Deconfinement and chiral transitions

Chiral crossover at physical quark masses

Chiral phase transition in 2+1 flavor QCD

Taylor expansion in chemical potential, fluctuations of conserved charges, Equation of State at non-zero baryon density

Deconfinement and chromo-electric screening in QCD

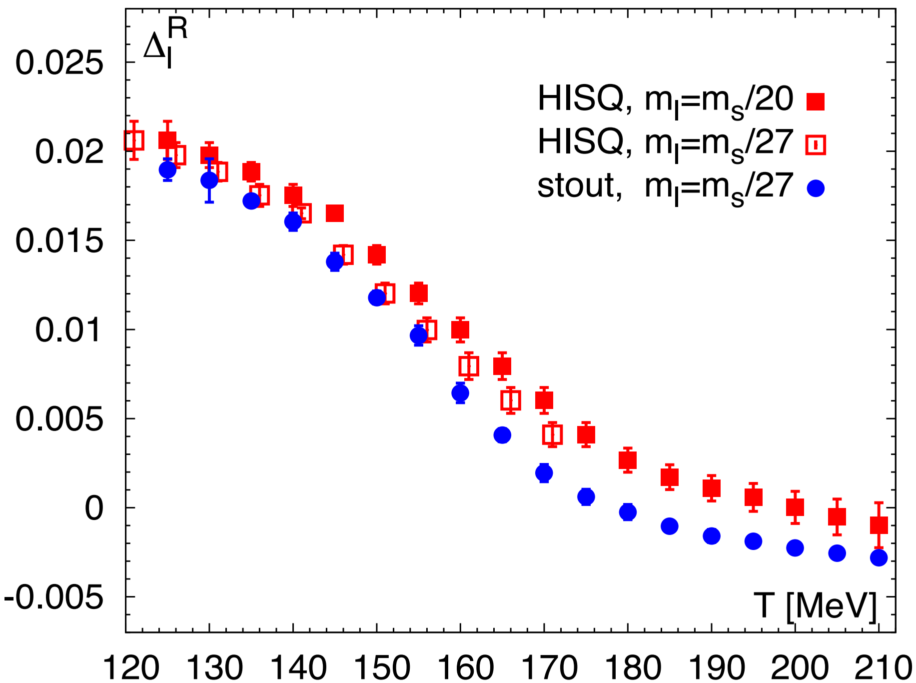
Chromo-magnetic screening, spatial string tension and EQCD

The chiral transition at non-zero temperature

Renormalized chiral condensate

$$\langle\bar\psi\psi\rangle \Rightarrow \Delta_l^R(T) = \\ = m_s r_1^4(\langle\bar\psi\psi\rangle_T - \langle\bar\psi\psi\rangle_{T=0}) + d, \\ d = m_s r_1^4\langle\bar\psi\psi\rangle_{T=0}^{m_q=0}, \quad r_1 = 0.3106\text{fm}$$

Bazavov et al (HotQCD), PRD85 (2012) 054503;
Bazavov et al, PRD 87(2013)094505,
Borsányi et al, JHEP 1009 (2010) 073

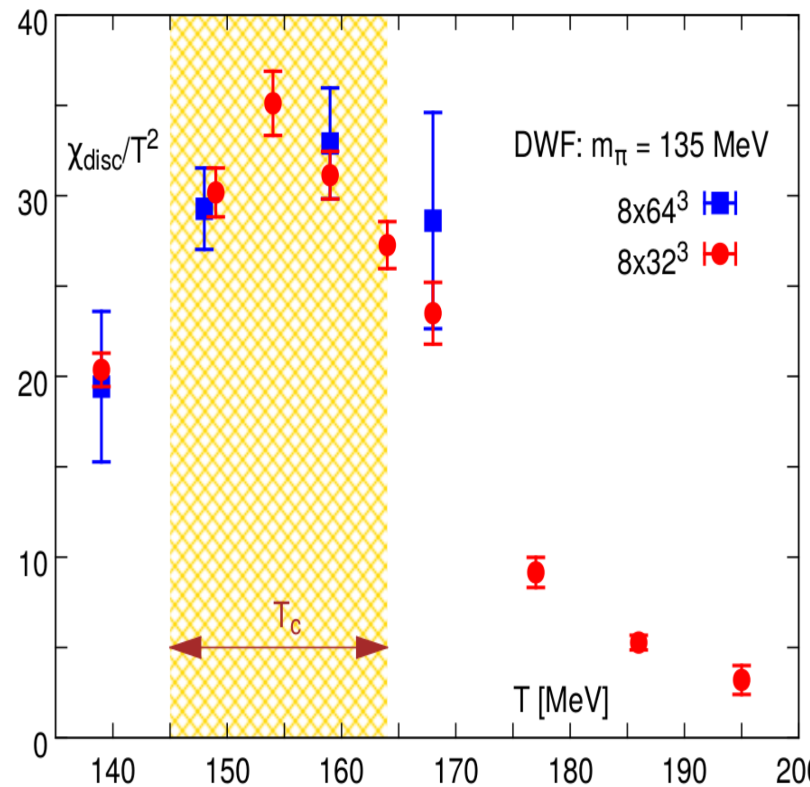


$T_c = (154 \pm 8 \pm 1(scale))$MeV

Fluctuations of the order parameter:

$$\chi_{disc} = VT^{-1}\left(\langle(\bar\psi\psi)^2\rangle - \langle\bar\psi\psi\rangle^2\right)$$

Bhattacharya et al (HotQCD), PRL 113 (2014)082001



$T_c = (155 \pm 8 \pm 1)$MeV

No increase with the volume
⇒ Crossover transition

O(N) scaling and the chiral transition temperature

For sufficiently small m_l and in the vicinity of the transition temperature:

$$f(T,m_l) = -\frac{T}{V}\ln Z = f_{reg}(T,m_l)+f_s(t,h),\ t=\frac{1}{t_0}\left(\frac{T-T_c^0}{T_c^0}+\kappa\frac{\mu_q^2}{T^2}\right),\ H=\frac{m_l}{m_s}, h=\frac{H}{h_0}$$

governed by universal *O(4)* scaling $M=-\frac{\partial f_s(t,h)}{\partial H}=h^{1/\delta}f_G(z),\ z=t/h^{1/\beta\delta}$

$$\langle q\bar{q}\rangle = T(\partial \ln Z)/\partial m_f$$

T_c^0 is critical temperature in the mass-less limit, h_0 and t_0 are scale parameters

$SU_A(2)\sim O(4)$

Pseudo-critical temperatures for non-zero quark mass are defined as peaks in the response functions (susceptibilities) :

$$\chi_{m,l}=\frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l^2}\sim m_l^{1/\delta-1} \quad \chi_{t,l}=\frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_l \partial t}\sim m_l^{\frac{\beta-1}{\beta\delta}} \quad \chi_{t,t}=\frac{T}{V}\frac{\partial^2 \ln Z}{\partial t^2}\sim |t|^{-\alpha}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$T_{m,l}$ = $T_{t,l}$ = $T_{t,t}$ = T_c^0

in the zero quark mass limit

$$\frac{\chi_{l,m}}{T^2}=\frac{T^2}{m_s^2}\left(\frac{1}{h_0}h^{1/\delta-1}f_\chi(z)+reg.\right)$$

universal scaling function has a peak at $z=z_p$

Caveat : staggered fermions O(2)

$m_l\to 0,\ a>0,$

proper limit $a\to 0$, before $m_l\to 0$

$$T_c(H)=T_{m,l}=T_c^0+T_c^0\frac{z_p}{z_0}H^{1/(\beta\delta)}+\ldots$$

The chiral cross-over temperature for physical masses

Chiral order parameter:

$$\Sigma = \frac{1}{f_K^4}\left[m_s\langle \bar{u}u + \bar{d}d\rangle - (m_u + m_d)\langle \bar{s}s\rangle\right] \qquad \langle q\bar{q}\rangle = T(\partial \ln Z)/\partial m_f$$

and the corresponding susceptibilities:

$$\chi^{\Sigma} = m_s\left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\Sigma \qquad \chi = \frac{m_s^2}{f_K^4}\left[\langle(\bar{u}u + \bar{d}d)^2\rangle - (\langle \bar{u}u\rangle + \langle \bar{d}d\rangle)^2\right]$$

For non-zero chemical potential we use Taylor expansion

$$\Sigma(T,\mu_X) = \sum_{n=0}^{\infty}\frac{C_{2n}^{\Sigma}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \quad \chi(T,\mu_X) = \sum_{n=0}^{\infty}\frac{C_{2n}^{\chi}(T)}{(2n)!}\left(\frac{\mu_X}{T}\right)^{2n} \qquad C_0^{\Sigma} = \Sigma \quad C_0^{\chi} = \chi$$

Derivatives in μ_X^2 are similar to derivatives in T *e.g.* $\partial_T C_0^{\chi} \sim C_2^{\chi}$

$\Rightarrow$ the following quantities will peak at T_c

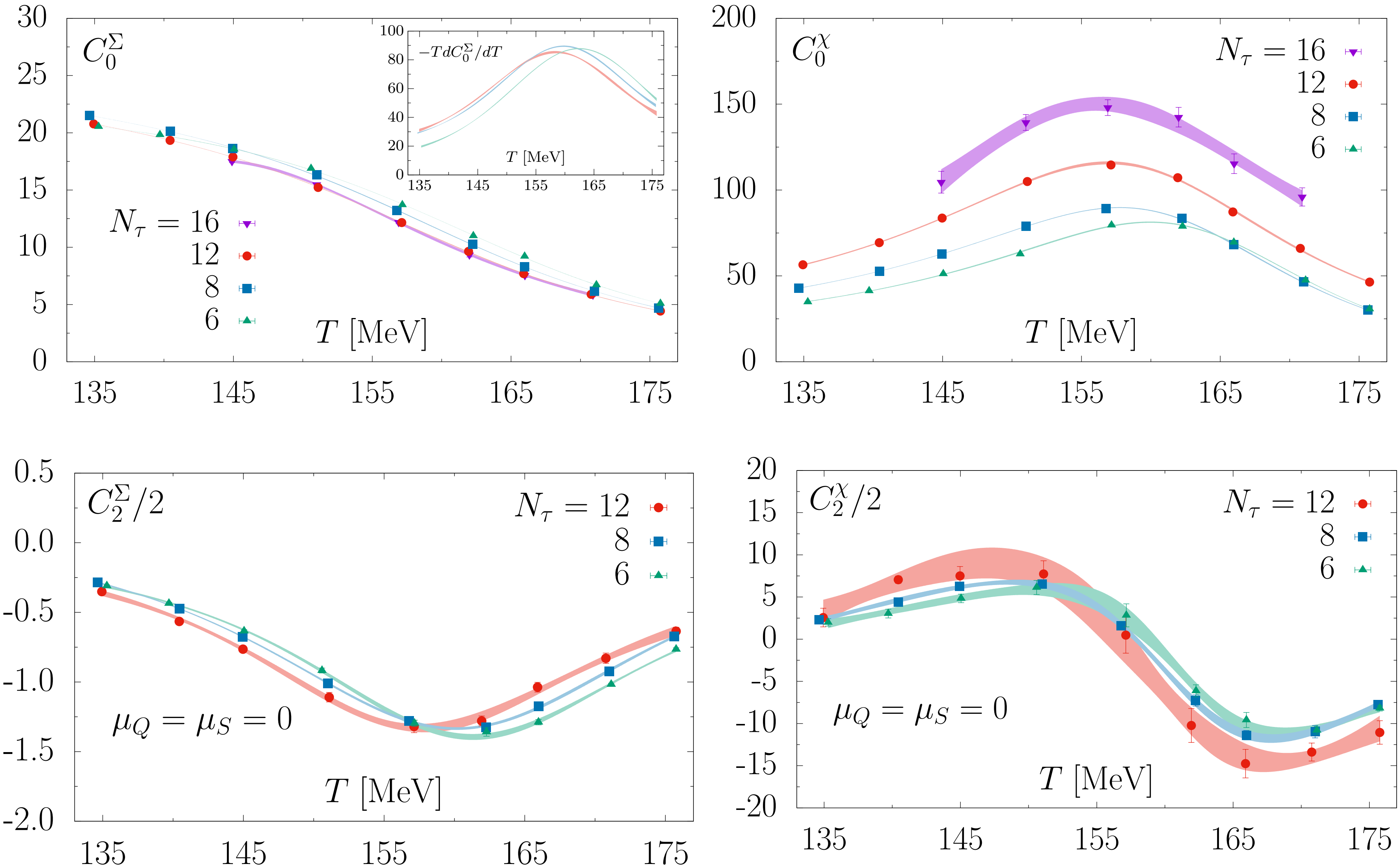
$\chi^{\Sigma},\ C_0^{\chi}(T) \sim \chi_{l,m} \qquad \partial_T C_0^{\Sigma},\ C_2^{\Sigma}(T) \sim \chi_{t,m}$ HotQCD, PLB795 (2019) 15

5 different definitions of T^{pc}:

$$\partial_T C_0^{\chi} = 0,\ \partial_T C_0^{\Sigma} = 0\ , C_2^{\chi} = 0 \qquad \partial_T^2 C_0^{\Sigma} = 0,\ \partial_T C_2^{\Sigma} = 0$$

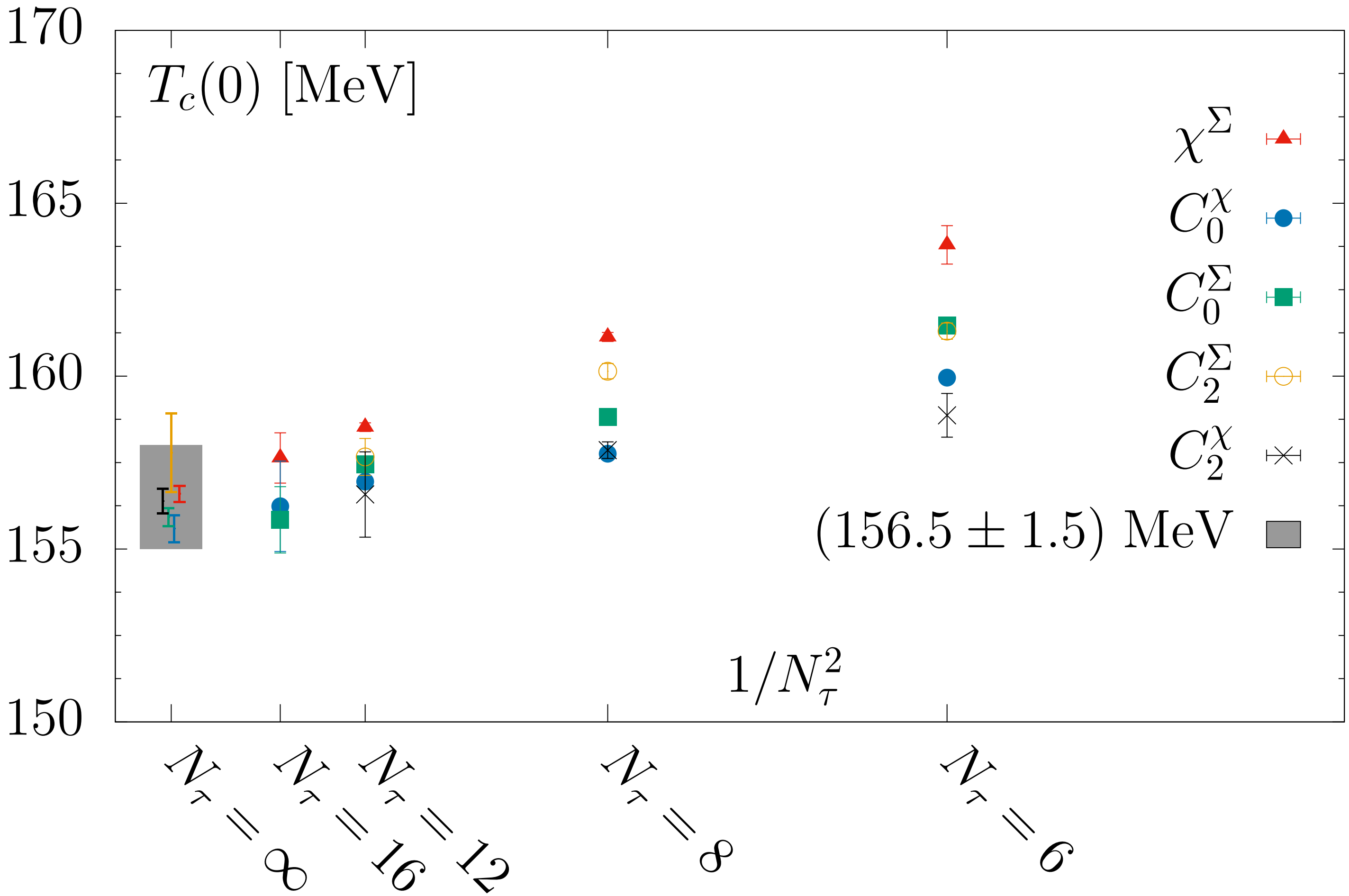
The 5 different T_c values reduce to $T_{l,m}$ and T_{let} if regular part is zero

Lattice calculations based on 100K - 500 K configurations, $N_\tau = 6 - 12$, and 4K configurations for $N_\tau = 16$



HotQCD, PLB795 (2019) 15

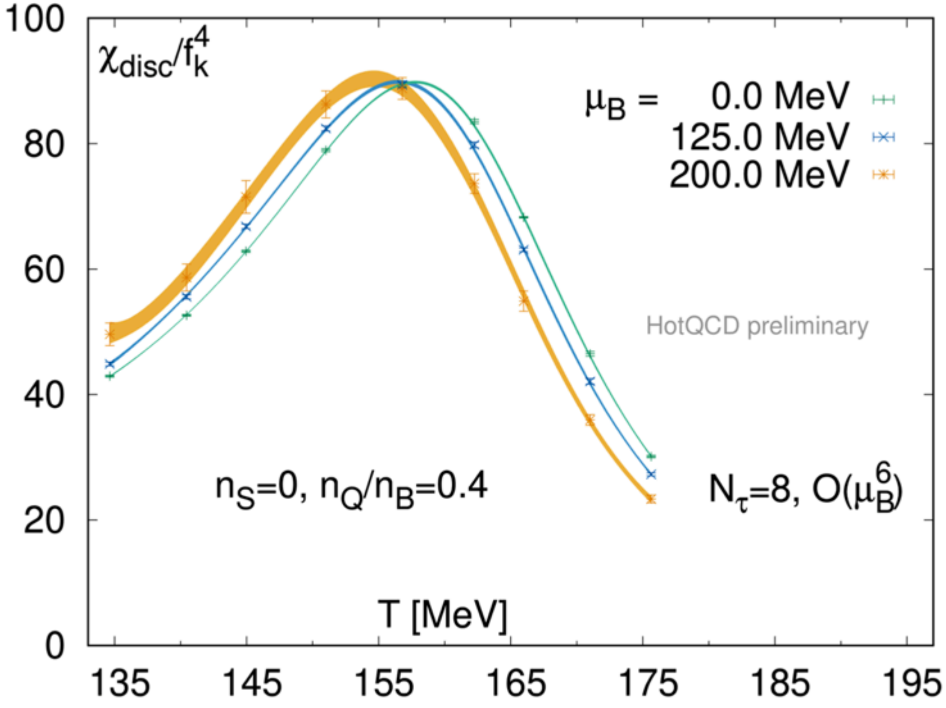
Different definitions of T_c surprisingly agree in the continuum limit and we for zero chemical potential we get $T_c = 156 \pm 1.5$ MeV

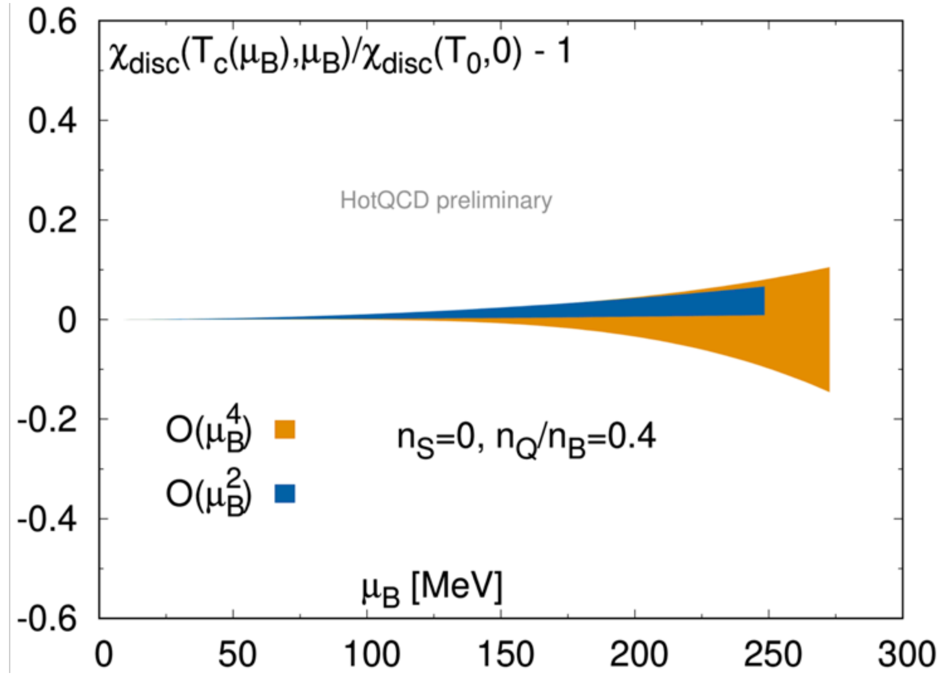


HotQCD, PLB795 (2019) 15

The chiral susceptibility at baryon density non-zero density

Conditions in heavy ion collisions: $n_B > 0$, $n_S = 0$, $n_Q = 0.4 n_B$ (for Au, Pb)

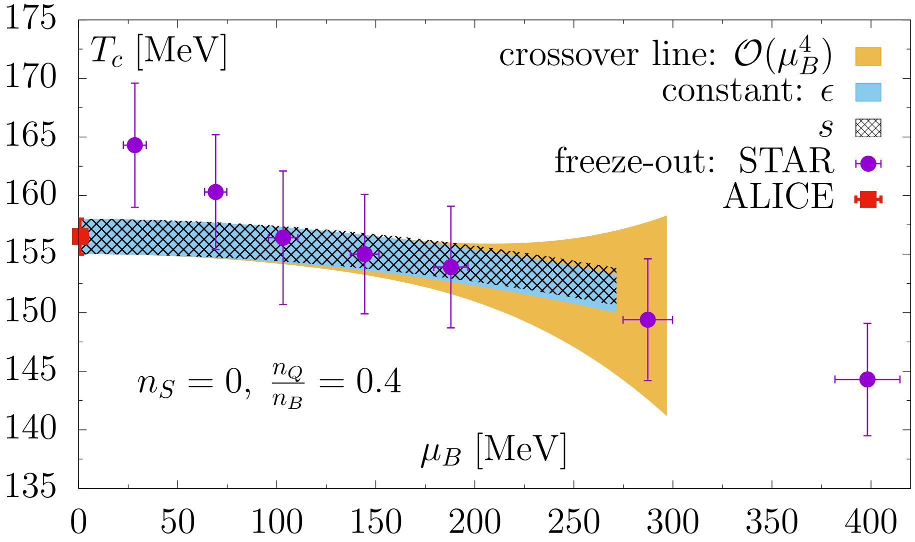




little change in peak-height & width with increasing baryon chemical potential: no indication of a stronger transition becoming stronger

The chiral cross-over temperature at non-zero density

$$T_c(\mu_B) = T_c(0)\left[1 - \kappa_2^B\left(\frac{\mu_B}{T_c(0)}\right)^2 - \kappa_4^B\left(\frac{\mu_B}{T_c(0)}\right)^4\right]$$

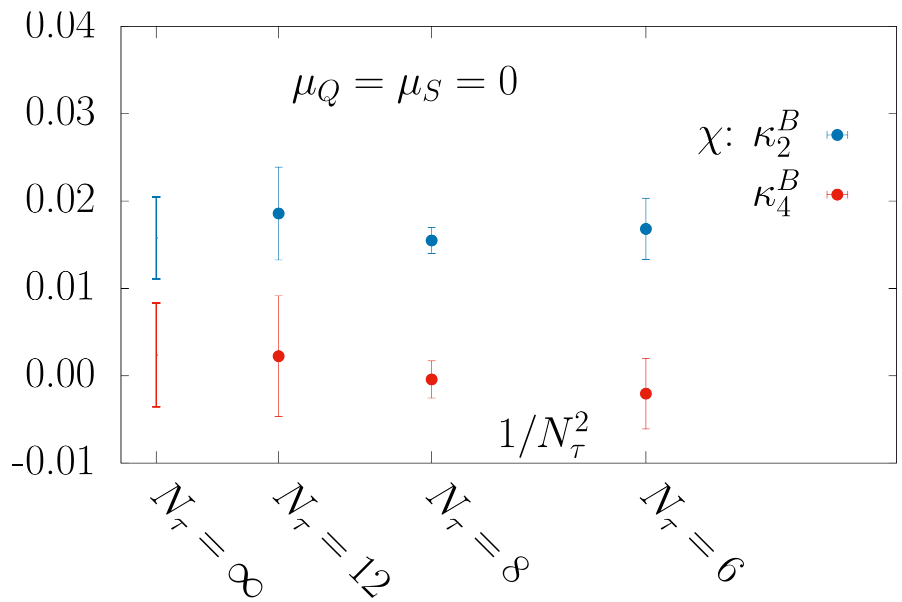


The freeze-out condition corresponding to constant energy density or constant entropy density agrees with the crossover line within errors

The μ_B dependence of T_c is small

$$\kappa_2^{B,\chi} \simeq \kappa_2^{B,\Sigma}$$

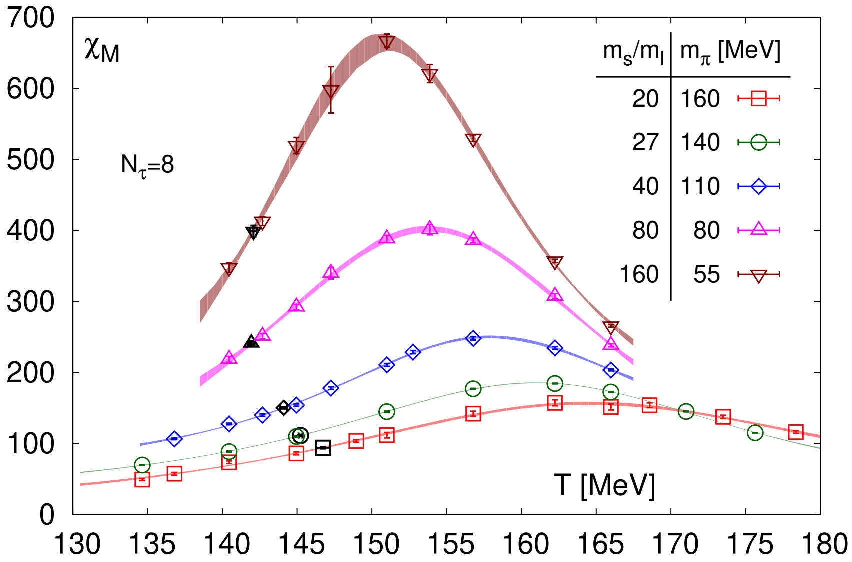
HotQCD, PLB795 (2019) 15

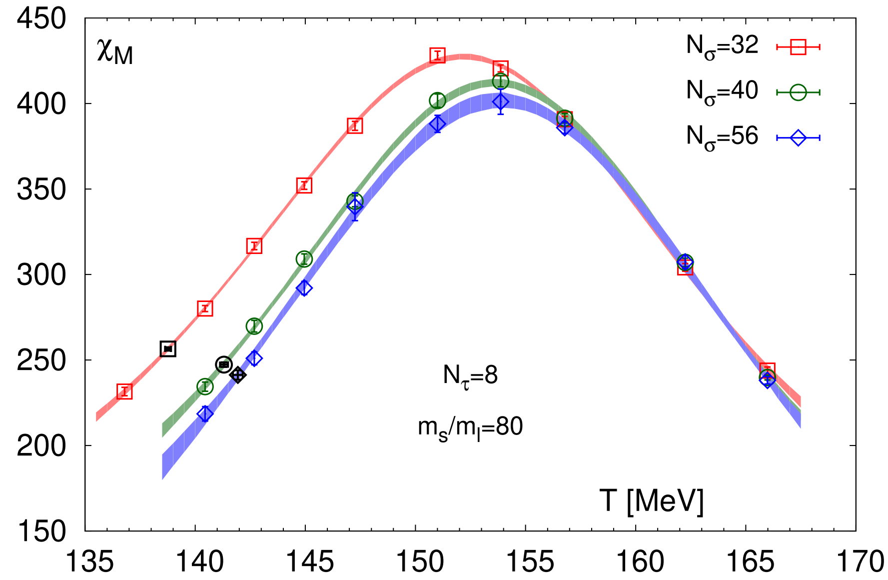


Chiral phase transition in 2+1 flavor QCD

What is the nature of the chiral transition in 2+1 flavor QCD for fixed m_s and $m_l \to 0$?

HotQCD, arXiv:1903.04801



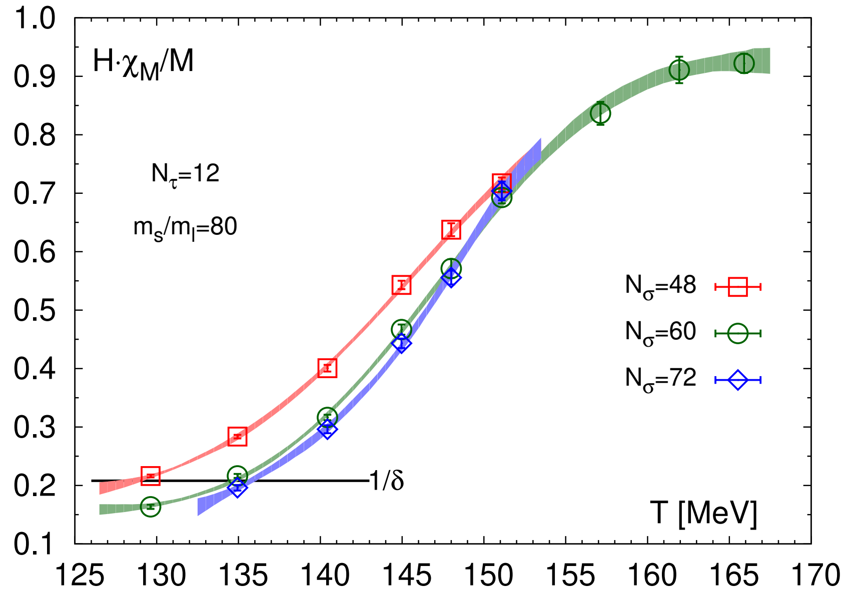


$$M = h^{1/\delta} f_G(z, z_L) + f_{sub}(T, H, L) \ ,$$

$$\chi_M = h_0^{-1} h^{1/\delta - 1} f_\chi(z, z_L) + \tilde{f}_{sub}(T, H, L) \ .$$

$$T_p(H, L) = T_c^0 \left(1 + \frac{z_p(z_L)}{z_0} H^{1/\beta\delta}\right) \ + \ \text{sub leading}$$

$$T_X(H, L) = T_c^0 \left(1 + \left(\frac{z_X(z_L)}{z_0}\right) H^{1/\beta\delta}\right)$$

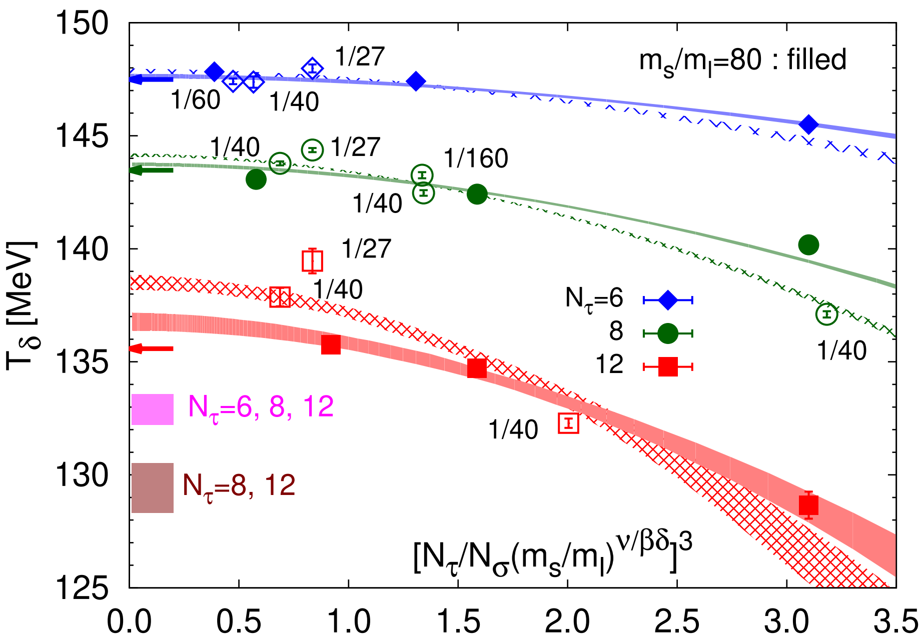


$$\frac{H\chi_M(T_\delta,H,L)}{M(T_\delta,H,L)} = \frac{1}{\delta}\ ,$$

$$\chi_M(T_{60},H) = 0.6\chi_M^{max}\ .$$

$$T_X(H,L) = T_c^0\left(1+\left(\frac{z_X(z_L)}{z_0}\right)H^{1/\beta\delta}\right)$$
$$+c_X H^{1-1/\delta+1/\beta\delta}\ ,\quad X=\delta,\ 60$$

$$z_{60}\simeq z_\delta \simeq 0$$



Use $O(4)$ fits for m_l and volume dependence

Continuum extrapolations:

$$T_c^0 = 132^{+3}_{-6}\ \text{MeV}$$

HotQCD, arXiv:1903.04801

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T,\mu_B,\mu_Q,\mu_S,\mu_C)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{BQSC} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k \left(\frac{\mu_C}{T}\right)^l$$ hadronic

$$\frac{p(T,\mu_u,\mu_d,\mu_s,\mu_c)}{T^4} = \sum_{ijkl} \frac{1}{i!j!k!l!} \chi_{ijkl}^{udsc} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k \left(\frac{\mu_c}{T}\right)^l$$ quark

$$\chi_{ijkl}^{abcd} = T^{i+j+k+l} \frac{\partial^i}{\partial \mu_b^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^i} \frac{\partial^l}{\partial \mu_d^l} \ln Z(T,V,\mu_a,\mu_b,\mu_c,\mu_d) \,|_{\mu_a=\mu_b=\mu_c=\mu_d=0}$$

Taylor expansion coefficients give the susceptibilities, i.e. the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3}\left(\langle X^2\rangle - \langle X\rangle^2\right) \qquad \chi_{11}^{XY} = \frac{1}{VT^3}\left(\langle XY\rangle - \langle X\rangle\langle Y\rangle\right)$$

⬇

information about carriers of the conserved charges (hadrons or quarks)

⬇

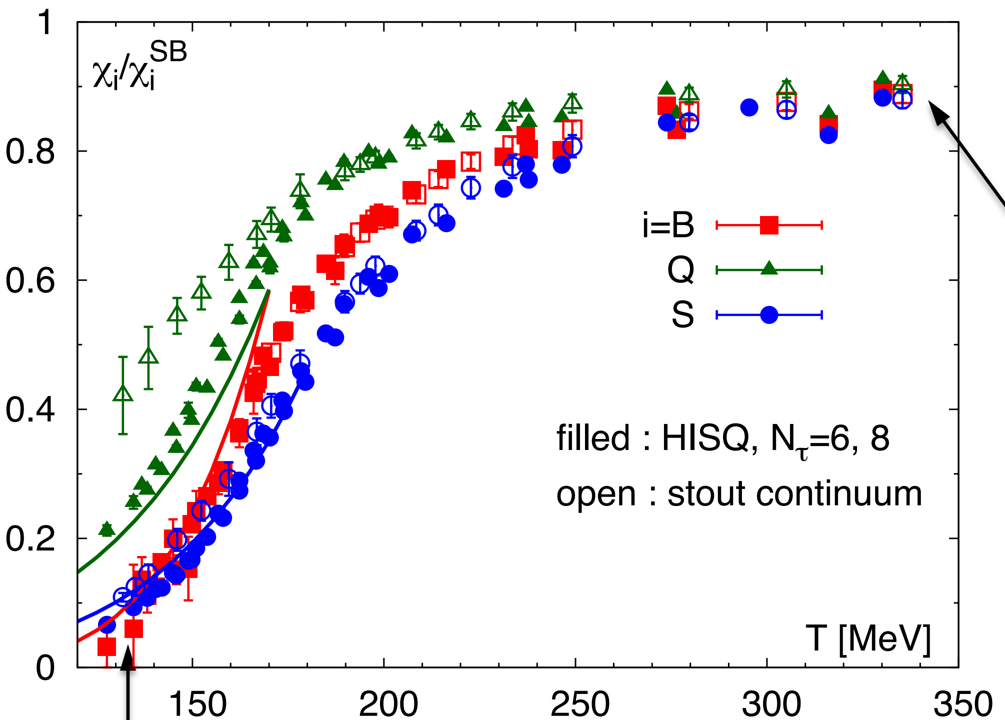
probes of deconfinement

Deconfinement : fluctuations of conserved charges

$$\chi_B = \frac{1}{VT^3}\left(\langle B^2\rangle - \langle B\rangle^2\right)$$ baryon number

$$\chi_Q = \frac{1}{VT^3}\left(\langle Q^2\rangle - \langle Q\rangle^2\right)$$ electric charge

$$\chi_S = \frac{1}{VT^3}\left(\langle S^2\rangle - \langle S\rangle^2\right)$$ strangeness



conserved charges are carried by massive hadrons

Ideal gas of massless quarks :

$$\chi_B^{SB} = \frac{1}{3} \qquad \chi_Q^{SB} = \frac{2}{3}$$

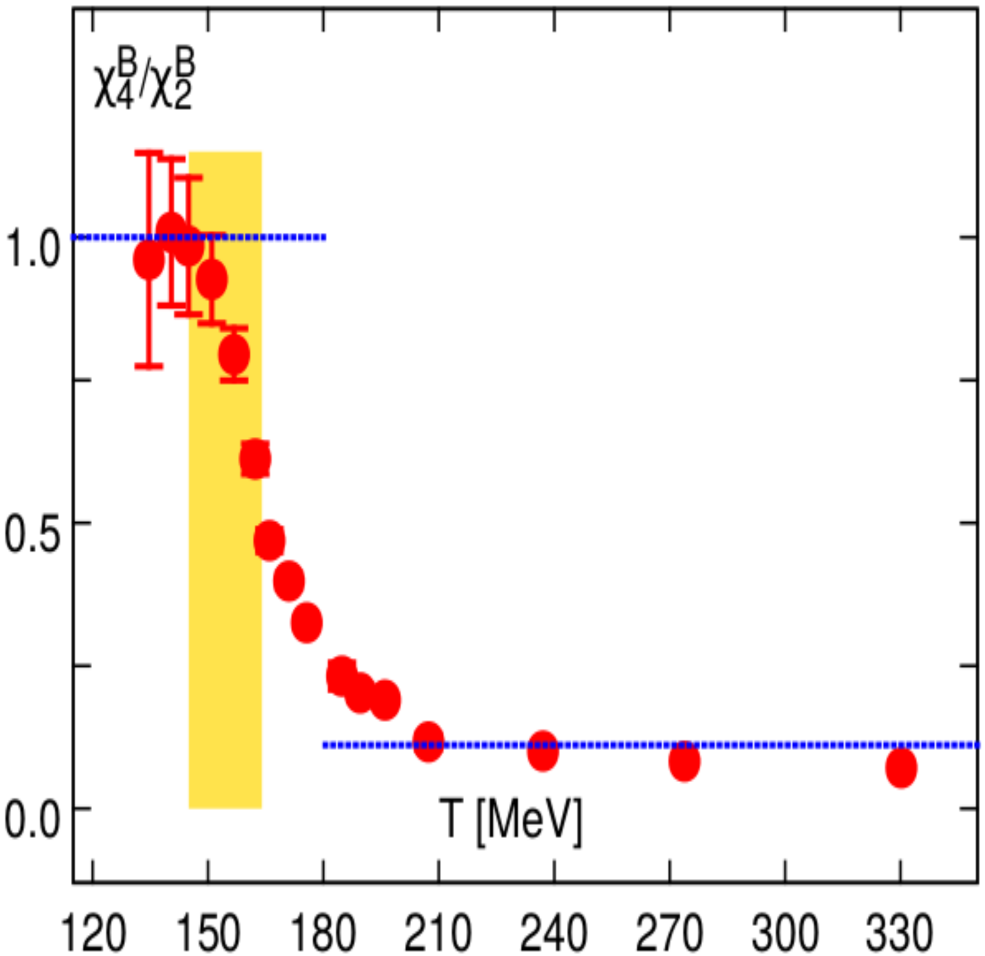
$$\chi_S^{SB} = 1$$

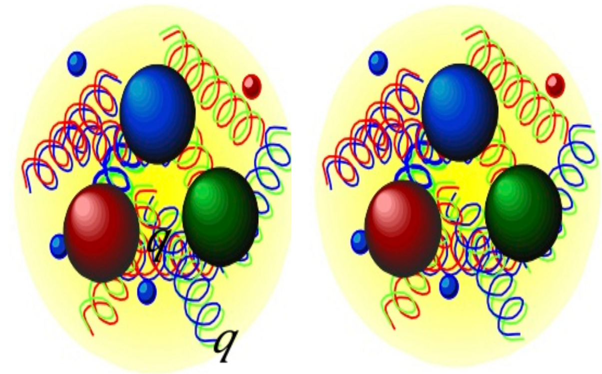
conserved charges carried by light quarks

HotQCD: PRD86 (2012) 034509

BW: JHEP 1201 (2012) 138,

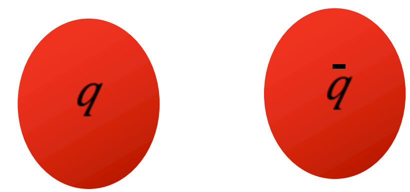
Deconfinement : fluctuations of conserved charges





B=1, -1

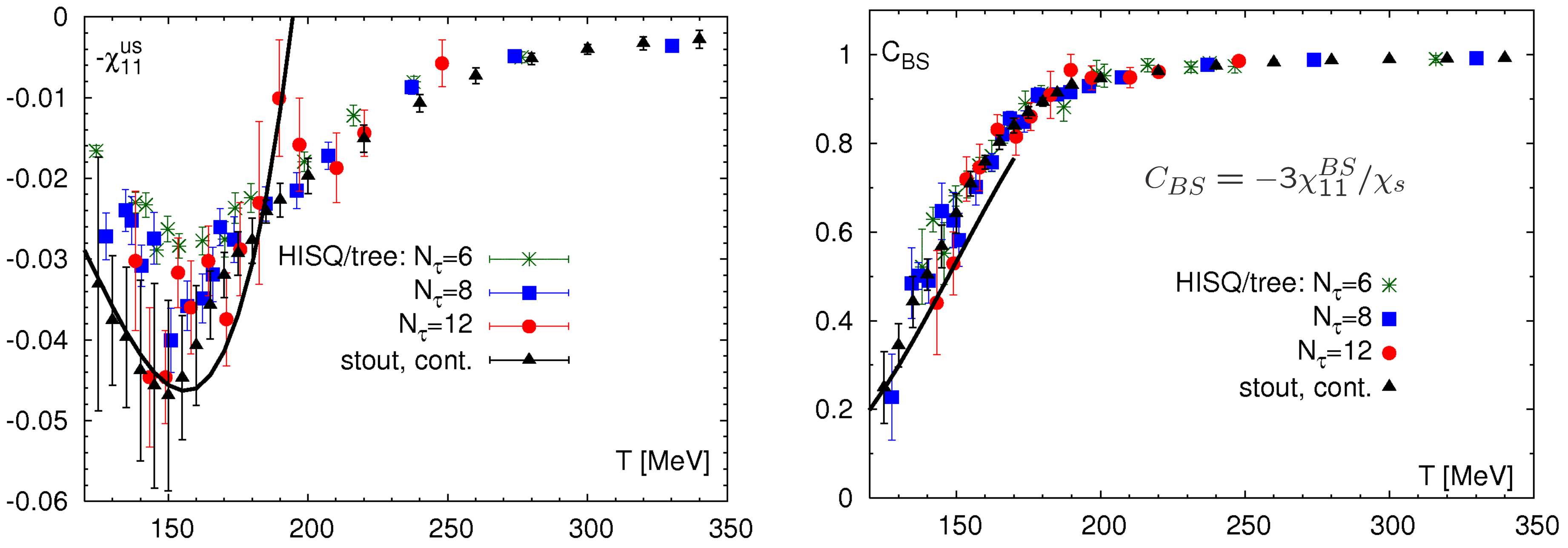
$$\chi_2^B = \langle B^2 \rangle = 1, \ \chi_4^B = \langle B^4 \rangle = 1$$



B=1/3, -1/3

$$\chi_2^B = \langle B^2 \rangle = \frac{1}{9}, \ \chi_4^B = \langle B^4 \rangle = \frac{1}{81}$$

Correlations of conserved charges



P.P. J.Phys. G39 (2012) 093002

- Correlations between strange and light quarks at low *T* are due to the fact that strange hadrons contain both strange and light quarks but very small at high *T (>250* MeV*)*
=> weakly interacting quark gas

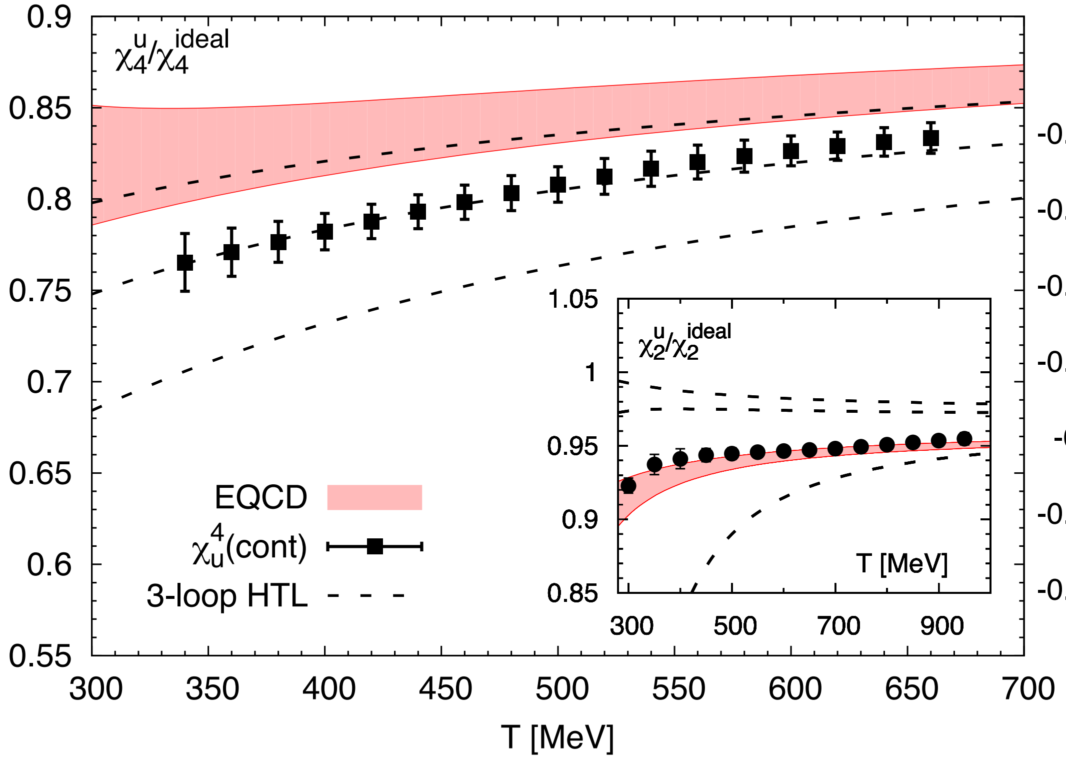
- For baryon-strangeness correlations HISQ results are close to the physical HRG result, at *T>250* MeV these correlations are very close to the ideal gas value

- The transition region where degrees of freedom change from hadronic to quark-like is broad ~ (100-150) MeV

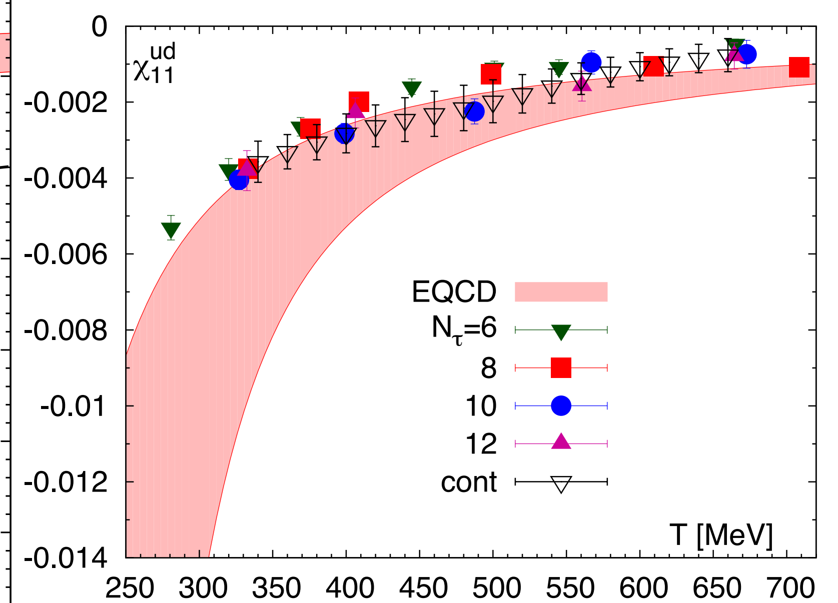
Quark number fluctuations at high T

At high temperatures quark number fluctuations can be described by weak coupling approach due to asymptotic freedom of QCD

quark number fluctuations



quark number correlations

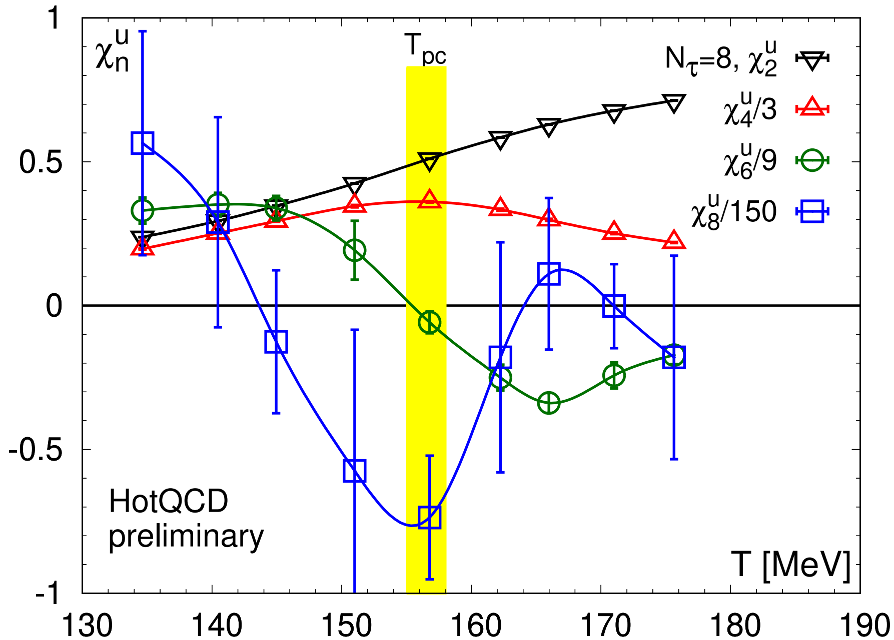


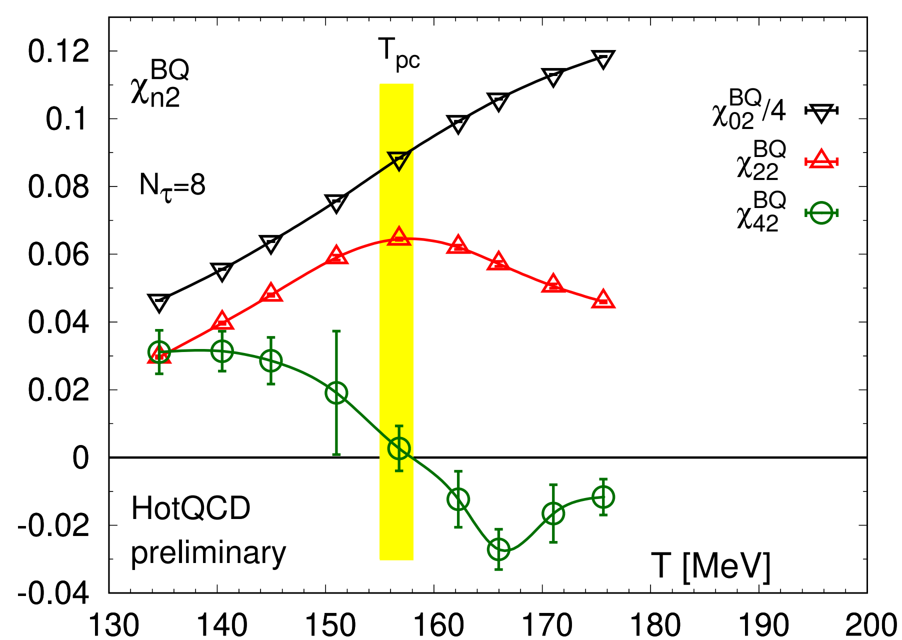
- Good agreement between continuum extrapolated lattice results and the weak coupling approach
- Quark number correlations vanish at any loop order but can be calculated in EQCD and the EQCD calculations agree with the continuum extrapolated lattice results

Bazavov et al, PRD88 (2013) 094021, Ding et at, PRD92 (2015) 074043

Temperature dependence of Taylor expansion coefficients

Karsch, arXiv:1905.03936



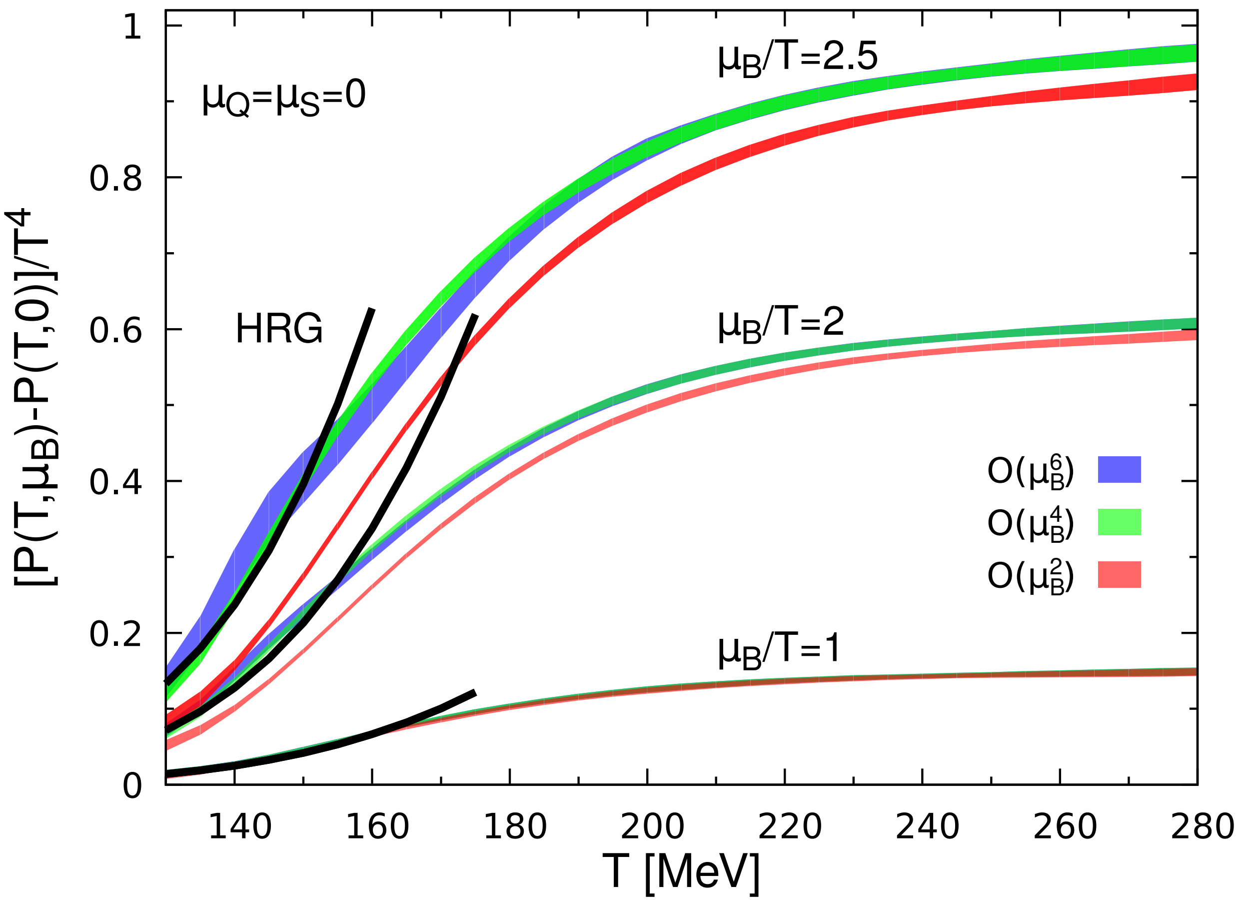


Derivatives in T are very similar to derivatives in μ_B^2; expected if $O(4)$ scaling holds $t = (T - T_c^0)/T_c^0 + \kappa_B(\mu_B/T_c^0)^2$, $\partial_T \sim \partial_t \sim \partial_{\mu_B^2}$

Higher order Taylor expansion coefficients are likely to be negative for $T > 130$ MeV $\Rightarrow$ the only singularity the expansion coefficients are sensitive to is $O(4)$ transition $\Rightarrow T_c^{CEP} < T_c^0 < 132$ MeV

Thermodynamics at non-zero net baryon density

6^{th} order Taylor expansion, BNL-Bielefeld-CCNU Coll., PRD 95 (2017) 054504

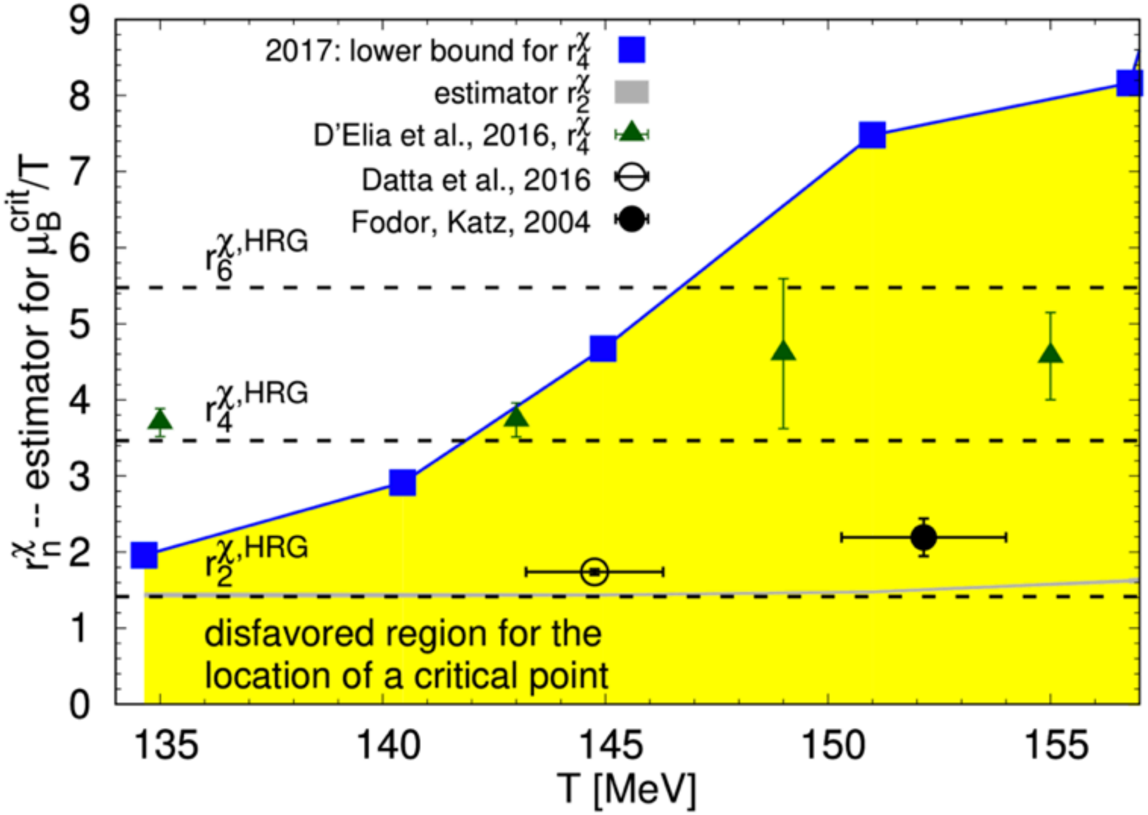


Truncation errors of the 6^{th} order Taylor expansions are small for $\mu_B/T<2.5$

Radius of convergence of Taylor series and critical point

$$\frac{P(T,\mu_B)-P(T,0)}{T^4}=\sum_{n=1}^{\infty}\frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n} \qquad \chi_2^B(T,\mu_B)=\sum_{n=1}^{\infty}\frac{\chi_{2n+2}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$

$T<T_c$: $\chi_n^B>0 \Rightarrow$ convergence of the Taylor expansion
is limited by a similarity on the real axis $\mu_B=\mu_B^c$ (critical point).

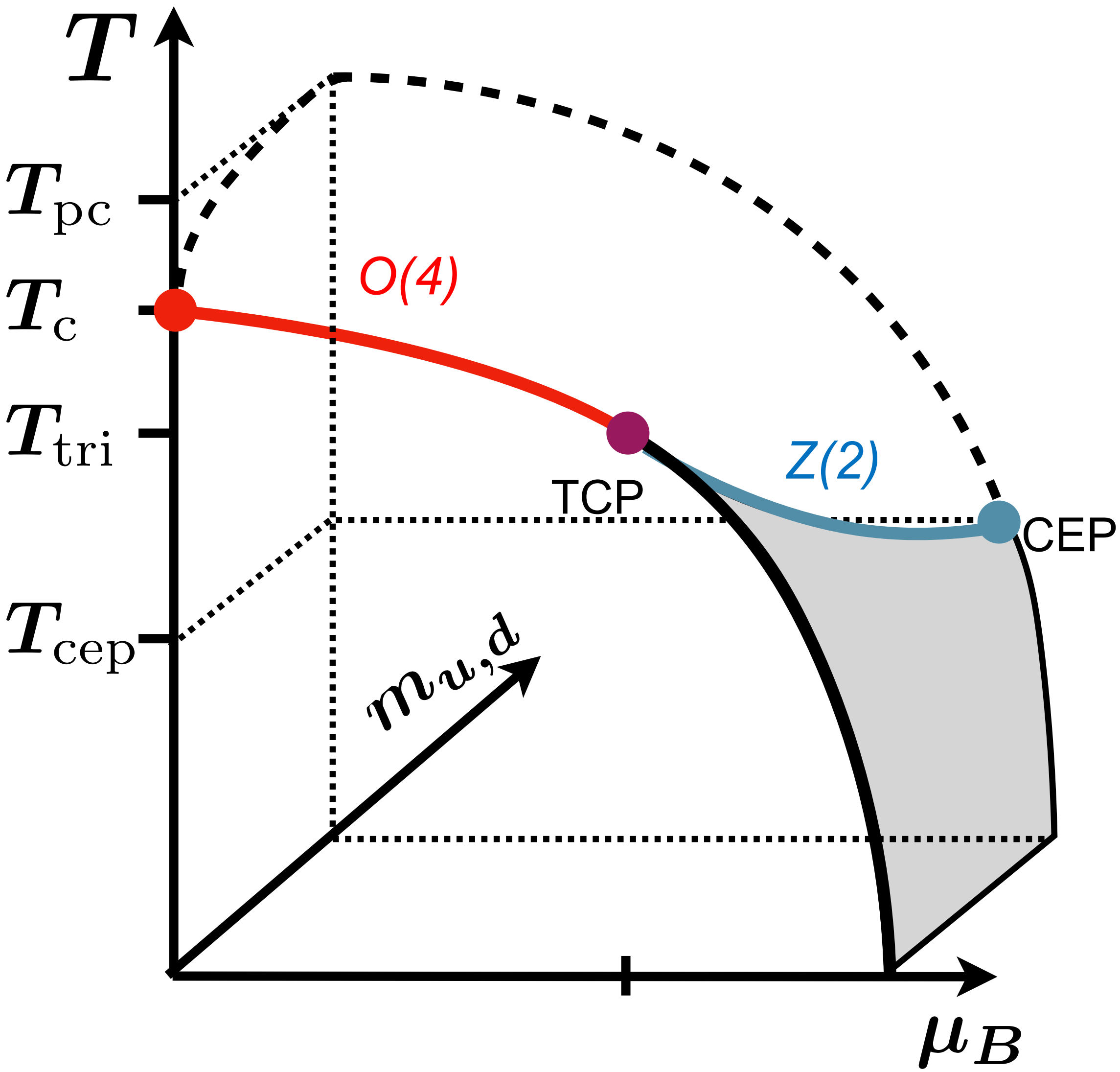


Estimator for radius of convergence:

$$r_n^\chi=\left|\frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B}\right|^2$$

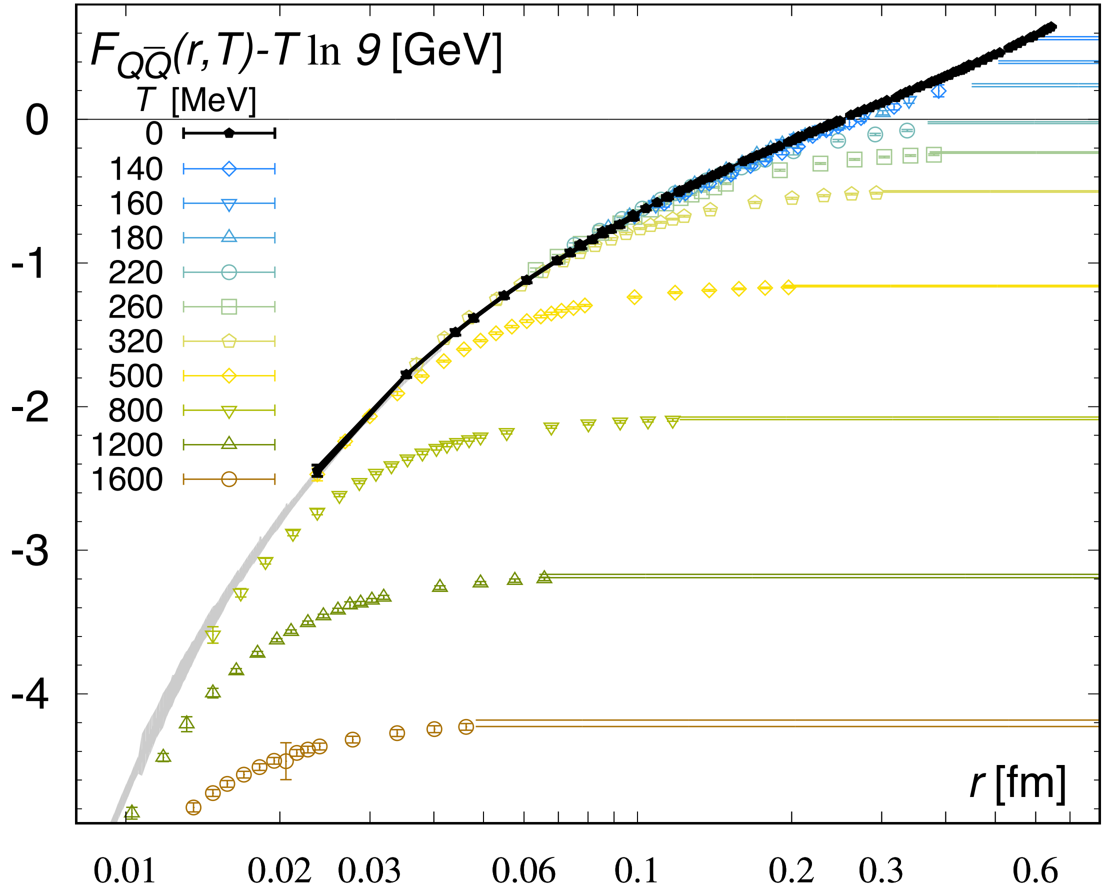
Critical point is disfavored for $\mu_B < 300$ MeV

Conjectured QCD phase diagram



Deconfinement and color screening in QCD

2+1 flavor QCD, continuum extrapolated, TUMQCD, PRD 98 (2018) 054511

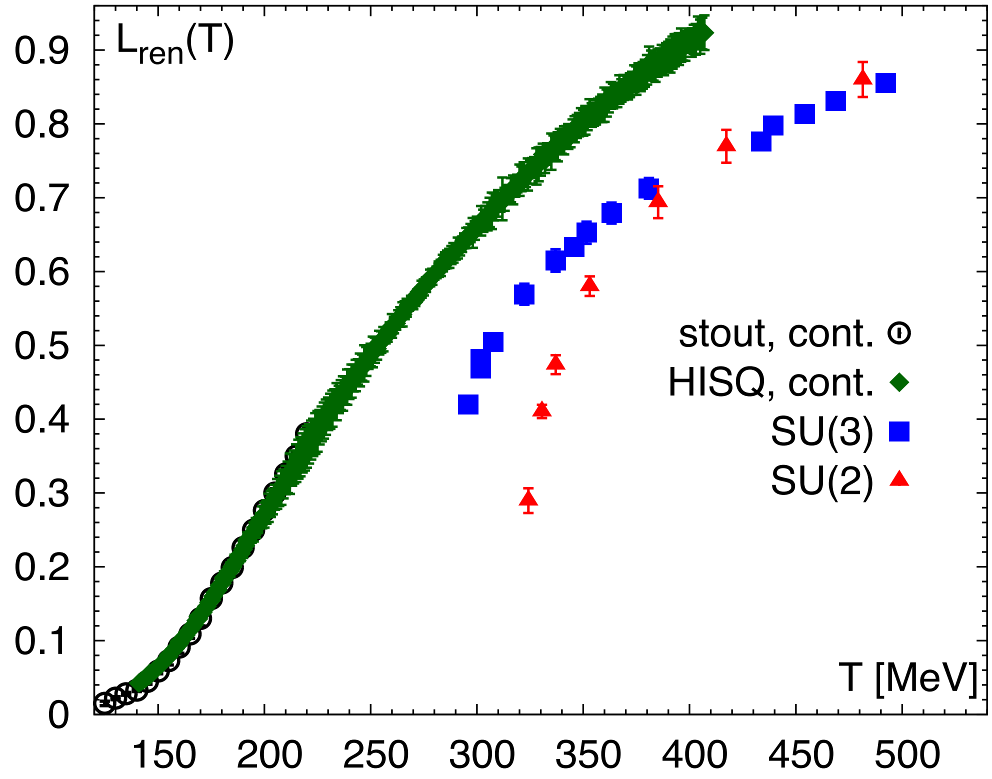


The free energy of static quark anti-quark pair agrees with the $T=0$ potential for $r \ll 1/T$

The free energy of static quark anti-quark pair is screened for $r>r_{scr}$ at any temperature

$r_{scr} \sim 1/T$ => Debye screening

Deconfinement and color screening in QCD (cont'd)



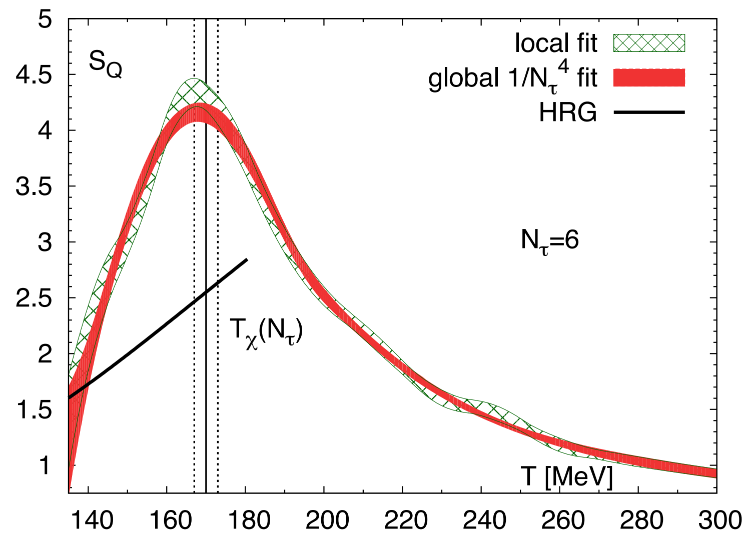
Pure glue ≠ QCD !

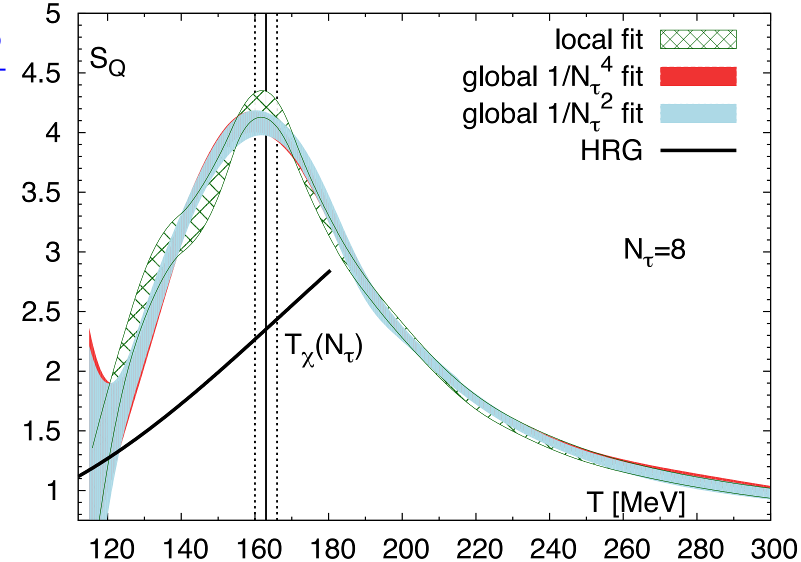
Deconfinement transition happens at lower temperature but the Polyakov loop behaves smoothlyaround T_c , *Z(3) symmetry plays no apparent role*

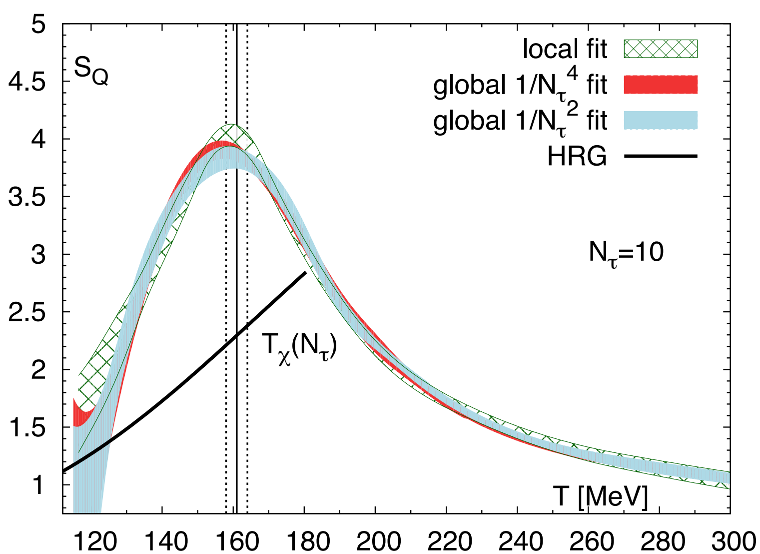
How to define deconfinement transition in QCD ?

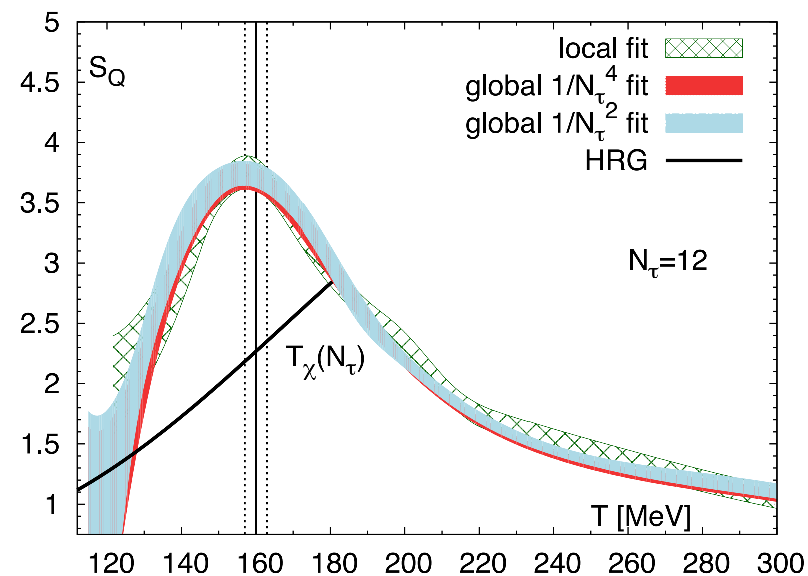
The entropy of static quark

$$S_Q = -\frac{\partial F_Q}{\partial T}$$









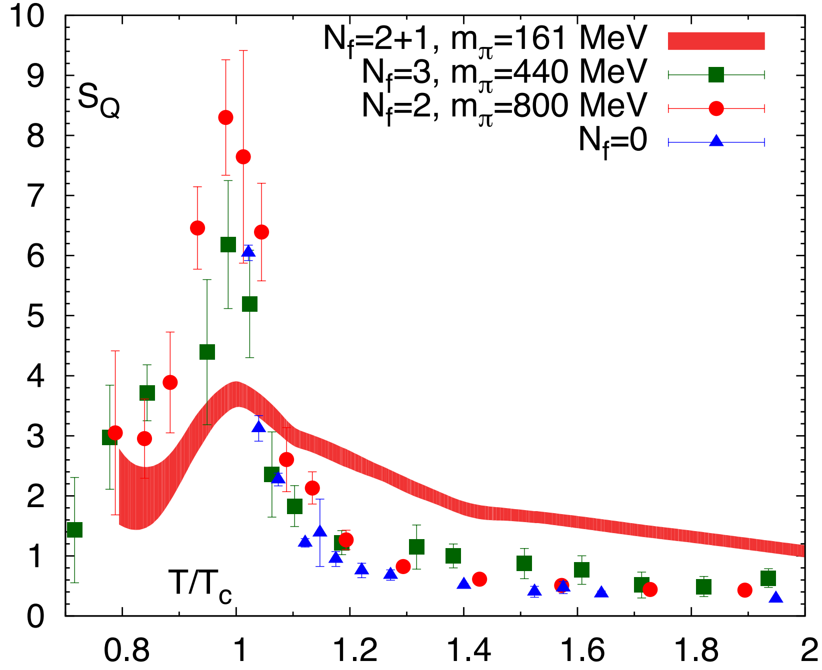
TUMQCD, PRDD93 (2016) 114502

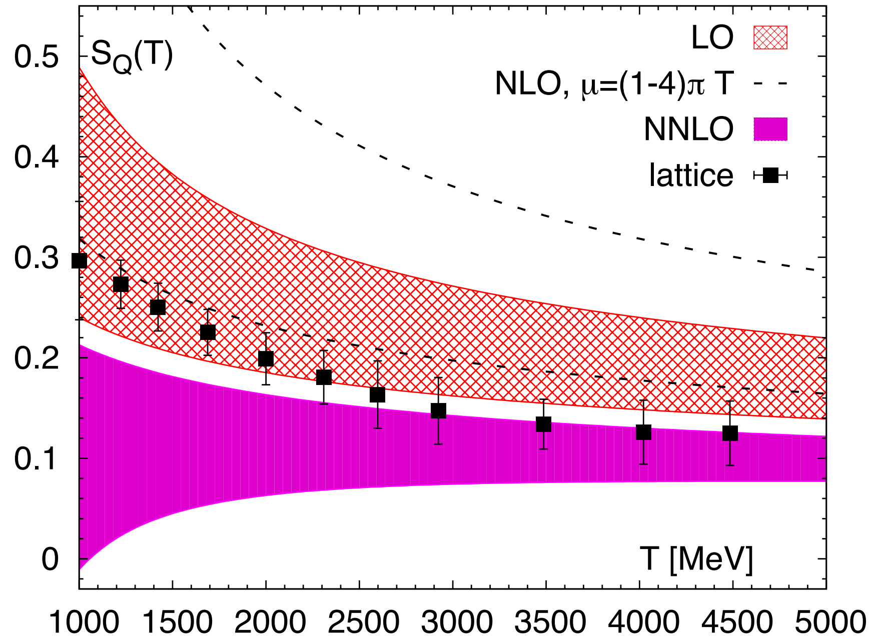
The onset of screening corresponds to peak is S_Q and its position coincides with T_c

The entropy of static quark

TUMQCD, PRDD93 (2016) 114502

$$S_Q = -\frac{\partial F_Q}{\partial T}$$





At low T the entropy S_Q increases reflecting the increase of states the heavy quark can be coupled to; at high temperature the static quark only "sees" the medium within a Debye radius, as T increases the Debye radius decreases and S_Q also decreases

The peak in the entropy is broader and smaller for smaller quark mass

Weak coupling (EQCD) calculations work for $T > 1500$ MeV
Berwein et al, PRD 93 (2016) 034010

Free energy of a static quark anti-quark pair at high T

The work to separate the $Q\bar{Q}$ pair from distance r_1 to r_2 : $F_{Q\bar{Q}}(r_2) - F_{Q\bar{Q}}(r_1)$

Leading order in perturbation theory:

$$F_{Q\bar{Q}}(r,T) = -\frac{1}{9}\frac{\alpha_s^2}{r^2 T}\exp(-2m_D r) + F_\infty$$

In QED at leading order:

$$F_{Q\bar{Q}}(r,T) = -\frac{\alpha}{r}\exp(-m_D r) + F_\infty$$

Conjecture: in QCD the work is reduced due to cancelation of color singlet and octet contribution

Fierz idenity: $\delta_{ij}\delta_{lk} = \frac{1}{N_c}\delta_{ik}\delta_{lj} + 2T^a_{ik}T^a_{lj}$

$$e^{-F_{Q\bar{Q}}(r,T)/T} = \frac{1}{9}e^{-F_S(r,T)/T} + \frac{8}{9}e^{-F_O(r,T)/T}$$

$$e^{-F_S(r,T)/T} = \frac{1}{N_c}\langle \mathrm{tr} W(r) W^\dagger(0)\rangle,\ W(r) = \prod_{\tau=0}^{N_\tau - 1} U_0(\mathbf{r},\tau)$$

LO:

$$F_S = -\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r} + F_\infty,\ F_O = +\frac{1}{6}\frac{\alpha_s}{r}e^{-m_D r} + F_\infty$$

➡ The LO result for $F_{Q\bar{Q}}$ is recovered

Free energy of a static quark anti-quark pair at high T (cont'd)

The definition of F_S requires gauge fixing. Is it possible to have a gauge invariant decomposition of $F_{Q\bar{Q}}$ into singlet and octet contributions ?

Yes, for $r \ll 1/T$ using pNRQCD

$$e^{-F_{Q\bar{Q}}(r,T)/T} = \langle S(r,1/T)S(r,0)\rangle + L_A \langle O^a(r,1/T)O^a(r,0)\rangle$$

L_A is Polyakov loop in the adjoint representation

$$e^{-F_{Q\bar{Q}}(r,T)/T} = \frac{1}{9}e^{-f_s(r,T)/T} + \frac{8}{9}e^{-f_o(r,T)/T},$$

$$f_s = V_s(r) + \mathcal{O}(\alpha_s^2 r T^2),\ f_o = V_o(r) - \frac{N_c \alpha_s m_D}{2} + \mathcal{O}(\alpha_s^2 r T^2)$$

Brambilla et al, PRD 82 (2010) 074019

$$F_s = f_s + \mathcal{O}(\alpha_s^3 T) + \mathcal{O}(\alpha_s^2 r T^2),\ F_O = f_o + \mathcal{O}(\alpha_s^3 T) + \mathcal{O}(\alpha_s^2 r T^2)$$

Berwein et al, PRD 96 (2017) 014025

The naïve and the pNRQCD decomposition into singlet and octet agree

To calculate $F_{Q\bar{Q}}(r,T)$ and $F_S(r,T)$ for $r \sim 1/m_D$ another EFT, namely EQCD, should be used

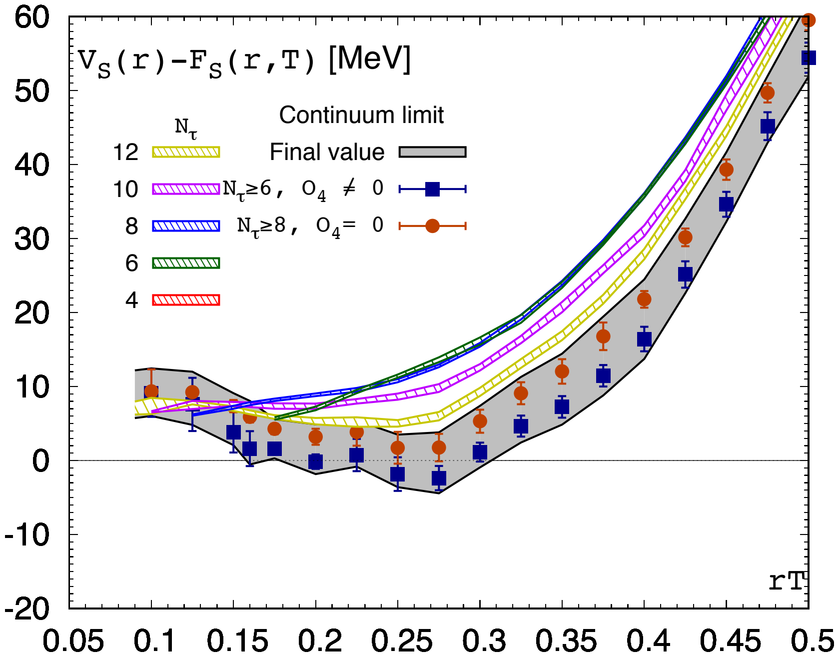
Free energy at short distances: lattice results vs. pNRQCD

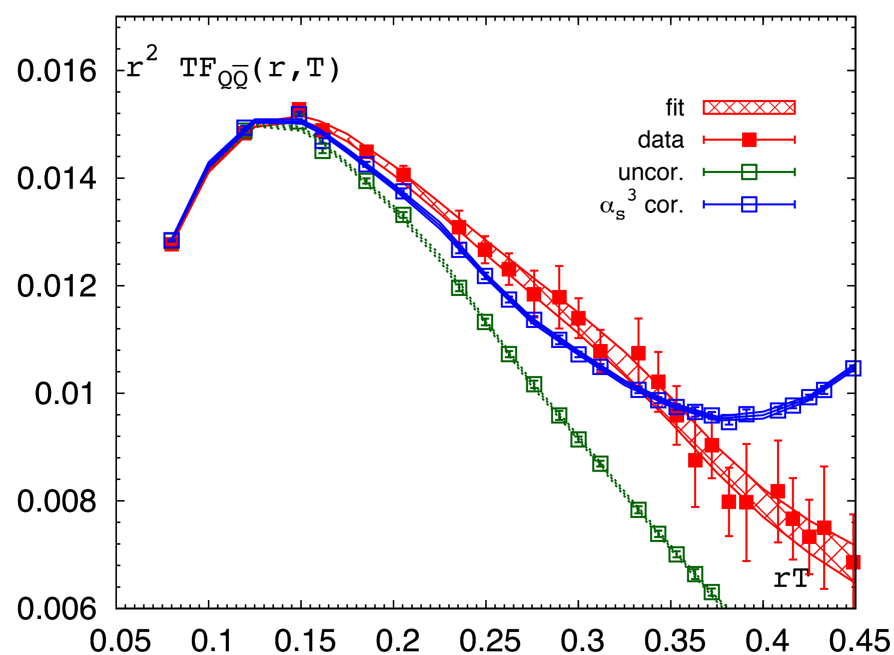
The difference between V_s and F_S is small as expected for $rT < 0.3$

Construct pNRQCD prediction for $F_{Q\bar{Q}}$ using the lattice data for F_S and V_s as proxy for f_s together with the relation:

$$f_o = -\frac{1}{8} f_s + \frac{3\alpha_s^3}{8r}\left(\frac{\pi^2}{4} - 3\right)$$

works !





T=407 MeV

TUMQCD, PRD 98 (2018) 054511

The interaction of static Q and $\bar{Q}$ is vacuum like for $rT < 0.3$

Free energy in the screening regime: lattice vs. weak coupling

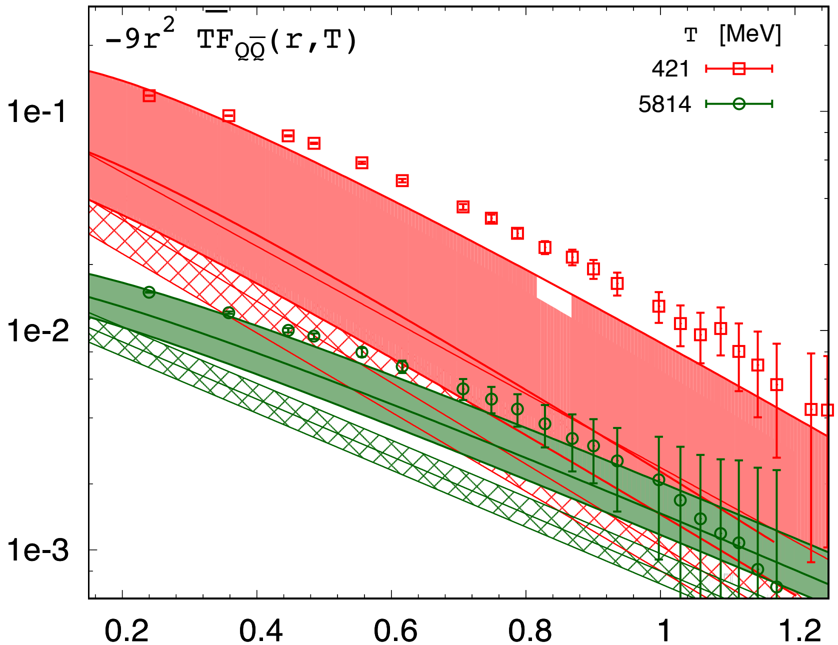
NLO in EQCD results are available for $\bar{F}_i(r,T) = F_i(r,T) - F_\infty(T)$

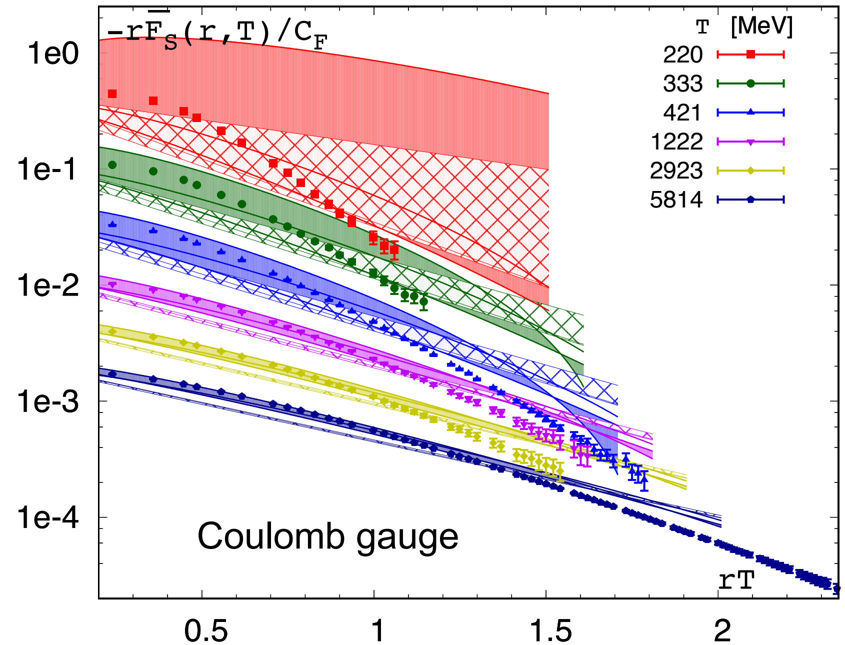
Nadkarni, PRD 33 (1986) 3738

$$F_{Q\bar{Q}}(r,T) = -\frac{\alpha_s^2 e^{-2m_D r}}{9r^2 T}(1+\alpha_s(\delta Z_1(\mu)+rTf(rm_D)))$$

Burnier et al, JHEP 01 (2010) 054

$$F_S(r,T) = -\frac{4\alpha_s e^{-m_D r}}{3r}(1+\alpha_s(\delta Z_1(\mu)+rTf_1(rm_D)))$$





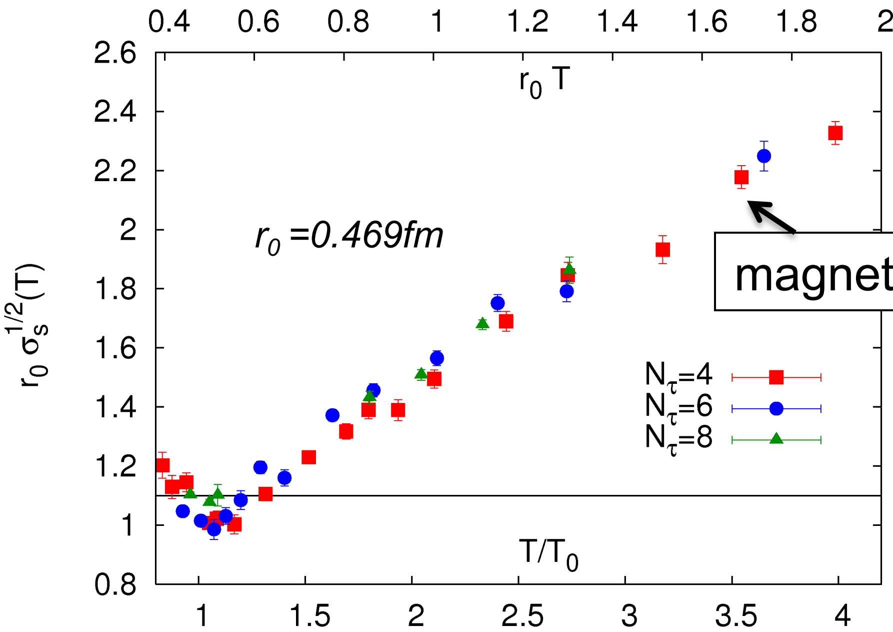
TUMQCD, PRD 98 (2018) 054511

Lattice results are in reasonable agreement with NLO weak coupling result for *rT<0.6*, at larger distances, non-pertubtative effects (due to chromo-magnetic sector) become important

Spatial string tension at $T>0$ and dimensional reduction

$$W(r(x,y),z) \sim \exp(-\sigma_s(T)\cdot r\cdot z)$$
$$\sigma_s(T) \simeq \sigma(T=0),\ T<T_c$$

Cheng et al., PRD 78 (2008) 034506



magnetic screening

EQCD :

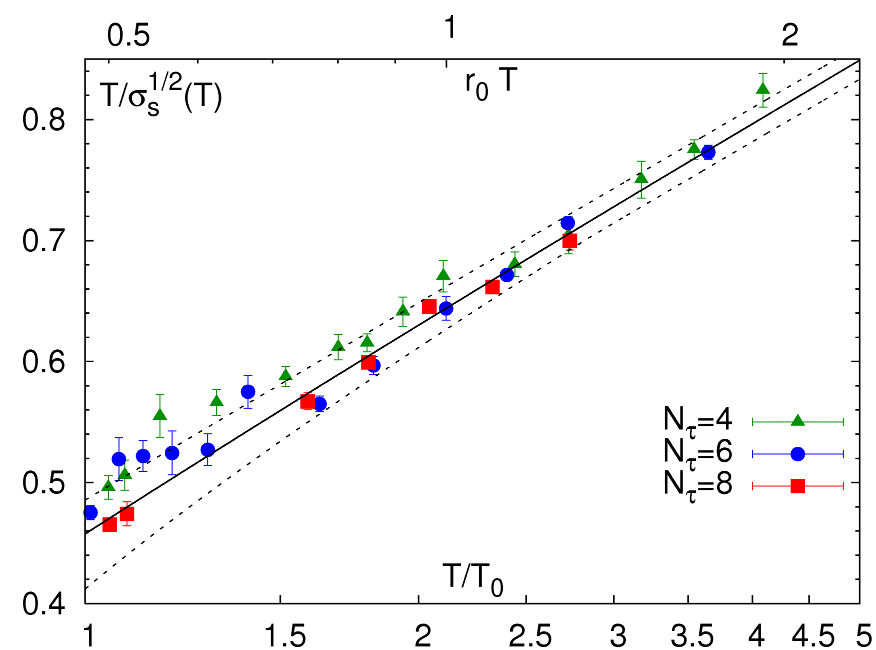
$$\sigma_s(T) = c_M \cdot g_3^4(T) \sim T^2,\ T \gg T_c$$

non-perturbative

$$c_M = 0.55(1)$$
$$g_3^2(T) = g^2(T)T(1 + c_1(N_f, T)g^2(T) + ...)$$

Laine, Schröder, JHEP0503 (2005) 067

Calculated perturbatively



The T-dependence of spatial string tension is perturbative for $T > 1.5T_c$!

BACKUP SLIDES

Relativistic viirial expansion and Hadron Resonance Gas

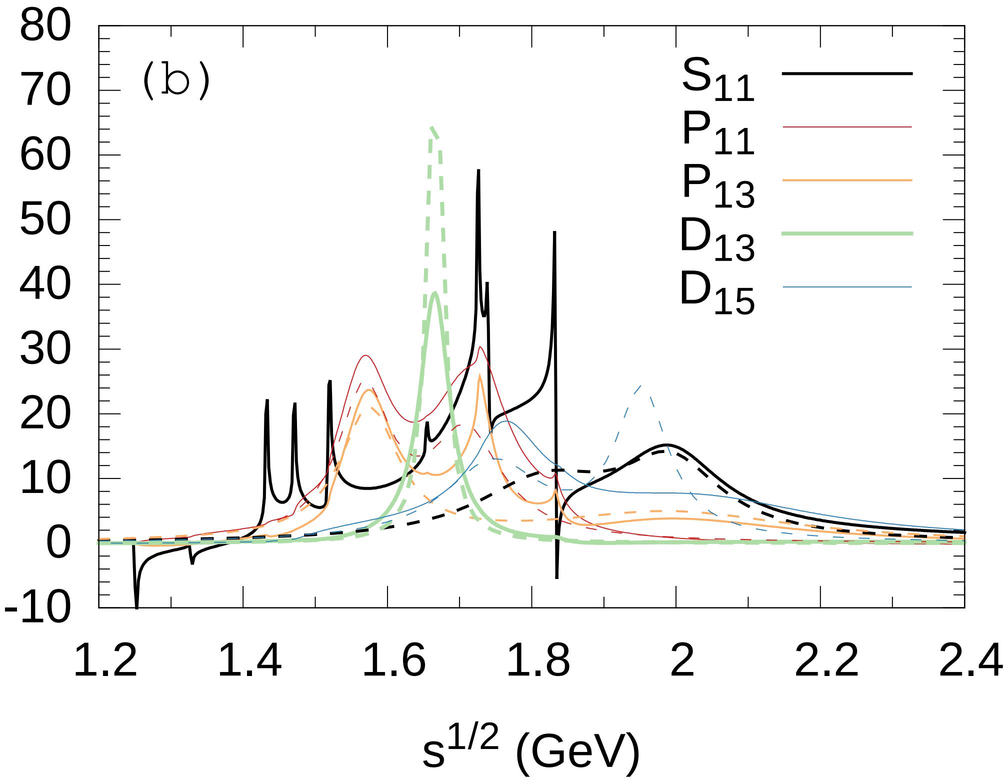
There are significant non-resonant meson-baryon interactions and overlapping resonances for example in the strange baryon sector

Fernandez-Ramirez, Lo, PP, PRC98 (2018) 044910

$$\Delta P_{int} = \sum_l \int dE E^2 T^2 K_2(E/T) \frac{d\delta_l(E)}{dE}$$

δ_l from coupled channel PWA analysis by JPAC

Fernandez-Ramirez et al, PRD 93 (2016) 034029

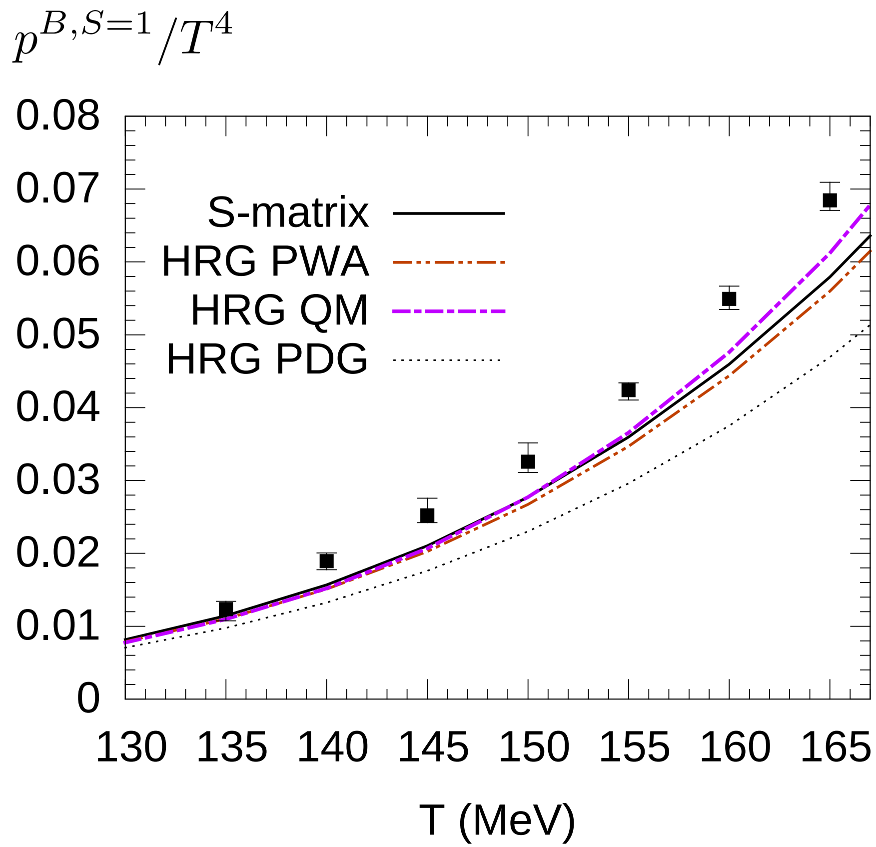


Partial pressures of
i=1 strange baryons in $10^{-3}\, T^4$

$I=1$			
	S-mat.	HRG	B-W
S_{11}	1.018	0.282	0.532
P_{11}	1.681	1.275	1.465
P_{13}	1.868	1.857	2.406
D_{13}	0.964	0.995	1.052
D_{15}	1.478	1.219	1.793
F_{15}	0.514	0.503	1.119
F_{17}	0.556	0.238	0.603
G_{17}	0.169	0.095	0.310

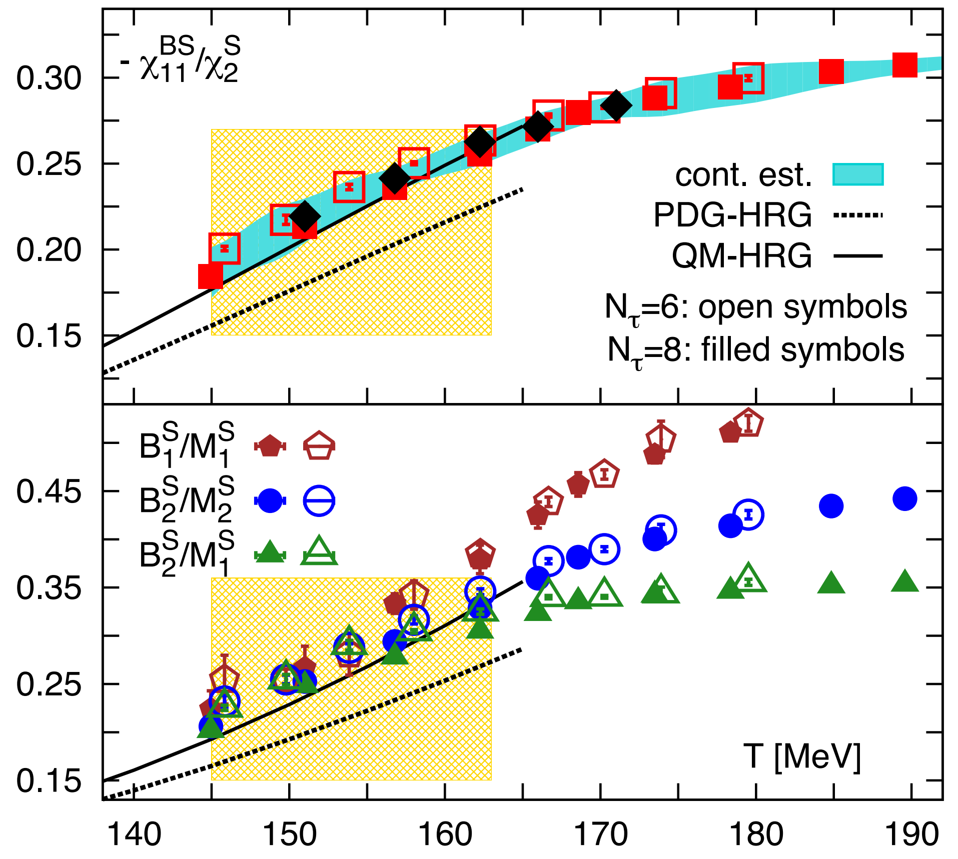
Yet the total strange baryon pressure is well approximated by HRG

Thermodynamics of strange hadrons and missing states



Partial pressure of S=1 baryons from S-matrix based virial expansion agrees with HRG that includes additional hyperon resonances in addition to PDG

Use excited strange hadrons from quark model to calculate the pressure from HRG



Lattice results agree with HRG that includes the "missing" states

Bazavov, PRL 113 (2014) 072001