Physics beyond the Standard Model and lattice calculations:

Higgs physics, the origin of mass and lattice field theory

Lecture 1

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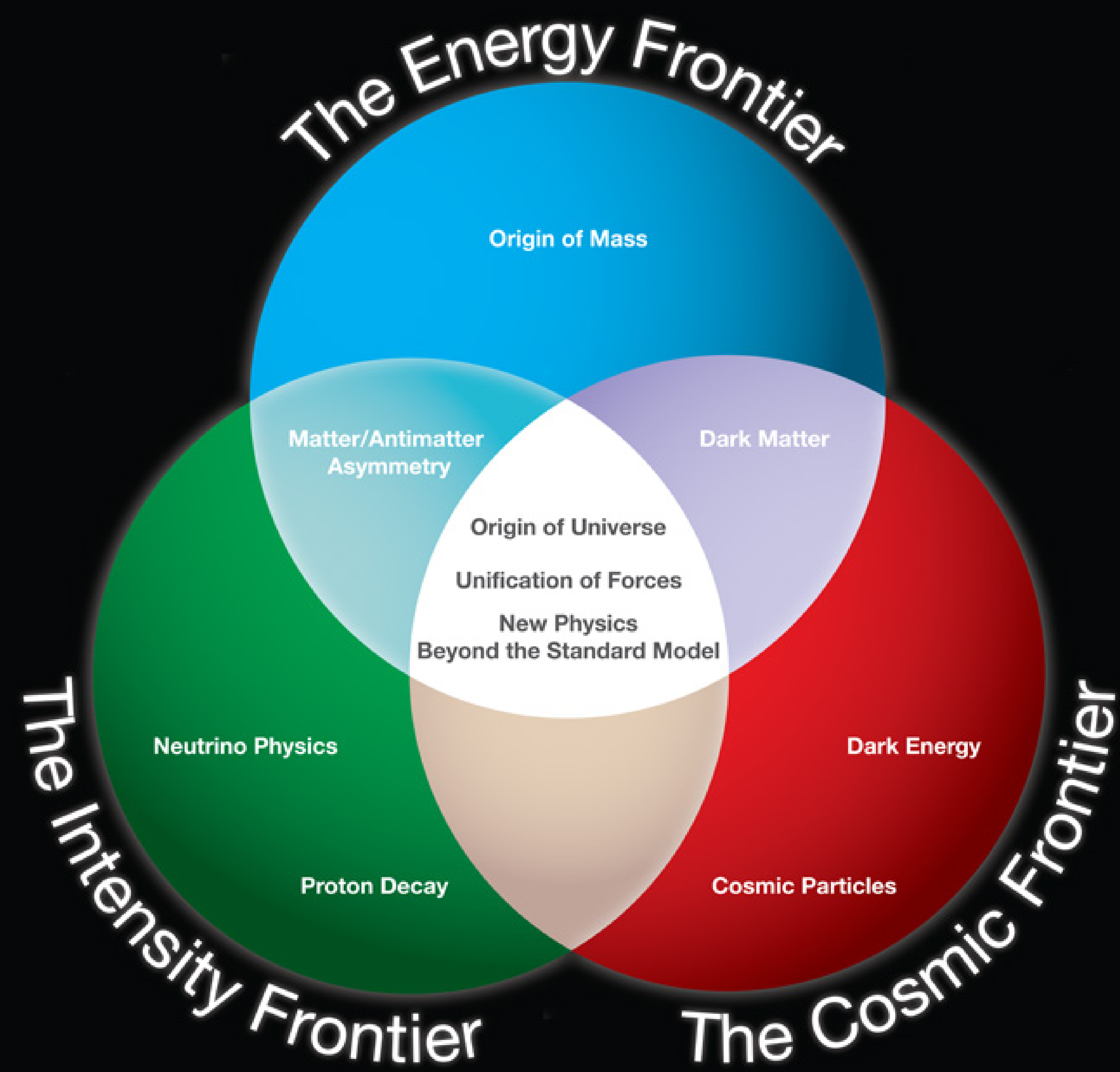


Figure taken from arXiv:1205.2671

Goal of this course

★ Motivate the study and strategy

★ Introduce the methods

★ Describe the basics of possibly viable scenarios

➡ Focus on dilaton Higgs and Goldstone Higgs (with partial compositeness)

★ Formulate relevant field-theory questions

➡ Questions we can realistically help to answer using LFT

Disclaimer

Results presented here may not be the most up-to-date, and they are selected for the purpose of illustration

Quest for BSM physics

A Wilsonian, non-perturbative point of view

What the LHC revealed to us hitherto

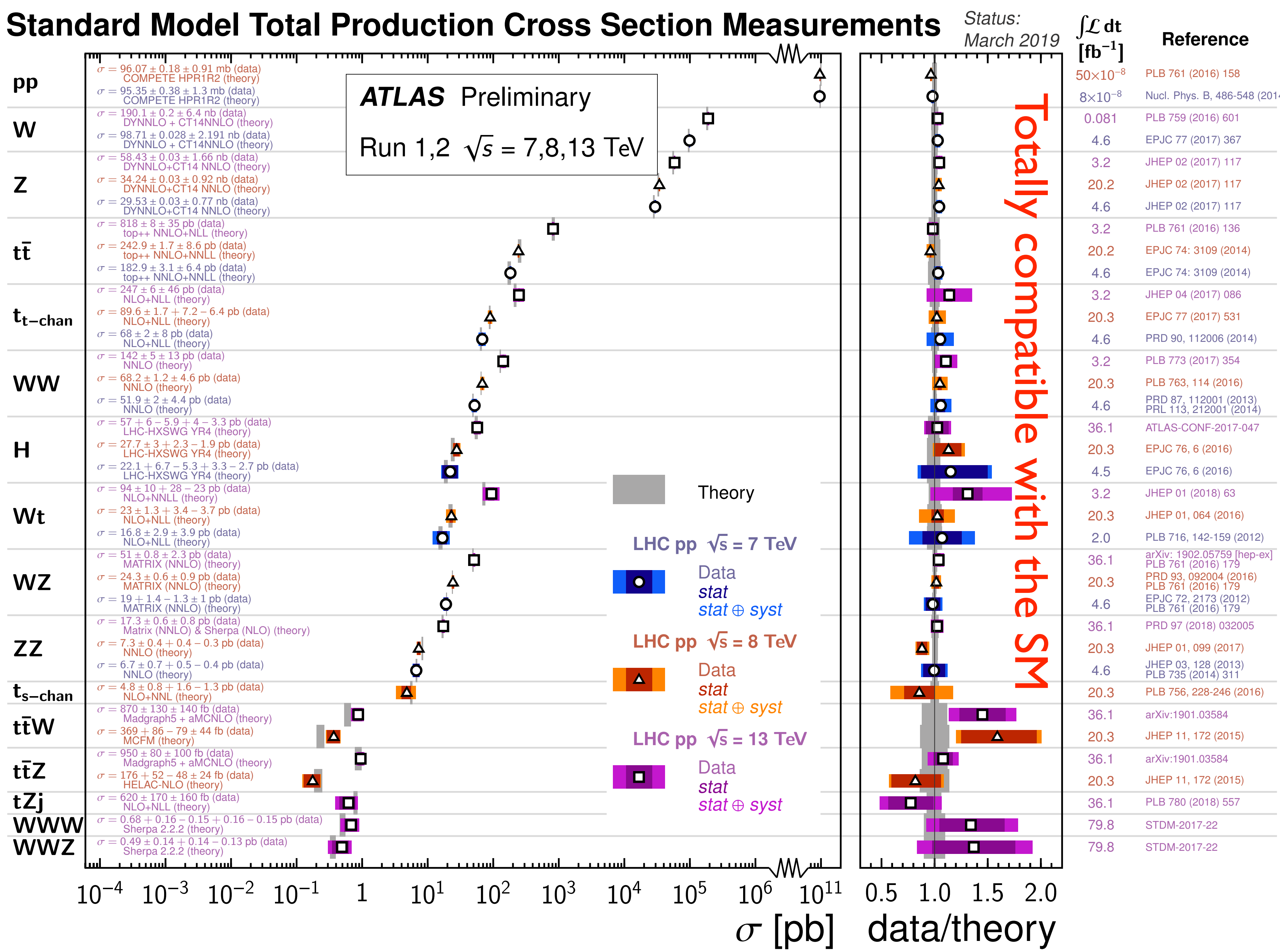
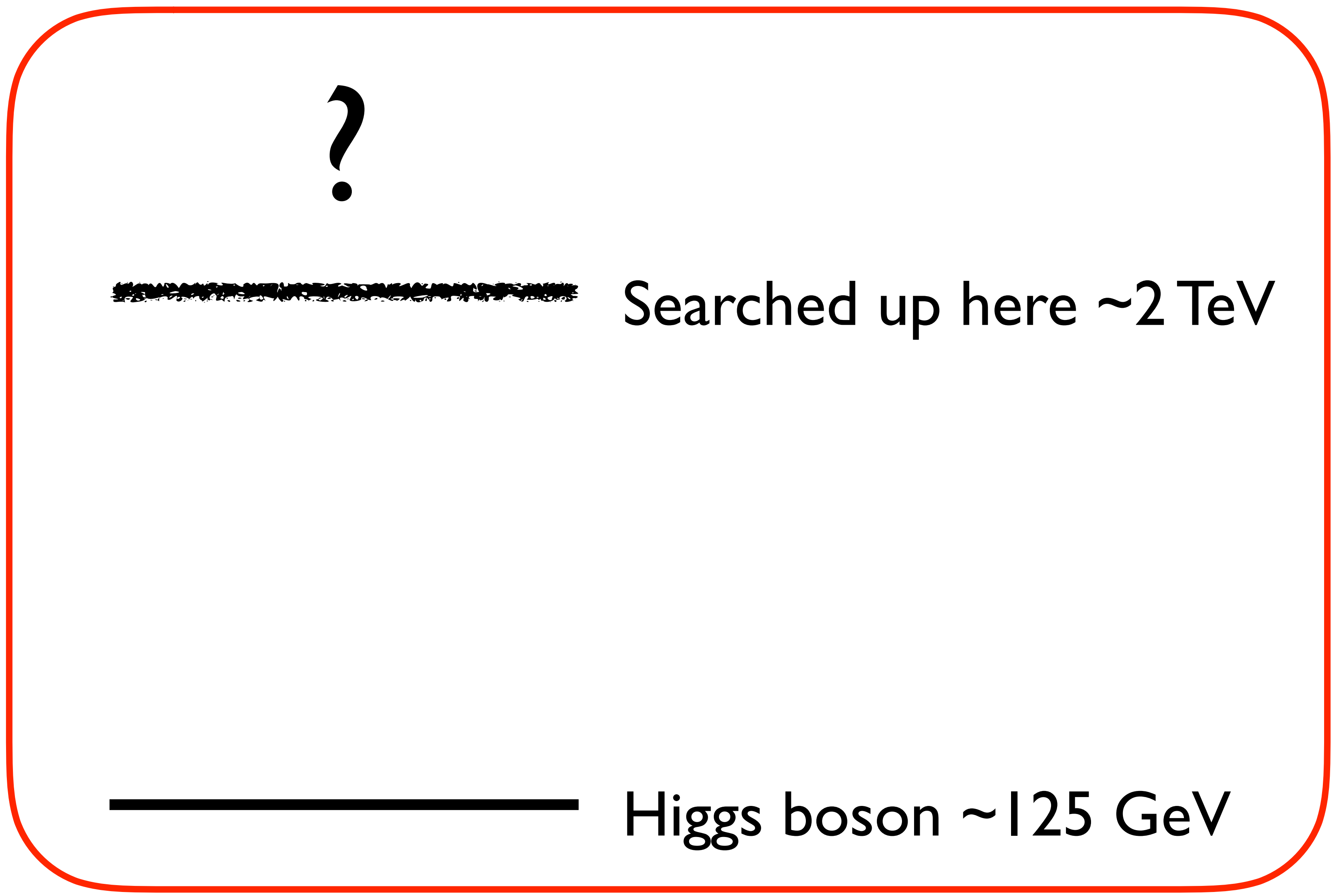


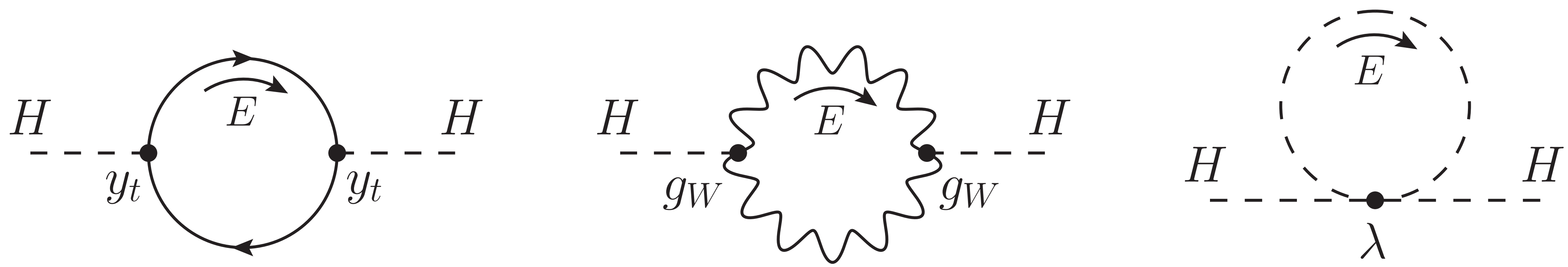
Figure taken from ATL-PHYS-PUB-2019-010

What the LHC revealed to us hitherto



The Higgs boson is light

Naturalness: one-loop perturbation theory



$E < \Lambda_{\rm UV}$

- ★ No symmetry protection for the Higgs mass term
  - ➡ Results in an additive Higgs mass renormalisation $\Lambda_{\rm UV}^2 H^\dagger H$
- ★ Assuming $\Lambda_{\rm UV} \sim M_{\rm GUT}$
  - ➡ Need to fine-tune $c \sim 10^{28}$ in $cM_H^2 H^\dagger H$

Fine-tuning: Is it a problem?

In principle, it is *not* a problem

- ★ Power-law dependence on $\Lambda_{\rm UV}$ hints nonperturbative nature
  - ➨ Recall that Dim-Reg only gives logarithmic scale dependence
  - ➨ Logarithmic scale dependence is a signature of perturbation theory
- ★ Lessons from Wilson fermions on the lattice
  - ➨ Chiral symmetry ensures fermion mass is multiplicatively renormalised
  - ➨ The Wilson term breaks chiral symmetry
  - ➨ Additive fermion mass renormalisation $m_c \sim 1/a$
  - ➨ Nonperturbative subtraction, and the continuum limit can be taken

Fine-tuning is fine

- ★ What is fine-tuned is a bare parameter
  - ➡ It may be practically challenging, but so what?
- ★ The cut-off can still be removed
- ★ Low-energy physics is insensitive to that at the cut-off scale
- ★ So why are we searching for BSM physics?

Triviality: hints from perturbation theory

★ The 1-loop quartic-coupling beta function of scalar theory

$$\beta(\lambda) = \mu \frac{d\lambda}{d\mu} = A_+ \lambda^2 \,, \ \text{where } A_+ > 0$$

➡ λ increases with μ

★ “Inconsistency” upon integrating the RGE

$$\int_{\lambda_{\rm IR}}^{\lambda_{\rm UV}} \frac{d\lambda}{A_+ \lambda^2} = \ln\left(\frac{\Lambda_{\rm UV}}{\mu_{\rm IR}}\right)$$

cut-off

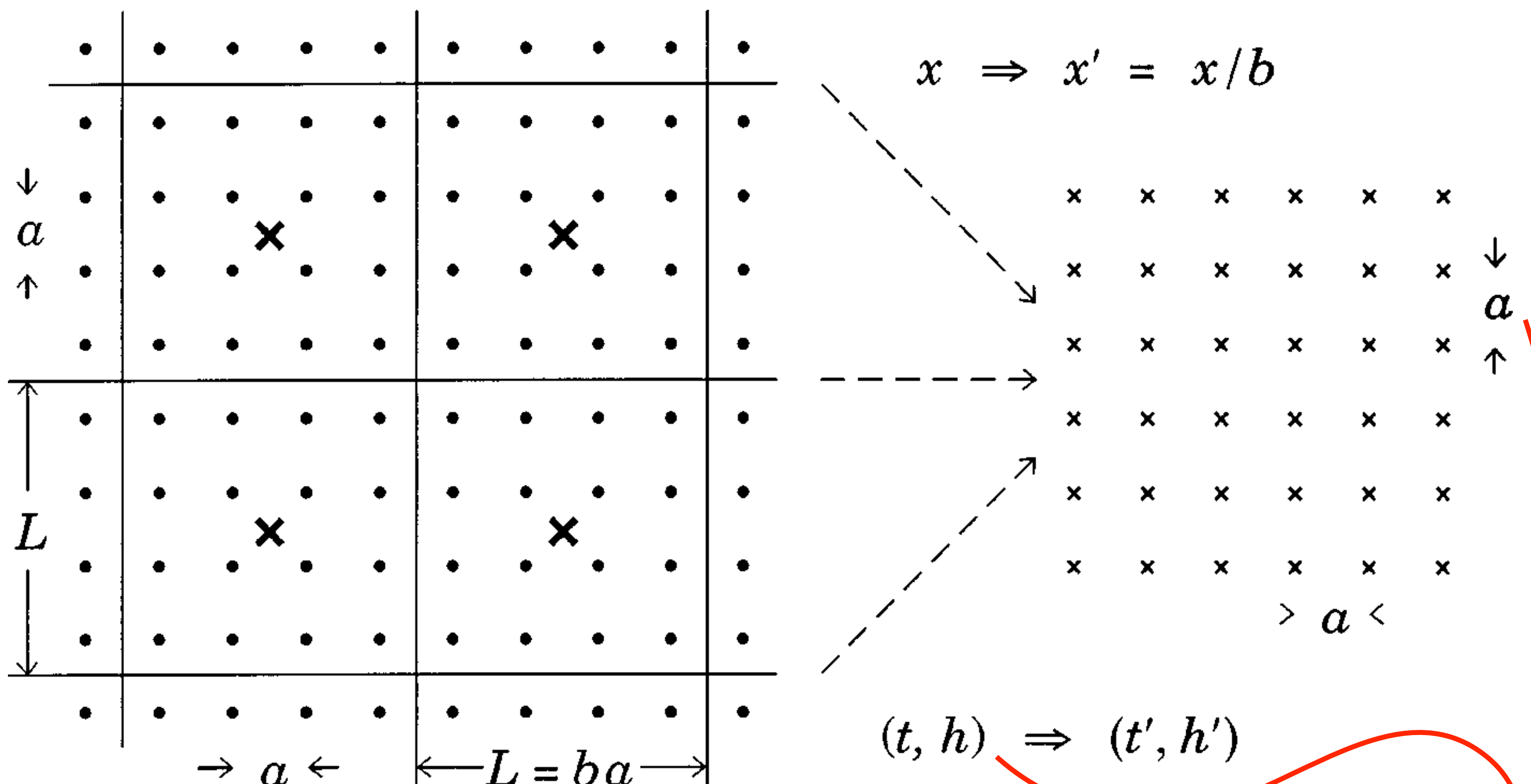
➡ Either keep the cut-off finite (EFT)

➡ Or the renormalised coupling vanishes at all IR scales (Triviality)

A digression: RG and critical phenomena

Coarse-graining and rescaling

Figure from M. Fisher, Rev. Mod. Phys. 70 (1998)



★ *c.f.,*
$$\Gamma[p_i, g_i(\mu), \mu] = \exp\left(-\int_{\mu/b}^{\mu} \gamma_\Gamma \; d\ln\mu'\right) \Gamma[p_i, g_i(\mu/b), \mu/b]$$
$$= b^{-d_\Gamma} \exp\left(-\int_{\mu/b}^{\mu} \gamma_\Gamma \; d\ln\mu'\right) \Gamma[bp_i, b^{d_i} g_i(\mu/b), \mu]$$

A digression: RG and critical phenomena

★ $\{K\} \overset{\text{RG}}{\Longrightarrow} \{K'\} \overset{\text{RG}}{\Longrightarrow} \{K''\} \ldots$ ➡ Criticality: $\{K\} \overset{\text{RG}}{\Longrightarrow} \{K\} \equiv \{K^*\}$

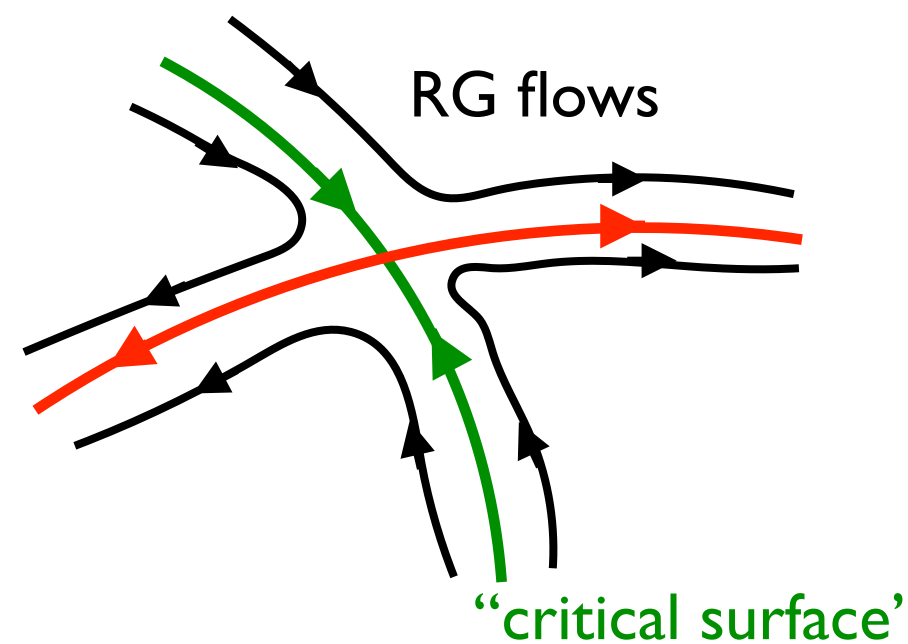
★ Linearisation: $K'_\alpha - K^*_\alpha \sim \sum_\beta \left(\frac{\partial K'_\alpha}{\partial K_\beta} \right)_{K=K^*} (K_\beta - K^*_\beta) = \sum_b T_{\alpha\beta} \, (K_\beta - K^*_\beta)$

★ From $\sum_\alpha \phi^i_\alpha T_{\alpha\beta} = b^{y_i} \phi^i_\beta$ define $u_i \equiv \sum_\alpha \phi^i_\alpha (K_\alpha - K^*_\alpha)$

★ Scaling variables: $u'_i = \sum_\alpha \phi^i_\alpha (K' - K^*_a) = \sum_{\alpha,\beta} \phi^i_\alpha T_{\alpha\beta} (K_\beta - K^*_\beta) = b^{y_i} u_i$

relevant	$y_i > 0$
irrelevant	$y_i < 0$
marginal	$y_i = 0$

"scheme" independent



A digression: RG and critical phenomena

Critical phenomena	(4d) Field theory fixed point
$\xi/a \to \infty$	$\Lambda_{\rm UV}/\Lambda_{\rm IR} \to \infty$
$\{K\} \stackrel{\rm RG}{\Longrightarrow} \{K'\} = \{K\} \equiv \{K^*\}$	$g_i(\mu) \stackrel{\rm RG}{\Longrightarrow} g_i(b\mu) = g_i(\mu) \equiv g_i^*$
Scaling variable	Operator ($\Lambda_{\rm UV}^{d_{\mathcal{O}}-4} \times \mathcal{O}$ in the action)
Irrelevant	$d_{\mathcal{O}} + \gamma_{\mathcal{O}} > 4$
Relevant	$d_{\mathcal{O}} + \gamma_{\mathcal{O}} < 4$
Marginal	$d_{\mathcal{O}} + \gamma_{\mathcal{O}} = 4$

Nonperturbative statement for triviality

- ★ Consider a scalar theory $(\hat{m} = am,\ \hat{\phi} = a\phi\ )$

$$S = \int d^4x\ \frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{1}{2}m_0^2\phi^2 + \lambda_0\phi^4$$

$$\xrightarrow{\text{latticise}} \sum_n \left[\sum_{\hat{\mu}} \frac{1}{2}\hat{\phi}(n)\left(\hat{\phi}(n+\hat{\mu}) + \hat{\phi}(n-\hat{\mu}) - 2\hat{\phi}(n)\right) + \frac{1}{2}\hat{m}_0\hat{\phi}^2(n) + \lambda_0\hat{\phi}^4(n)\right]$$

- ★ The limit $\Lambda_{\rm UV} \to \infty$ is the continuum limit, $a \to 0$
  - ➡ Occurs at 2nd order bulk phase transition
  - ➡ $\lambda_0 \neq 0$ and $\hat{m}_0 \neq 0$ while $\lambda_{\rm R} \to 0$ and $\hat{m}_{\rm R} \to 0$

logarithmically

- ★ "Scanning" required
  - ➡ Compared to QCD continuum limit: $g_0 \to 0$ and $\hat{m}_f \to 0$

Triviality in scalar theories

★ There is much reliable evidence for triviality in 4d scalar theories

M. Aizenman, PRL. 47 (1981)

J. Fröhlich, NPB 200 (1982)

M. Luscher and P. Weisz, PLB212 (1988), NPB 290 (1987), 295 (1988), 318 (1989)

M. Hoogervorst and U. Wolff, NPB 855 (2012)

J. Sievert and U. Wolff, PLB 733 (2014)

T. Korzec and U. Wolff, PoS LATTICE2014 (2015)

⋮

★ Presently there is very little doubt about it

The issue is:

No relevant interaction in the scalar sector

Physics beyond the SM: higher-dim operators

★ Keeping Λ_{UV} finite

➡ Higher-dim (irrelevant) operators can contribute

★ Many operators…

d	quantum numbers	$n_g = 1$	$n_g = 3$
5	$(\Delta L = 2) + \text{h.c.}$	$1 + 1$	$6 + 6$
6	$\Delta B = \Delta L = 0$	$76 = 53_+ + 23_-$	$2499 = 1350_+ + 1149_-$
6	$(\Delta B = \Delta L = 1) + \text{h.c.}$	$4 + 4$	$273 + 273$

★ How can the lattice help?

➡ Challenging to precisely constrain all couplings from experiments

➡ Lattice study of qualitative features, for example…

Physics beyond the SM: higher-dim operators

★ Consider a scalar field theory

$$S^{\rm cont}[\bar{\psi},\psi,\varphi] = \int d^4x \left\{ \frac{1}{2}\left(\partial_\mu\varphi\right)^\dagger\left(\partial^\mu\varphi\right) + \frac{1}{2}m_0^2\varphi^\dagger\varphi + \lambda\left(\varphi^\dagger\varphi\right)^2 + \lambda_6\left(\varphi^\dagger\varphi\right)^3 \right\}$$

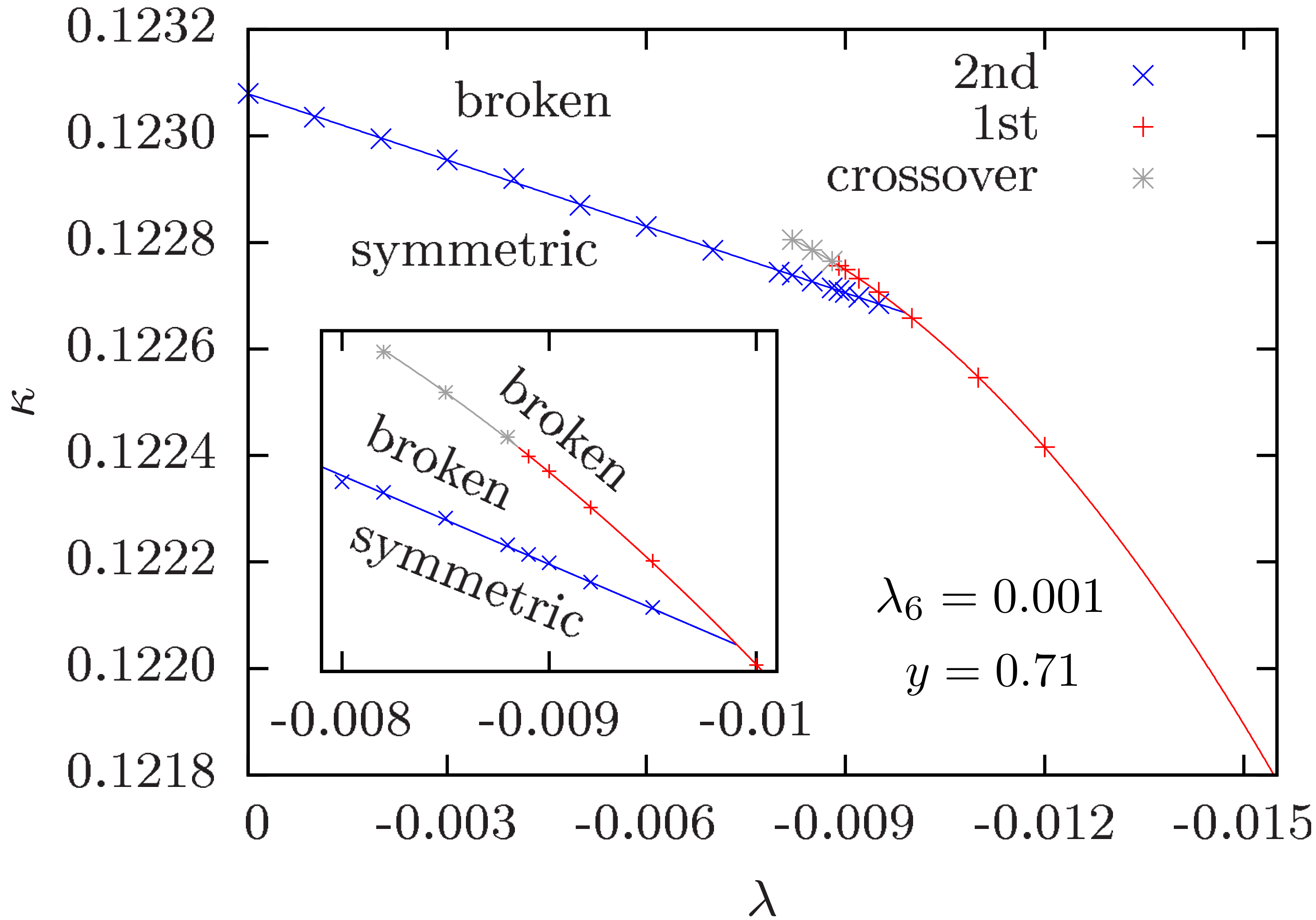
$$+ \int d^4x \left\{ \bar{t}\partial\!\!\!/ t + \bar{b}\partial\!\!\!/ b + y\left(\bar{\psi}_L\varphi\, b_R + \bar{\psi}_L\tilde{\varphi}\, t_R\right) + h.c. \right\}$$

$$\tilde{\varphi} = i\tau_2\varphi^*$$

$$\varphi = \sqrt{2\kappa}\begin{pmatrix} \Phi^2 + i\Phi^1 \\ \Phi^0 - i\Phi^3 \end{pmatrix}, \quad a^2 m_0^2 = \frac{1-2\hat{\lambda}-8\kappa}{\kappa}, \quad \lambda = \frac{\hat{\lambda}}{4\kappa^2}$$

Physics beyond the SM: higher-dim operators

D.Y.-J. Chu, K. Jansen, B. Knippschild, C.-J. D. Lin, A. Nagy, PLB 744 (2015)



★ Notice: Negative quartic coupling

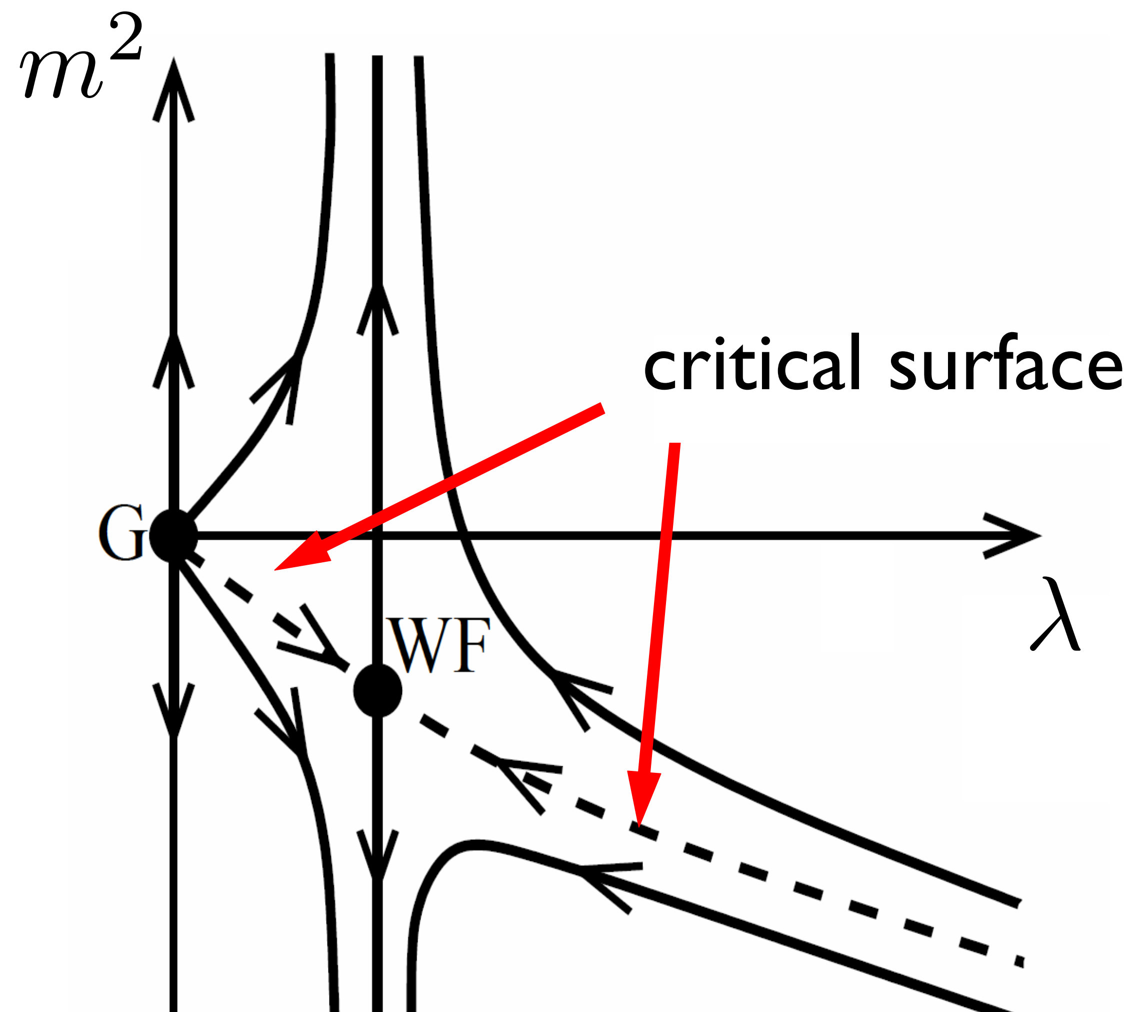
➡ Presence of first order phase transitions; finite temperature?

Searching for relevant interaction:

Looking inside the Higgs-Yukawa sector of the SM

Scalar theories are not always trivial

The Wilson-Fisher fixed point in 3-dimensional scalar field theory



Triviality in the Higgs-Yukawa sector

★There is much reliable evidence for triviality in 4d scalar theories

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T. Korzec and U. Wolff, PoS LATTICE2014 (2015)

⋮

★Evidence for 4d Higgs-Yukawa model is not a the same level

Early works by Lee, Shigemitsu, Shock, Haenfratz, Jansen,…

N. Butt, S. Catterall, D. Schaich, PRD98(2018)

D.Y.-J. Chu, K. Jansen, B. Knippschild, C.-J. D. Lin, JHEP 1901(2019)

Phase structure of Higgs-Yukawa sector

$$a_y \equiv \frac{y^2}{(4\pi)^2}\ , \qquad a_\lambda \equiv \frac{\lambda}{(4\pi)^2}$$

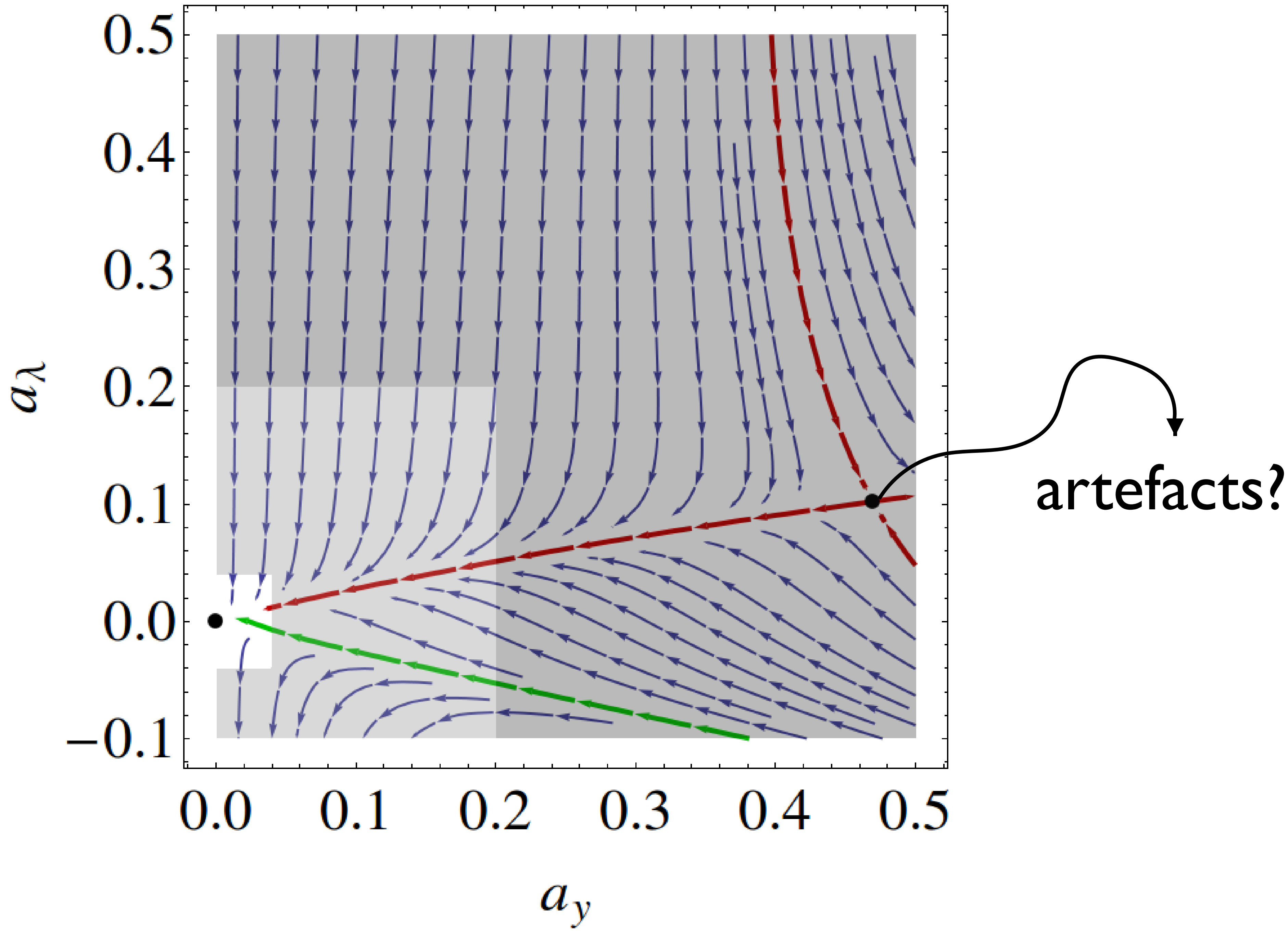
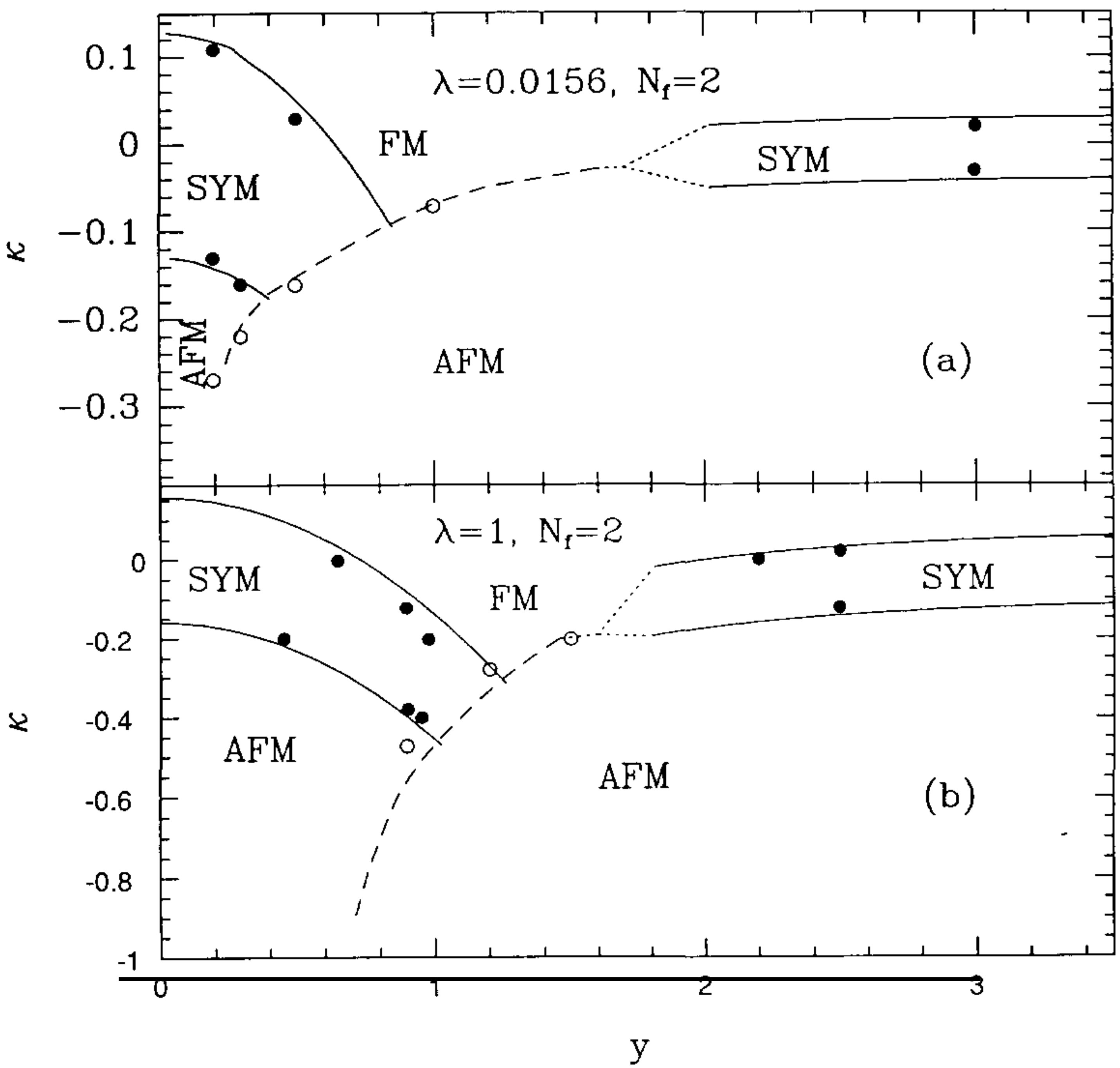


Figure from E. Molgaard and R. Shrock, PRD 89 (2014)

Early work on the bulk phase structure



A. Hasenfratz, K. Jansen, Y. Shen, NPB 394 (1993)

Finite-size scaling *àla* renormalisation group

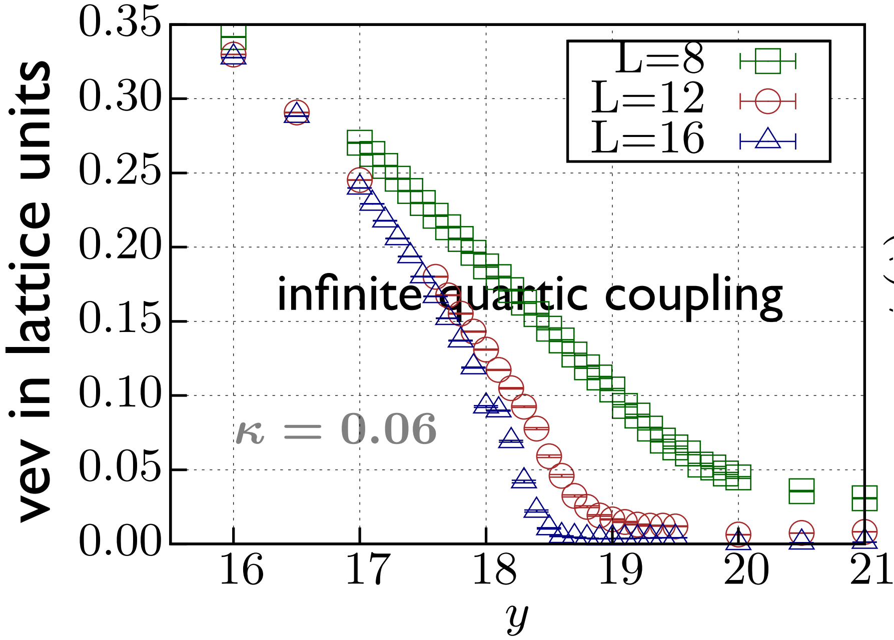
$$
\begin{aligned}
Z_M(\tilde{a},\tilde{l}) \times \tilde{M}_0[\tilde{m}_0^2,\{g_{0,i}\};\tilde{a},\tilde{L}] &= \tilde{M}[\tilde{m}^2(\tilde{l}),\{g_i(\tilde{l})\};\tilde{l},\tilde{L}] \\
&= \zeta_M(\tilde{l},\tilde{L}) \times \tilde{M}[\tilde{m}^2(\tilde{L}),\{g_i(\tilde{L})\};\tilde{L},\tilde{L}] \\
&= \zeta_M(\tilde{l},\tilde{L}) \times \tilde{L}^{-d_M} \times \tilde{M}[\tilde{m}^2(\tilde{L})\tilde{L}^2,\{g_i(\tilde{L})\};1,1]
\end{aligned}
$$

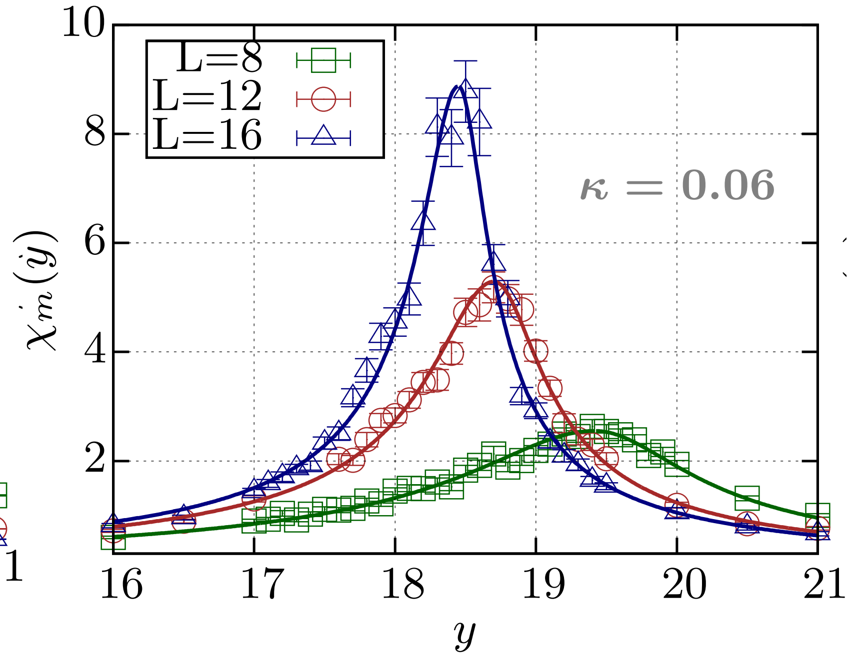
$$
\zeta_M(\tilde{l},\tilde{L}) = \exp\left[\int_{\tilde{l}}^{\tilde{L}} \gamma_M(\rho) d\ln\rho\right], \quad \tilde{m}^2(\tilde{L}) = \tilde{m}^2(\tilde{l}) \exp\left[\int_{\tilde{l}}^{\tilde{L}} \gamma(\rho) d\ln\rho\right]
$$

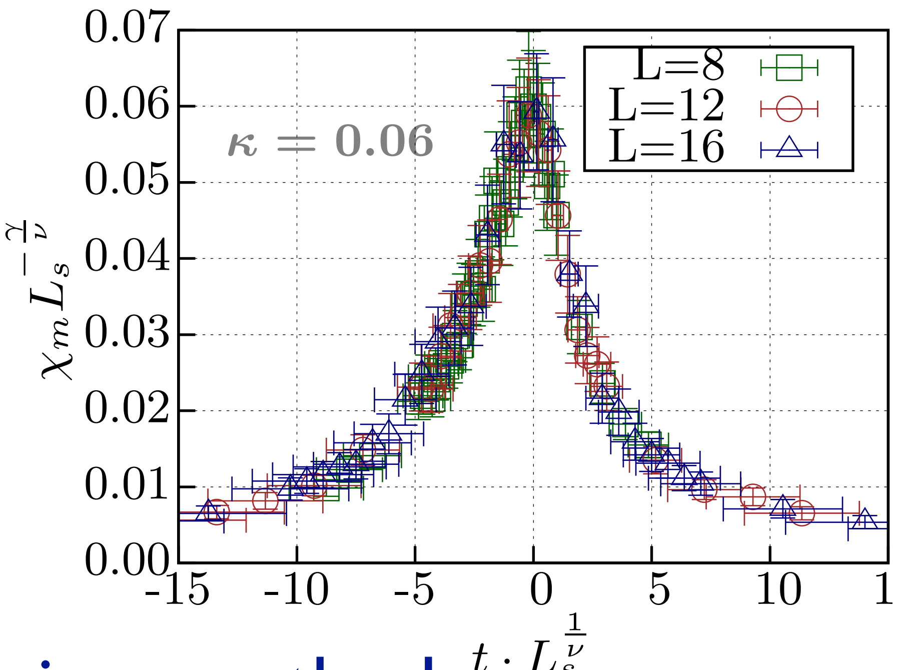
- ★ Near a fixed point, $g_i(\tilde{L}) \approx g_i^*$
  - ➡ γ_M and γ are constant
  - ➡ $\zeta_M = \left(\frac{\tilde{L}}{\tilde{l}}\right)^{\gamma_M^*}, \quad \tilde{m}^2(\tilde{L}) = \tilde{m}^2(\tilde{l})\left(\frac{\tilde{L}}{\tilde{l}}\right)^{1/\nu-2}$
  - ➡ $\tilde{L}^{d_M-\gamma_M} \times \tilde{M} = C_M F(\tilde{m}^2 L^{1/\nu})$ (unknown function in general)
- ★ Mean-field result: $\nu = 1/2$

What we found previously

notice: strong (bare) Yukawa coupling







The curve-collapsingg method

$t \sim \hat{m}_0^2 - \hat{m}_{\rm crit}^2$

Results from Binder's cumulant with a curve-collapse method

	$T_c^{(L=\infty)}$	ν	interval
$\kappa = 0.06$	18.147(24)	0.550(1)	17.4, 18.8
$\kappa = 0.00$	16.667(27)	0.525(6)	16.0, 17.2
O(4)	0.3005(34)	0.50000(3)	0.294, 0.314

J. Bulava *et al.*, AHEP 2013

mean-field? → logarithmic corrections needed

Finite-size scaling *àla* renormalisation group

Issues with Gaussian fixed point in 4 dimensions

★ Mean-field approach does not work

➡ Need to include logarithmic corrections

★ Exact mean-field scaling laws with leading-log corrections

➡ Pure scalar field theory

E. Brezin and J. Zinn-Justin, NPB 257 (1985)

➡ Higgs-Yukawa model

D.Y.-J. Chu, K. Jansen, B. Knippschild, C.-J. D. Lin, JHEP 1901(2019)

Mean-field FSS for 4d scalar O(N) model

E. Brezin and J. Zinn-Justin, NPB 257 (1985)

O(N) scalar field theory with quartic coupling

$$\Phi^T = (\phi_1, \cdots, \phi_N), \quad Z = \int D\Phi \, \exp \, (-S[\Phi])$$

$$\phi_a = \varphi_a + \chi_a$$

zero mode

$$Z = \int_{-\infty}^{\infty} \mathrm{d}^N \varphi_a \, \mathcal{N} \exp(-S_{eff}[\varphi_a]) = \Omega_{N-1} \int_0^{\infty} \mathrm{d}\varphi \, \varphi^{N-1} \mathcal{N} \exp(-S_{eff}[\varphi])$$

non-Gaussian modes of χ_a → decouples at one-loop

$$\exp(S_{eff}[\varphi]) = \underbrace{\det(M_B[\varphi])^{-1}}\exp\left(-sL^4 \frac{1}{2} M^2 \varphi^2 - sL^4 \lambda \varphi^4\right)$$

renormalises the couplings, and generates $\underbrace{\text{higher-dimensional operators}}$.

dropped in perturbation theory

anisotropy

Mean-field FSS for 4d scalar O(N) model

E. Brezin and J. Zinn-Justin, NPB 257 (1985)

$$\boxed{\text{Rescale: } \varphi \to \left(sL^4\lambda(L^{-1})\right)^{-1/4}\varphi \equiv S^{-1/4}\varphi}$$

$$Z = \mathcal{N}\Omega_{N-1}S^{-N/4}\int_0^\infty d\varphi\,\varphi^{N-1}\exp\left(-\frac{1}{2}z\varphi^2-\varphi^4\right)$$

$$\equiv \mathcal{N}\Omega_{N-1}S^{-N/4}\bar{\varphi}_{N-1}(z),$$

Logarithms

➡ The scaling variable: $z = \sqrt{s}\,\overbrace{\hat{M}^2(L^{-1})\hat{L}^2\lambda(L^{-1})}\,^{-1/2}$

$$\bar{\varphi}_0 = \frac{\pi}{8}\exp\left(\frac{z^2}{32}\right)\sqrt{|z|}\left[I_{-1/4}\left(\frac{z^2}{32}\right)-\mathrm{Sgn}(z)\,I_{1/4}\left(\frac{z^2}{32}\right)\right],$$

$$\bar{\varphi}_1 = \frac{\sqrt{\pi}}{8}\exp\left(\frac{z^2}{16}\right)\left[1-\mathrm{Sgn}(z)\,\mathrm{Erf}\left(\frac{|z|}{4}\right)\right],\quad \bar{\varphi}_{n+2} = -2\frac{\mathrm{d}}{\mathrm{d}z}\bar{\varphi}_n$$

➡ Can compute $\langle\varphi^k\rangle$.

D.Y.-J. Chu, K. Jansen, B. Knippschild, C.-J. D. Lin, JHEP 1901(2019)

Then one has to numerically discern log scaling

That will be a challenging task ahead