Lecture 2:
Large-Momentum Effective Theory

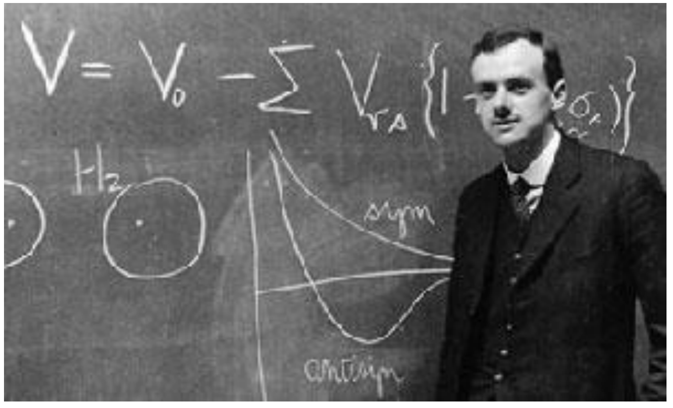
Struggles with light-cone physics

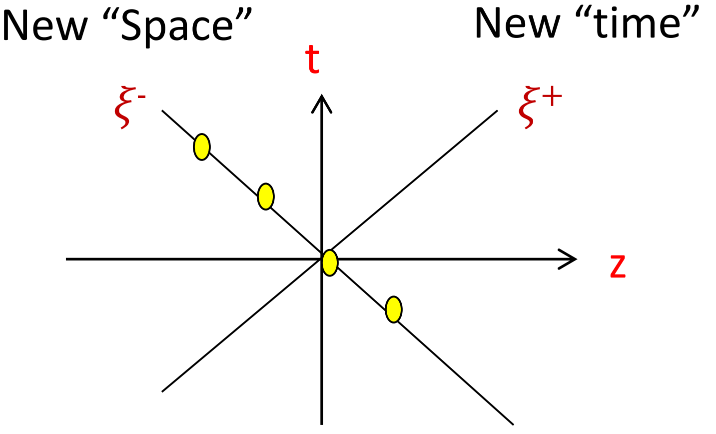
PDFs and light-cone physics

- Since late 1960's, due to the work by Gell-Mann, Weinberg, Feynman, Bjorken and Drell, among others, it is widely accepted that parton physics is about light-cone correlations.
- Because of PDFs involving real time, it is impossible to calculate them directly in lattice QCD, because it is an obvious NP-hard problem, just like the finite density case (D. Kaplan) .
- This lead K. Wilson to start light-cone quantization approach to solve QCD in late 1980's.

Light-cone quantization

- P.A.M. Dirac (1949)





- Choose a new system of coordinates with ξ^+ as the new "time" (light-cone time) and ξ^- as the new "space" (light-cone space)

LCQ and Partons

- In LCQ, Light-cone correlation becomes "equal-time" correlation.
- Parton physics is manifest through light-cone quantization (LCQ)

$$\psi_+(\xi^+ = 0, \xi^-, \xi_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{2k^+}\sum_\lambda \Big[b_\lambda(k)u(k\lambda)e^{-i(k^+\xi^- - \vec{k}_\perp\cdot\vec{\xi}_\perp)}$$
$$+d^\dagger_\lambda(k)v(k\lambda)e^{i(k^+\xi^- - \vec{k}_\perp\cdot\vec{\xi}_\perp)}\Big] \ .$$

- Wave functions of a bound state are directly expressed in terms of partons.

The Hamiltonian approach

- Despite many years of efforts, light-front quantization has not yielded a systematic approach to calculating non-perturbative parton physics.
- Not even an approximate calculation of parton distributions using QCD LF Hamiltonian has appeared.

Wilson’s unsolved problem

- Wilson published 44 papers in his life time.
- His h-index is 33, with average citation 300.
- He published 14 paper after 1993, among which 10 was on LCQ

Nonperturbative QCD: A weak-coupling treatment on the light front

K8. **Nonperturbative QCD: A Weak coupling treatment on the light front**
Kenneth G. Wilson, Timothy S. Walhout, Avaroth Harindranath, Wei-Min Zhang, Robert J. I
U.), Stanislaw D. Glazek (Warsaw U.). Jan 1994. 70 pp.
Published in **Phys.Rev. D49 (1994) 6720-6766**
DOI: 10.1103/PhysRevD.49.6720
e-Print: **hep-th/9401153** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
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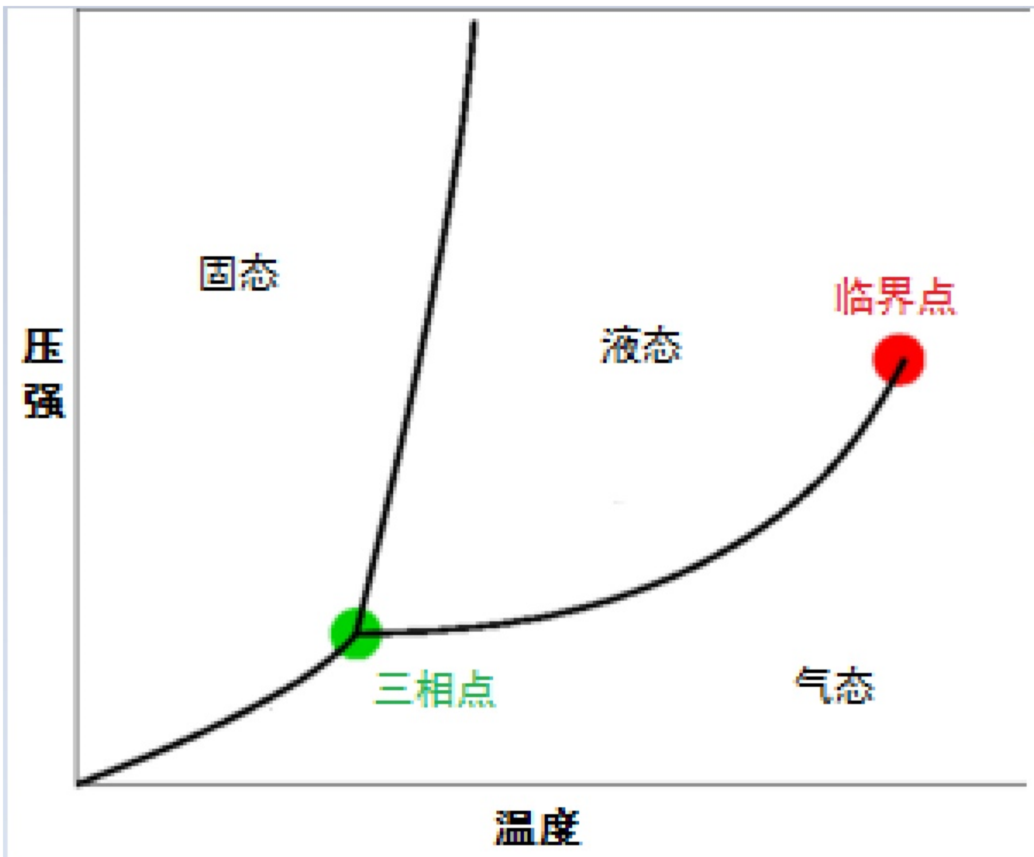
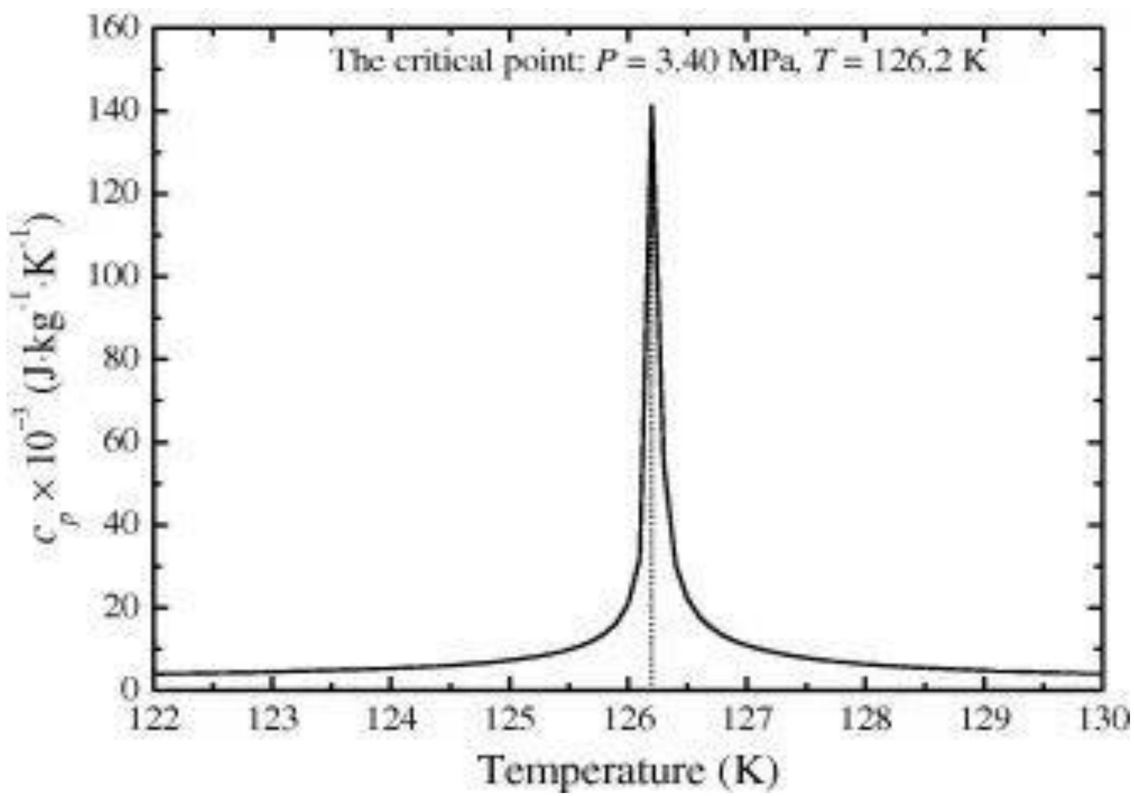
rather than the zero masses of the current picture. The use of constituent masses cuts off the growth

Critical point



Singular behavior of physical quantities at critical point



Parton physics on lattice?

- One can form local moments to get rid of the time-dependence $\langle x^n \rangle = \int q(x) x^n dx$ →matrix elements of local operators
- This equivalent to the Taylor expansion on the time-dependence,

  $f(t) = f(0) + tf'(0) + t^2 f''(0)/2 + t^3 f'''(0)/6 + \ldots$

  light-cone operator becomes sum of local ones.

Twist-2 operators

- Consider a quark operator with n-indices

$$O^{\mu_1 \ldots \mu_n} = \bar{\psi}\,\gamma^{(\mu_1} iD^{\mu_2} \ldots iD^{\mu_n)}\,\psi - \text{trace},$$

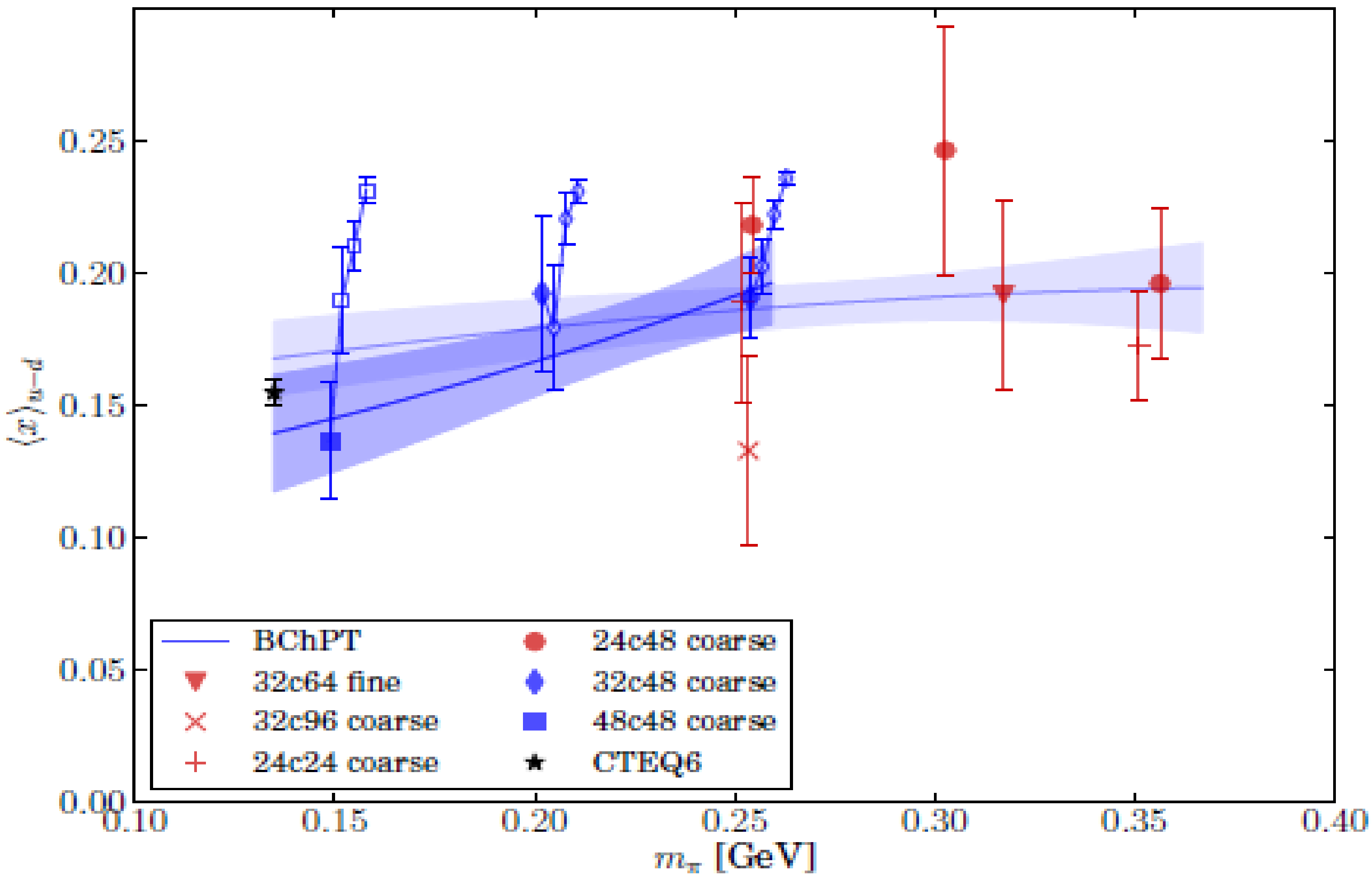
The maximum “spin” representation is the symmetric and traceless combination, which is called the twist-two operator.

- Its matrix element

$$\langle P|O^{\mu_1 \ldots \mu_n}(\mu^2)|P\rangle = 2a_n(\mu^2)(P^{\mu_1} \ldots P^{\mu_n} - \text{trace}),$$

yields moments of parton distributions.

First moment of parton distribution



J. R. Green et al, BMW colver-improved Wilson action and lowest $m_\pi = 149\text{MeV}$

Problems with moment approach

- One can only calculate lowest few moments in practice.
- Higher moments quickly become noisy (NP-hard)
- Many other parton properties cannot be related to local operators, e. g.
  - Transverse-Momentum-Dependent (TMDs） parton distribution,
  - Soft functions
  - Jet functions etc.

One must find alternative approach.

Large-momentum effective theory

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Parton Physics on a Euclidean Lattice

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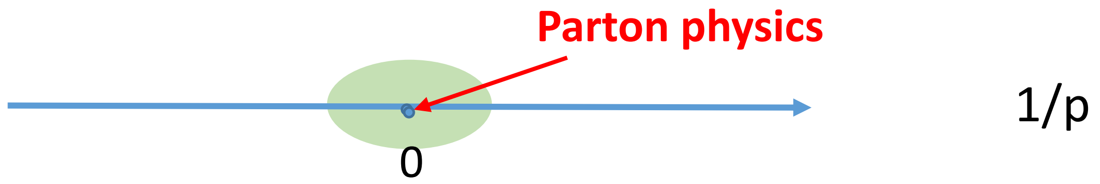
I show that the parton physics related to correlations of quarks and gluons on the light cone can be studied through the matrix elements of frame-dependent, equal-time correlators in the large momentum limit. This observation allows practical calculations of parton properties on a Euclidean lattice. As an example, I demonstrate how to recover the leading-twist quark distribution by boosting an equal-time correlator to a large momentum.

DOI: 10.1103/PhysRevLett.110.262002 PACS numbers: 12.38.Gc, 13.60.Hb, 14.20.Dh

Basic Idea

- P=∞ is a UV fixed point, near which QCD is a scale-invariant theory



- CM-Momentum renormalization group can relate physical quantities at different large p.
- P=∞ （parton） physics can be obtained using large momentum physics.

Hadron CM-momentum renormalization group

- QCD is an asymptotic-free theory. As such, once there is a large scale in the problem, such a scale dependence can be studied in pert. theory.
- One can establish a momentum renormalization group eq.

$$dO(p,\mu)/dlnp \; = \gamma_o(p,\; \mu)O(p,\mu)$$

γ is a perturbative expansion

in the strong interaction coupling constant.

Photon spin in an electron

Now we show that the matrix element of $\hat{S}_\gamma^{\rm inv} = (\vec{E} \times \vec{A}_\perp)^3$ depends on the choice of frames dynamically. By "dynamically" we mean that the frame dependence cannot be obtained from Lorentz transformation, and is a function of dynamic details. Let us consider the example of photon spin in a free electron state $|p, s\rangle$. A simple perturbative calculation of Fig. 1 yields,

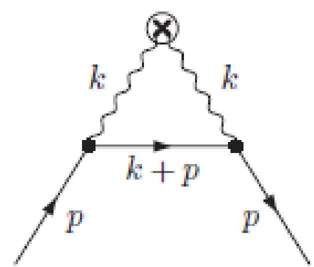


FIG. 1: Matrix element of $\vec{E} \times \vec{A}_\perp$ in an asymptotic electron state.

$$\langle p, s|\hat{S}_\gamma^{\rm inv}|p, s\rangle = \frac{\alpha_{\rm em}}{4\pi}\left[\frac{5}{3}D + \frac{31}{9} + 2\int_0^1 dx\sqrt{1-x}\ln\left(1 + x\frac{\vec{p}^{\,2}}{m^2}\right)\right]\bar{u}(p, s)\Sigma^3 u(p, s)\ . \quad (11)$$

where $p^\mu = (p^0, 0, 0, p^3)$, $D = 1/\epsilon - \gamma_E + \ln 4\pi + \ln(\mu^2/m^2)$, and m is the mass of the electron

In concrete

- When LHC proton has a velocity

  v=0.999999999c

According to Feynman， one can use v=c proton to approximate. However, this is singular point.

- In LaMET， one can use

  v=0.9c

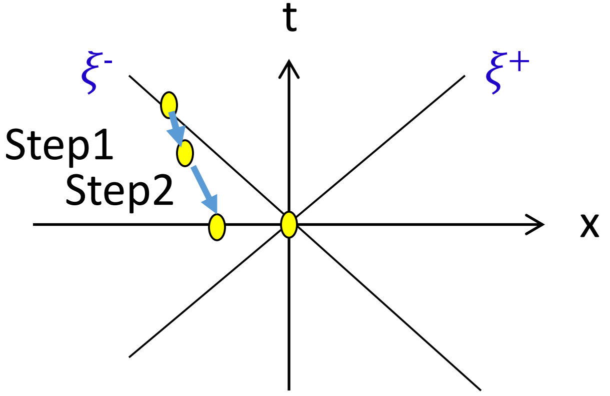
Proton to approximate：

v=0.9c can be calculated on lattice.

The difference between v=0.9c and v=c can be solved by RG equation.

Another angle: two steps

- Step 1: Light-cone correlations can be approximated by “off but near” light-cone “space-like” correlations
- Step 2: Any space-like separation can be made simultaneous by suitably choosing the Lorentz frame.



Justification for step 1

- Light-cone correlations arise from when hadrons travel at the speed-of-light or infinite momentum
- A hadron travels at large but finite momentum should not be too different from a hadron traveling at infinite momentum.
  - The difference is related to the size of the momentum P
  - But physics related to the large momentum P can be calculated in perturbation theory according to asymptotic freedom.

Consequences for step 2

- Equal time correlations can be calculated using Wilson's lattice QCD method.
- Thus all light-cone correlations can in the end be tackled with Monte Carlo simulations.

Effective field theory language

- Consider the ligh-cone physical observable, o(μ)
- Construct a Euclidean observable that depends large momentum P, O(P,a) with UV cut-off.
- The EFT matching condition or factorization theorem,

$$O(P,a) = Z(\frac{\mu}{P})o(\mu) + \frac{c_2}{P^2} + \frac{c_4}{P^4} + \cdots$$

where Z is perturbatively calculable.

μ is a renormalization scale.

Analogy with HQET

- In physical processes involving heavy quarks, it is often useful to take $m_Q \to \infty$ first before UV cutoff is applied, although the actual m_Q is finite and UV cutoff must be larger than m_Q in physical case.
- This result in a HQET in which the symmetric properties are manifest.
- In this sense, the QFT partons are an effective field theory description of high-energy scattering.

Step 1: Deconstructing the light-cone operators

- Consider parton physics described by light-cone operators, o. Construct a Euclidean quasi-operator "O" such that in the IMF limit, O becomes a light-cone (light-front, parton) operator o.

$$O_1 = A^0 \;\rightarrow\; \mathrm{o} = \Lambda\, A^+$$

- There are many operators leading to the same light-cone operator.

$$O_2 = A^3 \;\rightarrow\; \mathrm{o} = \Lambda\, A^+$$

$$O_3 = \alpha A^0 + (1-\alpha) A^3 \;\rightarrow\; \mathrm{o} = \Lambda\, A^+$$

Step 2: Lattice calculations

- Compute the matrix element of Euclidean operator O on a lattice in a nucleon state with large momentum P.
- The matrix element may depend on the momentum of the hadron P, O(P,a), and also on the details of the lattice actions (UV specifics).

Step 3: Extracting the light-cone physics from the lattice ME

- Extract light-front physics o(μ) from O(P,a) at large P through a EFT matching condition or factorization theorem,

$$O(P,a) = Z(\frac{\mu}{P})o(\mu) + \frac{c_2}{P^2} + \frac{c_4}{P^4} + \cdots$$

Where Z is perturbatively calculable.

Why factorization exists?

- When taking $p \to \infty$ first, before a UV regularization imposed, one recovers from O, the light-cone operator. [This is done through construction.]
- The lattice matrix element is obtained at large P, with UV regularization (lattice cut-off) imposed first.

Infrared factorization

- Infrared physics of O(P,a) is entirely captured by the parton physics o(μ). In particular, it contains all the collinear divergence when P gets large.
- Z contains all the lattice artifact (scheme dependence), but only depends on the UV physics, can be calculated in perturbation theory
- Factorization can be proved to all orders in perturbation theory

Universality class

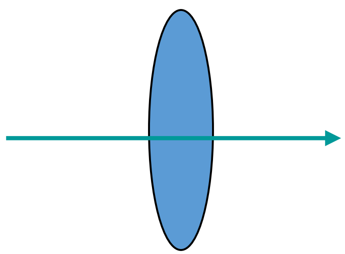
- Just like the same PDF can be extracted from different hard scattering processes, the same light-cone physics can be extracted from different lattice operators.
- All operators that yield the same light-cone physics form a universality class.
- Universality class allows one exploring different operator O so that a result at finite P can be as close to that at large P as possible.

Parton physics on lattice

- Parton distribution functions (PDFs)
- Gluon helicity distribution and total gluon spin
- Generalized parton distributions
- Transverse-momentum dependent parton distributions
- Light-cone wave functions
- Higher twist observables

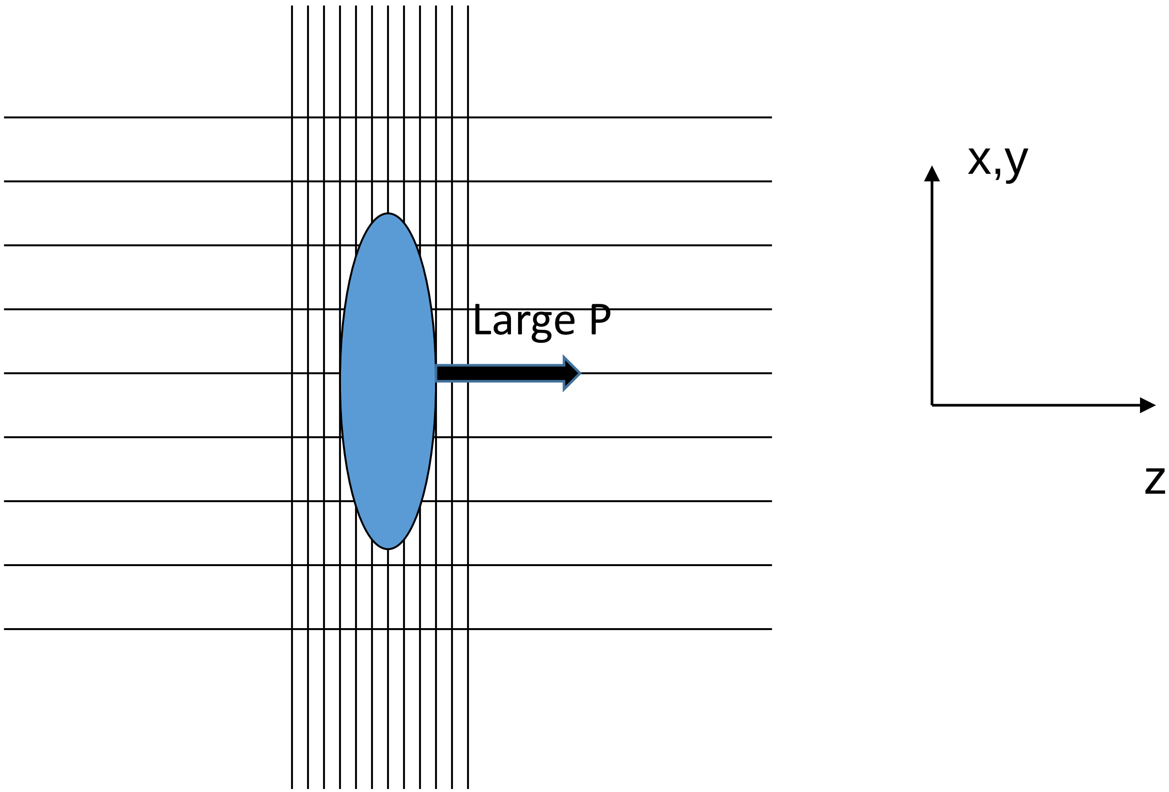
Practical considerations

- For a fixed x, large P_z means large k_z, thus, as P_z gets larger, the valence quark distribution in the z-direction get Lorentz contracted, $z \sim 1/k_z$.



- Thus one needs increasing resolution in the z-direction for a large-momentum nucleon. Roughly speaking: $a_L/a_T \sim \gamma$

$\gamma=4$



MS-bar scheme

- Parton physics is usually defined in MS-bar scheme.
- Lattice calculations are done in the lattice scheme.
- One has to match two different schemes.
- The difference is perturbative.