Rare b decays and lattice QCD

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1. The $b \to s\ell^+\ell^-$ "anomalies"
2. $\Lambda_b \to \Lambda\ell^+\ell^-$
3. Toward $B \to K\pi\ell^+\ell^-$

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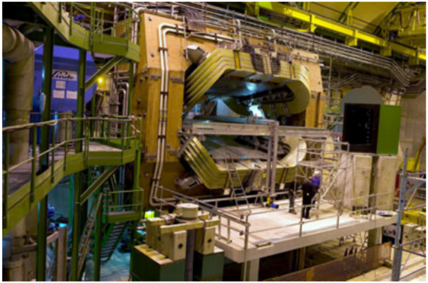
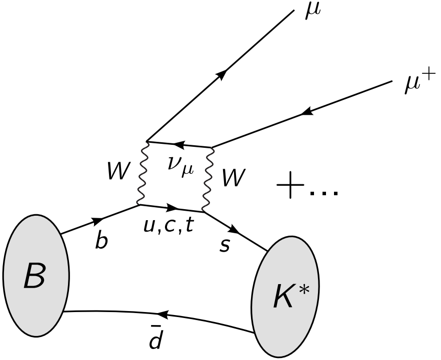
Has LHCb spotted physics beyond the Standard Model?

Aug 2, 2013 12 comments



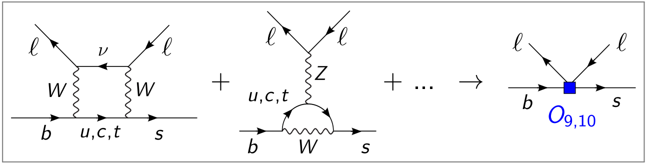
Is physics beyond the Standard Model lurking in LHCb?

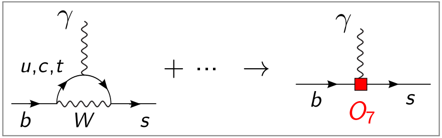
An analysis of data from the LHCb experiment at the CERN particle-physics lab suggests that the B-meson could decay in a way not predicted by the Standard Model of particle physics, according to theoretical physicists in Spain and France. The researchers believe that the deviation from the Standard Model has been measured with a confidence of 4.5σ – which is approaching the gold standard of 5σ required for a discovery in particle physics.

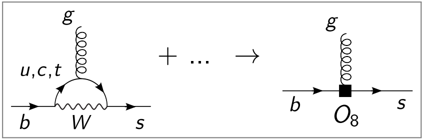


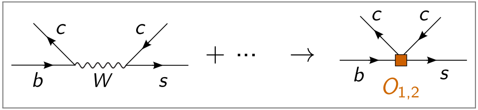
μ^-
μ^+
W
ν_μ
W
$+\cdots$
b
u,c,t
s
B
K^*
$\bar{d}$

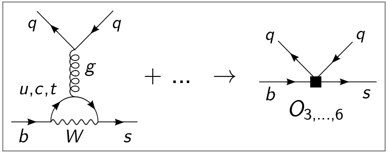
Effective weak Hamiltonian for $b \to s\ell^+\ell^-$ decays











[B. Grinstein, M. J. Savage, M. B. Wise, Nucl. Phys. B 319, 271 (1989)]

Effective weak Hamiltonian for $b \to s\ell^+\ell^-$ decays

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i$$

with

$$\begin{aligned}
O_1 &= \bar{c}^b \gamma^\mu b_L^a \;\; \bar{s}^a \gamma_\mu c_L^b, \\
O_2 &= \bar{c}^a \gamma^\mu b_L^a \;\; \bar{s}^b \gamma_\mu c_L^b, \\
O_7 &= \frac{e\, m_b}{16\pi^2} \; \bar{s}\sigma^{\mu\nu} b_R \;\; F_{\mu\nu}^{(\text{e.m.})}, \\
O_9 &= \frac{e^2}{16\pi^2} \; \bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \ell, \\
O_{10} &= \frac{e^2}{16\pi^2} \; \bar{s}\gamma^\mu b_L \;\; \bar{\ell}\gamma_\mu \gamma_5 \ell, \\
&\dots
\end{aligned}$$

In the Standard Model, $\overline{\text{MS}}$ scheme, at $\mu = 4.2$ GeV,

C_1	C_2	C_7	C_9	C_{10}	...
−0.288	1.010	−0.336	4.275	−4.160	...

[Computed using EOS, `https://eos.github.io/`]

Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays

For a generic decay $H_b \to H_s\ell^+\ell^-$:

Contributions from O_7, O_9, O_{10}:

$$\langle H_s(p')|\, \bar{s}\Gamma b \,|H_b(p)\rangle$$

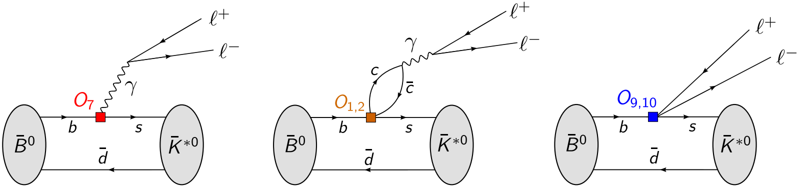
These local matrix elements can be calculated in lattice QCD.

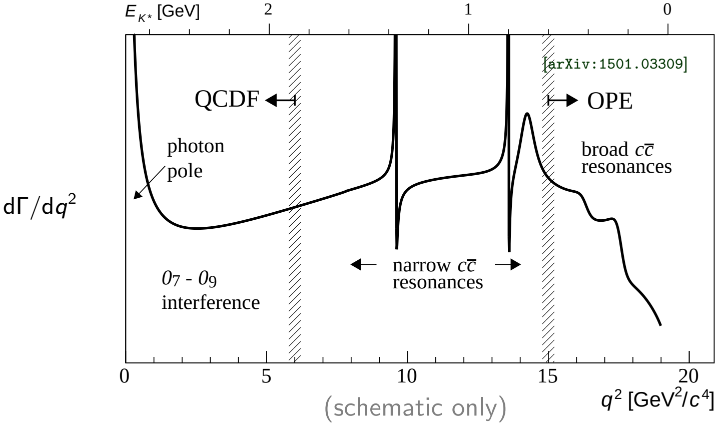
Contributions from $O_{1,\ldots,6}$, O_8:

$$\int \mathrm{d}^4x \;\, e^{iq\cdot x} \, \langle H_s(p')|\, \mathrm{T}\; O_i(0)\; J^{\mu}_{\mathrm{e.m.}}(x) \,|H_b(p)\rangle \qquad\qquad (q = p - p')$$

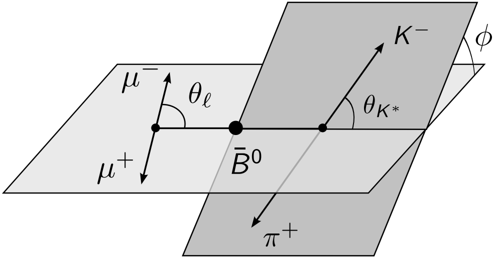
Calculating these nonlocal matrix elements in lattice QCD is extremely difficult because of the use of imaginary time. In this case there are many lighter intermediate states that would come with growing exponentials (see the lectures by Chris Sachrajda).

Hadronic matrix elements for exclusive $b \to s\ell^+\ell^-$ decays





$\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\mu^+\mu^-$ angular distribution



(the different angles are defined in different reference frames)

$\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\mu^+\mu^-$ angular distribution

In the narrow-width approximation,

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K^*}\, d\phi} = \frac{9}{32\pi}\Big[I_1^s \sin^2\theta_{K^*} + I_1^c \cos^2\theta_{K^*}$$

$$+ (I_2^s \sin^2\theta_{K^*} + I_2^c \cos^2\theta_{K^*}) \cos 2\theta_\ell$$

$$+ I_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi$$

$$+ I_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi$$

$$+ I_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi$$

$$+ (I_6^s \sin^2\theta_{K^*} + I_6^c \cos^2\theta_{K^*}) \cos\theta_\ell$$

$$+ I_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi$$

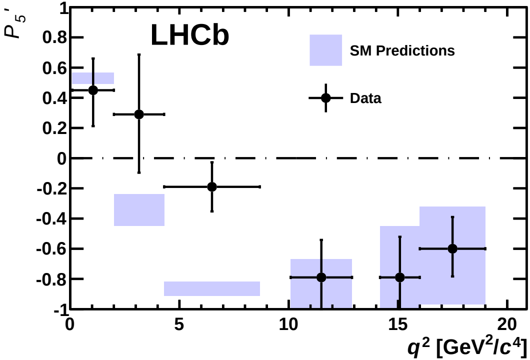
$$+ I_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi$$

$$+ I_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi \Big],$$

where the functions $I_i^{(a)}$ depend only on q^2 [F. Krüger *et al.*, arXiv:hep-ph/9907386/PRD 2000]

2013: The "P'_5 anomaly" in $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\mu^+\mu^-$

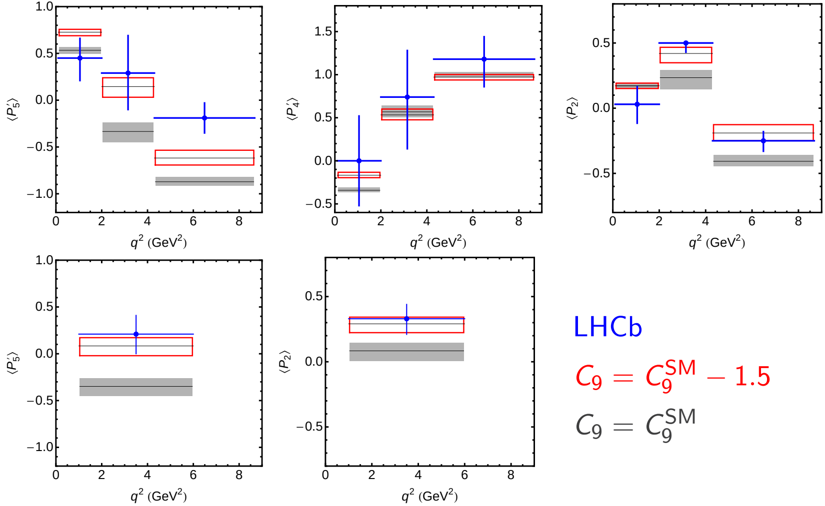
$$P'_5 = \frac{I_5}{2\sqrt{-I_2^s\, I_2^c}}$$



[LHCb Collaboration, arXiv:1308.1707/PRL 2013]

- SM predictions: [S. Descotes-Genon, T. Hurth, J. Matias, J. Virto, arXiv:1303.5794]
- Hadronic matrix elements calculated using QCD factorization and light-cone sum-rules (good at low q^2, i.e., high K^* momentum)

2013: A negative shift in the Wilson coefficient C_9?

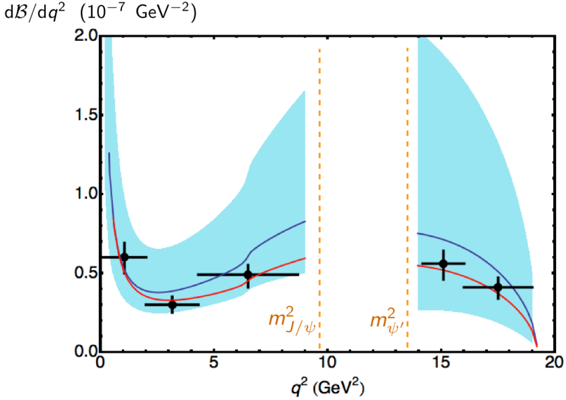


[S. Descotes-Genon, J. Matias, J. Virto, arXiv:1307.5683/PRD 2013]

(NB: normalization of observables defined differently)

2013: $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\mu^+\mu^-$ differential branching fraction calculated using light-cone sum rules

$$\frac{d\mathcal{B}}{dq^2} = \tau_B \frac{d\Gamma}{dq^2}$$



$C_9 = C_9^{SM}$

$C_9 = C_9^{SM} - 1.5$

Form factor uncertainty from light cone sum rules

[S. Descotes-Genon, J. Matias, J. Virto, arXiv:1311.3876]

$B \to K^*$ form factors

For now, let's treat the K^* as if it were a stable particle.
This is not correct; more on this later.

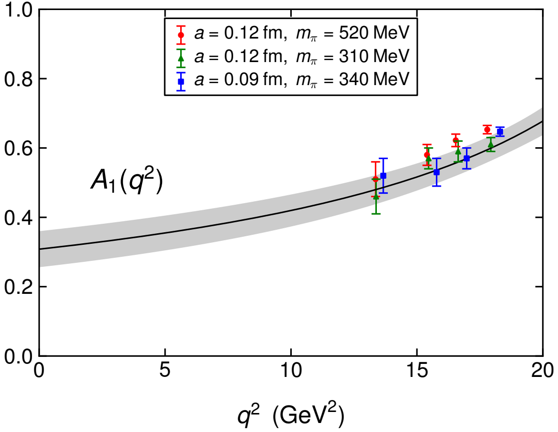
$$
\begin{aligned}
\langle K^*(p',\varepsilon)|\bar{s}\gamma^\mu b|B(p)\rangle &= \frac{2i\,V(q^2)}{M_B+M_{K^*}}\,\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu\,p'_\rho p_\sigma, \\
\langle K^*(p',\varepsilon)|\bar{s}\gamma^\mu\gamma_5 b|B(p)\rangle &= 2M_{K^*}\,A_0(q^2)\,\frac{\varepsilon^*\cdot q}{q^2}\,q^\mu \\
&\quad +(M_B+M_{K^*})\,A_1(q^2)\left[\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}\,q^\mu\right] \\
&\quad -A_2(q^2)\,\frac{\varepsilon^*\cdot q}{M_B+M_{K^*}}\left[p^\mu+p'^\mu-\frac{M_B^2-M_{K^*}^2}{q^2}\,q^\mu\right], \\
q^\nu\langle K^*(p',\varepsilon)|\bar{s}\sigma_{\mu\nu}b|B(p)\rangle &= 4\,T_1(q^2)\epsilon_{\mu\rho\kappa\sigma}\varepsilon^{*\rho}p^\kappa p'^\sigma, \\
q^\nu\langle K^*(p',\varepsilon)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle &= -2i\,T_2(q^2)\left[\varepsilon^*_\mu(M_B^2-M_{K^*}^2)-(\varepsilon^*\cdot q)(p+p')_\mu\right] \\
&\quad -2i\,T_3(q^2)(\varepsilon^*\cdot q)\left[q_\mu-\frac{q^2}{M_B^2-M_{K^*}^2}(p+p')_\mu\right].
\end{aligned}
$$

2013: $B \to K^*$ form factors from lattice QCD

First unquenched lattice calculation (using NRQCD b quarks and staggered light quarks)

[R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, arXiv:1310.3722/PRD 2014]

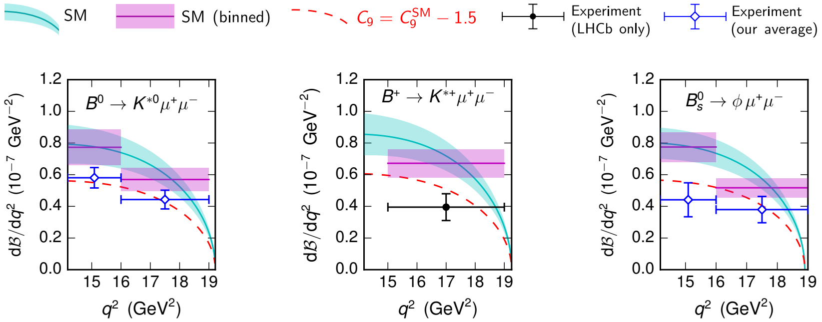
Example:



2013: $B \to K^*\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ differential BF's calculated using lattice QCD

Contributions from $O_{1\ldots6;8}$ treated with OPE ($\sim$ 10% effect) – see extra slide

[B. Grinstein and D. Pirjol, arXiv:hep-ph/0404250/PRD 2004]



[R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, arXiv:1310.3887/PRL 2014]

2014: Ratio of $B \to K\mu^+\mu^-$ and $B \to Ke^+e^-$ branching fractions

$$R_K \equiv \frac{\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} \frac{\mathrm{d}\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathrm{d}q^2}\mathrm{d}q^2}{\int_{1\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} \frac{\mathrm{d}\mathcal{B}(B^+ \to K^+e^+e^-)}{\mathrm{d}q^2}\mathrm{d}q^2},$$

Standard Model:

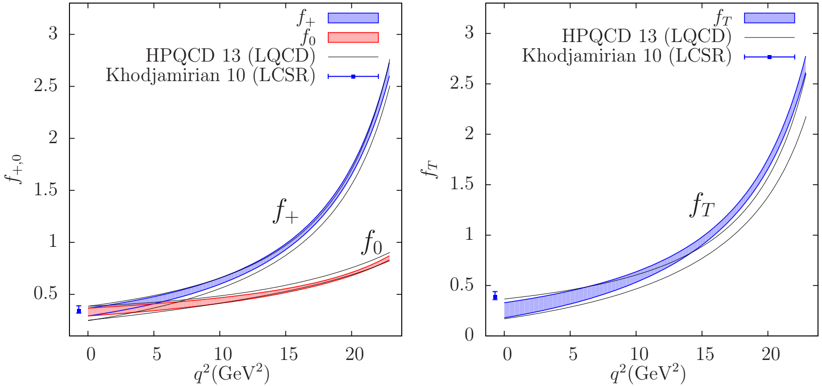
$$R_K = 1 + \mathcal{O}(10^{-3})$$

2014 LHCb measurement [arXiv:1406.6482/PRL 2014]:

$$R_K = 0.745^{+0.090}_{-0.074}(\mathrm{stat}) \pm 0.036(\mathrm{syst})$$

Note: even R_K and related ratios become dependent on hadronic form factors in the presence of new physics that breaks lepton flavor university!

2013, 2015: $B \to K$ form factors from lattice QCD

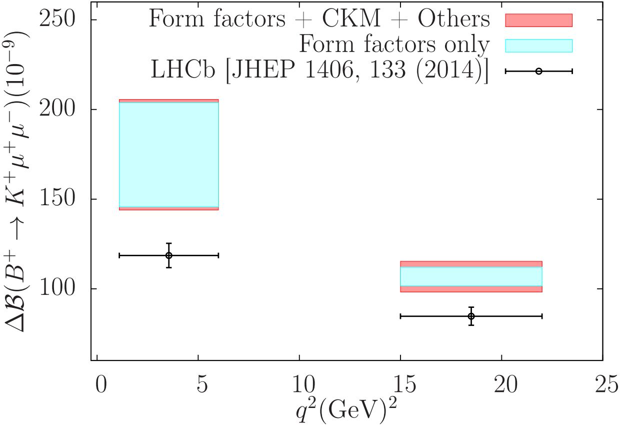


[J. Bailey *et al.* (Fermilab Lattice and MILC Collaborations), arXiv:1509.06235/PRD 2016]

[C. Bouchard, G. P. Lepage, C. Monahan, H. Na, J. Shigemitsu (HPQCD Collaboration), arXiv:1306.2384/PRD 2013]

2015: $B \to K\mu^+\mu^-$ differential BF calculated using lattice QCD

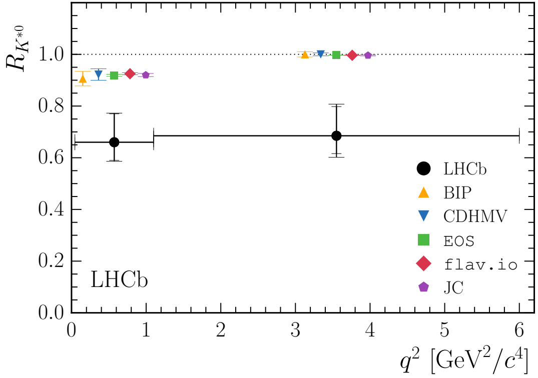
Contributions from $O_{1\ldots6;8}$ treated with OPE and QCD factorization



[D. Du *et al.* (Fermilab Lattice and MILC Collaborations), arXiv:1510.02349/PRD 2016]

2017: ratios of $B \to K^* \mu^+\mu^-$ and $B \to K^* e^+e^-$ BF's

$$R_{K^*} \equiv \frac{\int \frac{d\mathcal{B}(B^0 \to K^{*0}\mu^+\mu^-)}{dq^2} dq^2}{\int \frac{d\mathcal{B}(B^0 \to K^{*0}e^+e^-)}{dq^2} dq^2},$$



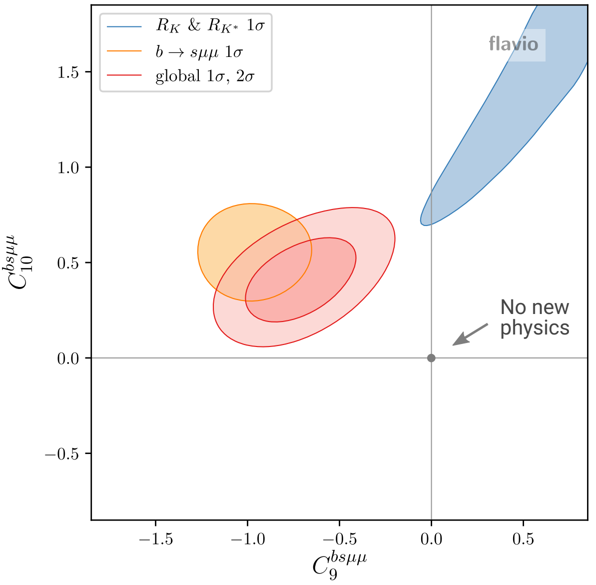
[LHCb Collaboration, arXiv:1705.05802/JHEP 2017]

2019: latest fits of NP contributions to Wilson coefficients

Coeff.	best fit	1σ	2σ	pull
$C_9^{bs\mu\mu}$	-0.95	$[-1.10, -0.79]$	$[-1.26, -0.63]$	5.8σ
$C_9'^{bs\mu\mu}$	$+0.09$	$[-0.07, +0.24]$	$[-0.23, +0.39]$	0.5σ
$C_{10}^{bs\mu\mu}$	$+0.73$	$[+0.59, +0.87]$	$[+0.46, +1.01]$	5.6σ
$C_{10}'^{bs\mu\mu}$	-0.19	$[-0.30, -0.07]$	$[-0.41, +0.04]$	1.6σ
$C_9^{bs\mu\mu} = C_{10}^{bs\mu\mu}$	$+0.20$	$[+0.05, +0.35]$	$[-0.09, +0.51]$	1.4σ
$C_9^{bs\mu\mu} = -C_{10}^{bs\mu\mu}$	-0.53	$[-0.62, -0.45]$	$[-0.70, -0.36]$	6.5σ

[J. Aebischer, W. Altmannshofer, D. Guadagnoli, M. Reboud, P. Stangl, D. Straub, `arXiv:1903.10434`]

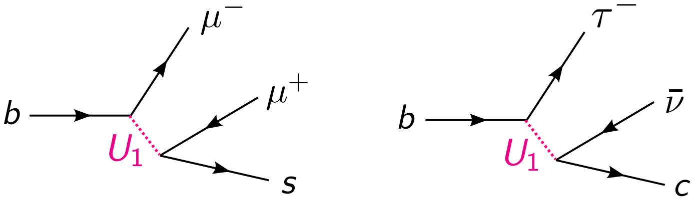
2019: latest fits of NP contributions to Wilson coefficients



[J. Aebischer, W. Altmannshofer, D. Guadagnoli, M. Reboud, P. Stangl, D. Straub, arXiv:1903.10434]

Possible combined explanation of the $b \to s\ell^+\ell^-$ and $b \to c\tau^-\bar{\nu}$ [1] anomalies

Vector leptoquark "U_1": $SU(3)$ triplet, $SU(2)$ singlet, $U(1)$ hypercharge $2/3$



[See, for example, D. Buttazzo, A. Greljo, G. Isidori, D. Marzocca, arXiv:1706.07808/JHEP 2017]

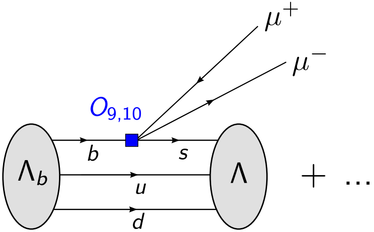
[1] R_D, R_{D^*}, $R_{J/\psi}$ – see the lectures by Chris Sachrajda and Weongjong Lee

1 The $b \to s\ell^+\ell^-$ "anomalies"

2 $\Lambda_b \to \Lambda\ell^+\ell^-$

3 Toward $B \to K\pi\ell^+\ell^-$

$\Lambda_b \to \Lambda \ell^+ \ell^-$

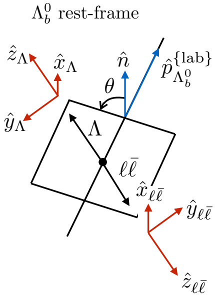


- Like the K^*, the Λ has nonzero spin (which is useful to probe all O_i in $\mathcal{H}_{\text{eff}}$).
- Like the K, the Λ is stable under the strong interactions (which simplifies the lattice calculation). The Λ is observed through its weak decay $\Lambda \to p^+ \pi^-$.
- Unlike the B, the initial Λ_b also has nonzero spin; if polarized, this gives access to even more observables.

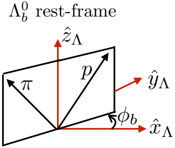
$\Lambda_b \to \Lambda(\to p^+\pi^-)\ell^+\ell^-$ decay angles

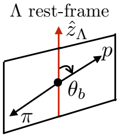
$\hat{n} = \hat{p}_{\Lambda_b} \times \hat{p}_{\text{beam}}$

$$\hat{z}_\Lambda = \hat{p}_\Lambda^{\{\Lambda_b^0\}}$$
$$\hat{y}_\Lambda = \hat{n} \times \hat{p}_\Lambda^{\Lambda_b^0}$$

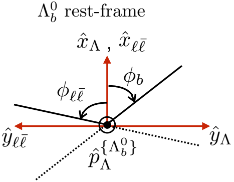


$$\hat{z}_{\ell\bar{\ell}} = \hat{p}_{\ell\bar{\ell}}^{\{\Lambda_b^0\}}$$
$$\hat{y}_{\ell\bar{\ell}} = \hat{n} \times \hat{p}_{\ell\bar{\ell}}^{\{\Lambda_b^0\}}$$





$$\hat{z}_\Lambda^{\{\Lambda\}} = -\hat{p}_{\ell\bar{\ell}}^{\{\Lambda\}}$$



[T. Blake, M. Kreps, arXiv:1710.00746/JHEP 2017]

$\Lambda_b \to \Lambda(\to p^+\pi^-)\ell^+\ell^-$ angular distribution

$$
\begin{aligned}
\frac{d^6\Gamma}{dq^2\, d\vec{\Omega}} = \frac{3}{32\pi^2}\Big(& \Big(K_1 \sin^2\theta_l + K_2 \cos^2\theta_l + K_3 \cos\theta_l\Big) + \\
& \Big(K_4 \sin^2\theta_l + K_5 \cos^2\theta_l + K_6 \cos\theta_l\Big) \cos\theta_b + \\
& (K_7 \sin\theta_l \cos\theta_l + K_8 \sin\theta_l) \sin\theta_b \cos(\phi_b + \phi_l) + \\
& (K_9 \sin\theta_l \cos\theta_l + K_{10} \sin\theta_l) \sin\theta_b \sin(\phi_b + \phi_l) + \\
& \Big(K_{11} \sin^2\theta_l + K_{12} \cos^2\theta_l + K_{13} \cos\theta_l\Big) \cos\theta + \\
& \Big(K_{14} \sin^2\theta_l + K_{15} \cos^2\theta_l + K_{16} \cos\theta_l\Big) \cos\theta_b \cos\theta + \\
& (K_{17} \sin\theta_l \cos\theta_l + K_{18} \sin\theta_l) \sin\theta_b \cos(\phi_b + \phi_l) \cos\theta + \\
& (K_{19} \sin\theta_l \cos\theta_l + K_{20} \sin\theta_l) \sin\theta_b \sin(\phi_b + \phi_l) \cos\theta + \\
& (K_{21} \cos\theta_l \sin\theta_l + K_{22} \sin\theta_l) \sin\phi_l \sin\theta + \\
& (K_{23} \cos\theta_l \sin\theta_l + K_{24} \sin\theta_l) \cos\phi_l \sin\theta + \\
& (K_{25} \cos\theta_l \sin\theta_l + K_{26} \sin\theta_l) \sin\phi_l \cos\theta_b \sin\theta + \\
& (K_{27} \cos\theta_l \sin\theta_l + K_{28} \sin\theta_l) \cos\phi_l \cos\theta_b \sin\theta + \\
& \Big(K_{29} \cos^2\theta_l + K_{30} \sin^2\theta_l\Big) \sin\theta_b \sin\phi_b \sin\theta + \\
& \Big(K_{31} \cos^2\theta_l + K_{32} \sin^2\theta_l\Big) \sin\theta_b \cos\phi_b \sin\theta + \\
& \Big(K_{33} \sin^2\theta_l\Big) \sin\theta_b \cos(2\phi_l + \phi_b) \sin\theta + \\
& \Big(K_{34} \sin^2\theta_l\Big) \sin\theta_b \sin(2\phi_l + \phi_b) \sin\theta \Big)
\end{aligned}
$$

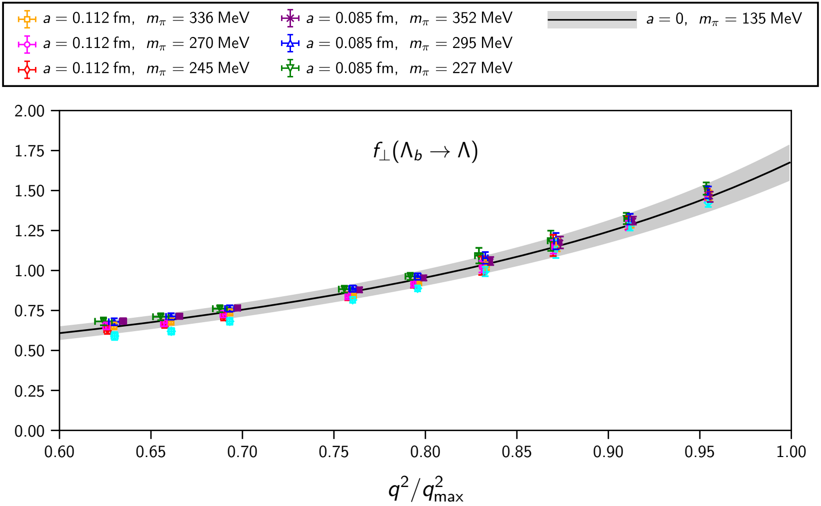
[T. Blake, M. Kreps, arXiv:1710.00746/JHEP 2017]

$\Lambda_b \to \Lambda$ form factors

$$\begin{aligned}
\langle \Lambda | \bar{s}\gamma^{\mu} b | \Lambda_b \rangle &= \bar{u}_{\Lambda} \Big[f_0(q^2)\,(m_{\Lambda_b} - m_{\Lambda})\frac{q^{\mu}}{q^2} \\
&\quad + f_{+}(q^2)\frac{m_{\Lambda_b} + m_{\Lambda}}{s_+}\left(p^{\mu} + p'^{\mu} - (m_{\Lambda_b}^2 - m_{\Lambda}^2)\frac{q^{\mu}}{q^2}\right) \\
&\quad + f_{\perp}(q^2)\left(\gamma^{\mu} - \frac{2m_{\Lambda}}{s_+}p^{\mu} - \frac{2m_{\Lambda_b}}{s_+}p'^{\mu}\right)\Big] u_{\Lambda_b}, \\
\langle \Lambda | \bar{s}\gamma^{\mu}\gamma_5 b | \Lambda_b \rangle &= -\bar{u}_{\Lambda}\,\gamma_5 \Big[g_0(q^2)\,(m_{\Lambda_b} + m_{\Lambda})\frac{q^{\mu}}{q^2} \\
&\quad + g_{+}(q^2)\frac{m_{\Lambda_b} - m_{\Lambda}}{s_-}\left(p^{\mu} + p'^{\mu} - (m_{\Lambda_b}^2 - m_{\Lambda}^2)\frac{q^{\mu}}{q^2}\right) \\
&\quad + g_{\perp}(q^2)\left(\gamma^{\mu} + \frac{2m_{\Lambda}}{s_-}p^{\mu} - \frac{2m_{\Lambda_b}}{s_-}p'^{\mu}\right)\Big] u_{\Lambda_b}, \\
iq_{\nu}\,\langle \Lambda | \bar{s}\sigma^{\mu\nu} b | \Lambda_b \rangle &= -\bar{u}_{\Lambda} \Big[h_{+}(q^2)\frac{q^2}{s_+}\left(p^{\mu} + p'^{\mu} - (m_{\Lambda_b}^2 - m_{\Lambda}^2)\frac{q^{\mu}}{q^2}\right) \\
&\quad + h_{\perp}(q^2)\,(m_{\Lambda_b} + m_{\Lambda})\left(\gamma^{\mu} - \frac{2m_{\Lambda}}{s_+}\,p^{\mu} - \frac{2m_{\Lambda_b}}{s_+}\,p'^{\mu}\right)\Big] u_{\Lambda_b}, \\
iq_{\nu}\,\langle \Lambda | \bar{s}\sigma^{\mu\nu}\gamma_5 b | \Lambda_b \rangle &= -\bar{u}_{\Lambda}\,\gamma_5 \Big[\widetilde{h}_{+}(q^2)\,\frac{q^2}{s_-}\left(p^{\mu} + p'^{\mu} - (m_{\Lambda_b}^2 - m_{\Lambda}^2)\frac{q^{\mu}}{q^2}\right) \\
&\quad + \widetilde{h}_{\perp}(q^2)\,(m_{\Lambda_b} - m_{\Lambda})\left(\gamma^{\mu} + \frac{2m_{\Lambda}}{s_-}\,p^{\mu} - \frac{2m_{\Lambda_b}}{s_-}\,p'^{\mu}\right)\Big] u_{\Lambda_b},
\end{aligned}$$

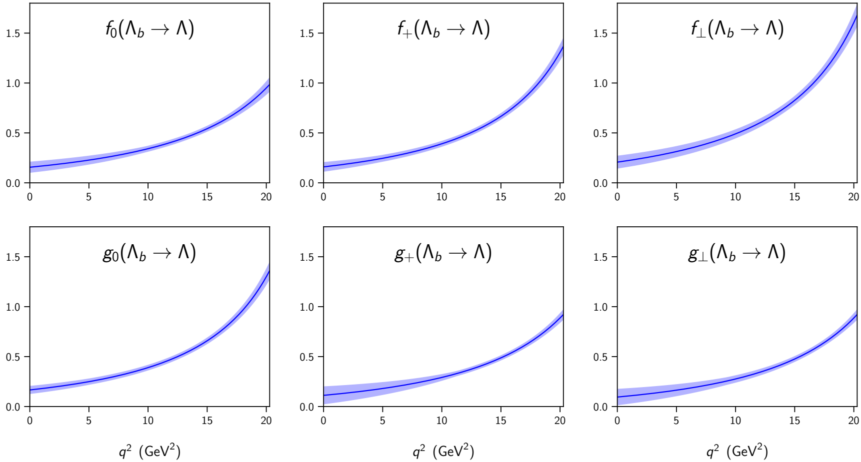
where $s_{\pm} = (m_{\Lambda_b} \pm m_{\Lambda})^2 - q^2$.

$\Lambda_b \to \Lambda$ form factors from lattice QCD



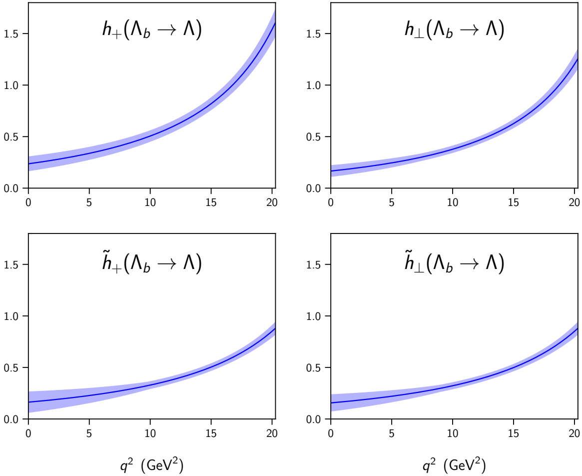
[S. Meinel, W. Detmold, arXiv:1602.01399/PRD 2016]

$\Lambda_b \to \Lambda$ form factors from lattice QCD



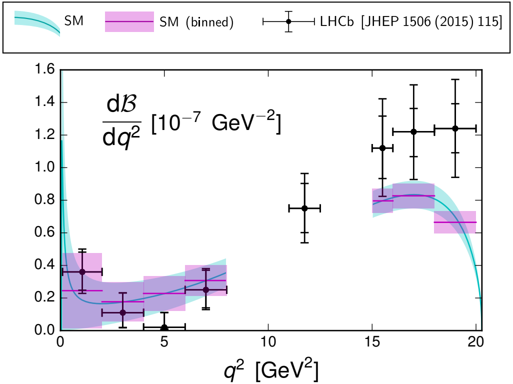
[S. Meinel, W. Detmold, arXiv:1602.01399/PRD 2016]

$\Lambda_b \to \Lambda$ form factors from lattice QCD



[S. Meinel, W. Detmold, arXiv:1602.01399/PRD 2016]

$\Lambda_b \to \Lambda\mu^+\mu^-$ differential branching fraction



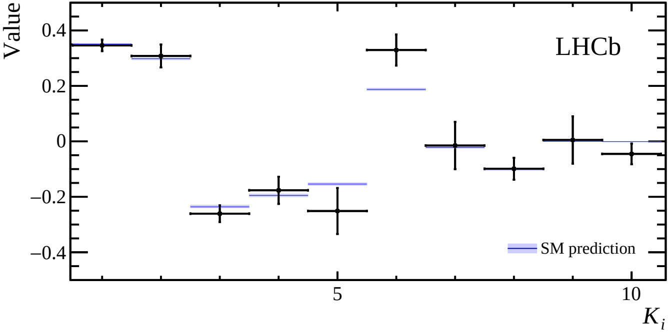
Experimental results: LHCb Collaboration [arXiv:1503.07138/JHEP 2015]

NB: possible systematic error in normalization; using f_{Λ_b} from Tevatron only (instead of Tevatron+LEP average) would give factor 2 lower BF

SM predictions: using lattice QCD [S. Meinel, W. Detmold, arXiv:1602.01399/PRD 2016]

$\Lambda_b \to \Lambda(\to p^+\pi^-)\mu^+\mu^-$ angular observables $K_{1\text{ - }10}$

In the bin $15 < q^2 < 20\ \mathrm{GeV}^2$



Note: $\frac{3}{2}K_3 = A_{\mathrm{FB}}^{\ell}$, $K_4 + \frac{1}{2}K_5 = A_{\mathrm{FB}}^{h}$, $\frac{3}{4}K_6 = A_{\mathrm{FB}}^{\ell h}$

Experimental results: LHCb Collaboration [arXiv:1808.00264/????]

SM predictions: using lattice QCD [S. Meinel, W. Detmold, arXiv:1602.01399/PRD 2016]

Fitting Wilson coefficients to $\Lambda_b \to \Lambda(\to p^+\pi^-)\mu^+\mu^-$ data

In 2016, we published fits of Wilson coefficients to $\Lambda_b \to \Lambda(\to p^+\pi^-)\mu^+\mu^-$ [S. Meinel and D. van Dyk, arXiv:1603.02974/PRD 2016].

However, it later turned out that the LHCb result for $A_{\rm FB}^{\ell}$ used in these fits was incorrect (they had accidentally measured the CP asymmetry in $A_{\rm FB}^{\ell}$).

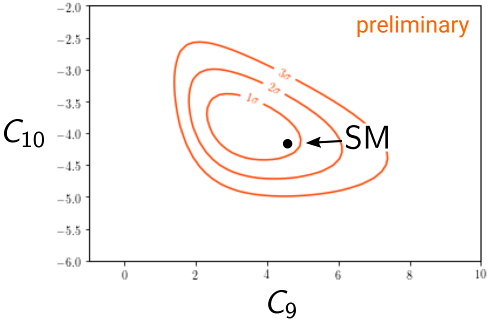
Also, BESIII published a new precise measurement of the Λ polarization parameter α_Λ that is $(17 \pm 3)\%$ higher than the previous world average [arXiv:1808.08917/Nature Phys. 2019]

Fitting Wilson coefficients to $\Lambda_b \to \Lambda(\to p^+\pi^-)\mu^+\mu^-$ data

A new fit of C_9 and C_{10} to

- The 2018 LHCb data for all 33 $\Lambda_b \to \Lambda(\to p^+\pi^-)\mu^+\mu^-$ angular observables
- The world average of $\mathcal{B}(B_s \to \mu^+\mu^-)$

gives the following [T. Blake and D. van Dyk, private communication]:

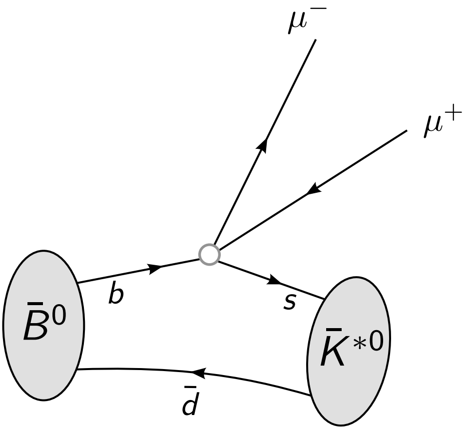


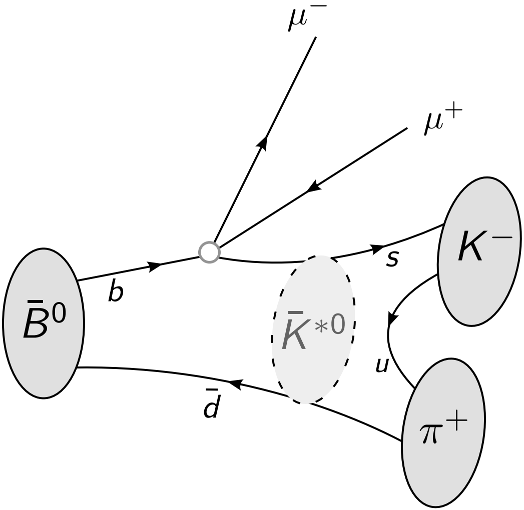
This is perfectly consistent with the “anomaly” seen in the other processes.

1. The $b \to s\ell^+\ell^-$ "anomalies"

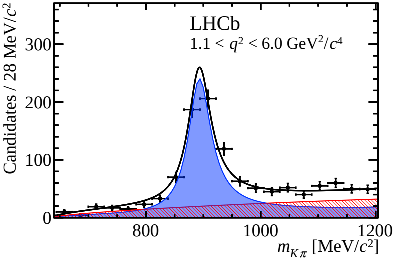
2. $\Lambda_b \to \Lambda\ell^+\ell^-$

3. Toward $B \to K\pi\ell^+\ell^-$


μ^-
μ^+
b
s
$\bar{B}^0$
$\bar{K}^{*0}$
$\bar{d}$



μ^-
μ^+
K^-
s
b
$\bar{B}^0$
$\bar{K}^{*0}$
u
$\bar{d}$
π^+



[LHCb Collaboration, arXiv:1606.04731/JHEP 2016]

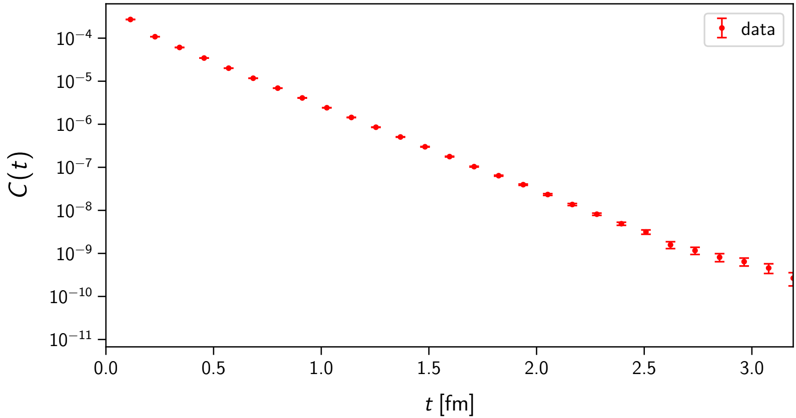
How NOT to treat the K^* on the lattice

We know the K^{*0} has flavor content $\bar{s}d$ and spin 1, so let's try

$$O_{K^*}(\mathbf{P}, t) = \sum_{\mathbf{x}} \bar{s}(\mathbf{x}, t)\gamma^j d(\mathbf{x}, t) e^{i\mathbf{P}\cdot\mathbf{x}}$$

How NOT to treat the K^* on the lattice

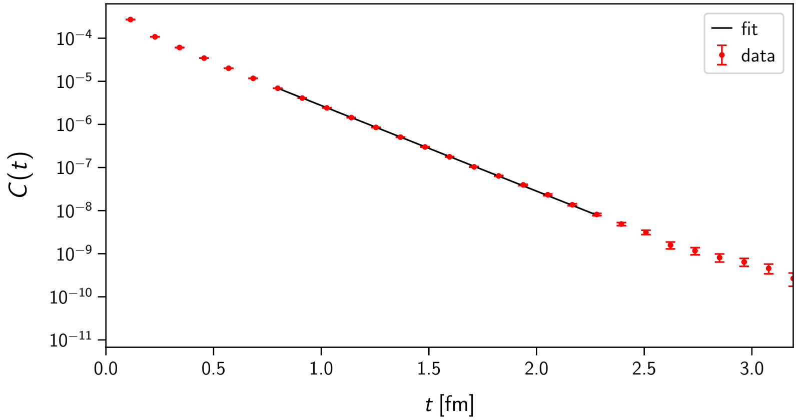
$$C(t) = \langle O_{K^*}(\mathbf{0}, t) O^\dagger_{K^*}(\mathbf{0}, 0) \rangle_L$$



$(5.5\ \text{fm})^3 \times (11\ \text{fm})$ lattice, physical m_π and m_K (RBC/UKQCD ensemble)

How NOT to treat the K^* on the lattice

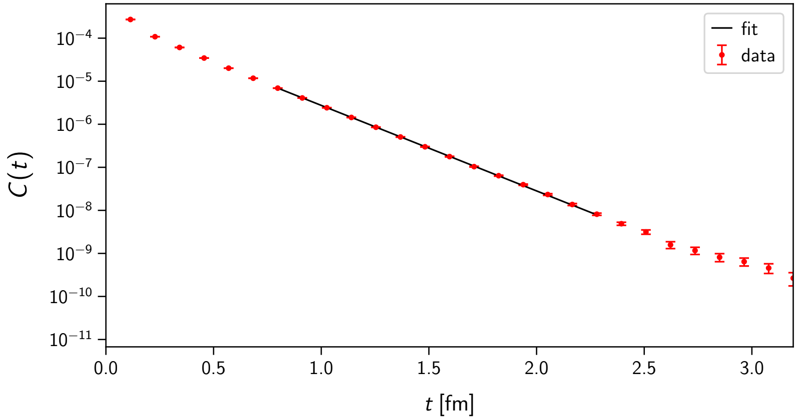
$$C(t) = \langle O_{K^*}(\mathbf{0}, t) O^\dagger_{K^*}(\mathbf{0}, 0) \rangle_L$$



Fit: $C(t) = A\, e^{-E\,t}$, χ^2/d.o.f. $= 0.99$, $E = 902 \pm 4$ MeV

How NOT to treat the K^* on the lattice

$$C(t) = \langle O_{K^*}(\mathbf{0}, t) O^\dagger_{K^*}(\mathbf{0}, 0) \rangle_L$$

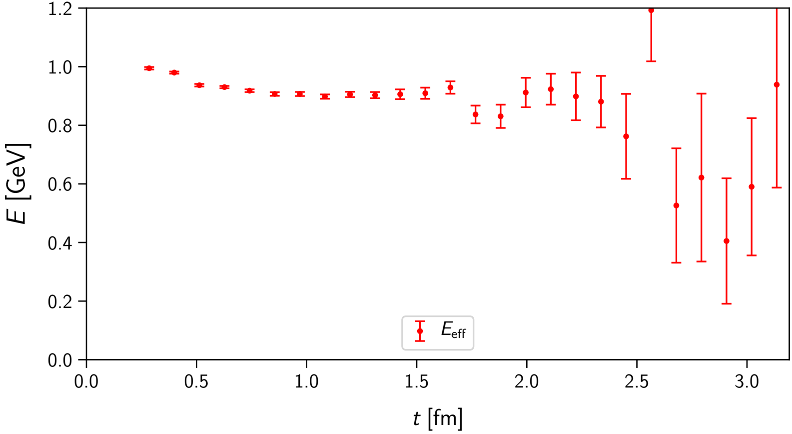


Fit: $C(t) = A\, e^{-E\, t}$, $\chi^2/\text{d.o.f.} = 0.99$, $E = 902 \pm 4$ MeV

(Particle Data Group: $m_{K^{*0}} = 895.6 \pm 0.2$ MeV)

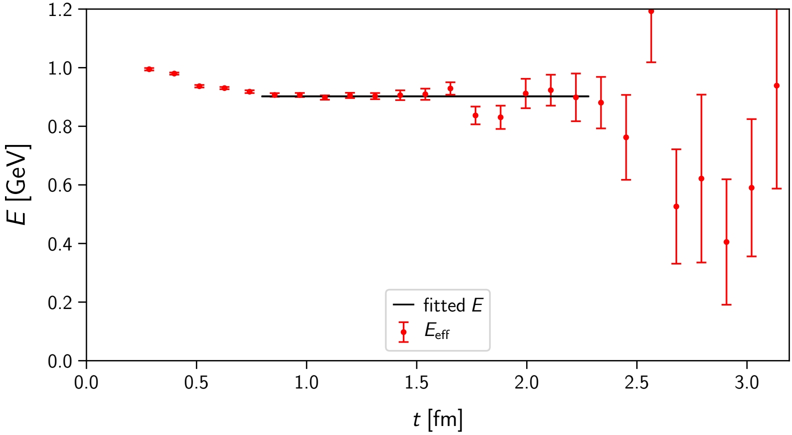
How NOT to treat the K^* on the lattice

$$E_{\text{eff}}(t) = a^{-1} \log[C(t)/C(t+a)]$$



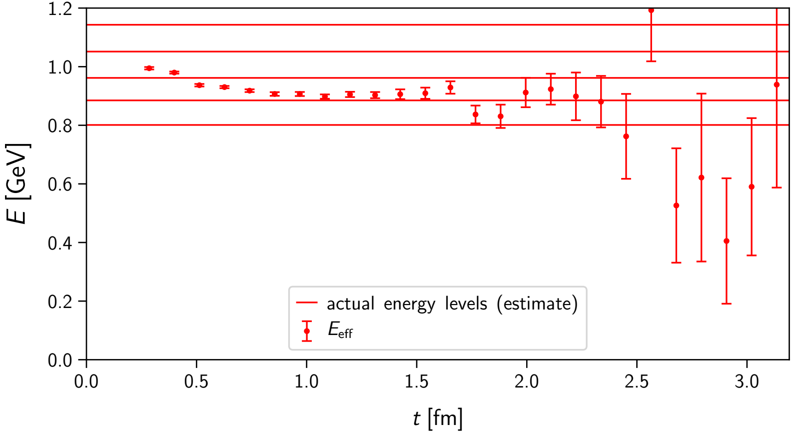
How NOT to treat the K^* on the lattice

$$E_{\text{eff}}(t) = a^{-1} \log[C(t)/C(t+a)]$$



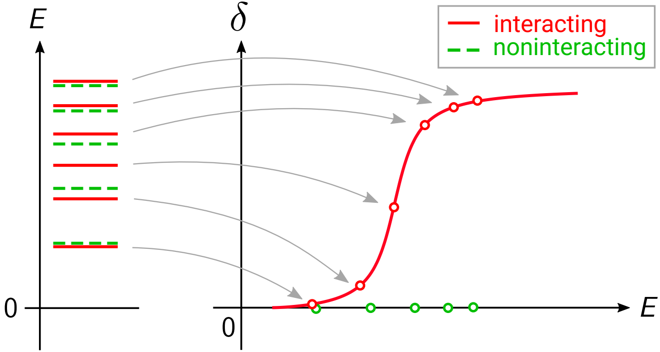
How NOT to treat the K^* on the lattice

$$E_{\text{eff}}(t) = a^{-1} \log[C(t)/C(t+a)]$$



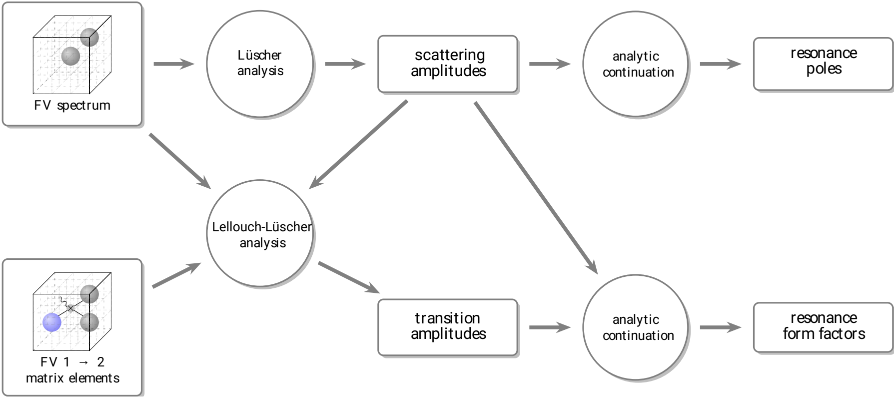
How to correctly treat the K^* on the lattice

Use the Lüscher method to study $K\pi$ scattering!
See the colloquium by Carsten Urbach.



$1 \to 2$ transition matrix elements on the lattice

We need to calculate $\langle K\pi|J^\mu|B\rangle$ infinite-volume transition matrix elements. They can be obtained from the finite-volume matrix elements computed on the lattice through the Lellouch-Lüscher formalism.



[L. Lellouch, M. Lüscher, arXiv:hep-lat/0003023/CMP 2001;

R. A. Briceño, M. T. Hansen, A. Walker-Loud, arXiv:1406.5965/PRD 2015; ...]

$1 \to 2$ transition matrix elements on the lattice

We are working on $\langle K\pi | J^\mu | B \rangle$, but have not yet completed the analysis.

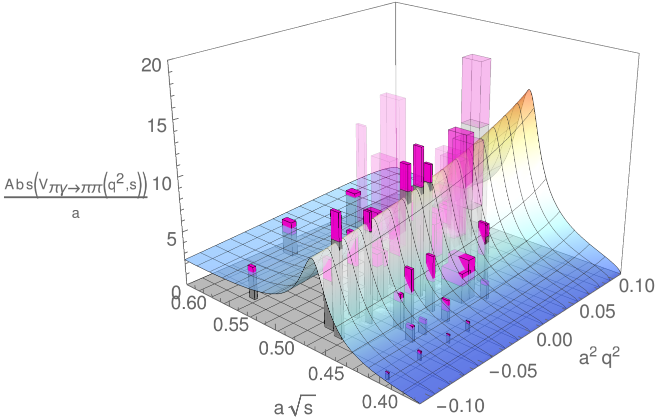
We have already completed a similar analysis for $\langle \pi\pi | J^\mu | \pi \rangle$, where J^μ is the electromagnetic current, for the $L = 1$, $I = 1$ final state.

[C. Alexandrou, L. Leskovec, S. Meinel, J. Negele, S. Paul, M. Petschlies, A. Pochinsky, G. Rendon, S. Syritsyn, arXiv:1807.08357/PRD 2018]

See also: [R. A. Briceno, J. J. Dudek, R. G. Edwards, C. J. Shultz, C. E. Thomas, D. J. Wilson, arXiv:1507.06622/PRL 2015; arXiv:1604.03530/PRD 2016]

$1 \to 2$ transition matrix elements on the lattice

The infinite-volume $\pi\gamma^* \to \pi\pi$ amplitude we have calculated is given by the form factor $V_{\pi\gamma^*\to\pi\pi}(q^2, s)$.



[C. Alexandrou, L. Leskovec, S. Meinel, J. Negele, S. Paul, M. Petschlies, A. Pochinsky, G. Rendon, S. Syritsyn, arXiv:1807.08357/PRD 2018]

$1 \to 2$ transition matrix elements on the lattice

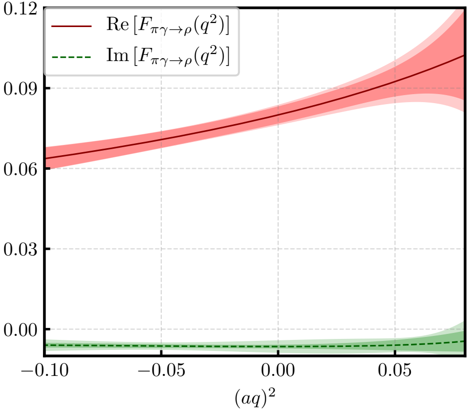
Let's define the a new function $F(q^2, s)$ through

$$V_{\pi\gamma^* \to \pi\pi}(q^2, s) = \frac{F(q^2, s)}{m_\rho^2 - s - i\sqrt{s}\,\Gamma(s)} \sqrt{\frac{16\pi s \Gamma(s)}{k}}.$$

The $\pi\gamma^* \to \rho$ resonance form factor, if desired, can be obtained from analytic continuation:

$$F_{\pi\gamma^* \to \rho}(q^2) = F(q^2,\, m_\rho^2 - i m_\rho \Gamma_\rho).$$

$1 \to 2$ transition matrix elements on the lattice



[C. Alexandrou, L. Leskovec, S. Meinel, J. Negele, S. Paul, M. Petschlies, A. Pochinsky, G. Rendon, S. Syritsyn, arXiv:1807.08357/PRD 2018]

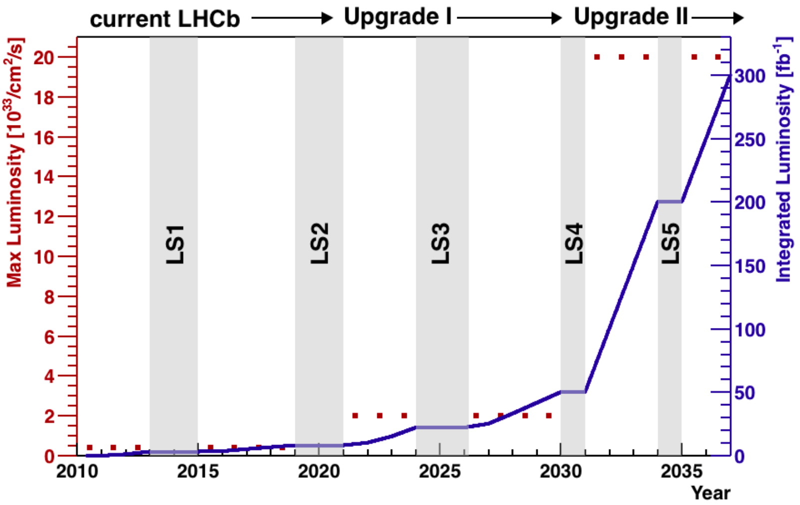
$1 \to 2$ transition matrix elements on the lattice

Stay tuned for $\langle K\pi | J^\mu | B \rangle$!

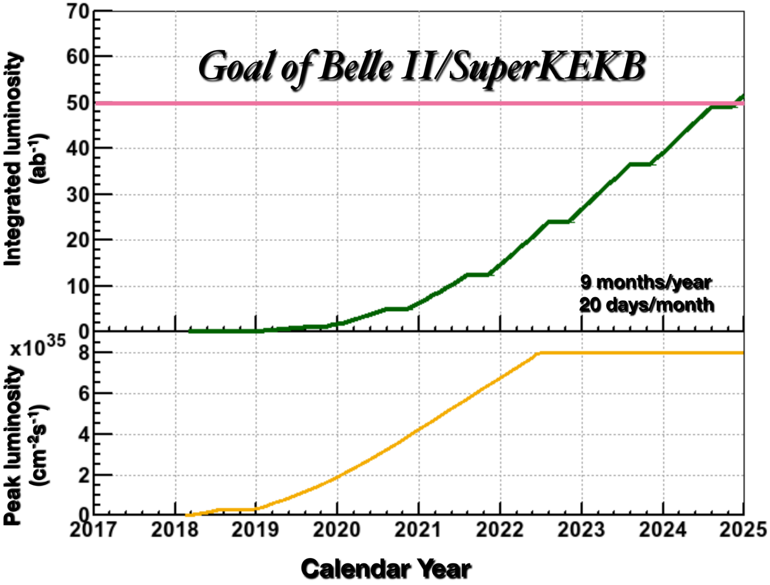
Gumaro Rendon has shown his analysis of $I = 1/2$ $K\pi$ scattering, including both $L = 0$ and $L = 1$, at Lattice 2019:

https://indico.cern.ch/event/764552/contributions/3420497

Outlook



[LHCb Collaboration, "Physics case for an LHCb Upgrade II", arXiv:1808.08865]



[Belle II Collaboration, "The Belle II Physics Book", arXiv:1808.10567]

See also:

“Opportunities for lattice QCD in quark and lepton flavor physics”

(USQCD whitepaper)

C. Lehner, S. Meinel, T. Blum, N. Christ, A. El-Khadra, M. Hansen, A. Kronfeld, J. Laiho, E. Neil, S. Sharpe, R. Van de Water

arXiv:1904.09479

Extra slide: OPE for nonlocal matrix elements at high q^2

$$
\begin{aligned}
\mathcal{T}_\mu &= \frac{-16 i \pi^2}{q^2} \sum_{i=1\ldots6;8} C_i \int \mathrm{d}^4 x \; e^{iq\cdot x} \, \langle H_s | \, \mathrm{T} \; O_i(0) \; J_\mu^{\mathrm{e.m.}}(x) \, | H_b \rangle \\
&= -T_7(q^2) \; \frac{2 m_b}{q^2} \; q^\nu \langle H_s | \, \bar{s} \, i \sigma_{\mu\nu} b_R \, | H_b \rangle \\
&\quad + T_9(q^2) \; \langle H_s | \bar{s} \gamma_\mu b_L | H_b \rangle \\
&\quad + \mathcal{O}(\Lambda^2/m_b^2, \;\; m_c^4/q^4, \;\; \alpha_s \Lambda/m_b),
\end{aligned}
$$

where

$$
\begin{aligned}
T_7(q^2) &= -(1/3)\Big[C_3 + 4\,C_4/3 + 20\,C_5 + 80\,C_6/3\Big] \\
&\quad + \alpha_s/(4\pi)\Big[(C_1 - 6\,C_2)\,A(q^2) - C_8 F_8^{(7)}(q^2)\Big], \\
T_9(q^2) &= (4/3)\Big[C_3 + (16/3) C_5 + (16/9) C_6\Big] \\
&\quad + h(0, q^2)\Big[4\,C_1/3 + C_2 + 11\,C_3/2 - 2\,C_4/3 + 52\,C_5 - 32\,C_6/3\Big] \\
&\quad + (8\,m_c^2/q^2)\Big[4\,C_1/9 + C_2/3 + 2\,C_3 + 20\,C_5\Big] \\
&\quad - h(m_b, q^2)\Big[7\,C_3/2 + 2\,C_4/3 + 38\,C_5 + 32\,C_6/3\Big] \\
&\quad + \alpha_s/(4\pi)\Big[C_1\,(B(q^2) + 4\,C(q^2)) - 3\,C_2\,(2\,B(q^2) - C(q^2)) - C_8 F_8^{(9)}(q^2)\Big].
\end{aligned}
$$

[B. Grinstein and D. Pirjol, `arXiv:hep-ph/0404250`/PRD 2004;
see also M. Beylich, G. Buchalla, T. Feldmann, `arXiv:1101.5118`/EPJC 2011]