Physics beyond the Standard Model and lattice calculations:

Higgs physics, the origin of mass and lattice field theory

Lecture II

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The issue is:

No relevant interaction in the scalar sector

Searching for relevant interaction:

The walking technicolour scenario

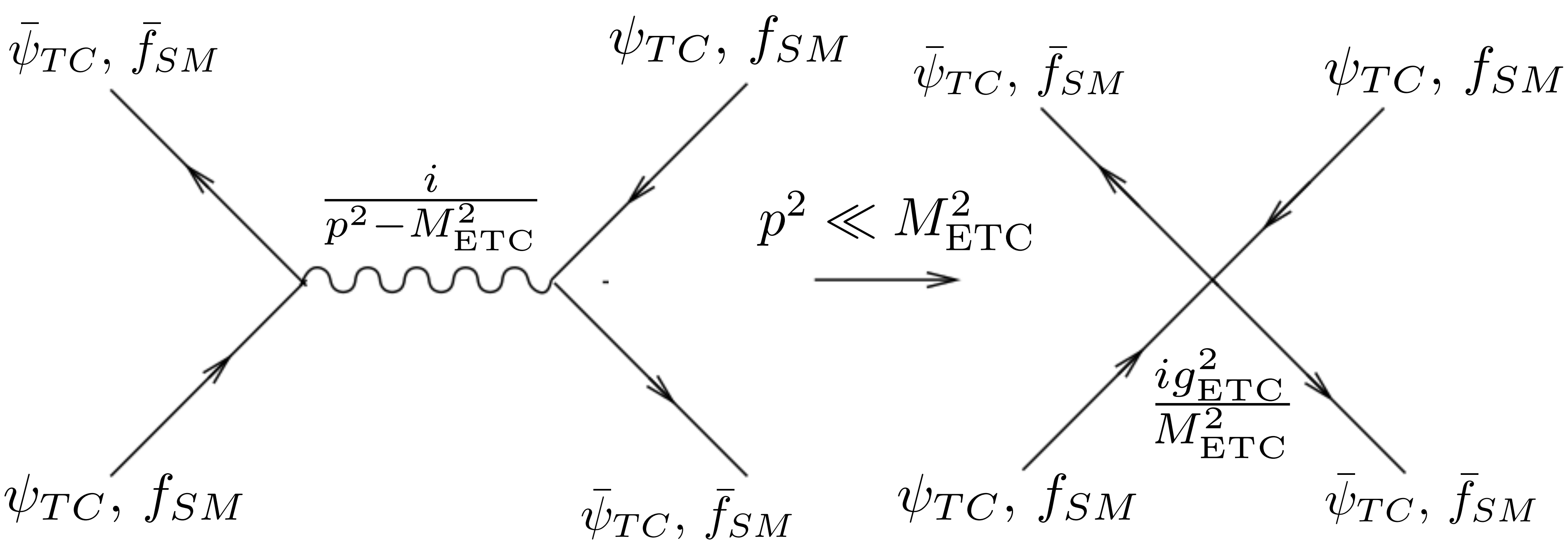
The idea of technicolour

Introduce a novel strong-interaction sector

- ★ Technicolour gauge group, e.g., $G_{TC} = SU(N_{TC})$
- ★ Technifermions are introduced in the following way:
  - ➨ The theory remains asymptotically free
  - ➨ Technifermions are preferably in a complex irrepn of G_{TC}
    - ◆ Chiral symmetry is broken *via* $SU(N_{TF})_L \otimes SU(N_{TF})_R \to SU(N_{TF})_V$
  - ➨ They are left doublet and right singlet under $[SU(2) \otimes U(1)]_{EW}$
    - ◆ $M_W \propto f_{\pi_{TC}} \sim (\langle \bar{\psi}_{TC} \psi_{TC} \rangle)^{1/3} \sim \Lambda_{\rm TC}$
- ★ This new sector looks just like a "novel QCD"

Flavour physics and extended technicolour

- Extended technicolour gauge group $G_{ETC} \supset G_{TC}$
- At scale Λ_{ETC}, the breaking $G_{ETC} \to G_{TC}$ occurs



$$\Lambda_{\rm ETC} \approx M_{\rm ETC} \gg \Lambda_{\rm TC}$$

Failure of QCD-like (extended) technicolour

- ★ The S parameter is too large — M. Peskin and T. Takeuchi, 1992
  - ➡ Define $\delta_{ab}\Pi_{\mu\nu}(q) = \int dq\; e^{iq\cdot x}\;\{\langle V_\mu^a(x)V_\nu^b(0)\rangle - \langle A_\mu^a(x)A_\nu^b(0)\rangle\}$

    $$= \delta_{ab}(q^2\delta_{\mu\nu} - q_\mu q_\nu)\Pi_{LR}(q^2) + \delta_{ab}q_\mu q_\nu\bar{\Pi}_{LR}(q^2)$$

  - ➡ The S parameter is extracted from $\frac{d\Pi_{LR}(q^2)}{dq^2}$ at $q^2 = 0$

- ★ No stable light Higgs
  - ➡ No such a scalar state in QCD

- ★ The FCNC problem
  - ➡ Explained on the next slide

FCNC problem in ETC models

$\frac{C(\mu)}{\Lambda_{ETC}^2}\bar{\psi}_{TC}\psi_{TC}(\mu)\bar{f}_{SM}f_{SM}$ $\frac{1}{\Lambda_{ETC}^2}\bar{f}_{SM}f_{SM}\bar{f}_{SM}f_{SM}$

Λ_{ETC}

$$m_{f_{SM}} = \frac{C(\mu)}{\Lambda_{ETC}^2}\langle\bar{\psi}_{TC}\psi_{TC}(\mu)\rangle$$

$$C(\Lambda_{ETC}) = 1$$

log running → $\langle\bar{\psi}_{TC}\psi_{TC}(\Lambda_{ETC})\rangle \sim \langle\bar{\psi}_{TC}\psi_{TC}(\Lambda_{TC})\rangle$

$\langle\bar{\psi}_{TC}\psi_{TC}(\Lambda_{TC})\rangle \sim M_W^3$

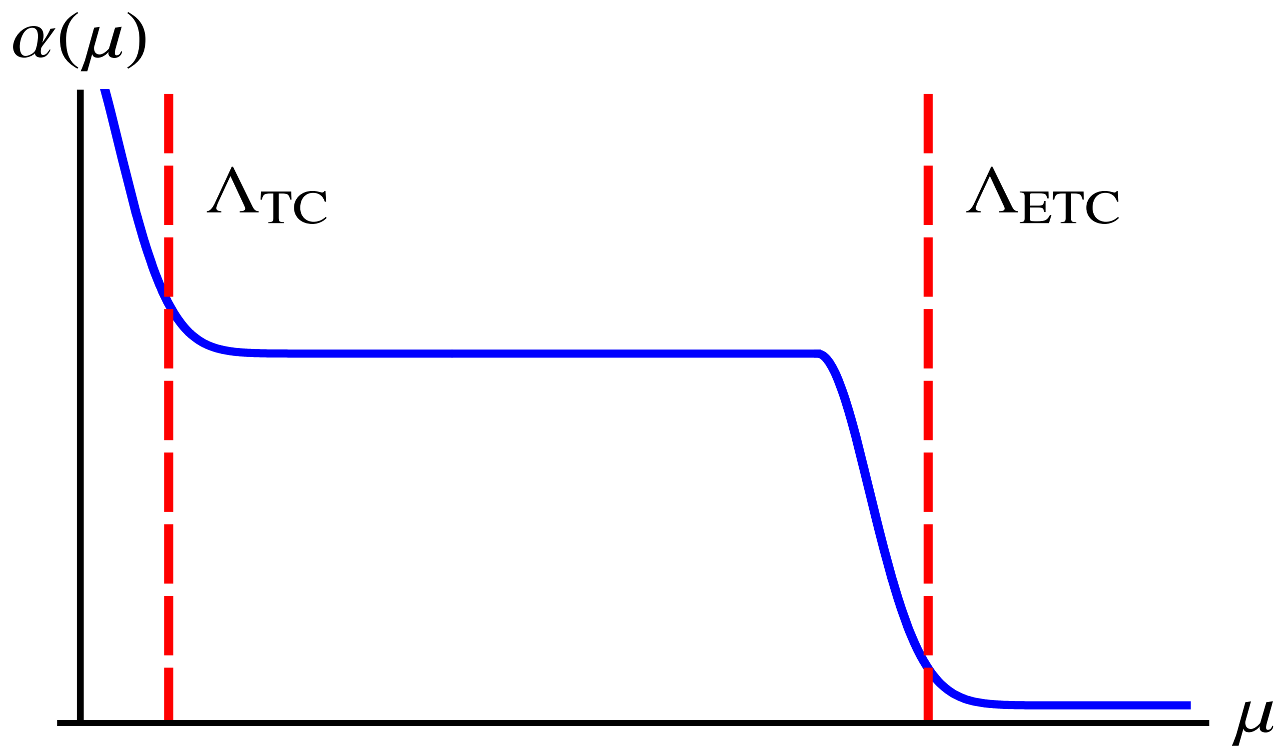
FCNC constraints

$\Lambda_{ETC} \sim 1\ \text{TeV}$ $\Lambda_{TC} \sim \Lambda_{EW}$ $\Lambda_{ETC} \sim 10^3\ \text{TeV}$

★ Can we enhance the running of the condensate?

→ If so, can lift Λ_{ETC} estimated from SM fermion mass

Dynamical solution from walking technicolour



- ★ Less significant chiral symmetry breaking effects
  - ➡ Smaller S parameter
- ★ Quasi scale invariance
  - ➡ Light Higgs as the dilaton
- ★ Almost power-law running behaviour
  - ➡ Ease the tension between SM fermion masses and FCNC

Dynamical solution to the FCNC problem

$\frac{C(\mu)}{\Lambda_{ETC}^2}\bar{\psi}_{TC}\psi_{TC}(\mu)\bar{f}_{SM}f_{SM}$ — Λ_{ETC} — $\frac{1}{\Lambda_{ETC}^2}\bar{f}_{SM}f_{SM}\bar{f}_{SM}f_{SM}$

$$m_{f_{SM}} = \frac{C(\mu)}{\Lambda_{ETC}^2}\langle\bar{\psi}_{TC}\psi_{TC}(\mu)\rangle$$

$C(\Lambda_{ETC}) = 1$

power-law running → anomalous dimension about unity

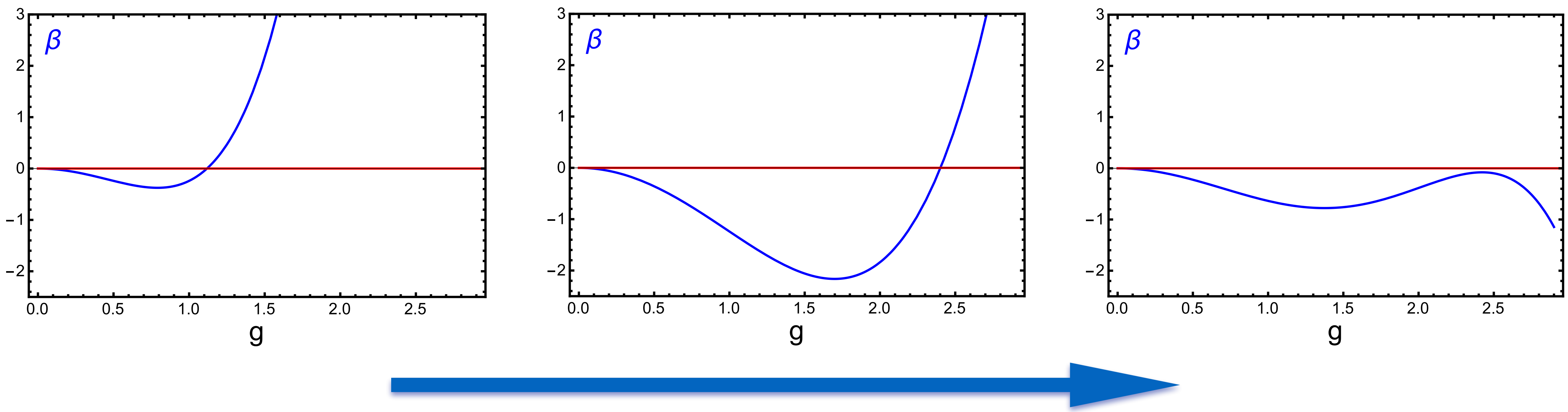
$\langle\bar{\psi}_{TC}\psi_{TC}(\Lambda_{TC})\rangle \sim M_W^3$ — FCNC constraints

$\Lambda_{ETC} \sim 10^3$ TeV — $\Lambda_{TC} \sim \Lambda_{EW}$ — $\Lambda_{ETC} \sim 10^3$ TeV

★ Enhance the running of the condensate

→ Estimated with an anomalous dimension close to unity

Looking for candidate theories



Decreasing N_f or $\dim[D\,(G_{TC})]$

★ The key issues are

Given a gauge group and a fermion repn

➡ What is the critical number of flavours?

➡ Is the theory just blow this number viable?

The “conformal windows”

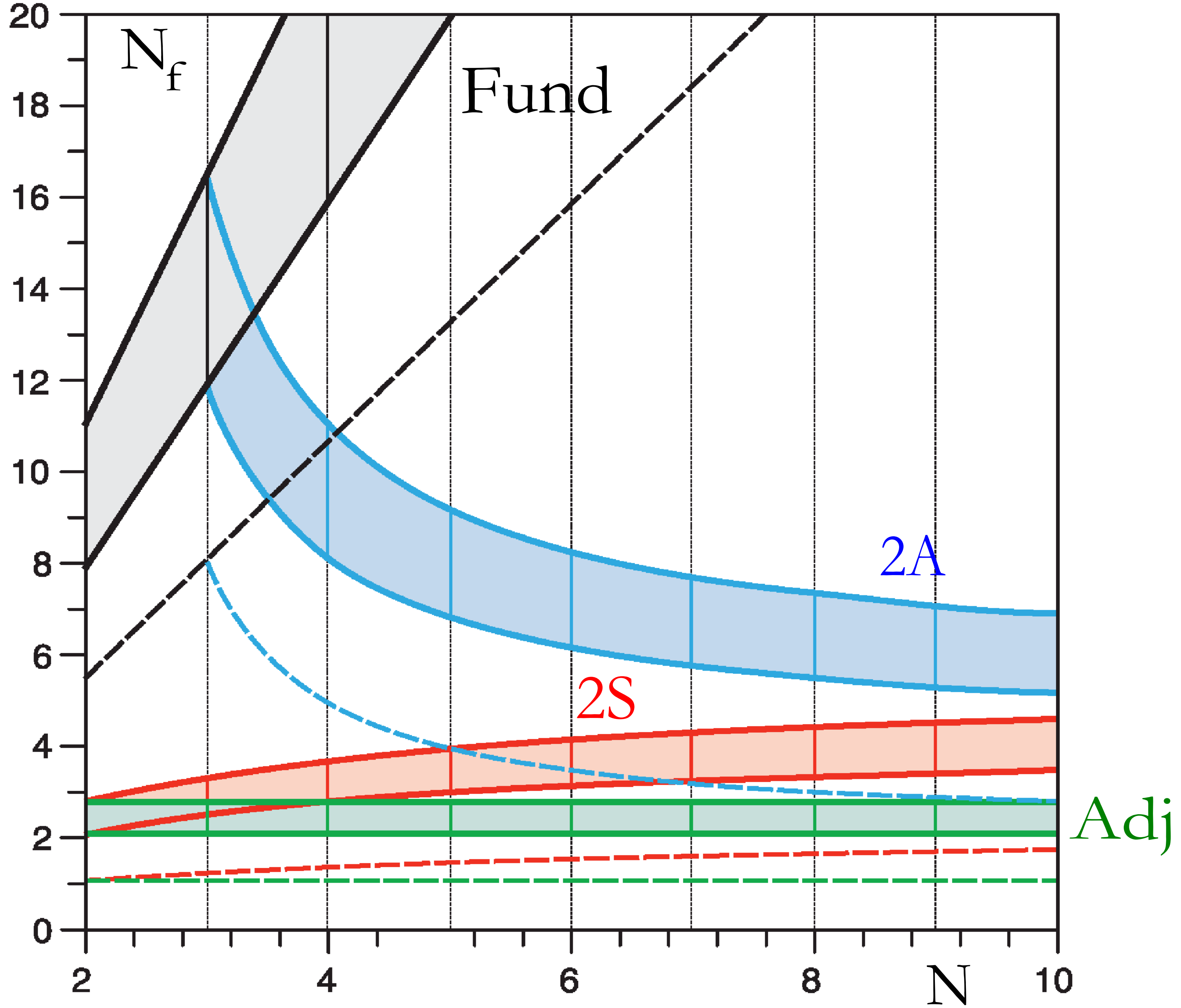


Figure credit: F. Sannino

Where is the lower conformal windowsill?

We may want the theory just below it!

Studies of the running coupling

The gradient flow

M.Luscher, 2010; M.Luscher, P.Weisz, 2011.

$$\dot{B}_\mu = D_\nu G_{\nu\mu}, \qquad B_\mu|_{t=0} = A_\mu,$$

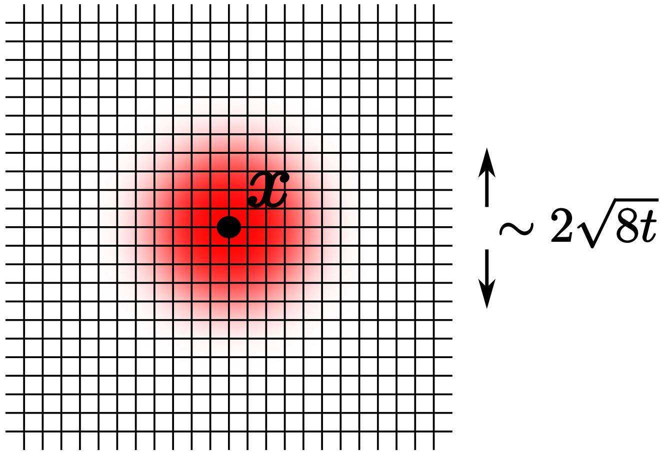
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu], \qquad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

↓ Wilson flow on the lattice (or improved flows)

$$\dot{V}_t(x,\mu) = -g_0^2\,\{\partial_{x,\mu} S_{\rm w}(V_t)\}\,V_t(x,\mu), \quad V_t(x,\mu)|_{t=0} = U(x,\mu),$$

$$S_{\rm w}(U) = \frac{1}{g_0^2}\sum_p \mathrm{Re\,tr}\{1 - U(p)\}$$

- Gauge field becomes renormalised at $t > 0$.
- A diffusion process.
- Extra dimension.

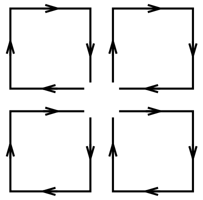


The Gradient Flow coupling

Z.Fodor et al., 2012.

- The quantity, $\langle E(t)\rangle = \frac{1}{4}\langle G_{\mu\nu}(t)G_{\mu\nu}(t)\rangle$, is finite when expressed in terms of renormalised coupling at positive flow time.
- With appropriate boundary condition, define,

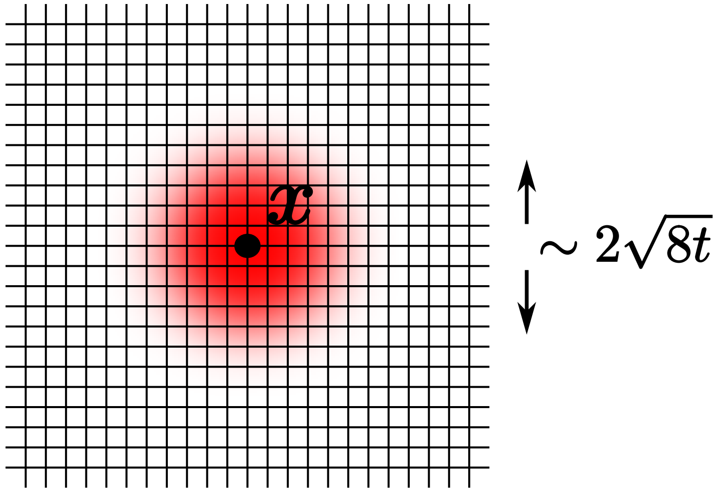
$$\bar{g}^2_{\rm GF}(L) = \mathcal{N}^{-1} t^2 \langle E(t)\rangle = \bar{g}^2_{\rm MS} + \mathcal{O}(\bar{g}^4_{\rm MS}) \quad ,$$

  with tree-level improvement.
- Use the clover operator,  ,

  as well as the plaquette, to extract $\langle E(t)\rangle$.
- The "lattice renormalised coupling" $\bar{g}^2_{\rm latt}$.

The renormalisation scheme

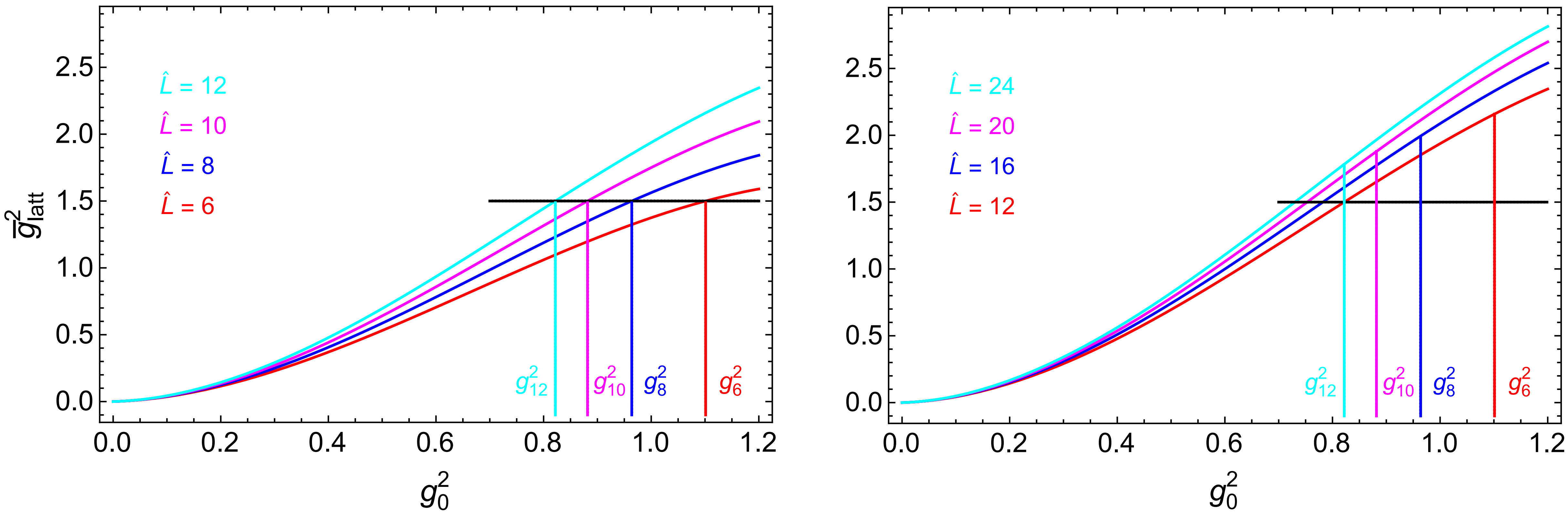
- The "flow-time" can be regarded as a renormalisation scale.
- The diffusion equation leads to a "gauge-field averaging radius.



- Step scaling at fixed $c_\tau = \frac{\sqrt{8t}}{L}$.
- One value of c_τ corresponds to one scheme.

Cartoon of the step-scaling method in practice

(no real lattice data shown on this slide)



- ★ Choose a value of the renormalised coupling
- ★ Read off the values of the bare coupling
- ★ Increase the lattice size and take the continuum limit

Running coupling and the β-function

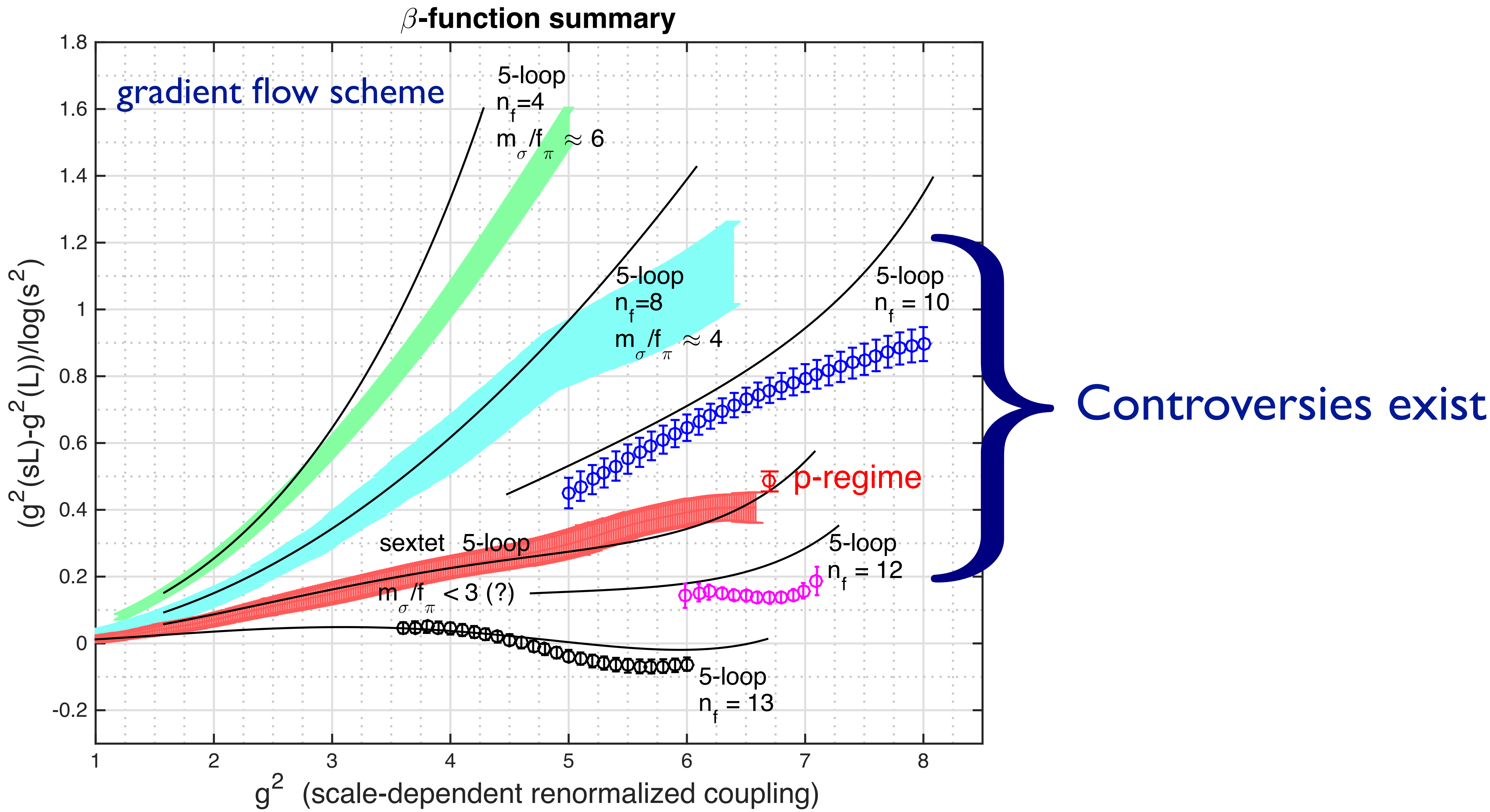


Figure from Kieran Holland, LATTICE 2019

Running coupling and the β-function

12-flavour QCD

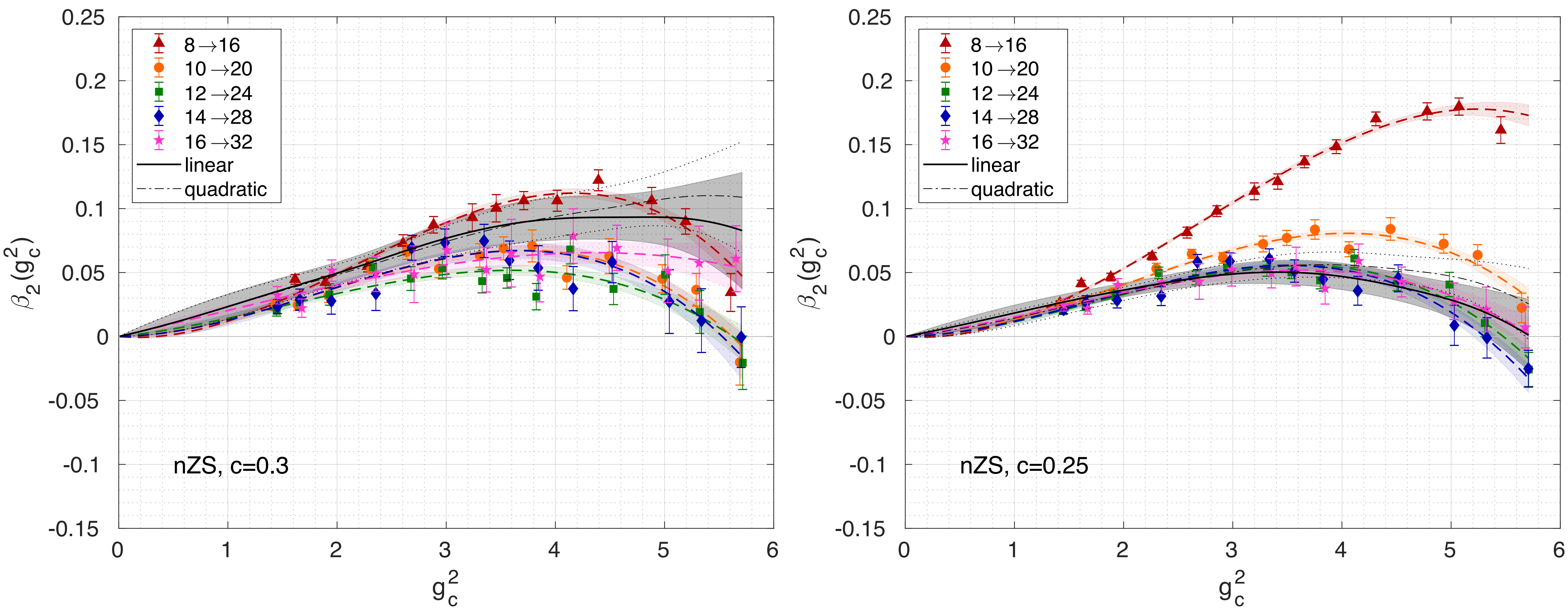


Figure from A. Hasenfratz, C. Rebbi, O. Witzel, arXiv:1810.0517 [hep-lat] (LATTICE 2018)

Running coupling and the β-function

12-flavour QCD

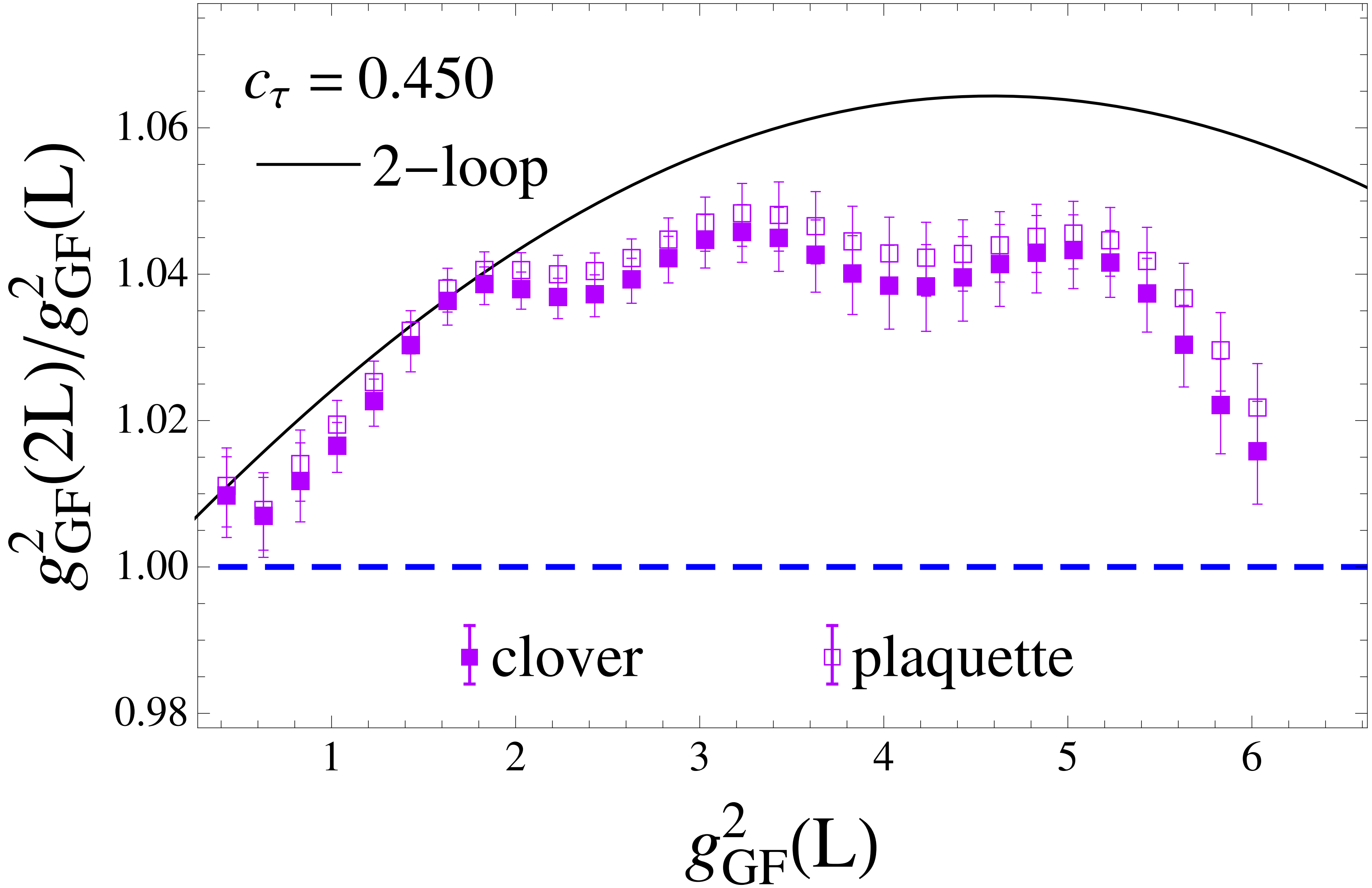
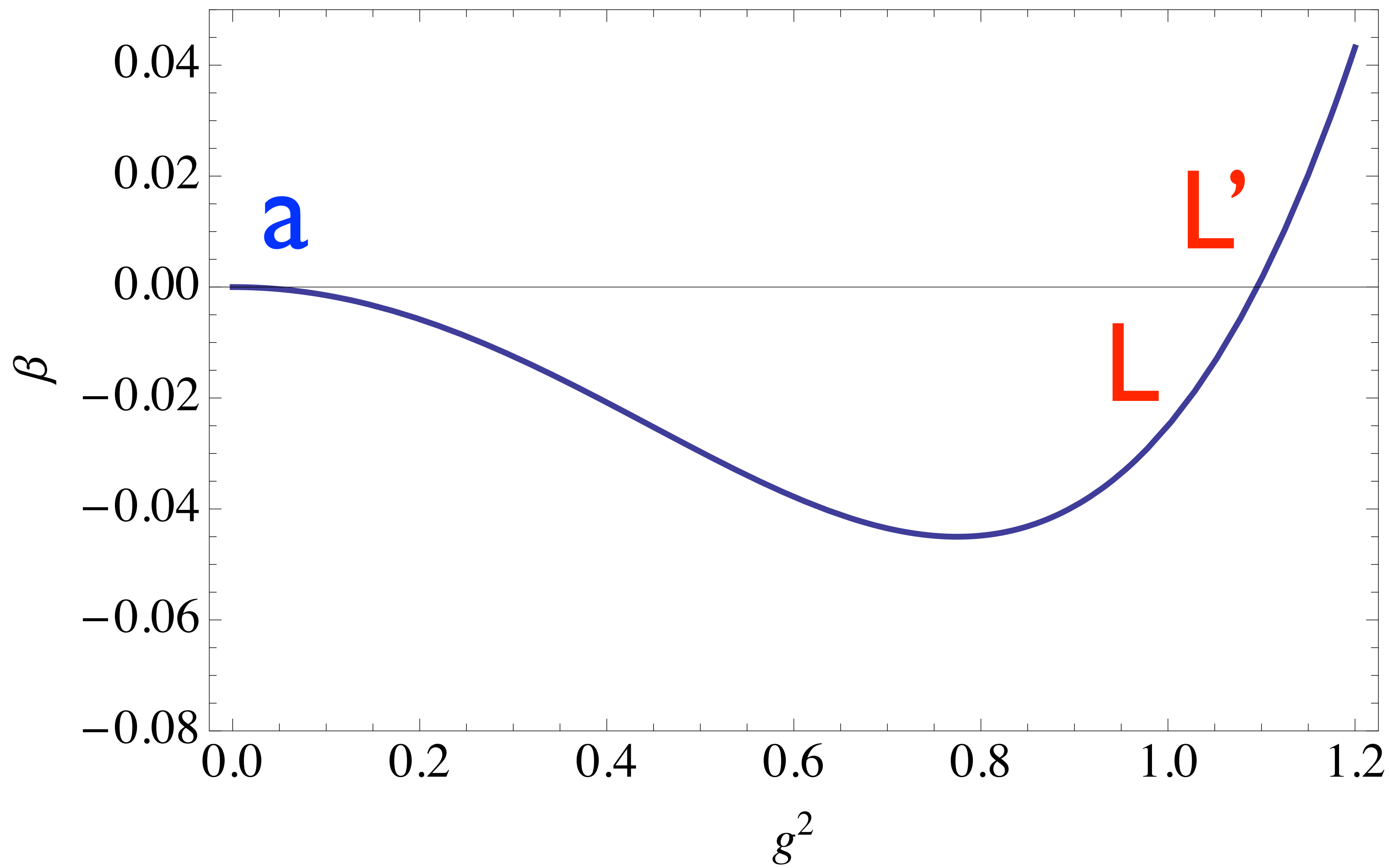


Figure from C.-J.D.L., K. Ogawa, A. Ramos, JHEP 12 (2015)

“The” continuum limit as desired



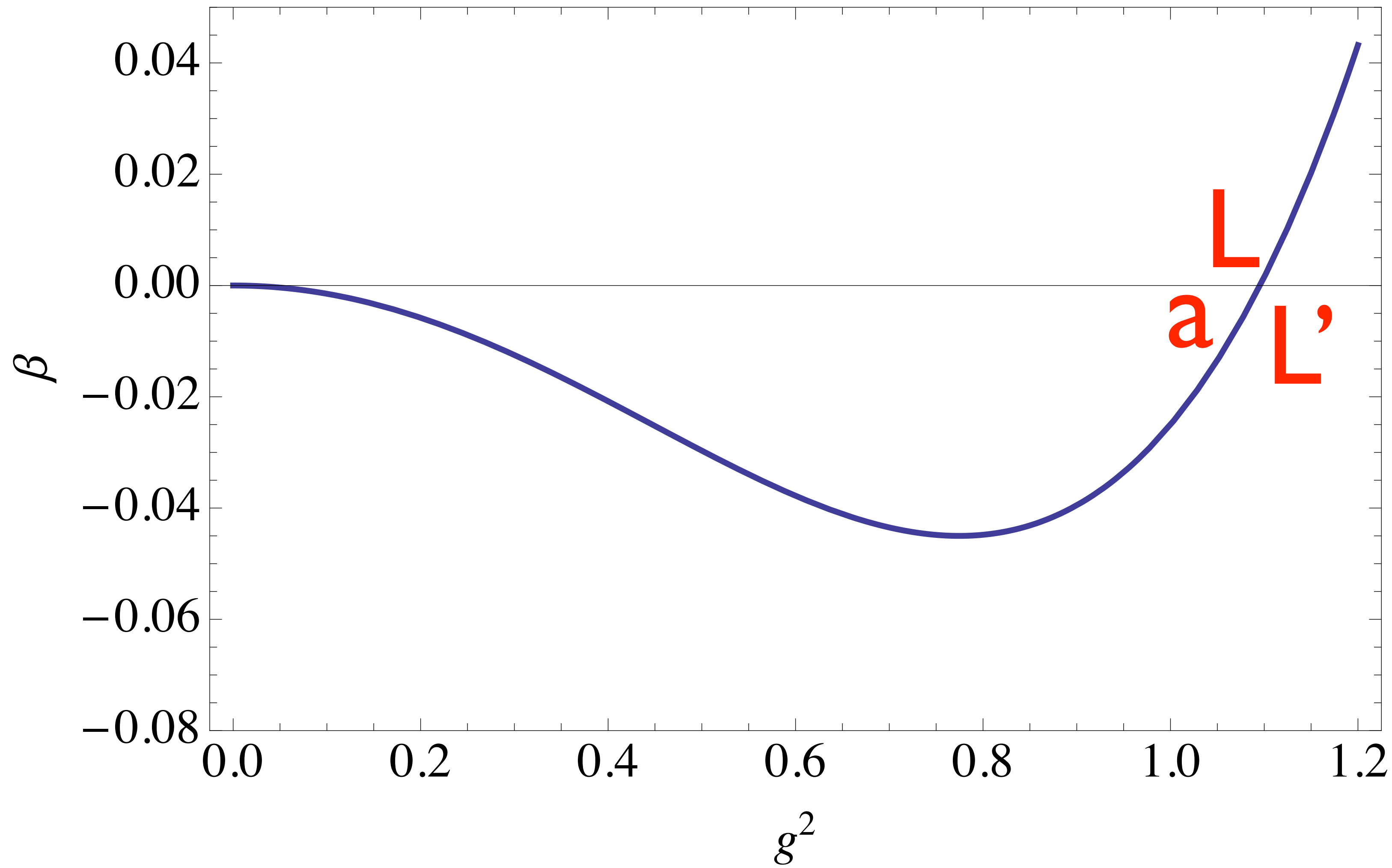
★ Scaling violation through the inclusion of the irrelevant operators.

★ “Symanzik-type” continuum extrapolation.

$$\mathcal{M}_{\mathrm{latt}} = \mathcal{M}_0 + \sum_{n=1}^{\infty} \sum_{i=1}^{N_{\mathrm{IR}}} \mathcal{M}_{n,i} \left(a\Lambda_i\right)^n$$

The continuum limit we may actually deal with

Small beta function and practical choices of L/a

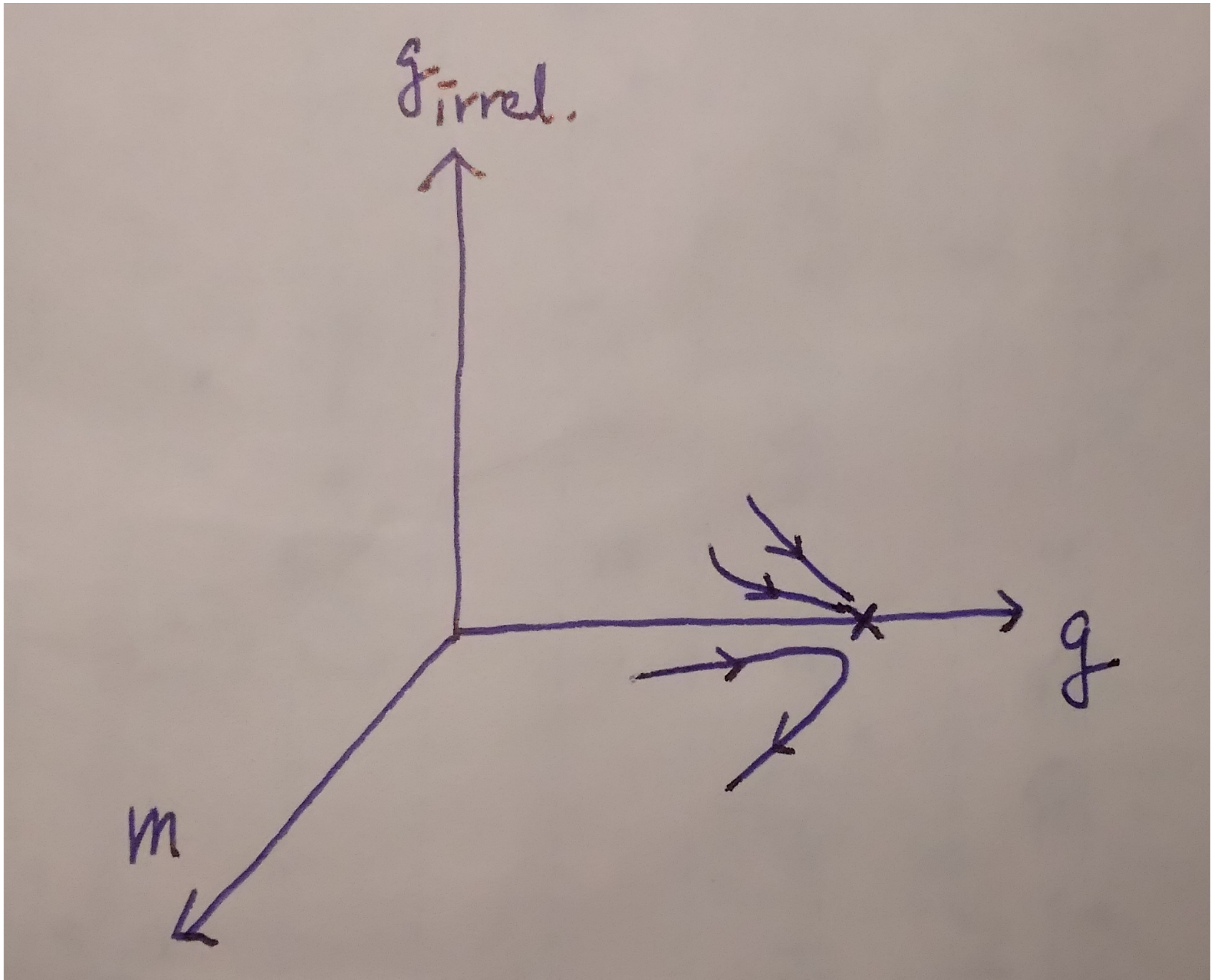


The theory is engineered to be very close to the strongly-coupled IR fixed point.

- Assume to be on the critical surface,

→ "Continuum extrapolation" with the scaling behaviour near the IRFP.

In other words, what we discretise is this….



Close enough to the IRFP...

C.-J.D.L., K. Ogawa, A. Ramos, JHEP 12 (2015)

★ The beta function is well approximated by the linearised form

➡ $$\beta\left(g_{\mathrm{R}}^{2}\right) \equiv -\rho \frac{\mathrm{d} g_{\mathrm{R}}^{2}}{\mathrm{d}\rho} = \gamma_{*}\left(g_{\mathrm{R}}^{2} - g_{*}^{2}\right)$$

➡ $$g_{\mathrm{R}}^{2}(l_{2}) = g_{*}^{2} + \left[g_{\mathrm{R}}^{2}(l_{1}) - g_{*}^{2}\right]\left(\frac{l_{1}}{l_{2}}\right)^{\gamma_{*}}$$

★ Introduce a reference length scale, $L > L_{\mathrm{ref}} > a$

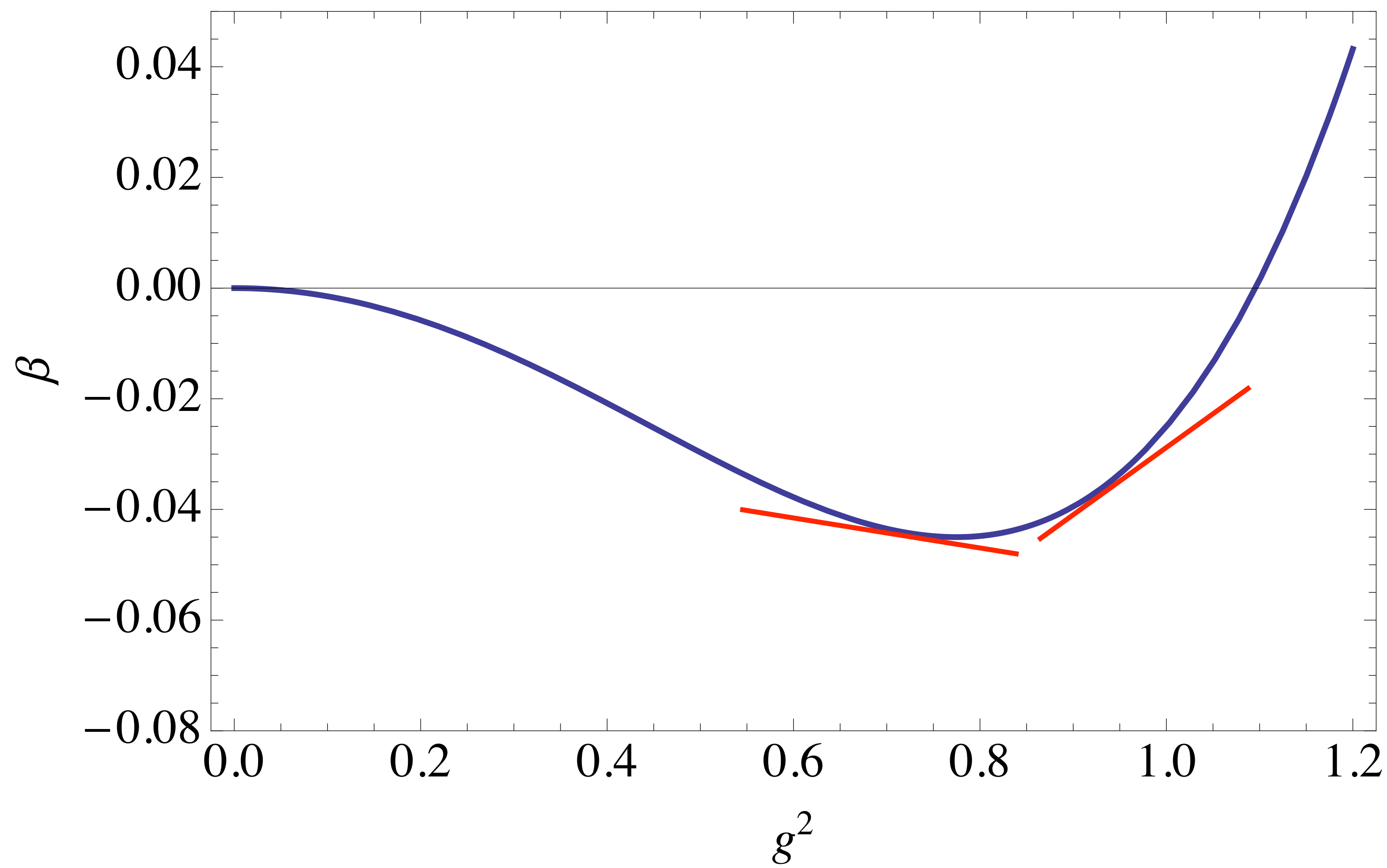
➡ $$\bar{g}_{\mathrm{latt}}^{2}(g_{0}^{2}, \hat{L}) = g_{*}^{2} + \left[\bar{g}_{\mathrm{latt}}^{2}(g_{0}^{2}, \hat{L}_{\mathrm{ref}}) - g_{*}^{2}\right]\left(\frac{\hat{L}_{\mathrm{ref}}}{\hat{L}}\right)^{\gamma_{*}}$$

★ Check that the lattice artefacts are small

➡ Done in practice using more than one discretisation

A practical strategy

The "locally linearised" beta function



$$\bar{g}^2_{\mathrm{latt}}(g_0^2, \hat{L}) = g_{\mathrm{l}}^2(g_{\mathrm{ref}}) + \left[g^2_{\mathrm{ref}} - g_{\mathrm{l}}^2(g_{\mathrm{ref}})\right] \left(\frac{\hat{L}_{\mathrm{ref}}}{\hat{L}}\right)^{\gamma(g_{\mathrm{ref}})}$$

(Good approximation for small beta function)

→ $\gamma(g_{\mathrm{ref}})$ plateaus to the value γ_* near the IRFP.

Results of the scaling test

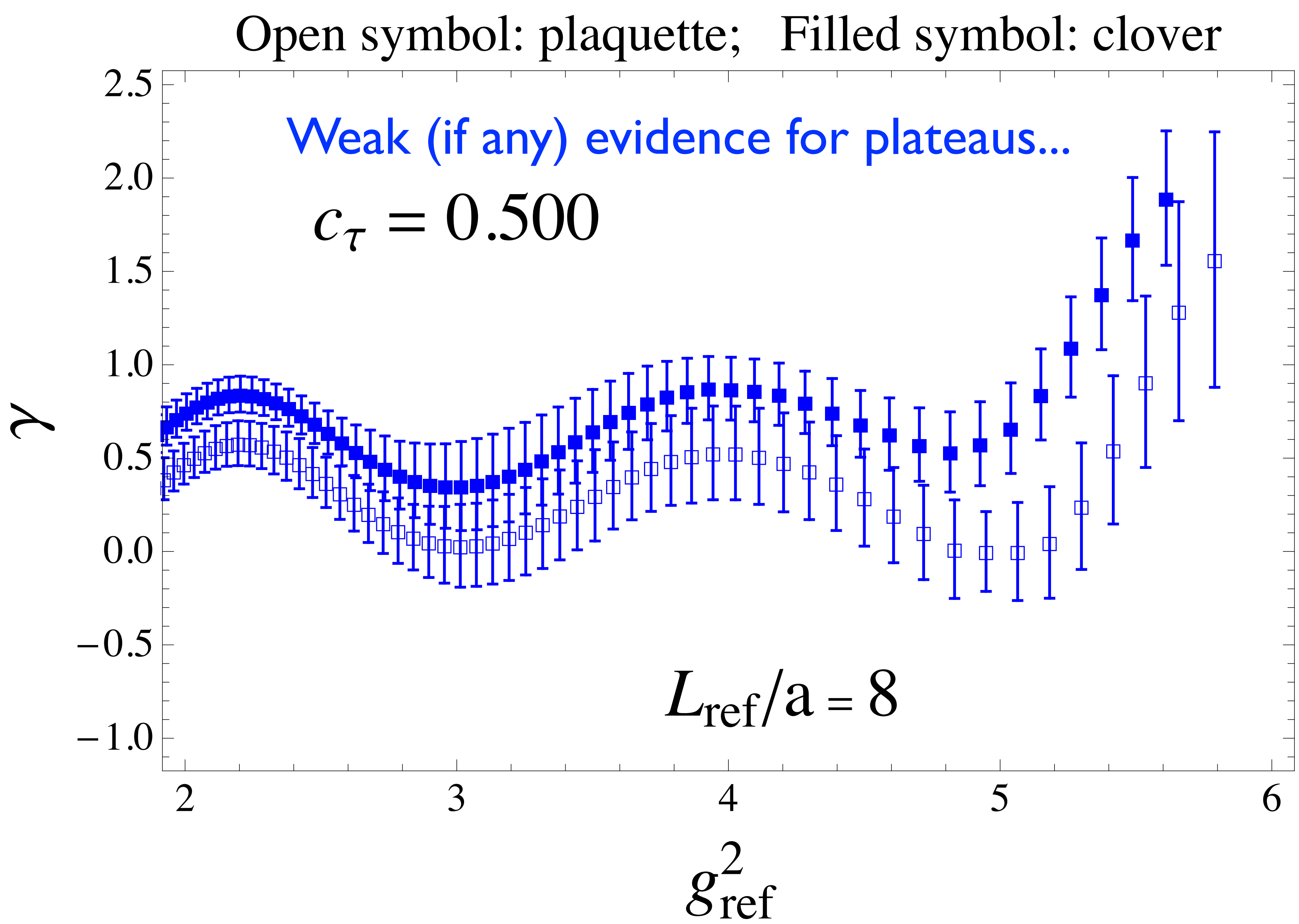
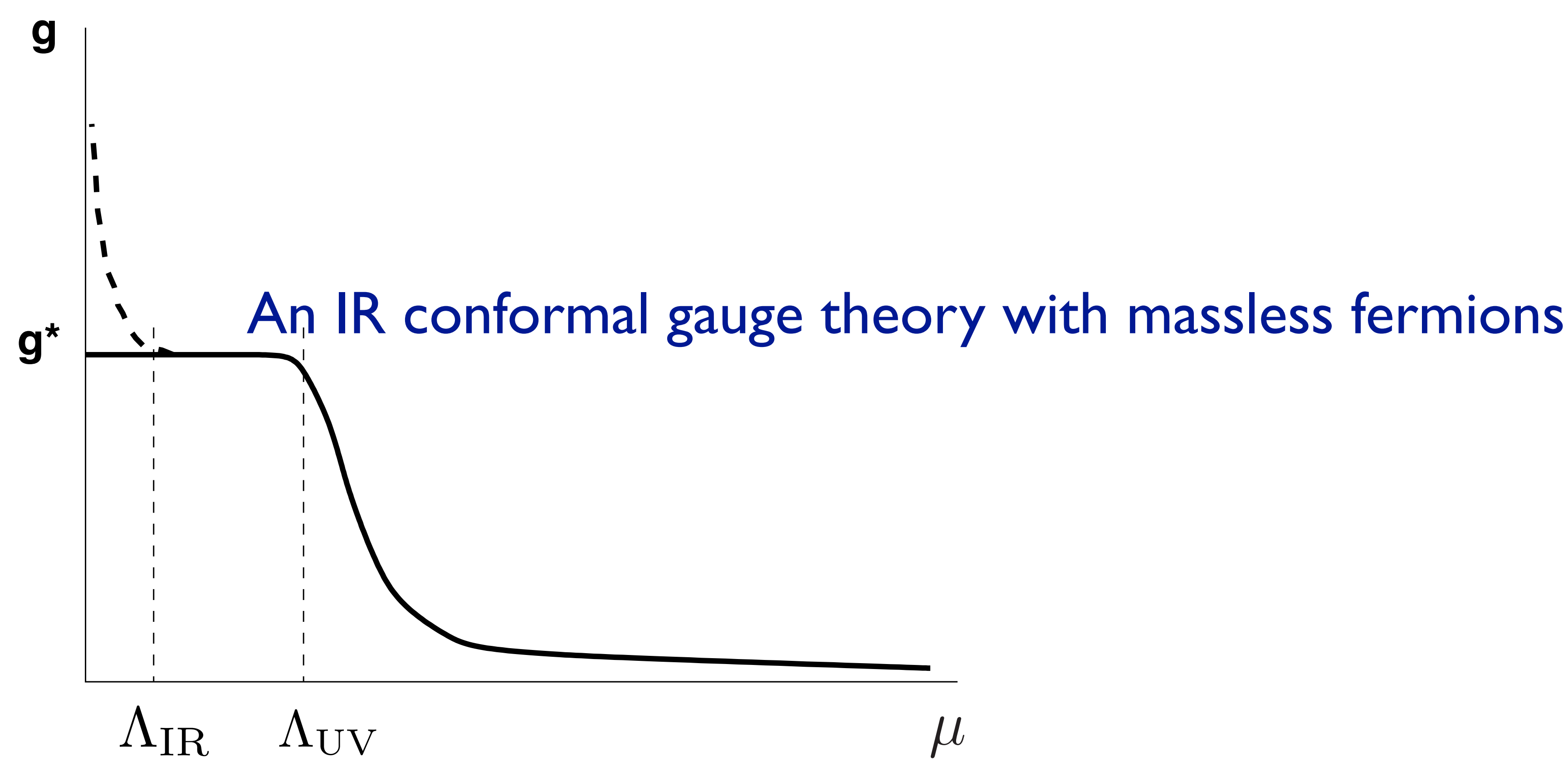


Figure from C.-J.D.L., K. Ogawa, A. Ramos, JHEP 12 (2015)

Deformation and scaling

Deformation of a strongly-coupled IRFP

L. Del Debbio and R. Zwicky, PRD 82, 2010



★ Deform the theory by introducing a relevant operator

- ➡ Break IR scale invariance at the scale $\Lambda_{\rm IR}$

★ A popular approach is "mass deformation"

- ➡ Introduce the operator $m\bar{\psi}\psi$
- ➡ Deformation scale $\Lambda_{\rm IR} = m$ ➡ Integrated out at $\mu < m$

Hyperscaling near a strongly-coupled IRFP

L. Del Debbio and R. Zwicky, PRD 82, 2010

★ Study the correlator near the mass-deformed IRFP

$$C_H(t; g, \hat{m}, \mu) = \int d^3x \, \langle H(t,x) H(0)^\dagger \rangle \big|_{g,\hat{m},\mu} \sim e^{-M_H t}, \quad \hat{m}(\mu) = m(\mu)/\mu.$$

★ The deformation operator is the fermion bilinear

$$\mu \frac{d}{d\mu} \left(\bar{\psi}\psi\right)_\mu \approx -\gamma_* \left(\bar{\psi}\psi\right)_\mu , \quad \Delta_{\bar{\psi}\psi} = 3 - \gamma_*, \; y_m = 1 + \gamma_*$$

★ Under RG $\mu = b\mu'$ near the IRFP $\hat{m}' = b^{y_m} \hat{m}$

$$C_H(t; g_*, \hat{m}, \mu) = b^{-2\gamma_H} C_H(t; g_*, \hat{m}', \mu') = b^{3-2\gamma_H-2d_H} C_H(tb^{-1}; g_*, \hat{m}', \mu)$$

★ Choosing b such that $\hat{m}' = 1$

➡ $C_H(t; \hat{m}, \mu) = \mathcal{C}_H F(t\hat{m}^{1/(1+\gamma_*)}, \mu)$ ➡ $M_H \simeq c_H \mu \, \hat{m}^{\frac{1}{1+\gamma_*}}$

Hyperscaling near a strongly-coupled IRFP

L. Del Debbio and R. Zwicky, PRD 82, 2010

★ All "hadron" masses vanish in the chiral limit

➡ And they all vanish in a "universal" way

★ Can derive similar scaling relations for other quantities

➡ Exercise: derive hyperscaling formula for the pion decay constant

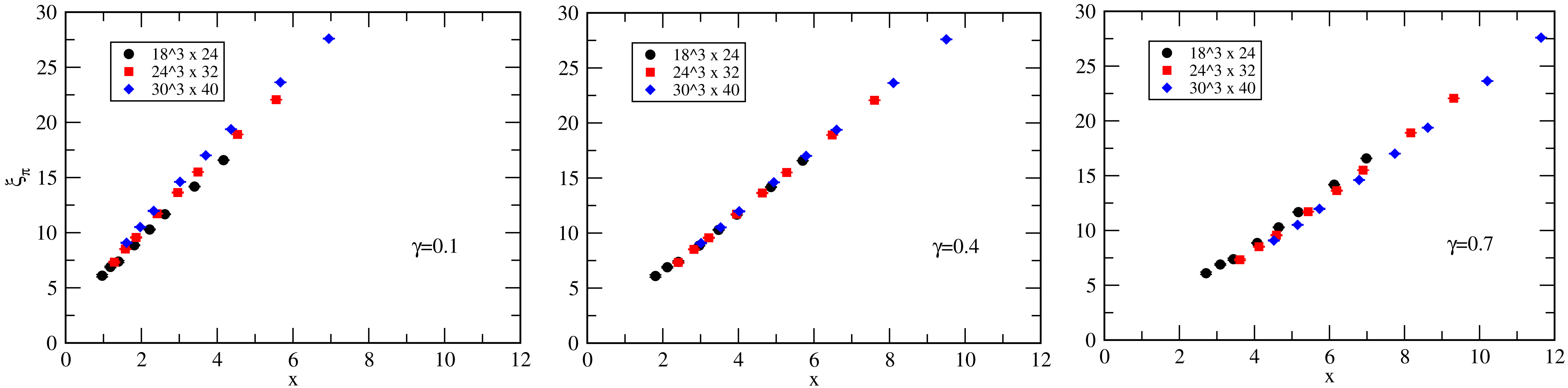
★ Can derive relations for finite-size scaling

➡ The scaling variable is $x = \hat{L}\hat{m}^{1/y_m}$

Numerical tests for hyperscaling

12-flavour QCD

LatKMI Collaboration, PRD 86, 2012



$$\xi_\pi = LM_\pi,\ x = \hat{L}\hat{m}^{(1/1+\gamma_*)}$$

★ Evidence of hyperscaling with a small anomalous dimension

→ Similar value of γ_* found for other quantities and at another lattice spacing

~15%

What we know for sure:

12-flavour QCD has a very small β-function

(This is the origin of the challenge)

The purpose of such research programme

★ Must we know the exact location of the conformal window?

- It is definitely an interesting field-theory question
- It is very challenging to deal with theories with small β-functions
- Deserves further hard work and new ideas

★ Which features would be interesting to WTC model builders?

- A scalar state that is much lighter than others
- Large anomalous dimension for $\bar{\psi}_{TC}\psi_{TC}$
- Look for theories containing the dilaton

Which theories for spectrum studies?

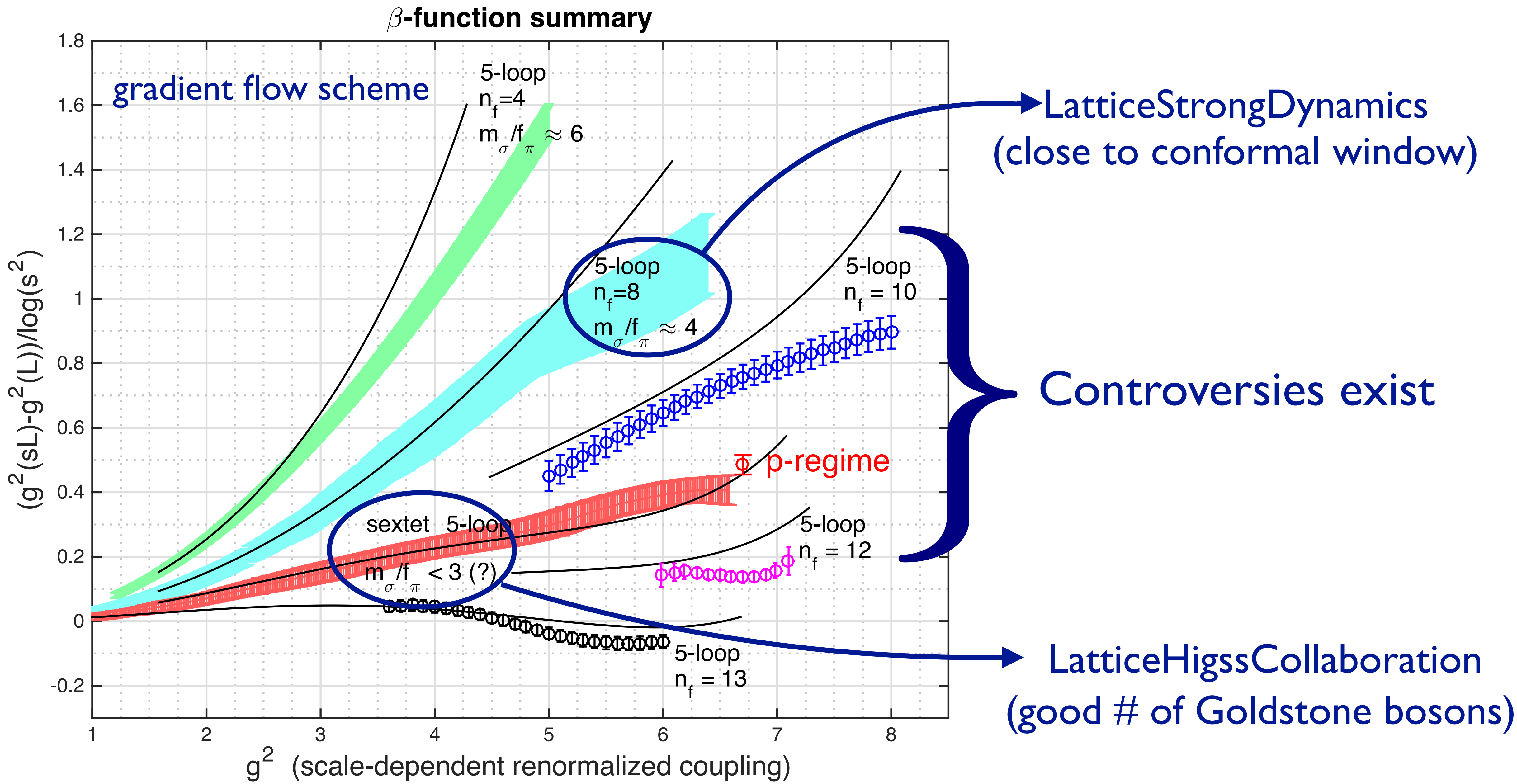
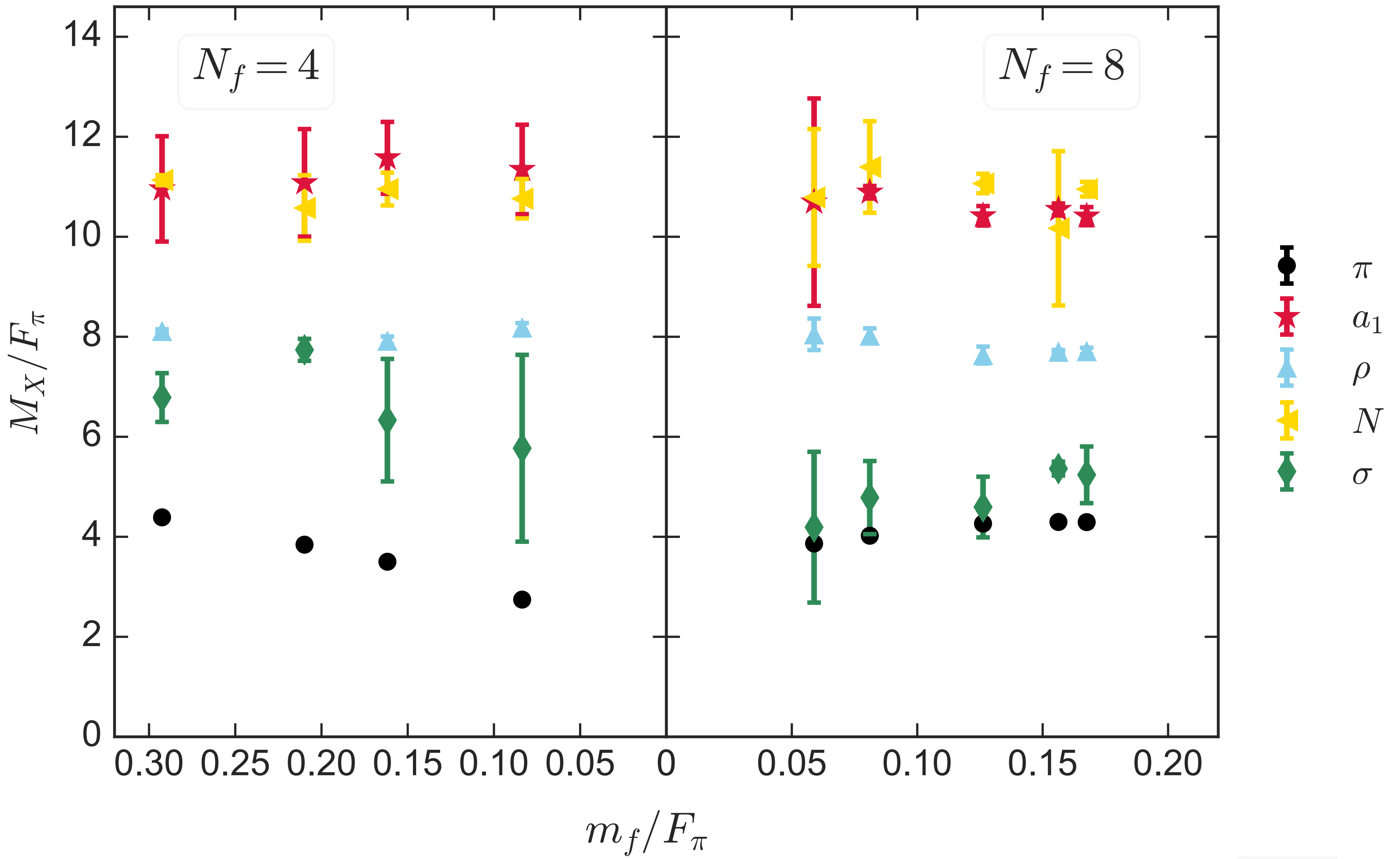


Figure from Kieran Holland, LATTICE 2019

Latest spectrum results

4- and 8-flavour QCD

LSD Collaboration, PRD 99, 2019

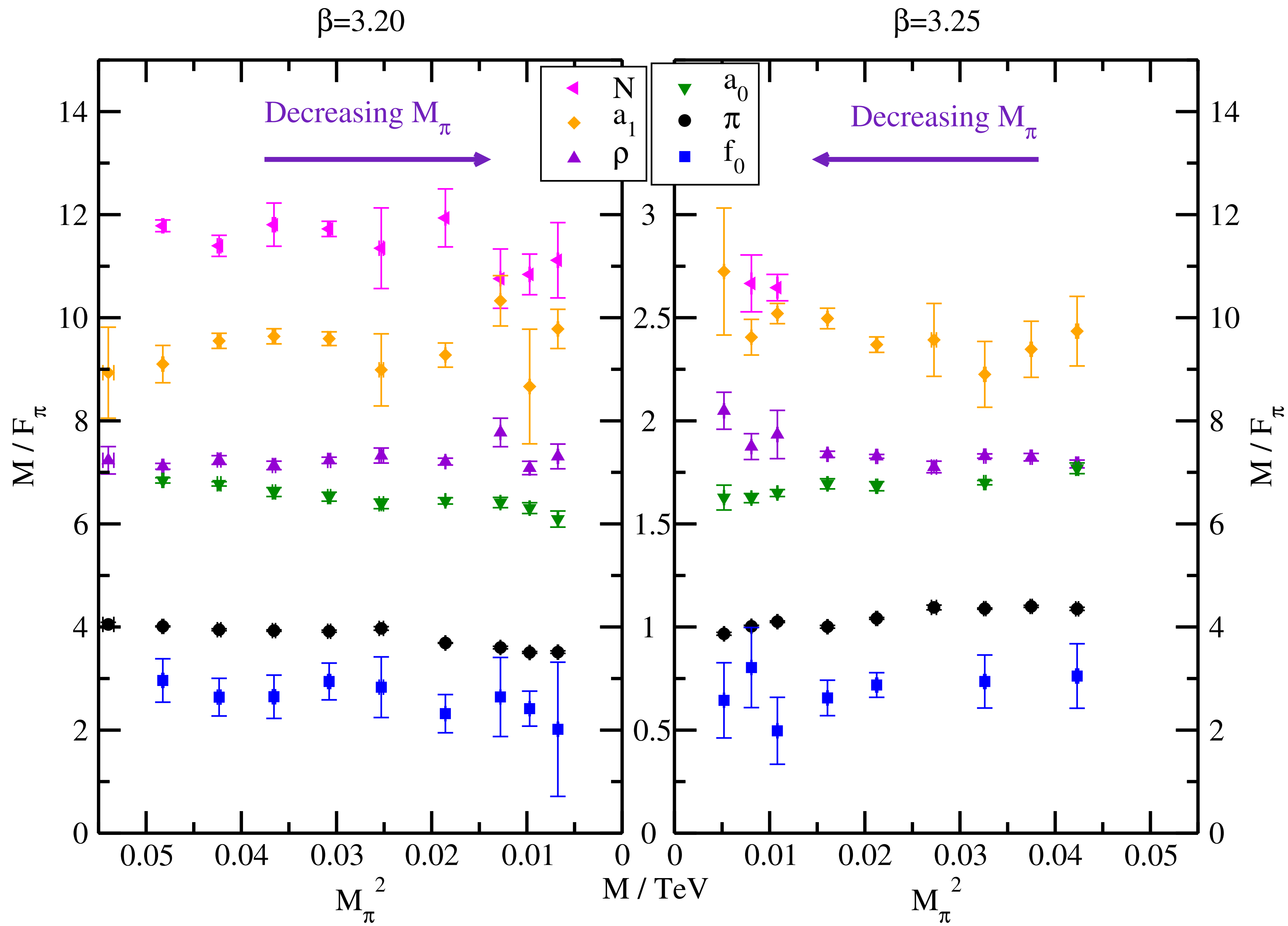


★Evidence for scalar getting lighter at increasing N_f

Latest spectrum (preliminary) results

SU(3) gauge theory with 2 flavours of sextet fermions

LHC Collaboration, arXiv:1605.08750 (LATTICE2015)



★ Evidence for light scalar

Spectrum and EFT

Including the light scalar in the EFT