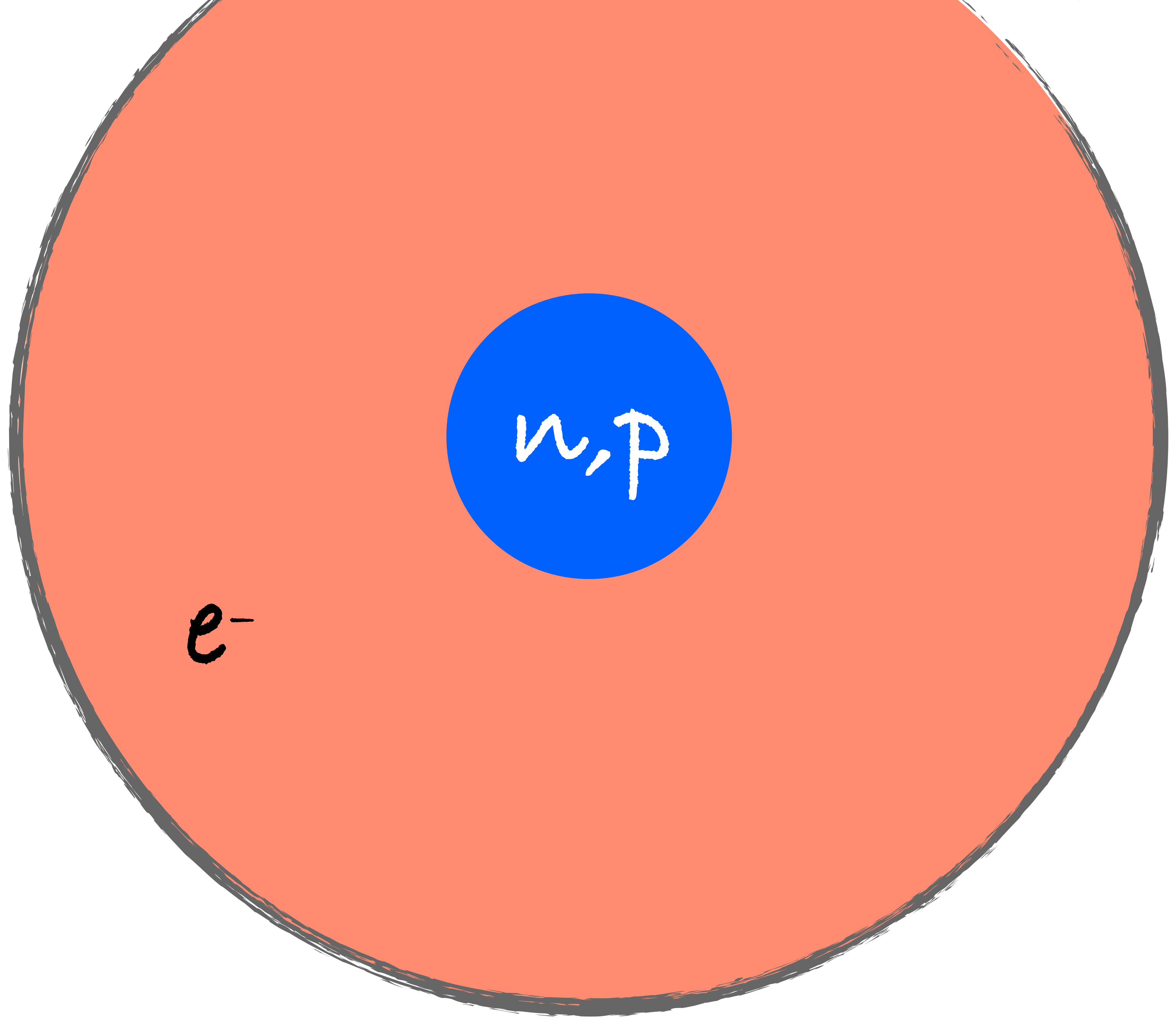
nuclei with "excess" neutrons or protons are unstable to weak interactions.

$$\hat{\mu} = \mu_n - \mu_p = \frac{\partial\ \mathrm{B.E}(A,Z)}{\partial Z} + (m_n - m_p)$$
$$= 4\alpha_{\mathrm{sym}}\ (1 - 2\frac{Z}{A}) - 2\alpha_C\ \frac{Z}{A^{1/3}} + (m_n - m_p)$$

Problem : Show that for A=100, Z=44 is the most stable nucleus.

Use: $\alpha_{\mathrm{sym}} = 28\ \mathrm{MeV}$ $\quad \alpha_C = 0.697\ \mathrm{MeV}$

Nuclei immersed in a dense electron gas



Beta Equilibrium:

$$e^- + p \rightarrow n + \nu_e, \quad n \rightarrow p + e^- + \bar{\nu}_e$$

$$\mu_n - \mu_p = \mu_e \simeq 4\ \alpha_{\rm sym}(1 - 2\ x_p)$$

$$\boxed{x_p \simeq \frac{1}{2}\left(1 - \frac{\mu_e}{4\ \alpha_{\rm sym}}\right)\left(1 + \frac{\alpha_{\rm C}\ A^{2/3}}{4\ \alpha_{\rm sym}}\right)^{-1}}$$

Neutron Fermi levels rise : $\mu_n \simeq -\alpha_{\rm bulk} + 2\alpha_{\rm sym}[(1 - 2x_p) - \frac{1}{2}(1 - 2x_p)^2]$

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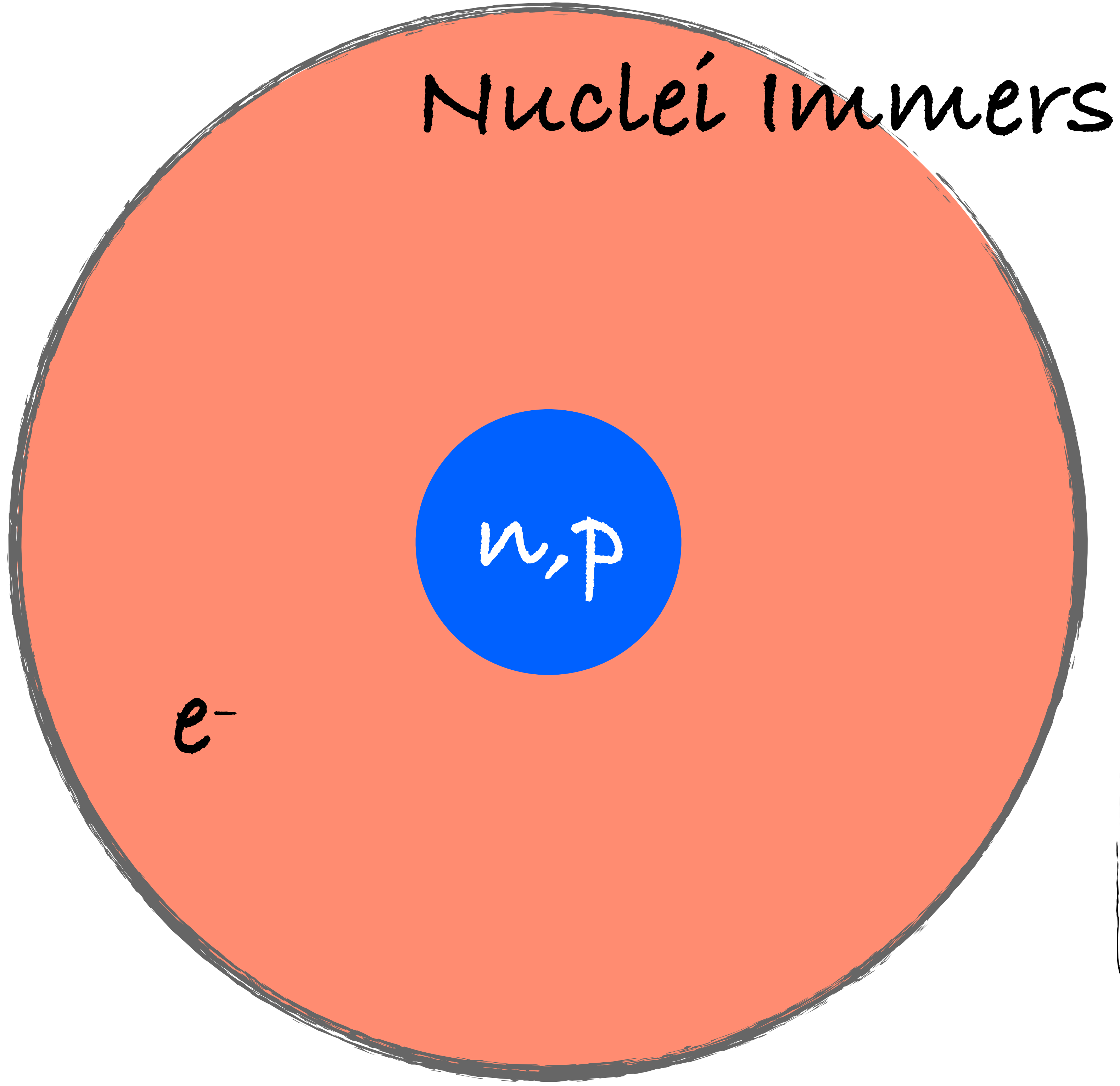

Beta Equilibrium:

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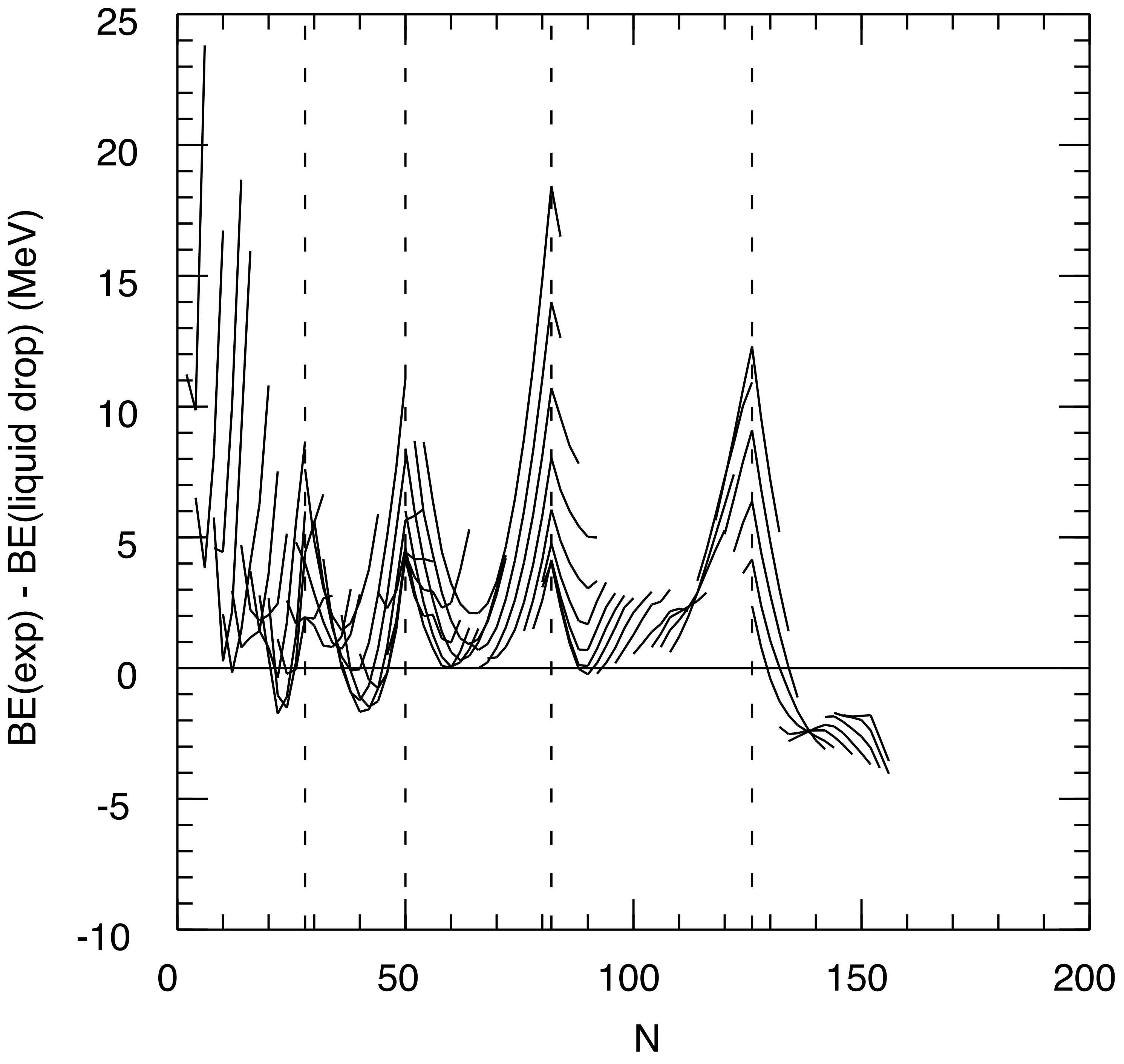
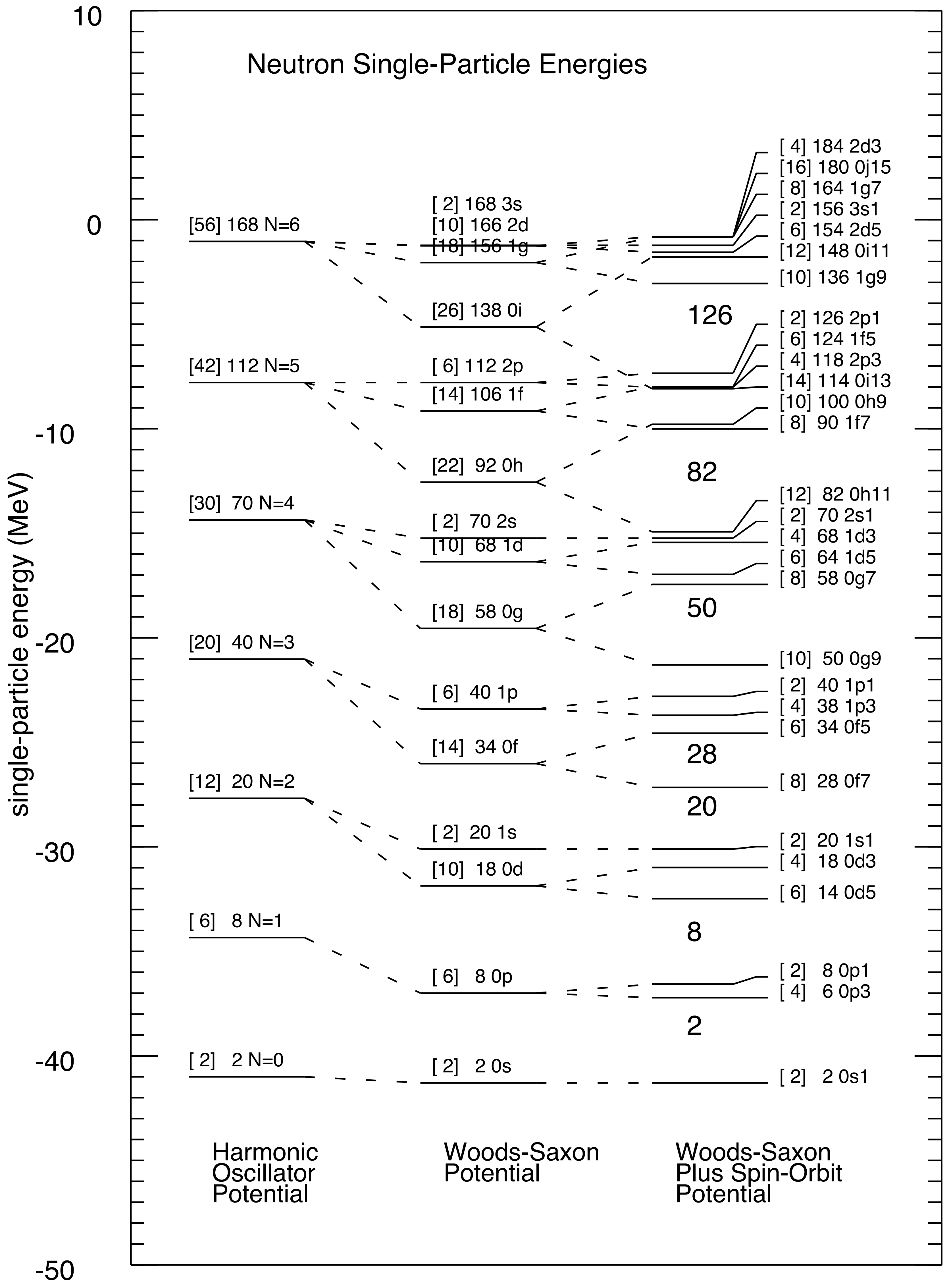
Neutron Fermi levels rise : $\mu_n \simeq -\alpha_{\rm bulk} + 2\alpha_{\rm sym}[(1-2x_p) - \frac{1}{2}(1-2x_p)^2]$

Neutrons drip at : $x_p \simeq \frac{1}{2}\sqrt{1 - \frac{\alpha_{\rm bulk}}{\alpha_{\rm sym}}} \approx 0.34$

What have we ignored thus far ?

- Shell structure - Magic numbers

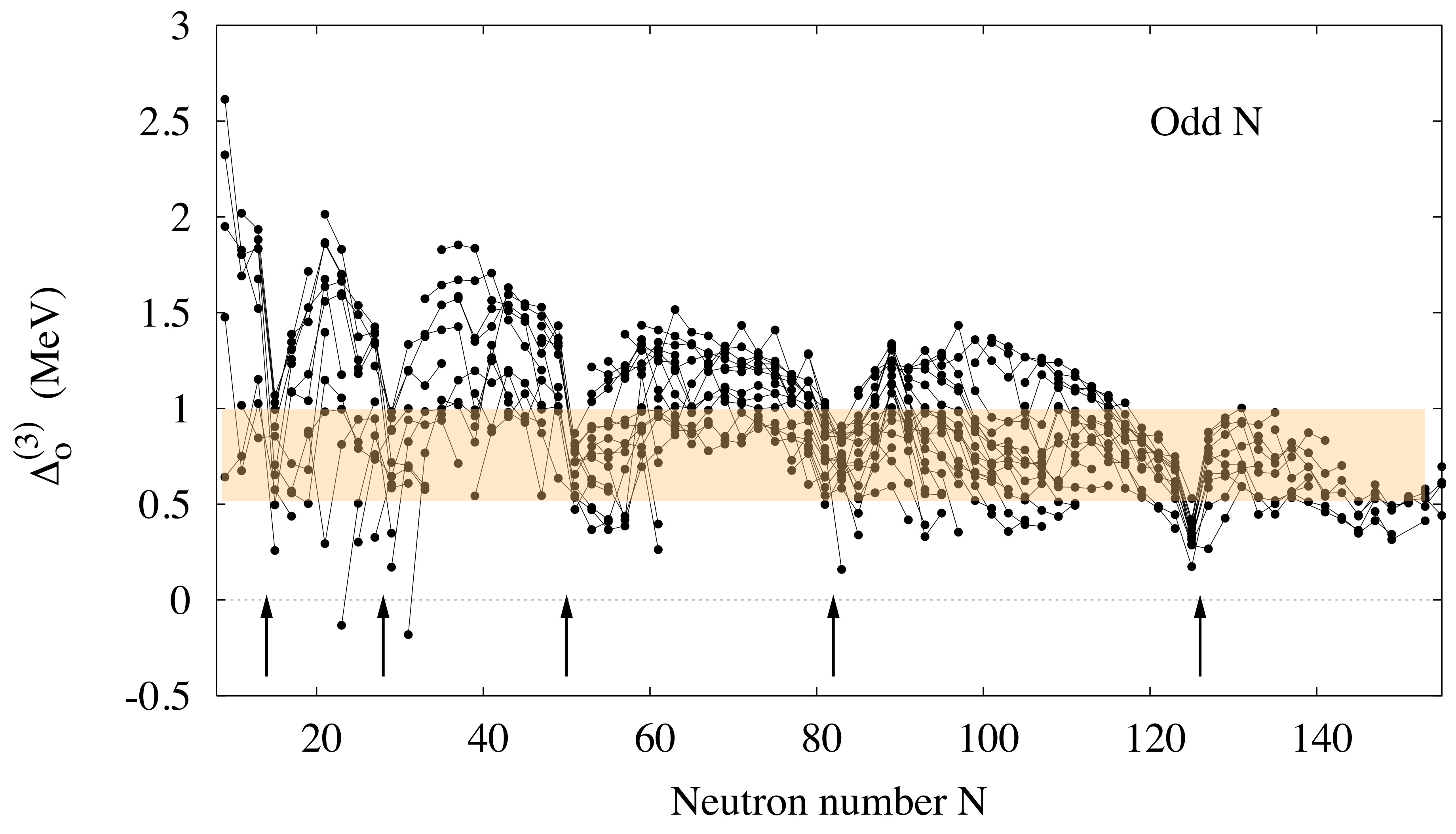




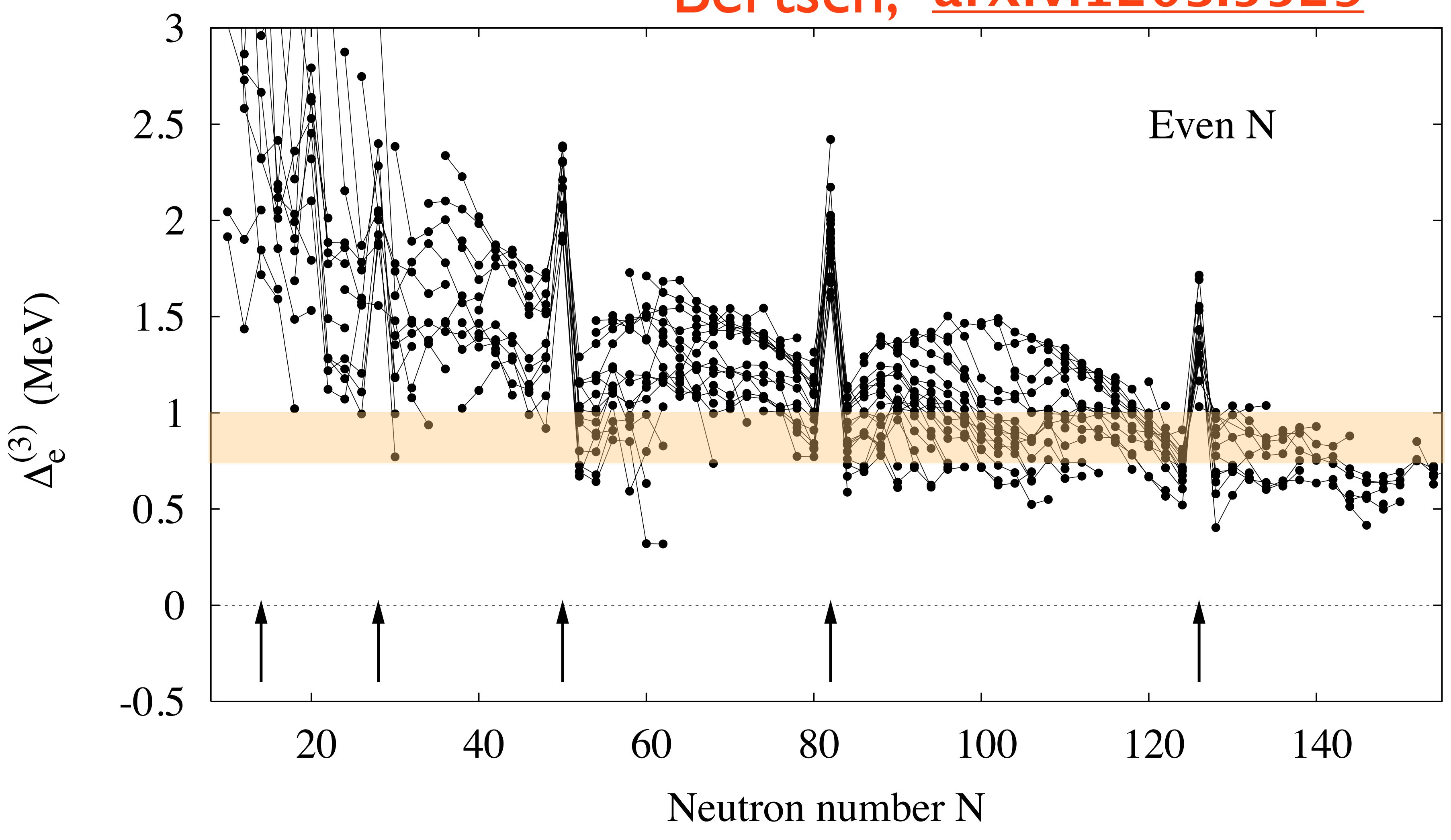
Figures: http://www.nscl.msu.edu/~brown/Jina-workshop/BAB-lecture-notes.pdf

Pairing

Bertsch, arXiv:1203.5529



$$\Delta^{(3)}_{o,Z}(N) = \frac{1}{2}(E_b(Z,N+1) - 2E_b(Z,N) + E_b(Z,N-1))$$



$$\Delta^{(3)}_{e,Z}(N) = -\frac{1}{2}(E_b(Z,N+1) - 2E_b(Z,N) + E_b(Z,N-1))$$

There is a gap in the single particle spectrum
Systems with odd number of neutrons or protons have lower relative binding energy.

Sequence of nuclei encountered in the neutron star outer crust.

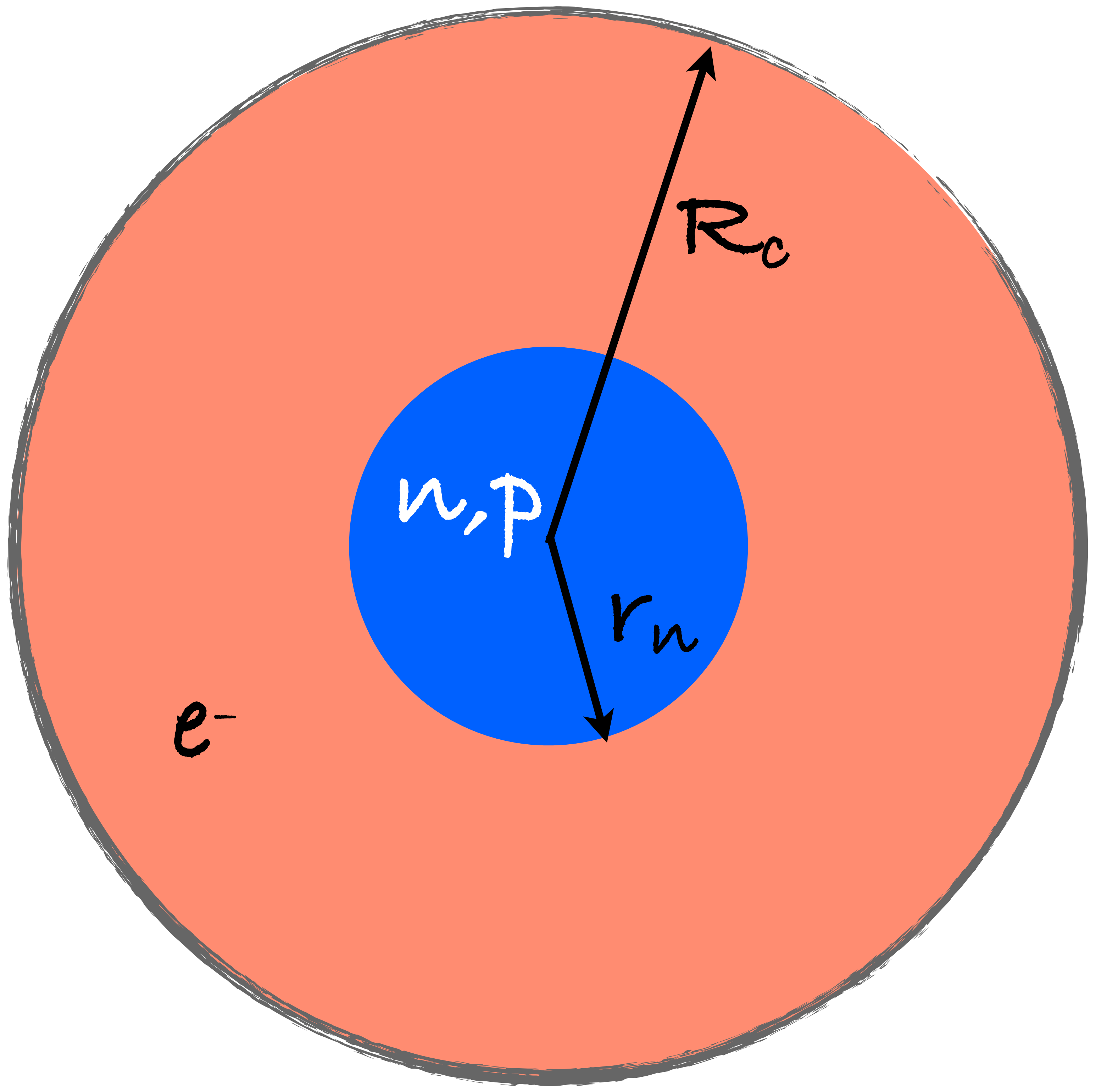
From ^{56}Fe to ^{118}Kr

Table 1 Nuclides in the ground state of cold matter as a function of density, from Haensel & Pichon (21)

Element	Z	N	Z/A	ρ^{a}_{max} (g cm^{-3})	μ^{b}_{e} (MeV)	$\Delta\rho/\rho^{c}$ (%)
Using experimental nuclear masses						
^{56}Fe	26	30	0.4643	7.96 10^{6}	0.95	2.9
^{62}Ni	28	34	0.4516	2.71 10^{8}	2.61	3.1
^{64}Ni	28	36	0.4375	1.30 10^{9}	4.31	3.1
^{66}Ni	28	38	0.4242	1.48 10^{9}	4.45	2.0
^{86}Kr	36	50	0.4186	3.12 10^{9}	5.66	3.3
^{84}Se	34	50	0.4048	1.10 10^{10}	8.49	3.6
^{82}Ge	32	50	0.3902	2.80 10^{10}	11.44	3.9
^{80}Zn	30	50	0.3750	5.44 10^{10}	14.08	4.3
^{78}Ni	28	50	0.3590	9.64 10^{10}	16.78	4.0
From the mass formula of Möller (1992), unpublished results						
^{126}Ru	44	82	0.3492	1.29 10^{11}	18.34	3.0
^{124}Mo	42	82	0.3387	1.88 10^{11}	20.56	3.2
^{122}Zr	40	82	0.3279	2.67 10^{11}	22.86	3.4
^{120}Sr	38	82	0.3167	3.79 10^{11}	25.38	3.6
^{118}Kr	36	82	0.3051	(4.33 10^{11})^{d}	(26.19)	

[a] ρ_{max} is the maximum density at which the nuclide is present.
[b] μ_e is the electron chemical potential (including electron rest mass) at that density.
[c] $\Delta\rho/\rho$ is the fractional increase in the mass density in the transition to the next nuclide.
[d] The lines with ρ_{max} in parentheses correspond to the neutron drip point.

Electron-nucleus Interaction and Lattice Energy



unit cell

To good approximation, electron charge distribution is uniform.

$$E_C = \frac{3}{5}\frac{Z^2\alpha}{r}\left(1 - \frac{3}{2}\frac{r}{R} + \frac{1}{2}\frac{r^3}{R^3}\right)$$

Nucleus becomes unstable to deformations when

$$E_C^0 = \frac{3}{5}\frac{Z^2\alpha}{r} > 2\ E_S$$

or $\left(1 - \frac{3}{2}\frac{r}{R} + \frac{1}{2}\frac{r^3}{R^3}\right) < \frac{1}{4}$ or $\frac{r}{R} > 0.56$

Bohr-Wheeler (1938)

Non-spherical nuclei or Pasta

For spherical nuclei $$E_{\mathrm{C}} = \frac{3}{5}\frac{Z^2\alpha}{r}\left(1 - \frac{3}{2}\frac{r}{R} + \frac{1}{2}\frac{r^3}{R^3}\right)$$

For "d" dimensional structures:

$$\boxed{\frac{E_C}{V} = 2\pi\ \alpha\ n_p^2\ r^2\ u\ f_d(u)}$$

$$\boxed{\frac{E_S}{V} = \frac{d\ u\ \sigma}{r}}$$

where:

$$f_3(u) = \frac{1}{5}(2 - 3u^{1/3} + u) \simeq \frac{2}{5},$$

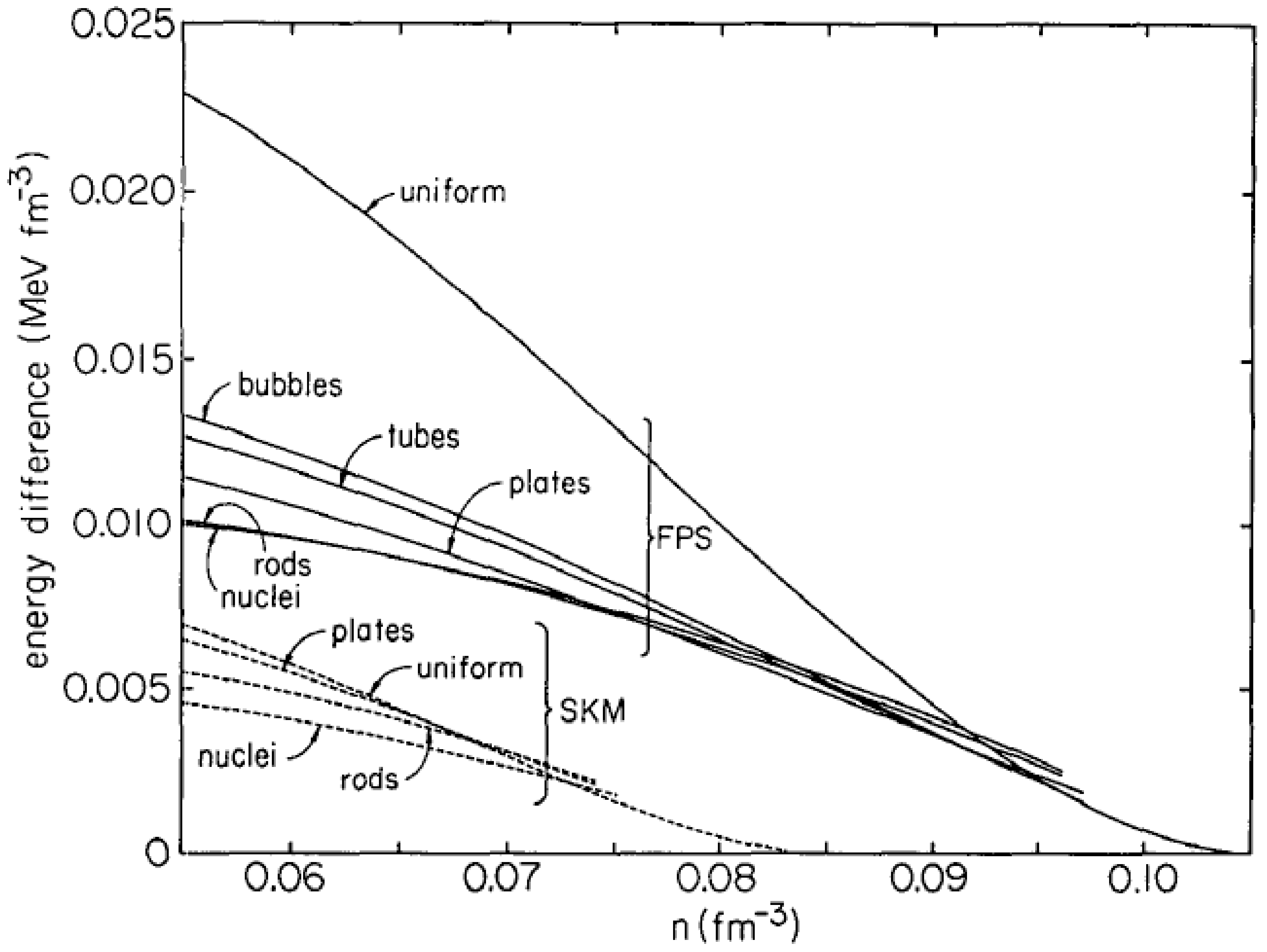
$$f_2(u) = \frac{1}{4}\left(\ln\frac{1}{u} - 1 + u\right) \simeq \frac{1}{4}\ln\frac{1}{eu}$$

$$f_1(u) = \frac{1}{3}\left(\frac{1}{u} - 2 + u\right) \simeq \frac{1}{3u}.$$

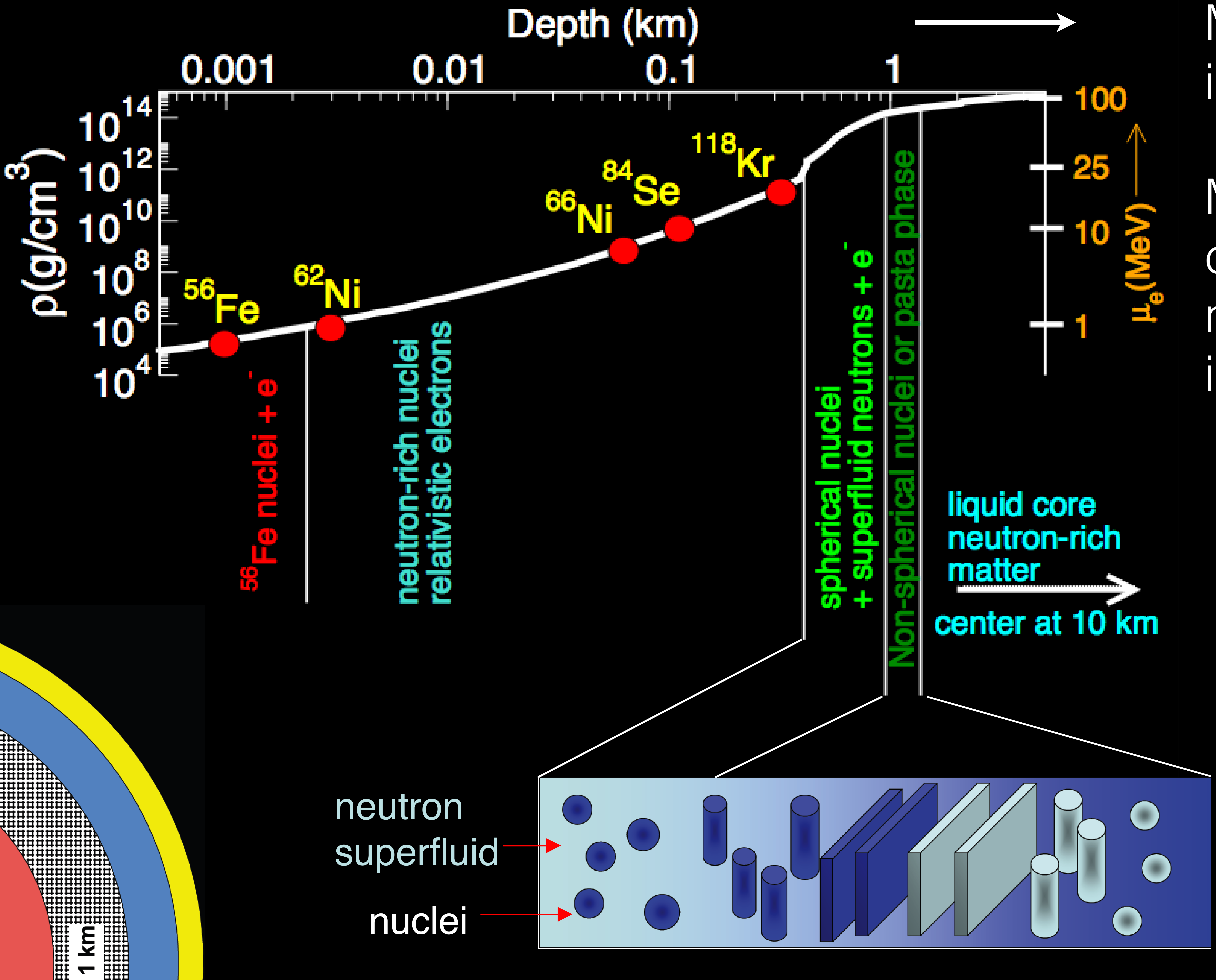
For small surface tension Pasta is favored.

Baym, Bethe, Pethick (1971)

Energy gain is modest and model dependent



Baym, Bethe, Pethick (1971)



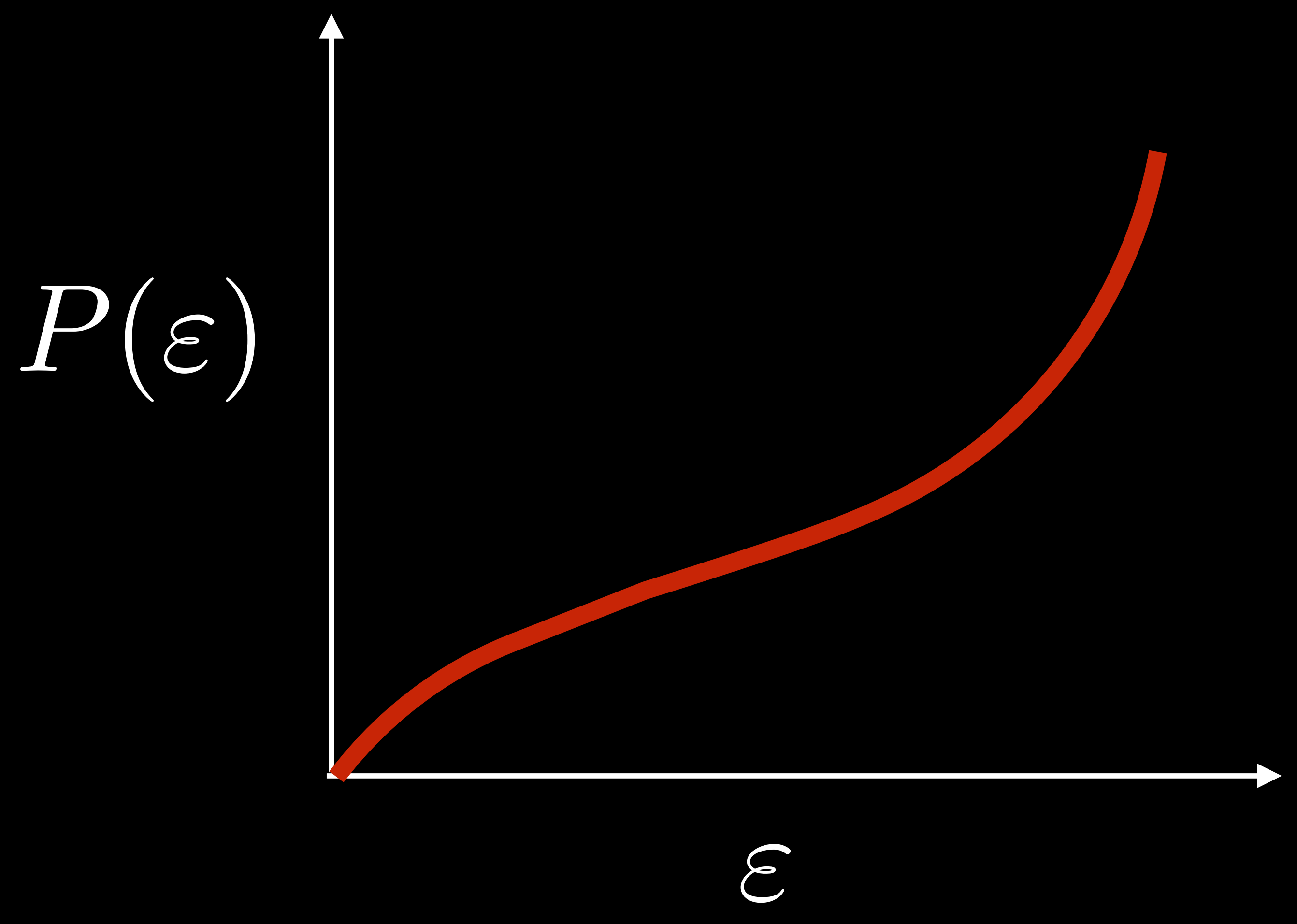
Mass contained in the crust is small ~ few percent.

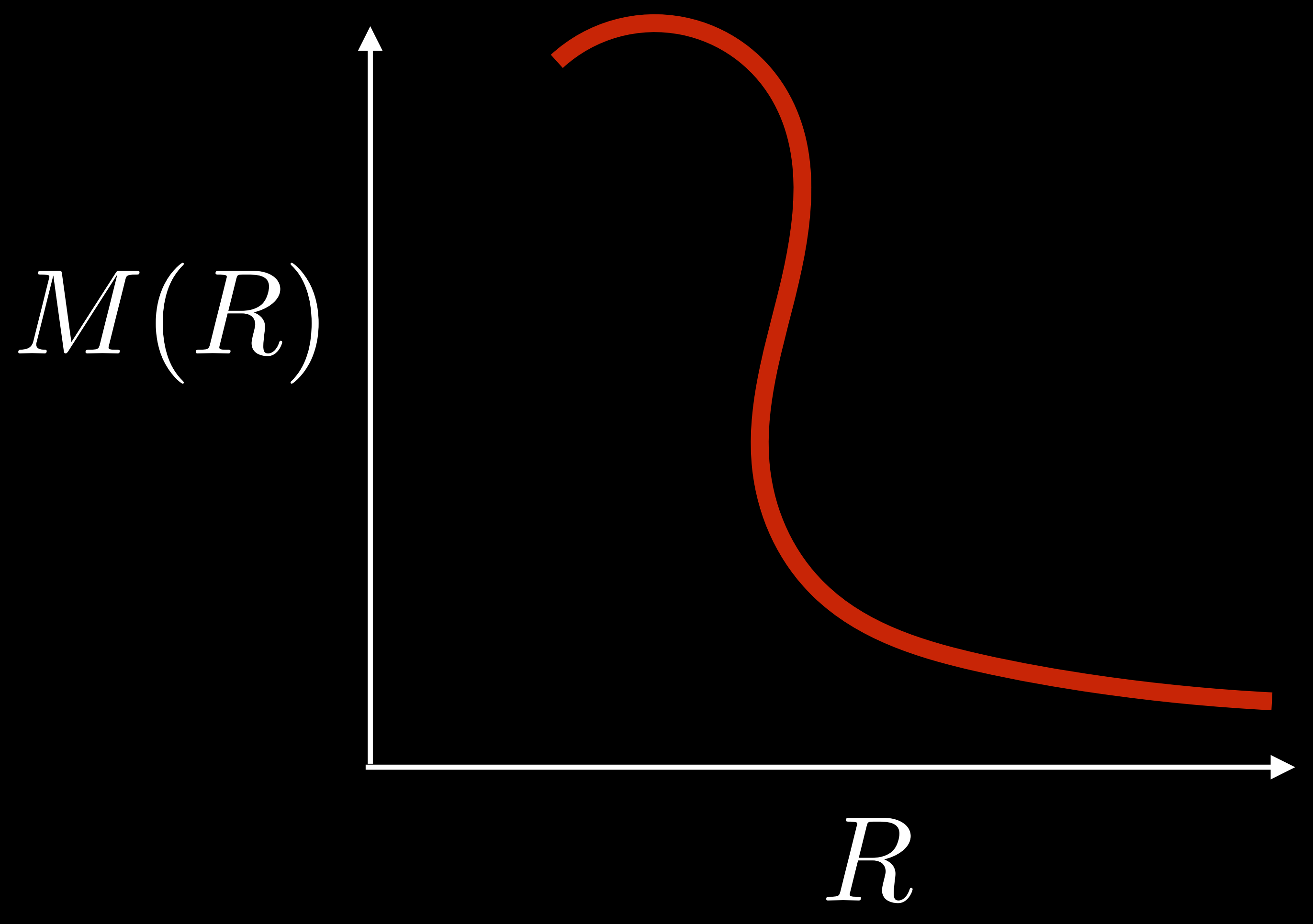
Most of it is in the inner-crust as either spherical or non-spherical nuclei immersed in a neutron fluid.

Neutron Fraction:

Outer Crust < 70%.
Inner Crust ~ 90%.
Outer Core: > 90%

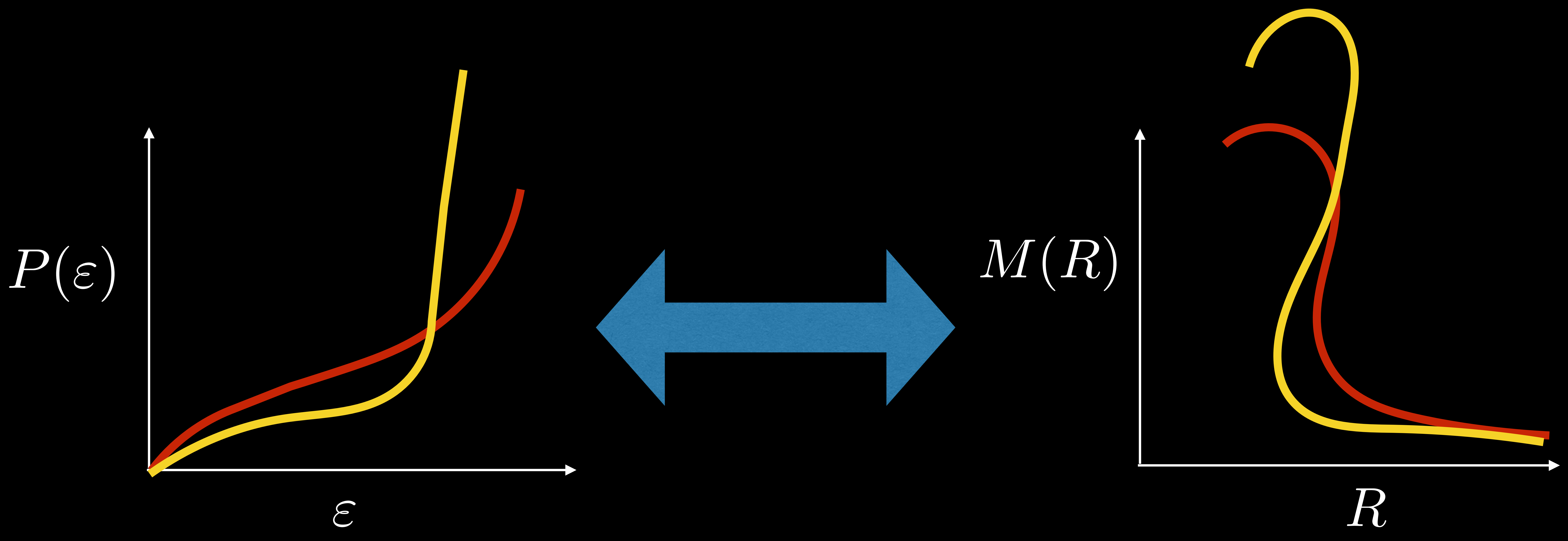
Equation of State and Neutron Star Structure





$$P(\varepsilon) + \text{Gen.Rel.} = M(R)$$

Equation of State and Neutron Star Structure



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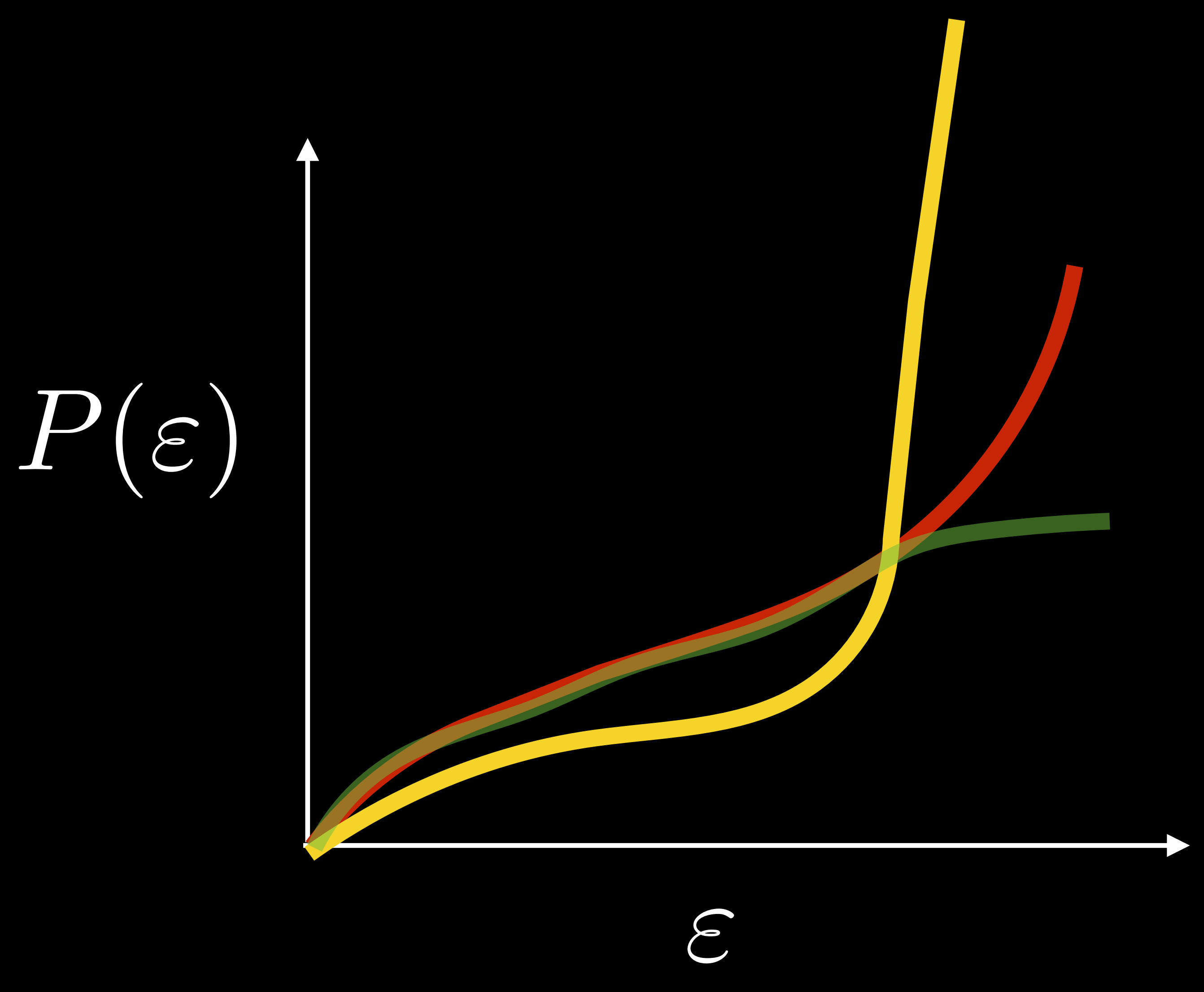

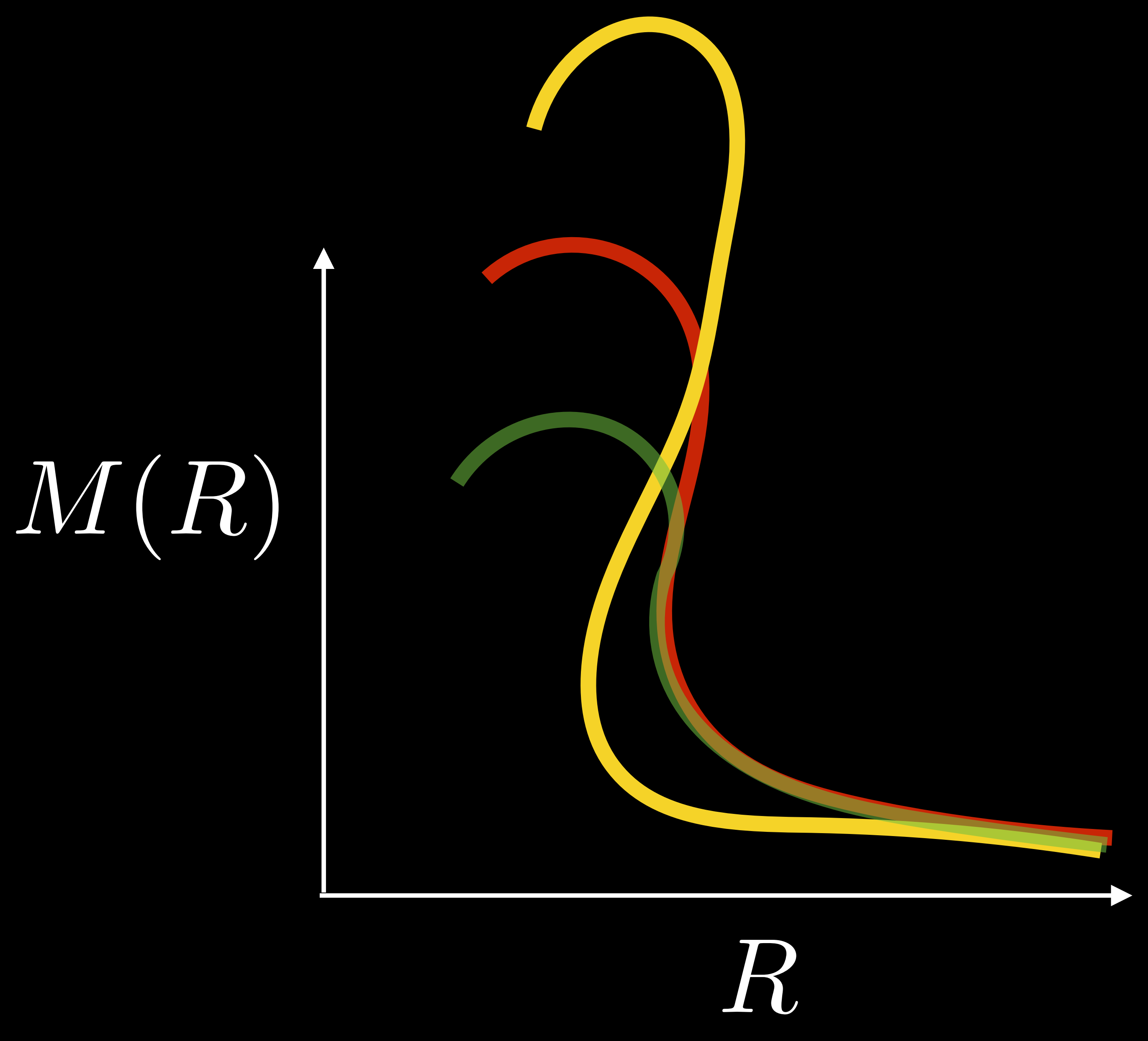

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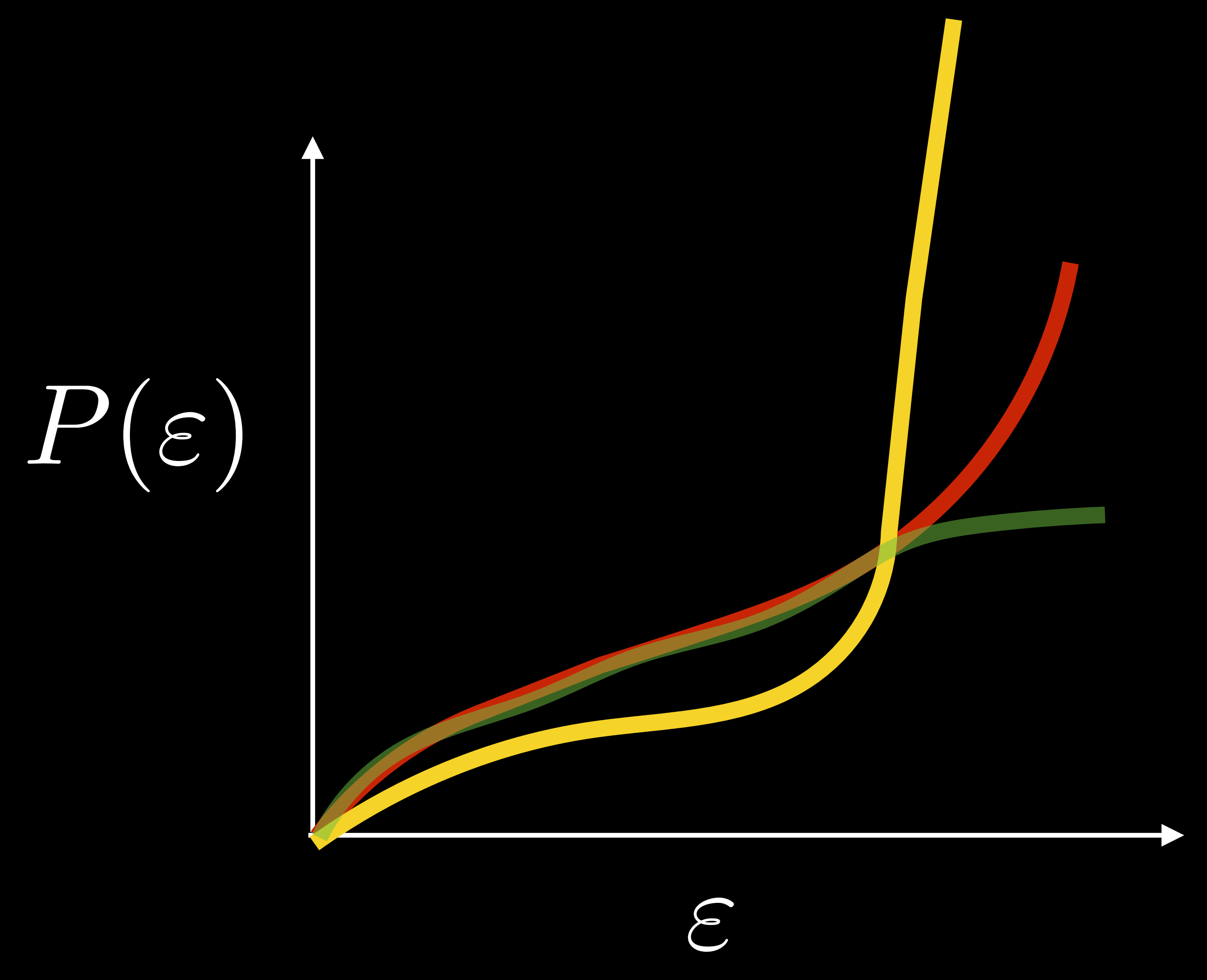
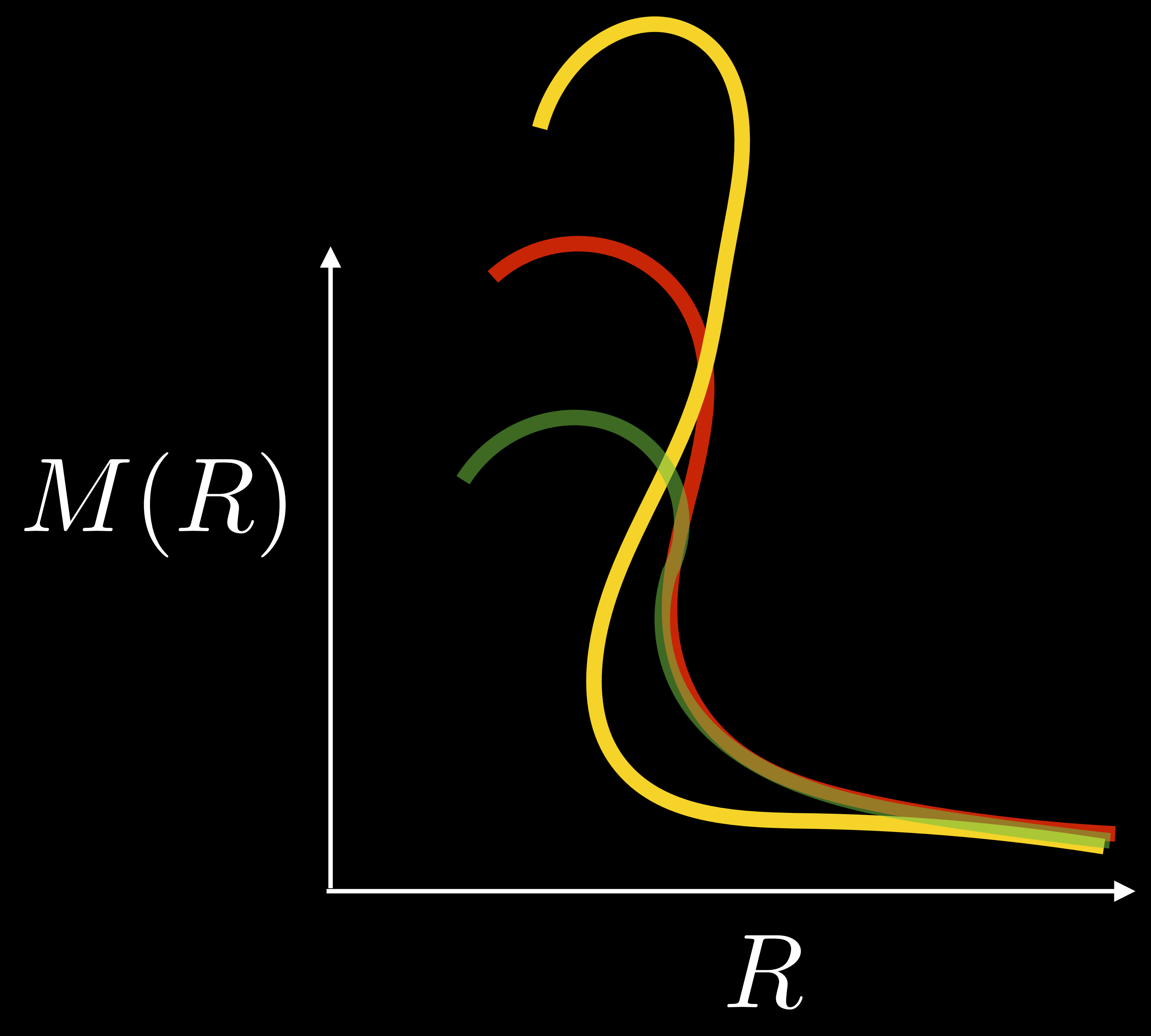
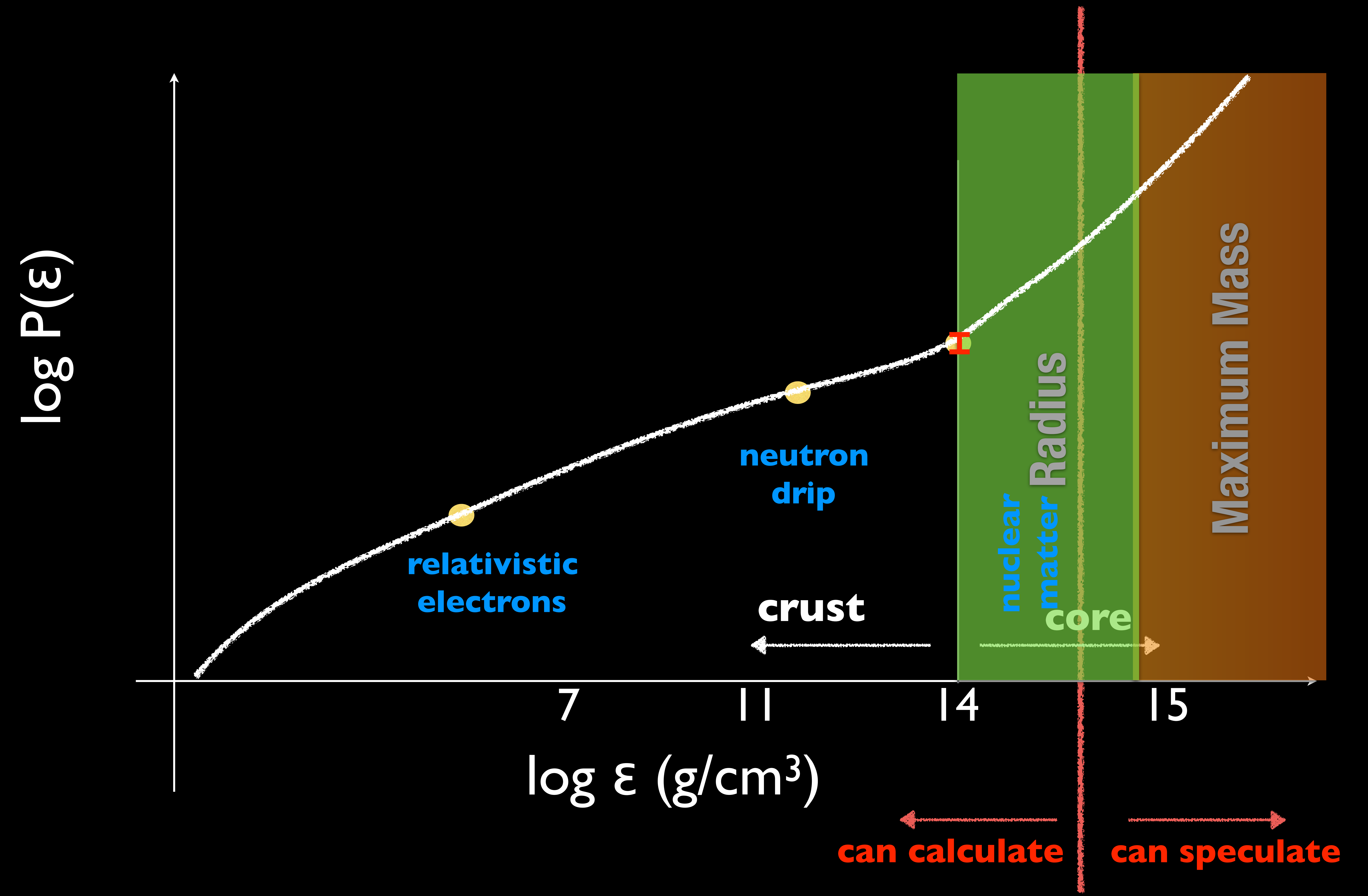




$$P(\varepsilon) + \text{Gen.Rel.} = M(R)$$

A small radius and large maximum mass implies a rapid transition from low pressure to high pressure with density.

Constraints on the Equation of State



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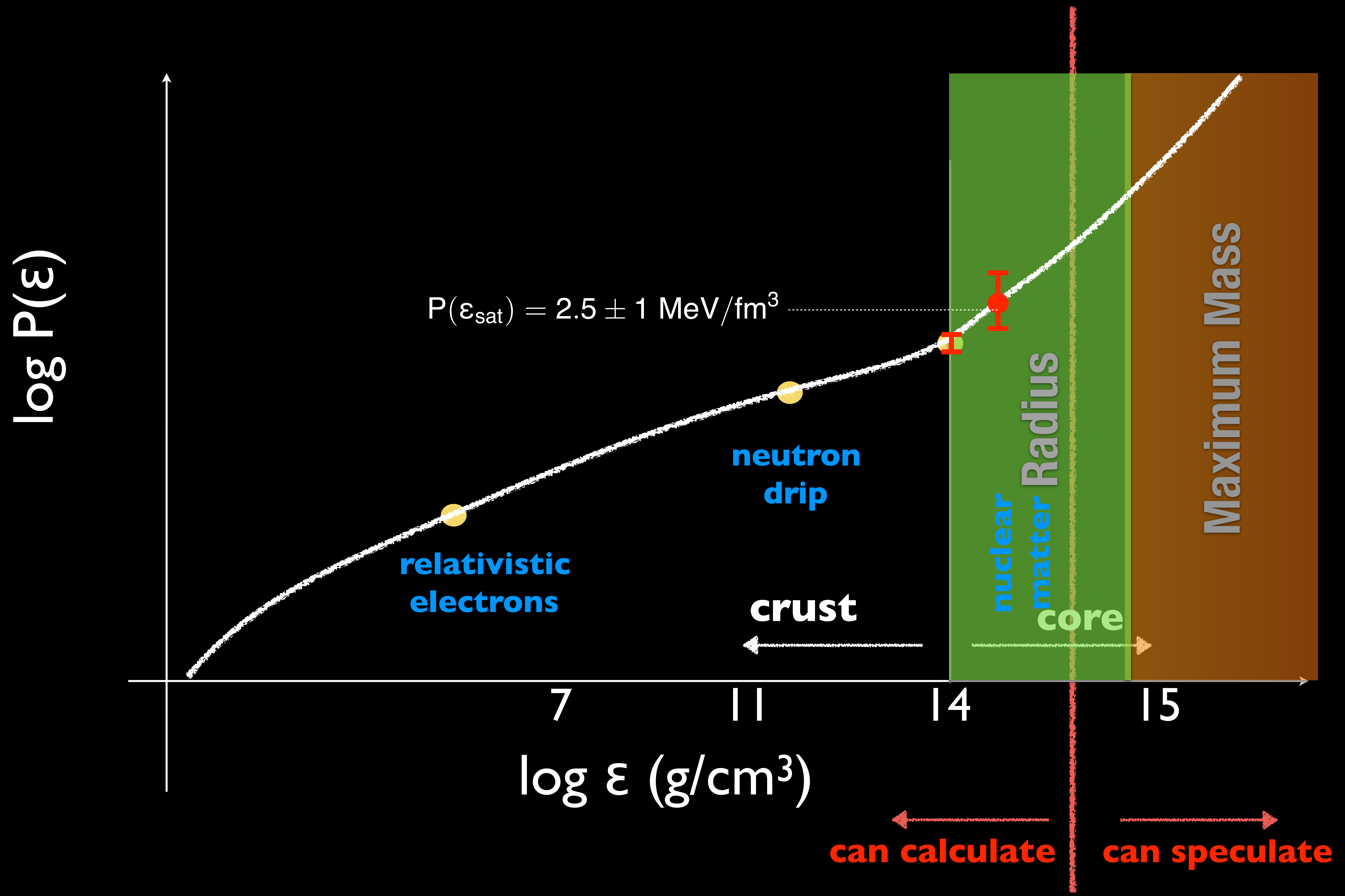

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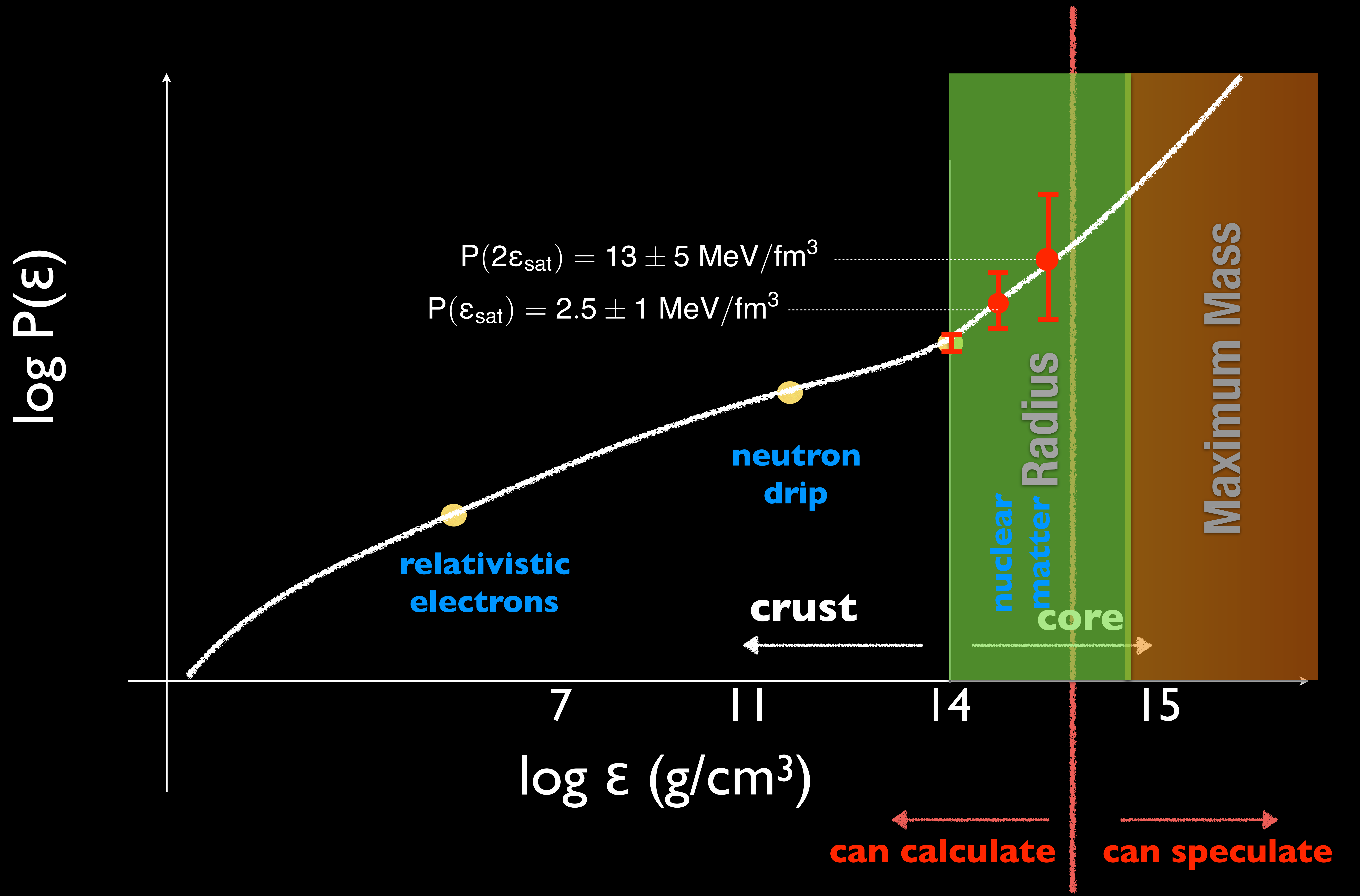

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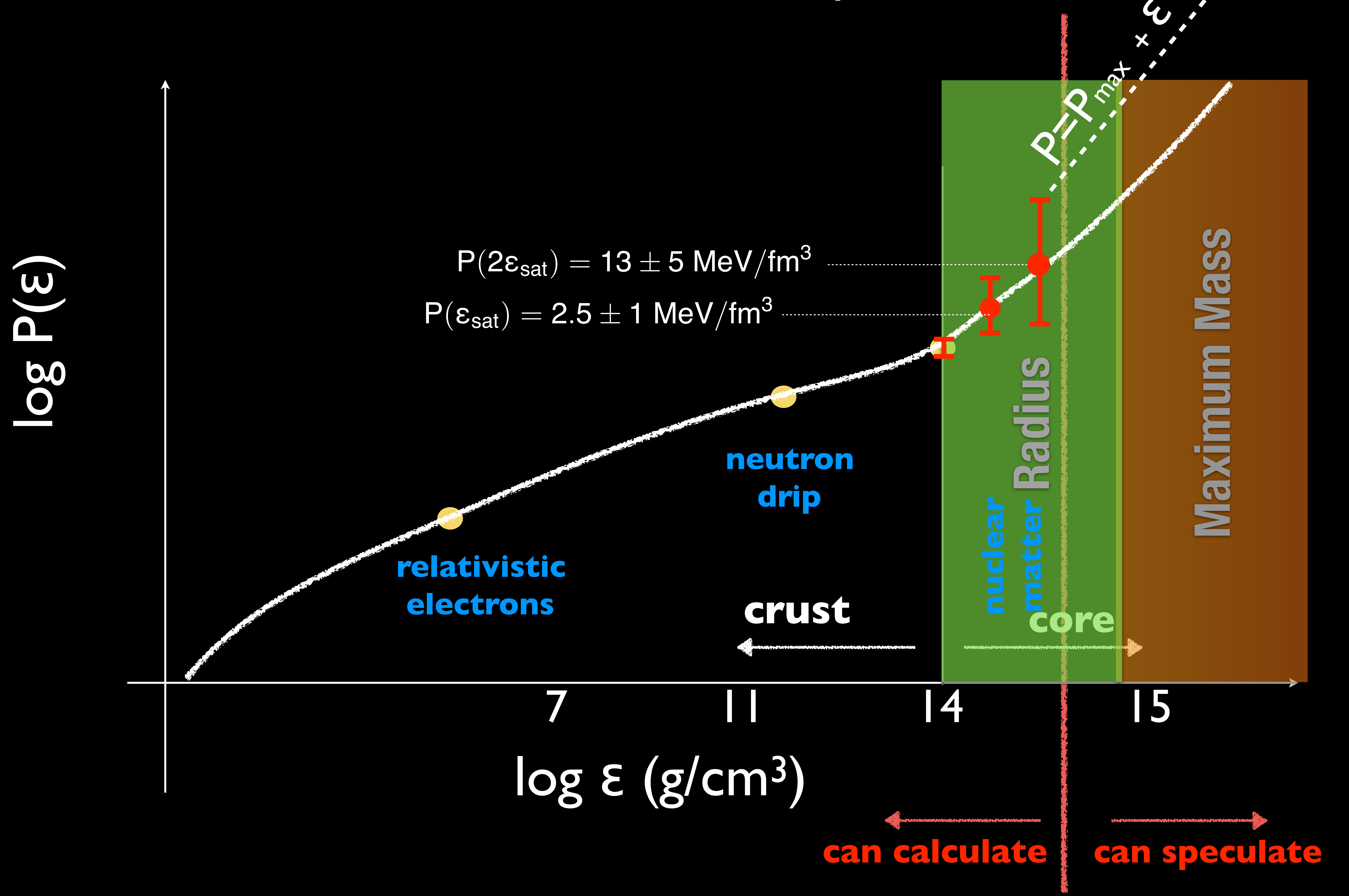

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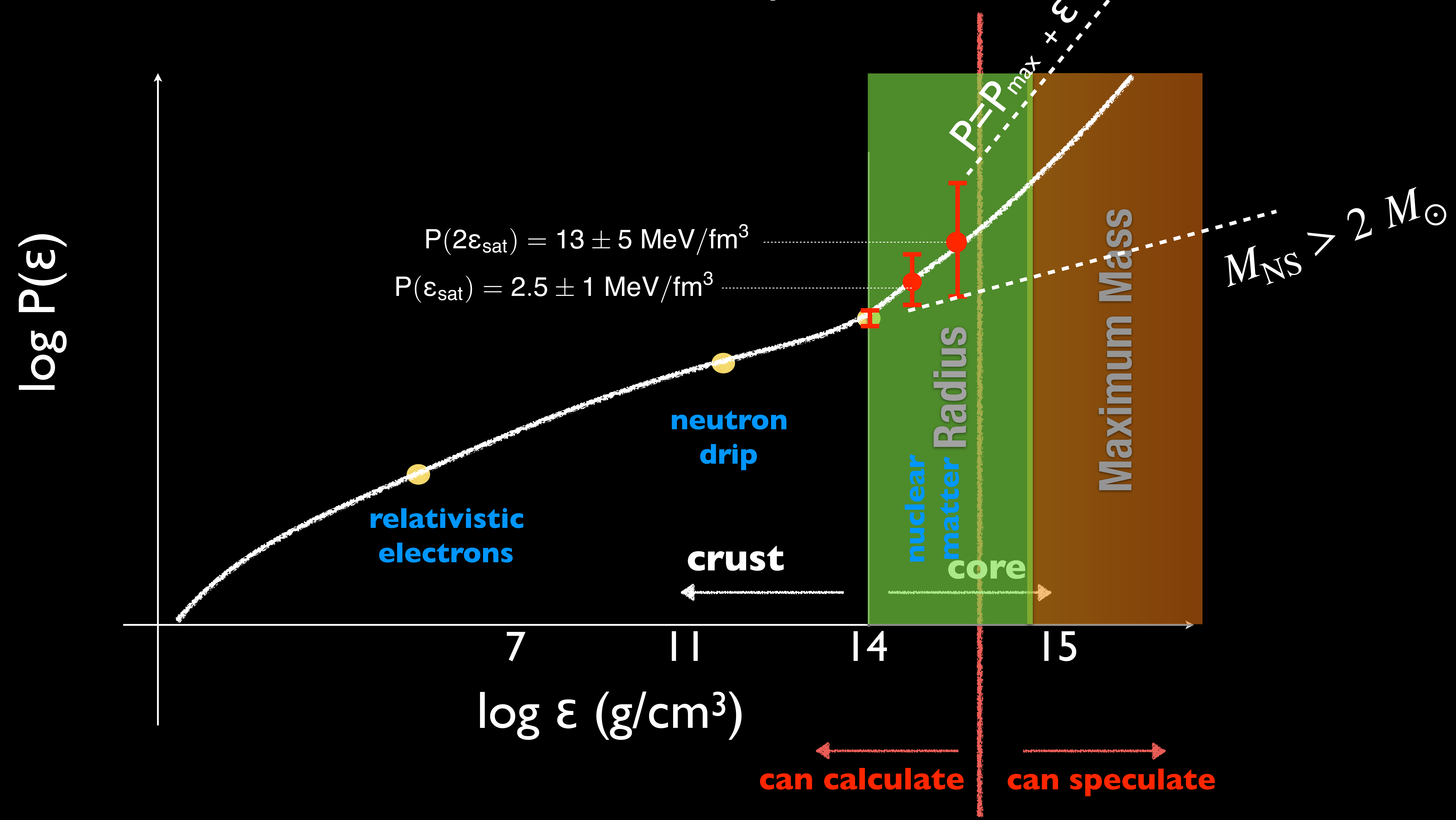
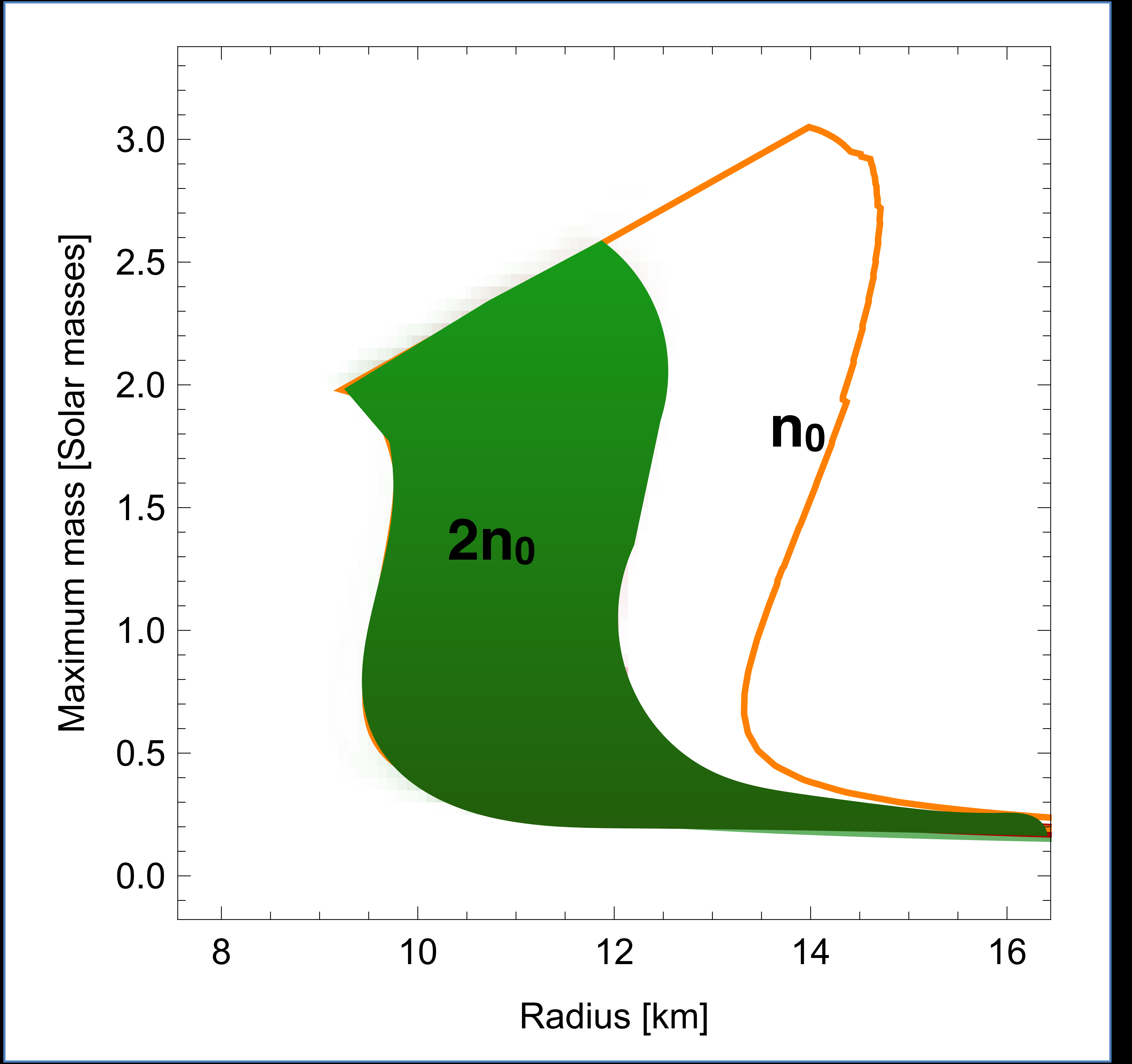


Dense matter EOS and NS structure

Neutron matter calculations and a sound speed at higher density constrained by 2 solar mass NS and causality provide useful constraints on the NS properties.

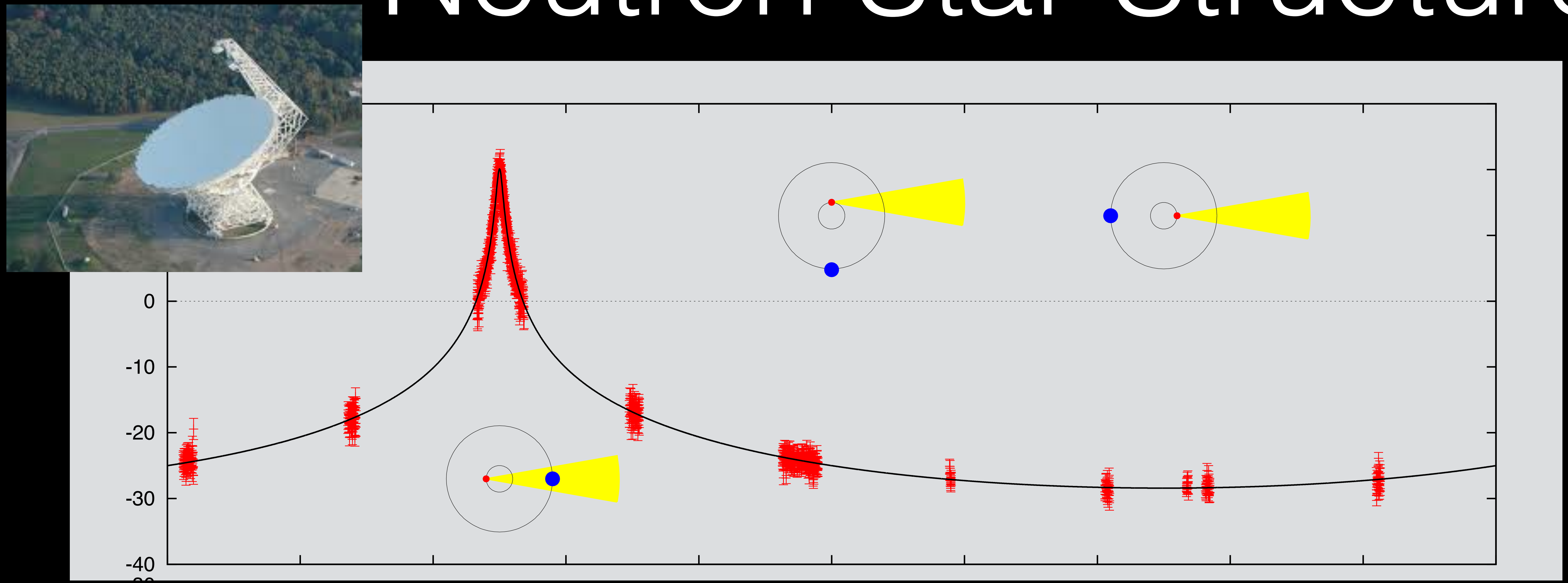
$R_{1.4} = 9.5 - 12.5$ km

$M_{max} = 2.0 - 2.5\ M_{solar}$



Tews, Gandolfi, Carlson, Reddy (2018), Tews, Margueron, Reddy (2018)
Hebeler, Schwenk, Lattimer and Pethick (2010,2013) and Carlson, Gandolfi, Reddy (2012)

Neutron Star Structure: Observations



2 $M_\odot$ neutron stars exist.

PSR J1614-2230: M=1.93(2)

Demorest et al. (2010)

PSR J0348+0432: M=2.01(4) $M_\odot$

Anthoniadis et al. (2013)

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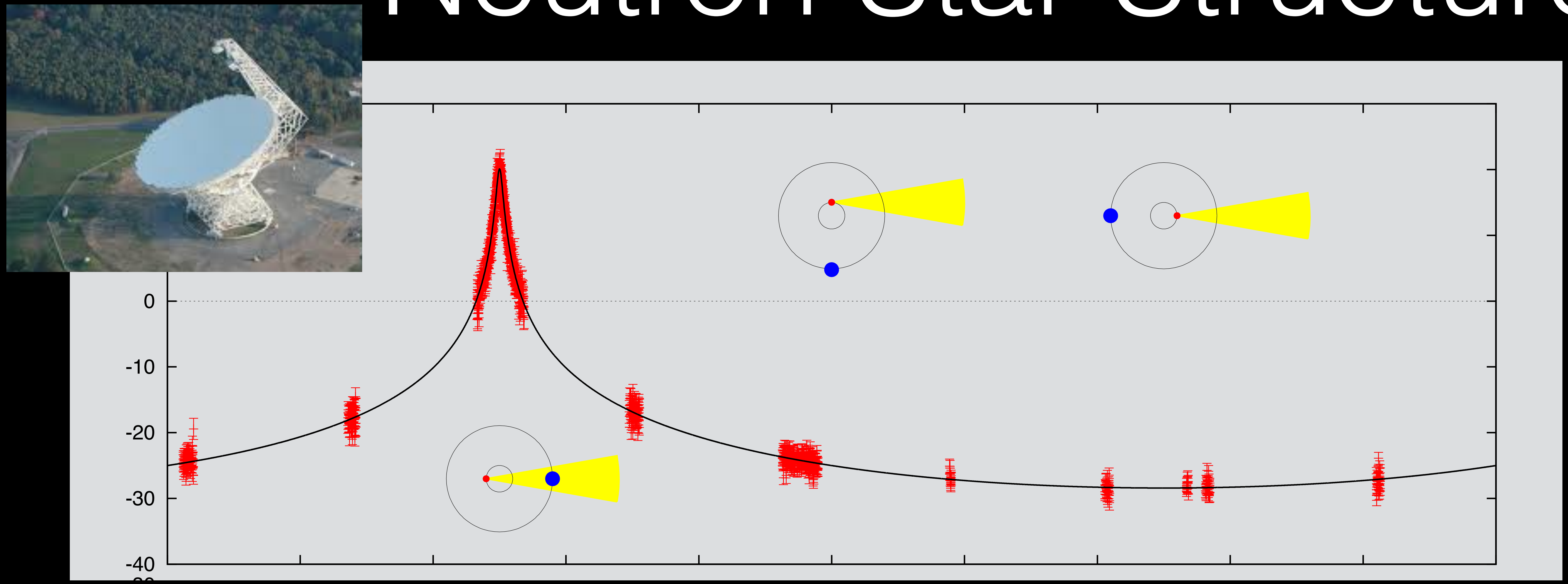

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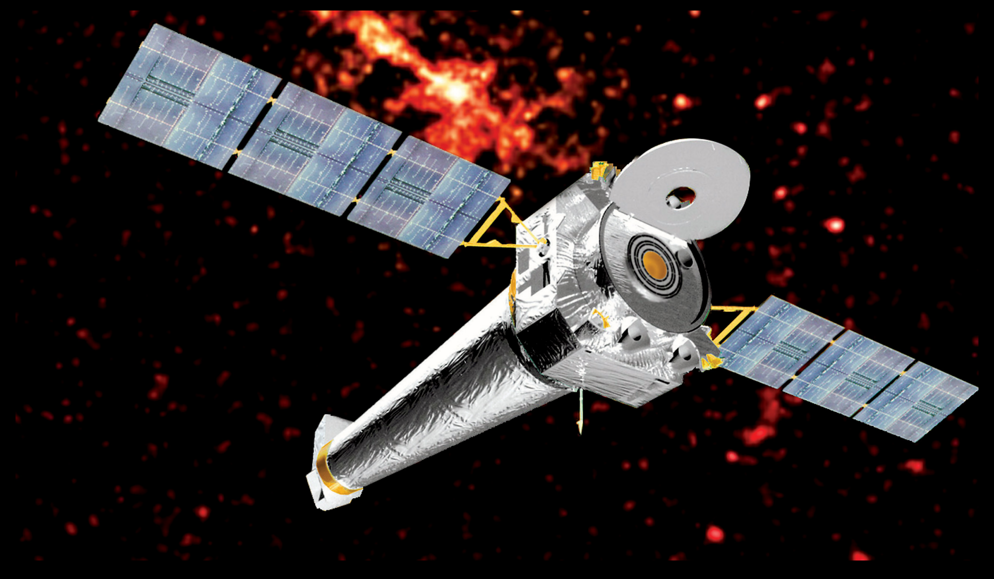
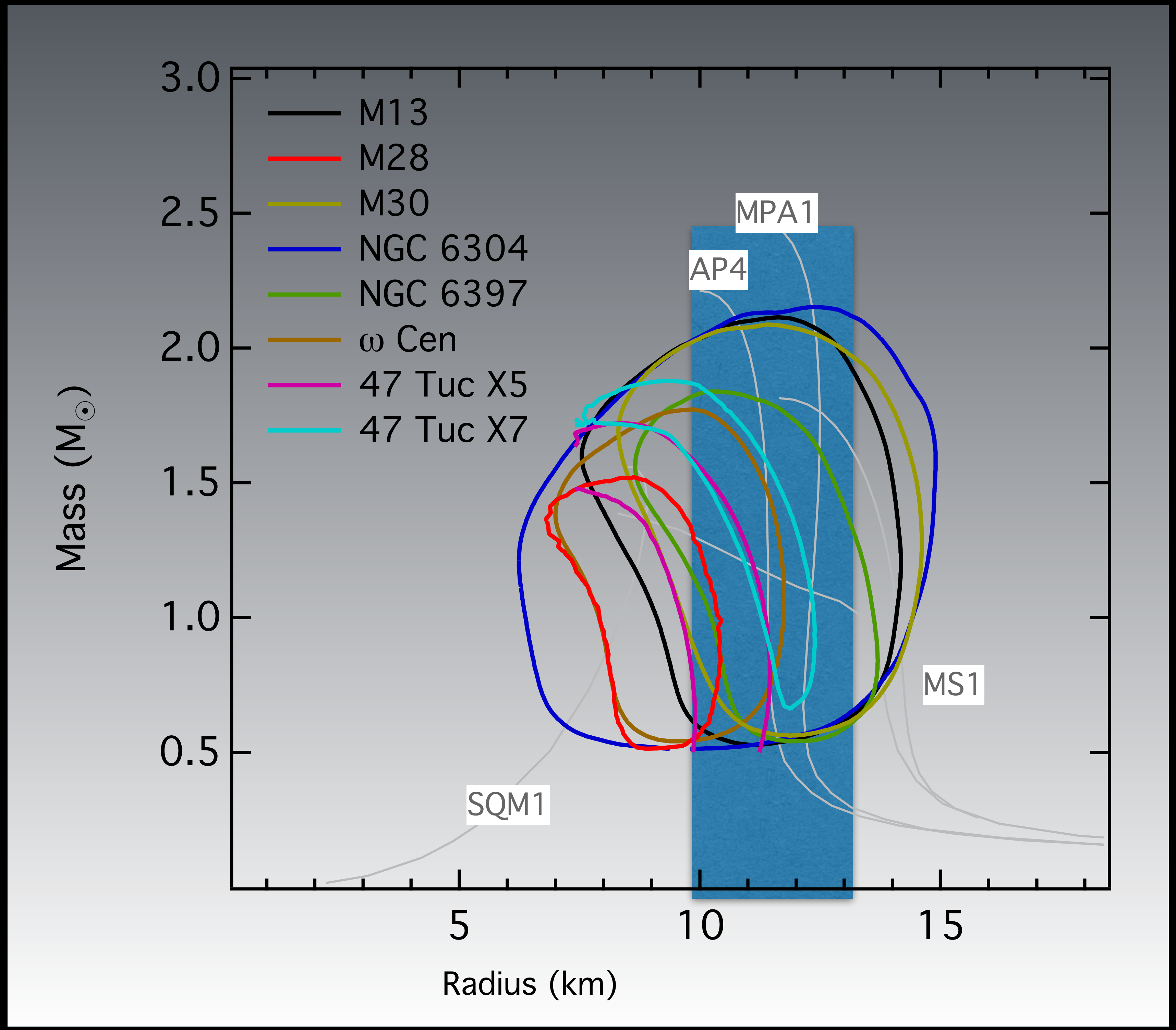


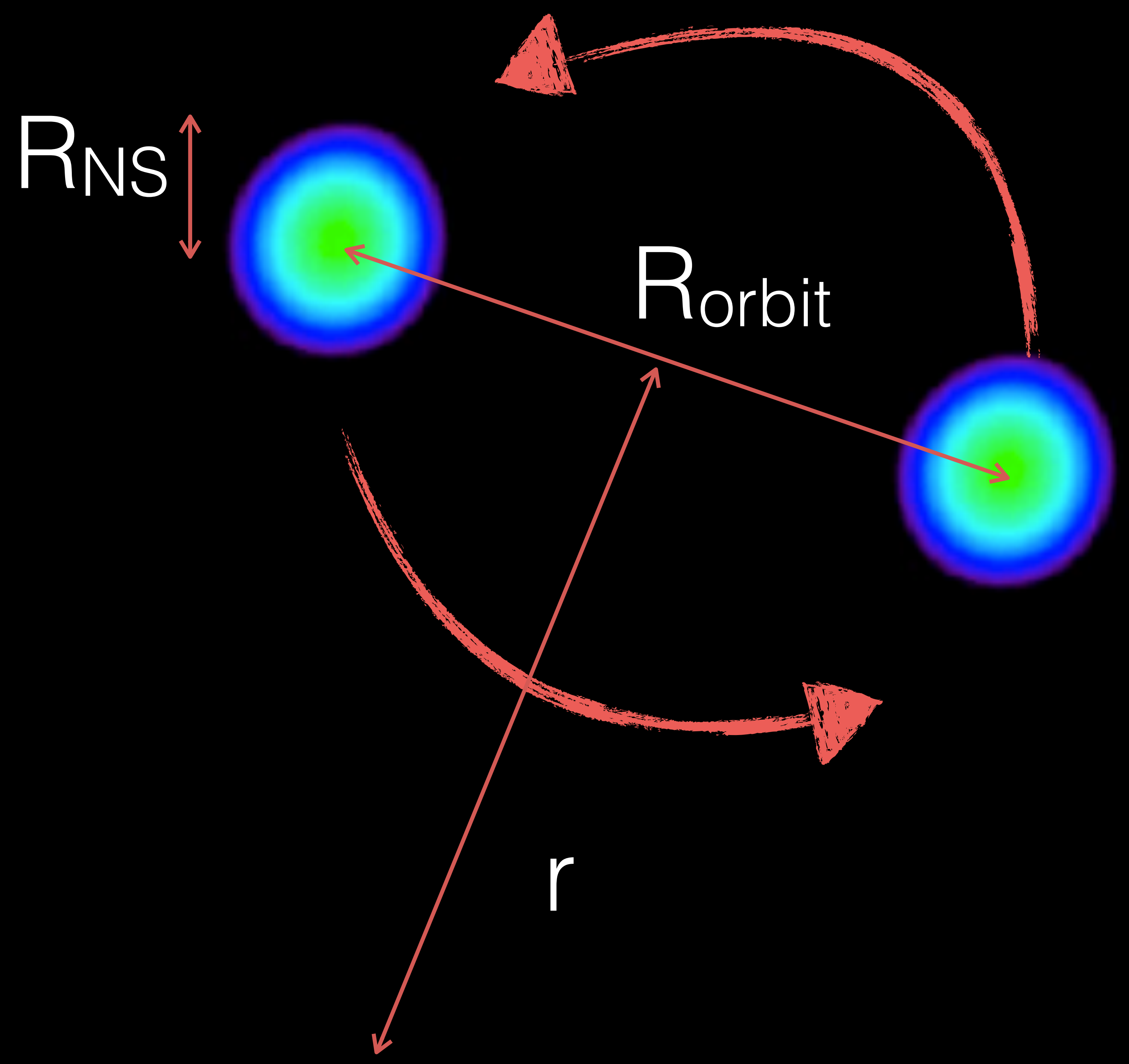
Inferred NS radii are small.

Despite poorly understood systematic errors, x-ray observations suggest R ~ 9-13 km. Perhaps even preferring a smaller range R~ 10-12 km.

Ozel & Freire (2016)



Binary Inspiral and Gravitational Waves



GWs are produced by fluctuating quadrupoles.

$$h_{\mu\nu}(r,t) = \frac{2G}{r}\,\ddot{I}_{ij}(t_R)$$

$$\ddot{I}_{ij}(t) \approx M\,R_{orbit}^2\,f^2 \approx M^{5/3}\,f^{2/3}$$

$$h \approx 10^{-23}\left(\frac{M_{NS}}{M_\odot}\right)^{5/3}\left(\frac{f}{200\text{ Hz}}\right)^{2/3}\left(\frac{100\text{ Mpc}}{r}\right)$$

- Advanced LIGO can detect GWs from binary neutron stars out to about 200 Mpc at design sensitivity. Detection rate ~ 1- 50 per year.

GW170817: Gravitational Waves from Neutron Stars!

Selected for a Viewpoint in *Physics*
PRL **119,** 161101 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

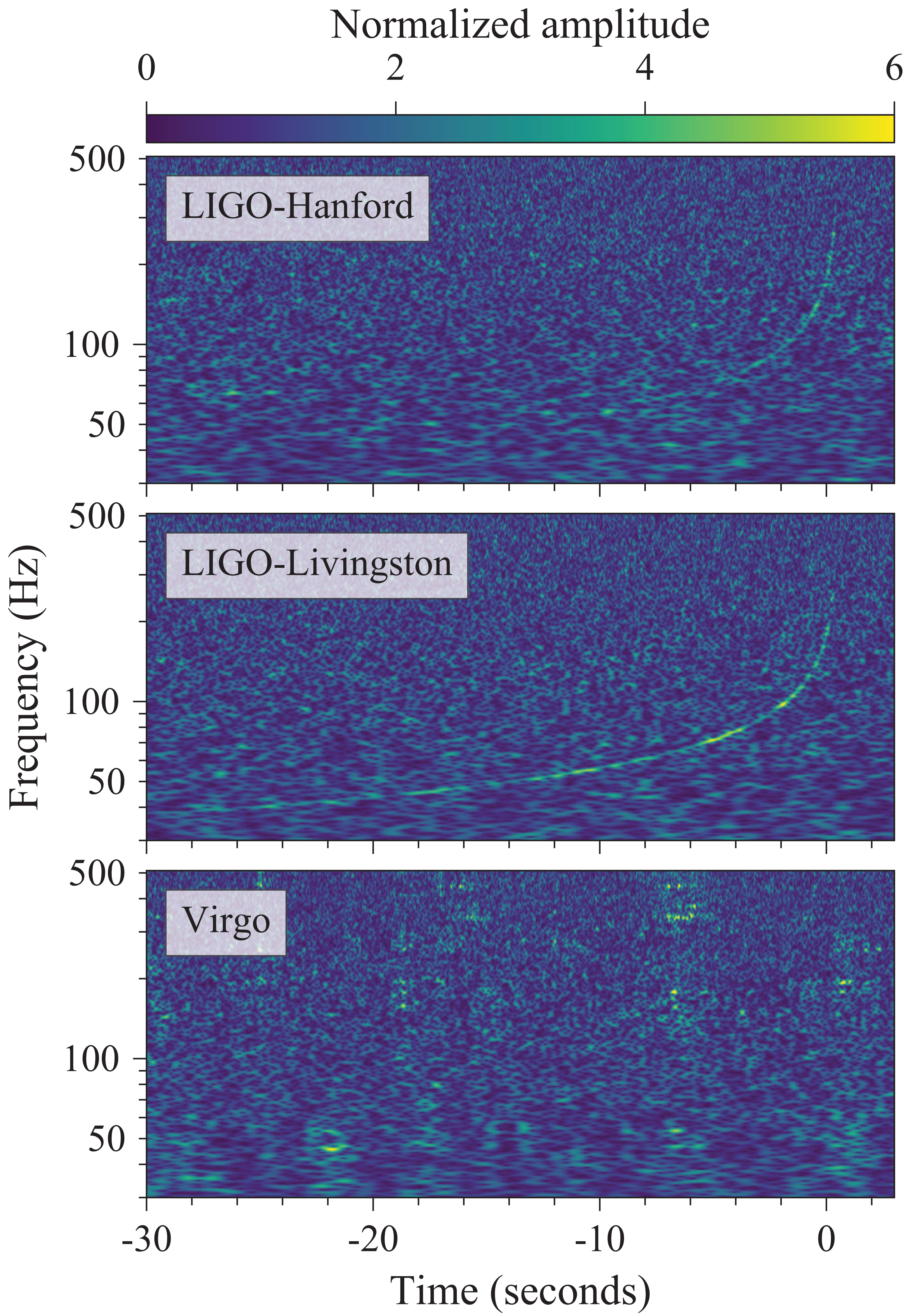
B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 26 September 2017; revised manuscript received 2 October 2017; published 16 October 2017)

Component masses:

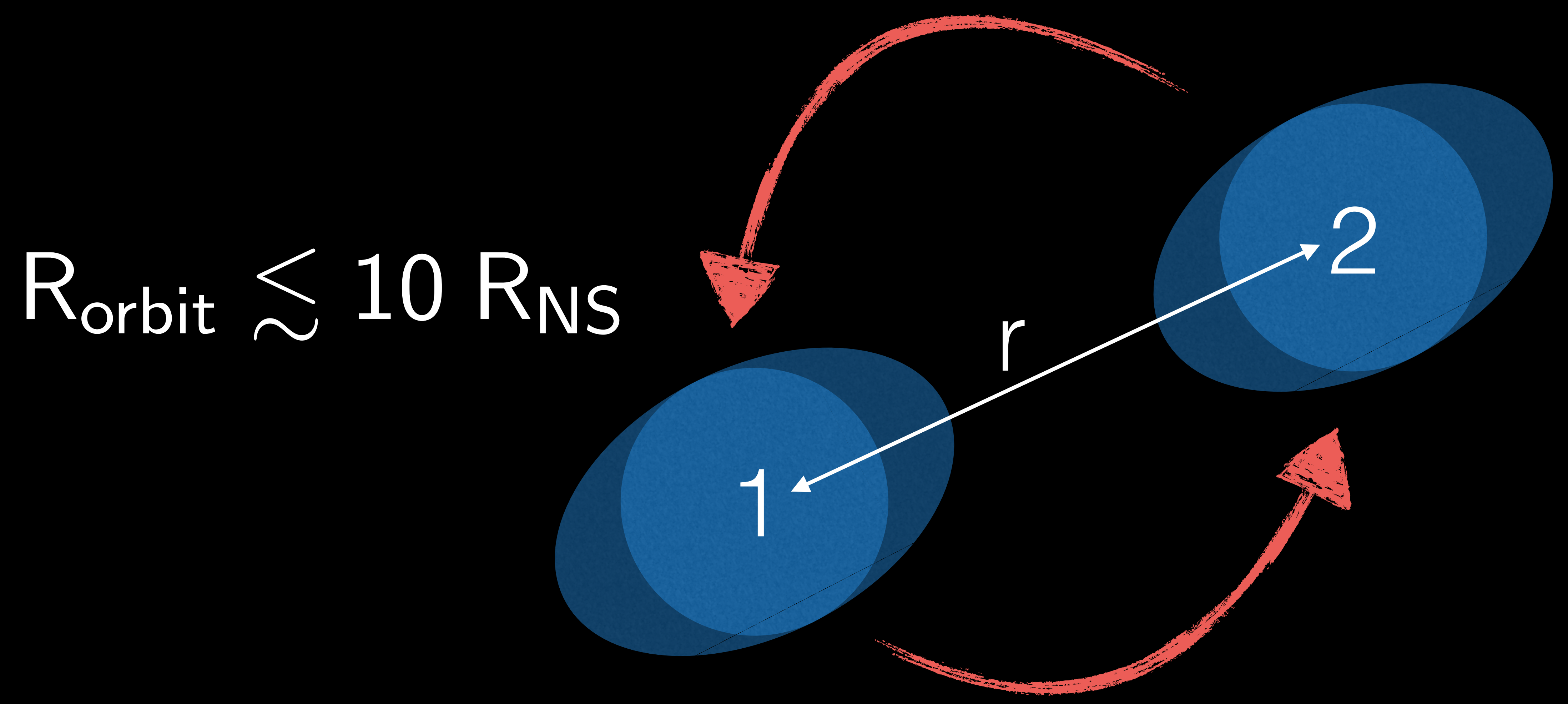
$$m_1 = 1.47 \pm 0.13\ M_\odot$$
$$m_2 = 1.17 \pm 0.09\ M_\odot$$

Chirp Mass: $$\mathscr{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_s)^{1/5}} = 1.188^{+0.004}_{-0.002}\ M_\odot$$

Total Mass: $$M = m_1 + m_2 = 2.74^{+0.04}_{-0.01}\ M_\odot$$



Tidal Deformation: Measuring the Neutron Star Radius



Tidal forces deform neutron stars.
Induces a quadrupole moment.

$$Q_{xy} = \lambda\ E_{xy} \qquad E_{xy} = -\frac{\partial^2 V_G}{\partial x \partial y}$$

tidal deformability

external field

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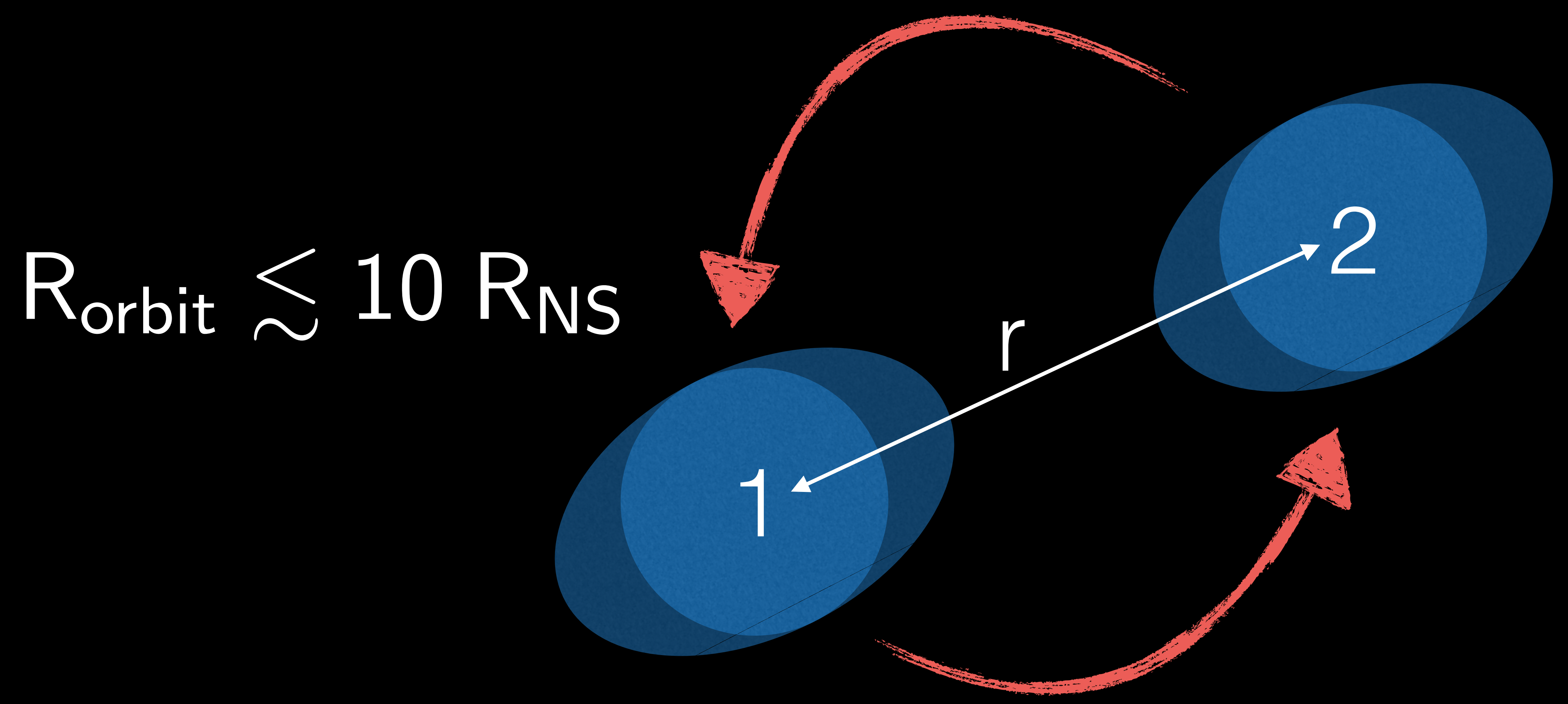

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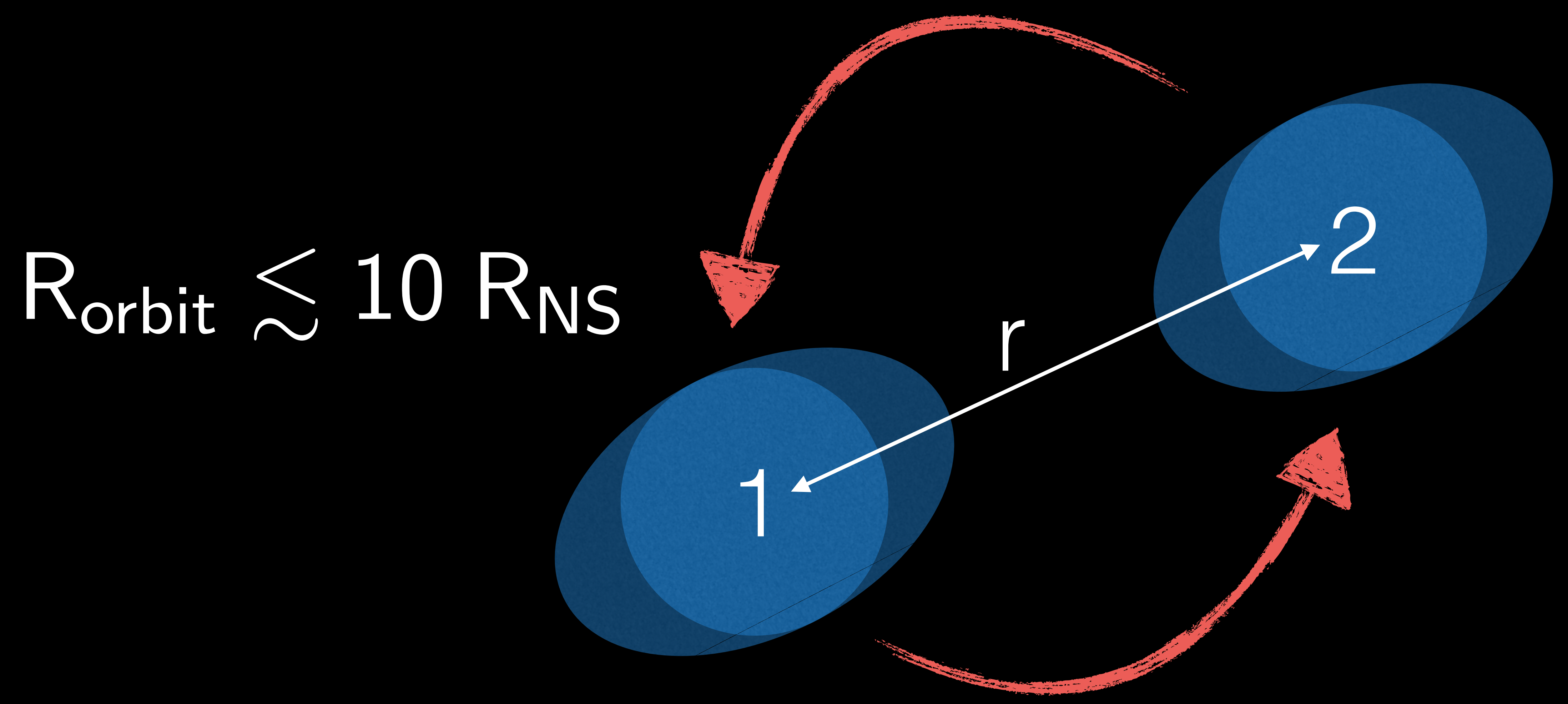

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Dimensionless binary tidal deformability: $\tilde{\Lambda} = \frac{16}{13}\left(\left(\frac{M_1}{M}\right)^5 \left(1 + \frac{M_2}{M_1}\right)\Lambda_1 + 1 \leftrightarrow 2\right)$

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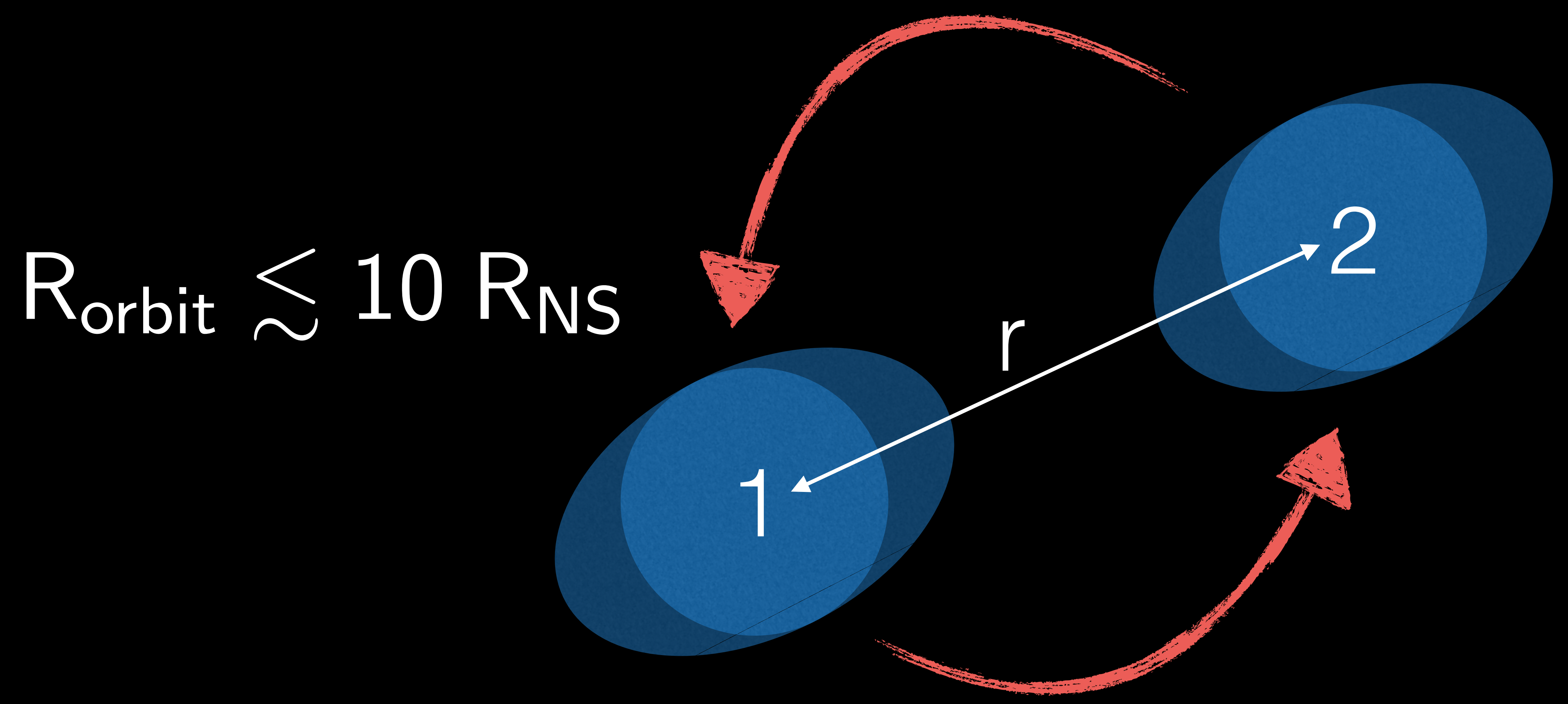

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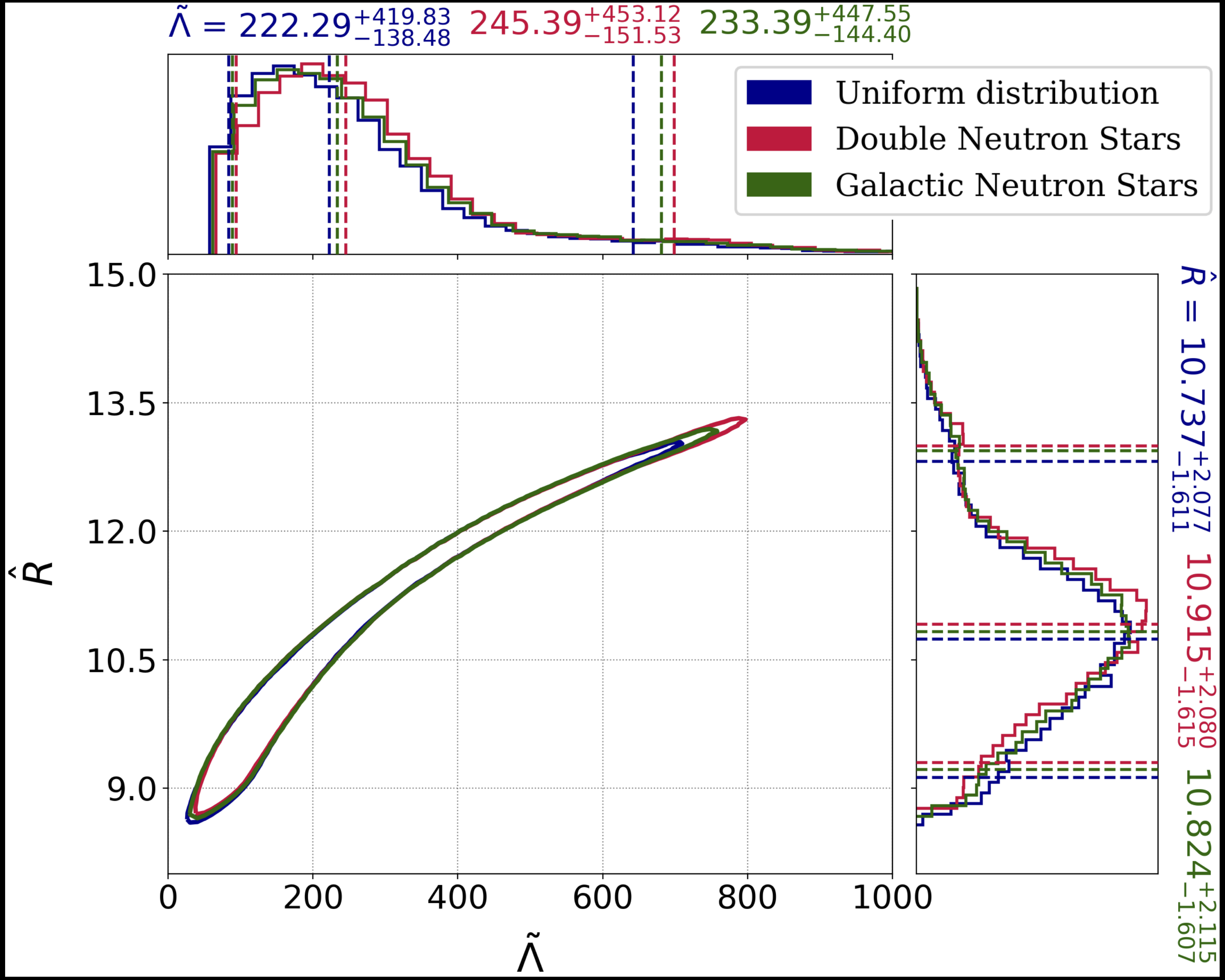
Tidal deformations are large for a large NS: $\Lambda_i = \frac{\lambda_i}{M_i^5} = k_2 \frac{R_i^5}{M_i^5}$

Tidal Effects at Late Times

B. Lackey, L. Wade. PRD 91, 043002 (2015)

Neutron Stars are Small



Tidal deformations observed in GW170817 are small and suggests that the NS radius:

R < 13 km

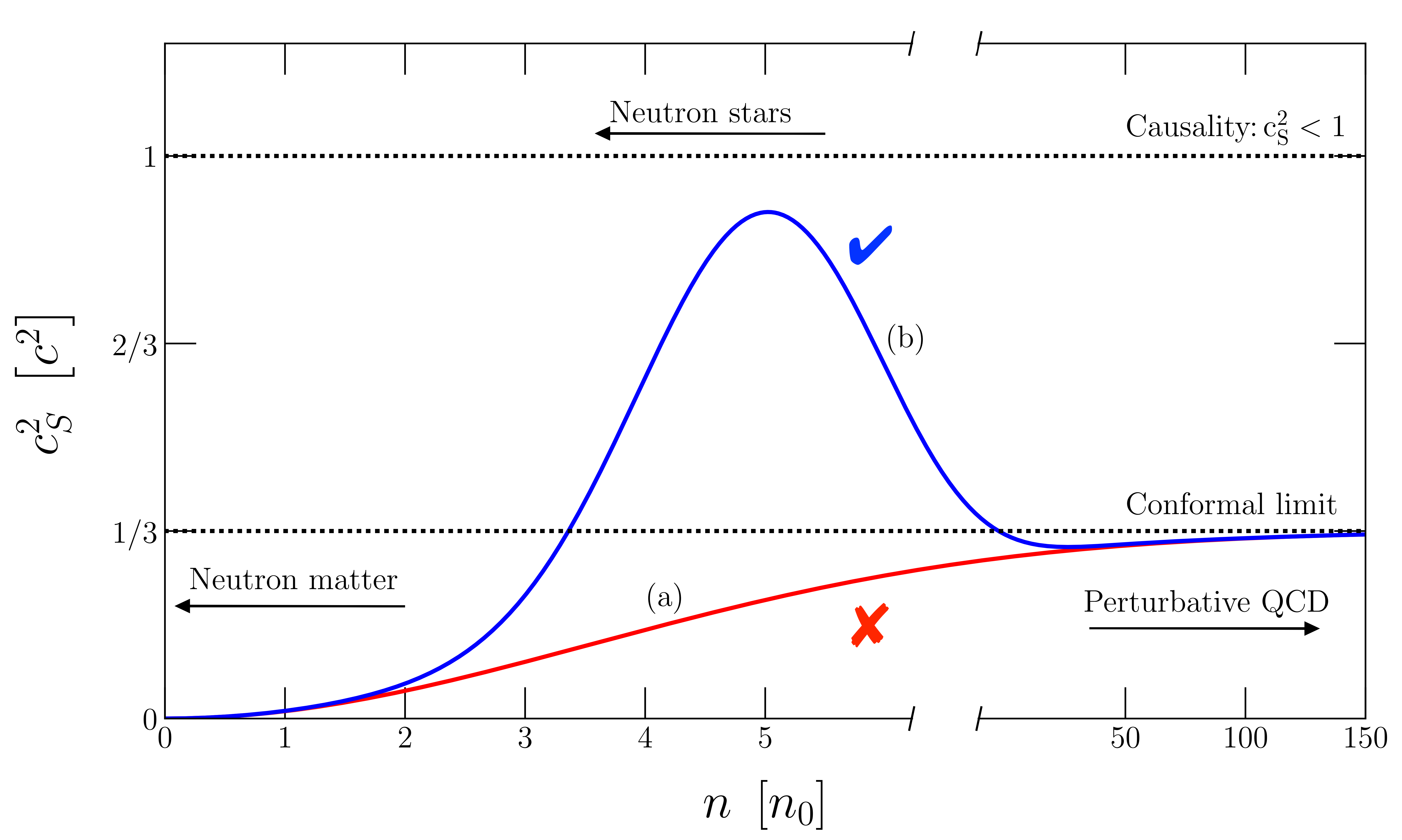
Requiring a maximum mass greater than 2 M_{sun} **implies:**

R > 9 km

De et al. PRL (2018)
See also LIGO and Virgo Scientific Collaboration arXiV:1805.11581v1

Speed of Sound in Dense Matter

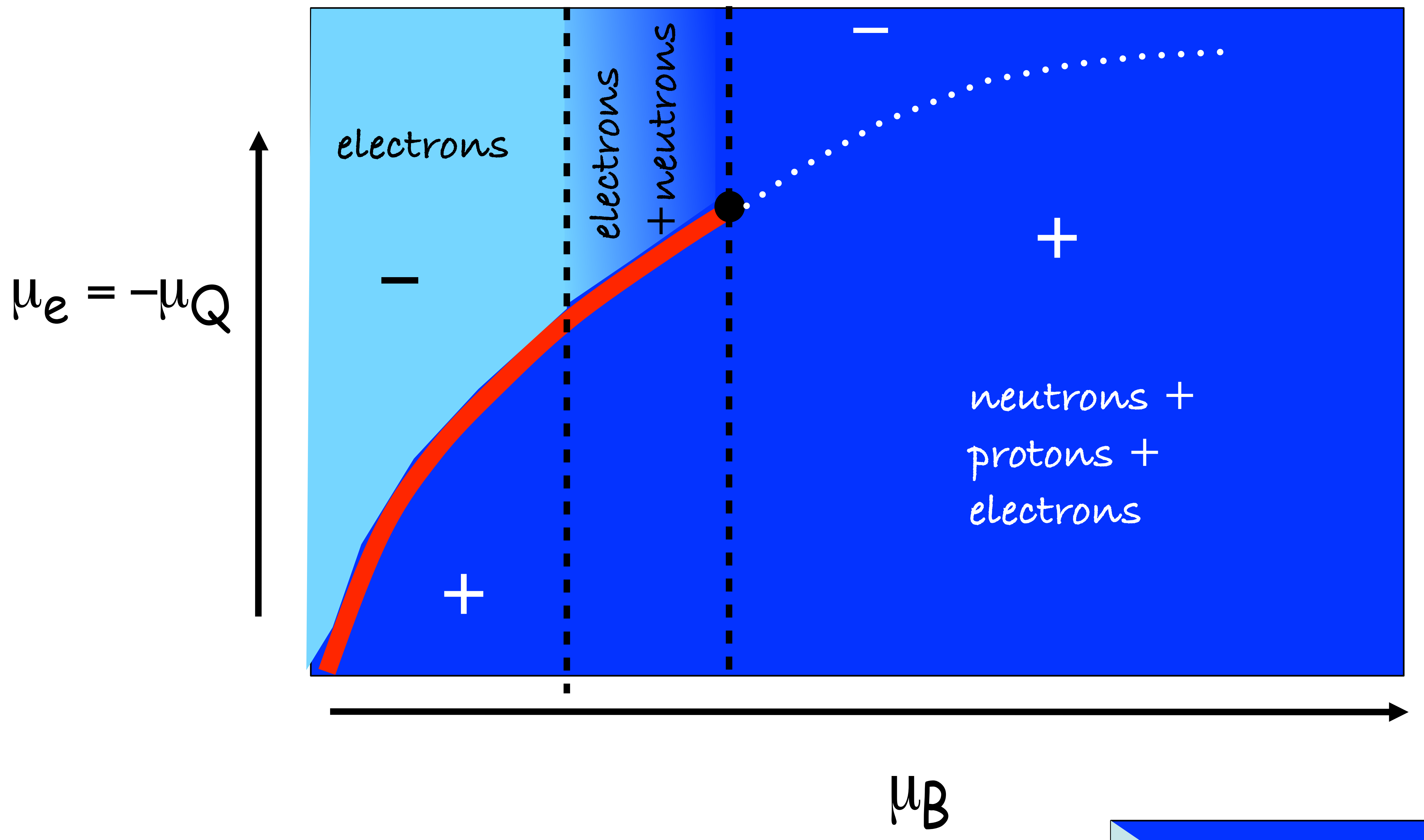
Large observed maximum mass combined with small radius and neutron matter calculations suggests a rapid increase in pressure in the neutron star core. Implies a large and non-monotonic sound speed in dense QCD matter.



Tews, Carlson, Gandolfi and Reddy (2018), Steiner & Bedaque (2016)

Summary

Mixed Phase are generic to first-order transitions



Vacuum
to
nuclear matter

Nuclear matter
to
Exotic Phase