Physics beyond the Standard Model and lattice calculations:

Higgs physics, the origin of mass and lattice field theory

Lecture III

10/07/2019 ~ 12/07/2019
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Spectrum and dilaton EFT

Including the light scalar in the EFT

Including the dilaton in the EFT

★ Basic ideas of "conformal compensator" explained in

S. Coleman, "*Aspects of Symmetry*"

★ Various similar methods formulated over time

➡ Incomplete list of works include

Migdal and Shifman, PLB 114 (1982)

⋮

W. Goldberger, B. Grinstein, W. Skiba, PRL 100 (2008)

⋮

S. Matsuzaki and K. Yamawaki, PRL 113 (2014)

M. Golterman and Y. Shamir, PRD 94 (2016)

T. Appelquist, J. Ingoldby and M. Piai, JHEP 1707 (2017), JHEP 1803 (2018)

⋮

★ I will follow the approach of Appelquist, Ingoldby and Piai

Including the dilaton in the EFT

General strategy

- Under the scale transformation $x^\mu \to e^\alpha x^\mu$
  - Operator with mass dimension d_i: $\mathcal{O}_i(x) \to e^{\alpha d_i}\mathcal{O}_i(e^\alpha x)$
  - Scalar field $\chi(x)$ in 4-d: $\chi(x) \to e^\alpha \chi(e^\alpha x)$
- Non-linear parameterisation: $\chi(x) = f_d\ e^{\sigma(x)/f_d}$
  - f_d is a low-energy constant
  - f_d is the value of $\chi(x)$ at the minimum of the potential $V(\chi)$
  - Can expand around the minimum
- $\mathcal{L}_\chi = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi$ is scale-invariant
  - The potential, $V(\chi)$, incorporates effects of scale-invariance breaking
- Conformal compensator for $\mathcal{L} = \sum_i g_i(\mu)\mathcal{O}_i(x)$
  - $g_i(\mu) \to g_i(\mu\chi/f_d)(\chi/f_d)^{4-d_i}$ works in the same way as spurions

Including the dilaton in the EFT

Dilaton chiral perturbation theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \mathcal{L}_\pi - \mathcal{L}_M - V(\chi) + \ldots$$

- ★ $\mathcal{L}_\pi = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^2 \mathrm{Tr}\left[\partial_\mu\Sigma(\partial^\mu\Sigma)^\dagger\right]$, $\Sigma = \exp\left[2i\pi/f_\pi\right]$, $\pi = \sum_a \pi^a T^a$
- ★ $\mathcal{L}_M = \frac{m_\pi^2 f_\pi^2}{4}\left(\frac{\chi}{f_d}\right)^y \mathrm{Tr}\left[\Sigma + \Sigma^\dagger\right]$, $m_\pi^2 = 2mB_\pi$, $B_\pi = \langle\bar\psi\psi\rangle/2f_\pi^2$
  - ➡ y is the scaling dimension of $\langle\bar\psi\psi\rangle$
- ★ $V(\chi)$: effects of breaking scale invariance
  - ➡ e.g., $V_1 = \frac{m_d^2}{2f_d^2}\left(\frac{\chi^2}{2} - \frac{f_d^2}{2}\right)$ (SM), $V_2 = \frac{m_d^2}{16 f_d^2}\chi^4\left(4\ln\frac{\chi}{f_d} - 1\right)$
- ★ Study the dependence on $m_\pi^2 = 2mB_\pi$ in physical quantities
  - ➡ in terms of 4 parameters: f_π, f_d, m_d^2, y

Use of the dilaton ChPT

T. Appelquist, J. Ingoldby and M. Piai, JHEP 1707 (2017), JHEP 1803 (2018)

★ Expand the nonlinear pion field

- $\mathcal{L}_M = \frac{N_f m_\pi^2 f_\pi^2}{2} \left(\frac{\chi}{f_d}\right)^y - \frac{m_\pi^2}{2} \left(\frac{\chi}{f_d}\right)^y \pi^a \pi^a + \ldots$

★ This modifies the dilaton potential

- $W(\chi) = V(\chi) - \frac{N_f m_\pi^2 f_\pi^2}{2} \left(\frac{\chi}{f_d}\right)^y$

★ Denote the value of χ at to minimum of $W(\chi)$ as F_d

- Need details of $V(\chi)$ to determine F_d in the EFT
- But it can also be related to the dilaton decay constant
- Computed from lattice to test EFT's.

Use of the dilaton ChPT

Information extracted without details of the dilaton potential

T. Appelquist, J. Ingoldby and M. Piai, JHEP 1707 (2017), JHEP 1803 (2018)

★ Expand around the minimum of $W(\chi)$: $\chi = F_d + \bar{\chi}$

➡ $\mathcal{L}_\pi = \frac{f_\pi^2}{4}\left(\frac{\bar{\chi}^2+2\bar{\chi}F_d+F_d^2}{f_d^2}\right)\mathrm{Tr}\left[\partial_\mu\Sigma(\partial^\mu\Sigma)^\dagger\right] = \frac{f_\pi^2}{4}\left(\frac{F_d^2}{f_d^2}\right)\mathrm{Tr}\left[\partial_\mu\Sigma(\partial^\mu\Sigma)^\dagger\right] + \ldots$

➡ Compare with $\mathcal{L}_\pi = \frac{F_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu\Sigma(\partial^\mu\Sigma)^\dagger\right] + \ldots$

➡ $\frac{F_\pi^2}{f_\pi^2} = \frac{F_d^2}{f_d^2}$ (1)

★ Can do the same to $\mathcal{L}_M$

➡ $\frac{M_\pi^2}{m_\pi^2} = \left(\frac{F_d^2}{f_d^2}\right)^{y/2-1}$ (2)

★ (1) and (2) gives $M_\pi^2\left(F_\pi^2\right)^{(1-y/2)} = Cm\,,\ C = 2B_\pi(f_\pi^2)^{(1-y/2)}$

➡ No reference to details of $V(\chi)$, can be used to determine y

Use of the dilaton ChPT

Extracting more information

T. Appelquist, J. Ingoldby and M. Piai, JHEP 1803 (2018)

★ Use (1), (2) and the following

- $\left(\frac{\partial W}{\partial \chi}\right)_{\chi=F_d} = 0, \ \left(\frac{\partial^2 W}{\partial \chi^2}\right)_{\chi=F_d} = M_d^2$ (dilaton mass)
- Phenomenological assumption: $V(\chi) \propto \chi^p$

★ Can easily derive

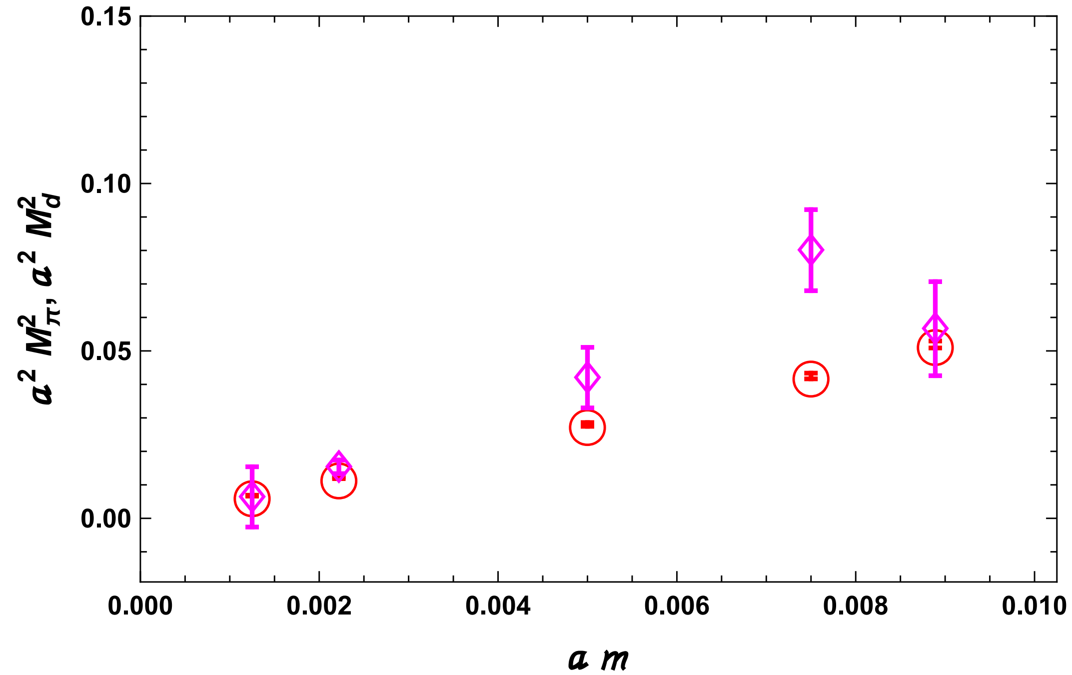
- $M_\pi^2 = BF_\pi^{p-2}$
- $M_d^2 = \frac{yN_f f_\pi^2}{2f_d^2}(p-y)BF_\pi^{p-2}$

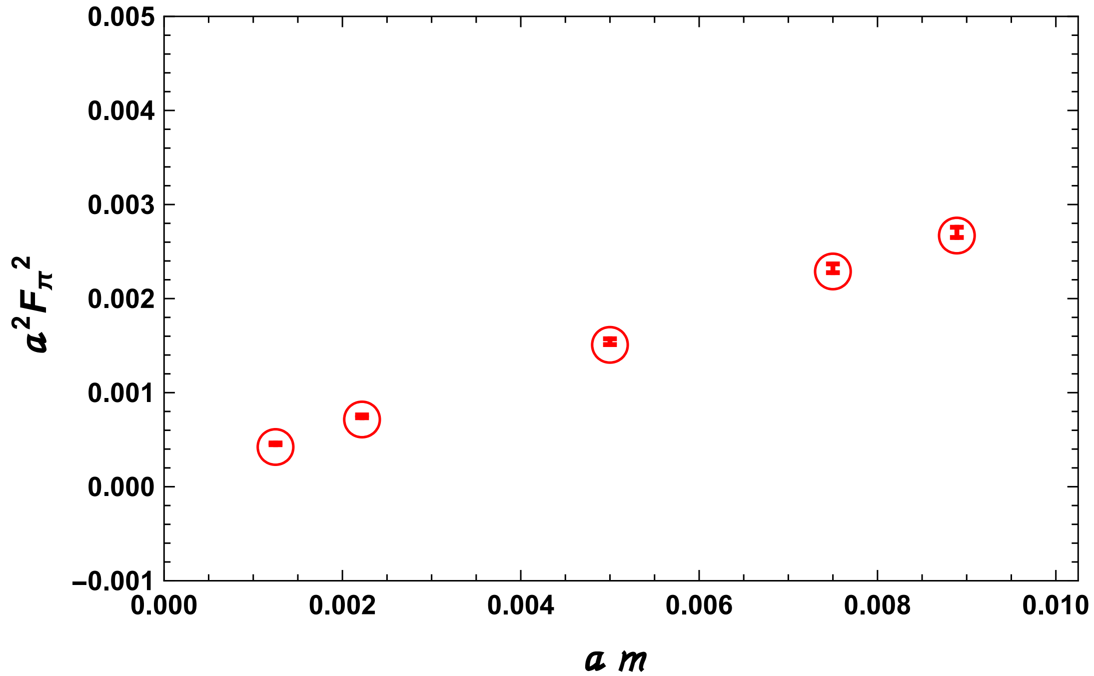
} same B

★ Can further fit p and $f_\pi^2/f_d^2 = F_\pi^2/F_d^2$

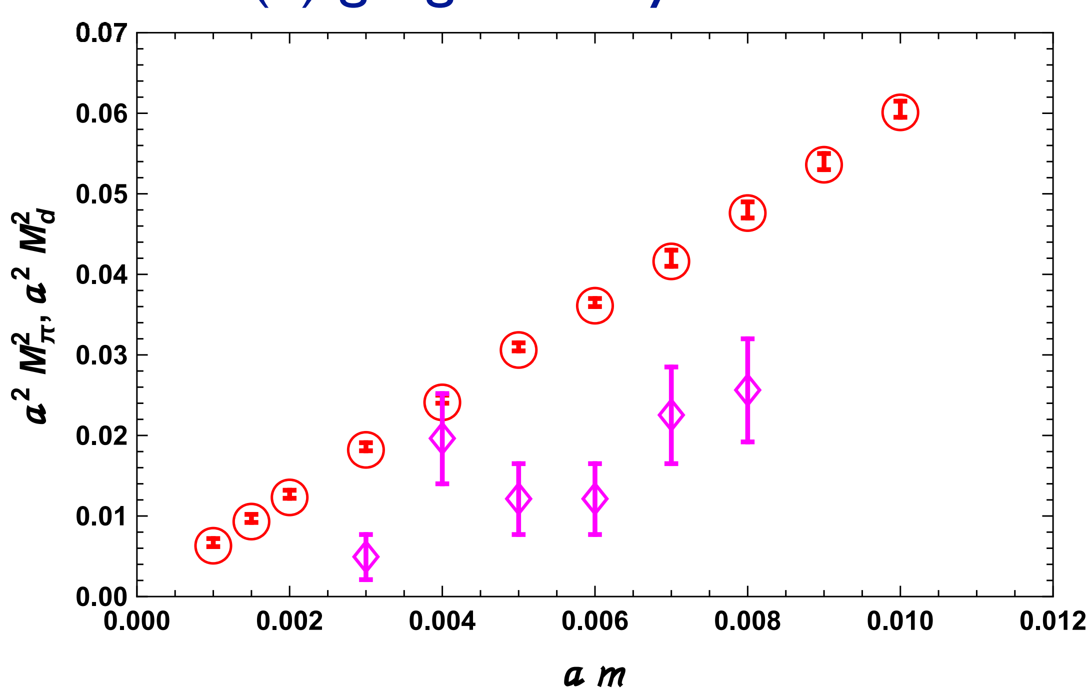
Data used in the AIP analysis

SU(3) gauge theory with 8 fund fermions, LSD collaboration, 2016





SU(3) gauge theory with 2 sextet fermions, LHC collaboration, 2012 ~ 2016



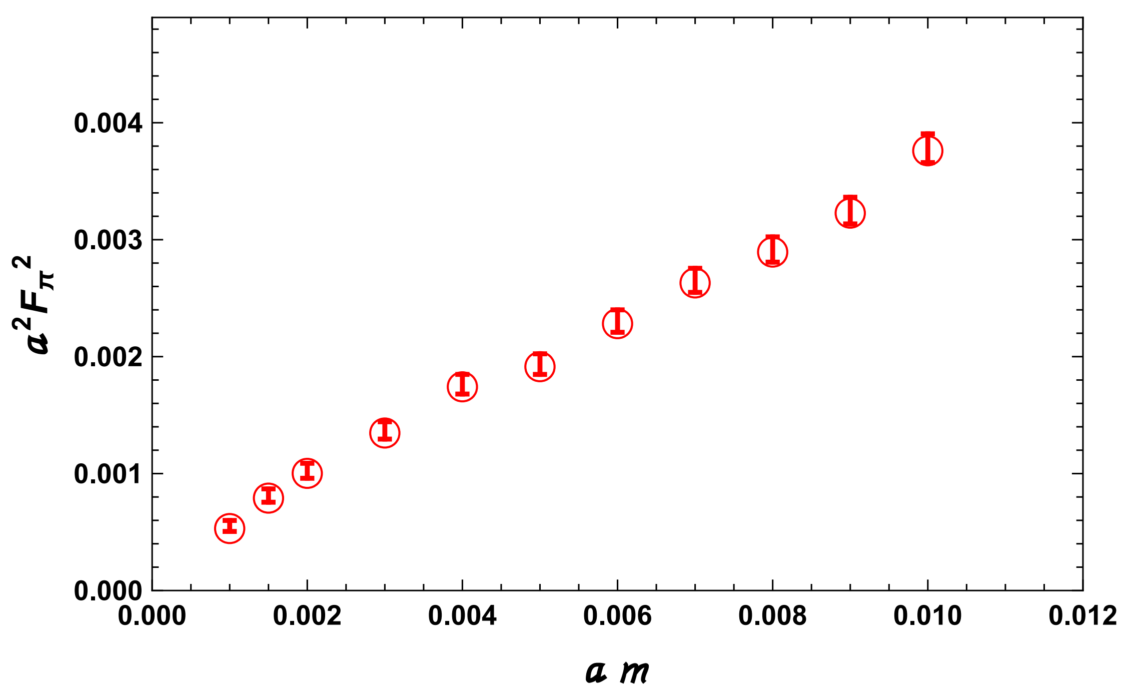


Figure from T. Appelquist, J. Ingoldby and M. Piai, JHEP 1803 (2018)

Use of the dilaton ChPT

T. Appelquist, J. Ingoldby and M. Piai, JHEP 1803 (2018)

★ SU(3) with 8 fundamental fermions

$$y = 2.1 \pm 0.1$$
$$p = 4.3 \pm 0.2$$
$$\frac{f_\pi^2}{f_d^2} = 0.08 \pm 0.04$$

★ SU(3) with 2 sextet fermions

$$y = 1.9 \pm 0.1$$
$$p = 4.4 \pm 0.3$$
$$\frac{f_\pi^2}{f_d^2} = 0.09 \pm 0.06$$

} surprisingly similar

★ $F_d \sim 3F_\pi$: prediction, can be tested with future lattice result

Searching for relevant interaction:

Composite Higgs and partial compositeness

Basic idea *àla* CCWZ, naturally light Higgs

D.B. Kaplan, H. Georgi, M. Dugan, S. Dimopoulos,… *circa* 1985

- ★ Global symmetry G/H, Higgs doublet $\in$ Goldsone
- ★ SM $SU(2)_L \times SU(2)_R \subset H$
- ★ EW symm breaking induced by additional interactions
  - ➡ Vacuum misalignment
  - ➡ $v << f \sin\langle\theta\rangle,\ f = |\vec{F}| \sim \Lambda_{HC}$
- ★ Fermion masses generated *via* partial compositeness
  - ➡ Spin-1/2 bound states mixing with the top

D.B. Kaplan, 1991

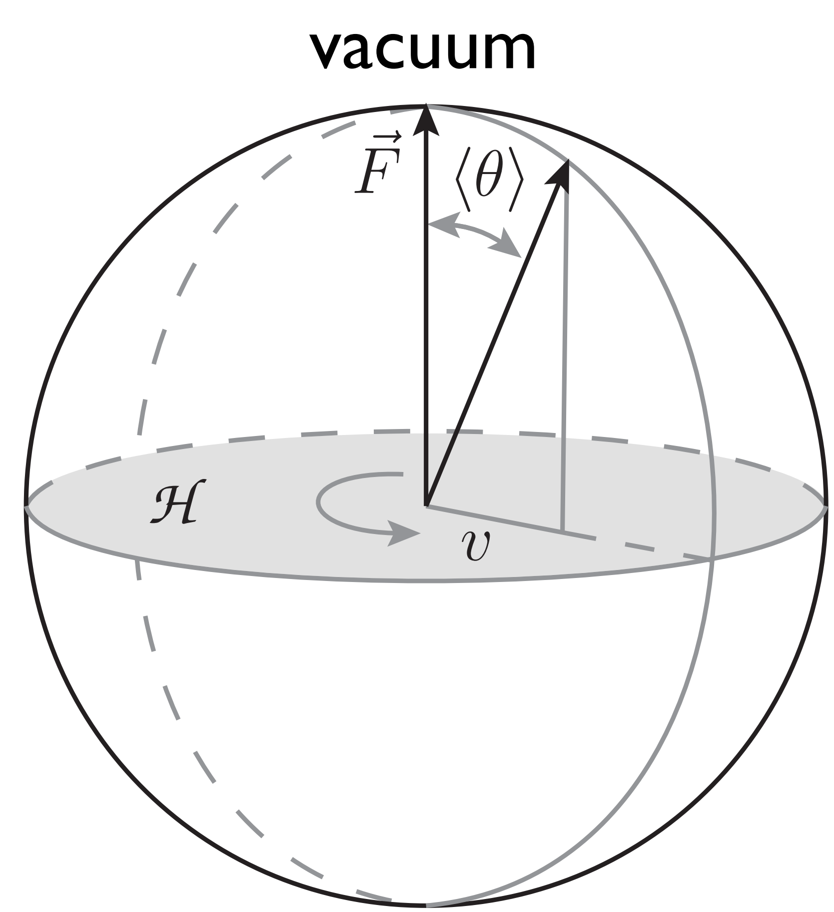


Figure in G. Panico and A. Wulzer, arXiv:1506.01961

Main difference between TC and CH

Global symmetry breaking G $\longrightarrow$ H

- ★ Technicolour: Higgs $\in$ H
  - ➡ Challenge: Have a light Higgs state
- ★ Technicolour: Higgs $\in$ G/H
  - ➡ Challenge: Obtain the correct Higgs mass

Hierarchy of scales in composite Higgs models

An analogy: QCD plus weak interaction

Λ	Flavour scale, symmetry G
$\Lambda_{HC} \sim f$	Composite scale, $G \to H$
$v_{EW} \sim v$	EW scale, EWSB

★ The relevant operator in the UV completion above Λ_{HC}

➡ Details of the misalignment is model-dependent

UV completion and lattice calculation

Name	Gauge group	ψ	χ	Baryon type
M1	$SO(7)$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$\psi\chi\chi$
M2	$SO(9)$	$5 \times \mathbf{F}$	$6 \times \mathbf{Spin}$	$\psi\chi\chi$
M3	$SO(7)$	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$\psi\psi\chi$
M4	$SO(9)$	$5 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$\psi\psi\chi$
M5	$Sp(4)$	$5 \times \mathbf{A}_2$	$6 \times \mathbf{F}$	$\psi\chi\chi$
M6	$SU(4)$	$5 \times \mathbf{A}_2$	$3 \times (\mathbf{F}, \overline{\mathbf{F}})$	$\psi\chi\chi$
M7	$SO(10)$	$5 \times \mathbf{F}$	$3 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$\psi\chi\chi$
M8	$Sp(4)$	$4 \times \mathbf{F}$	$6 \times \mathbf{A}_2$	$\psi\psi\chi$
M9	$SO(11)$	$4 \times \mathbf{Spin}$	$6 \times \mathbf{F}$	$\psi\psi\chi$
M10	$SO(10)$	$4 \times (\mathbf{Spin}, \overline{\mathbf{Spin}})$	$6 \times \mathbf{F}$	$\psi\psi\chi$
M11	$SU(4)$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$6 \times \mathbf{A}_2$	$\psi\psi\chi$
M12	$SU(5)$	$4 \times (\mathbf{F}, \overline{\mathbf{F}})$	$3 \times (\mathbf{A}_2, \overline{\mathbf{A}_2})$	$\psi\psi\chi$, $\psi\chi\chi$

Table from D. Franzosi and G. Ferretti, arXiv:1905.08273 [hep-ph]

★ Before starting lattice simulations

➡ Choose a model wisely before you'll spend years on the simulation

Gauge group repn and global coset

M. Peskin, 1980

- ★ Real : $(T^a)^* = (T^a)^{\mathrm{T}} = -S^{-1} T^a S, \qquad SS^* = 1.$
- ★ Pseudoreal : $(T^a)^* = (T^a)^{\mathrm{T}} = -S^{-1} T^a S, \qquad SS^* = -1.$

$$\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} \equiv \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\beta}} \end{pmatrix} = \begin{pmatrix} \psi_\alpha \\ (\chi^\beta)^* \end{pmatrix}, \quad \bar{\Psi}\Psi = \epsilon^{\alpha\beta} \chi_\beta^{ia} \psi_{\alpha ia} + \text{h.c.}$$

gauge repn	condensate	global symmetry
Complex	$\epsilon^{\alpha\beta} \psi_\beta^{i(\bar{r})} \psi_{\alpha i}^{(r)} + \text{h.c.}$	$SU(N_f) \times SU(N_f) \to SU(N_f)$
Real	$\epsilon^{\alpha\beta} \psi_\beta^{ia} \psi_{\alpha i}^{b} S_{ab}^{-1}$	$SU(2N_f) \to SO(2N_f)$
Pseudoreal	$\epsilon^{\alpha\beta} \psi_\beta^{ia} \psi_\alpha^{jb} S_{ab}^{-1} E_{ij}$	$SU(2N_f) \to Sp(2N_f)$

On-going lattice projects

★ Colorado - Tel Aviv (TACO) collaboration 

V. Ayyar, T. Degrand, D.C. Hacket, W.I. Jay, E. Neil, Y. Shamir, B. Svetiteky

➡ $G_{HC} = SU(4),\ G/H = \{SU(4) \times [SU(2)_L \times SU(2)_R]\}/\{SO(4) \times SU(2)_V\}$

★ CP3-Origin - Plymouth collaboration

V. Drach, M. Hansen, T. Janowski, C. Pica, J. Rantaharju, F. Sannino,

➡ $G_{HC} = SU(2),\ G/H = SU(4)/Sp(4)$

★ Hsinchu - Pusan - Swansea collaboration

E. Bennet, D.K. Hong, J.-W. Lee, C.J.D.L., B. Lucini, M. Piai, D. Vadacchino

➡ $G_{HC} = Sp(4),\ G/H = \{SU(4) \times SU(6)\}/\{Sp(4) \times SO(6)\}$

★ Edinburgh - Torino collaboration

G. Cossu, L. Del Debbio, M. Panero, D. Preti

➡ $G_{HC} = SU(4),\ G/H = \{SU(4) \times [SU(2)_L \times SU(2)_R]\}/\{SO(4) \times SU(2)_V\}$

Composite Higgs with Sp(4) gauge group

J. Barnard, T. Gherghetta, T.S. Ray, 2014

Field	$Sp(4)$ gauge	$SU(4)$ global
A_μ	10	1
ψ	4	4

★ Two Dirac fermions in the fundamental repn

pseudoreal

★ The Higss doublet in the coset $SU(4)/Sp(4)$

★ The SM $SU(2)_L \times SU(2)_R$ in the unbroken global $Sp(4)$

Idea and implementation of partial compositeness

D.B. Kaplan, 1991; J. Barnard, T. Gherghetta and T.S. Ray, 2014

Field	$Sp(4)$ gauge	$SU(4)$ global	$SU(6)$ global
A_μ	10	1	1
ψ	4	4	1
χ	5	1	6

★ Two Dirac fermions in the fundamental repn
pseudoreal

★ Three Dirac fermions in the antisymmetric repn
real

★ $SU(6) \to SO(6) \supset SU(3)$

★ Gauge the $SU(3)$ to be the QCD colour group

The top partner and the top mass

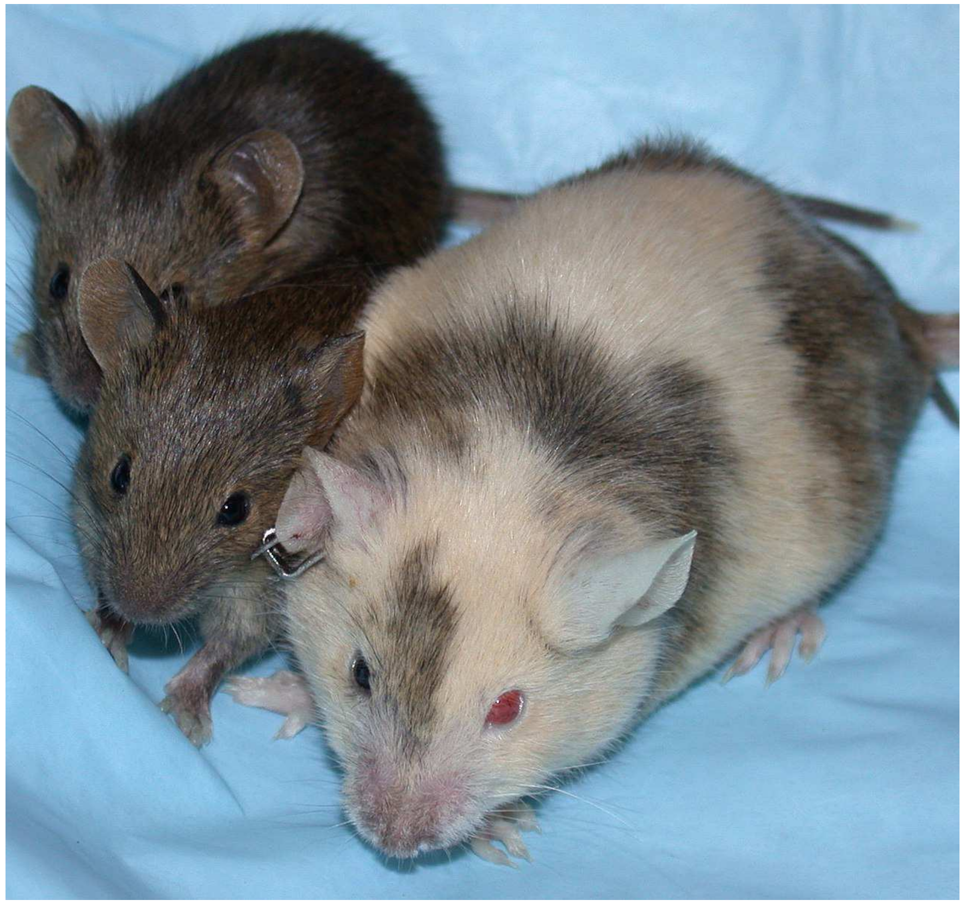
$$\hat{\Psi}^{\alpha}_{ij} = (\psi_i \chi^{\alpha} \psi_j), \quad \hat{\Psi}^{c,\alpha}_{ij} = (\psi_i \chi^{c,\alpha} \psi_j)$$

QCD colour

global SU(4) indices

- ★ Neutral under gauge-Sp(4) colour
- ★ Carry QCD colour and mix with the top
- ★ Operators emerge at the flavour scale
  - → Global SU(4) instead of Sp(4)
- ★ Called the chimera baryon by the TACO collab.

Chimera



Pictures from Ben Svetitsky, LATTICE 2019

The top partner and the top mass

$$\hat{\Psi}_{ij}^{\alpha} = (\psi_i \chi^{\alpha} \psi_j), \ \hat{\Psi}_{ij}^{c,\alpha} = (\psi_i \chi^{c,\alpha} \psi_j)$$

$$\mathcal{L}^{\rm mix} = -\frac{1}{2}\left\{\lambda_1 M_* \left(\frac{M_*}{\Lambda}\right)^{d_\Psi - 5/2} \Psi_1^T \tilde{C} t^c + \lambda_2 M_* \left(\frac{M_*}{\Lambda}\right)^{d_{\Psi^c} - 5/2} t^T \tilde{C} \Psi_2^c + \right.$$
$$\left. + \lambda M_* \left[\Psi_1^T \tilde{C} \Psi_1^c + \Psi_2^T \tilde{C} \Psi_2^c\right] + y v_W \left[\Psi_1^T \tilde{C} \Psi_2^c + \Psi_2^T \tilde{C} \Psi_1^c\right]\right\} + \text{h.c.}$$

$$m_t^2 \simeq \frac{\lambda_1^2 \lambda_2^2 y^2 \left(\frac{M_*}{\Lambda}\right)^{2d_\Psi + 2d_{\Psi^c} - 10} v_W^2 M_*^4}{m_1^2 m_2^2}$$

where

$$m_1^2 \simeq \left(\lambda^2 + \lambda_1^2 \left(\frac{M_*}{\Lambda}\right)^{2d_\Psi - 5}\right) M_*^2 ,$$

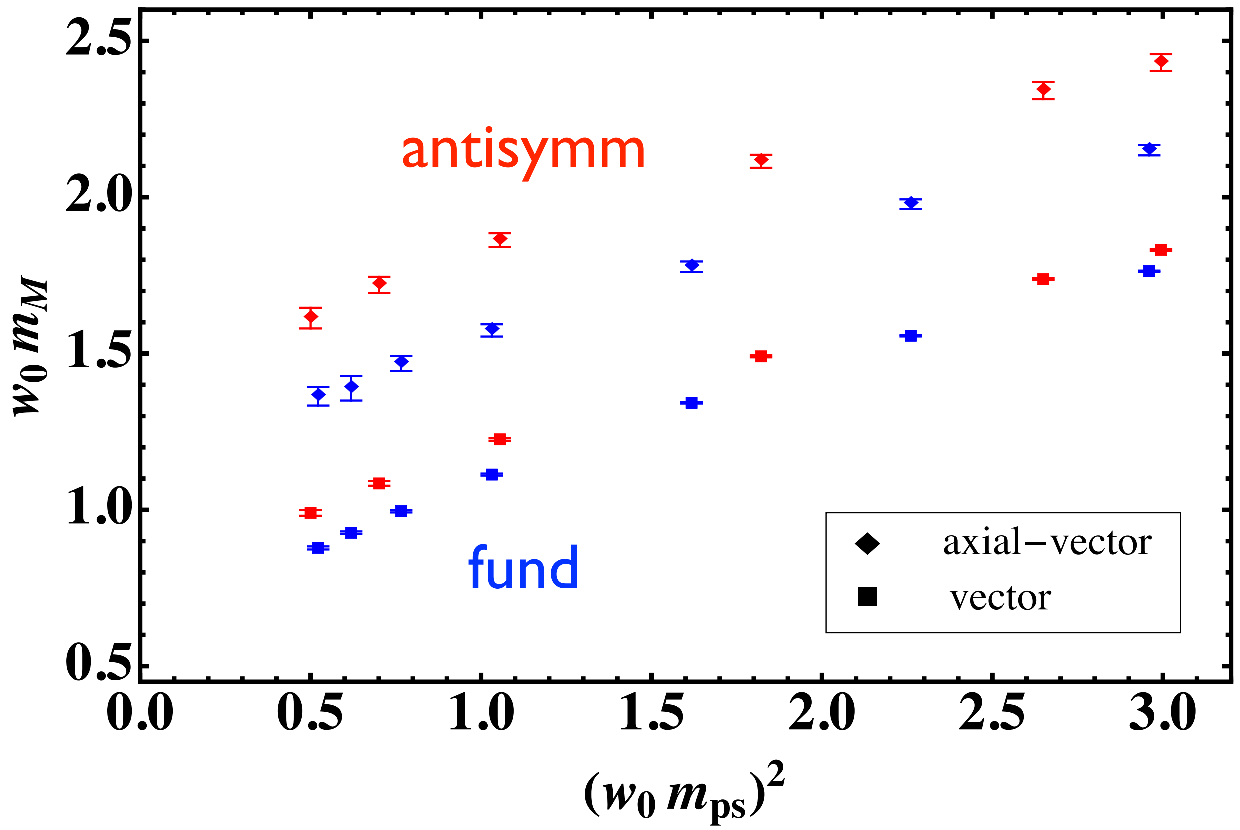
$$m_2^2 \simeq \left(\lambda^2 + \lambda_2^2 \left(\frac{M_*}{\Lambda}\right)^{2d_{\Psi^c} - 5}\right) M_*^2$$

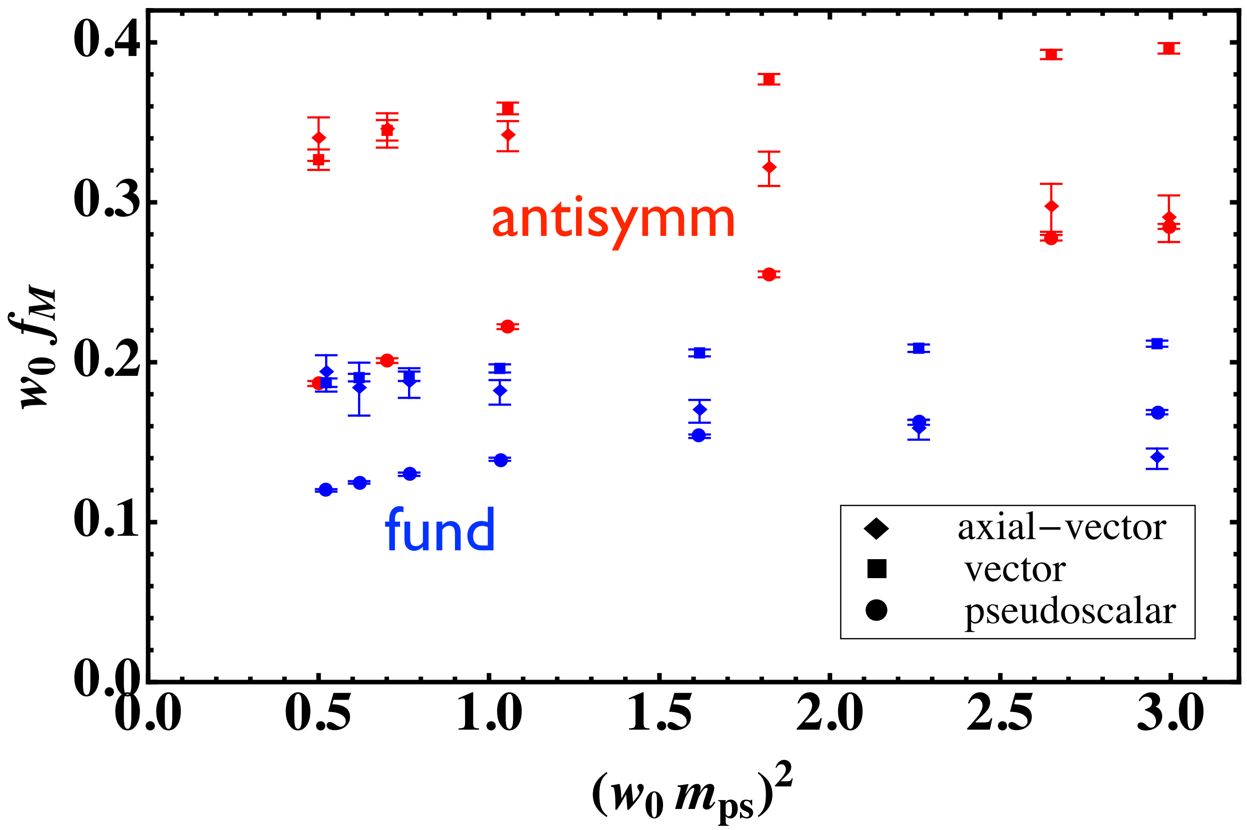
- ★ Prefer $d_\Psi = d_{\Psi^c} \lesssim 5/2$, *ie*, large anomalous dimension
  - ➡ $\lambda_{1,2}$ estimated to be small in ~M6 [TACO collab., PRD99 (2019)]
  - ➡ IR conformality with more fermion flavours?

Spectrum results

Sp(4) gauge, two fundamental Dirac fermions in the action

J.-W. Lee *et al.*, arXiv:1811.00276 [hep-lat] (LATTICE 2018)

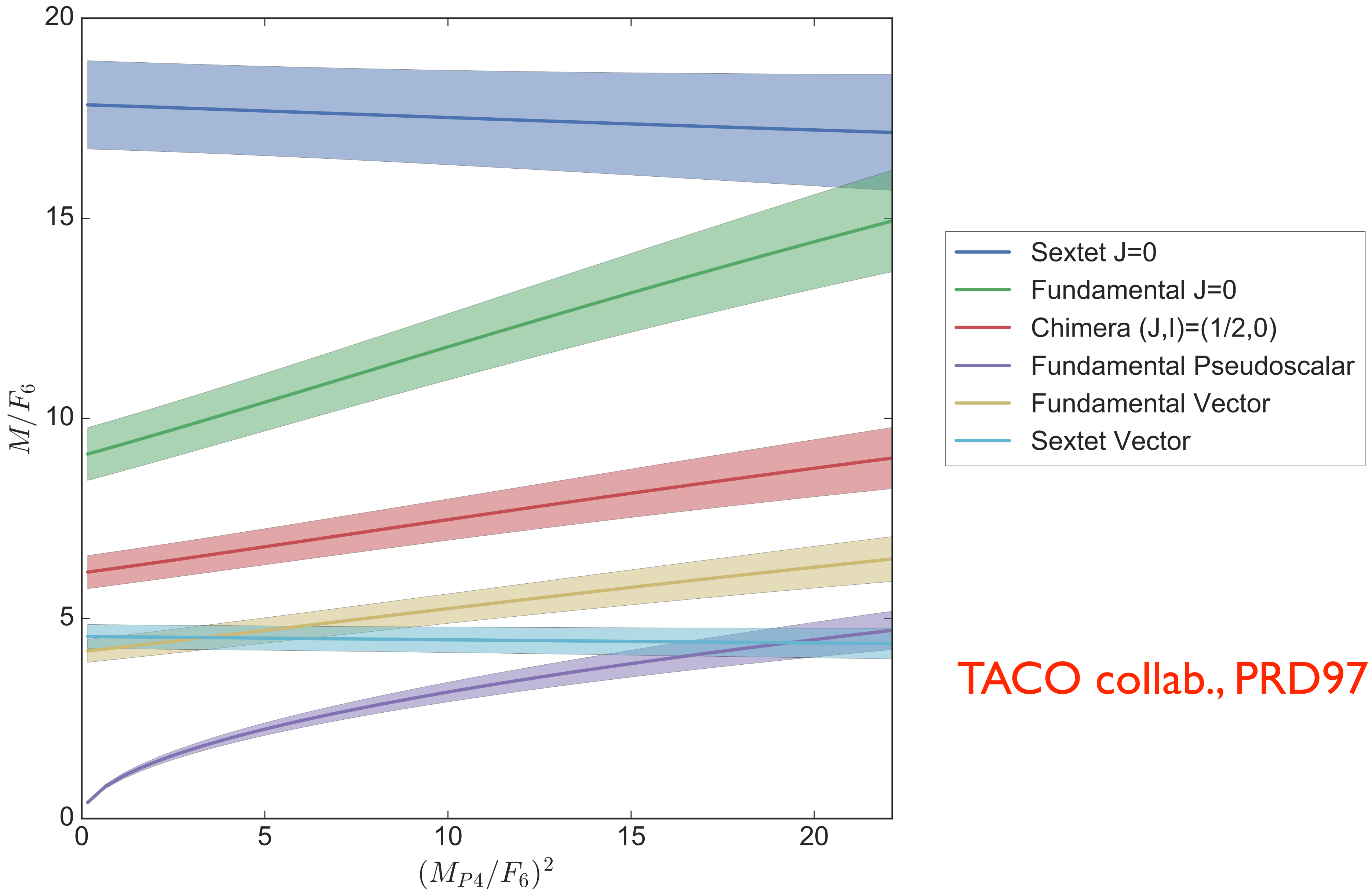




★Typically higher scale for antisymmetric repn

Spectrum results

SU(4) gauge, two fundamental and two sextet Dirac fermions in the action



TACO collab., PRD97 (2018)

What would be interesting for future work?

The Higgs potential

★ The gauge contribution near the origin

- Analogous to the $M_{\pi^+} - M_{\pi^0}$ difference in QCD+QED
- Computed in M6 and obtained the expected positive contribution

TACO collab., PRD99 (2019)

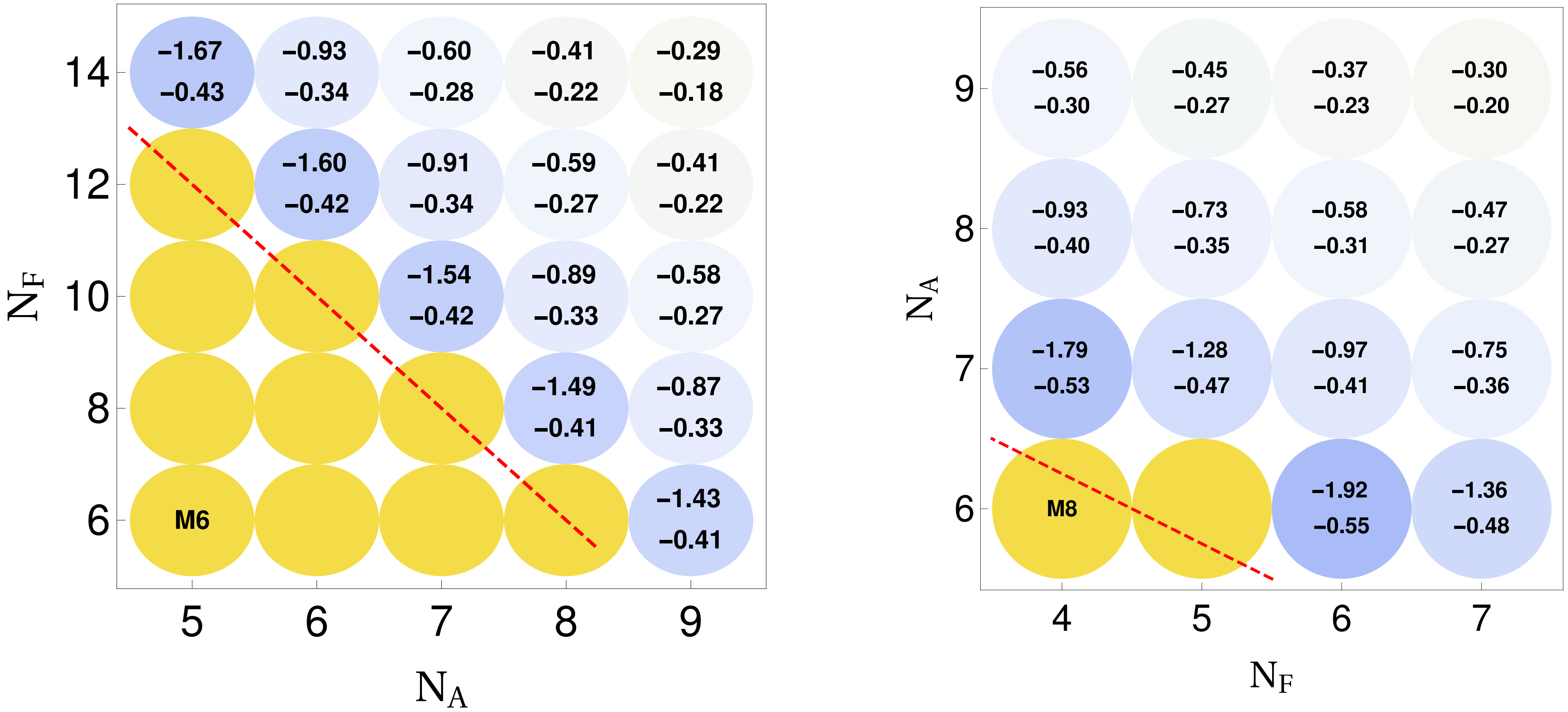
★ The top-quark contribution near the origin

G. Cacciapaglia *et al.*, Golterman and Shamir, recent years

- It can be negative
- Crucial for breaking electroweak symmetry

Where to search for large anomalous dimension

D.B. Franzosi and G. Ferretti, arXiv:1905.08273 [hep-ph]



★ To facilitate partial compositeness

Summary and outlook

- ★ Looking for BSM physics is inevitable
- ★ Lattice BSM programmes are part of the effort
  - ➨ Deliver useful information for phenomenologists
  - ➨ Study interesting topics in strongly-coupled field theory
- ★ Recent lattice works on other scenarios
  - ➨ S. Capitani *et al.*, arXiv: 1901.09872 [hep-th]
  - ➨ V. Afferrante, A. Maas, P. Torek, arXiv: 1906.11193 [hep-lat]

⋮

Thank you, Xu!

Also thanks to the organisers and all the participants