

Neutron stars and the properties of matter under extreme conditions:

General introduction

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University of Illinois, Urbana

8th Huada School on QCD
Central China Normal University

Wuhan 武汉
May 6-10, 2019



华中师范大学
CENTRAL CHINA NORMAL UNIVERSITY

THE NEUTRON STAR NEWS

Special Wuhan Edition

FROM QUARKS TO THE COSMOS

- Since 1934

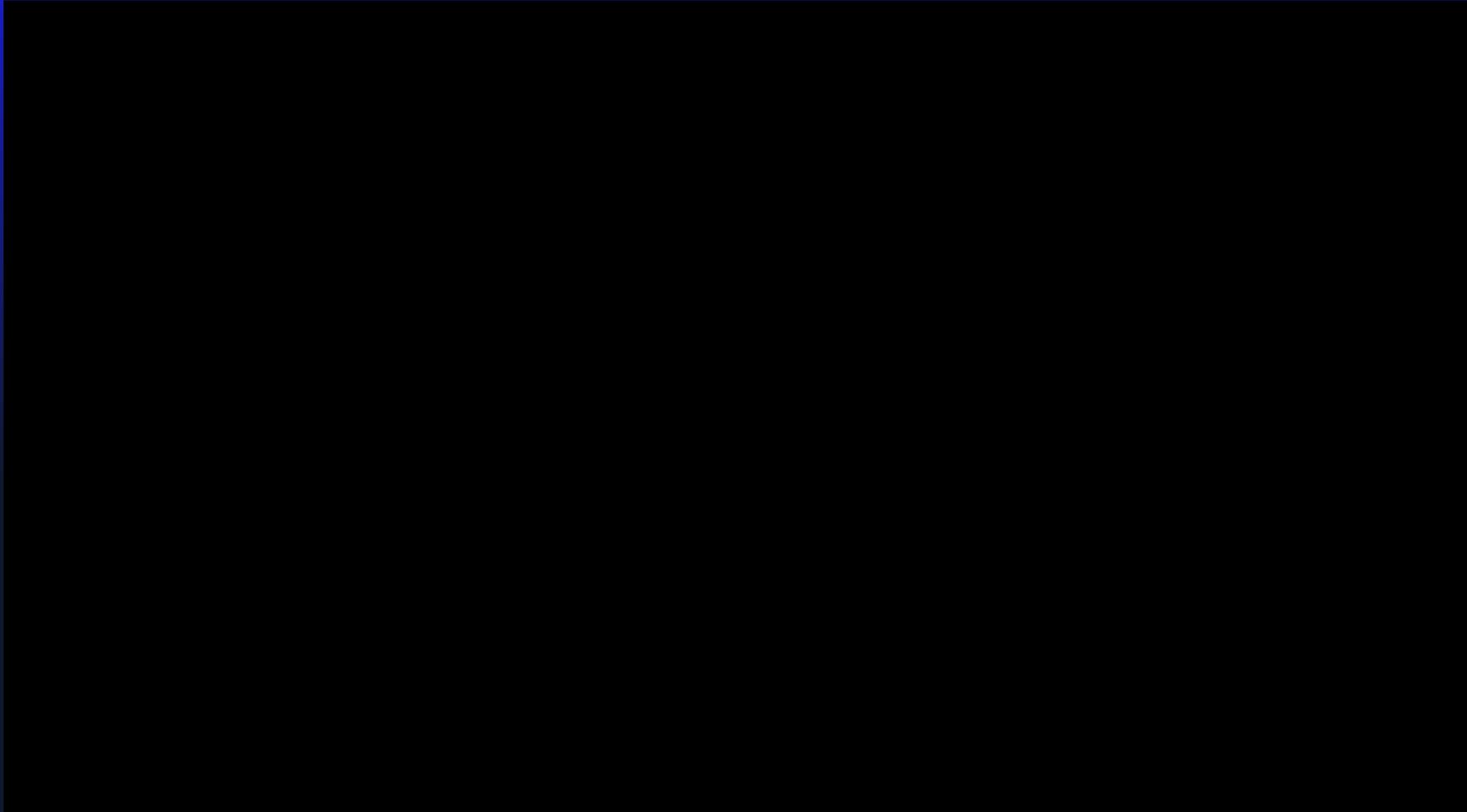
DIRECT DETECTION OF GRAVITATIONAL RADIATION FROM A BINARY NEUTRON STAR MERGER GW170817 – MULTI-MESSENGER ASTRONOMY –PRODUCING GOLD AND OTHER HEAVY ELEMENTS. **TWO MERGER REPORTS APRIL 25 AND 26!!**

DIRECT OBSERVATIONS OF 2.0 SOLAR MASS STARS AND **ONE OF 2.17 ON APRIL 14**

ONGOING DETERMINATIONS OF MASSES AND RADII CONSTRAINING NEUTRON STAR INTERIORS: **NICER** EXPERIMENT, ON INTERNATIONAL SPACE STATION

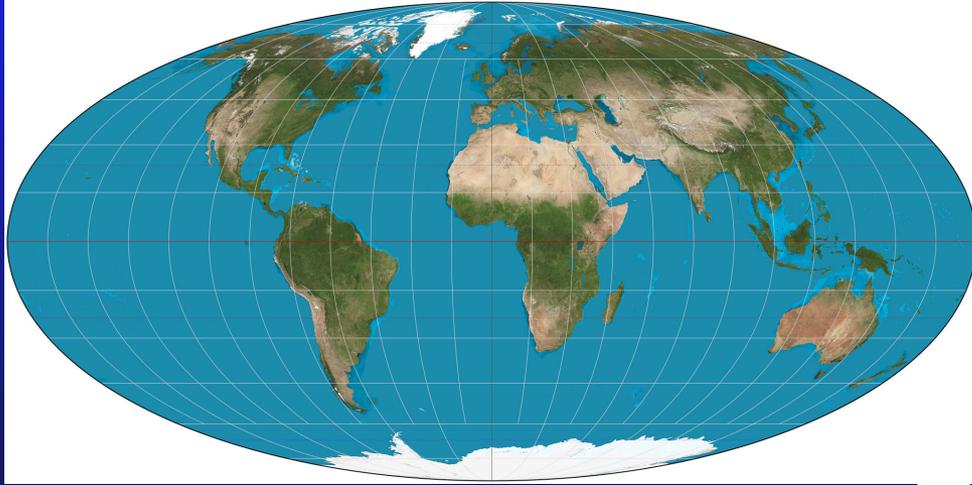
EMERGING UNDERSTANDING IN QCD OF HOW NUCLEAR MATTER TURNS INTO DECONFINED QUARK MATTER AT HIGH BARYON DENSITIES
(complementary to studying dense matter in ultrarelativistic heavy ion collision experiments at RHIC and the LHC)

Neutron star – neutron star merger observed on 17 Aug. 2017
by LIGO and Virgo (gravitational radiation), FERMI (gamma ray
telescope) + ~ 70 other electromagnetic observatories. **GW170817**

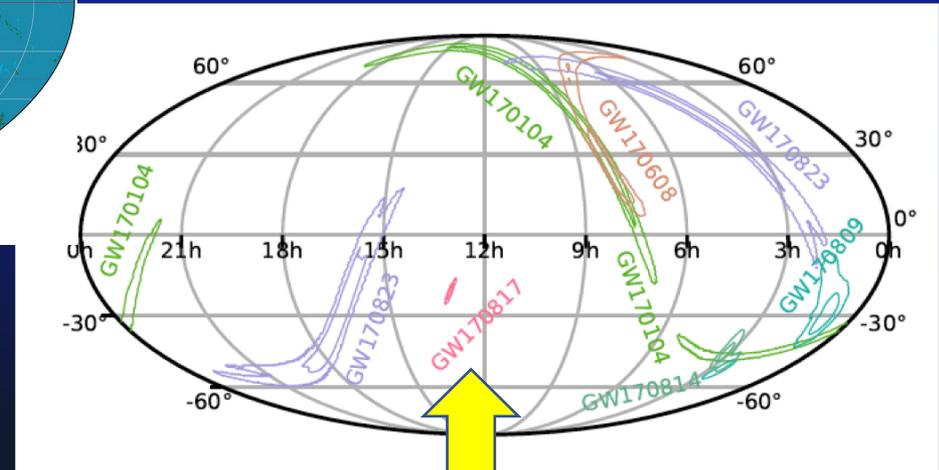


Merger was 130,000,000 light years = 40 MPc away





Mollweide projection

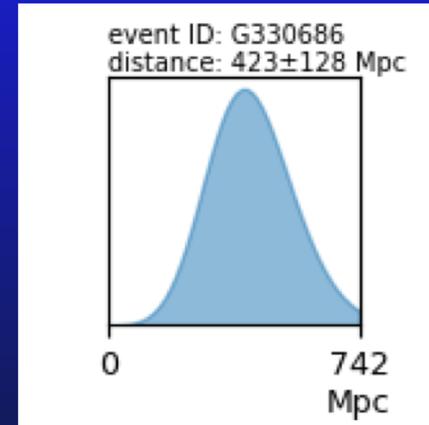
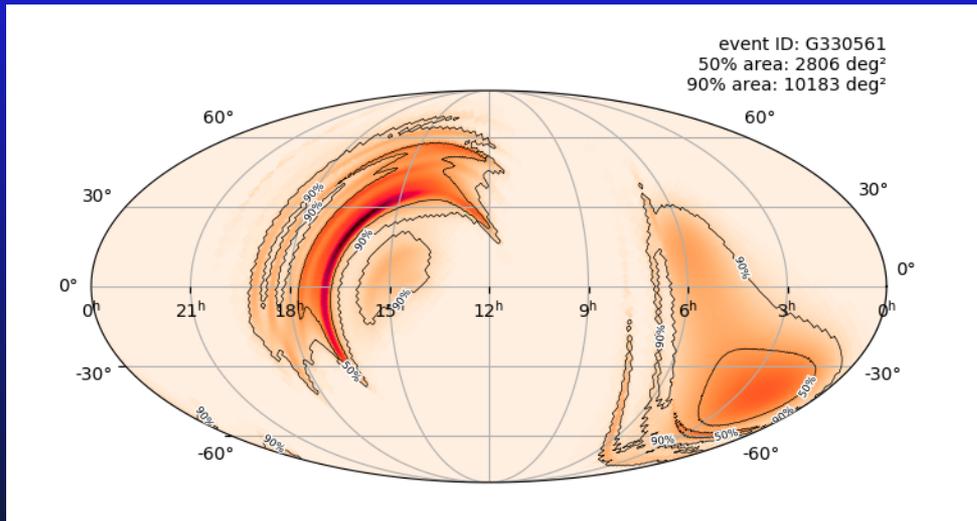


GW170817

Two binary neutron star merger candidates in LIGO/Virgo O3 run

S190425z LIGO (Livingston) + Virgo. 042519

Hanford detector of LIGO was down for 40 minutes when event occurred.

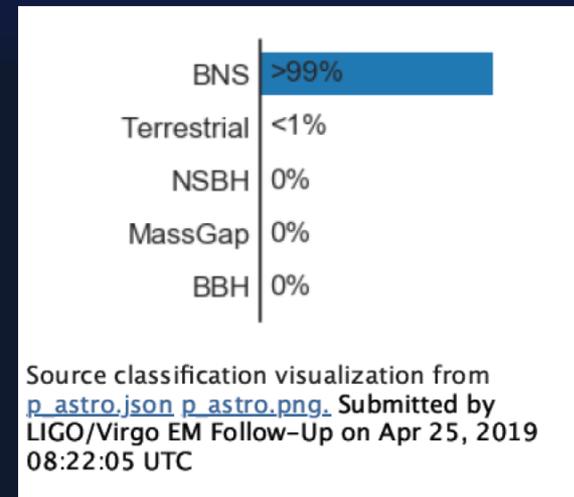


~1380 ly

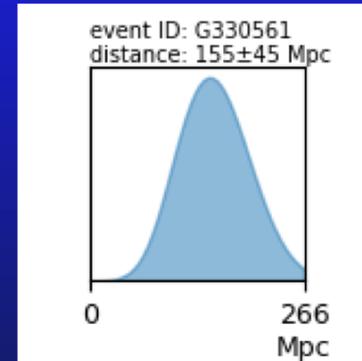
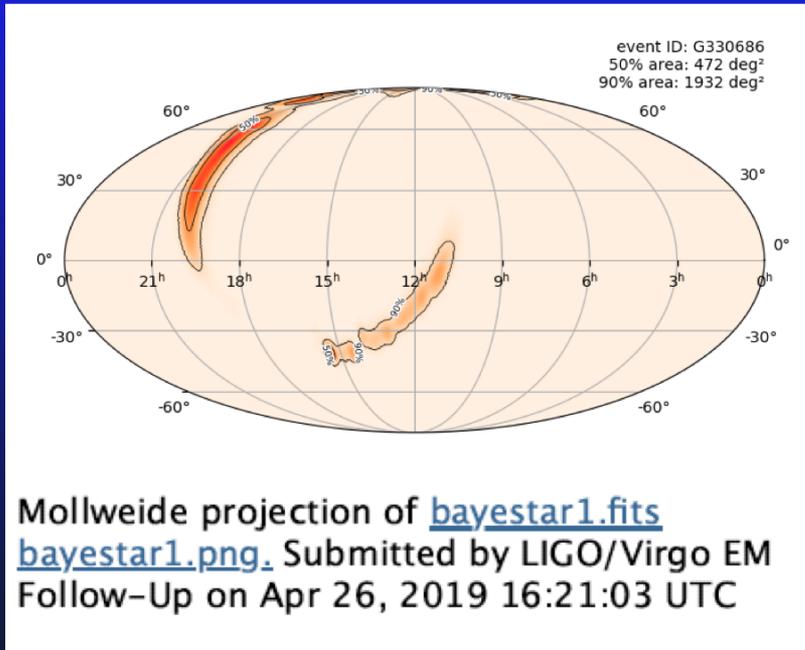
False alarm rate: 1 per 69834 years

Neutron star-neutron star merger most likely.

Very poor localization in sky => difficult to carry out electromagnetic observations.



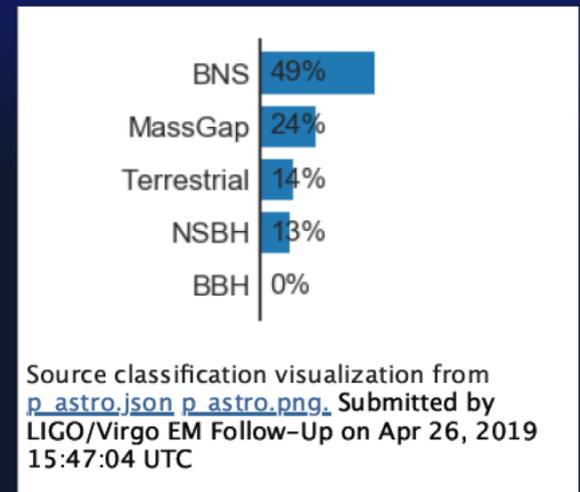
S190426c LIGO (H + L) + Virgo. 042619



~500 ly

Low significance,
False alarm rate: 1 per 1.6276 years
15% chance of being spurious

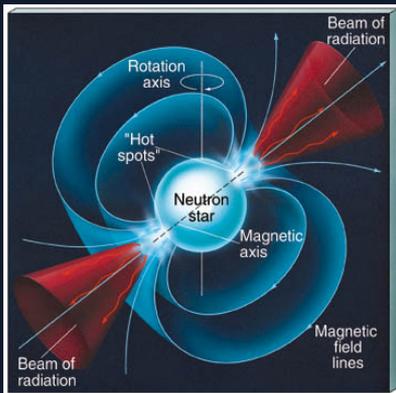
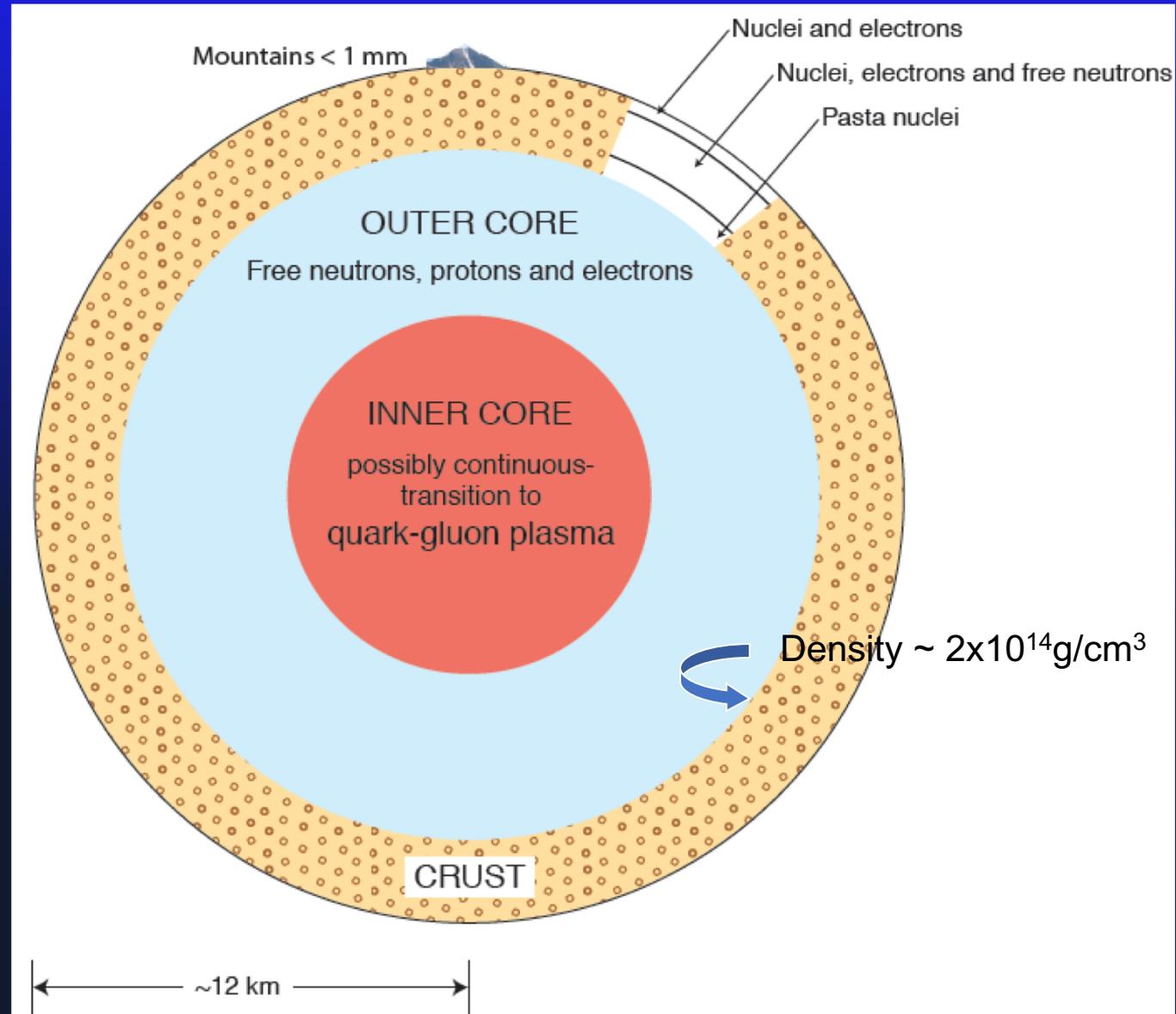
Binary neutron star merger?
Neutron star-black hole merger?
No electromagnetic observations yet!



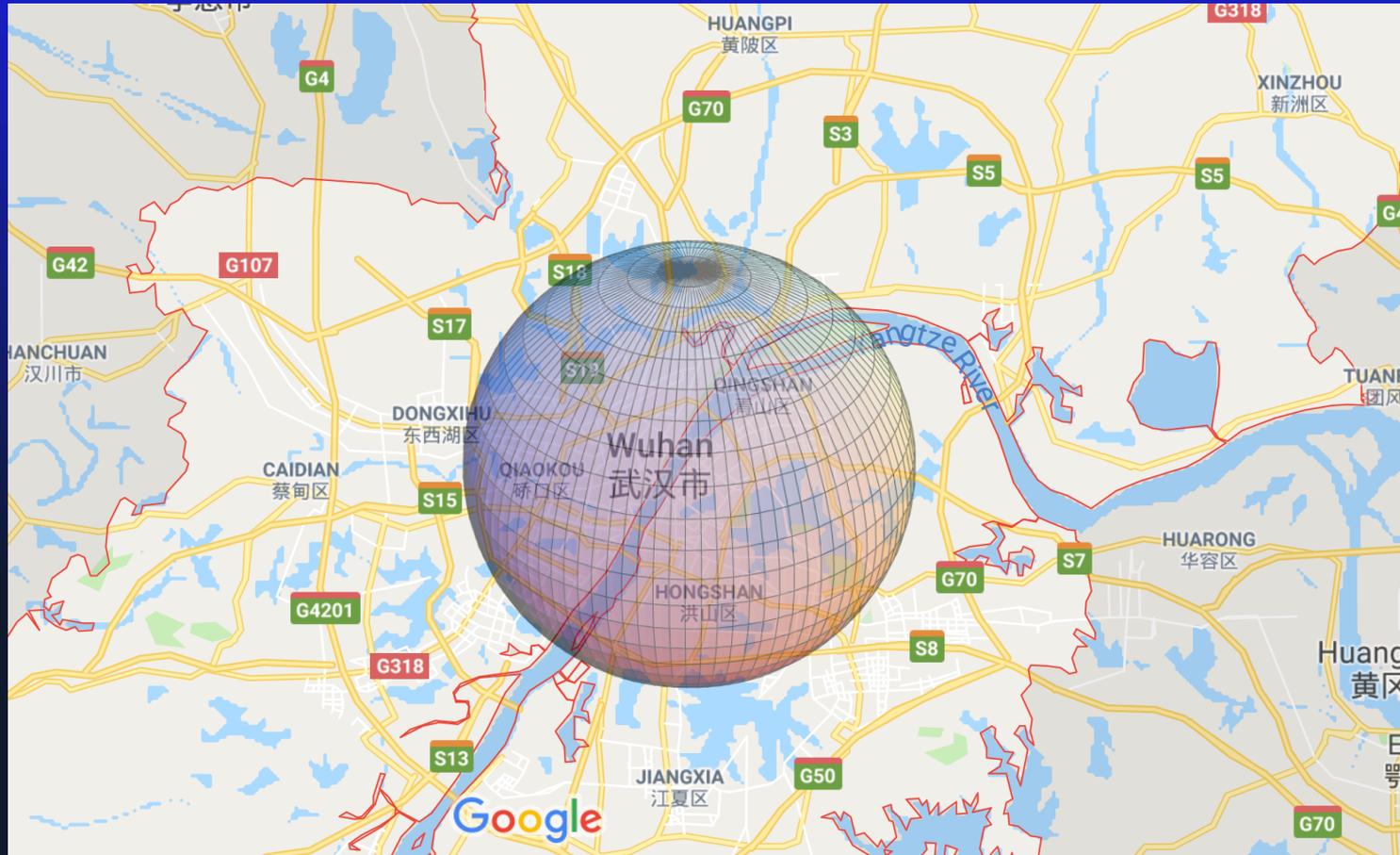
Neutron star interior

Mass $\sim 1.4\text{-}2 M_{\text{sun}}$
Radius $\sim 10\text{-}12 \text{ km}$
Temperature
 $\sim 10^6\text{-}10^9 \text{ K}$

Surface gravity
 $\sim 10^{11}$ that of Earth
Surface binding
 $\sim 1/10 mc^2$



Neutron star over Wuhan



Masses $\sim 1-2 M_{\odot}$

Baryon number $\sim 10^{57}$

Radii $\sim 10-12$ km

Magnetic fields $\sim 10^6 - 10^{15}$ G

Made in gravitational collapse of massive stars (supernovae)

Central element in variety of compact energetic systems:

pulsars, binary x-ray sources, soft gamma repeaters

Merging neutron star-neutron star and neutron star-black hole

sources of gamma ray bursts and **gravitational waves**

Matter in neutron stars is densest in universe:

baryon density n up to $\sim 5-10 n_0$ ($\rho_0 = 3 \times 10^{14}$ g/cm³ = density of matter in atomic nuclei = 0.16 /fm³) 1 fm = 10⁻¹³ cm

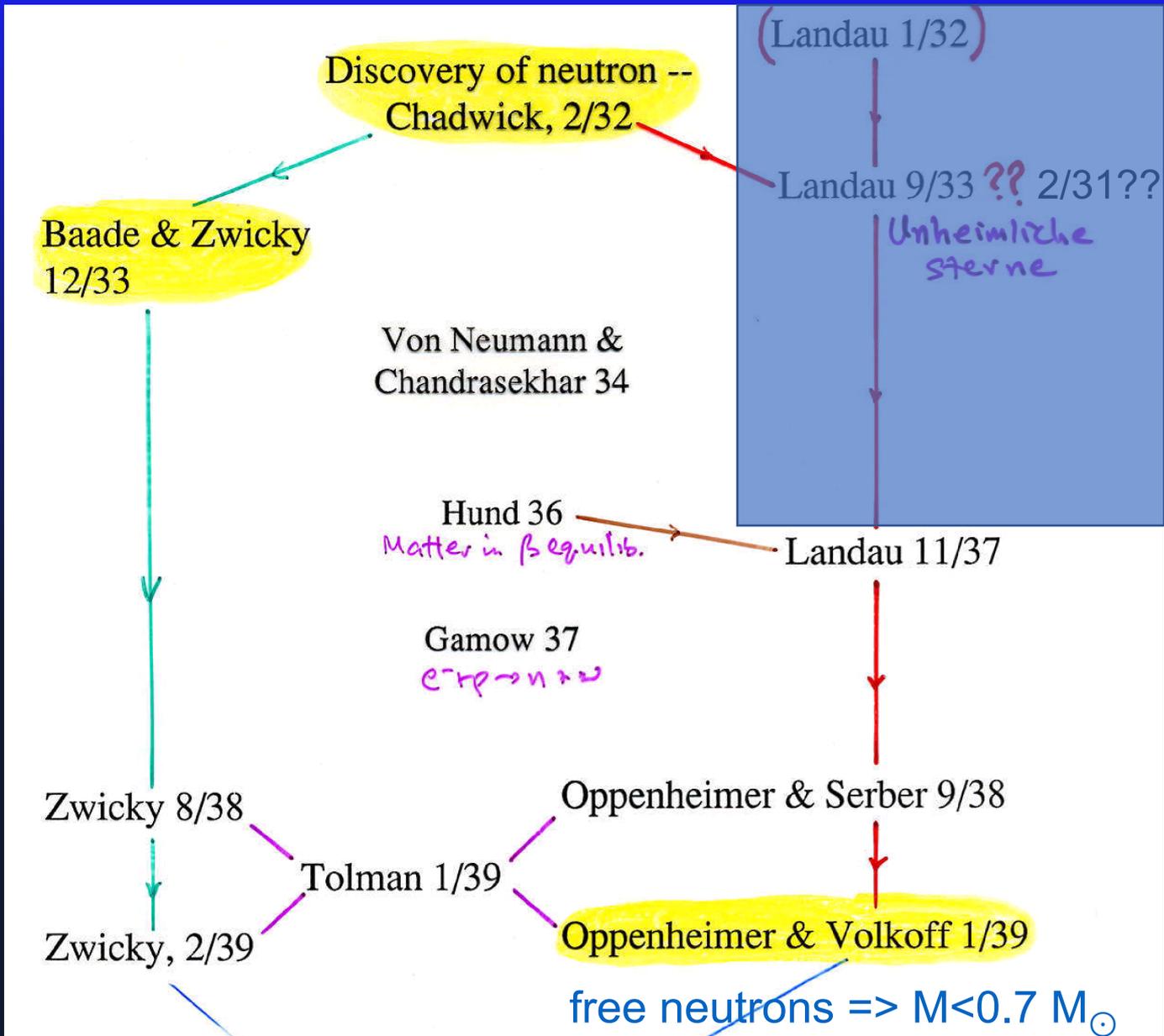
[cf. white dwarfs: $\rho \sim 10^5-10^9$ g/cm³]

Supported against gravitational collapse by nucleon degeneracy pressure

Astrophysical laboratory for study of high density matter

What are states in interior? Existence of quark stars?

Early History of Neutron Stars





Chadwick to Bohr, 24 Feb. 1932 announcing the discovery of the neutron



Cavendish Laboratory,
Cambridge.

24 February 1932.

Dear Bohr,

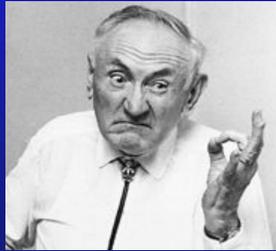
I enclose the proof of a letter I have written to "Nature" and which will appear either this week or next. I thought you might like to know about it beforehand.

The suggestion is that α particles split from beryllium (and also from boron) particles which have no net charge, and which probably have a mass ^{equiv} $\frac{1}{2}$ that of the proton. As you will see, I put this forward rather cautiously, but I think the evidence is really rather strong. Whatever the radiation from Be may be, it has most remarkable properties. I have made many experiments which I do not mention in the

letter to "Nature" and they can all be interpreted readily on the assumption that the particles are neutrons. Feather has taken some pictures in the dispersion chamber and we have already found about 20 cases of recoil atoms. About $\frac{1}{4}$ of these show an abrupt bend ^{or fork} (and it is almost certain that ~~this~~ one arm of this fork represents a recoil atom and the other some other particle, probably an α particle. The α are disintegrations due to the capture of the neutron by N_{14} or O_{16}). I enclose two photographs one of which shows the simple recoil atom, and the other what we suppose is a disintegration. The photographs are not very good but they were printed in a hurry.

With best regards
Yours sincerely
J. Chadwick

W. Baade and F. Zwicky



Stanford APS Meeting,
15-16 Dec. 1933
Phys. Rev. 45, 138 (1934)

38. Supernovae and Cosmic Rays. W. BAADE, *Mt. Wilson Observatory*, AND F. ZWICKY, *California Institute of Technology*.--Supernovae flare up in every stellar system (nebula) once in several centuries. The lifetime of a supernova is about twenty days and its absolute brightness at maximum may be as high as $M_{\text{vis}} = -14^M$. The visible radiation L_v of a supernova is about 10^4 times the radiation of our sun, that is, $L_v = 3.78 \times 10^{41}$ ergs/sec. Calculations indicate that the total radiation, visible and invisible, is of the order $L_r = 10^7 L_v = 3.78 \times 10^{48}$ ergs/sec. The supernova therefore emits during its life a total energy $E_r \cong 10^5 L_r = 3.78 \times 10^{53}$ ergs. If supernovae initially are quite ordinary stars of mass $M < 10^{34}$ g, E_r/c^2 is of the same order as M itself. In the supernova process mass in bulk is annihilated. In addition the hypothesis suggests itself that cosmic rays are produced by supernovae. Assuming that in every nebula one supernova occurs every thousand years, the intensity of the cosmic rays to be observed on the earth should be of the order $\sigma = 2 \times 10^{-3}$ erg/cm² sec. The observational values are about $\sigma = 3 \times 10^{-3}$ erg/cm² sec. (Millikan, Regener). With all reserve we advance the view that supernovae represent the transitions from ordinary stars into *neutron stars*, which in their final stages consist of extremely closely packed neutrons.

Landau to Bohr, 11 November 1937

5/XI 37

Lieber Herr Bohr,

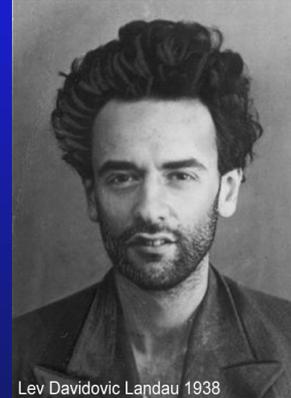
Ich sende Ihnen einen Artikel
über Sternenergie, den ich gedichtet
habe. Schreiben Sie dem einen physikalischen
Sinn zu, so schicken Sie in Bitte an
die „Nature“. Macht es Ihnen nicht zu
viel Mühe, so würde ich mich sehr freuen
Ihre Meinung darüber zu erfahren.

Mit vielen herzlichen Grüßen

Ihr

L Landau

am 13. XI. an Nature geschrieben.
o. Brief an Kapitza!



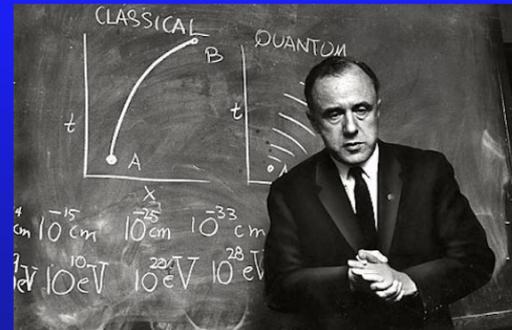
Lev Davidovic Landau 1938

I send you an article about stellar energy, which I have written. If you think it makes physical sense, please send it to “Nature”. Do not put too much work into it, and it would make me very happy to learn what you think about it.

Izvestia to Bohr

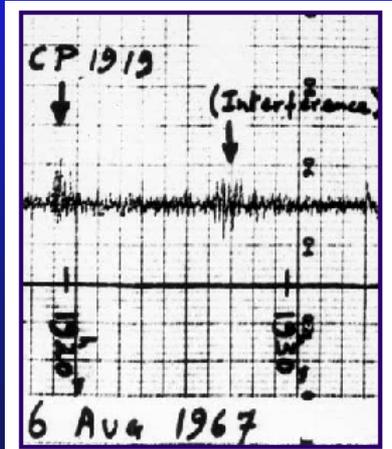
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<p>WE DESIRE PUBLISH YOUR OPINION IN OUR NEWSPAPER IN CONNECTION WITH DISCUSSIONS OF LANDAUS LAST WORK REGARDING</p> <p>NEUTRON CORE IN SCIENTIFIC ORGANIZATIONS =</p> <p>IZVESTIA EDITORIAL OFFICE ☞</p>							
Vedtagne Forkortelser: T. 4 1/2 (C 5)		D Telegram. RP Svaz betalt.	IC Kollationsdag. Overst Udleveres saabent.	MP Udlev. egenhaendigt. PS Eftersendes.	PC Modtagelsesbetalt. XP Bud betalt.		

J. Wheeler
Ann. Rev. Astronomy
and Astrophysics, 1966

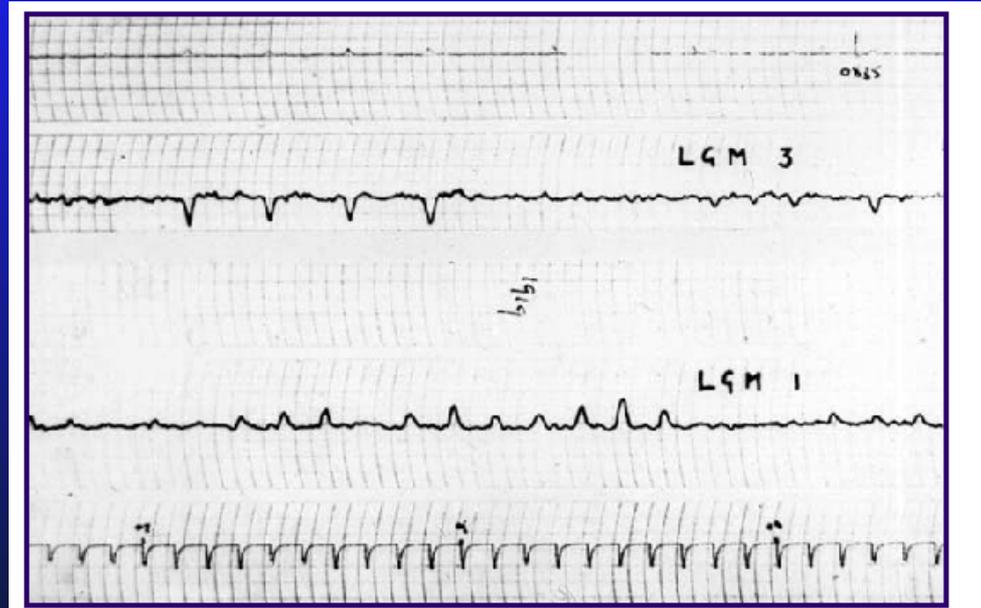


“So far no superdense star has been identified. Moreover, a ‘cool’ superdense star -- with a radius of 10 km, with a surface temperature (after $\sim 10^6$ years of cooling) of 10^6 °K, and at a distance of 10pc, comparable to the distance of near-by stars -- is fainter than the 19th magnitude and therefore hardly likely to be seen. The rapidity of cooling makes detection even more difficult.”

1967 (November) First pulsar detection: 1919+21 *Bell & Hewish*



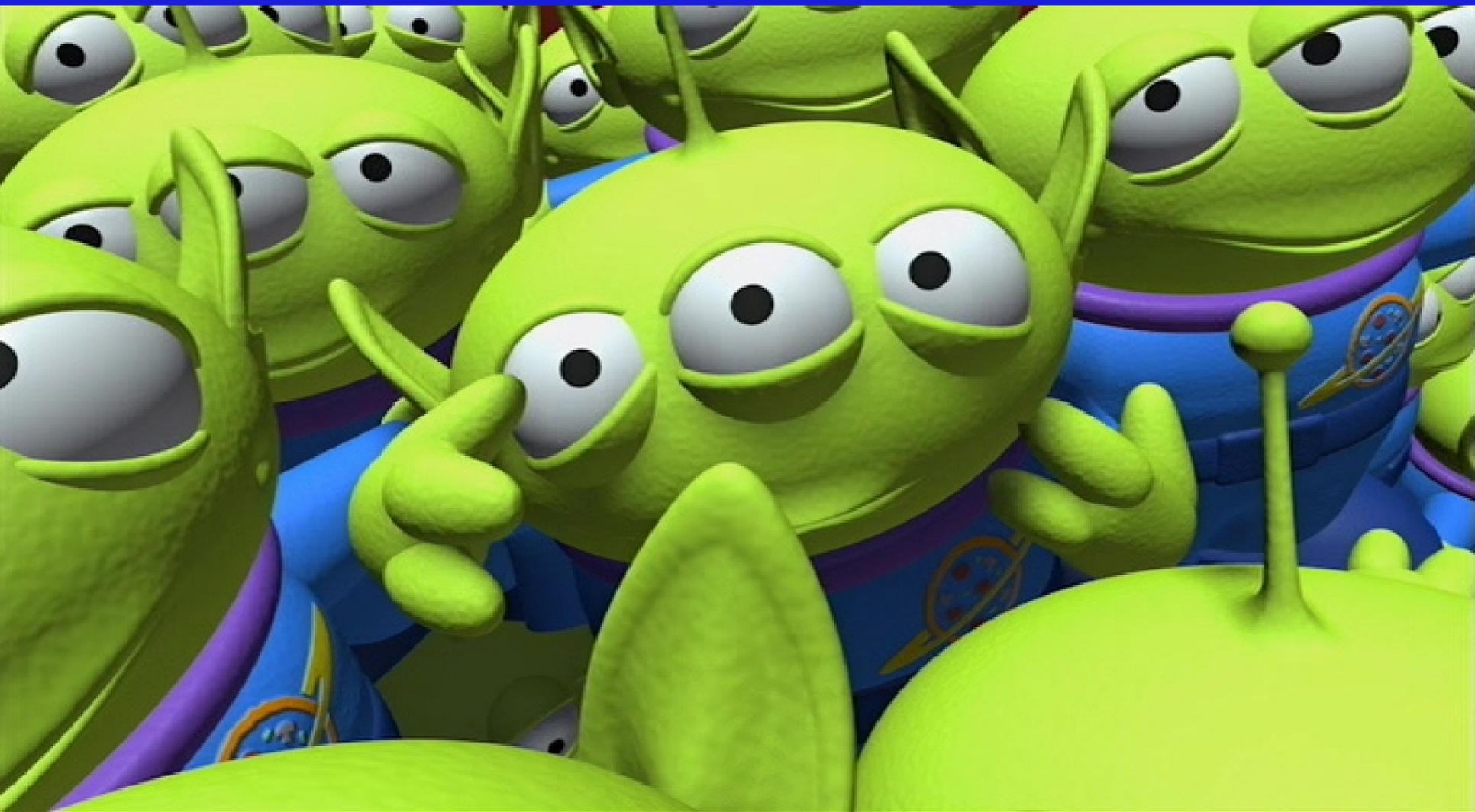
2: The first detection of the first pulsar, occupying about one-quarter inch of chart paper. About five minutes later is a short burst of low level interference. This signal has been high-pass filtered, to remove the telescope's interference pattern. (Mullard Radio Astronomy Observatory.)



3: The bottom trace is of broadcast one-second time pips. The middle trace shows the first recording to reveal the pulsed nature of the pulsar PSR1919. The top trace is of the third pulsar discovered, PSR0834.

“It was highly unlikely that there would be two lots of little green men on opposite sides of the universe both deciding to signal at the same time to a rather inconspicuous star on a rather curious frequency.” Jocelyn Bell

1968 (Spring): Pulsars identified as rotating neutron stars by Tommy Gold and Franco Pacini



Detection of neutron stars in x-ray

Early 1960's: observations of x-ray sources (Sco X-1) by balloon flights

1972: x-ray pulsars in binary stars: UHURU
(Ricardo Giacconi et al.)

Observed neutron stars

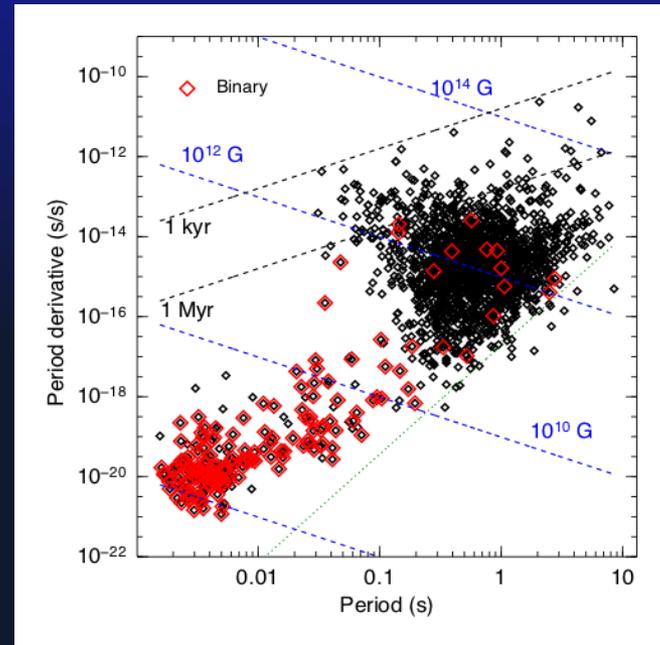
- > 1500 ns in isolated rotation-powered radio pulsars
~ 400 millisecond pulsars ($1/716 < P < 1/30$ sec.)

<https://apatruno.wordpress.com/about/millisecond-pulsar-catalogue/>

- > 100 ns in accretion-powered x-ray binaries
~ 50 x-ray pulsars
intense x-ray bursters
(thermonuclear flashes)

Short (10-100 s) gamma-ray bursts
(ns-ns, ns-bh? mergers)

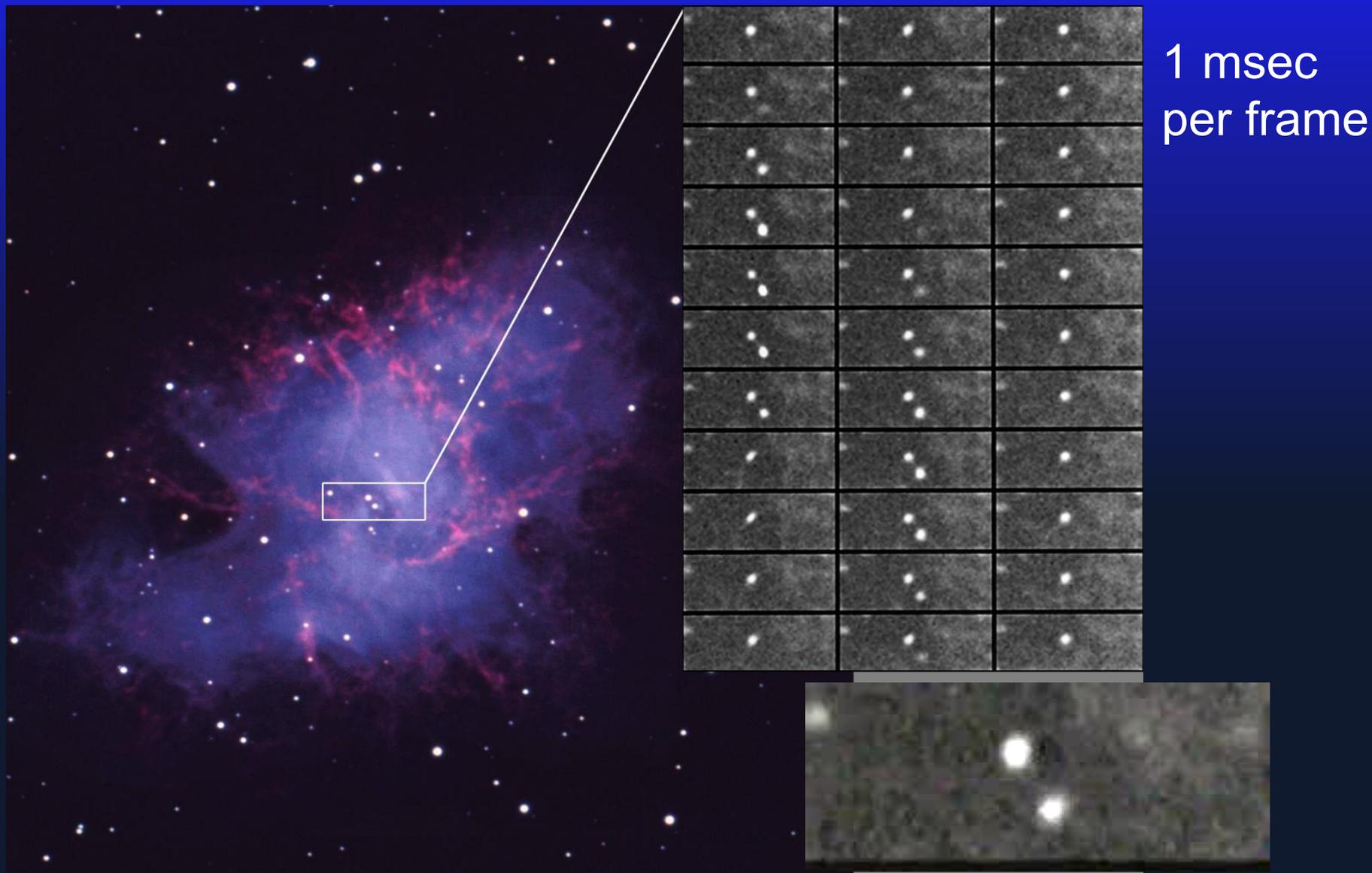
Soft gamma-ray repeaters
-- magnetars ($B \sim 10^{14}$ - 10^{15} G)



Period derivative vs period for known pulsars
Shown is dipolar magnetic field needed to give dP/dt .

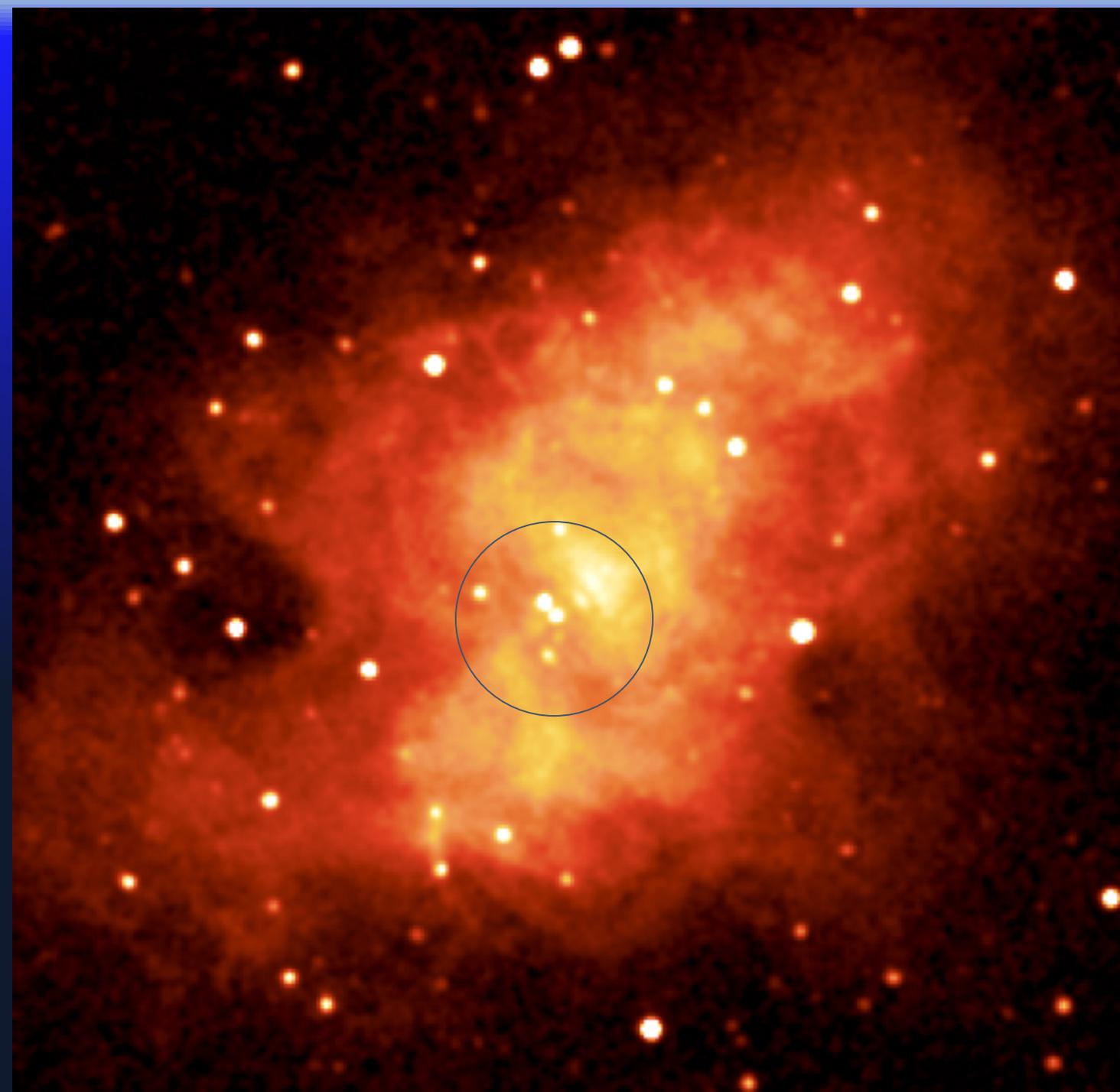
Crab Pulsar (period = 33msec)

Supernova July 4, 1054



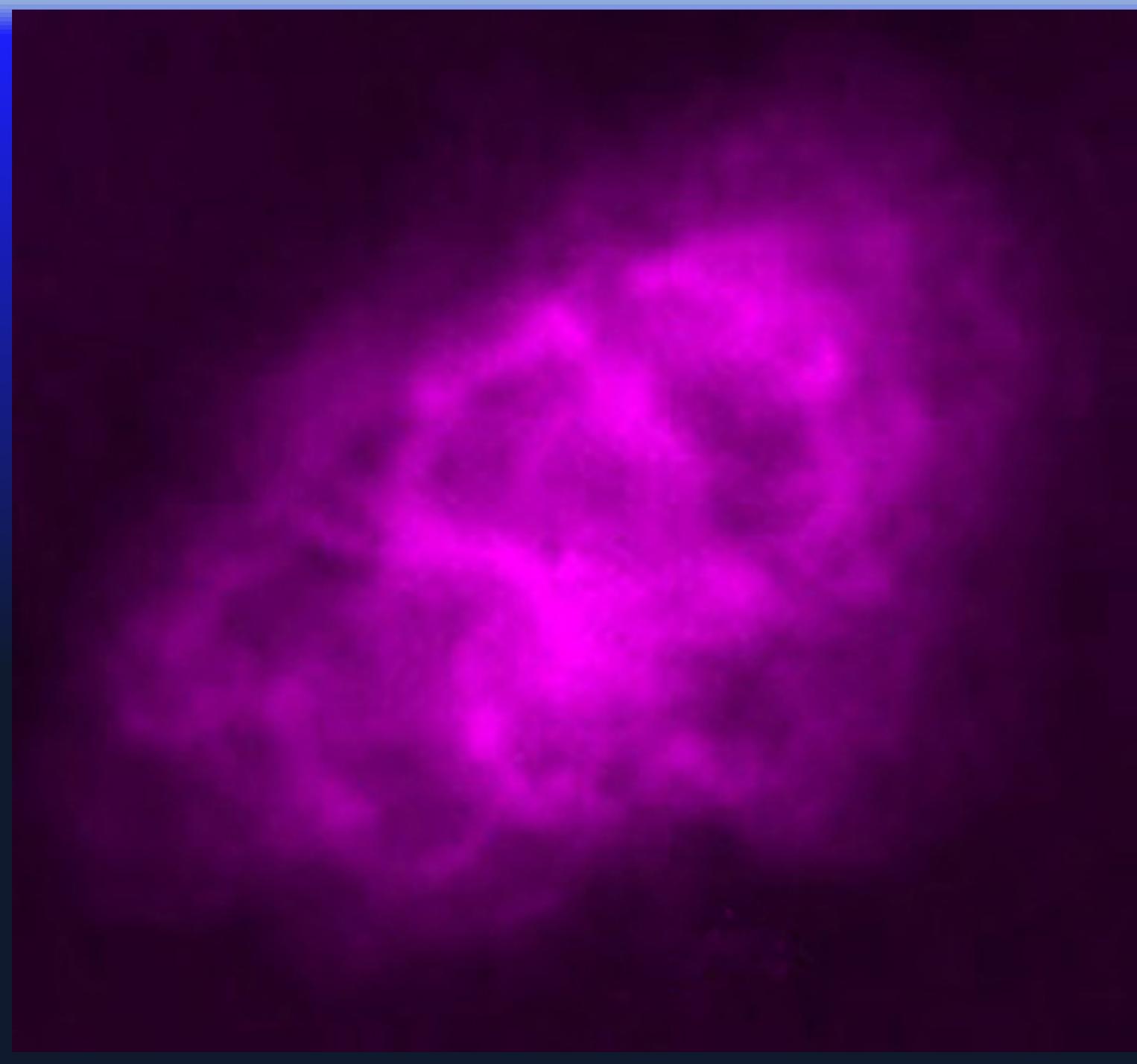


Crab nebula
in optical

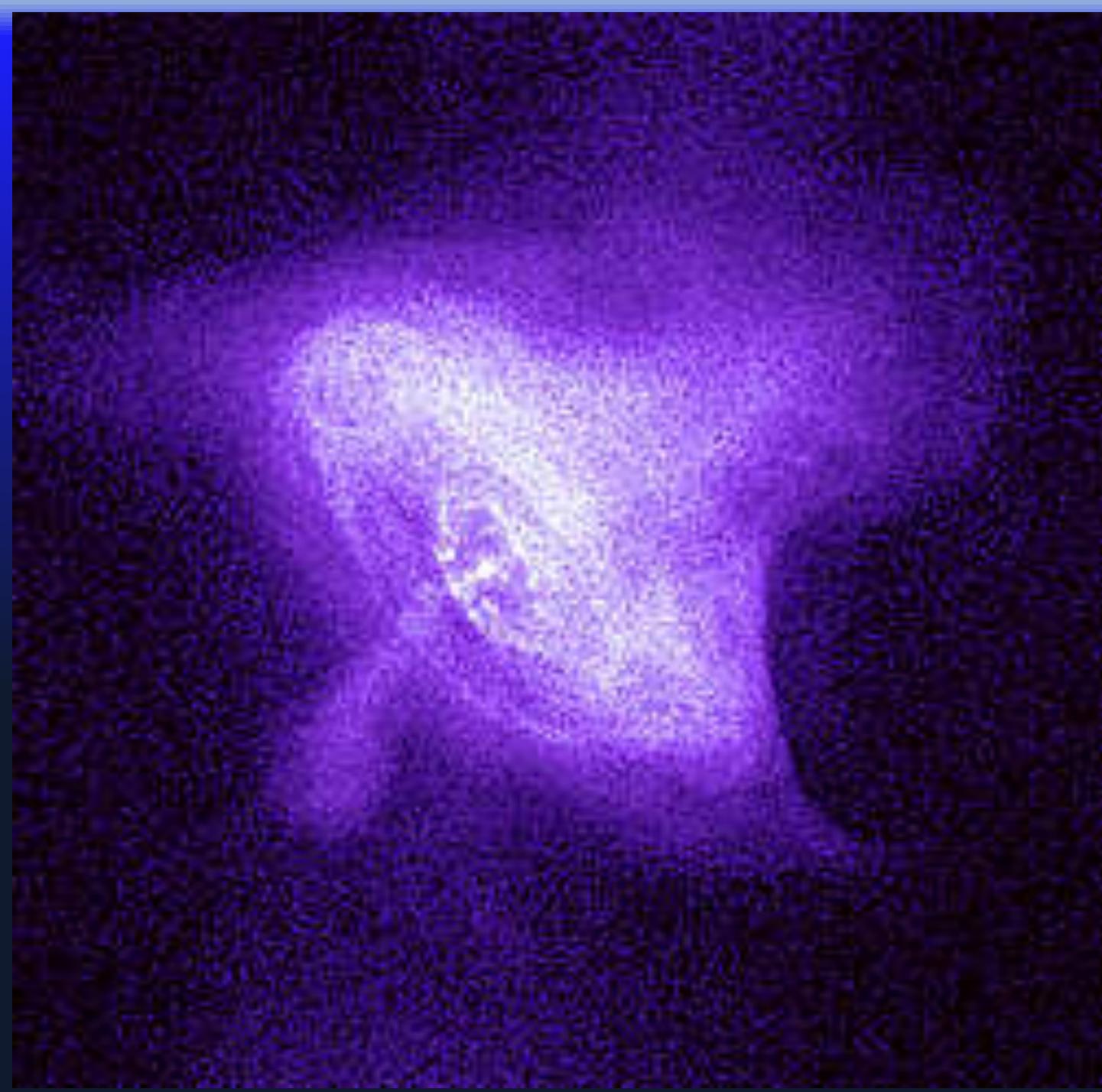


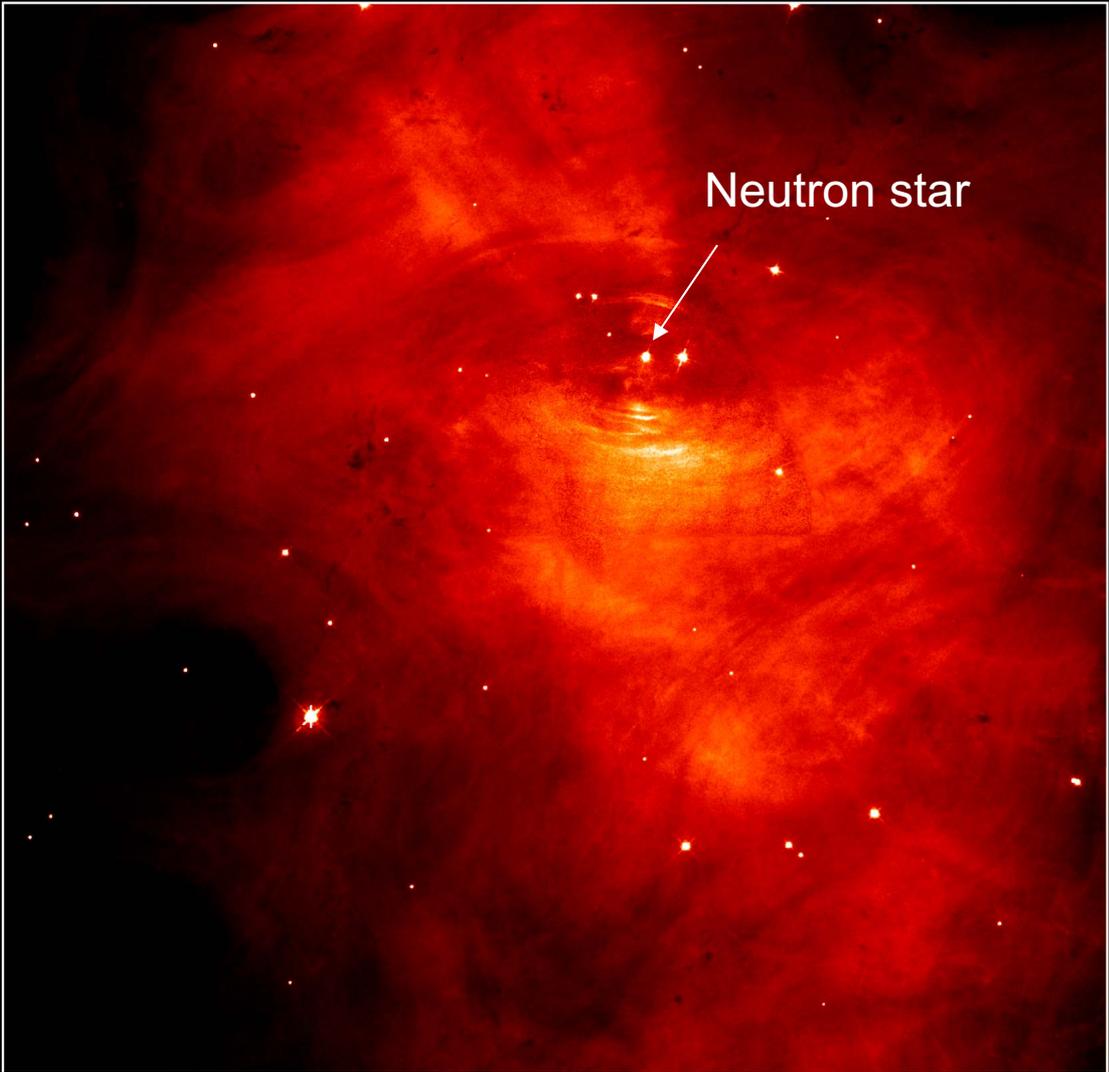
Crab nebula
in infrared

Crab nebula
in radio
(VLA)

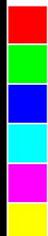


Crab nebula
in x-ray
(Chandra)





Palomar

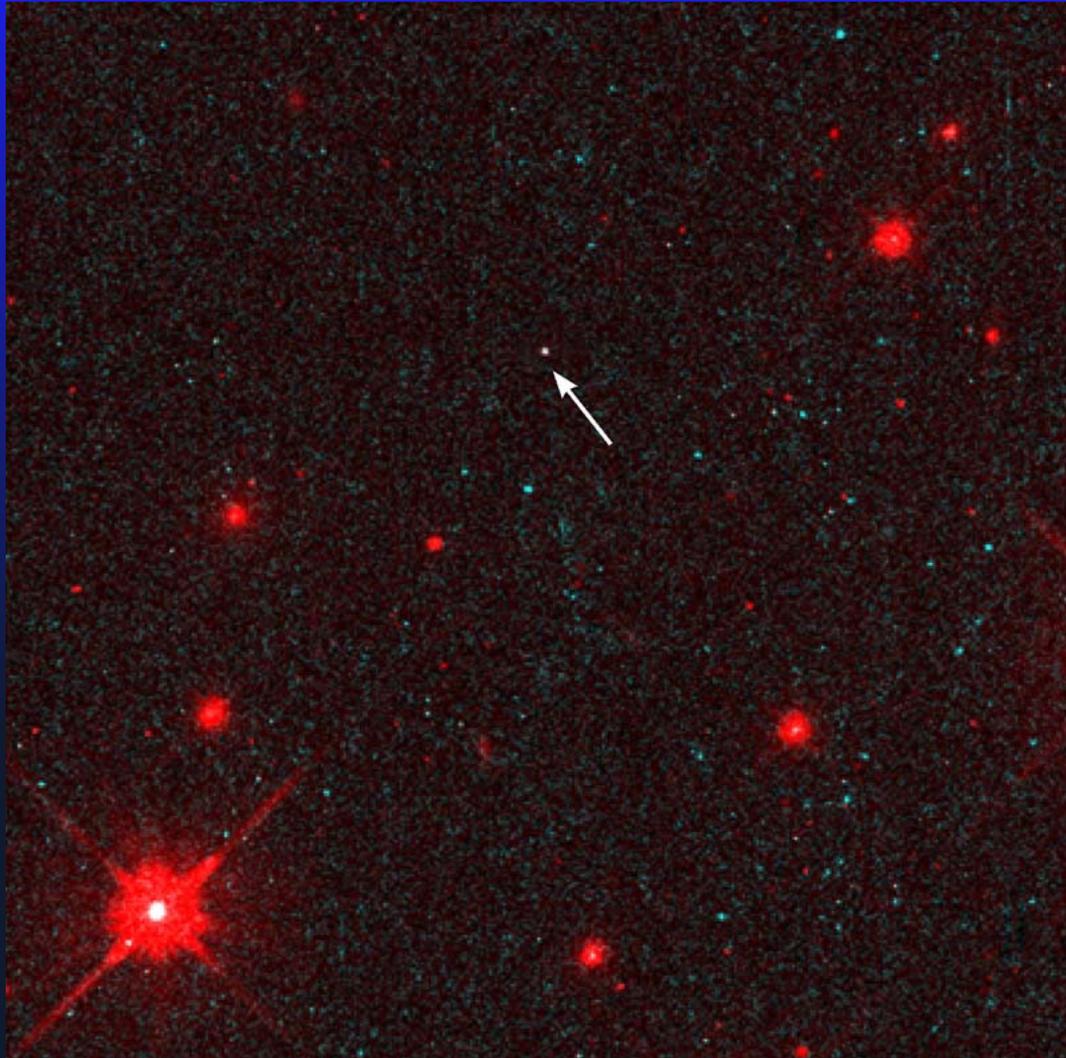


Crab Nebula

Hubble Space Telescope · Wide Field Planetary Camera 2



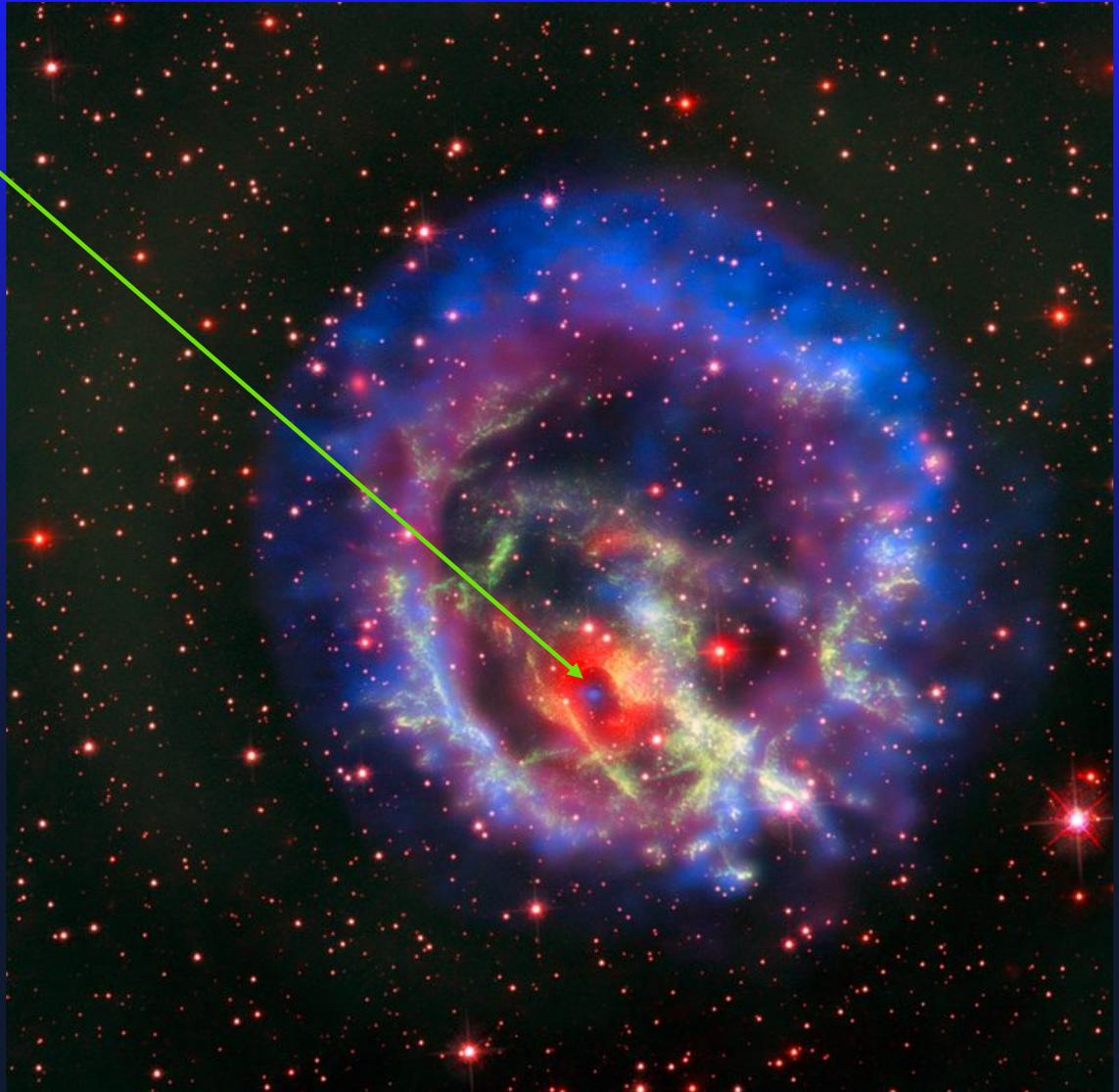
Isolated neutron star RX J185635-3742 HST WFPC2



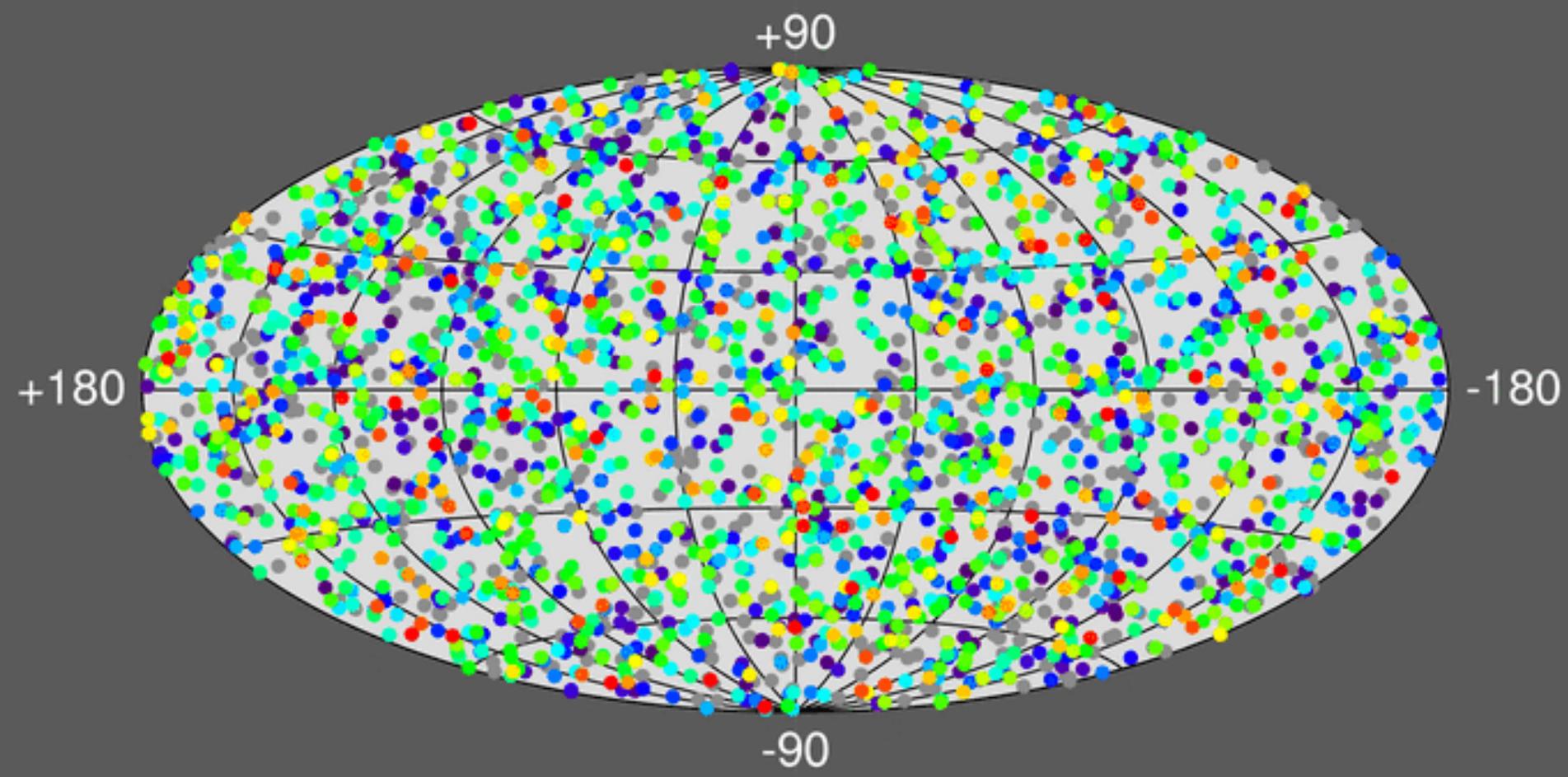
Distance ~ 117 pc ~ 400 l.y.

Radius of emitting region ~ 17 km

Isolated neutron
star in supernova
remnant (green)
1E 0102.2-7219
in the Small
Magellanic Cloud

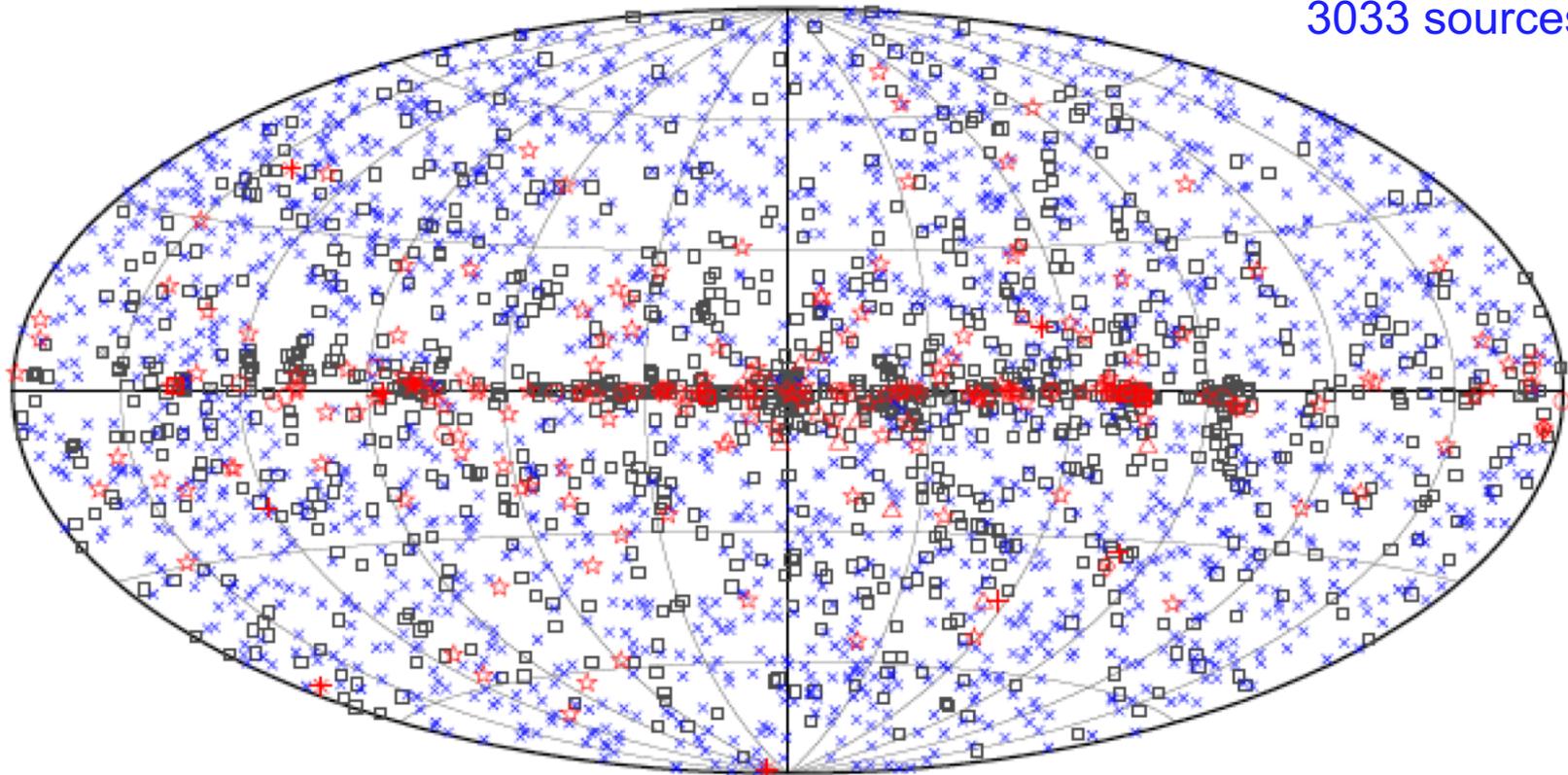


2704 BATSE Gamma-Ray Bursts



Fermi Gamma-ray LAT burst catalog 3FGL (2015)

3033 sources



□ No association	□ Possible association with SNR or PWN	× AGN
☆ Pulsar	△ Globular cluster	+ Starburst Galaxy
◻ Binary	+ Galaxy	◊ PWN
★ Star-forming region	○ SNR	+ Nova

PSR = pulsar, PWN = pulsar wind nebula, AGN = active galactic nebula

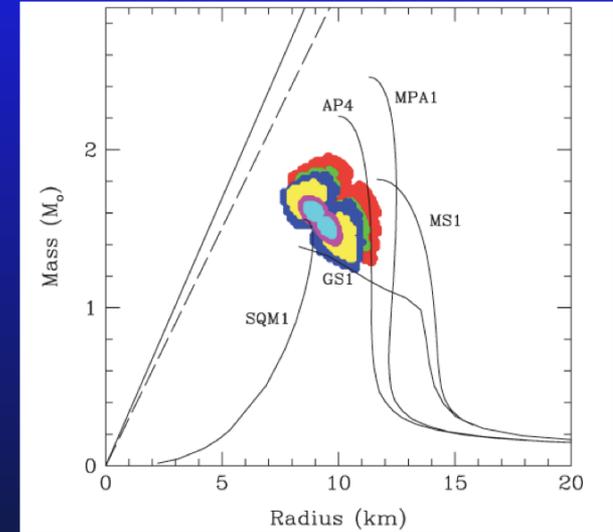
Learning about dense matter from neutron star observations



Challenges to nuclear theory!!

Learning about dense matter from neutron star observations

Masses and radii of neutron stars
 Binary systems: stiff e.o.s
 Thermonuclear bursts in X-ray binaries => Mass vs. Radius, strongly constrains eq.of state.



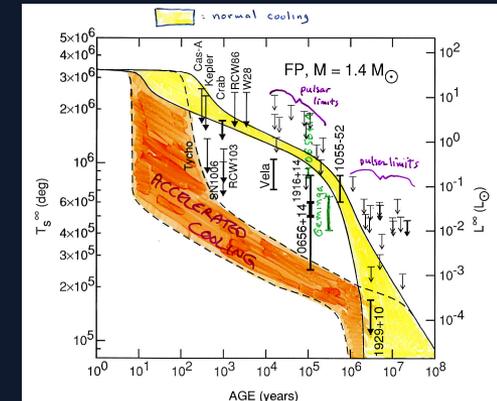
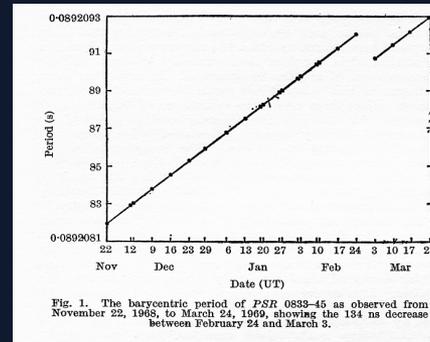
NICER to measure M/R directly

Gravitational waves from ns-ns and ns-bh mergers
 explore masses, radii, and tidal deformabilities

Glitches: probe n,p
 superfluidity and crust

Cooling of n-stars: search
 for exotica

Measuring equation of state in crust



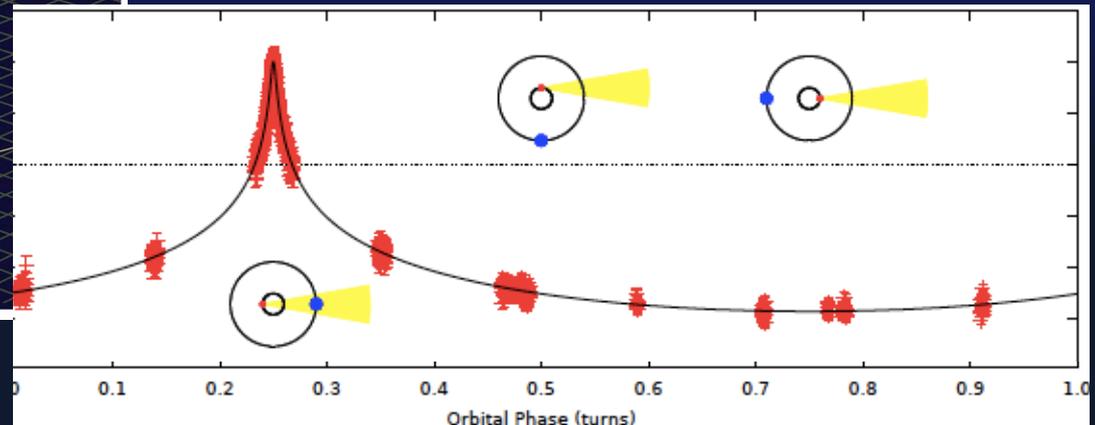
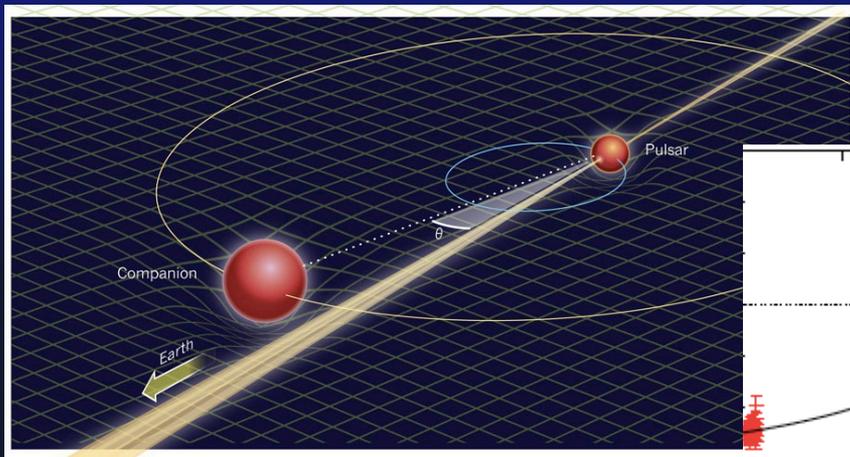
First firm high mass neutron star, PSR J1614-2230 -- in neutron star-white dwarf binary

Demorest et al., Nature 467, 1081 (2010); E. Fonseca et al., ApJ. 832, 16 (2016).

Spin period = 3.15 ms; orbital period = 8.7 day

Inclination = $89:17^\circ \pm 0:02^\circ$: edge on

$M_{\text{neutron star}} = 1.928 \pm 0.017 M_\odot$; $M_{\text{white dwarf}} = 0.500 \pm 0.006 M_\odot$



(Gravitational) Shapiro delay of light from pulsar
when passing the companion white dwarf

Highest mass neutron star, PSR J0740+6620 -- in neutron star-white dwarf binary

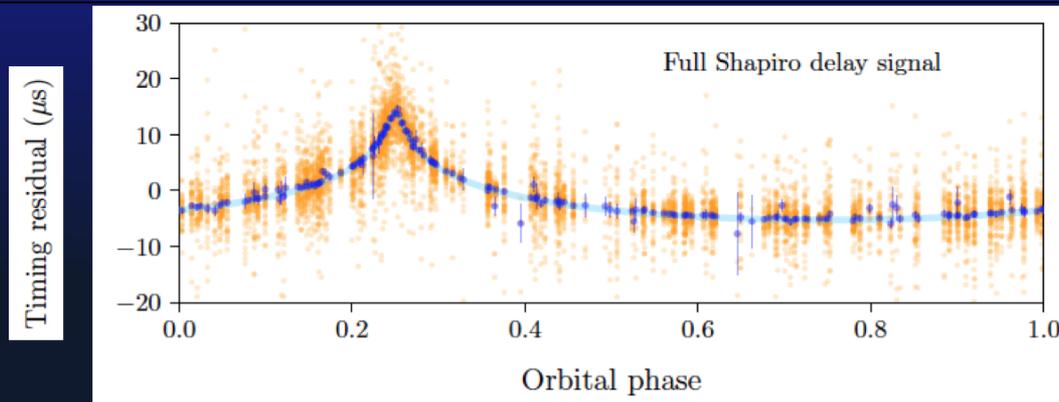
Chromartie et al., arXiv:1904.06759. (April 14, 2019)

Spin period = 2.89 ms; orbital period = 4.77 day

Nearly circular orbit (eccentricity = 5×10^{-6})

Inclination = $87.44^\circ \pm 0:01^\circ$: edge on

$M_{\text{neutron star}} = 2.17 \pm 0.1 M_\odot$; $M_{\text{white dwarf}} = 0.26 \pm 0.01 M_\odot$



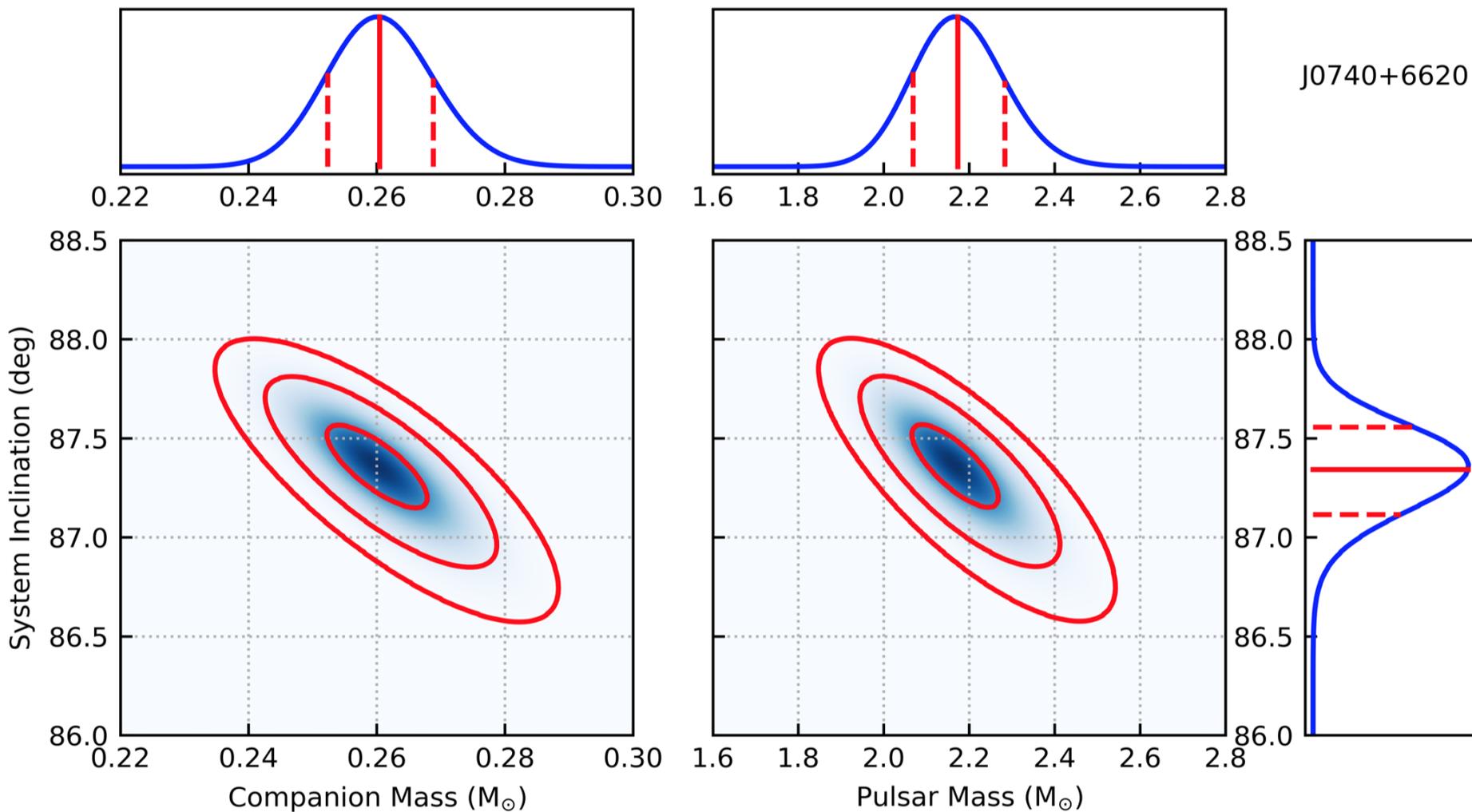
(Gravitational) Shapiro delay of light from pulsar

NICER is also observing J0740+6620 in the X-ray

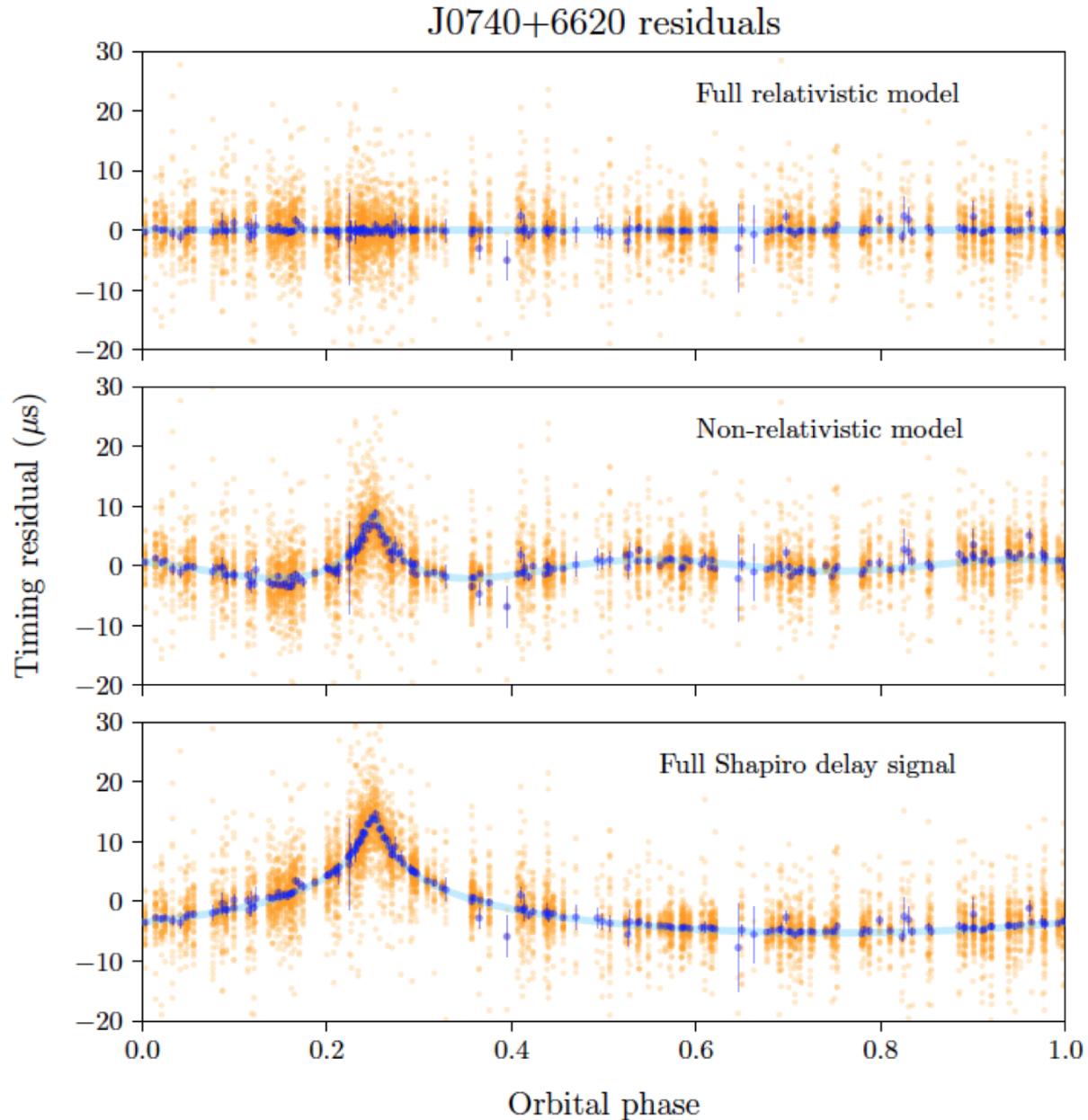
Firm test of the equation of state!!!

Mass distribution

J0740+6620



Timing Residuals



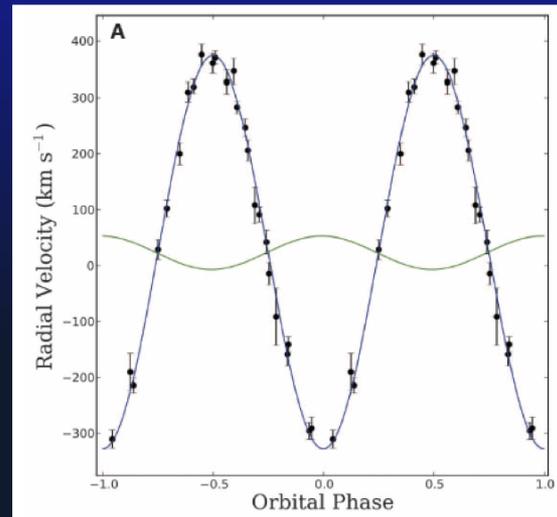
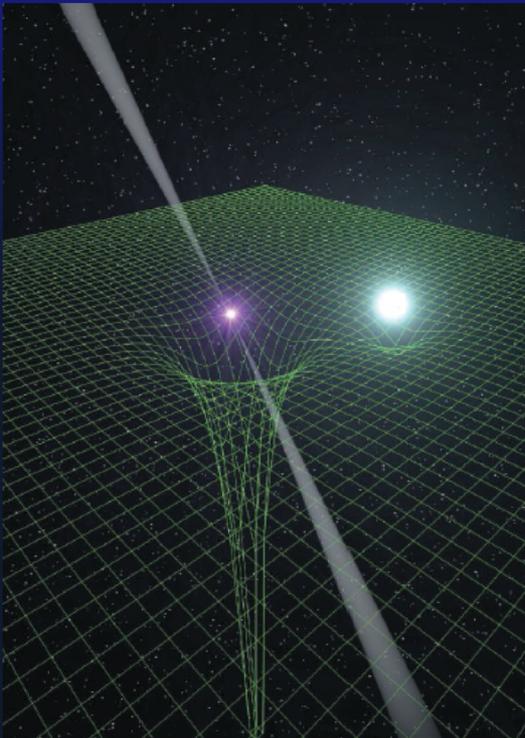
Second highest mass neutron star, PSRJ0348+0432 -- in neutron star-white dwarf binary

Antonidas et al., Science 340 1233232 (2013)

Spin period = 39 ms; orbital period = 2.46 hours

Inclination = 40.2°

$M_{\text{neutron star}} = 2.01 \pm 0.04 M_\odot$; $M_{\text{white dwarf}} = 0.172 \pm 0.003 M_\odot$



Significant gravitational radiation

$$\dot{P}/\dot{P}_{\text{GR}} = 1.05 \pm 0.18$$

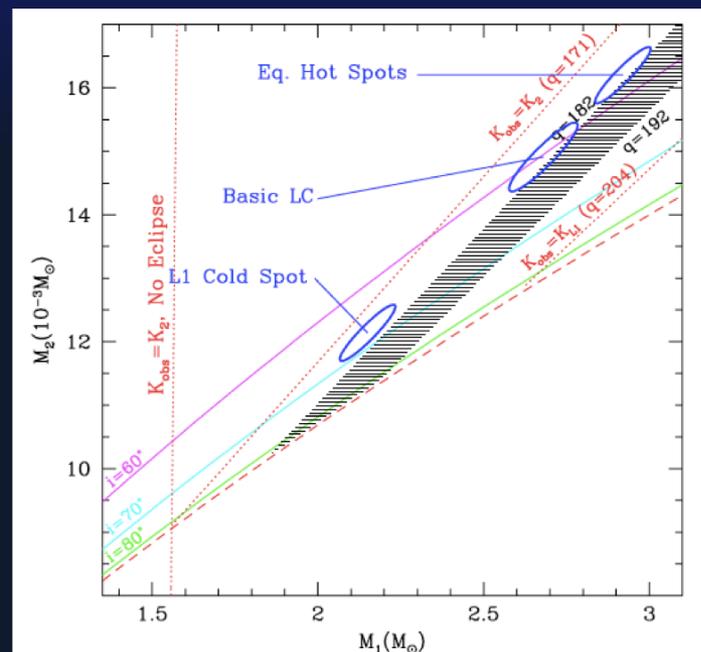
400 Myr to coalescence! AAAALigo

Possible high mass neutron stars ($< 2.7M_{\odot}$) in extreme “black widow pulsars” **J1311-3430, B1957+20, J2215+5135** **(neutron star - He star binaries)**

Romani et al. (1311-3430), Ap. J. Lett., 760:L36 (2012), Ap. J. 804:115R (2015),
 van Kerkwijk, Breton, & Kulkarni (B1957+20),” Ap.J. 728, 95 (2011),
 Schroeder & Halpern (J2215+5135), Ap. J. 793, 78 (2014).

$$M_{\text{ns-J1311}} \sim 1.8 - 2.7 M_{\odot} ; \quad M_{\text{companion}} \sim 0.01M_{\odot}$$

Uncertainties arise from incomplete modeling of heating of the companions by the neutron stars.



Neutron star masses

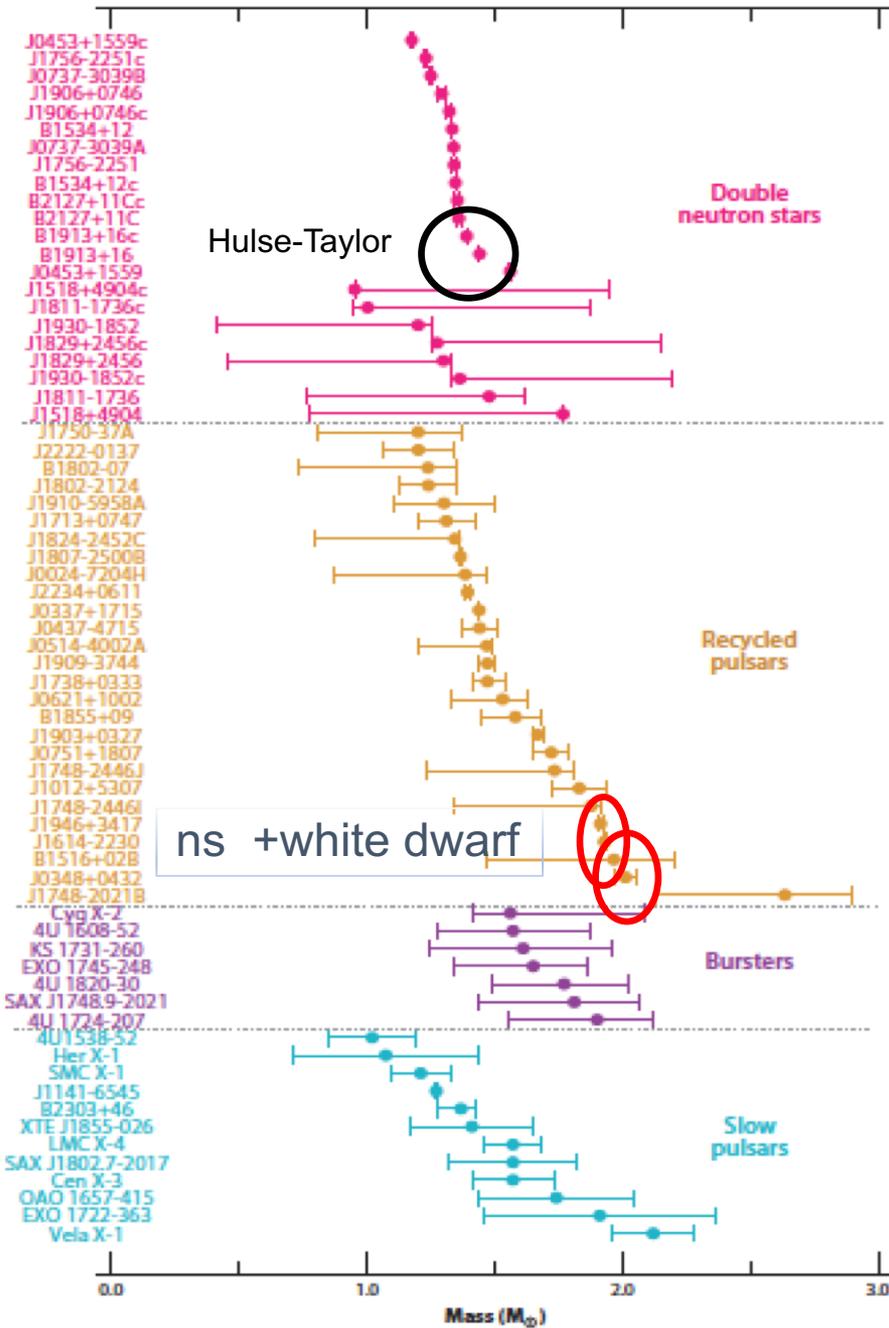
Özel & Freire, *Ann Rev AA* (2016)

PSR J1614-2230 :

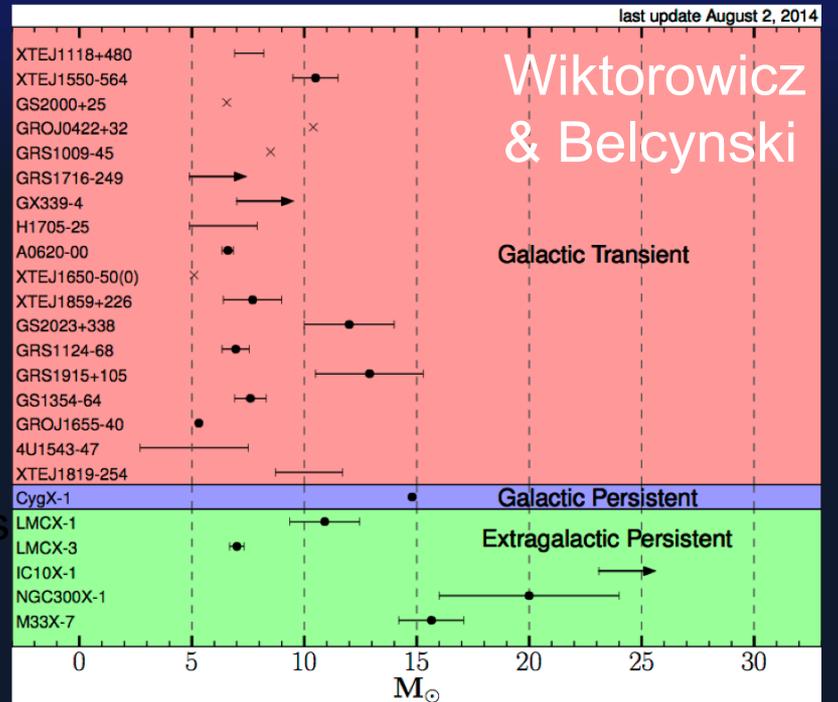
$$M_{\text{nstar}} = 1.928 \pm 0.017 M_{\odot}$$

PSR J0348+0432:

$$M_{\text{nstar}} = 2.01 \pm 0.04 M_{\odot}$$

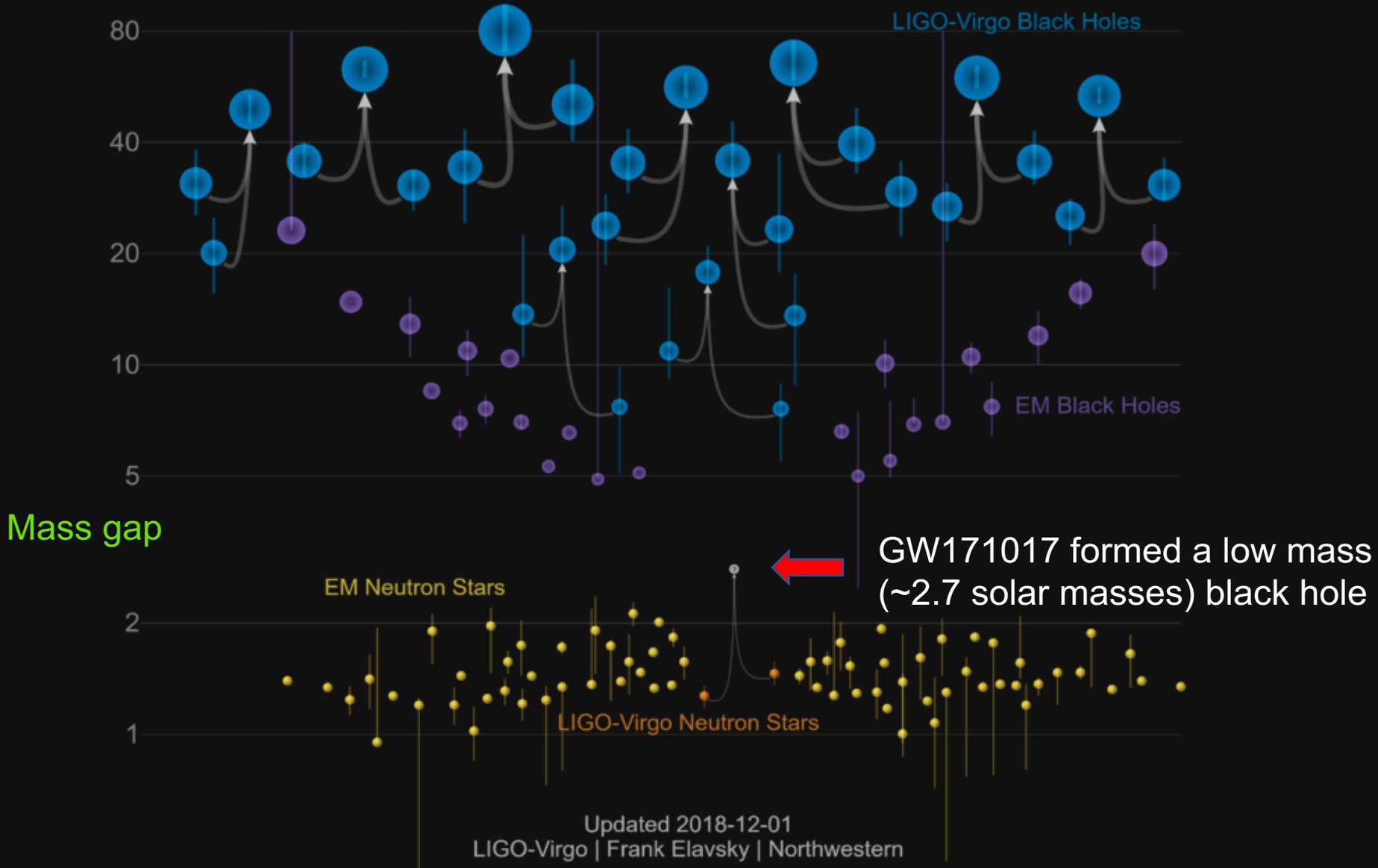


Galactic black hole masses

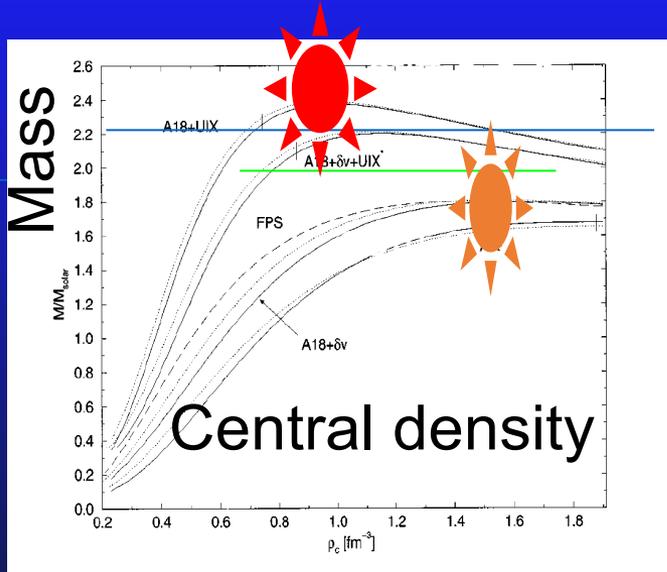


Masses in the Stellar Graveyard

in Solar Masses



The equation of state is very stiff



Softer equation of state =>
lower maximum mass and
higher central density

Binary neutron stars $\sim 1.4 M_{\odot}$: consistent with soft eq. of state

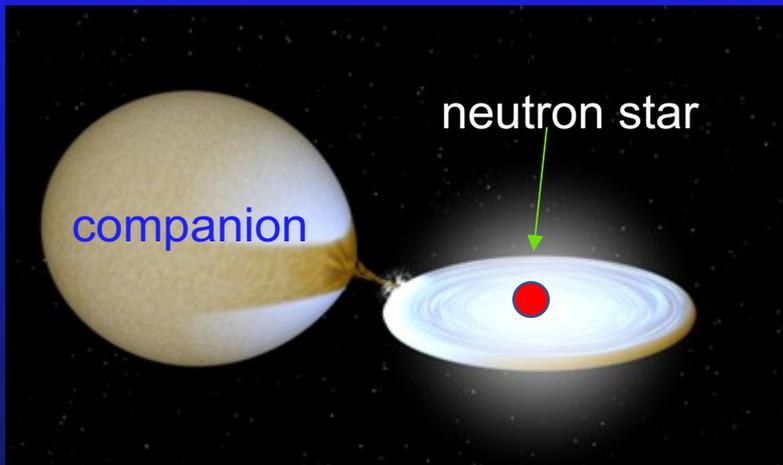
$$\text{PSR J1614-2230 : } M_{\text{neutron star}} = 1.93 \pm 0.02 M_{\odot}$$

$$\text{PSR J0348+0432: } M_{\text{neutron star}} = 2.01 \pm 0.04 M_{\odot}$$

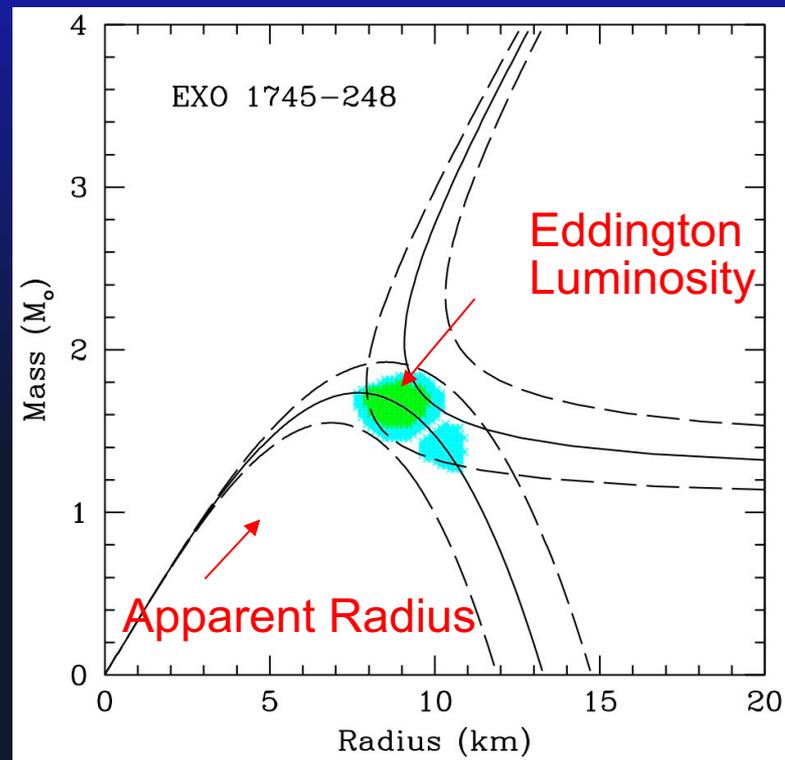
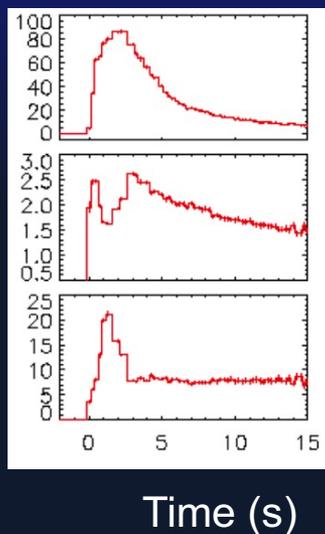
$$\text{PSR J0740+6620 : } M_{\text{neutron star}} = 2.17 \pm 0.1 M_{\odot}$$

require very stiff equation of state! How possible?

Measuring masses and radii of neutron stars in thermonuclear bursts in X-ray binaries



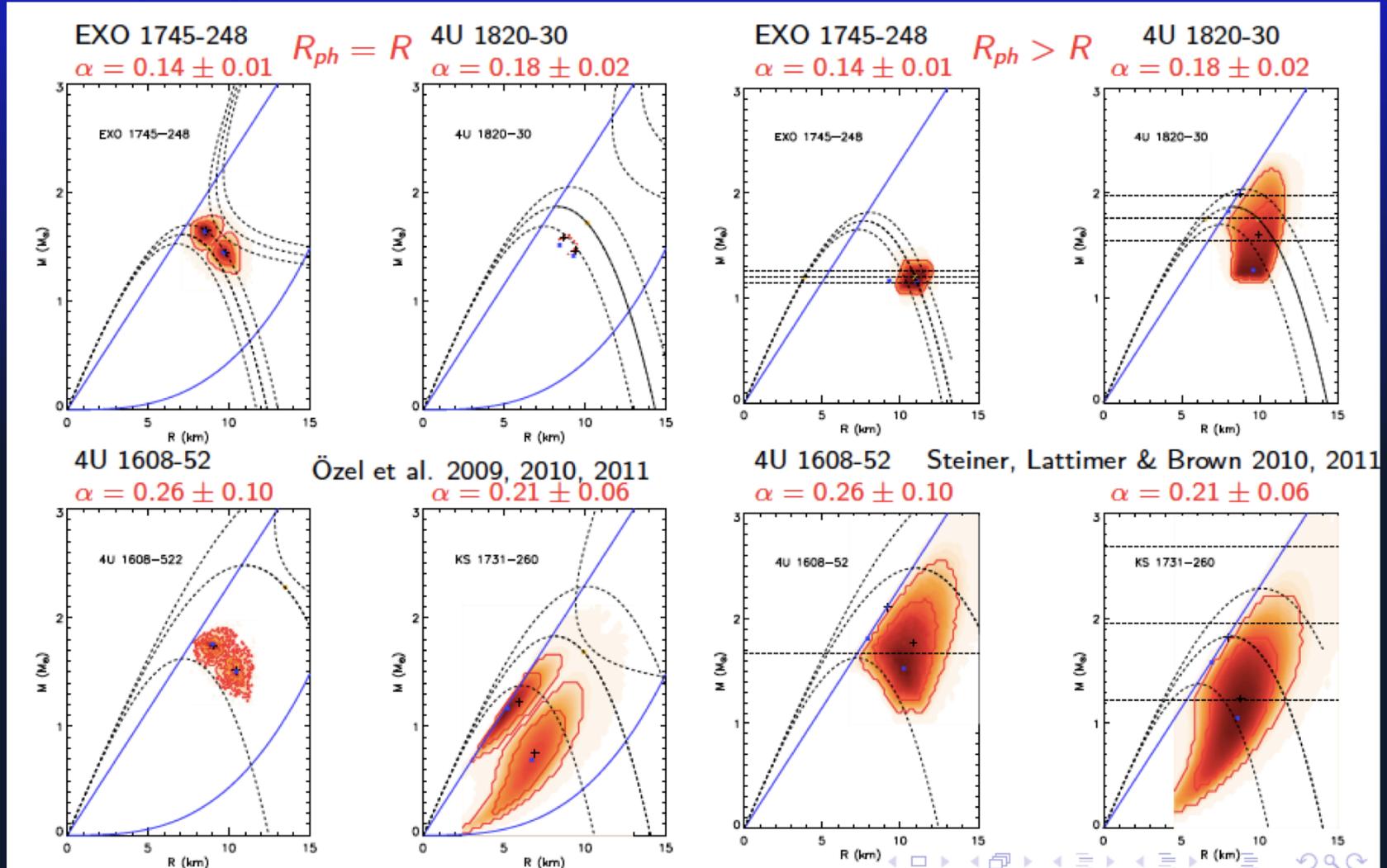
F. Özel, GB., T. Güver, *PRD* 82, 101301 (2010);
J.M. Lattimer & A. W. Steiner, *Ap. J.*, 784, 123
(2014). F. Özel, D. Psaltis, T. Guver, GB, C.
Heinke, & S. Guillot, *Ap. J.* 820, 28:1 (2016).



Measurements of *apparent* surface area, flux at Eddington limit (radiation pressure = gravity), combined with distance to star, constrains M and R .

Mass vs. radius determination of neutron stars in burst sources (low mass x-ray binaries). $R \sim 9-13$ km

F. Özel, GB., T. Güver, *PRD* 82, 101301 (2010); J.M. Lattimer and A. W. Steiner, *Ap J*, 784, 123 (2014). F. Özel, D. Psaltis, T. Güver, GB, C. Heinke, and S. Guillot, *Ap. J.* 820, 28:1 (2016).



NICER = Neutron star Interior Composition ExploreR

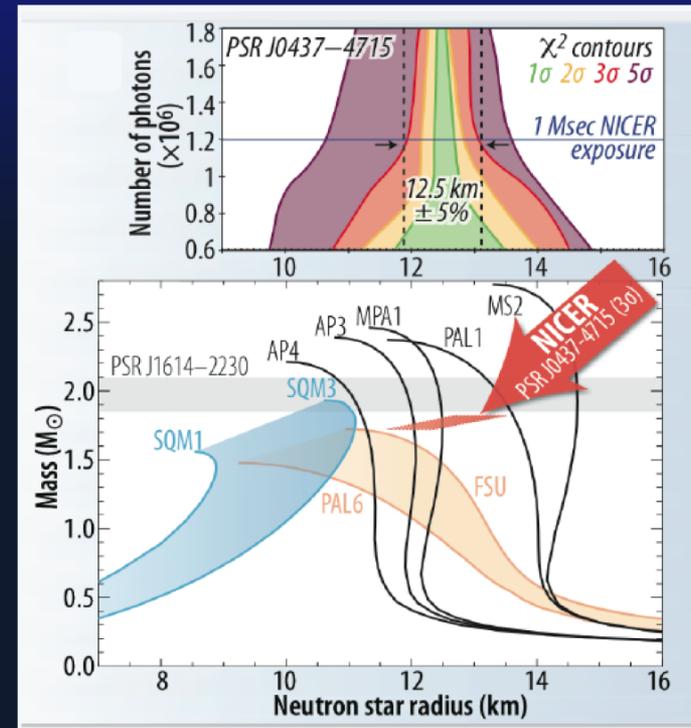
X-ray timing (GPS to 300 nsec)
& spectroscopy (0.12-12 KeV)

Measure masses and radii (5%) by
monitoring X-ray pulse profiles of
nearby neutron stars (J0437, ...)

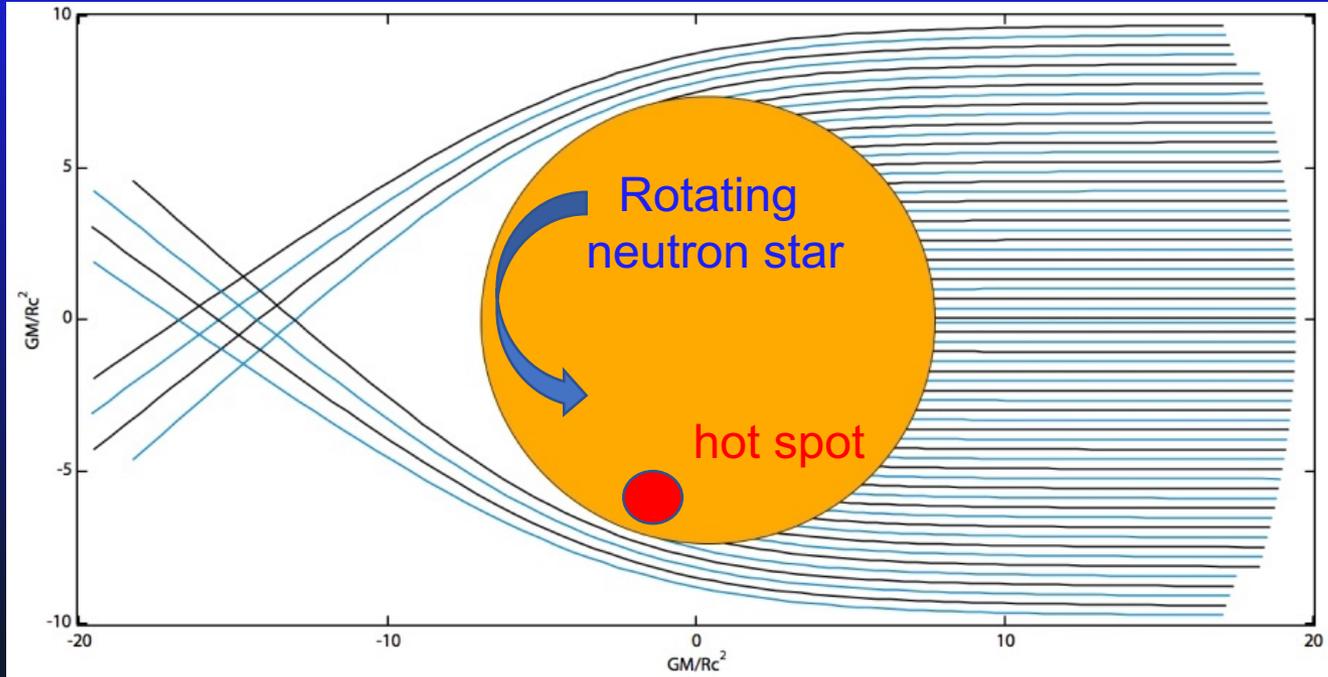
Properties of ns crusts via
astroseismology

Periodic pulsations from
transient & steady systems

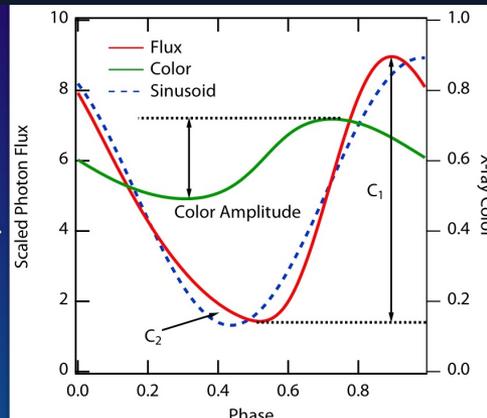
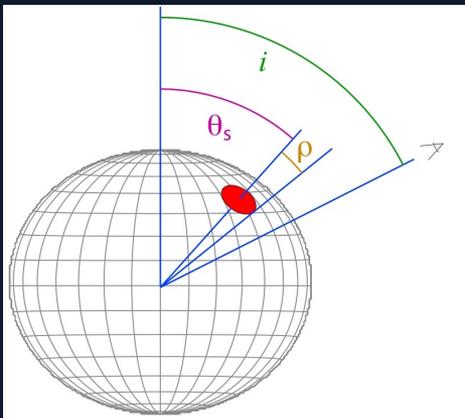
Now taking data!



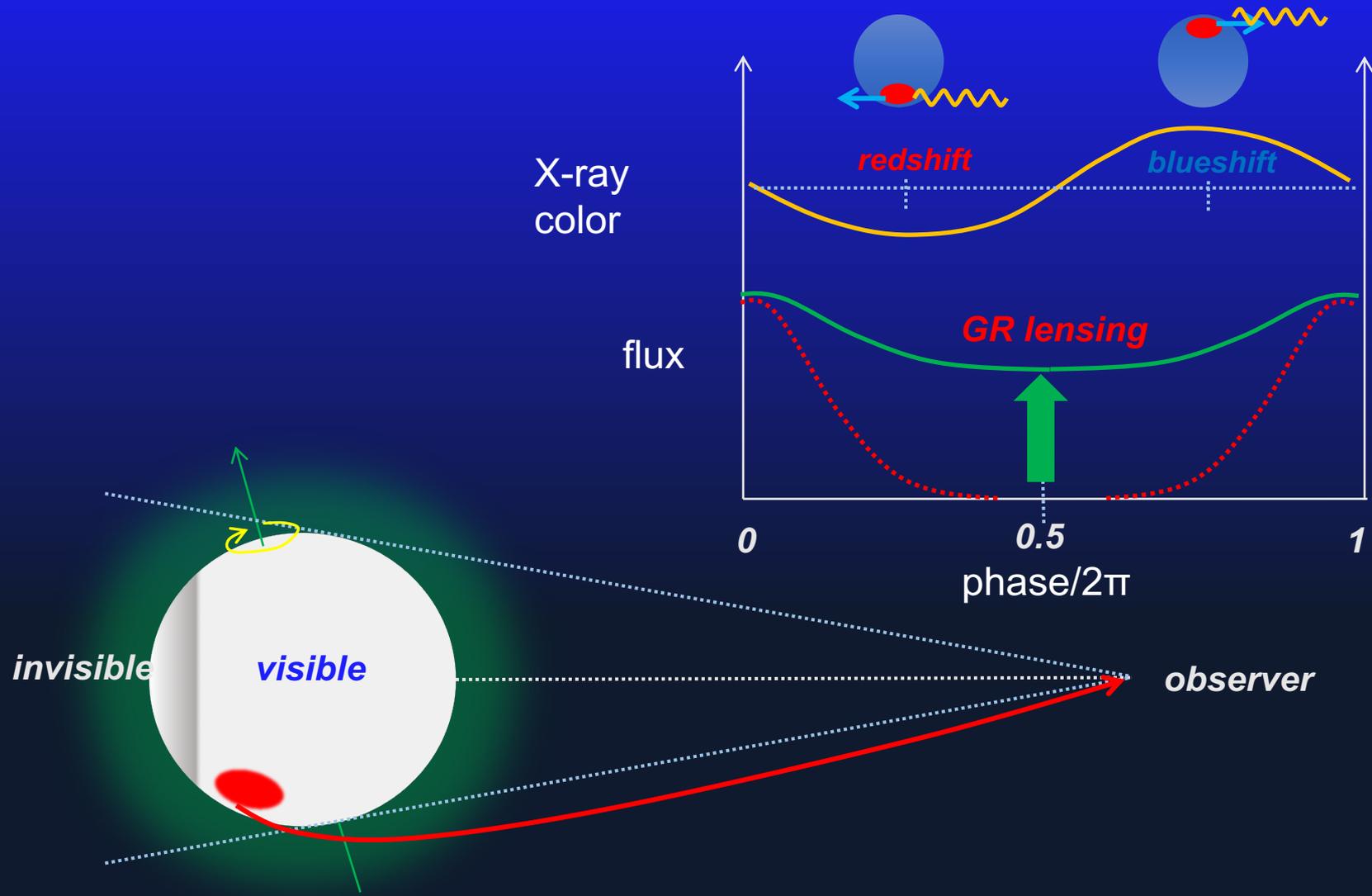
Track hot spots on neutron star. Light bending by star enables one to see spot “behind” star. Bending depends on M and R .

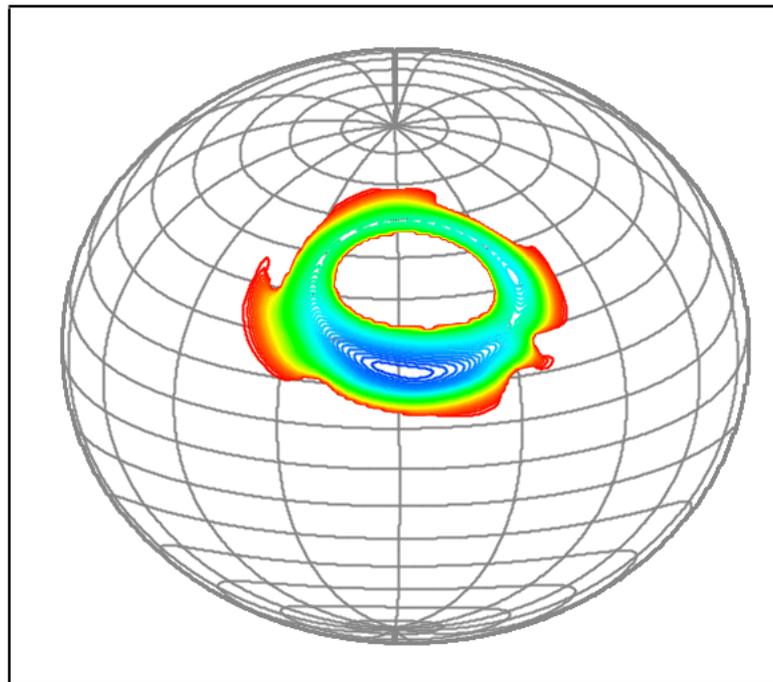
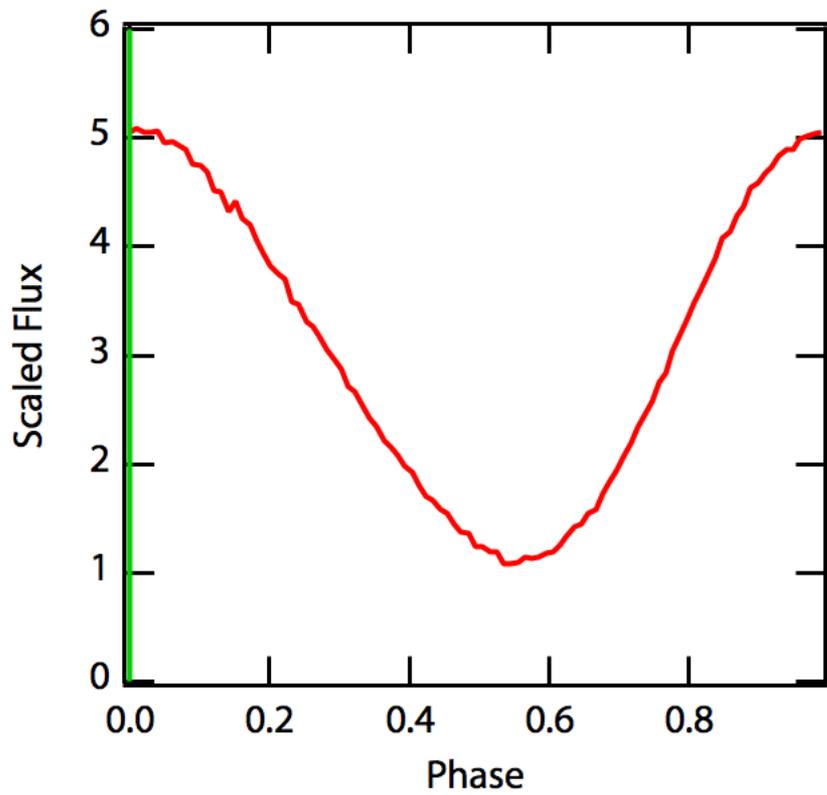


Observer = NICER



Measure amplitudes and phases in different colors (frequencies)





NICER sources being studied:

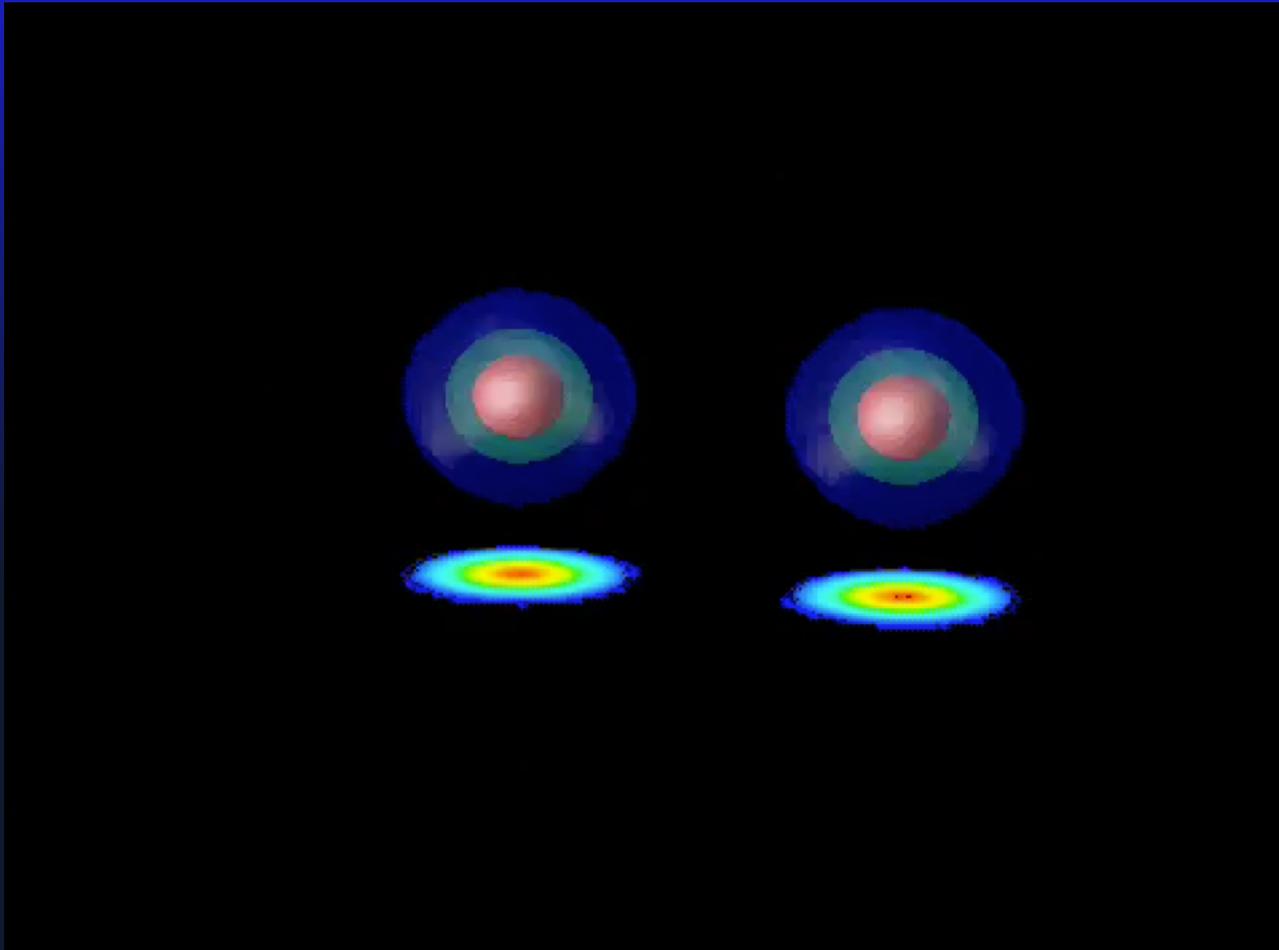
PSR J0030+0541 d=244pc age= 7580Myr
P = 4.865 ms

PSR J0437-4715 156pc 1590Myr
P = 5.75 ms
closest & brightest ms. pulsar known

PSR J1231-1411 440pc
P=3.684 ms
3rd suitable millisecond pulsar has a waveform
apparently simpler to analyze

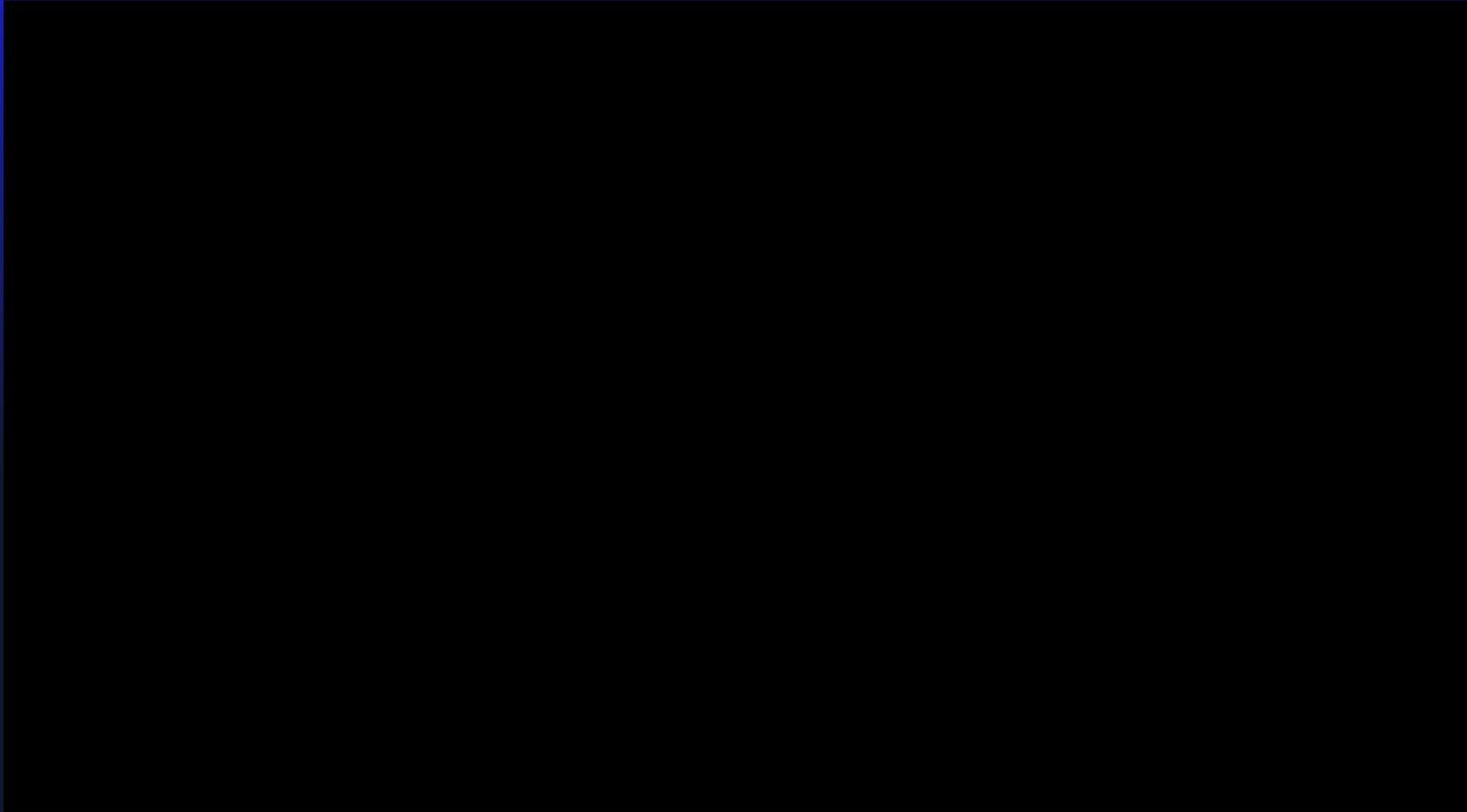
Neutron star - neutron star binary inspiral

(Fred Rasio, 2005)



$$\frac{dE}{dt}_{\text{grav rad}} = - \left(\frac{G}{5c^5} \right) h \left(\frac{d^3Q}{dt^3} \right)^2 \sim m^2 R^4 \omega^6$$

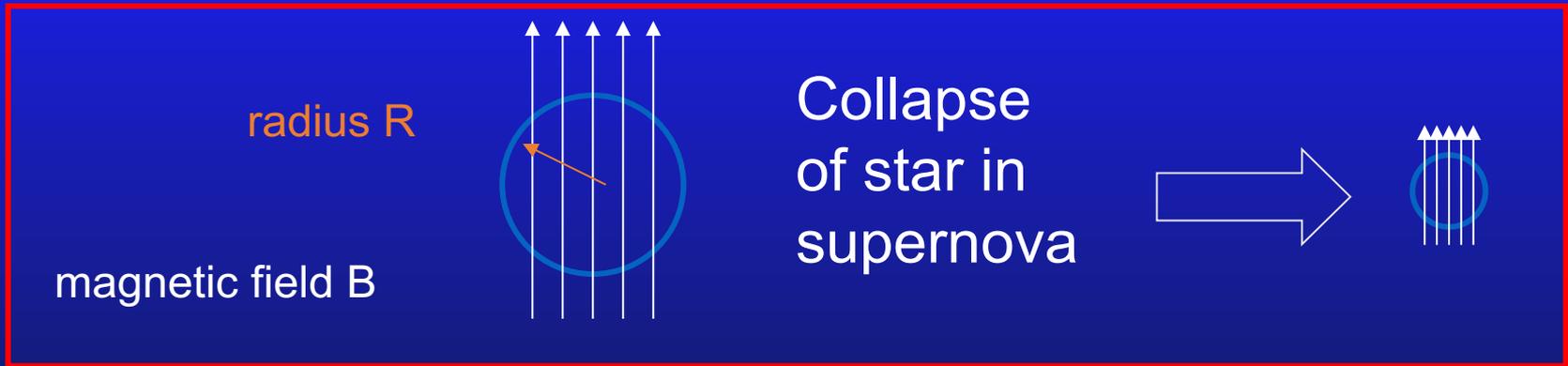
Neutron star – neutron star merger observed on 17 Aug. 2017
by LIGO and Virgo (gravitational radiation), FERMI (gamma ray
telescope) and ~ 70 other electromagnetic observatories.



Merger was 130,000,000 light years = 40 MPc away



Scaling of magnetic fields and periods



Conservation of flux $\Rightarrow BR^2 = \text{constant}$

$$R: 10^{11} \text{ cm} \Rightarrow 10^6 \text{ cm}$$

$$B: 10^2 \text{ G} \Rightarrow 10^{12} \text{ G}$$

Conservation of angular momentum

$$\Rightarrow I \Omega \sim R^2 \Omega = \text{constant}$$

$$\text{Period} = 2\pi/\Omega : 10^7 \text{ s} \Rightarrow 10^{-3} \text{ s}$$

Mass and radius scales of neutron stars

$$\frac{\partial P}{\partial R} = -\frac{G\rho(r)M(r)}{r^2}$$

Non-relativistic equation of hydrostatic balance.

$$M(r) \sim \frac{4\pi}{3}\rho_c r^3$$

Mass within radius r
 ρ_c = central mass density

$dP = n d\mu + S dT \Rightarrow n d\mu$ at zero temperature T
 μ = baryon chemical potential

$$\frac{1}{\rho} \frac{\partial P}{\partial r} = \frac{n}{\rho} \frac{\partial \mu}{\partial r} = \frac{1}{m} \frac{\partial \mu}{\partial r} \sim \frac{4\pi}{3} G\rho_c r \quad m = \text{nucleon mass}$$

$$\mu(r) \sim \mu(0) - \frac{2\pi}{3} G\rho_c r^2$$

R = neutron star radius

At surface, $\mu(R) = m \Rightarrow$

$$\mu(0) \sim m + \frac{2\pi}{3} G\rho_c R^2$$

$$\mu(r) \sim m + \frac{2\pi}{3} G\rho_c (R^2 - r^2)$$

Newton's gravitational constant

$$G = 6.67 \times 10^{-8} \text{ cgs.}$$

Electrodynamic fine structure constant: $\frac{e^2}{\hbar c} = \frac{1}{137}$

Gravitational fine structure constant:

$$\frac{Gm_p^2}{\hbar c} \equiv \alpha_G \simeq 0.589 \times 10^{-38} \equiv N_0^{-2/3}$$

m_p = proton mass

$$N_0 = 2.21 \times 10^{57}, \quad m_n N_0 = 1.189 M_\odot = \text{solar mass}$$

N_0 sets the scale of the masses of stars

Virial theorem for gravitating systems:

kinetic energy = -potential energy/2

Potential energy: $-\frac{M^2 G}{R}$

Classical stars: Kinetic energy $\sim NT$

Total energy = -kinetic energy \Rightarrow as stars lose energy they heat up. Negative specific heat!!! Burning fuel \Rightarrow cooling.

Negative feedback keeps star stable as it burns fuel.

$$M = m_n N \quad N \sim n_c R^3 \quad \longrightarrow \quad N^{2/3} \sim \frac{T}{\hbar c n_c^{1/3}} \alpha_G^{-1}$$

$$\hbar c = 2 \text{ KeV } \text{\AA}$$

$$N \sim \frac{(T[\text{KeV}])^{3/2}}{(n_c[\text{\AA}^{-3}])^{1/2}} \alpha_G^{-3/2}$$

Scale of stellar masses $\sim m_n \alpha_G^{-3/2}$

Neutron stars with free degenerate neutron Fermi gas

Mass density $\rho = \frac{mp_F^3}{3\pi^2\hbar^3}$ Chemical potential $\mu = m + \frac{p_f^2}{2m}$

Neutron star structure: $\mu(0) \sim m + \frac{2\pi}{3}G\rho_c R^2$



$$R \sim \frac{3\sqrt{\pi}}{2} N_0^{1/3} \frac{\hbar}{m_n c} \left(\frac{mc}{p_F} \right)^{1/2} = \frac{12 \text{ km}}{\sqrt{p_F (\text{fm}^{-1})}}$$

$$1 \text{ fm} = 10^{-13} \text{ cm}$$

Typically, $n = \frac{p_F^3}{3\pi^2\hbar^3} \sim 1/\text{fm}^3 \implies \frac{p_F}{mc} \sim 0.6$

$$M \sim \frac{4\pi}{3} \rho_c R^3 \sim m N_0 \left(\frac{p_F}{mc} \right)^{3/2} \sim M_\odot$$

$$\frac{M}{M_\odot} \left(\frac{R}{10 \text{ km}} \right)^3 \sim 1$$

Neutron star surface

Surface gravity: $g^* = MG/R^2 \sim 10^{14} \text{ cm/sec}^2 \sim 10^{11} g_{\text{earth}}$

Gravitational binding energy at surface:

$$E_{\text{grav}}(m) = -mMG/R = -mc^2 R_s/2R \sim -mc^2 (1.5 \text{ km}/R) (M/M_{\odot})$$

($R_s = 2M G/c^2 =$ Schwarzschild radius = 2.94 km for Sun)

$$E_{\text{grav}}(\text{proton}) > 100 \text{ MeV}, \quad m_p c^2 = 938 \text{ MeV}$$

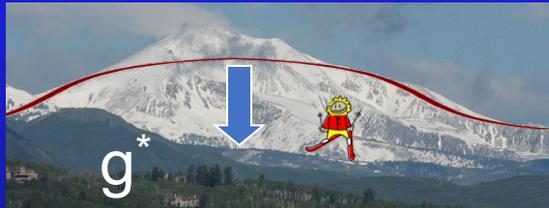
Neutron star atmosphere

$$\frac{\partial P}{\partial r} = -\rho g^* \quad P = \text{pressure} = T \rho/m_{\text{atom}}, \quad m_{\text{atom}} = A m_p$$

$$\text{Scale height of atmosphere, } \ell: \quad \frac{1}{\rho} \frac{\partial \rho}{\partial r} \equiv \frac{1}{\ell} \quad \rho(h) = \rho(0) e^{-h/\ell}$$

$$\ell = T/m_{\text{atom}} g^* \sim (10/A) \text{ cm} \quad (\text{compared with } \ell_{\text{earth}} \sim 7 \text{ km})$$

Mountain on a neutron star



Mountain of mass M , area A , height h

$$\text{stress} = \text{force/area} \sim Mg^*/A \sim \rho g^* h, \quad \rho = m_{\text{atom}} n_{\text{atom}}$$

$$\text{stress} = \nu \times \text{strain}, \quad \nu = \text{shear modulus} \sim E_{\text{binding}} n_{\text{atom}}$$

$$\text{strain} \sim m_{\text{atom}} g^* h / E_{\text{bind}}$$

When strain $\sim 10^{-2}$, mountain breaks

$$h \sim 10^{-2} E_{\text{bind}} / A m g^* \sim T_m / A m g^* \quad T_m \sim 10^{-2} E_{\text{bind}}$$

= scale height at melting temperature, T_m



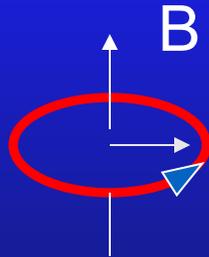
$$h_{\text{max}} \ll 1 \text{ cm}$$

$$h_{\text{earth}} \sim T_m / m_{\text{rock-atoms}} g_{\text{earth}} \sim 20 \text{ km}$$

Mountains are source of low level gravitational radiation – so far unseen

Magnetic fields can make deformations $\sim 10^{-6} (B/10^{15} \text{G})^2$

Atoms in strong magnetic fields

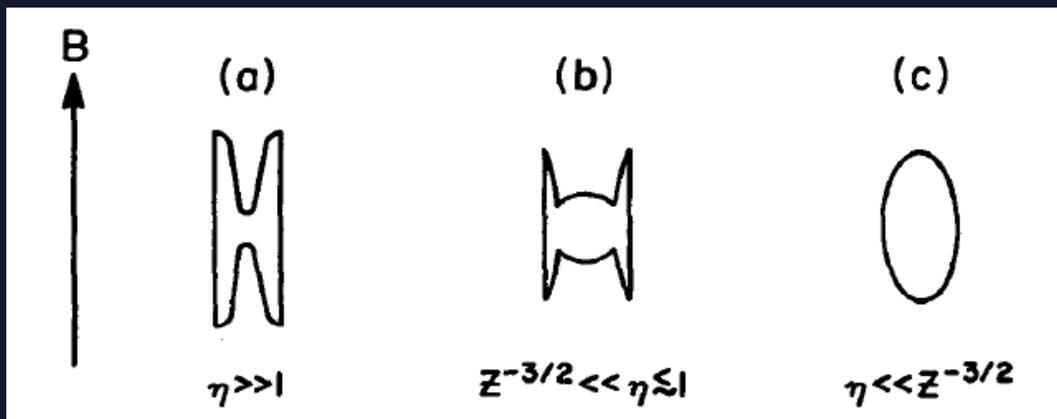


$$\rho = (\hbar / m_e B)^{1/2} = \text{cyclotron radius}$$

$$\omega_c = eB/m_e c$$

In atom, last filled orbit at $\rho_z \sim (2Z)^{1/2} \rho$; $Z = \text{proton no.}$

Atomic shape determined by competition between atomic (Bohr) orbits in Coulomb potential and Landau orbits in magnetic field



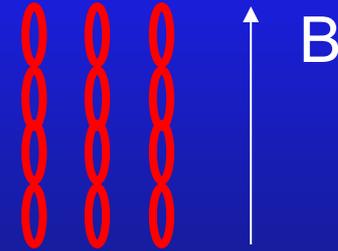
$$\eta \equiv a_0 / Z \rho_z$$

$$= (B / 4.6 \times 10^9 \text{ G})^{1/2} / Z^{3/2}$$

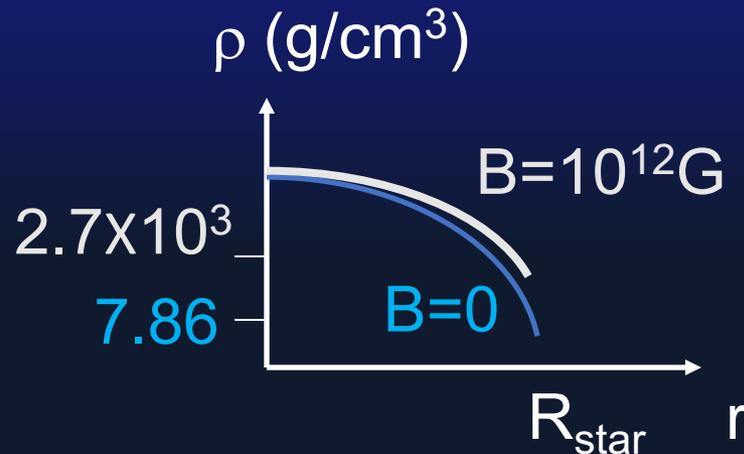
$$a_0 = \hbar / m_e e^2$$

Surface effects of strong magnetic fields

Atoms form covalently bonded chains parallel to magnetic field. Chains electrostatically bonded to each other.



Surface density is raised considerably:



Neutron stars are not black bodies. Difficult to absorb or emit radiation polarized perpendicular to magnetic field. Very important in interpretation of detected x-ray emission from neutron stars.



Outline

General structure of neutron stars (with a little history).

Neutron star models, masses, and observations

Nuclear physics of the crust

Liquid interior: equation of state.

Quark matter/ g_V and H

Glitches, superfluidity and vortices

dark matter in neutron stars -- Sanjay