N_f dependence and decoupling

- NP computations typically have only 2+1 (or 2+1+1) quark flavors
- What about charm, bottom, top?
  - “decoupling”
  - “matching at thresholds”
  - “active flavors”
  - “flavor number schemes”
- let us discuss what this means

Decoupling

basic example

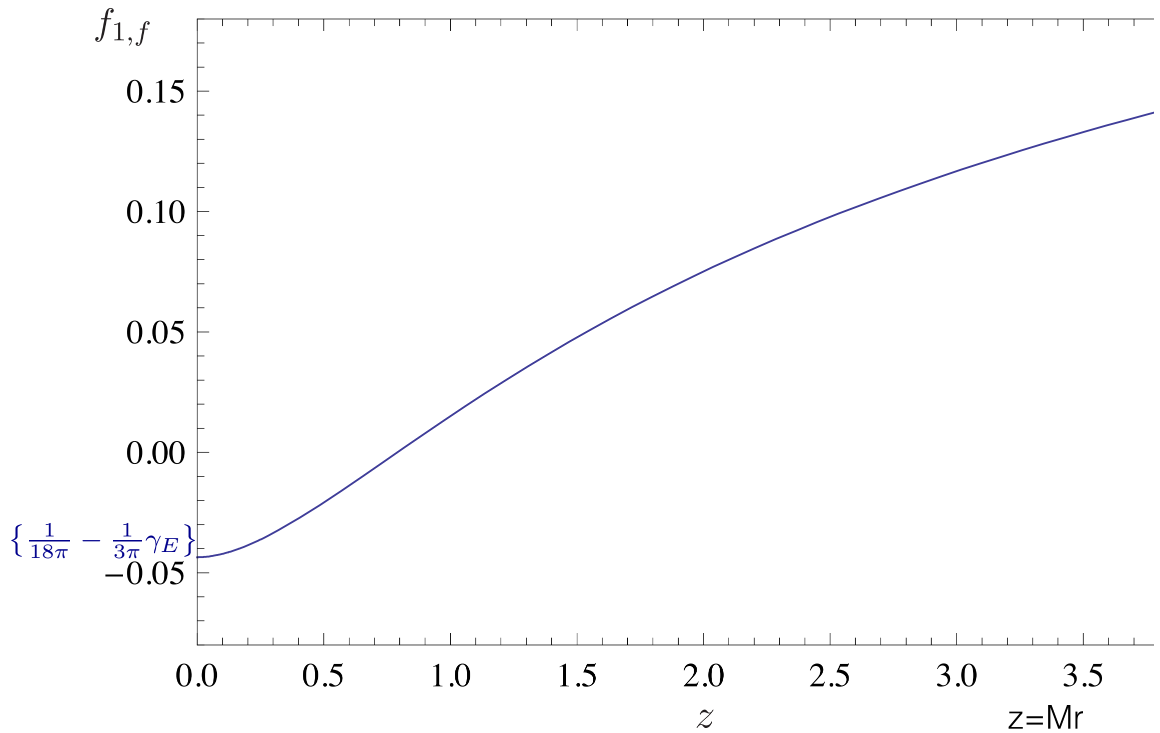
$$
\begin{aligned}
\frac{1}{C_\mathrm{F}} r^2 F(r) &= \alpha_\mathrm{qq}(1/r, \{z_i\}) \\
&= \alpha_{\overline{\mathrm{MS}}}(1/r) + [f_{1,g} + \sum_{i=1}^{N_f} f_{1,f}(z_i)]\alpha^2_{\overline{\mathrm{MS}}}(1/r) + \mathrm{O}(\alpha^3_{\overline{\mathrm{MS}}})\,, \\
& \quad z_i = \overline{m}_i r
\end{aligned}
$$

$$
\begin{aligned}
f_{1,g} &= \frac{3}{\pi}\left[-\tfrac{35}{36} + \tfrac{11}{6}\gamma_E\right], \\
f_{1,f}(z) &= \tfrac{1}{2\pi}\left[\tfrac{1}{3}\log(z^2) + \tfrac{2}{3}\int_1^\infty \mathrm{d}x\, \tfrac{1}{x^2}\sqrt{x^2-1}\,\left(1+\tfrac{1}{2x^2}\right)(1+2zx)\,e^{-2zx}\right],
\end{aligned}
$$

- Naively one may think a heavy quark does not matter for large z and

$$\lim_{z\to\infty} f_{1,f}(z) = 0$$

but



$$f_{1,f}(z) \overset{z\gg 1}{\sim} \frac{1}{6\pi}\log(z^2)$$

- This is because a mass-independent renormalization scheme is used. Large mass physics and small mass physics enter together. Consider only $r \gg 1/m$ physics:

$$r\partial_r \bar{g}^2_{\rm qq}(1/r,\{z_i\})\Big|_{m_i} \quad = \quad 4\pi r\partial_r \alpha_{\rm qq}(1/r,\{z_i\})\big|_{m_i}$$

Decoupling

basic example, continued

- Consider only $r \gg 1/m$ physics: $f_{1,f}(z) \overset{z\gg 1}{\sim} \frac{1}{6\pi}\log(z^2)$

$$
\begin{aligned}
r\partial_r \bar{g}^2_{\rm qq}(1/r,\{z_i\})\big|_{m_i} &= -2\bar{g}_{\overline{\rm MS}}(1/r)\beta_{\overline{\rm MS}}(\bar{g}_{\overline{\rm MS}}(1/r)) + \bar{g}^4_{\overline{\rm MS}}(1/r)\, r\partial_r \sum_{i=1}^{N_f} \frac{f_{1,f}(z_i)}{4\pi} \\
&\quad + {\rm O}(g^6) \\
&= 2b_0\bar{g}^4_{\overline{\rm MS}} + \bar{g}^4_{\overline{\rm MS}}\, r\partial_r \sum_{i=1}^{N_f} \frac{f_{1,f}(z_i)}{4\pi} + {\rm O}(g^6)
\end{aligned}
$$

now: $z_i = 0, \quad i = 1,\ldots,N_\ell = N_{\rm f}-1\,, \quad z_{N_{\rm f}} \gg 1$

$$
\begin{aligned}
&= 2\bar{g}^4_{\overline{\rm MS}}\,\frac{1}{(4\pi)^2}\,[11 - \tfrac{2}{3}(N_{\rm f}-1)] + {\rm O}(g^6) \\
&= -2\bar{g}_{\overline{\rm MS}}(1/r)\beta^{(N_{\rm f}-1)}_{\overline{\rm MS}}(\bar{g}_{\overline{\rm MS}}(1/r, m=0)) + \ldots
\end{aligned}
$$

effectively

physics at $r \gg 1/m_{N_{\rm f}} =: M$: $N_\ell = N_{\rm f} - 1$ flavor QCD = EFT

- Consider only $r \gg 1/m$ physics: $f_{1,f}(z) \overset{z\gg 1}{\sim} \frac{1}{6\pi}\log(z^2)$

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effectively

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Decoupling

basic example, continued

- Consider only $r \gg 1/m$ physics: $f_{1,f}(z) \overset{z\gg 1}{\approx} \frac{1}{6\pi}\log(z^2)$

$$r\partial_r \bar{g}^2_{\rm qq}(1/r,\{z_i\})\big|_{m_i} \quad = \quad -2\bar{g}_{\overline{\rm MS}}(1/r)\beta^{(N_{\rm f}-1)}_{\overline{\rm MS}}(\bar{g}_{\overline{\rm MS}}(1/r, m=0)) + \ldots$$

effectively

physics at $r \gg 1/m_{N_{\rm f}} =: M$: $N_\ell = N_{\rm f} - 1$ flavor QCD = EFT

- The contribution from $f_{1,f}(z) \overset{z\gg 1}{\approx} \frac{1}{6\pi}\log(z^2)$ is exactly necessary such that the heavy quark decouples at large r
- From the discussion at this order it is not clear how the coupling of the $N_\ell = N_{\rm f} - 1$ effective theory is related to the fundamental one $\bar{g}^{(N_{\rm f})}_{\overline{\rm MS}}$, but it is clear they are related. We will come back to how.

- In original (fundamental) theory the perturbative expression has
$$\bar{g}^{2n}[\log(z)]^m$$
terms. It is unreliable/useless for $z \gg 1$.
- need to resum: done by EFT ($\approx$ renormalisation group improvement for the leading order in $1/m^2$)
- note that the need for resummation is a problem of perturbation theory only
- EFT description expected to hold beyond PT
Weinberg theorem (unproven but established)

  local effective Lagrangian $N_\ell \neq 1$, M mass of the heavy quark

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{{\rm QCD}_{N_\ell}} + \frac{1}{M^2}\sum_i \omega_i \Phi_i + \ldots$$

$$\Phi_1 = \frac{1}{g_0^2}\,\mathrm{tr}\,(D_\mu F_{\nu\rho} D_\mu F_{\nu\rho})\,, \quad \Phi_2 = i\sum_{r=1}^{N_\ell} m_r \overline{\psi}_r \sigma_{\mu\nu} F_{\mu\nu} \psi_r\,, \quad \ldots$$

- $\mathcal{S} \in \{q, 1/r, \Lambda\}\,, \quad \mathcal{S} \ll M : \mathcal{L}_{\rm eff} = \mathcal{L}_{{\rm QCD}_{N_\ell}}$
up to small $(\mathcal{S}/M)^2$ corrections; drop them

Matching at "thresholds"

- Leading order EFT $\mathcal{L}_{\rm eff} = \mathcal{L}_{{\rm QCD}_{N_\ell}}$
- neglecting light masses, only parameter is $\bar{g}^{\ell}_{\overline{\rm MS}}$
- it has to be a function of $\bar{g}^{\rm f}_{\overline{\rm MS}}$ and $\overline{m}_{N_{\rm f}} =: \overline{m}$
- it is

$$[\bar{g}^{\ell}_{\overline{\rm MS}}(m_\star)]^2 = [\bar{g}^{\rm f}_{\overline{\rm MS}}(m_\star)]^2 \times C(\bar{g}^{\rm f}_{\overline{\rm MS}}(m_\star)).$$

with

$$C(x) = 1 + c_2 x^4 + c_3 x^6 + \ldots$$

$c_1 = 0$ due to choice $\mu = m_\star$ with $\overline{m}_{\overline{\rm MS}}(m_\star) = m_\star$

- clearly

$$\begin{array}{ccc} \bar{g}^{\ell}_{\overline{\rm MS}} & \Longleftrightarrow & \bar{g}^{\rm f}_{\overline{\rm MS}} \\ \Updownarrow & & \Updownarrow \\ \Lambda^{\ell}_{\overline{\rm MS}} & \Longleftrightarrow & \Lambda^{\rm f}_{\overline{\rm MS}} \end{array}$$

- therefore with $M = \phi_m(\bar{g}(\mu))\overline{m}(\mu)$:

$$\Lambda^{\ell}_{\overline{\rm MS}} = \Lambda^{\ell}_{\overline{\rm MS}}(M, \Lambda^{\rm f}_{\overline{\rm MS}}) = P_{\ell,{\rm f}}(M/\Lambda^{\rm f}_{\overline{\rm MS}})\,\Lambda^{\rm f}_{\overline{\rm MS}}$$

$$\Lambda^{\ell}_{\overline{\rm MS}} = P_{\ell,\rm f}(M/\Lambda^{\rm f}_{\overline{\rm MS}})\,\Lambda^{\rm f}_{\overline{\rm MS}}$$

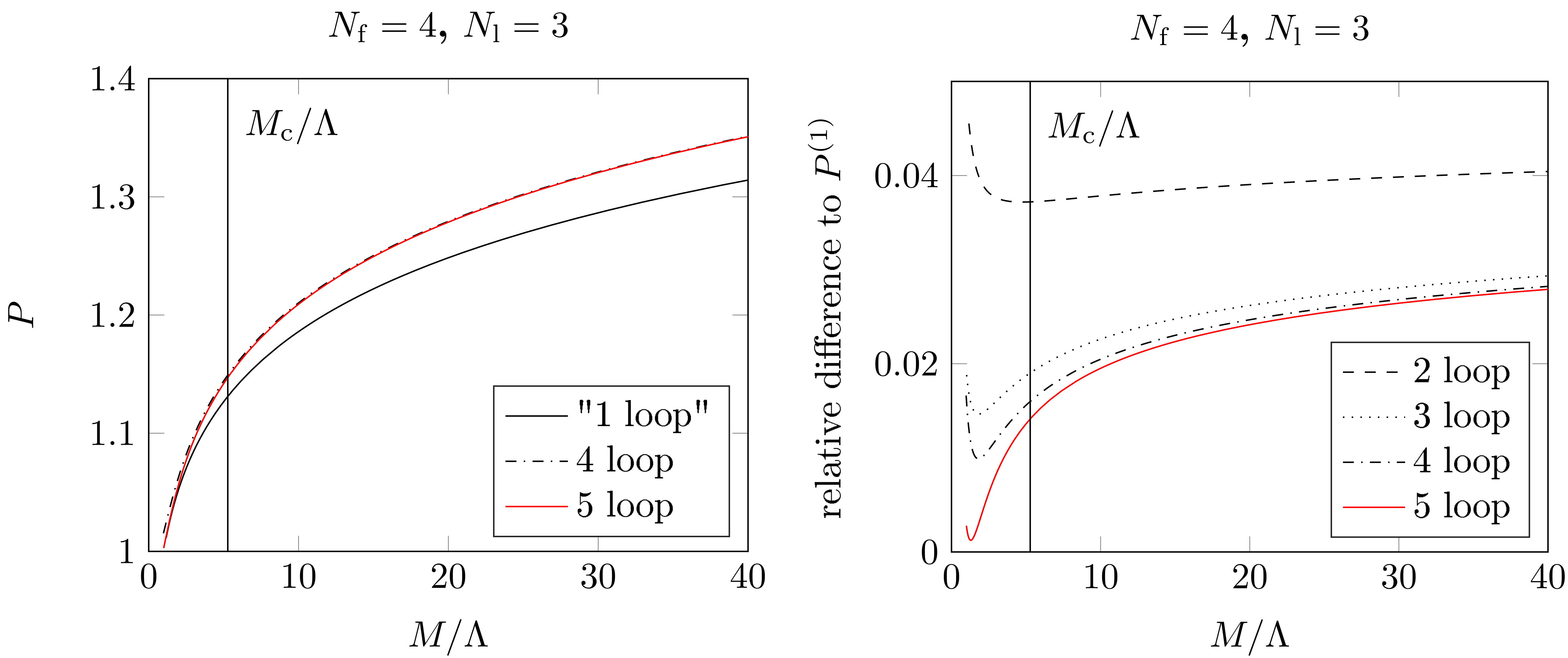
- For completeness the formula for $P_{\ell,\rm f}$ is

$$P_{\ell,\rm f}(M/\Lambda^{\rm f}_{\overline{\rm MS}}) = \frac{\varphi^{\ell}_{\overline{\rm MS}}\left(g_\star\sqrt{C(g_\star)}\right)}{\varphi^{\rm f}_{\overline{\rm MS}}(g_\star)}\,,$$

$g_\star = \bar{g}_{\overline{\rm MS}}(m_\star)$ as solution of

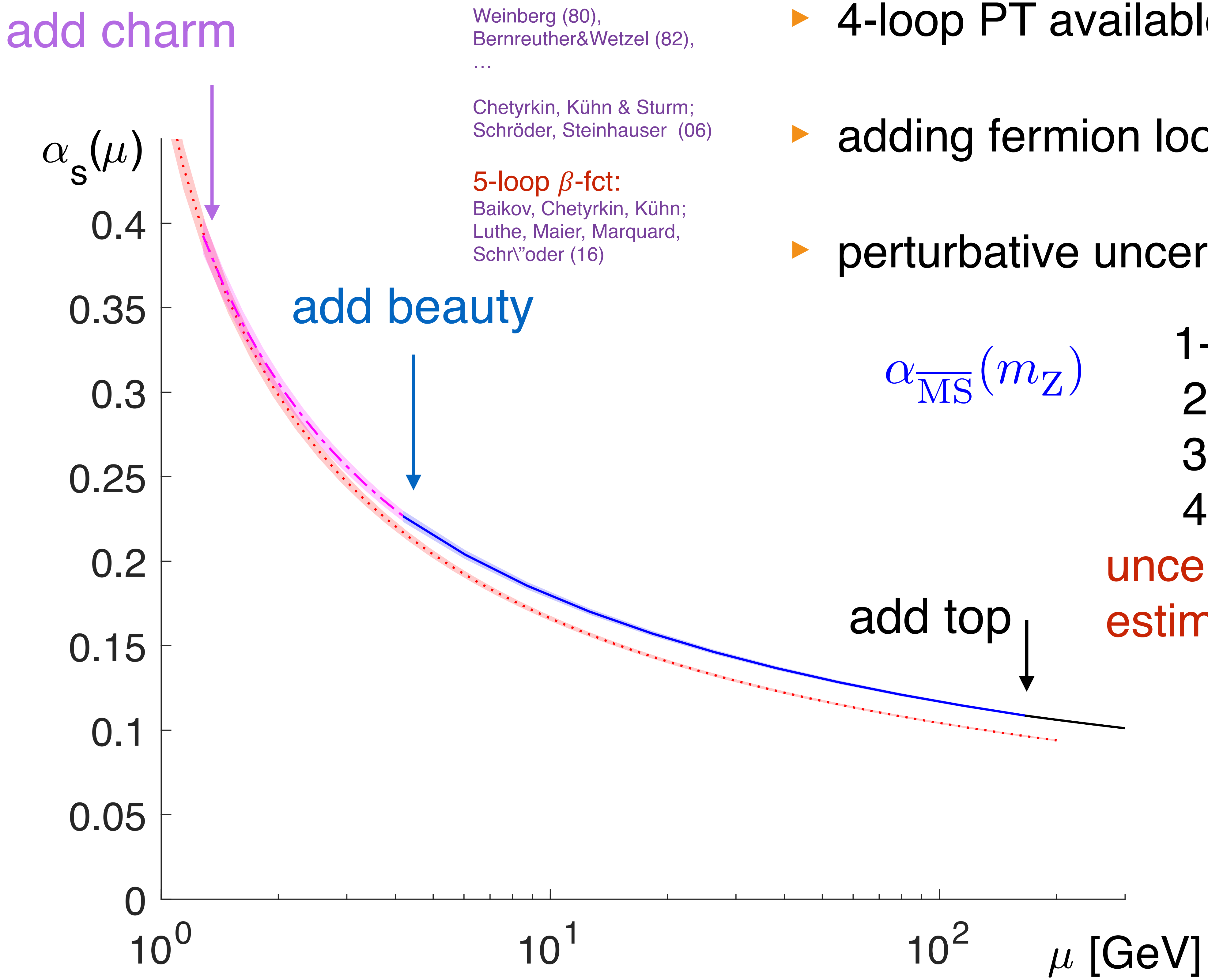
$$\frac{\Lambda^{\rm f}_{\overline{\rm MS}}}{M} = \frac{\left(b_0 g_\star^2\right)^{-b_1/(2b_0^2)}}{(2b_0 g_\star^2)^{-d_0/(2b_0)}}\,\mathrm{e}^{-1/(2b_0 g_\star^2)} \times \exp\left\{-\int_0^{g_\star(M/\Lambda)} \mathrm{d}x\,\left[\frac{1-\tau_{\rm f}(x)}{\beta_{\rm f}(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x} + \frac{d_0}{b_0 x}\right]\right\}$$

Accuracy of perturbation theory



- looking just at PT intrinsic error: 0.1% at charm
- for ratio of Λ-parameters
- But can PT be trusted at the charm? 1GeV
  - ➡ yes, for this case. I show a test later.

Therefore we can add in c, b, t - quarks by perturbation theory



Weinberg (80),
Bernreuther&Wetzel (82),
…

Chetyrkin, Kühn & Sturm;
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5-loop β-fct:
Baikov, Chetyrkin, Kühn;
Luthe, Maier, Marquard,
Schr\"oder (16)

- 4-loop PT available
- adding fermion loops, "only"
- perturbative uncertainties are tiny

$\alpha_{\overline{\rm MS}}(m_Z)$

1-loop:	0.11701
2	0.00128
3	0.00019
4	0.00006

uncertainty estimate= 0.00025

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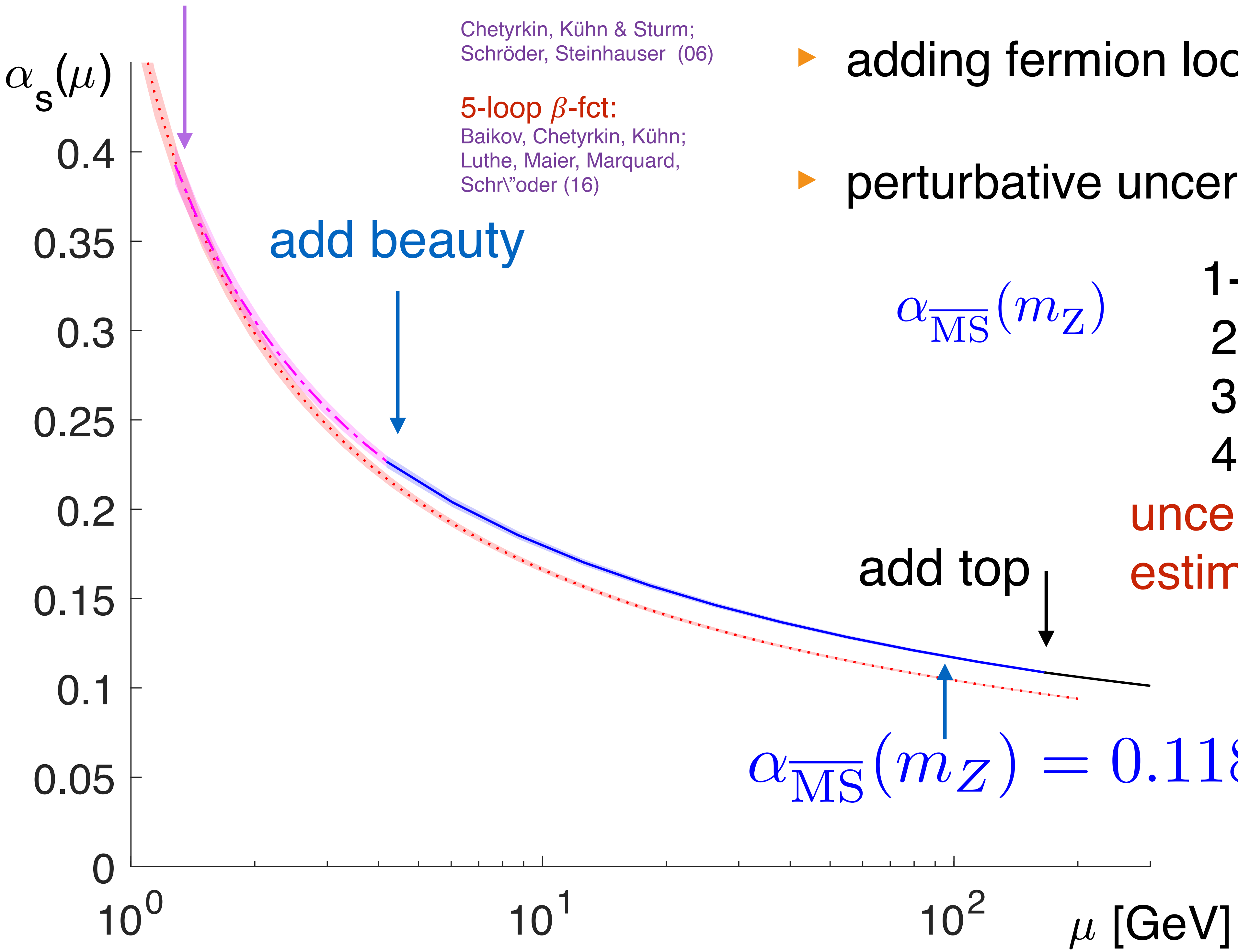

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Surprising factorization formula

Let us be very carful and put a mass-scale in to make things dimensionless.

$$P_{\ell,\mathrm{f}}(M/\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}})\,\frac{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{\mathrm{f}}(M)} = \frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{\ell}}$$

- multiply with $\frac{\mathcal{S}^{\mathrm{f}}(0)}{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}}$

  then

$$P_{\ell,\mathrm{f}}(M/\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}})\,\frac{\mathcal{S}^{\mathrm{f}}(0)}{\mathcal{S}^{\mathrm{f}}(M)} = \frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{\ell}}\,\frac{\mathcal{S}^{\mathrm{f}}(0)}{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}}$$

  or

$$\frac{\mathcal{S}^{\mathrm{f}}(M)}{\mathcal{S}^{\mathrm{f}}(0)} = Q^{\mathcal{S}}_{\ell,\mathrm{f}} \times P_{\ell,\mathrm{f}}(M/\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}})\,, \qquad Q^{\mathcal{S}}_{\ell,\mathrm{f}} = \frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}/\mathcal{S}^{\ell}}{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}/\mathcal{S}^{\mathrm{f}}(0)}$$

- Now take as an example $\mathcal{S} = m_{\mathrm{proton}}$, and $N_\ell = 3$, $m_4 = m_{\mathrm{charm}}$, then we conclude that the charm-mass-dependence of the proton mass can be computed perturbatively and is the same as e.g. the charm-mass dependence of, e.g., F_π.

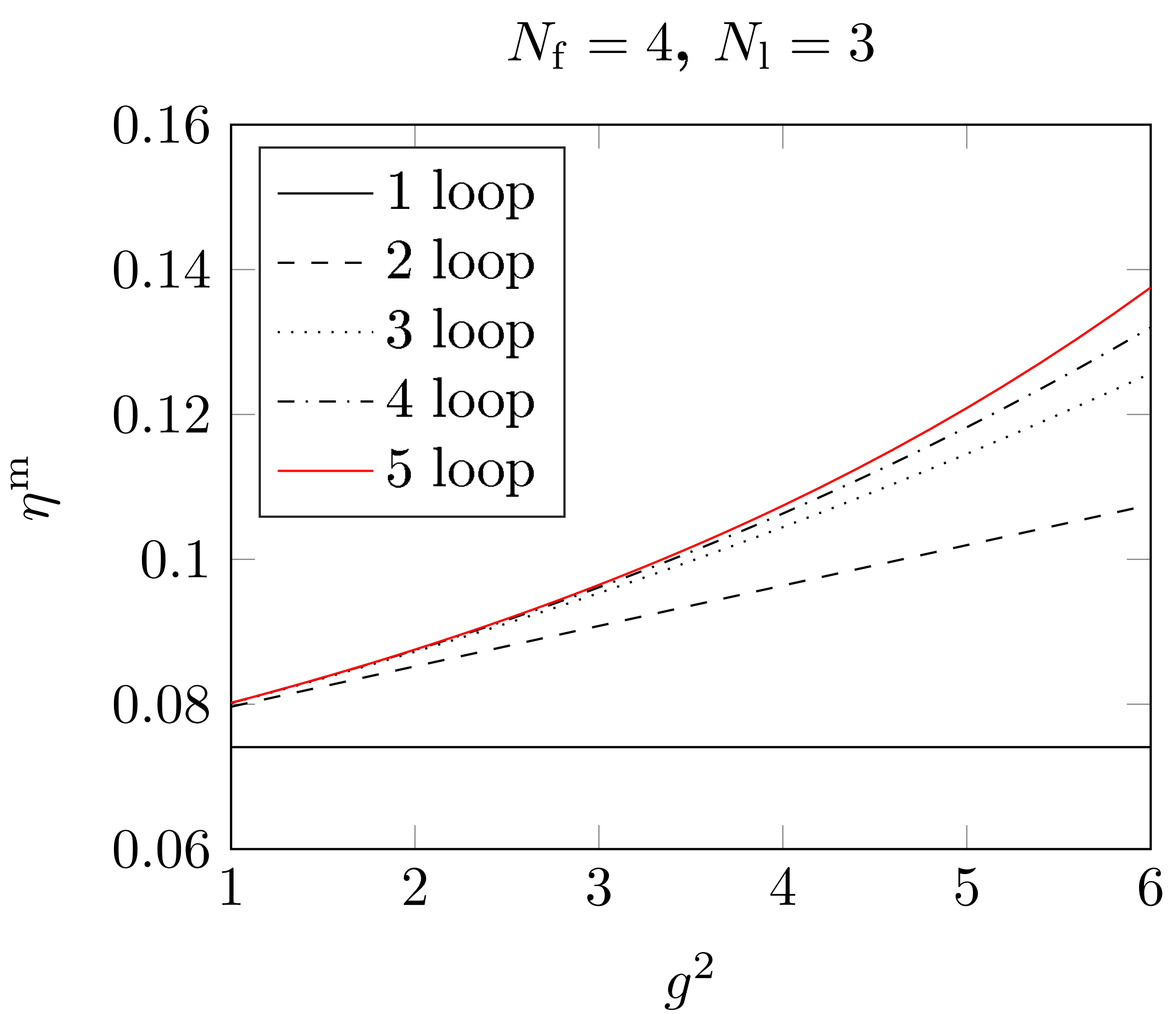
  A little algebra yields

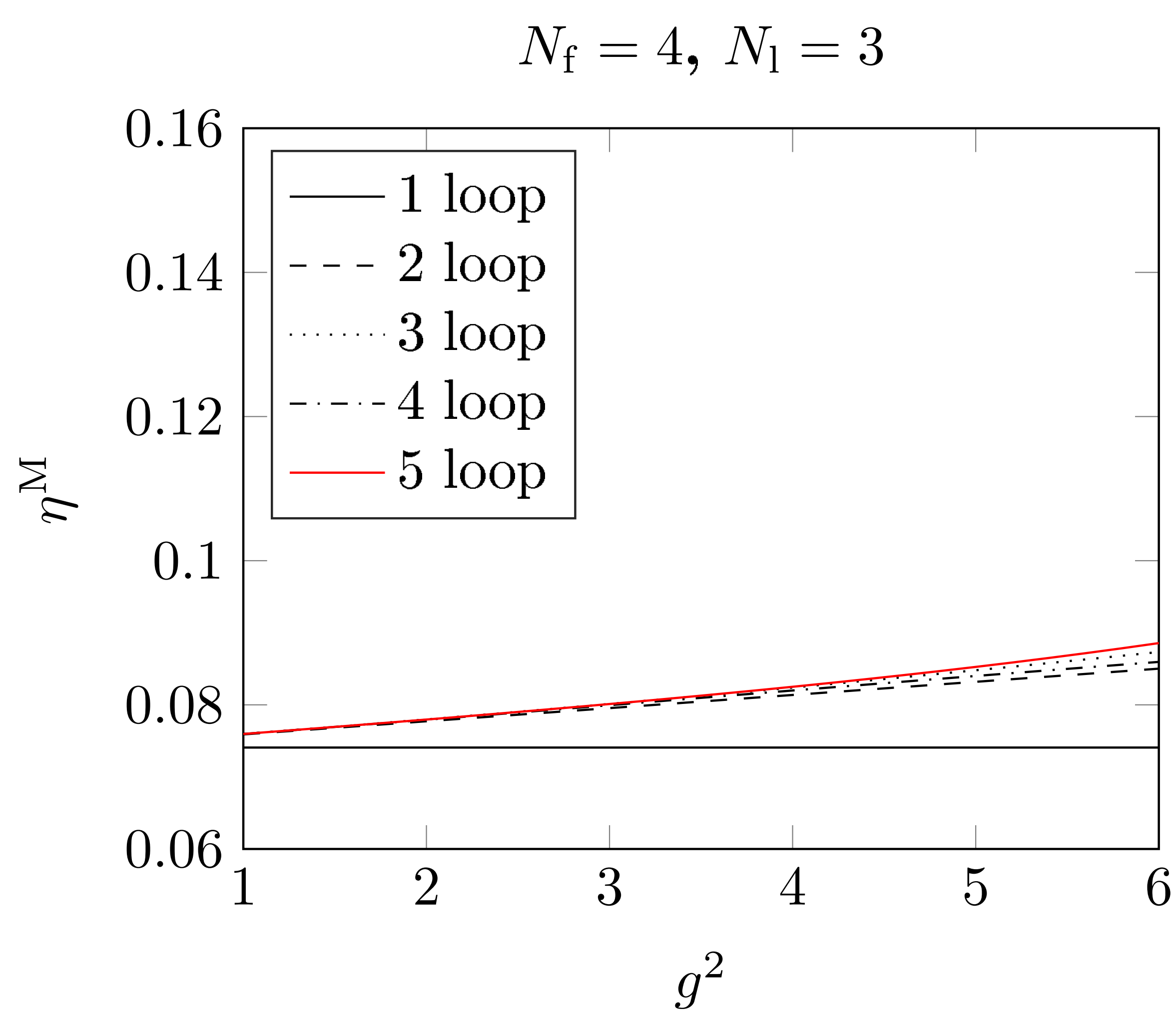
$$\eta \equiv \frac{M_{\mathrm{charm}}}{m_{\mathrm{proton}}}\,\frac{\partial m_{\mathrm{proton}}}{\partial M_{\mathrm{charm}}}\bigg|_{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}} = 1 - \frac{b_0(N_{\mathrm{f}})}{b_0(N_\ell)} + \mathrm{O}(g^2(M_{\mathrm{charm}})) = 0.074 + \mathrm{O}(g^2)\,,$$

Mass scaling function

$$\frac{M}{m_{\mathrm{f}}^{\mathrm{had}}(M)} \left. \frac{\partial m_{\mathrm{f}}^{\mathrm{had}}(M)}{\partial M} \right|_{\Lambda_{\mathrm{f}}} = \eta^{\mathrm{M}}$$

$$\eta^{\mathrm{M}}(M) \equiv \frac{M}{P} \left. \frac{\partial P}{\partial M} \right|_{\Lambda_{\mathrm{f}}} = \frac{M}{\Lambda_{\mathrm{f}}} \frac{P'}{P}$$





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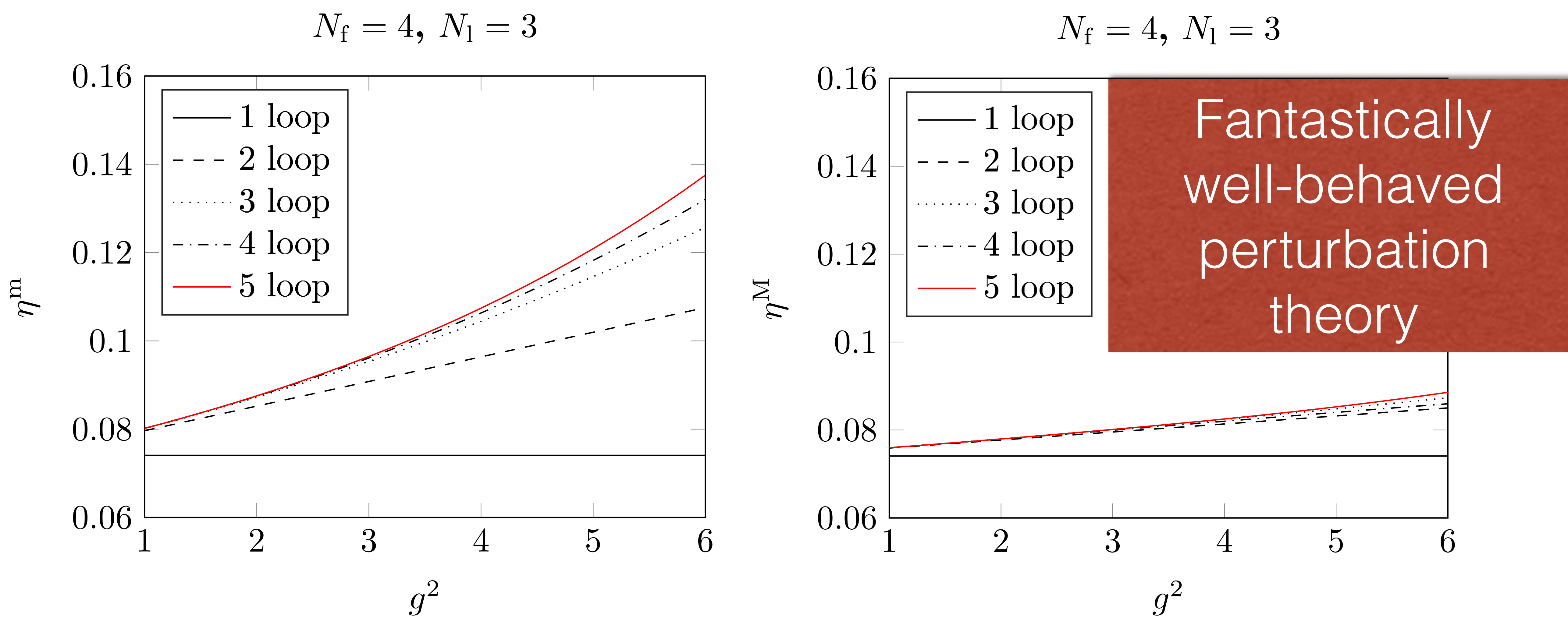


Mass scaling function evaluated by NP MC in a model: N_f=2 —> 0

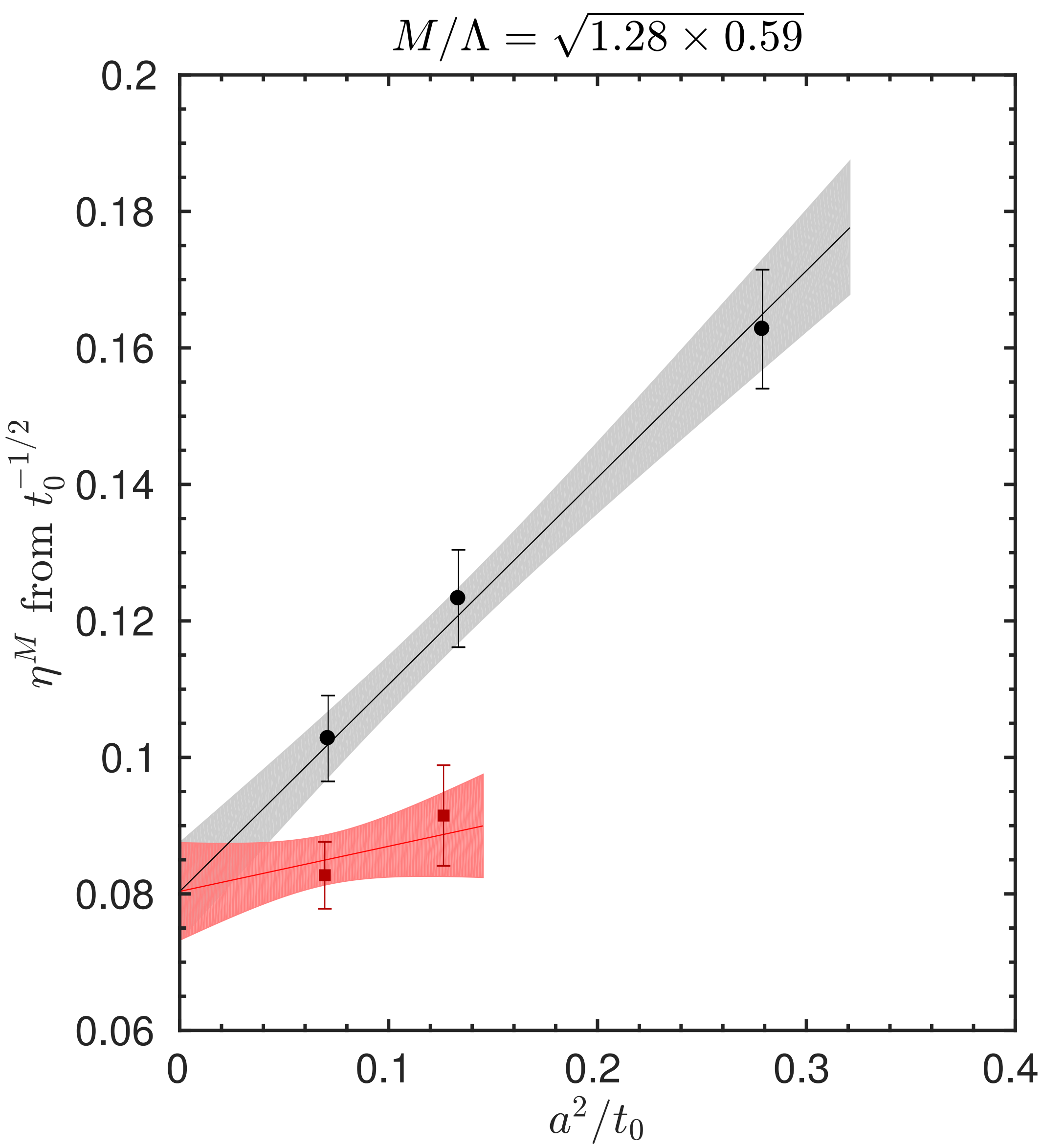
$$\eta^{\mathrm{M}}(\overline{M}) \approx \frac{\log(m^{\mathrm{had}}(M_2)/m^{\mathrm{had}}(M_1))}{\log(M_2/M_1)}$$

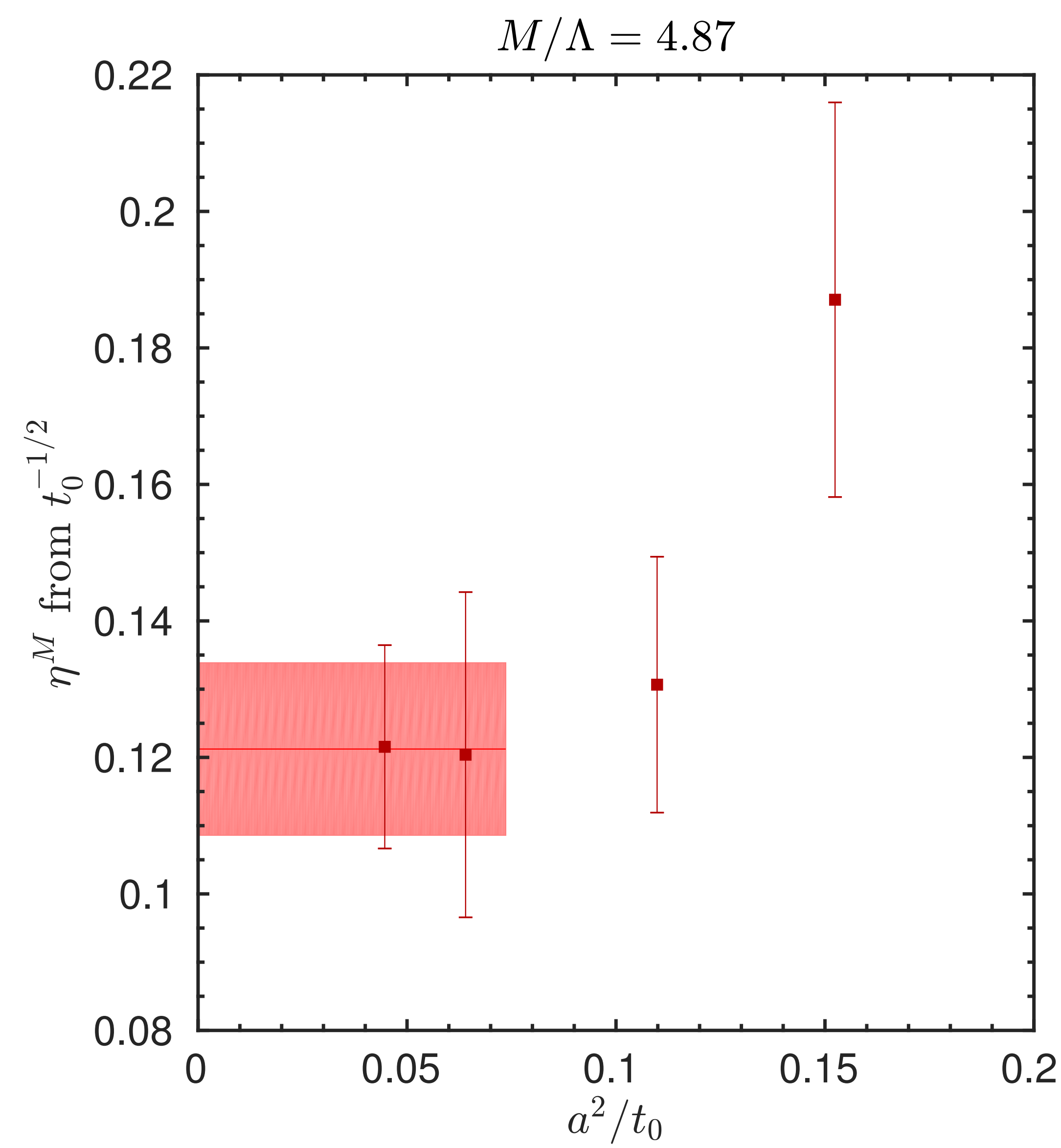
$$\overline{M} = \sqrt{M_2 M_1}$$

$$-\frac{\mu}{2t_0}\frac{\mathrm{d}t_0}{\mathrm{d}\mu} = \eta^{\mathrm{M}}(M)$$

M=Z μ

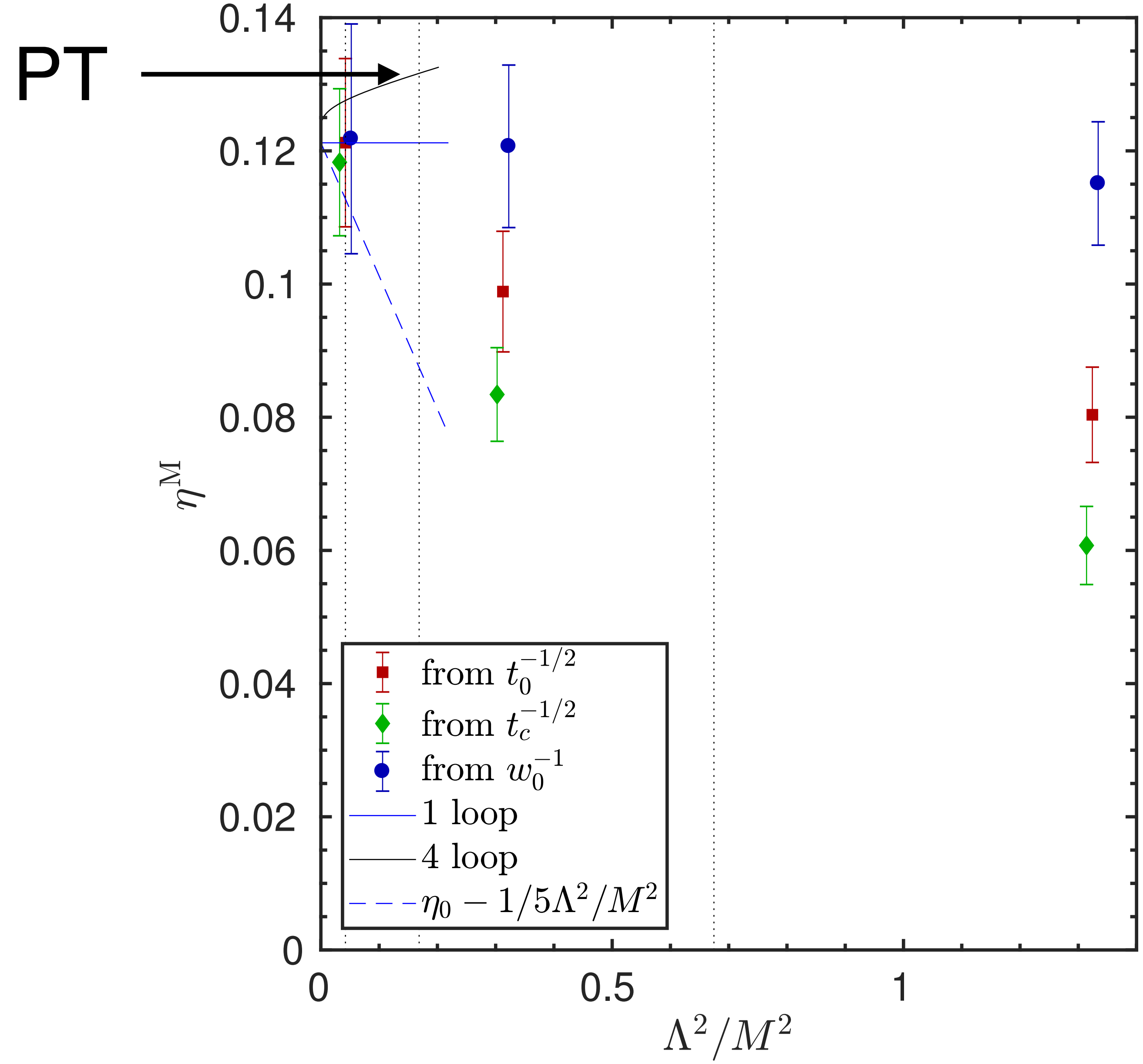
μ = twisted mass





Athenodorou, Finkenrath, Knechtli,Korzec, Leder, Marinkovic, S.,

Mass scaling function: result



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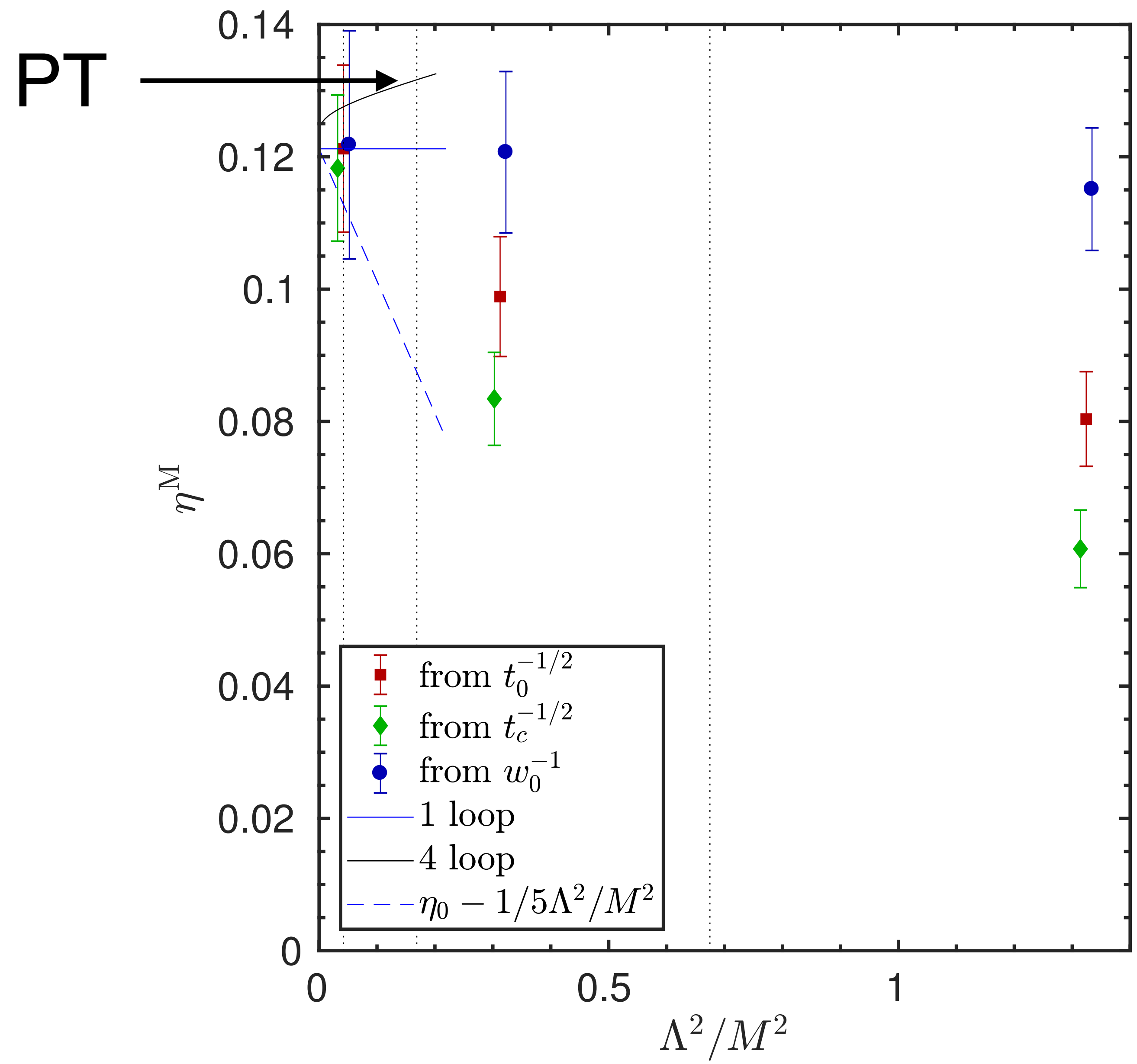

- precise confirmation of PT at charm

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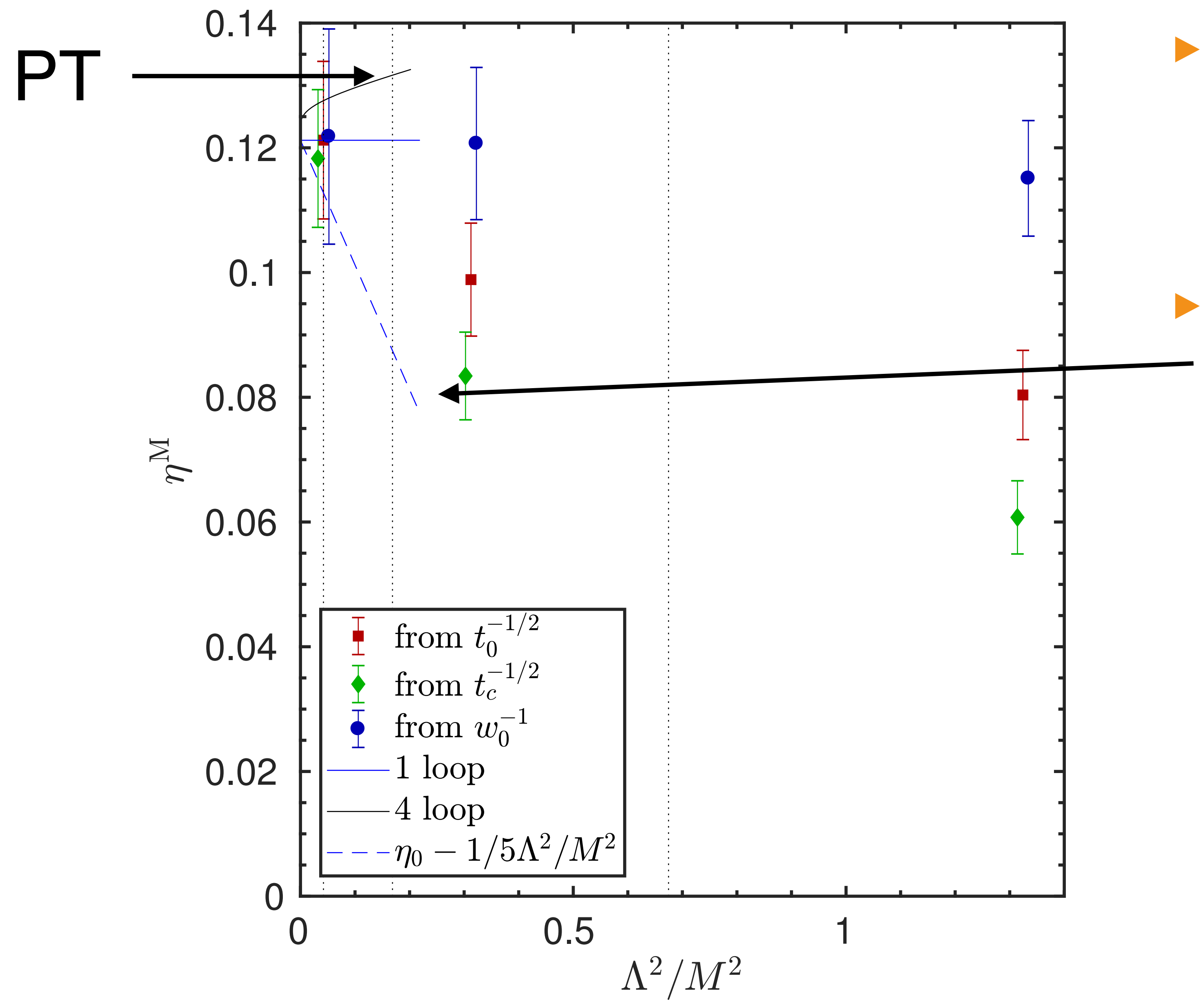
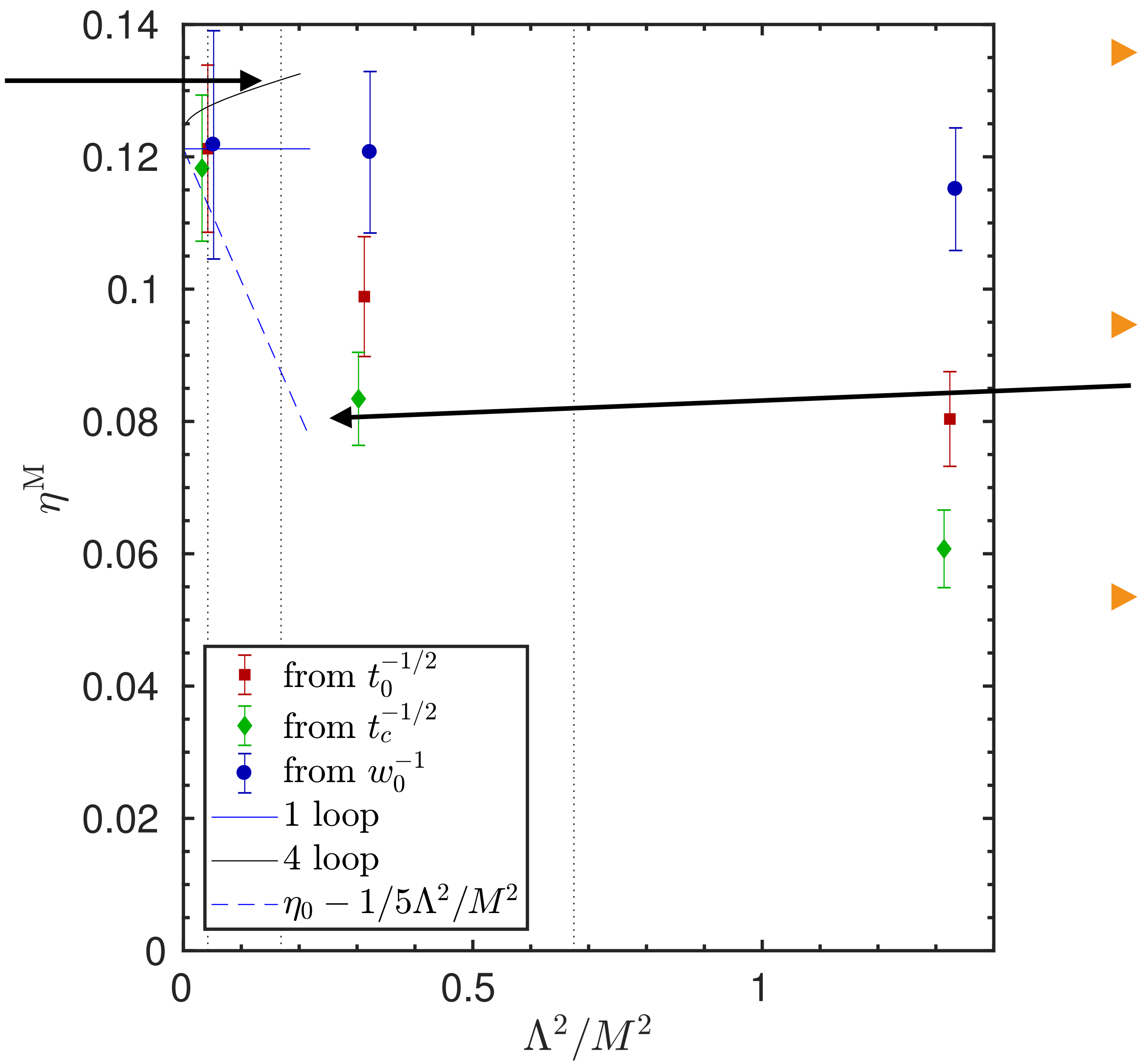


- precise confirmation of PT at charm
- estimate of (~maximal NP contribution)

Mass scaling function: result

PT



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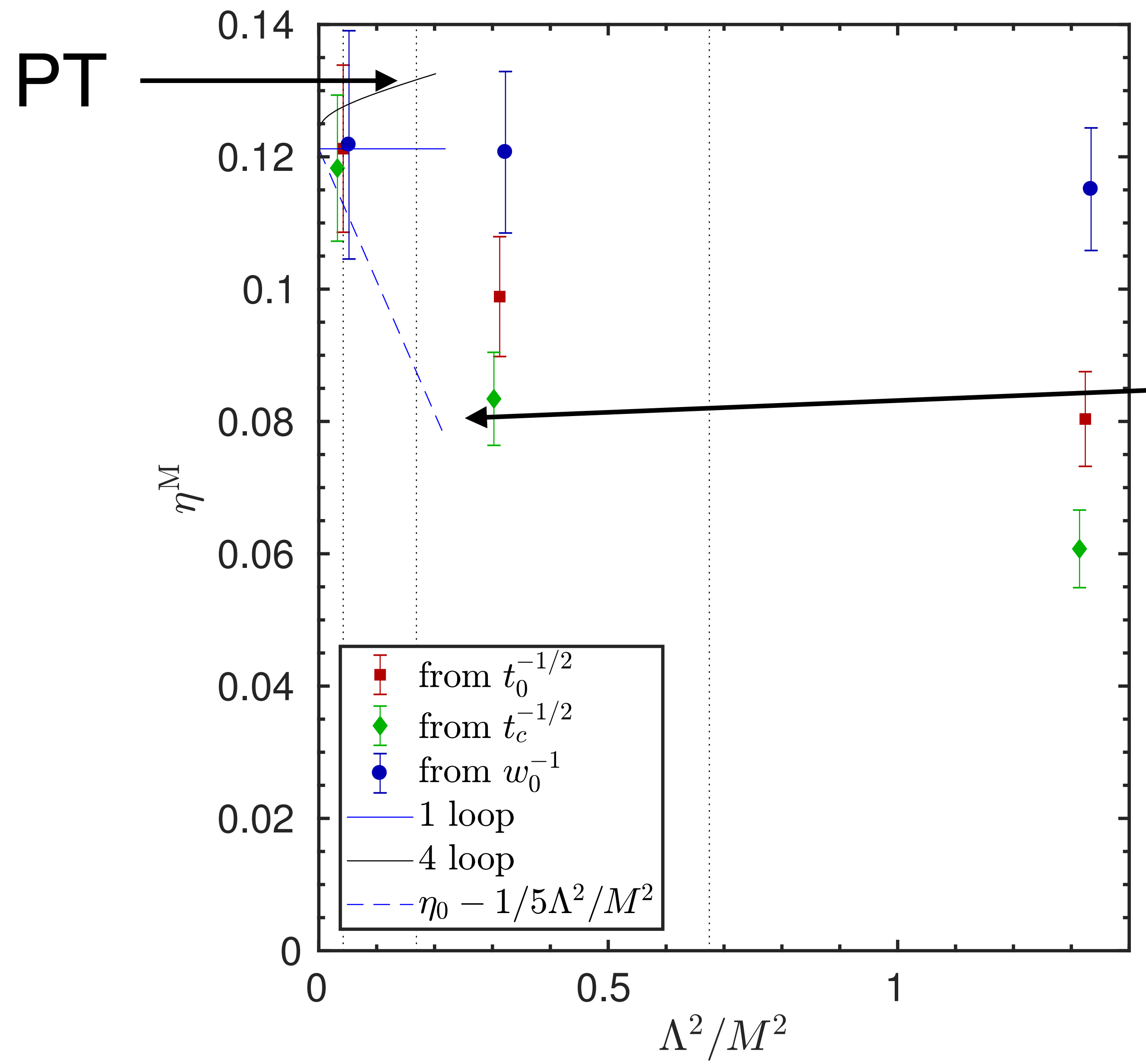
- leads to interesting statements:

$$\Delta \log\left[P_{\rm l,f}(M/\Lambda_{\rm f})\right] = 0.004.$$

Mass scaling function: result



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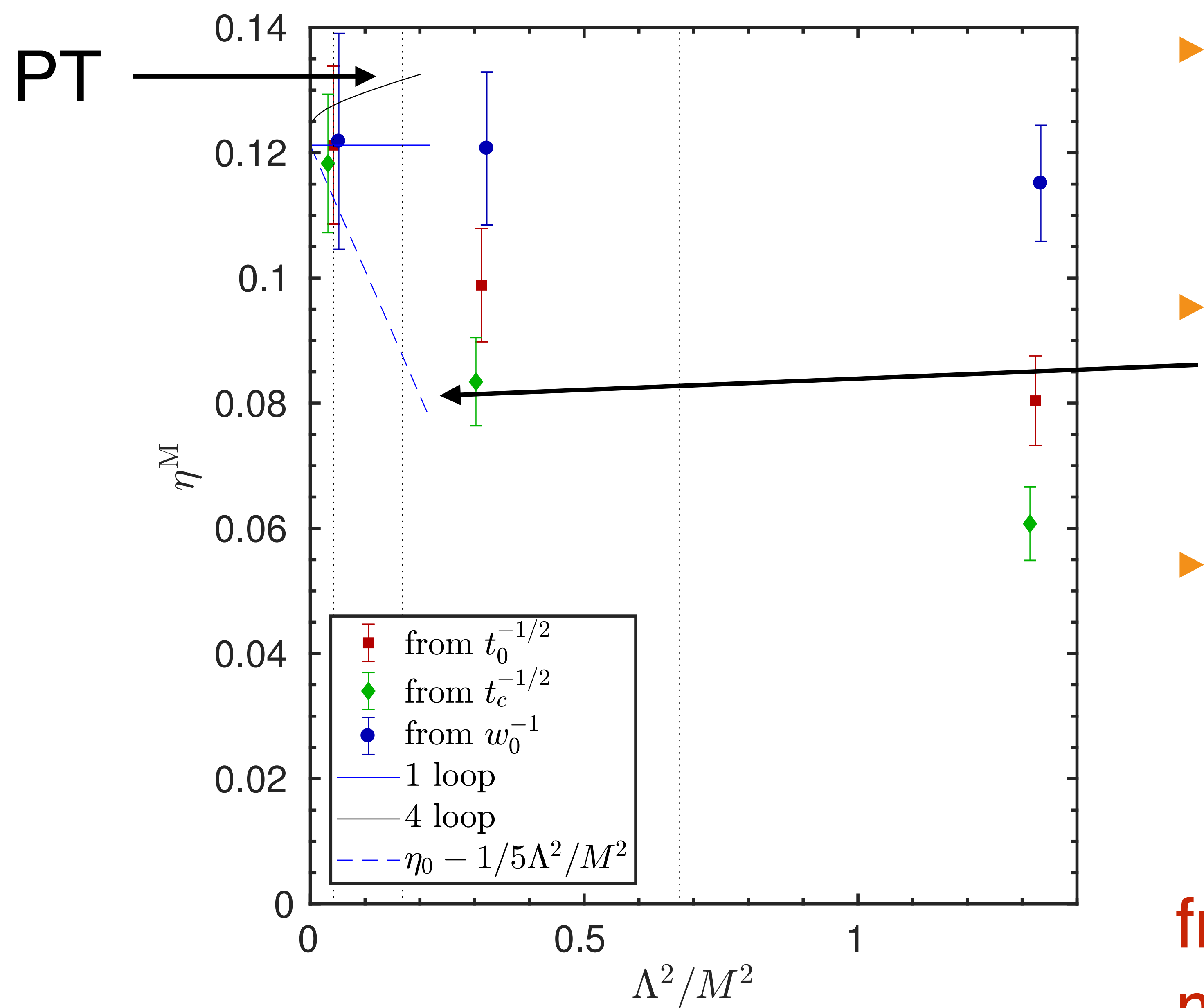
0.4% precision for Λ_4 / Λ_3 (decoupling two charm quarks)

~0.2% decoupling one charm q.

Mass scaling function: result



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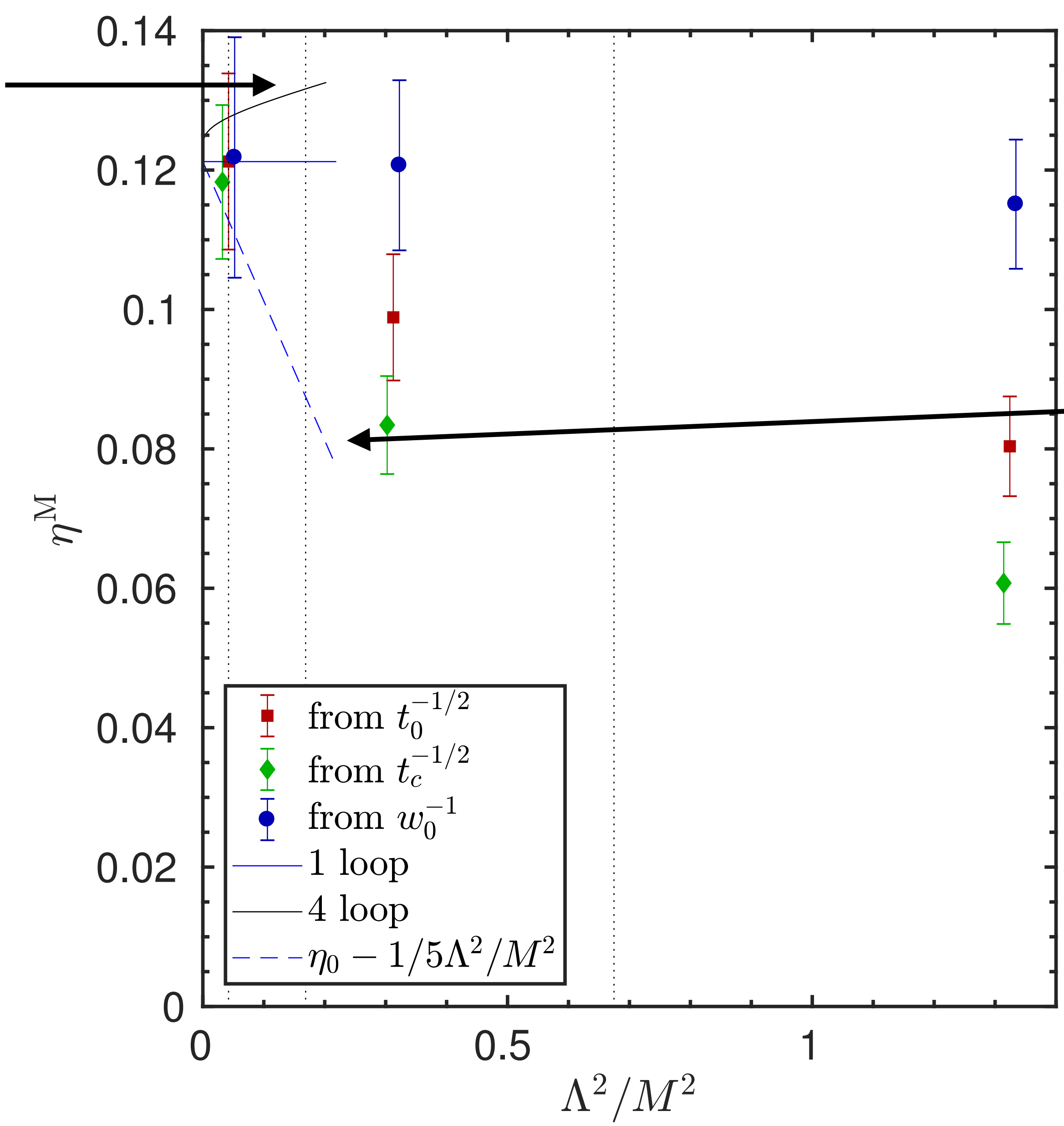
from the determined mass-effect in Nf=2 can predict Λ_2 / Λ_0 w. precision

$$\frac{\Lambda_{\overline{\mathrm{MS}}}\sqrt{t_0(0)}\Big|_{N_{\mathrm{f}}=2}}{\Lambda_{\overline{\mathrm{MS}}}\sqrt{t_0}\Big|_{N_{\mathrm{l}}=0}} = 1.134(17)$$

Mass scaling function: result

PT



- precise confirmation of PT at charm
- estimate of (~maximal NP contribution)
- leads to interesting statements:

$$\Delta \log\left[P_{\mathrm{l,f}}(M/\Lambda_{\mathrm{f}})\right] = 0.004.$$

What about power corrections?

from the determined mass-effect in Nf=2 can predict Λ_2 / Λ_0 w. precision

$$\frac{\Lambda_{\overline{\mathrm{MS}}}\sqrt{t_0(0)}\Big|_{N_\mathrm{f}=2}}{\Lambda_{\overline{\mathrm{MS}}}\sqrt{t_0}\Big|_{N_\mathrm{l}=0}} = 1.134(17)$$

Considered scales

- static potential

  force $F(r) = V'(r)$,

  r_0 defined by: $(r_0)^2 F(r_0) = 1.65$

  r_1 defined by: $(r_1)^2 F(r_1) = 1.0$

- Gradient flow observables: t_0, t_c, w_0

Simulations

NP O(a)-improved Wilson, standard mass term

$\frac{T}{a} \times \left(\frac{L}{a}\right)^3$	β	BC	κ	am	M/Λ	r_0/a	t_0/a^2	kMDU
64×32^3	5.3	p	0.13550	0.03405(8)	0.638(46)	5.903(36)	3.481(14)	1
64×32^3	5.3	p	0.13450	0.06979(7)	1.308(95)	5.193(20)	2.714(14)	2
64×32^3	5.3	p	0.13270	0.13873(8)	2.600(189)	4.270(6)	1.842(3)	2
120×32^3	5.5	o	0.136020	0.02467(4)	0.630(46)	8.49(12)	7.318(36)	8
120×32^3	5.5	o	0.135236	0.05022(3)	1.282(93)	7.580(44)	6.092(21)	8
96×48^3	5.5	p	0.133830	0.09614(2)	2.454(178)	6.787(19)	4.867(12)	4
192×48^3	5.7	o	0.136200	0.01691(2)	0.586(43)	11.48(24)	14.02(6)	4
192×48^3	5.7	o	0.135570	0.03683(2)	1.277(94)	10.53(12)	11.87(7)	4
192×48^3	5.7	o	0.134450	0.07209(2)	2.500(184)	9.50(5)	9.821(36)	8

Simulations

NP O(a)-improved Wilson, at maximal (mass) twist

(same lattice spacings as un-twisted)

$\frac{T}{a} \times \left(\frac{L}{a}\right)^3$	β	κ	$a\mu$	M/Λ	r_0/a	t_0/a^2	kMDU
120×32^3	5.300	0.136457	0.024505	0.5900	–	4.174(13)	4.3
120×32^3	5.500	0.1367749	0.018334	0.5900	8.77(15)	7.917(82)	8
192×48^3	5.700	0.136687	0.013713	0.5900	–	14.40(10)	5.8
120×32^3	5.500	0.1367749	0.039776	1.2800	8.010(62)	6.871(33)	8
192×48^3	5.700	0.136687	0.029751	1.2800	–	12.668(39)	16.2
120×32^3	5.500	0.1367749	0.077687	2.5000	7.392(62)	5.836(27)	8
192×48^3	5.700	0.136687	0.058108	2.5000	–	10.916(38)	9
192×48^3	5.600	0.136710	0.130949	4.8700	–	6.561(12)	16
120×32^3	5.700	0.136698	0.113200	4.8703	9.123(57)	9.104(36)	17.2
192×48^3	5.880	0.136509	0.087626	4.8700	11.946(55)	15.622(62)	23.1
192×48^3	6.000	0.136335	0.072557	4.8700	14.34(10)	22.39(12)	22.4
192×48^3	5.600	0.136710	0.155367	5.7781	–	6.181(11)	2.1
192×48^3	5.700	0.136687	0.1343	5.7781	–	8.565(31)	2.7
120×32^3	5.880	0.136509	0.103965	5.7781	–	14.916(93)	59.9

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NP O(a)-improved Wilson, at maximal (mass) twist

(same lattice spacings as un-twisted)

$\frac{T}{a} \times \left(\frac{L}{a}\right)^3$	β	κ	$a\mu$	M/Λ	r_0/a	t_0/a^2	kMDU
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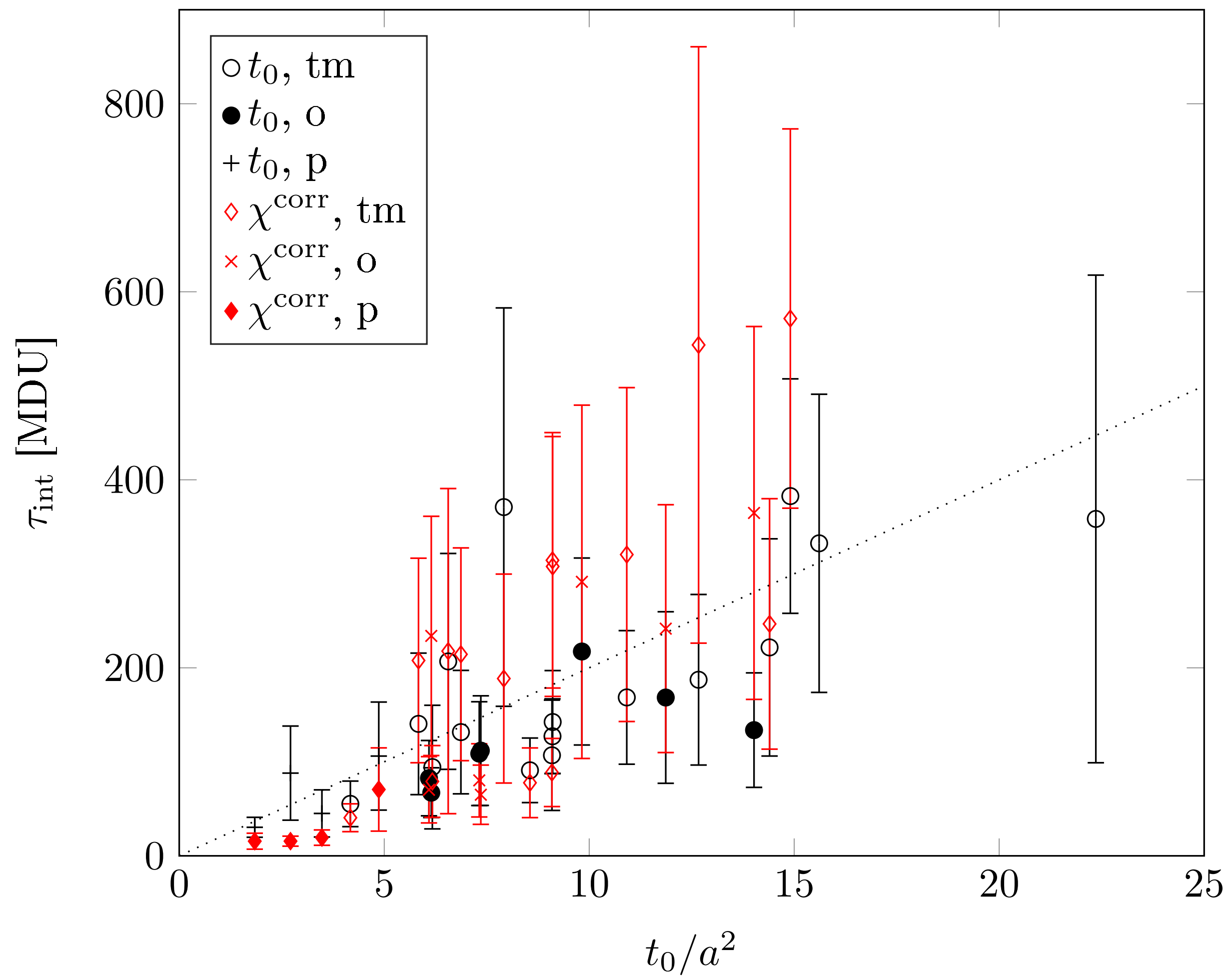
120×32^3	5.500	0.1367749	0.018334	0.5900	8.77(15)	7.917(82)	8
192×48^3	5.700	0.136687	0.013713	0.5900	–	14.40(10)	5.8
120×32^3	5.500	0.1367749	0.039776	1.2800	8.010(62)	6.871(33)	8
192×48^3	5.700	0.136687	0.029751	1.2800	–	12.668(39)	16.2
120×32^3	5.500	0.1367749	0.077687	2.5000	7.392(62)	5.836(27)	8
192×48^3	5.700	0.136687	0.058108	2.5000	–	10.916(38)	9
192×48^3	5.600	0.136710	0.130949	4.8700	–	6.561(12)	16
120×32^3	5.700	0.136698	0.113200	4.8703	9.123(57)	9.104(36)	17.2
192×48^3	5.880	0.136509	0.087626	4.8700	11.946(55)	15.622(62)	23.1
192×48^3	6.000	0.136335	0.072557	4.8700	14.34(10)	22.39(12)	22.4
192×48^3	5.600	0.136710	0.155367	5.7781	–	6.181(11)	2.1
192×48^3	5.700	0.136687	0.1343	5.7781	–	8.565(31)	2.7
120×32^3	5.880	0.136509	0.103965	5.7781	–	14.916(93)	59.9

Autocorrelations (for lattice (non)-experts)



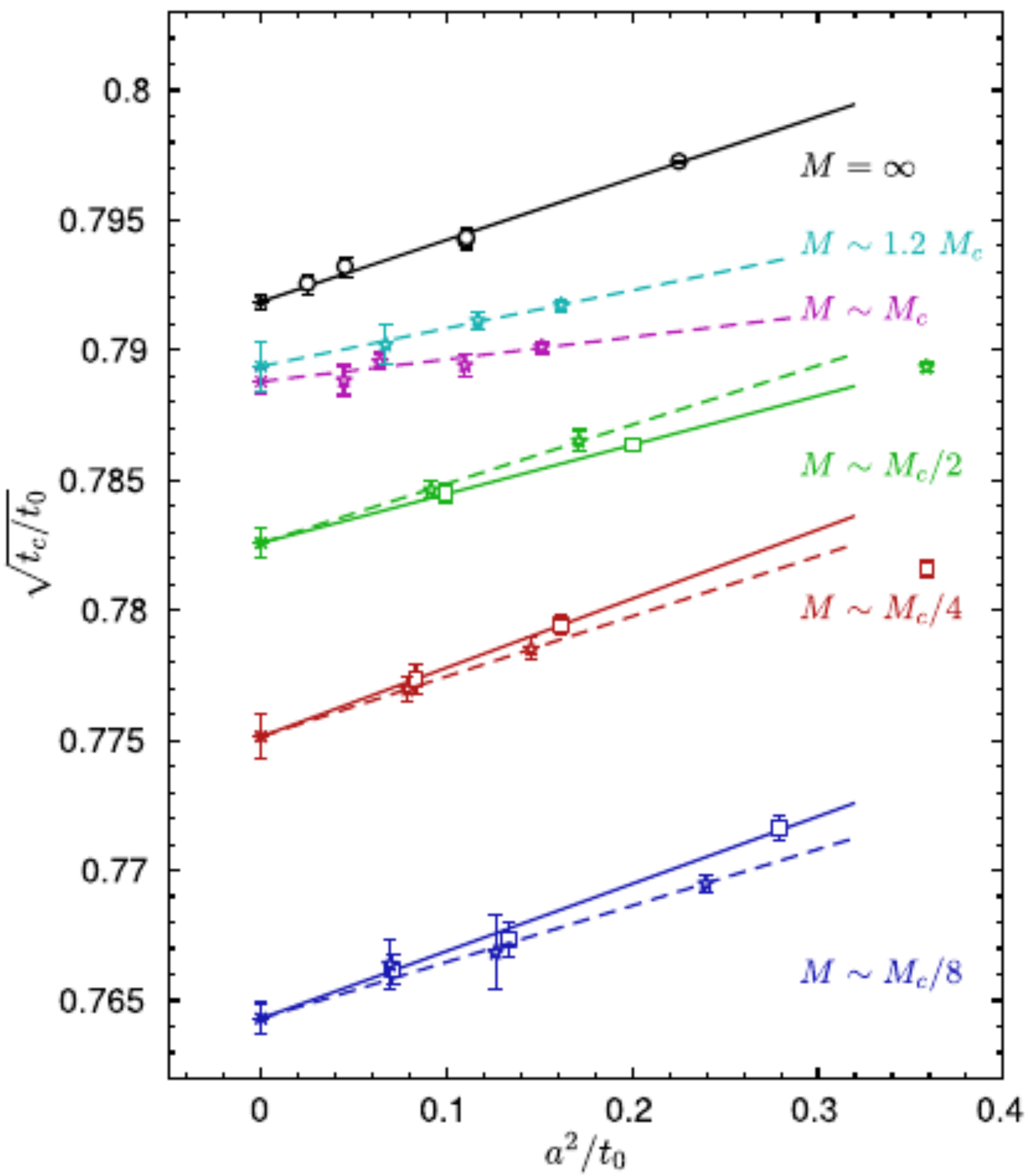
$t_0/a^2 > 5.5$: open boundary conditions [Lüscher and Schaefer, arXiv:1206.2809], using openQCD

with statistics of 1k MDU: 5-10 independent configurations —> doing 1 .. 4 … 20 …60 kMDU

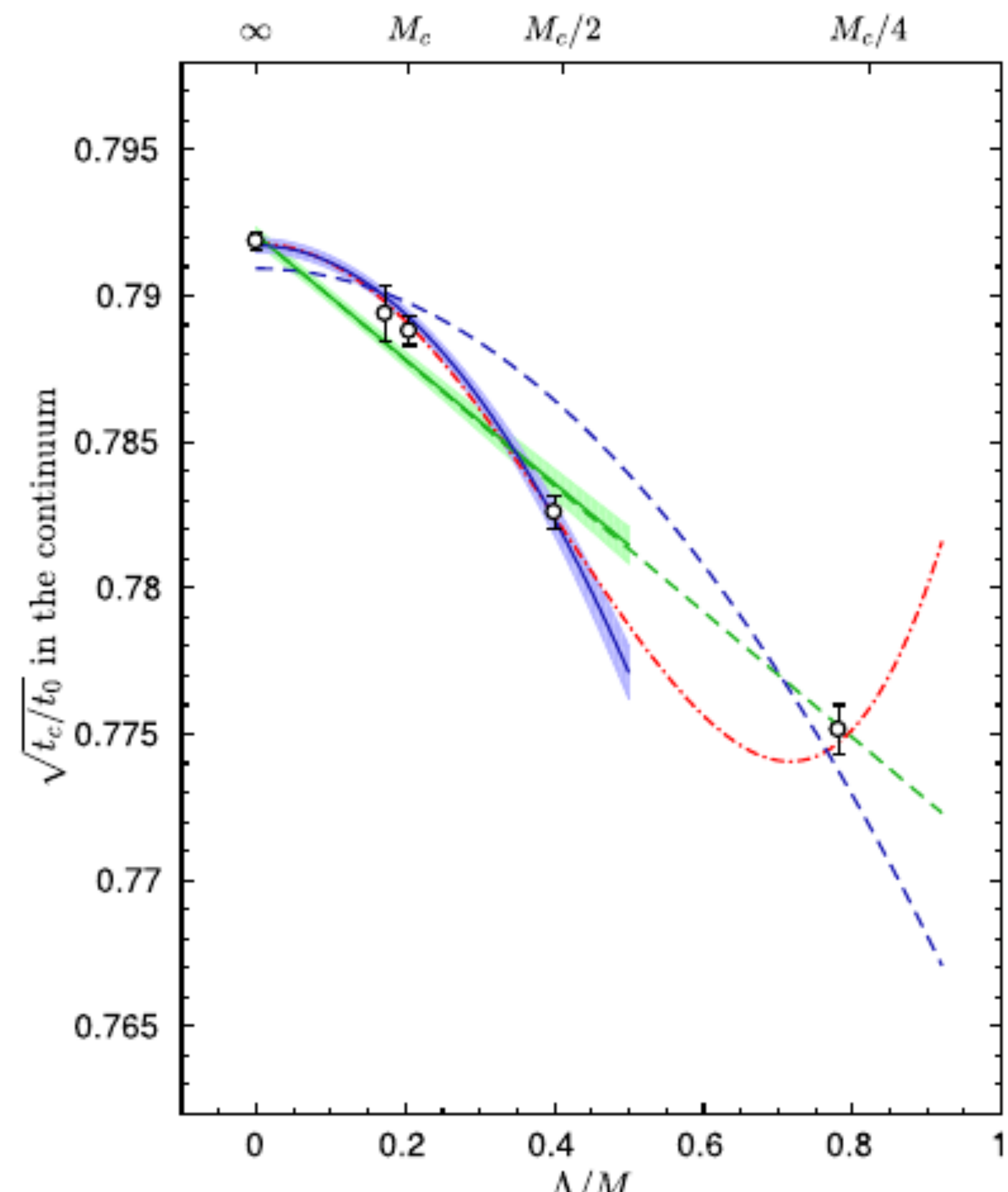
error analysis with τ_{exp} [Wolff, hep-lat/0306017;Schaefer, Sommer and Virotta, arXiv:1009:5228]

original study: Bruno, Finkenrath, Knechtli, Leder, Sommer, Phys.Rev.Lett. 114 (2015)
significant improvement: Knechtli, Leder, Korzec, Phys.Lett. B774 (2017)
(twisted + untwisted quarks, higher masses,)

continuum limits



at charm: ~0.2 % effects



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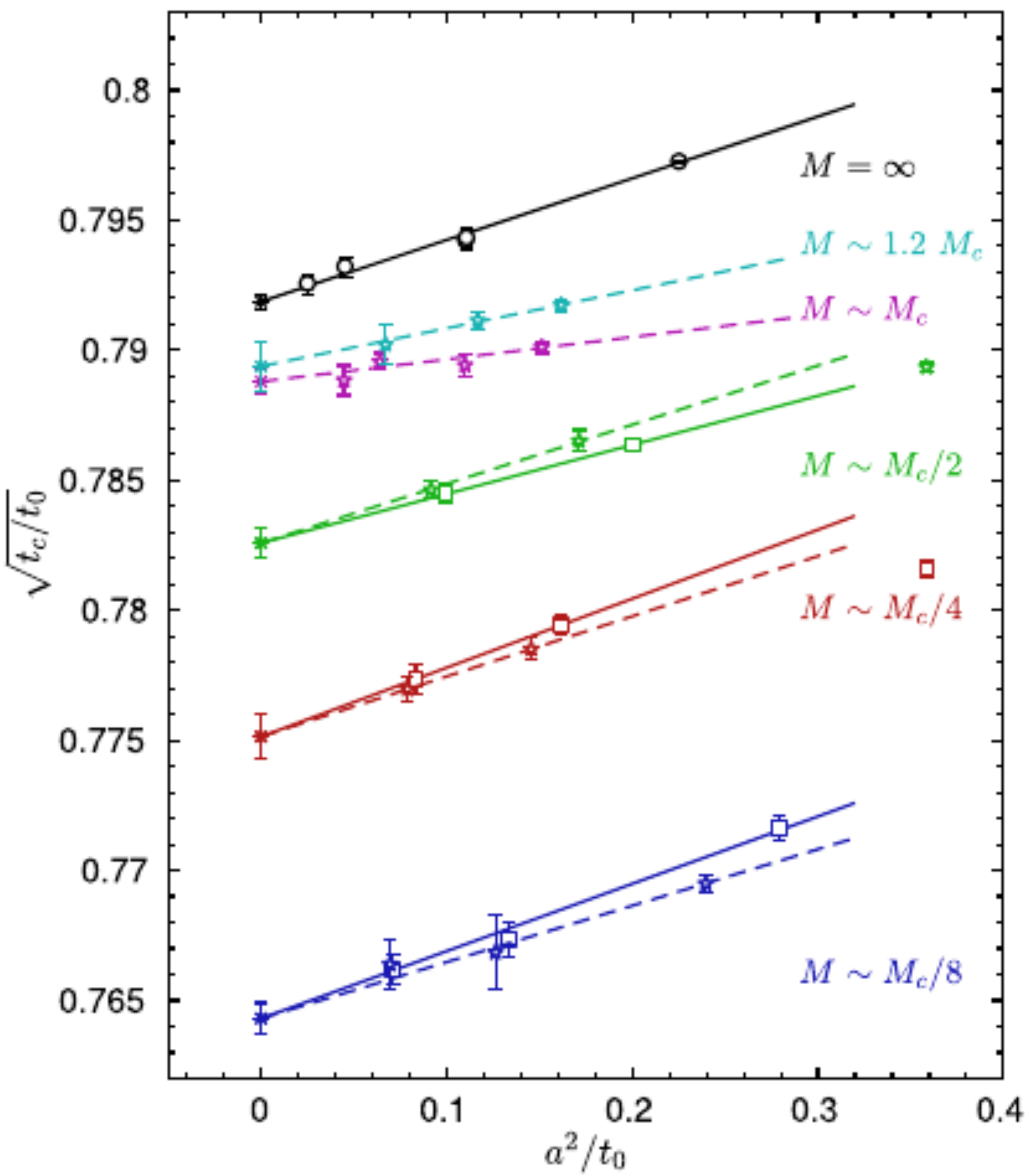
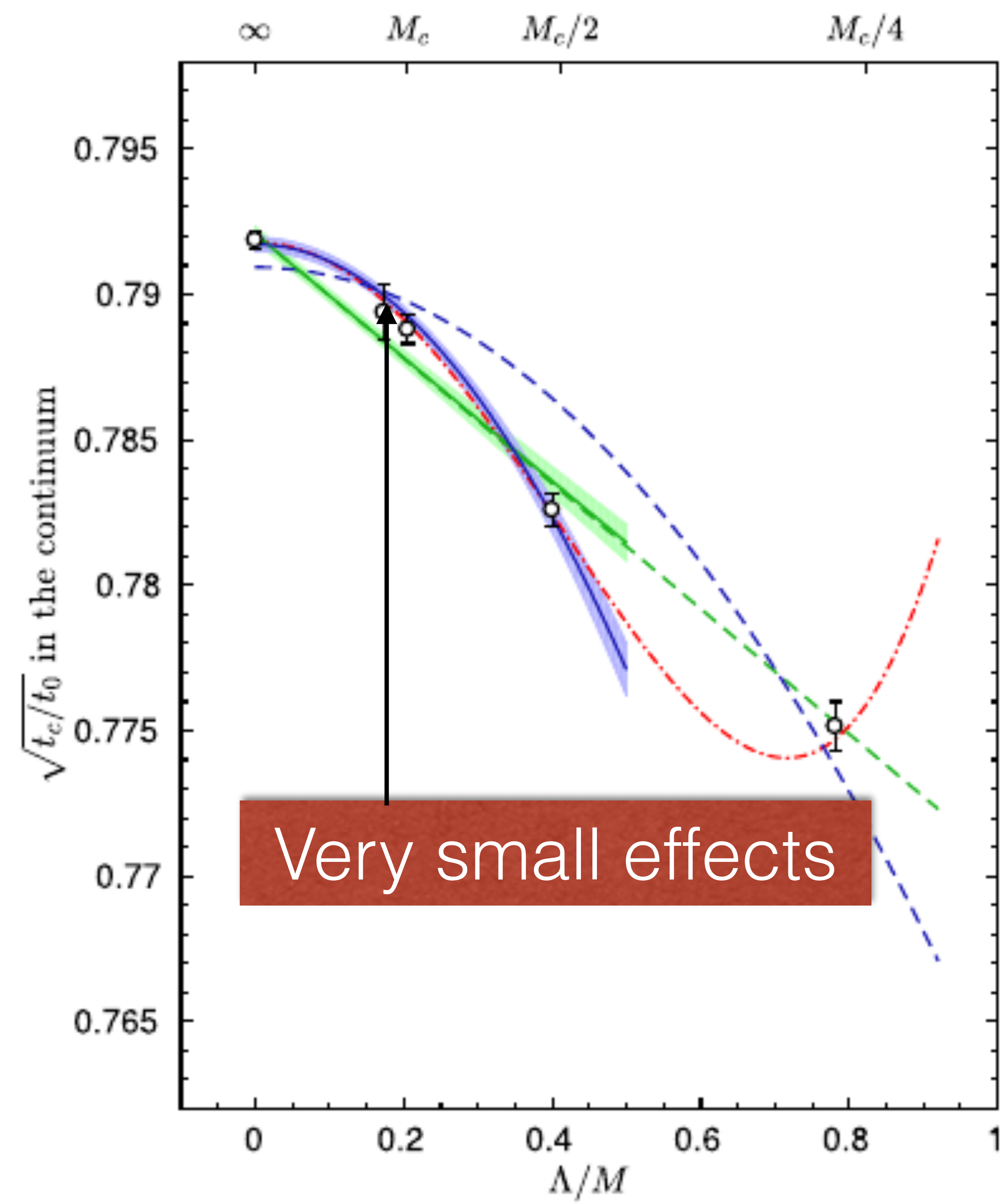


at charm: ~0.2 % effects



EFT prediction ~$1/M^2$ favored Knechtli, Leder, Korzec, Moir, 2017

mass dependence in continuum **at charm: ~0.2 % effects**

One can make use of this for obtaining higher precision in renormalization problems:

Decoupling as a tool. Presented in Wuhan:

Non-perturbative renormalization by decoupling

Alberto Ramos <alberto.ramos@maths.tcd.ie>

Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

ALPHA Collaboration

In collaboration with: M. Dalla Brida, T. Korzec, F. Knechtli, R. Höllwieser, S. Sint, R. Sommer

Decoupling as a tool

$$P_{\ell,\mathrm{f}}(M/\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}})\,\frac{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{\mathrm{f}}(M)} = \frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{\ell}}\,,$$

where $\mathcal{S}^{\ell} = \mathcal{S}^{\mathrm{f}}(M) + \mathrm{O}(1/M^2)$ is a mass-scale (e.g. $1/\sqrt{t_0}$))

It is very practical to define the scale by

$$\mathcal{S} = \mu_{\mathrm{dec}}\,,\ \text{with } [\bar{g}^{\mathrm{f}}_{\mathrm{GF}}(\mu_{\mathrm{dec}}, M/\mu_{\mathrm{dec}})]^2 = u_{\mathrm{M}}\,.$$

decoupling:

$$\bar{g}^{\ell}_{\mathrm{GF}}(\mu_{\mathrm{dec}})^2 = u_{\mathrm{M}}\,.$$

and rewrite ($\Lambda = \mu\varphi_g$):

$$\frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\mu_{\mathrm{dec}}} = \frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\Lambda^{\ell}_{\mathrm{GF}}}\,\varphi^{\ell}_{\mathrm{g,GF}}(\sqrt{u_{\mathrm{M}}})\,.$$

function which relates the coupling in the full theory with the massive quarks and the one with all massless ones,

$$u_{\mathrm{M}} = \Psi_{\mathrm{M}}(u_0, z)\,,\ \text{with } u_0 = [\bar{g}^{\mathrm{f}}_{\mathrm{GF}}(\mu, 0)]^2\,,\quad z = M/\mu\,,$$

$$P_{\ell,\mathrm{f}}(M/\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}})\,\frac{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{\mathrm{f}}(M)} = \frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\mathcal{S}^{\ell}}\,,$$

becomes

$$\underbrace{\rho P_{\ell,\mathrm{f}}(z/\rho)}_{\text{High order PT}} = \underbrace{\frac{\Lambda^{\ell}_{\overline{\mathrm{MS}}}}{\Lambda^{\ell}_{\mathrm{GF}}}}_{\text{1-lp exact}}\;\underbrace{\varphi^{\ell}_{\mathrm{GF}}}_{\text{YM}}(\underbrace{\sqrt{\Psi_{\mathrm{M}}(u_0,z)}}_{\text{full}}) \tag{1}$$

in terms of the dimensionless

$$\rho = \frac{\Lambda^{\mathrm{f}}_{\overline{\mathrm{MS}}}}{\mu_{\mathrm{dec}}}\,.$$

needed

- $N_\mathrm{f} = 3$: fix coupling at $M = 0$, determine coupling for $M \gg \mu_{\mathrm{dec}}$

$$u_\mathrm{M} = \Psi_\mathrm{M}(u_0, z)\,, \text{ with } u_0 = [\bar{g}^{\mathrm{f}}_{\mathrm{GF}}(\mu,0)]^2\,, \quad z = M/\mu\,,$$

- $N_\mathrm{f} = 0$: very precise running of couplings to very lage μ
  step scaling functions
  $\rightarrow \varphi^{\ell}_{\mathrm{GF}}$
  done by M.Dalla Brida and A. Ramos. do not discuss further

Decoupling as a tool for renormalization

- Choose μ_{dec} relatively low. Here Schroedinger Functional, $\mu_{\text{dec}} = 1/L = 0.8$GeV. Fixed by coupling in GF scheme, massless.

L/a	β	$\bar{g}^2(\mu_{\text{dec}}(M))\big\vert_{N_f=3,M=0,T=L}$	$\mu_{\text{dec}}(M)$ [GeV]
12	4.3020	3.9533(59)	0.789(15)
16	4.4662	3.9496(77)	0.789(15)
20	4.5997	3.9648(97)	0.789(15)
24	4.7141	3.959(50)	0.789(15)
32	4.90	3.949(11)	0.789(15)

Decoupling as a tool for renormalization

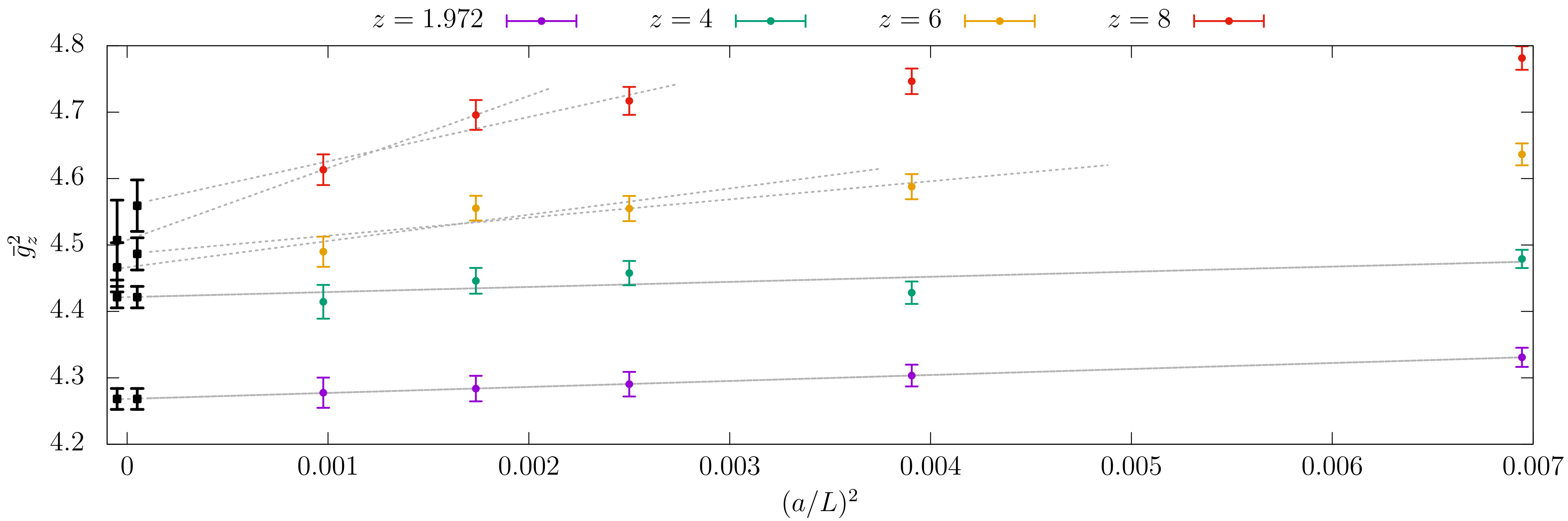
- Choose $\mu_{\rm dec}$ relatively low. Here Schroedinger Functional, $\mu_{\rm dec}$ =1/L = 0.8GeV. Fixed by coupling in GF scheme, massless.
- Turn on heavy masses, 1.6 GeV … 6.4 GeV (3 heavy degenerate quarks)
- Compute coupling with massive quarks

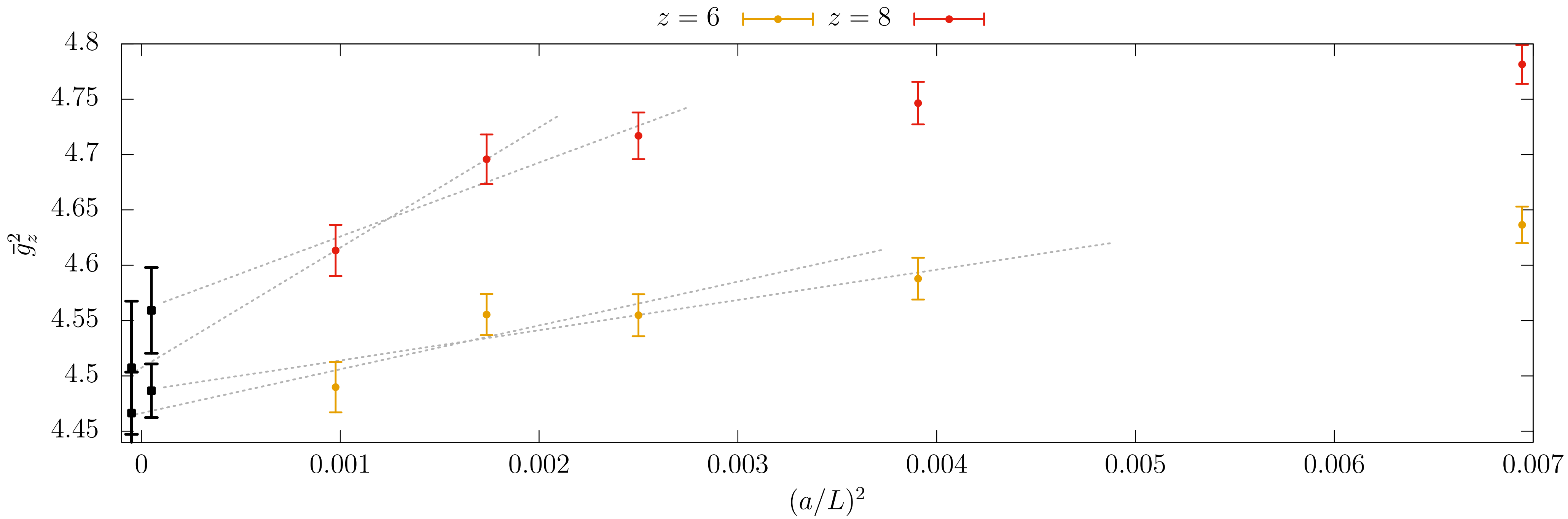
Example: $L/a = 20$

β	κ	$z = M/\mu_{\rm dec}(M)$	M [GeV]	$\bar{g}^2(\mu_{\rm low}(M))\big\vert_{N_{\rm f}=3,M,T=2L}$
4.5997	0.1352889	0	0	3.9648(97)
4.6083	0.133831710060	1.972(18)	1.6	4.290(15)
4.6172	0.132345249425	4.000(37)	3.2	4.458(14)
4.6266	0.130827894135	6.000(58)	4.7	4.555(14)
4.6364	0.129273827559	8.000(85)	6.3	4.717(14)

Continuum extrapolation

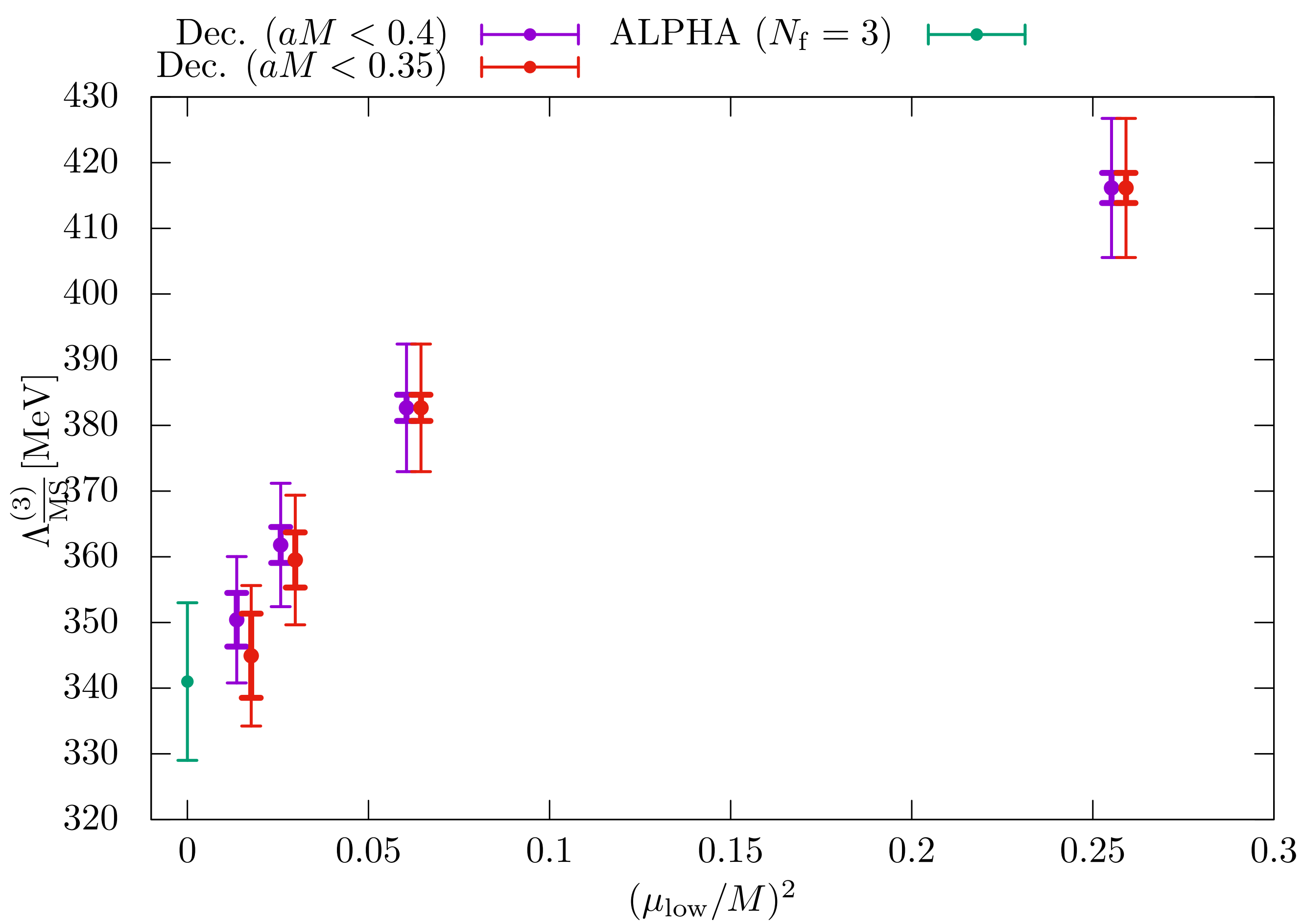
CONTINUUM EXTRAPOLATIONS WITH TWO CUTS: $aM < 0.40, 0.35$





Preliminary result

M [GeV]	$\mu_{\rm dec}(M)$ [GeV]	$\bar{g}^2(\mu_{\rm low}(M))\big\vert_{N_{\rm f}=3,M,T=2L}$	$\Lambda^{(0)}/\mu_{\rm low}$	$\frac{1}{P(\Lambda/M)}$	$\Lambda^{(3)}$ [MeV]
1.6	0.789(15)	-	0.689(11)	0.7662(44)	416(11)
3.2	0.789(15)	-	0.725(11)	0.6693(37)	382.7(96)
4.7	0.789(15)	-	0.741(12)	0.6198(34)	362.0(92)
6.3	0.789(15)	-	0.757(13)	0.5871(32)	350.3(92)



- nice and precise without that much effort —> improve further

Summary

- The Standard Model is (many feel: too) alive
- We need to push it to its limits in energy **and precision**
- Somewhat provocative but true: If we want a non-perturbative result, we need it renormalized non-perturbatively.
- The perturbative series is divergent, asymptotic (well understood! I recommend 't Hooft Erice lectures).
  When one uses it $\alpha(\mu)$ better is small.
- For scale dependent renormalizations, $\alpha(\mu)\; m_\mathrm{R}(\mu)$, $Z_\mathrm{LL}(\mu)$

  step scaling with finite volume schemes

  can be used to go to very large μ and connect to

  Renormalization Group Invariants
- On the other hand, RI-sMOM is more genaral (automatic) is mostly used and dominant discretization errors can be removed perturbatively

  Can the question of NP gauge fixing be better understood?

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- **There is a new trick: renormalization by decoupling**
- **There is also the Gradient flow –> Hiroshi Suzuki**

Finally

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chirally rotated Schroedinger Functional has been used to obtain very precise renormalization factors.
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- THANK YOU!