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Gluon Fragmentation into ${}^1S_0^{[1,8]}$ Quarkonia at NLO

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Outline

- Introduction
- LO calculation
- Real calculation
- Virtual calculation
- Discussion
- Conclusion and Summary

Introduction

Heavy quarkonium

- Simple bound states
  - Non-relativistic system: $v \ll 1$
  - Heavy mass: $m_Q \gg \Lambda_{QCD}$
  - Separated momentum scales: m_Q、$m_Q v$、$m_Q v^2$、Λ_{QCD}
- NRQCD factorization

Bodwin, Braaten, Lepage, *Phys. Rev.* **D51** (1995) 1125-1171

$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}+X} \langle \mathcal{O}^H(n) \rangle$$

  - color-octet mechanism: Fock state expansion
  - Short-distance part
    - perturbation calculable
    - Organized in powers of α_s
  - Long-distance matrix elements (LDMEs)
    - parametrize the non-perturbative part
    - Organized in powers of v
    - Universal (process independent)

Introduction

Fragmentation Function (FF)

- QCD Collinear Factorization

Collins, Soper, Sterman, *Adv. Ser. Direct. High Energy Phys.* **5** (1989) 1-91

$$d\sigma_{A+B\to H+X}(p_T) = \sum_i d\hat{\sigma}_{A+B\to i+X}\left(\frac{p_T}{z},\mu\right) \otimes D_{i\to H}(z,\mu) + \mathcal{O}\left(\frac{1}{p_T^2}\right)$$

- Evolution by the DGLAP evolution

Gribov, Lipatov, *Sov. J. Nucl. Phys.* **15** (1972) 438-450
Altarelli, Parisi, *Nucl. Phys.* **B126** (1977) 298-318
Dokshitzer, *Sov. Phys. JETP* **46** (1977) 641-653

$$\mu\frac{d}{d\mu}D_{g\to H}(z,\mu) = \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ig}\left(\frac{z}{\xi};\alpha_s(\mu)\right) D_{i\to H}(\xi,\mu)$$

- Compare with the NRQCD factorization

$$D_{i\to H}(z,\mu_0) = \sum_n d_n(z,\mu_0,\mu_f)\langle \mathcal{O}^H(n)\rangle$$

  - LDMEs: lattice, potential model, experiment
  - SDCs: match from the Full QCD calculation
    - Parton fragment into a free $Q\bar{Q}$ state.
    - Calculate the Feynman diagrams

Introduction

Present calculation (single parton FFs)

- Up to α_s^2-order
  - Summary

    Ma, Qiu, Zhang, *Phys.Rev.* **D89** (2014) 094029
    Ma, Qiu, Zhang, *JHEP* **06** (2015) 021

- α_s^3-order
  - $g \to Q\bar{Q}({}^3S_1^{[1]}) + gg$
    - Numerical

      Braaten, Yuan, *Phys. Rev. Lett.* **71** (1993) 1673-1676
      Braaten, Yuan, *Phys. Rev.* **D52** (1995) 1125-1171

    - Analytical

      Zhang, Ma, Chen, Chao, *Phys. Rev.* **D96** (2017) 094016

  - $g \to Q\bar{Q}({}^1P_1^{[1]}) + gg$
    - Analytical

      Sun, Jia, Liu, Zhu, *Phys. Rev.* **D98** (2018) 014039

  - $g \to Q\bar{Q}({}^1S_0^{[1]}) + X$ (NLO)
    - Numerical

      Artoisenet, Braaten, *JHEP.* **04** (2015) 121

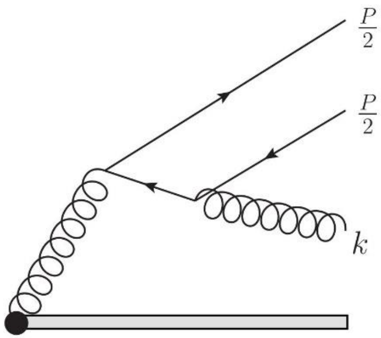
LO Calculation

- Feynman Diagram
- Form of SDC $\int \mathrm{d}\Phi_{\mathrm{Born}} \frac{1}{k \cdot P + a}$

  in which $a = 0, \frac{1}{2}$,

  and the phase space integral is

$$\int \mathrm{d}\Phi_{\mathrm{Born}} = \frac{1}{4\pi z(1-z)} \int \frac{\mathrm{d}^{D-2}k_\perp}{(2\pi)^{D-2}}$$



- Final results
  - $g \to Q\bar{Q}(^1S_0^{[1]}) + X$
  - $g \to Q\bar{Q}(^1S_0^{[8]}) + X$

$$d_{\mathrm{LO}}^{[1]}(z) = \frac{\alpha_s^2}{2(1-\epsilon)N_c m_Q^3} \left(\frac{\pi\mu_r^2}{m_Q^2}\right)^{\epsilon} d_{\mathrm{LO}}(z),$$

$$d_{\mathrm{LO}}^{[8]}(z) = \frac{\alpha_s^2(N_c^2-4)}{4(1-\epsilon)N_c(N_c^2-1)m_Q^3} \left(\frac{\pi\mu_r^2}{m_Q^2}\right)^{\epsilon} d_{\mathrm{LO}}(z)$$

$$d_{\mathrm{LO}}^{(0)}(z) = \lim_{\epsilon\to 0} d_{\mathrm{LO}}(z) = (3-2z)z + 2(1-z)\ln(1-z)$$

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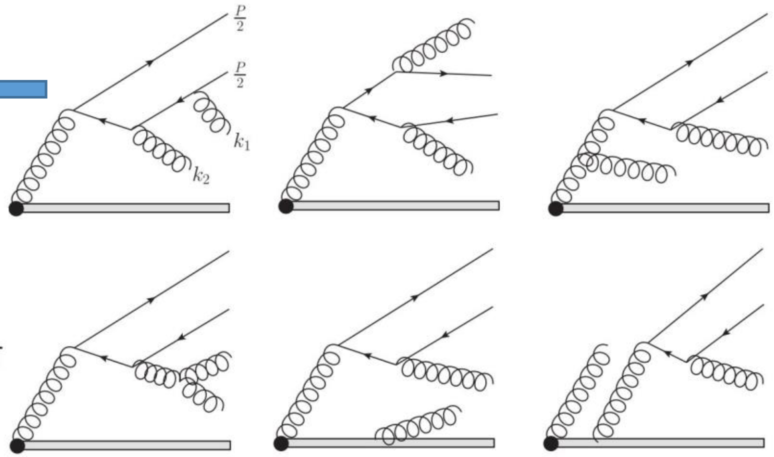
Real Calculation

- Feynman Diagram
- Form of SDCs

$S = 2$

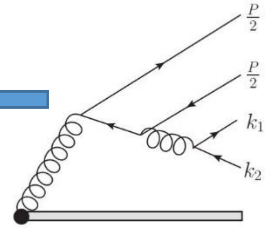


$$\int \mathrm{d}\Phi_{\text{real}} \prod_i \frac{1}{E_i^{a_i}}$$

$$= \frac{P \cdot n}{2z^2} \int \frac{\mathrm{d}^D k_1}{(2\pi)^{D-1}} \frac{\mathrm{d}^D k_2}{(2\pi)^{D-1}} \delta_+(k_1^2)\delta_+(k_2^2)\delta\left(k_1 \cdot n + k_2 \cdot n - \frac{1-z}{z} P \cdot n\right) \prod_i \frac{1}{E_i^{a_i}}$$

in which

$S = 1$



$$E_1 = k_1 \cdot k_2\,, \qquad E_2 = k_1 \cdot P\,, \qquad E_3 = k_2 \cdot P\,,$$

$$E_4 = 2\,k_1 \cdot P + 1\,, \qquad E_5 = 2\,k_2 \cdot P + 1\,,$$

$$E_6 = 2\,k_1 \cdot k_2 + k_1 \cdot P + k_2 \cdot P\,,$$

$$E_7 = 2\,k_1 \cdot k_2 + 2\,k_1 \cdot P + 2\,k_2 \cdot P + 1\,,$$

$$E_8 = k_1 \cdot n\,, \qquad E_9 = k_1 \cdot n + P \cdot n\,,$$

$$E_{10} = k_2 \cdot n\,, \qquad E_{11} = k_2 \cdot n + P \cdot n\,.$$

$$E_{12} = k_1^2\,,$$

$$E_{13} = k_2^2\,,$$

$$E_{14} = k_1 \cdot n + k_2 \cdot n - \frac{1-z}{z} P \cdot n$$

Real Calculation

- IBP reduction

Chetyrkin, Tkachov, *Nucl. Phys.* **A15** (1981) 159-204
Smirnov, *Comput. Phys. Commun.* **189** (2015) 182-191

  - For a family of Feynman integrals

$$F(a_1,\ldots,a_n)=\int\cdots\int\frac{\mathrm{d}^D l_1\ldots\mathrm{d}^D l_h}{D_1^{a_1}\ldots D_n^{a_n}}$$

D_i : linear functions with respect to the scalar product of loop momenta and external momenta.

  - IBP relations (integrated boundlessly)

$$\int\cdots\int \mathrm{d}^D l_1\ldots\mathrm{d}^D l_h\, p_j^\mu \frac{\partial}{\partial l_i^\mu}\frac{1}{D_1^{a_1}\ldots D_n^{a_n}}=0$$

  - If D_i are independent and complete

$$\sum c_i F(a_1+b_{i,1},\ldots,a_n+b_{i,n})=0 \quad b_{i,j}\in\{-1,0,1\}$$

  - Reduce to master integrals (MIs)

Real Calculation

- Transform of delta function

$$(2\pi)\delta(x) = \lim_{\eta\to 0}\left(\frac{i}{x+i\eta} - \frac{i}{x-i\eta}\right)$$

  - Transform $\delta(D_j)$ in phase space to $1/D_j$
  - Normally IBP relations independent with $i\eta$
  - Ignore the infinitesimal imaginary parts
  - Choose MIs with power of D_j no more than 1
  - Change $1/D_j$ back to $\delta(D_j)$
  - Ignore the MIs with power of D_j no more than 0

Real Calculation

Problem in real IBP reduction

- Unregularized rapidity divergence
  - For MI $\int \mathrm{d}\Phi_{\text{real}} \frac{1}{E_1 E_4}$ → $E_1 = k_1 \cdot k_2$, $E_4 = 2k_1 \cdot P + 1$
- integrated out k_1^-, k_2^-, k_2^+, we get

$$\frac{1}{(4\pi)^2 z^2}\int_0^1 \frac{\mathrm{d}z_1}{z_1}\int \frac{\mathrm{d}^{D-2}k_{1\perp}}{(2\pi)^{D-2}}\frac{\mathrm{d}^{D-2}k_{2\perp}}{(2\pi)^{D-2}}\frac{1}{(k_{2\perp}-k_{1\perp})^2\left(k_{1\perp}^2+\left(\frac{1-z}{z}\right)^2 z_1(1-z_1)+\frac{1-z}{z}(1-z_1)\right)}$$

- rapidity divergence
- unregularized in dimensional regularization
- Problems
  - IBP relation?
  - Value of MI?

Real Calculation

Problem in real IBP reduction

- Gluon mass regularization
  - Transform the phase space integral

$$d\Phi' = \frac{P\cdot n}{z^2 2!}\frac{d^D k_1}{(2\pi)^{D-1}}\frac{d^D k_2}{(2\pi)^{D-1}}\delta_+(k_1^2 - m_g^2)\delta_+(k_2^2 - m_g^2)\delta\left(k_1\cdot n + k_2\cdot n - \frac{1-z}{z}P\cdot n\right)$$

  - Take the limit of $m_g \to 0$
- Calculation of the MI
  - integrated out $k_{1\perp}, k_{2\perp}$, we get

$t = \frac{1-z}{z}$

$$(4\pi)^{-4+2\epsilon} m_g^{-2\epsilon}\Gamma(\epsilon)^2 z^{-2}\int_0^1 dz_1\, z_1^{-1+\epsilon}(1-2z_1+2z_1^2)^{-\epsilon}(t^2 z_1 + t + m_g^2/z_1)^{-\epsilon}$$

  - Only $z_1 \sim m_g^2$ values in the limit of $m_g \to 0$
  - The final result is

$$(4\pi)^{-4+2\epsilon}\Gamma(\epsilon)^2 z^{-2}\int_0^\infty dy\, y^{-1+\epsilon}(t+1/y)^{-\epsilon}$$
$$=(4\pi)^{-4+2\epsilon} z^{-2+2\epsilon}(1-z)^{-2\epsilon}\Gamma(2\epsilon)\Gamma(\epsilon)\Gamma(-\epsilon)\,.$$

Real Calculation

Problem in real IBP reduction

- Divide the origin express two parts
  - Integrals that can be regularized:
    naive IBP reduction (ignore $i\eta$ directly)
  - Integrals that can not be regularized:
    gluon mass regularization method
- The unregularized integrals cancelled finally
- Test the IBP reduction of $\int \mathrm{d}\Phi \frac{1}{E_1 E_4 E_7}$ with naive IBP method
  - One of MIs is $\int \mathrm{d}\Phi_{\text{real}} \frac{1}{E_1\, E_4}$, but IBP relation values once we take the gluon mass regularization method in the calculation of this MI
  - gluon mass regulator can indeed give correct result
  - naïve IBP reduction values once the initial integrals are regularized
- Finally obtain 95 MIs

Real Calculation

Calculation of MIs

- Set up differential equations (DEs)

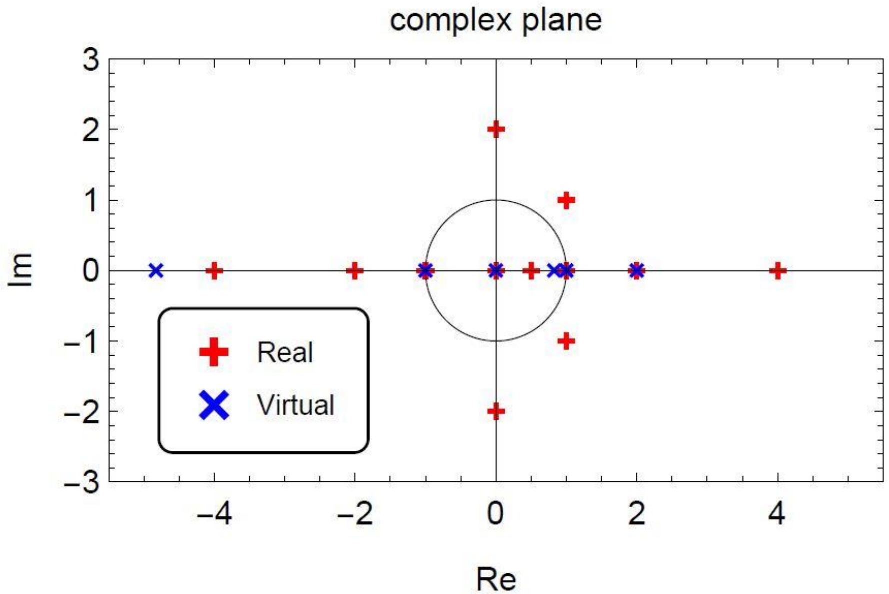
Henn, *J. Phys.* **A48** (2015) 153001

$$\frac{\mathrm{d}\boldsymbol{I}(\epsilon,z)}{\mathrm{d}z} = A(\epsilon,z)\boldsymbol{I}(\epsilon,z)$$

- Asymptotic expansions

$$I_k(z,\epsilon)|_{z_0} = \sum_s \sum_{i=0}^{n_s} (z-z_0)^s \ln^i(z-z_0) \sum_{j=0}^{\infty} I_k^{s\,i\,j}(\epsilon)(z-z_0)^j$$

- Singularities in DEs: 0, 1/2, 1



Real Calculation

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Henn, *J. Phys.* **A48** (2015) 153001

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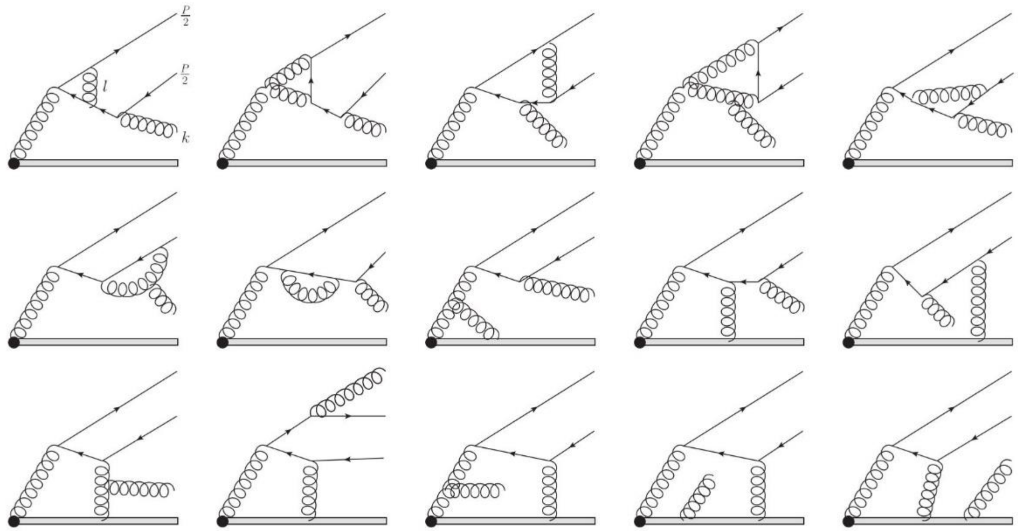
- Singularities in DEs: $0, 1/2, 1$
- estimate values of MIs in regions $0 \sim \frac{1}{4}, \frac{1}{4} \sim \frac{3}{4}, \frac{3}{4} \sim 1$ respectively by the asymptotic expansions of MIs at $z = 0,1/2,1$
- Coefficients at high order are related with those at lower order
- Calculate the boundary at $z \to 1$
    - Sector analyzation
    - Sector decomposition

Virtual Calculation

- Feynman Diagram
- Form of SDCs

$S = 1$



$$\int \mathrm{d}\Phi_{\mathrm{loop}} \int \frac{\mathrm{d}^D l}{(2\pi)^D} \prod_i \frac{1}{F_i^{a_i}}$$

$$= \frac{P \cdot n}{z^2} \int \frac{\mathrm{d}^D k}{(2\pi)^{D-1}} \frac{\mathrm{d}^D l}{(2\pi)^D} \delta_+(k^2) \delta\left(k \cdot n - \frac{1-z}{z} P \cdot n\right) \prod_i \frac{1}{F_i^{a_i}}$$

In which

$$F_1 = k \cdot P, \quad F_2 = 2k \cdot P + 1,$$

$$F_3 = l^2, \quad F_4 = (l+k)^2, \quad F_5 = (l+P)^2,$$

$$F_6 = (l + \frac{P}{2})^2 - \frac{1}{4}, \quad F_7 = (l - \frac{P}{2})^2 - \frac{1}{4}, \quad F_8 = (l + k + \frac{P}{2})^2 - \frac{1}{4},$$

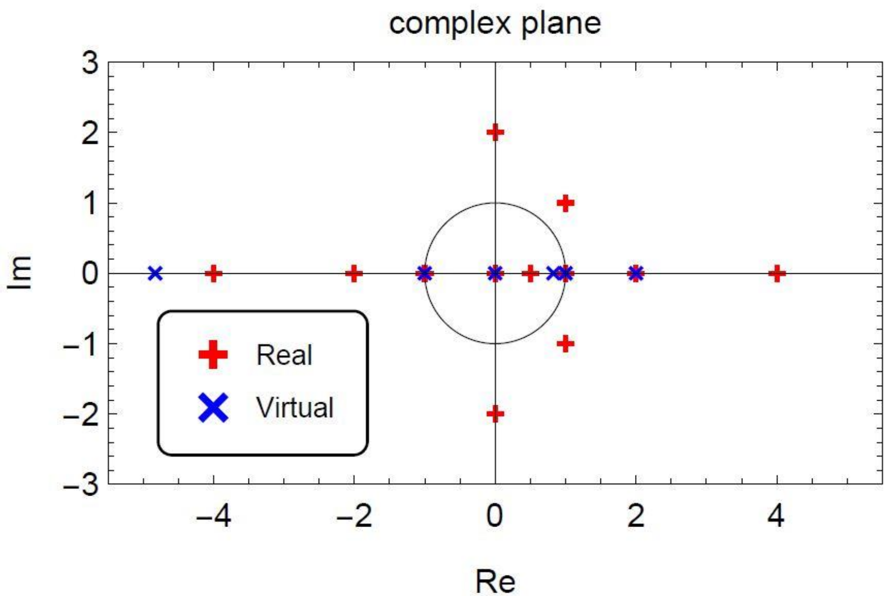
$$F_9 = (l+k+P)^2, \quad F_{10} = l \cdot n.$$

- Use naïve IBP reduction
- Obtain 66 MIs

Virtual Calculation

Calculations of MIs

- Calculate the asymptotic expansions at singularities
- Singularities in DEs: $0, 2(\sqrt{2}-1), 1$



- $z = 2(\sqrt{2}-1)$ does not affect the radius of convergence
- Boundaries at $z \to 1$ are difficult to calculate

Virtual Calculation

Calculations of MIs

Liu, Ma, Wang, *Phys. Lett.* **B779** (2018) 353-357

- Use $i\eta$ method to calculate boundaries at $z = z_0$
  - Add $i\eta$ in the denominators that contains loop momentum l
  - Set up the DEs over η
  - Calculate the boundaries of new MIs at $\eta \to \infty$ with $z = z_0$ in which z_0 is a analytical point
  - Sector analyzation
  - Calculate the values of new MIs at $\eta \to 0$ with DEs
  - Use these values as boundaries of initial MIs at $z = z_0$
- estimate values of MIs in regions $0 \sim \frac{1}{4}, \frac{1}{4} \sim \frac{3}{4}, \frac{3}{4} \sim 1$ respectively by the asymptotic expansions of MIs at $z = 0, 1/2, 1$

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Discussion

Final results

After renormalization

$$b_0 = \frac{11N_c - 2n_f}{6}$$

$$d^{(0)}_{\rm LO}(z) = \lim_{\epsilon\to 0} d_{\rm LO}(z) = (3-2z)z + 2(1-z)\ln(1-z)$$

$$d^{[1]}_{\rm NLO}(z) = \frac{\alpha_s^3}{2\pi N_c m_Q^3} \times \left(d^{[1]}(z) + \ln\left(\frac{\mu_r^2}{4m_Q^2}\right) b_0\, d^{(0)}_{\rm LO}(z) + \ln\left(\frac{\mu_f^2}{4m_Q^2}\right) f(z) \right),$$

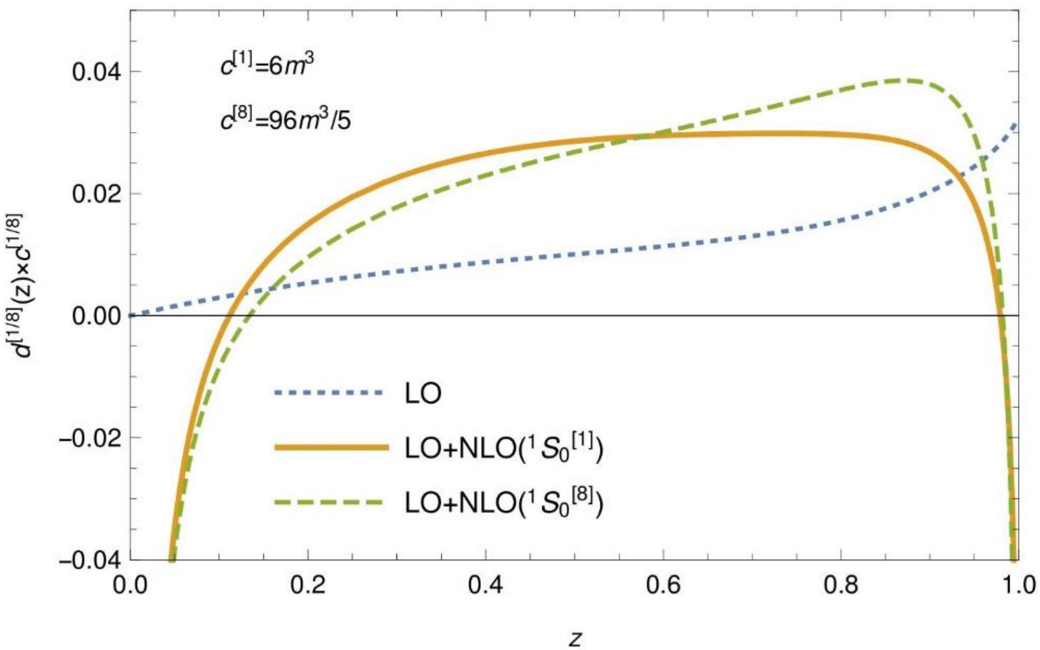
$$d^{[8]}_{\rm NLO}(z) = \frac{\alpha_s^3 (N_c^2-4)}{4\pi N_c (N_c^2-1) m_Q^3} \times \left(d^{[8]}(z) + \ln\left(\frac{\mu_r^2}{4m_Q^2}\right) b_0\, d^{(0)}_{\rm LO}(z) + \ln\left(\frac{\mu_f^2}{4m_Q^2}\right) f(z) \right),$$

$$f(z) = -\frac{n_f}{6} d^{(0)}_{\rm LO}(z) + N_c \Bigg(-2(z+2){\rm Li}_2(z) - 2(z-1)\ln^2(1-z) + 2(z-1)\ln(z)\ln(1-z) + (z-4)z\ln(z) - \frac{(2z+1)\left(9z^2-5z-6\right)\ln(1-z)}{6z} + \frac{46z^3 + \left(8\pi^2-3\right)z^2 + 4\left(\pi^2-9\right)z + 4}{12z} \Bigg),$$

$$d^{[1/8]}(z) = \begin{cases} -\dfrac{N_c}{2z} + \displaystyle\sum_{i=0}^{2}\sum_{j=0}^{\infty} \ln^i z\,(2z)^j \left(A^f_{ij}\, n_f + A^{[1/8]}_{ij} N_c + \frac{A^N_{ij}}{N_c} \right), & \text{for } 0 < z < \frac{1}{4} \\ \displaystyle\sum_{j=0}^{\infty} (2z-1)^j \left(B^f_j\, n_f + B^{[1/8]}_j N_c + \frac{B^N_j}{N_c} \right), & \text{for } \frac{1}{4} \le z \le \frac{3}{4} \\ \displaystyle\sum_{i=0}^{3}\sum_{j=0}^{\infty} \ln^i(1-z)(2-2z)^j \left(C^f_{ij}\, n_f + C^{[1/8]}_{ij} N_c + \frac{C^N_{ij}}{N_c} \right), & \text{for } \frac{3}{4} < z < 1 \end{cases}$$

Discussion

Numerical results



[1] Artoisenet, Braaten, JHEP **04** (2015) 121
[2] Artoisenet, Braaten, *JHEP* **01** (2019) 227
[3] Feng, Jia, arXiv: 1810.04138

- Agree with Ref [2,3] for $g \to Q\bar{Q}(^1S_0^{[8]}) + X$
- Agree with Ref [3] but different with Ref [1] for $g \to Q\bar{Q}(^1S_0^{[1]}) + X$ especially at $z \to 0$

Discussion

Divergence at the endpoints

- Divergence at $z \to 0$
  - Leading behavior: $1/z$
  - Partonic hard part behaves as $z^n (n \geq 4)$
  - small z region has negligible contributions to physical cross sections
- Divergence at $z \to 1$
  - Leading behavior: $\ln^2(1-z)$
  - At specific order, perturbative calculation in this region is not good
  - Resummation may help to solve this problem

Discussion

Estimate the cross section

$$\int_0^1 \mathrm{d}z\, z^n (d_{\mathrm{LO}}^{[1/8]}(z) + d_{\mathrm{NLO}}^{[1/8]}(z)) * c^{[1/8]}$$

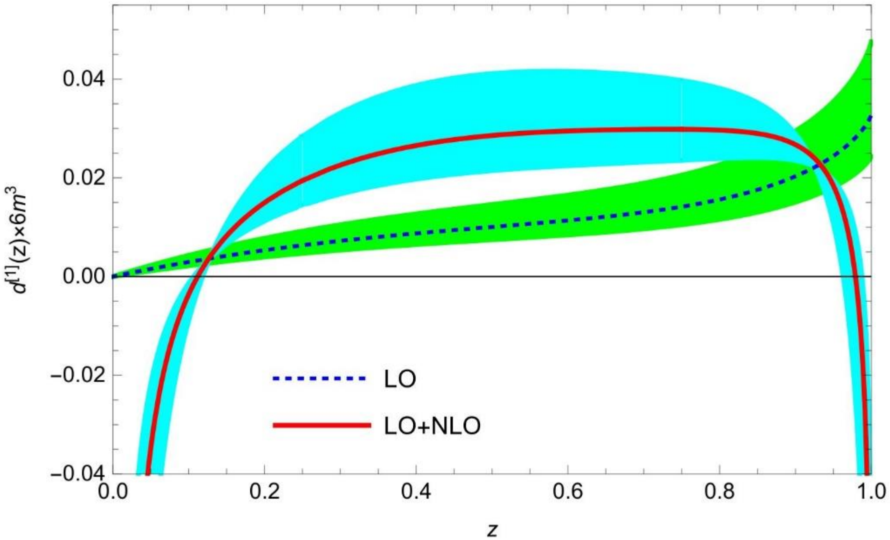
For 2nd,4th,6th moments, the integral results are

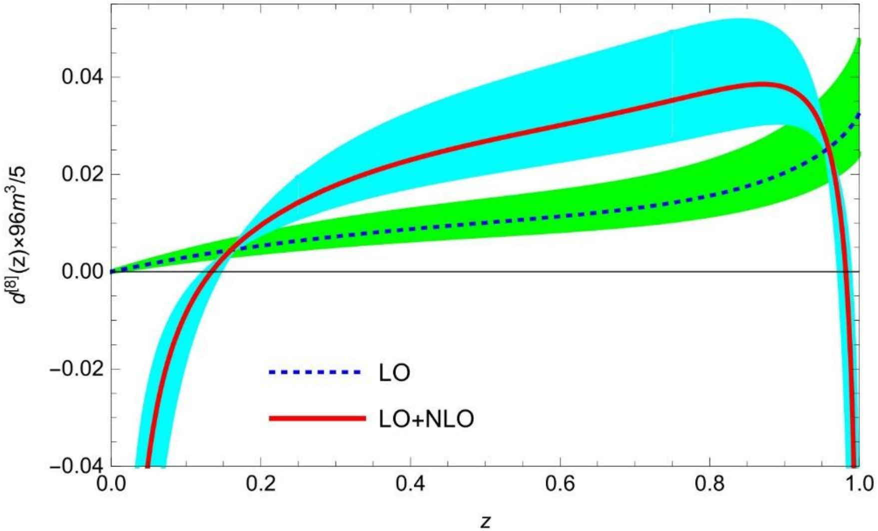
State	SDCs$*c^{[1/8]}$	z^2	z^4	z^6
	LO ($\times 10^{-3}$)	5.55116944444	3.85331761905	2.99519856859
$^1S_0^{[1]}$	LO+NLO ($\times 10^{-3}$)	7.54577896198	3.90413390636	2.31890675630
	K-factor	1.35931339108	1.01318767159	0.774208021001
$^1S_0^{[8]}$	LO+NLO ($\times 10^{-3}$)	8.94021475091	4.99398540596	3.12511443983
	K-factor	1.61051015293	1.29602225917	1.04337471064

- K-factor are moderate

Discussion

Sensitivities to renormalization scale





μ_r change from m_b to $4m_b$

- In NLO error bands, upper value are larger than lower value by a factor of 2 in the middle z region
- Large theoretical uncertainties

Conclusion and Summary

- Give a semi-analytical calculation of the FFs for processes $g \to Q\bar{Q}({}^1S_0^{[1,8]}) + X$ at NLO
  - NLO corrections are large
  - Theoretical uncertainties are large
  - K-factors are moderate for high moments integrals

- Discover a method to calculate FFs with high-loop integrals
  - IBP reduction
  - Set up DEs of the MIs
  - Calculate the boundaries of MIs
  - Solve the asymptotic series at singularities through DEs
  - Use the series to estimate the value at near region

Thank you!