QCD at finite temperature and density

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Study of the strongly interacting matter and its new "phases"

Theory:

Low T, low density:
EFT
(Chiral perturbation Theory) ,
Virial expansion

Lattice QCD and Super-computing

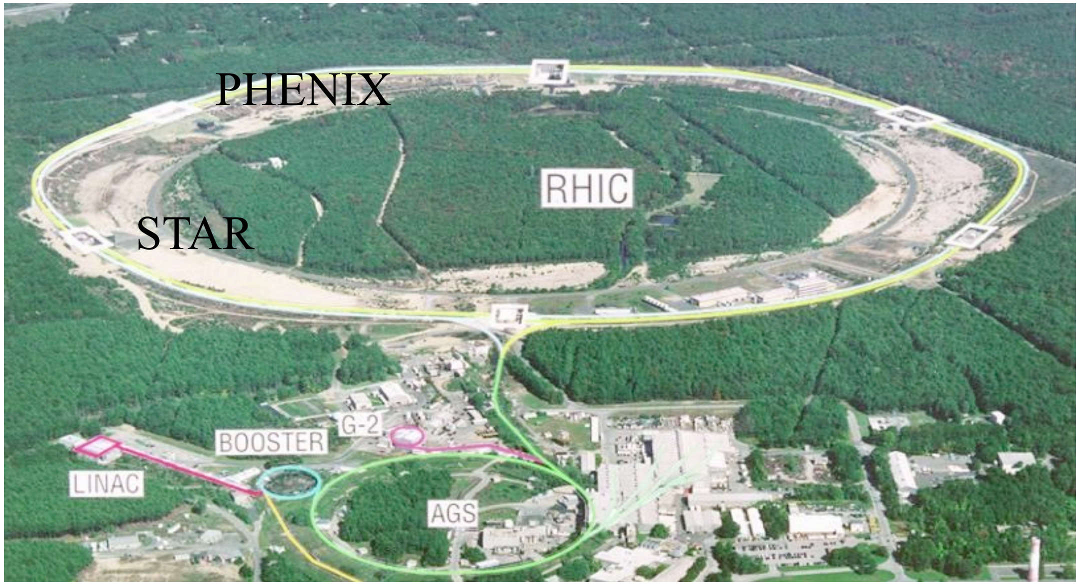
High T, high desnity:
Weak coupling methods,
Dimensionally reduced EFT

Experiment:

Past:
AGS, SPS, $E_{cm}=(1\text{-}17)$ GeV

Present :
RHIC, $E_{cm}=(5.5\text{-}200)$ GeV,
LHC, $E_{cm}=2.76$ TeV, 5.5 TeV

Future:
NICA, CBM@FAIR
$E_{cm}=(1\text{-}10)$ GeV



Relativistic Heavy Ion Collisions

RHIC: Au+ Au, Cu+Cu, Cu+Au, U+U

$\sqrt{s} = 5.5 - 200\text{GeV}$

$\epsilon \simeq 15 - 30\text{GeV/fm}^3$

LHC: ALICE, also HI in CMS, ATLAS and LHCb

Pb+Pb $\sqrt{s} = 2.76 - 5.5\text{TeV}$

$\epsilon \simeq 100\text{GeV/fm}^3$

Supercomputing and LQCD at *T>0*:

2013: TOP500 rank: 3

Titan, USA

TOP500 rank: 4

Sequoia, USA

TOP500 rank: 9

MIRA, USA

New states of strongly interacting matter ?

I. Ya. Pomeranchuk, Doklady Akad. Nauk. SSSR 78 (1951) 889

Because of finite size of hadrons hadronic matter cannot exist up to arbitrarily high temperature/density, hadron size has to be smaller than $1/T$

Hagedorn, Nouvo Cim. 35 (1965) 395

Exponentially increasing density of hadronic states => limiting temperature

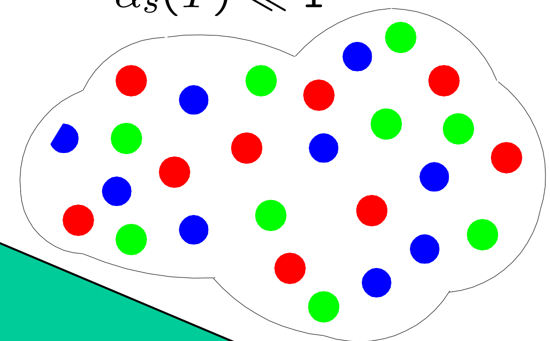
Cabbibo, Parisi, PLB 59 (1975) 67

Realization that at high temperature hadronic language is not appropriate
and reinterpretation of the limiting temperature as the phase transition temperature
to medium consisting of quarks and gluons

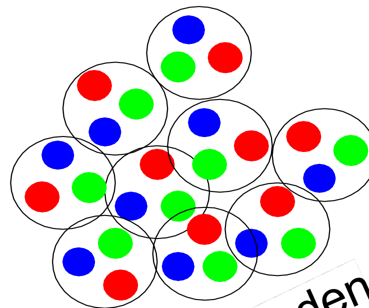
Collins and Perry, PRL 34 (1975) 1353
At very high density strongly interacting matter should consist of quarks due to assymptotic freedom

Deconfinement at high temperature and density

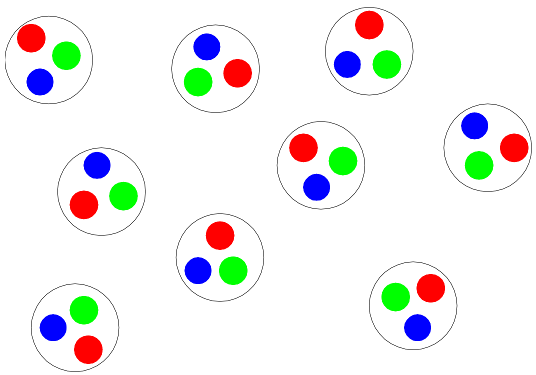
Quark Gluon Plasma (QGP)
$\alpha_s(T) \ll 1$

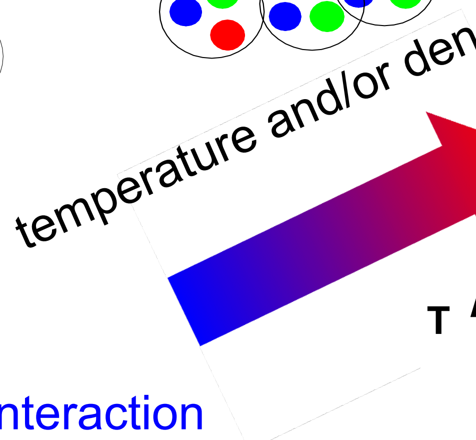


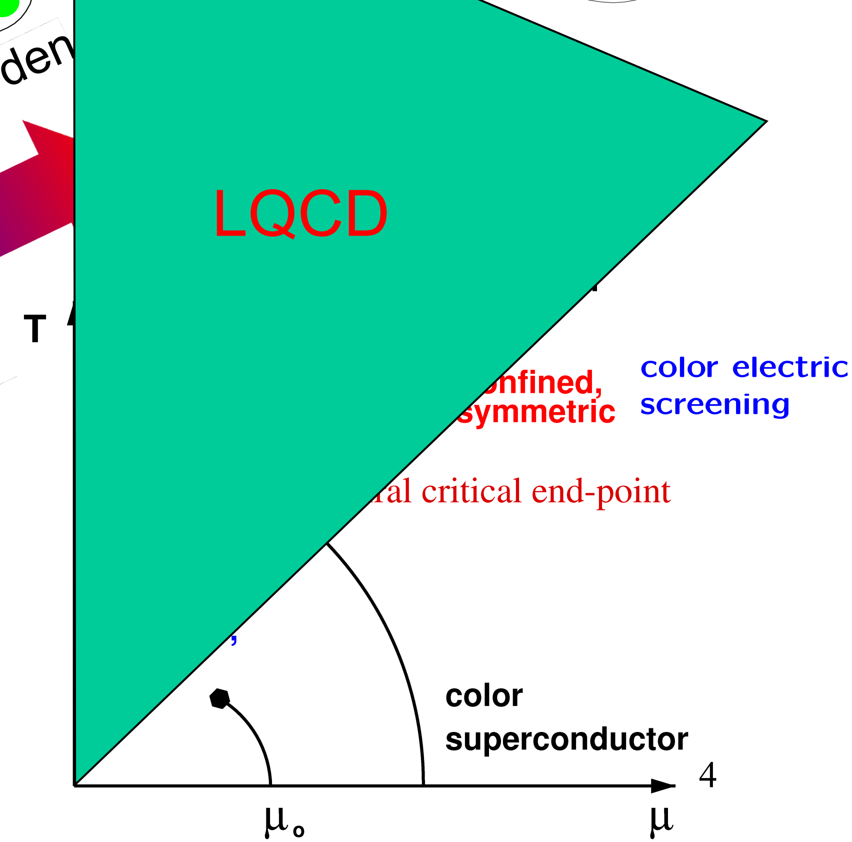
Transition



Hadron Gas







Why this is interesting ? :
basic properties of strong interaction

astrophysical (compact stars and transients):
boundaries of hadronic matter

cosmological consequences
(Early Universe few microseconds after Big Bang,
e.g. axion cosmology)

Symmetries of QCD in the vacuum at high T

$T \gg \Lambda_{QCD}$:

- Chiral symmetry : $m_{u,d} \ll \Lambda_{\rm QCD}$

$SU_A(2)$ rotation $\psi \to e^{i\phi T^a \gamma_5}\psi$ $\qquad \psi_{L,R} \to e^{i\phi_{L,R} T^a}\psi_{L,R}$

$$\langle\bar\psi\psi\rangle = \langle\bar\psi_L\psi_R\rangle + \langle\bar\psi_R\psi_L\rangle \neq 0$$

$\langle\bar\psi\psi\rangle = 0$

restored

spontaneous symmetry breaking or Nambu-Goldstone symmetry realization

2008

hadrons with opposite parity have very different masses,
interactions between hadrons are weak at low E

- Axial or $U_A(1)$ symmetry: invariance $\psi \to e^{i\phi\gamma_5}\psi$

Effectively restored ?

is broken by anomaly (ABJ) : $\langle\partial^\mu j^A_\mu\rangle = -\dfrac{\alpha_s}{4\pi}\langle\epsilon^{\alpha\beta\gamma\delta}F^a_{\alpha\beta}F^a_{\gamma\delta}\rangle$

topology

η' meson mass, π-a_0 mass difference

- Center (Z_3) symmetry : invariance under global gauge transformation

$\langle L\rangle \neq 0$

broken

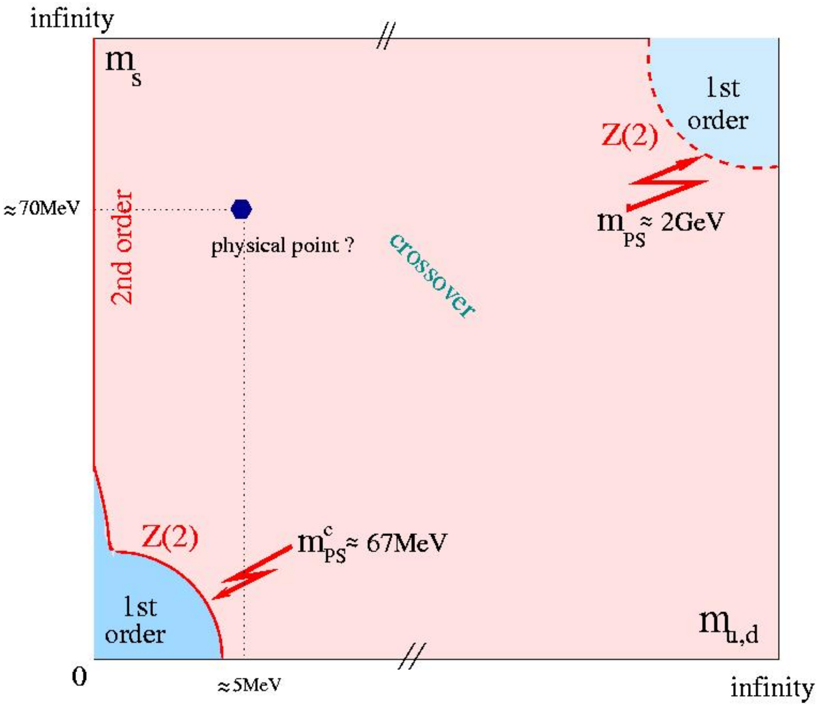
$$A_\mu(0,\mathrm{x}) = e^{i2\pi N/3}A_\mu(1/T,\mathrm{x}),\;\; N = 1,2,3$$

Exact symmetry for infinitely heavy quarks and the order parameter is the
Expectation value of the Polyakov loop:

$$L = \mathrm{tr}\mathcal{P}e^{ig\int_0^{1/T} d\tau A_0(\tau,\vec{x})} \qquad \langle L\rangle = 0$$

QCD phase diagram as function of the quark mass

Pisarski, Wilczek, PD29 (1984) 338



For very large quark masses there is a 1st order deconfining phase transition

Chiral transition:

• For vanishing u,d -quark masses the Chiral transition is either 1st order or 2nd order phase transition

• For physical quark masses there could be a 1st order phase transition or crossover

Evidence for 2nd order transition in the chiral limit
=> universal properties of QCD transition:

$SU_A(2) \sim O(4)$
relation to spin models

transition is a crossover
for physical quark masses

In these lectures:

1. Basics of filed theory at $T>0$ and weak coupling expansion
2. Dimensionally reduced EFT (EQCD)
3. Virial expansion at low T
4. Lattice QCD at $T>0$ basics
5. Deconfinement transition in absence of quarks (SU(N) gauge theories)
6. Chiral transition in QCD
7. Equation of State in QCD
8. Deconfinement and color electric screening in QCD
9. Chromo-magnetic screening and testing EQCD non-perturbatively
10. QCD at non-zero chemical potentials and Taylor expansion

Quantum Statistical mechanics

Transition amplitude in QM and its path integral represenation

$$F(q', t'; q, t) = \langle q' | e^{-i\hat{H}(t'-t)} | q \rangle$$

$t \to -i\tau, \; t' \to -i\tau$ (imaginary time)

$$F(q' - i\tau'; q, -i\tau) = \langle q' | e^{-\hat{H}(\tau'-\tau)} | q \rangle$$

$$\hat{H} = \frac{1}{2} p^2 + V(q)$$

$$F(q', -i\tau'; q, -i\tau) = \int \mathcal{D}q \exp\left[- \int_{\tau}^{\tau'} d\tau'' \left(\frac{1}{2} \dot{q}^2(\tau'') + V(q(\tau'')) \right) \right]$$

$$q(\tau) = q, \; q(\tau') = q'$$

Partition function in statistical mechanics:

$$Z(\beta) = \mathrm{Tr} e^{-\beta \hat{H}}, \; \beta = 1/T$$

$$Z(\beta) = \sum e^{-\beta E_n}, \quad \hat{H}|n\rangle = E_n |n\rangle$$

$$Z(\beta) = \int dq \langle q | e^{-\beta \hat{H}} | q \rangle$$

$$Z(\beta) = \int dq F(q, -i\beta; q, 0)$$

$$\Downarrow$$

$$Z(\beta) = \int \mathcal{D}q(\tau) \exp \left[- \int_0^\beta d\tau \left(\frac{1}{2} \dot{q}^2(\tau) + V(q(\tau)) \right) \right],$$

$$q(\beta) = q(0)$$

Euclidean action $S_E(\beta) = \int_0^\beta d\tau \left(\frac{1}{2} \dot{q}^2(\tau) + V(q(\tau)) \right)$

We can also calculate the generating functional

$$Z(\beta; j) = \int \mathcal{D}q \exp \left[-S_E(\beta) + \int_0^\beta j(\tau) q(\tau) d\tau \right]$$

$$\Delta(\tau_1, \tau_2) = \frac{1}{Z(\beta)} \frac{\delta^2 Z(\beta; j)}{\delta j(\tau_1) \delta j(\tau_2)} |_{j=0} = \frac{1}{Z(\beta)} \int \mathcal{D}q q(\tau_1) q(\tau_2) e^{-S_E(\beta)}$$

Thermodynamics of scalar field theory

Straightforward generalization to infinite number of degrees of freedom $q(t) \to \phi_x(t) \equiv \phi(t,x)$

$$L = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 - \frac{g^2}{4!}\phi^4$$

$$\Downarrow$$

$$S_E(\beta) = \int_0^\beta d\tau \int d^3x \left(\frac{1}{2}(\partial_\tau \phi)^2 + \frac{1}{2}(\partial_i \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{g^2}{4!}\phi^4 \right)$$

$$Z(\beta; j) = \int \mathcal{D}\phi \exp(-S_E(\beta) + \int_0^\beta d\tau \int d^3x j(\tau,x)\phi(\tau,x))$$

$$\phi(0,x) = \phi(\beta,x)$$

Free field limit ($g=0$):

$$Z(\beta; j) = \int \mathcal{D}\phi \exp\left[-\int d^4x_E \frac{1}{2}\phi(-\partial_\tau^2 - \nabla^2 + m^2)\phi + \int_0^\beta d^4x_E j(x_E)\phi(x_E) \right]$$

$$x_E = (\tau, x)$$

Gaussian integration:

$$Z_0(\beta;j) = Z(\beta)\exp\left[\int_0^\beta d^4x_E dy_E\; j(x_E)\Delta_0(x_E-y_E)j(y_E)\right]$$

$$Z(\beta) = (det\Delta_0)^{1/2} = \mathrm{Tr}\ln\Delta_0\,/2$$

$$\left[-\partial_\tau^2 - \nabla^2 + m^2\right]\Delta_0(x_E-y_E) = \delta(\tau_x-\tau_y)\delta(x-y)$$

$$\Downarrow$$

$$(\omega_n^2+k^2+m^2)\Delta_0(i\omega_n,k) = (\omega_n^2+\omega_k^2)\Delta_0(i\omega_n,k) = 1$$

$$\omega_n = 2\pi T n,\;\; \omega_k^2 = k^2+m^2$$

$$\Downarrow$$

$$\Delta_0(i\omega_n,k) = \frac{1}{\omega_n^2+\omega_k^2}$$ -Matsubara propagator

Mixed (Saclay) representation:

$$\Delta_0(\tau,k) = T\textstyle\sum_n e^{-i\omega_n\tau}\Delta_0(i\omega_n,k)$$

$$[-\partial_\tau^2+\omega_k^2]\Delta_0(\tau,k) = \delta(\tau_x-\tau_y),\;\; \Delta_0(\tau-\beta) = \Delta_0(\tau)$$

$$\rightarrow \Delta_0(\tau) = \tfrac{1}{2\omega_k}((1+f(\omega_k))e^{-\omega_k\tau}+f(\omega_k)e^{\omega_k\tau}), f(\omega_k) = (e^{\beta\omega_k}-1)^{-1}$$

$$\ln Z(\beta) = \frac{1}{2}\mathrm{Tr}\ln\Delta_0 = \frac{1}{2}\sum_{n=-\infty}^{\infty}\sum_{k}\ln\beta^2\Delta(i\omega_n,k) =$$

$$-\frac{1}{2}\sum_{n=-\infty}^{\infty} V\int\frac{d^3k}{(2\pi)^3}\ln\beta^2[\omega_n^2+\omega_k^2] =$$

$$\sum_n\frac{d}{d\omega_k^2}\ln[\omega_n^2+\omega_k^2] = \sum_n\frac{1}{\omega_n^2+\omega_k^2} = \beta\Delta_0(\tau=0,k) = \frac{\beta}{2\omega_k}(1+2f(\omega_k))$$

$$\Downarrow$$

$$\sum_n\ln\beta^2(\omega_n^2+\omega_k^2) = \beta\omega + 2\ln(1+e^{-\beta\omega_k}) + const$$

$$\ln Z(\beta) = -V\int\frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}\beta\omega_k + \ln(1-e^{-\beta\omega_k})\right]$$

$$F(T,V) = T\ln Z(\beta),\ p = -\partial F(T,V)/\partial V,\ S = -\frac{\partial F(T,V)}{\partial T}, U = F+TS$$

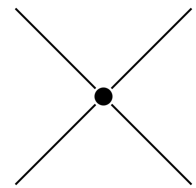
Massless case $(m=0\to\omega_k=k)$:

$$p = \int\frac{d^3k}{(2\pi)^3}\left[\frac{1}{2}k + T\ln(1-e^{-\beta k}))\right] = \frac{\pi^2T^4}{90}$$

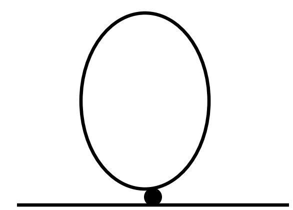
$$\epsilon(T) = U(T,V)/V = 3p,\ s(T) = S(T,V)/V = 4/3\epsilon(T)$$

Perturbative expansion:

$$e^{-S_E(\beta)} \simeq e^{-S_E^0(\beta)} \left(1 - \frac{g^2}{4!} \int d^4 x_E \phi^4(x_E)\right)$$



$$= \frac{g^2}{2} T \sum_n \int \frac{d^3k}{(2\pi)^3} \Delta_0(i\omega_n, k)$$

$\Pi =$ 

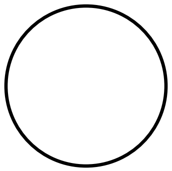
$$= \frac{g^2}{2} \int \frac{d^3k}{(2\pi)^3} \Delta_0(\tau = 0, k)$$

UV divergent part renormalized as at T=0

$$= \frac{g^2}{2} \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{2}\omega_k^{-1/2} + n(\omega_k)\right)$$

Massless case $(\omega_k = k)$: $\Pi = \dfrac{g^2 T^2}{24}$

Particles aquire a thermal mass !



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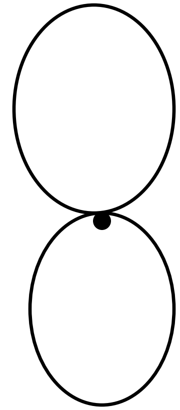
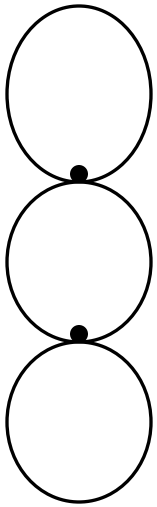


$$T\sum_n \int \frac{d^3k}{(2\pi)^3} \ln \Delta_0(i\omega_n,k) \quad + \quad \frac{g^2}{8}\left(T\sum_n \int \frac{d^3k}{(2\pi)^3} \Delta_0(i\omega_n,k)\right)^2$$

Massless case:

$$P = \frac{\pi^2 T^4}{90}\left(1 - \frac{5g^2}{64\pi^2}\right)$$

Infrared problems at finite temperature: the next-to-leading correction to the pressure is not of order λ^2 but $\lambda^{3/2}$ from $m = 0$!



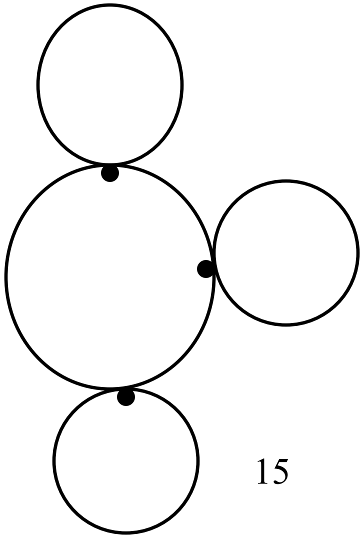
$$\frac{g^4}{16}\left(T\sum_l \int \frac{d^3p}{(2\pi)^3}\frac{1}{(p^2+\omega_l^2)^2}\right)\left(T\sum_n \int \frac{d^3k}{(2\pi)^3}\Delta_0(i\omega_n,k)\right)^2$$

the $l = 0$ term is IR divergent as $\int d^3p/p^4$

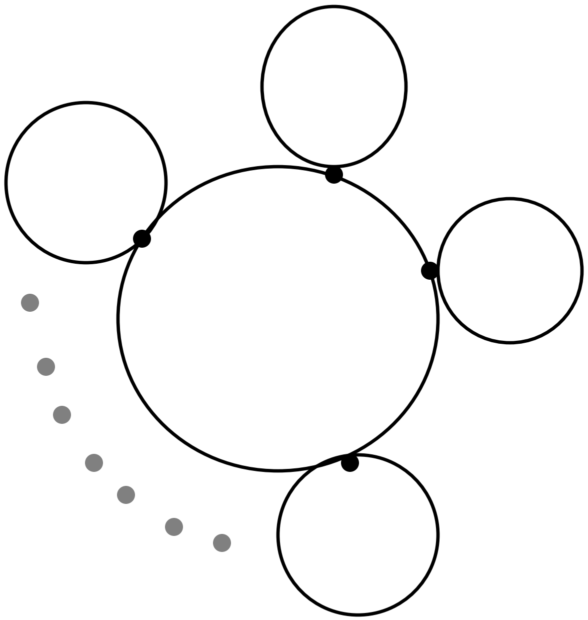
In the 4-loop diagram

$$\sim g^6\left(T\sum_l \int \frac{d^3p}{(2\pi)^3}\frac{1}{(p^2+\omega_l^2)^3}\right)\left(T\sum_n \int \frac{d^3k}{(2\pi)^3}\Delta_0(i\omega_n,k)\right)^3$$

the $l = 0$ term is IR divergent as $\int d^3p/p^6$



We need to resum all diagrams of the following type (ring diagrams)



$$= -\frac{1}{2}VT\sum_n \int \frac{d^3p}{(2\pi)^3} \sum_{N=2}^{\infty} \frac{1}{N}\left(\frac{(-\Pi)}{(\omega_n^2+p^2)}\right)^N$$

$$= -\frac{1}{2}VT\sum_n \int \frac{d^3p}{(2\pi)^3} \left(\ln\left(1+\frac{\Pi}{p^2+\omega_n^2}\right) - \frac{\Pi}{p^2+\omega_n^2}\right)$$

keeping only the contribution from (IR sensitive) $n=0$ and using $\Pi = g^2T^2/24$ we get

$$F_{ring} = \frac{VT^4}{12\pi}\left(\frac{g^2}{24}\right)^{3/2}$$

Collective effects in the medium have to be taken into account at all orders !

$$P = \frac{\pi^2T^4}{90}\left(1 - \frac{15}{8}\left(\frac{g^2}{24\pi^2}\right) + \frac{15}{2}\left(\frac{g^2}{24\pi^2}\right)^{3/2} + \ldots\right)$$

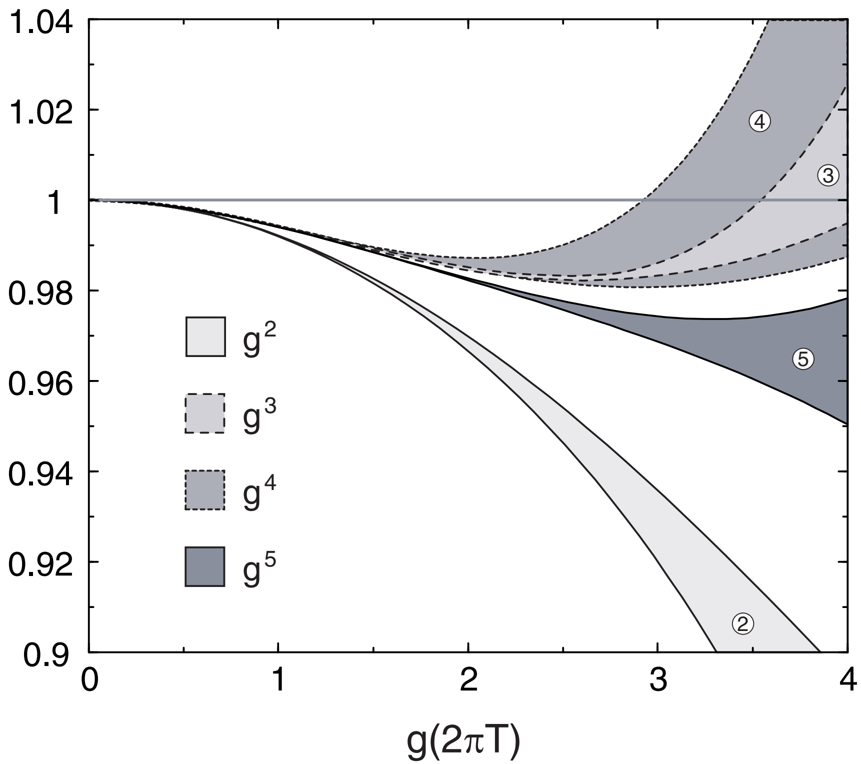
Resummation of ring diagrams corresponds to using massive propagators in *n=0* modes

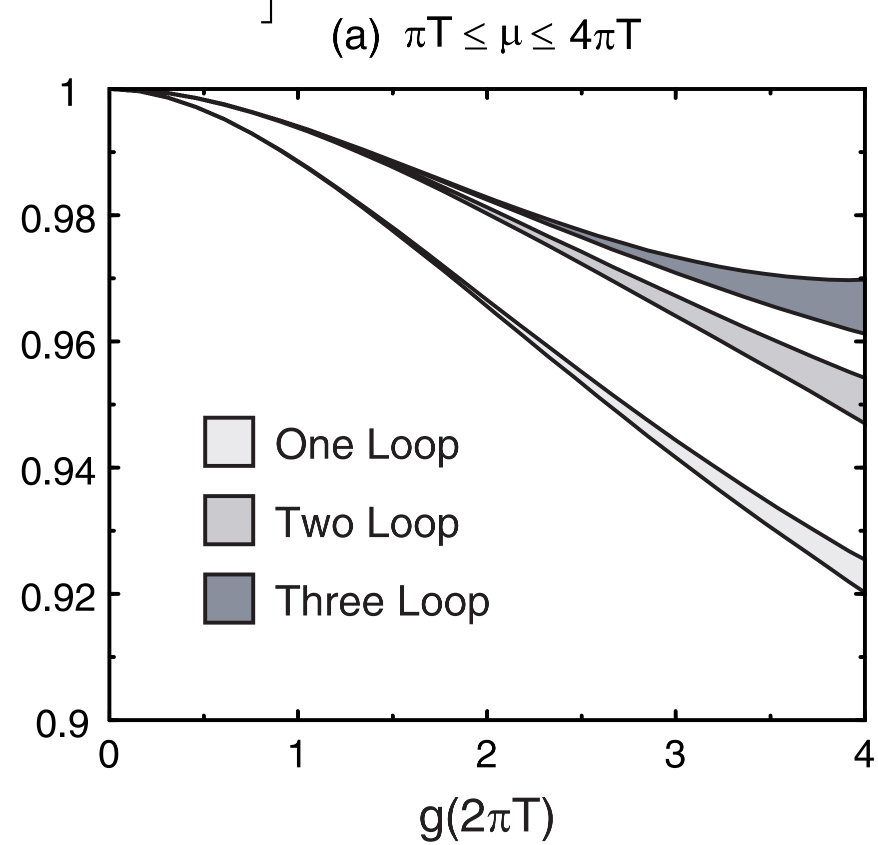
Resummation and screened perturbation theory

$$\mathcal{P} = \mathcal{P}_{\text{ideal}}\left[1 - \frac{5}{4}\alpha + \frac{5\sqrt{6}}{3}\alpha^{3/2} + \frac{15}{4}\left(\log\frac{\mu}{2\pi T} + 0.40\right)\alpha^2 \right.$$
$$\left. - \frac{15\sqrt{6}}{2}\left(\log\frac{\mu}{2\pi T} - \frac{2}{3}\log\alpha - 0.72\right)\alpha^{5/2} + \mathcal{O}(\alpha^3\log\alpha)\right]$$

Arnold, Zhai, PRD50 (94) 7603

$\alpha = g^2/(16\pi^2)$





Karsch, Patkos, PP, PLB 401 (97) 69 ,
Andersen, Braaten, Strickland PRD63 (01) 105008

Dirac Fields at finite temperature

Free Dirac Hamiltonian

$$\widehat{H} = \int d^3x \psi^\dagger \gamma_0 (-i\gamma \cdot \nabla + m)\psi(x)$$

$\widehat{Q} = \int d^3x \psi^\dagger \gamma^0 \psi$ -conserved charge

Canonical and grand canonical partition functions

$$Z_{can} = \mathrm{Tr} e^{-\beta \widehat{H}}, \quad Z = \mathrm{Tr} e^{-\beta \widehat{H} - \mu \widehat{Q}}$$

$\mathrm{Tr} A = \int \eta^* \eta e^{-\eta^* \eta} < -\eta | A | \eta >$

$$Z = \int \mathcal{D}(\psi^*_\alpha, \ \psi_\alpha) \exp\left(- \int_0^\beta d\tau \left[\psi_\alpha (\partial_\tau - \mu)\psi_\alpha + H(\psi^*_\alpha, \psi_\alpha) \right] \right)$$

fermion fields anticommute $\Rightarrow \psi_\alpha(\beta) = -\psi_\alpha(0)$

$\Rightarrow \quad \omega_n = (2n+1)\pi T, n = 0, \ \pm 1, \pm 2 \ldots$

$\prod_{i=1}^{N} \int \eta_i^* \eta_i e^{-\eta_j^* D_{ij} \eta_i} = detD$

$$Z = \mathrm{Tr} \ln \left[-i\beta((-i\omega_n + \mu) - \gamma^0 \gamma \cdot k - m\gamma_0) \right]$$

$$= 2 \sum_n \sum_k \ln \left[\beta^2 \left((\omega_n + i\mu)^2 + \omega_k^2 \right) \right]$$

$$2V \int \frac{d^3k}{(2\pi)^3} \left[\beta \omega_k + \ln(1 + e^{-\beta(\omega_k - \mu)}) + \ln(1 + e^{-\beta(\omega_k + \mu)}) \right]$$

Gauge field at finite temperature

$$Z(\beta) = \int \mathcal{D}(A^a_\mu, \eta_b, \eta_c) \exp\left[-\int_0^\beta d^4x_E \mathcal{L}_{eff}(x)\right]$$

$$\mathcal{L}_{eff}(x) = \frac{1}{4}F^a_{\mu\nu}(x)F^a_{\mu\nu}(x) + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + \bar{\eta}_a(x)\left[\partial^2\delta_{ab} + f_{abc}A^c_\mu\partial_\mu\right]\eta_b(x)$$

$$A_\mu(0,x) = A_\mu(\beta,x), \quad \eta_a(0,x) = -\eta_a(\beta,x)$$

$$\ln Z(\beta) = -\frac{1}{2} \times 4(N_c^2-1)\sum_n\sum_k \ln[\beta^2(\omega_n^2+k^2)] +$$

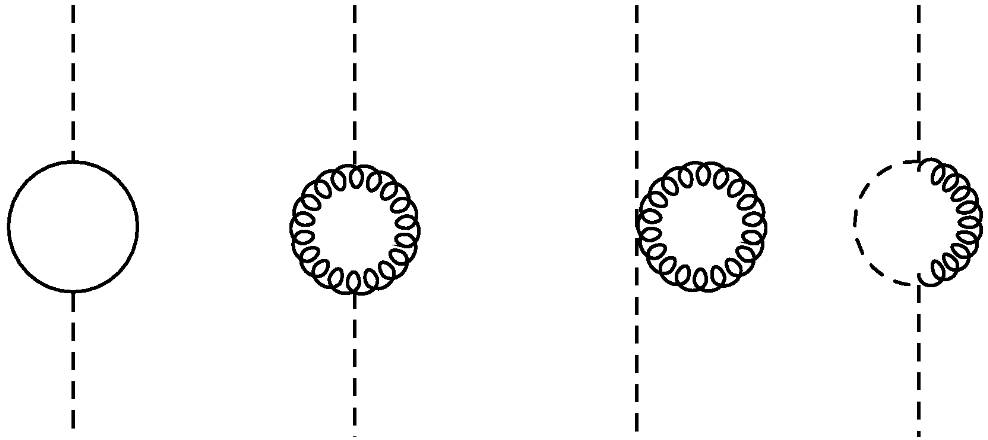
4 gluons

$$(N_c^2-1)\sum_n\sum_k \ln[\beta^2(\omega_n^2+k^2)]$$

ghosts

$$p(T) = 2(N_c^2-1)\frac{\pi^2 T^4}{90}$$

Gulon self energy and color screening in perturbation theory



Gluon self energy in the static limit:

$$\Pi_{00}(\omega_n = 0, k \to 0) = m_D^2 = (\frac{N_c}{3} + \frac{N_f}{6})g^2T^2$$

$$\Pi_{ii}(\omega_n = 0, k \to 0) = 0$$

$$V(r) \simeq -\frac{N_c^2 - 1}{2N_c} g^2 \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \frac{1}{k^2 + \Pi_{00}(k)} = -\frac{N_c^2 - 1}{2N_c} \alpha_s \frac{e^{-m_D r}}{r}$$

chromo-electric fields are screened but chromo-magnetic fields are not screened (at least in perturbation theory)

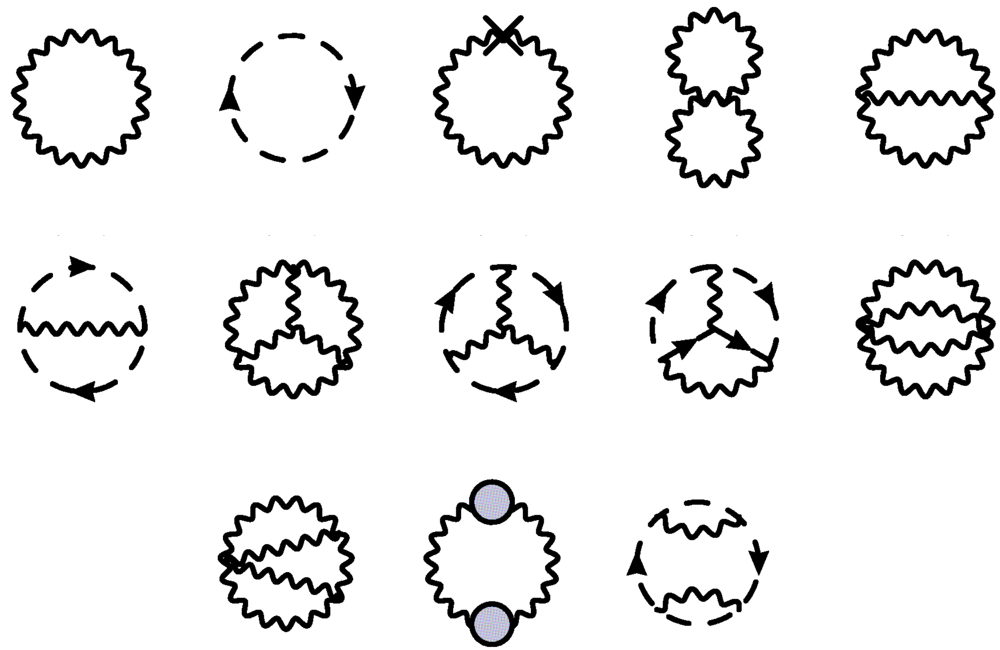
QCD at high temperatures

Because of asymptotic freedom thermodynamics quantities can be calculated in perturbation theory if $T \gg \Lambda_{QCD}$, at least in principle

Pressure has been calculated to 3-loop order

Arnold, Zhai, Phys.Rev. D51 (1995) 1906, Kastening, Zhai, Phys.Rev. D52 (1995) 7232

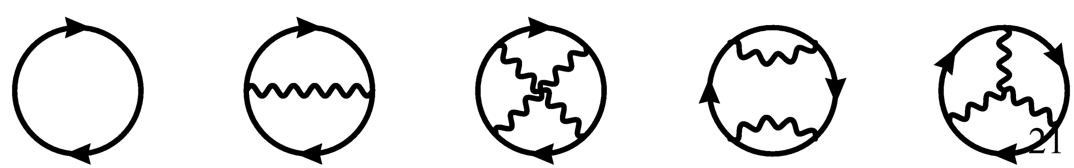
Bosonic contribution:



Static resummation:

$$\frac{1}{2} m_D^2 A_0^2 \delta_{\omega_n,0}$$

Fermionic contribution:



$$
\begin{aligned}
F \;=\; d_{\mathrm{A}} T^4 \frac{\pi^2}{9}\Big\{ & -\frac{1}{5}\left(1+\frac{7 d_{\mathrm{F}}}{4 d_{\mathrm{A}}}\right)+\left(\frac{g(\bar\mu)}{4\pi}\right)^2\left(C_{\mathrm{A}}+\tfrac{5}{2}S_{\mathrm{F}}\right) \\
& -48\left(\frac{g(\bar\mu)}{4\pi}\right)^3\left(\frac{C_{\mathrm{A}}+S_{\mathrm{F}}}{3}\right)^{3/2}-48\left(\frac{g}{4\pi}\right)^4 C_{\mathrm{A}}(C_{\mathrm{A}}+S_{\mathrm{F}})\ln\left(\frac{g}{2\pi}\sqrt{\frac{C_{\mathrm{A}}+S_{\mathrm{F}}}{3}}\right) \\
& +\left(\frac{g}{4\pi}\right)^4\Big[C_{\mathrm{A}}^2\left(\frac{22}{3}\ln\frac{\bar\mu}{4\pi T}+\frac{38}{3}\frac{\zeta'(-3)}{\zeta(-3)}-\frac{148}{3}\frac{\zeta'(-1)}{\zeta(-1)}-4\gamma_{\mathrm{E}}+\frac{64}{5}\right) \\
& \qquad +C_{\mathrm{A}}S_{\mathrm{F}}\left(\frac{47}{3}\ln\frac{\bar\mu}{4\pi T}+\frac{1}{3}\frac{\zeta'(-3)}{\zeta(-3)}-\frac{74}{3}\frac{\zeta'(-1)}{\zeta(-1)}-8\gamma_{\mathrm{E}}+\frac{1759}{60}+\frac{37}{5}\ln 2\right) \\
& \qquad +S_{\mathrm{F}}^2\left(-\frac{20}{3}\ln\frac{\bar\mu}{4\pi T}+\frac{8}{3}\frac{\zeta'(-3)}{\zeta(-3)}-\frac{16}{3}\frac{\zeta'(-1)}{\zeta(-1)}-4\gamma_{\mathrm{E}}-\frac{1}{3}+\frac{88}{5}\ln 2\right) \\
& \qquad +S_{2\mathrm{F}}\left(-\frac{105}{4}+24\ln 2\right)\Big] \\
& -\left(\frac{g}{4\pi}\right)^5\left(\frac{C_{\mathrm{A}}+S_{\mathrm{F}}}{3}\right)^{1/2}\Big[C_{\mathrm{A}}^2\left(176\ln\frac{\bar\mu}{4\pi T}+176\gamma_{\mathrm{E}}-24\pi^2-494+264\ln 2\right) \\
& \qquad +C_{\mathrm{A}}S_{\mathrm{F}}\left(112\ln\frac{\bar\mu}{4\pi T}+112\gamma_{\mathrm{E}}+72-128\ln 2\right) \\
& \qquad +S_{\mathrm{F}}^2\left(-64\ln\frac{\bar\mu}{4\pi T}-64\gamma_{\mathrm{E}}+32-128\ln 2\right) \\
& \qquad -144 S_{2\mathrm{F}}\Big]+O(g^6)\Big\},
\end{aligned}
$$

Running coupling:

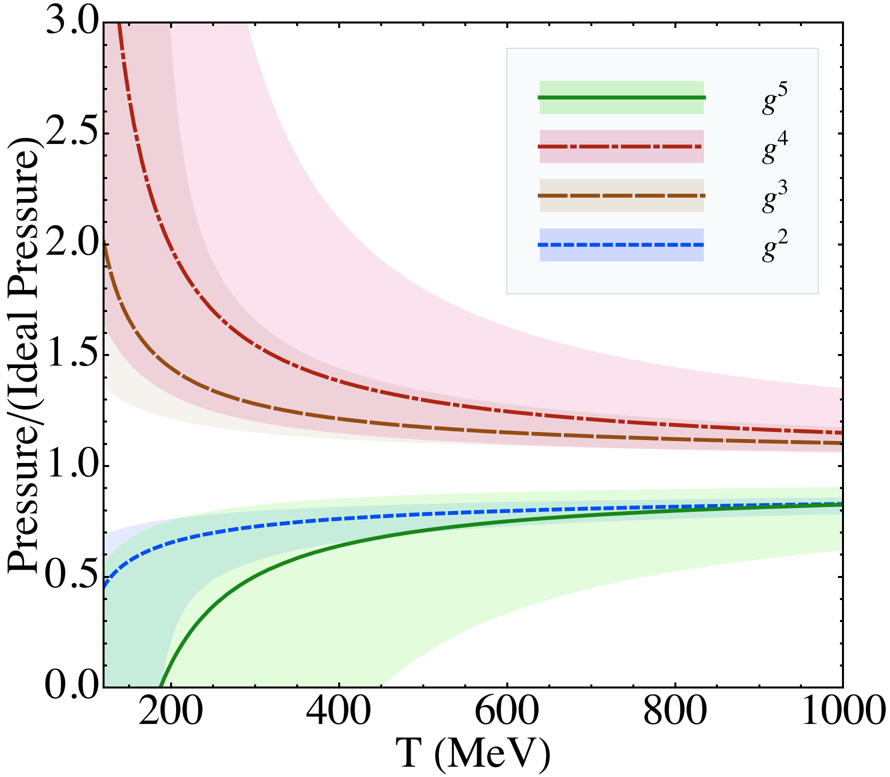
$$g^2(\mu) = g^2(1-\beta_0\ln(\mu/\mu_0))$$

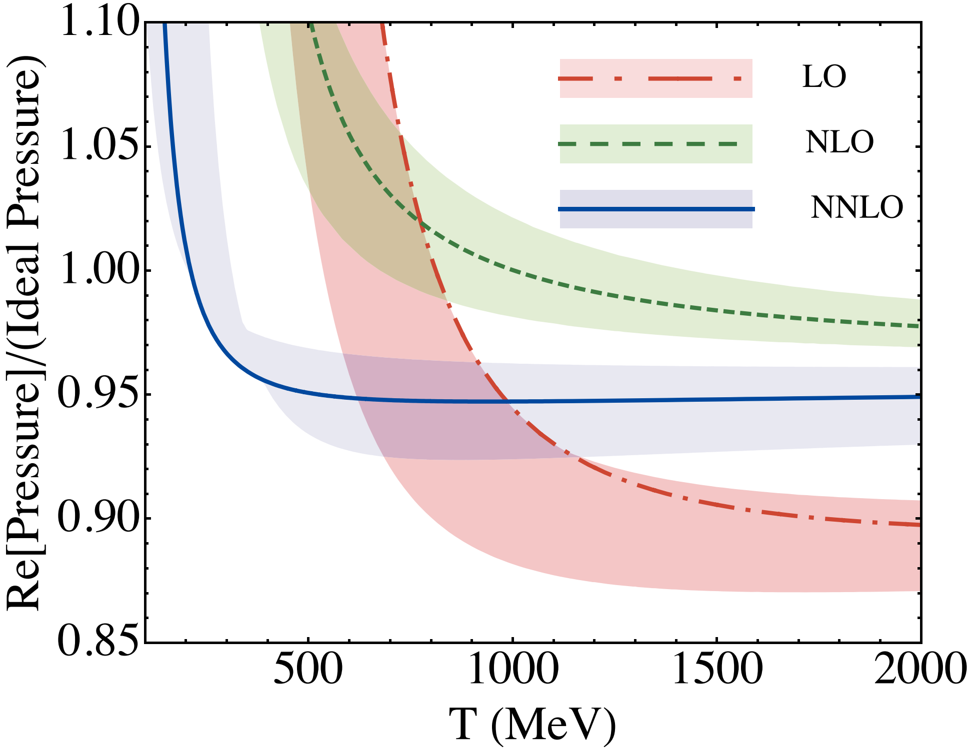
$$\beta_0 = \frac{1}{3(4\pi)^2}(11C_{\mathrm{A}}-2N_f)$$

$$d_{\mathrm{A}} = N_c^2-1\,,\quad C_{\mathrm{A}} = N_c\,,\quad d_{\mathrm{F}} = N_c N_{\mathrm{f}}\,,\quad S_{\mathrm{F}} = \frac{1}{2}N_{\mathrm{f}}\,,\quad S_{2\mathrm{F}} = \frac{N_c^2-1}{4N_c}N_{\mathrm{f}}\,.$$

Arnold, Zhai, Phys.Rev. D51 (1995) 1906, Kastening, Zhai, Phys.Rev. D52 (1995) 7232

Convergence of perturbation theory and HTL





Andersen, Leganger, Strickland, Su, JHEP 1108 (2011) 053

The same poor convergence of perturbative series for the pressure as in scalar field theory, the problem is largely due to odd powers in g

Hard Thermal Loop (HTL) resumed perturbation theory absorbs odd powers in g to lower order contributions

Pressure at order g^6 and magnetic mass

Infrated sensitive contribution to the partition function at $l+1$-loop order:

$$g^{2l}\left(T\int d^3p\right)^{l+1} p^{2l}(p^2+m_{mag}^2)^{-3l}$$

$$g^{2l}T^4, \quad l=1,2$$

$$g^6T^4\ln(T/m_{mag}), \quad l=3$$

$$g^6T^4(g^2T/m_{mag})^{l-3}, \quad l>3$$

$m_{mag}\sim g^2T \Rightarrow$ infinitely many diagramls contribute at g^6 order !

Confining nature of static chromomagnetic fields at high T

In practice g is not very small : $g(\mu=10^2\mathrm{GeV})=\sqrt{4\pi\alpha_s(\mu=10^2\mathrm{GeV})}\simeq 1$ $g(\mu=10^{16}\mathrm{GeV})\simeq 1/2$

Dimensional reduction at high temperatures

Decomposition in Matsubara modes

$$\phi(\tau,x)=\sum_n e^{i\omega_n\tau}\phi_n(x)$$

$$S_E=\int_0^\beta d\tau\int d^3x[(\partial_\mu\phi)^2+V(\phi)]\rightarrow\int d^3x(\sum_n(\partial_i\phi_n(x))^2+(2\pi Tn)^2\phi_n(x))+V(\phi_n))$$

integrate out all $n\neq 0$ modes ⇨ mass term for n=0 mode $\quad F_{\mu\nu}=D_\mu A_\mu-D_\nu A_\mu$

Effective hight T theory for QCD $2\pi T\gg gT\gg g^2T$: $\quad A_\mu\rightarrow\beta^{1/2}A_\mu$

$$S_{eff}=\int d^3x\left(\frac{1}{2}TrF_{ij}^2+Tr(D_iA_0)^2+m_D^2TrA_0^2+\lambda_3(TrA_0^2)^2\right)$$

$$F_{ij}=\partial_iA_j-\partial_jA_i+ig_3[A_i,A_j],\ \ D_iA_0=\partial_iA_0+ig_3[A_i,A_0]$$

EQCD

the parameters $g_3^2\sim g^2T$, $m_D\sim gT$ and $\lambda_3\sim g^4T$ can be computed perturbatively to any order.

The effective theory is confining and non-perturbative at momentum scales $< g_3^2$ but can be solved on the lattice to calculate the weak coupling expansion of the pressure and other quantities

Integrate out A_0

⇩

MQCD

Braaten, Nieto, PRD 51 (95) 6990, PRD 53 (96) 3421

Kajantie et al, NPB 503 (97) 357, PRD 67 (03) 105008

even powers in g $\quad$ odd powers in g $\quad$ 3d YM theory

$$F=F(non\text{-}static)+T\,F^{3d}$$

$$F^{3d}\sim g_3{}^6$$

Relativistic Virial Expansion and Hadron Resonance Gas

Density of hadrons is small at low temperature
Relativistic virial expansion : compute thermodynamic quantities in terms as a gas of non-interacting particles and S – matrix
Dashen, Ma, Bernstein, Phys. Rev. 187 (1969) 345

Free gas of stables hadros: π, K, N

interactions

$$\ln Z = \ln Z_0 + \sum_{i_1, i_2} e^{\mu_{i_1}/T} e^{\mu_{i_2}/T} b(i_1, i_2)$$

$$b(i_1, i_2) = \frac{V}{4\pi i} \int \frac{d^3p}{(2\pi)^3} \int dE e^{-(p^2+E^2)^{1/2}/T} \sum_{final} \left[AS(S^{-1}\frac{\partial S}{\partial E} - \frac{\partial S^{-1}}{\partial E} S) \right]$$

(anti) symmetrization (spin-statistics)

Elastic scattering dominates at low T (final state = initial state)

$$S(E) = \sum_{l,I}{}' (2l+1)(2I+1) \exp(2i\delta_l^I(E))$$

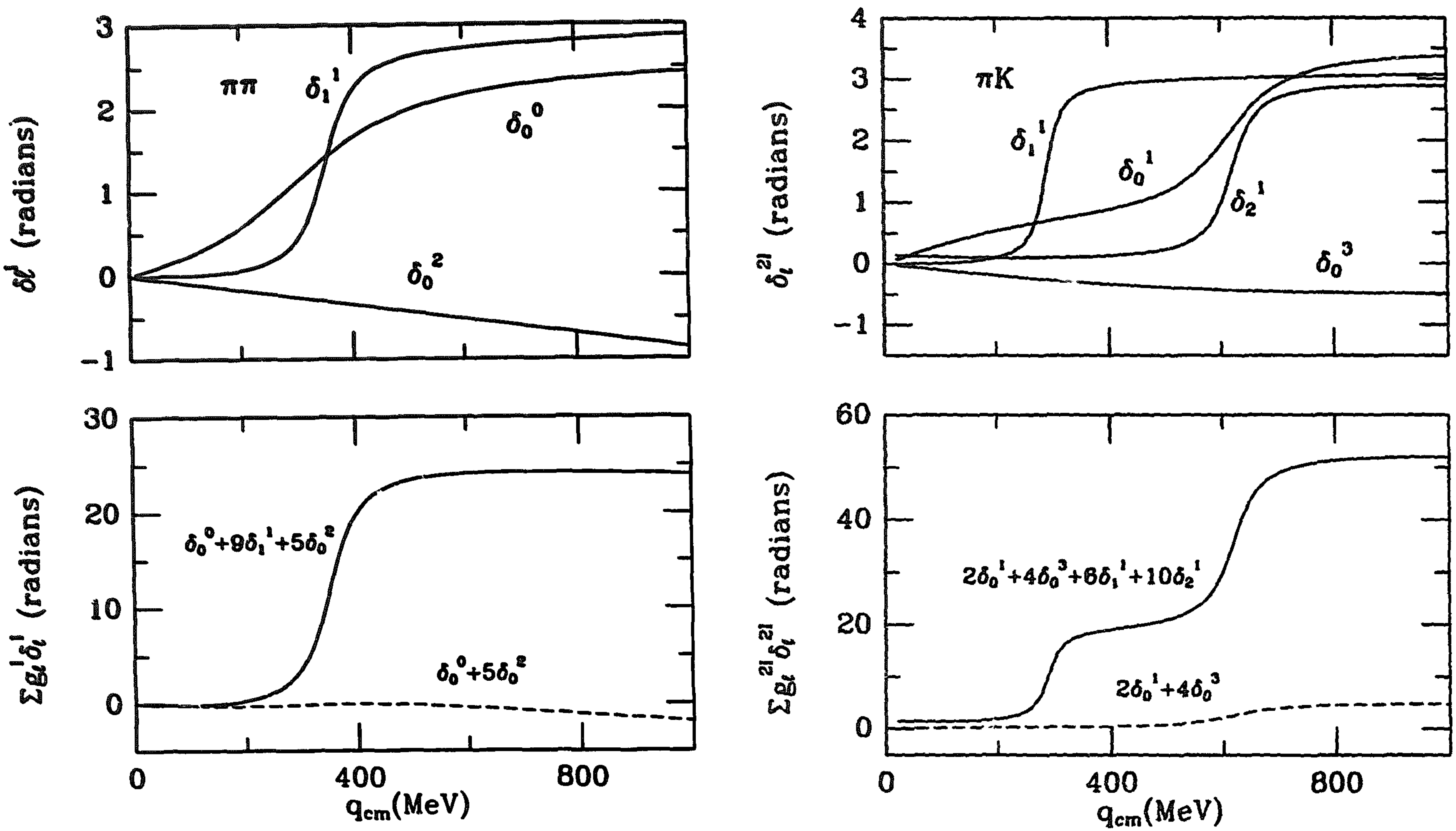
Partial wave decomposition

perform the integral over the 3-momentum

$$b_2 = \frac{T}{2\pi^3} \int_M^\infty dE E^2 K_2(E/T) \sum\nolimits'_{l,I} (2l+1)(2I+1) \frac{\partial \delta_l^I(E)}{\partial E}$$

of the pair at threshold invariant mass

Use experimental phase shifts to determine b_2 , Venugopalan, Prakash, NPA546 (1992) 718



After summing all the channels only resonance contributions survives in

$$\sum_{l,I}'(2l+1)(2I+1)\frac{\partial\delta_l^I(E)}{\partial E}$$

Interacting hadron gas = non-nteracting gas of hadrons and resonances

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D. Teaney, arXiv:0905:2433

P. Petreczky, arXiv:1203:5320

H.T. Ding, F. Karsch, S. Mukherjee, arXiv:1504.05274